

The Subprime Mortgage Crisis: Irrational Exuberance or Rational Error?

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Abstract

We present a model of the subprime market in which credit quality and loan performance are driven by a statistical process with idiosyncratic and aggregate shocks. Investors use portfolio performance to infer the weight of each shock. We show that low and stable default rates from 2002-2005 convinced investors that the aggregate shock weight was small. In late 2006, when default rates surged, the market collapsed abruptly as investors abandoned their low-weight beliefs. We examine various proposals to fix the mortgage market and find that policy intervention has limited effectiveness in our model.

JEL Classifications: G21, D10, R21

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1 Introduction

The subprime mortgage market is in free fall. Since the end of 2005, default rates on subprime mortgages have soared from 6.5% to 17%, while foreclosure rates have jumped from 2.5% to 9%. Future increases are seemingly inevitable, and the prospects for recovery are threatened by the ongoing turmoil in financial markets, and the curtailment of credit to distressed homeowners.

The precipitous nature of this deterioration, and its potential spillover to the broader economy, has provoked a public outcry for swift and decisive policy responses. Accordingly, Congress, the Executive Office of the President (EOP), the Treasury Department, and various federal and state mortgage regulators are all in the process of reviewing the rules under which new subprime mortgages may be issued, and renegotiating the terms under which previously-issued mortgages can default or be repaid. The EOP, in particular, has instructed the Federal Housing Administration to expand its insurance of mortgages in order to help creditworthy borrowers secure more favorable refinancing. Fannie Mae and Freddie Mac, on the other hand, have both increased their exposure to mortgage assets in an effort to shore up the housing market. This policy response assumes that subprime defaults are a result of *temporary* failures in both origination and securitization markets, and that government intervention is needed to correct and mediate these failures.

This paper provides an alternative story for the subprime crisis and hence differs in its policy prescriptions. Rather than focusing on correctable market failures, we characterize subprime developments as outcomes of rational errors in the risk perceptions of market participants. The model we develop rationally explains the pattern of mortgage default rates (both prime and subprime) from 2001 through 2007, and captures both the boom and the bust of the subprime market. While investors have long agreed on the riskiness of individual subprime mortgages, no such agreement exists over the risk in pools of subprime assets. Accordingly, the key element in our model is the evolution of investor beliefs over the importance of aggregate risk to subprime portfolios.

In our model, investors view mortgage borrowers as a dividend process, which pays a contractual stream of income while the mortgage remains current and a final payment when the mortgage either defaults or is paid off. The properties of the dividend process are governed by the evolution of the mortgage borrower's credit quality. Investors have knowledge over the initial value of the household's credit quality and understand the factors which determine its evolution. They do not, however, know how important aggregate shocks are to this evolution, and cannot directly observe credit quality after the initial date.

Initially, in 2002, investors draw random beliefs of the importance of aggregate risk in subprime portfolios. Investors who draw low-weight beliefs infer a very high Sharpe ratio for subprime assets. In contrast, investors who draw high-weight beliefs infer a low Sharpe ratio, and purchase very few, if any, subprime assets.

Investors subsequently use incoming information on aggregate defaults and early mortgage payoffs to update their beliefs. Low and stable default and prepayment rates in 2003 and 2004 lead investors to believe the weight on aggregate risk is lower than previously assumed. As a result, over time, the pool of potential subprime investors broadens, increasing the size of the subprime market, pushing down spreads and degrading underwriting standards.

We calibrate the model to replicate the aggregate default and prepayment rates observed in 2003. By varying only the initial average credit quality, we are able to match both subprime and prime mortgage default rates, as well the prepayment rates. In calibrating the model, we assign a high weight to the aggregate shock. However, model investors, who observe the low and stable defaults, move toward beliefs of a low weight on the aggregate shock.

We replicate default rates in the latter years of the housing market in two ways. In the first approach, we maintain our assumptions on all of the model parameters. In particular, we do not allow the variance of the shocks to change. We then compute the probability of observing the actual 2006 default levels under this assumption. We find that this probability

is essentially zero unless the weight on the aggregate shock is high. Second, we replicate default rates in 2005, 2006, and 2007 by gradually increasing the variance and the mean of the aggregate shock. As the variance rises, the number of defaults increases and the variability of these defaults rises.

Using either method, high default rates in 2006 and 2007 prompt investors to reassess their beliefs over the aggregate shock weight. As higher aggregate risk is priced into investment decisions, demand for subprime assets falls, and both subprime origination and securitization grind to a halt. At this point, all investors agree that subprime assets are too risky if the weight on the aggregate shock is high.

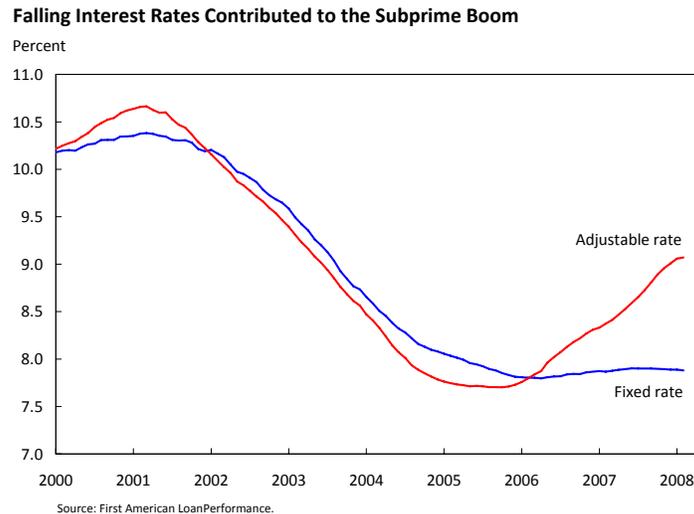
The rest of the paper is structured as follows. Section 2 presents some stylized facts about rise and fall of the subprime market, and highlights the fundamental shift in mortgage performance beginning in 2006. Section 3 reviews the relevant literature on the subprime crisis. Section 4 formally outlines our model. We present both the household and investor sides of the problem, and then describe the model's sensitivity to shifts in its underlying parameters. Section 5 explains the features of our data set. In section 6, we calibrate the model to match 2003 aggregate default and prepayment data, and in section 7, we use the calibrated version to run various simulations and gauge the degree to which the model can replicate post-2003 trends in the mortgage market. Section 8 examines various policy proposals for mitigating the subprime crisis, and tests their effectiveness within the confines of the model. Finally, section 9 concludes.

2 The Rise and Fall of the Subprime Market

Large-scale lending to subprime borrowers¹ is a relatively recent phenomenon. In 1995, subprime loan originations amounted to a modest \$65 billion, and accounted for roughly

¹Subprime borrowers are characterized by poor credit histories and limited capacity to repay their debt. Although there is no official definition of a subprime borrower, Fannie Mae's lending guidelines for conforming loans imply that a borrower is considered subprime if he has a FICO credit score below 620, a debt-to-income ratio (DTI) greater than 75%, and a combined loan-to-value (LTV) ratio of more than 90%.

Figure 2.1: Average Interest Rates on Subprime Mortgages



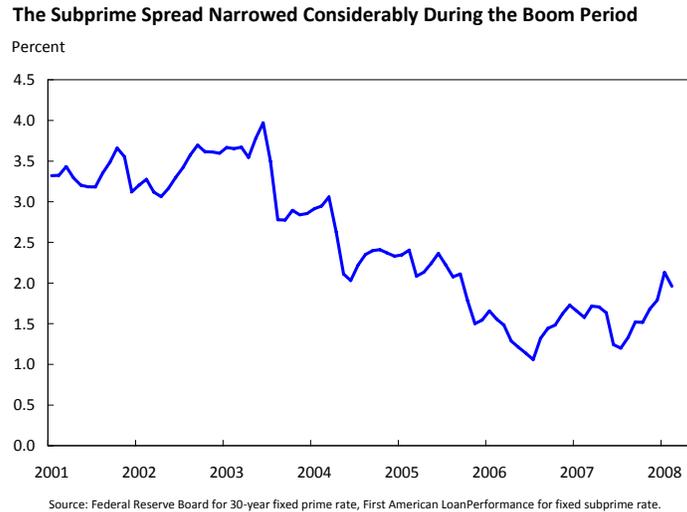
10% of originations in the residential mortgage market. Over the next 5 years, the subprime market share remained low even as both prime and subprime loans grew rapidly. Between 2000 and 2005, however, growth in subprime loans accelerated sharply, and by 2005, the volume of loans reached \$625 billion, with the market share doubling to 20%.

Several factors contributed to the rapid growth in subprime lending – low interest rates, rapid house price appreciation, easier credit, and new mortgage instruments. Between 2001 and 2005, interest rates on both fixed and adjustable-rate subprime mortgages declined by over 2 percentage points, considerably reducing the borrowing costs for prospective homebuyers (Figure 2.1). In addition, hybrid products such as 2/28 and 3/27 adjustable rate mortgages (ARMs) attracted a larger pool of borrowers by offering lower initial costs.

The drawback to these hybrids was that payments increased when the initial teaser rate reset.² One of the major debates in policy circles has been the extent to which households understood the payment risk embedded in these loans. Nevertheless, borrowers entered into these hybrid contracts in increasing numbers. By 2005, hybrid ARMs accounted for nearly

²For a 30-year 2/28 loan for example, a fixed interest rate is in place during first 2 years of the loan, after which a floating rate takes over for the remaining 28 years. This floating rate usually tracks market rates such as the London Interbank Offer Rate (LIBOR), the Cost of Savings Index, or the 11th District Cost of Funds.

Figure 2.2: Subprime-Prime Spread on Fixed Rate Mortgages



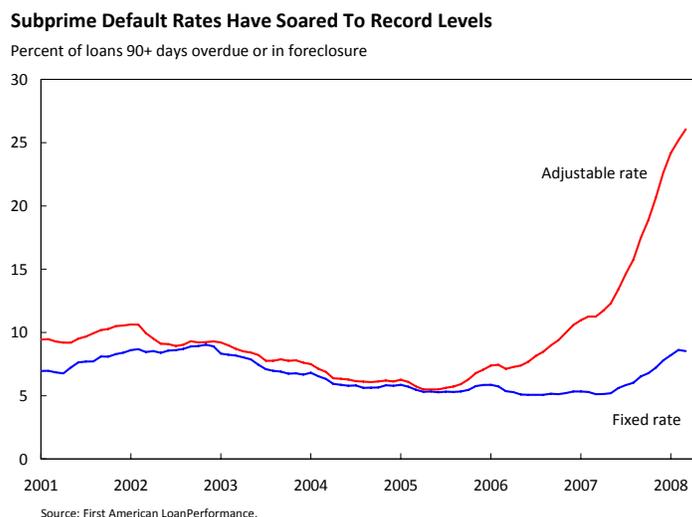
80% of all subprime loans.

During the period of rapid subprime expansion (2001-2005), national house prices rose roughly 50%, giving even high loan-to-value (LTV) borrowers a home equity cushion that would allow for easy refinancing in the event that monthly payments became unaffordable. High house price appreciation also allowed many borrowers to refinance out of the subprime pool.

This high appreciation seems to have tempted mortgage originators to loosen their underwriting standards, and to lend to borrowers with increasingly questionable credit histories and income documentation. Indeed, between 2000 and 2005, average loan-to-value (LTV) ratios on subprime mortgages rose from 75% to 85%, while the share of loans with *both* low FICO scores and high LTV ratios (a form of risk layering) ticked up from 3% to 10%. More importantly, the percent of borrowers presenting full documentation to secure their loans fell from 75% to 62%.

New entrants to the subprime market during this early period appeared convinced that they had managed to diversify much of the risk in their portfolios. Countrywide Financial, the largest mortgage lender in the United States, repeatedly highlighted the subprime market

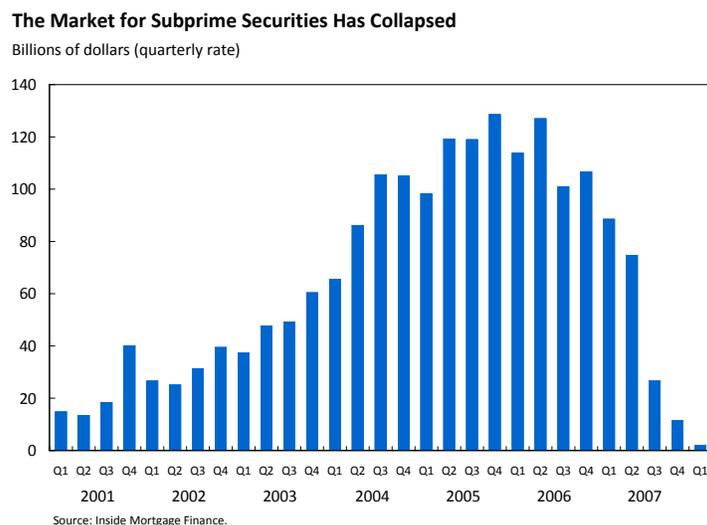
Figure 2.3: Default Rates on Subprime Mortgages



as a source of growth and profitability, and expanded its origination of subprime loans by over 700% between 2001 and 2005. Countrywide's sentiment seemed to permeate throughout the industry. The spread between subprime and prime mortgage interest rates, which can be interpreted as the extra risk compensation that lenders build into their subprime loans, narrowed considerably between 2001 and 2005 (Figure 2.2). Statements from influential financial authorities such as Federal Reserve Governor Edward Gramlich further reinforced these views: "Given the generally low level of serious delinquencies, a purely numerical analysis seems to suggest that significant net social benefits have resulted from the rise in (subprime) credit extensions and homeownership" (Gramlich 2004).

Ongoing technological advancements in credit scoring also seemed to convince investors that even the most intangible subprime borrower risks were being thoroughly considered by lenders and accurately priced by credit rating agencies (Greenspan 2002, 2005). As the perception that subprime portfolios carried high risk-adjusted returns became more inured among market participants, demand for assets backed by these portfolios soared. Incoming data on mortgage portfolio performance during this period appeared to confirm the market's assessment of declining subprime risk. Default rates on both fixed and adjustable-rate

Figure 2.4: Gross Issuance of Non-GSE Subprime Mortgage-Backed Securities



subprime mortgages remained fairly stable through the end of 2005 (Figure 2.3).

With the Government Sponsored Enterprises (GSE’s) – Fannie Mae and Freddie Mac – reluctant to undertake subprime securitization, a private-label market for subprime securities emerged to service the burgeoning demand for these assets. Accordingly, gross issuance of non-GSE subprime securities rose rapidly from 2001-2005 (Figure 2.4). Leading the charge were major Wall Street investment banks, many of whom launched subprime-specific investment arms. Bear Stearns, for example, touted its status as the top issuer of mortgage backed-securities (MBS). In its 2005 annual report, the company noted that it had “secured the top spot in the securitization of adjustable-rate mortgages.”³ Ironically, it is these same mortgages that experienced widespread defaults over the next two years, unwinding Bear Stearns’ portfolio and driving the firm to the brink of bankruptcy.

The subprime tide turned in 2006. Default rates on subprime ARMs, which by then comprised the majority of the subprime market, increased markedly, reaching 27% by May 2008. As lenders responded to this unanticipated elevation in risk, interest rates on subprime ARMs crept up, and the spread between subprime and prime rates increased. Issuance of

³Bear Stearns, *2005 Annual Report*, p. 11.

Figure 2.5: Cumulative Defaults on Subprime Mortgages

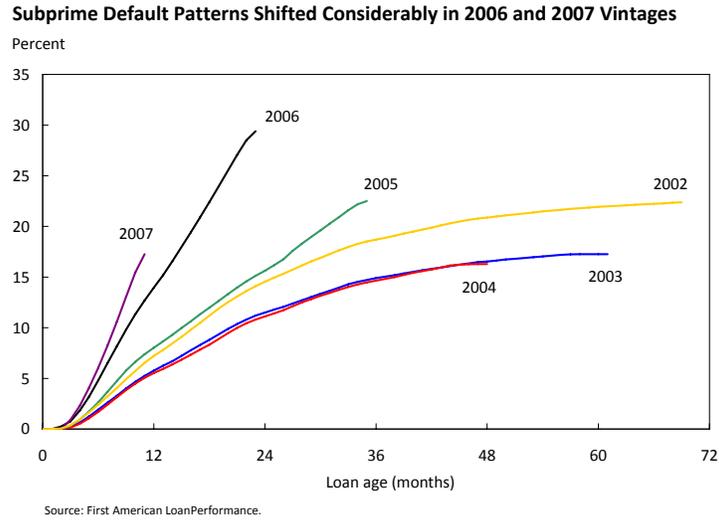
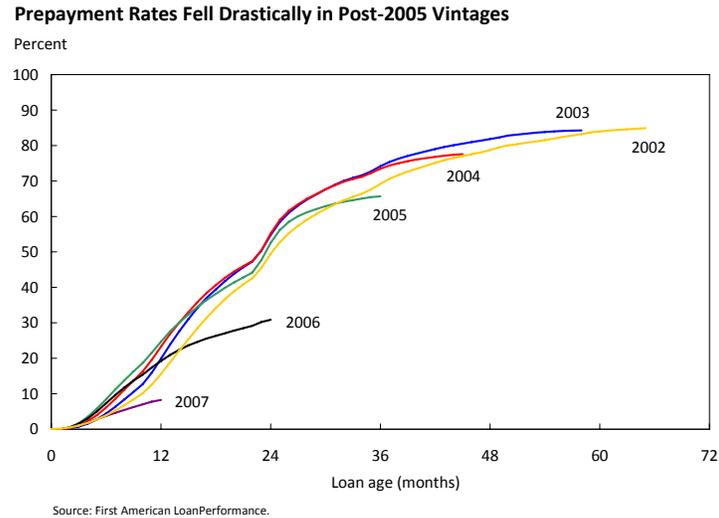


Figure 2.6: Cumulative Prepayments on Subprime Mortgages



subprime MBS fell to near-zero levels by the first quarter of 2008.

Mortgage performance by vintage year shows that the most deterioration occurred in 2006 and 2007 mortgages (Figure 2.5). The same trend is observed in the prepayment rates of the different vintages (Figure 2.6). For example, for 2005 vintages, roughly 50% of subprime borrowers had paid off their loan by the second year, either by selling their house or refinancing into a new mortgage. In 2006, this prepayment rate dropped to 30%.

Previous authors (i.e. Dell’Ariccia, Igan and Laeven (2008)) have reasonably attributed the deterioration in mortgage performance to looser underwriting standards. Under this hypothesis, the uptick in default rates should have been concentrated in the subset of mortgages with lax underwriting standards, such as low documentation or bad risk grades. However, as Figure 2.7 demonstrates, 2006 and 2007 mortgage vintages performed significantly worse than their predecessors, regardless of their interest rate regime, level of documentation, or risk grade. The same is true of the prepayment performance of 2006 and 2007 vintages, as shown in Figure 2.8. Demyank and Van Hemert (2008) document this observation in more detail. Accordingly, we do not believe that the breakdown the subprime market can be attributed solely to a deterioration in credit standards. In subsequent sections, we will show that macro-level shocks are the more significant driving force of the shift in subprime performance beginning in 2006.

Figure 2.7: Cumulative Prepayment Rates on Different Classes of Subprime Mortgages

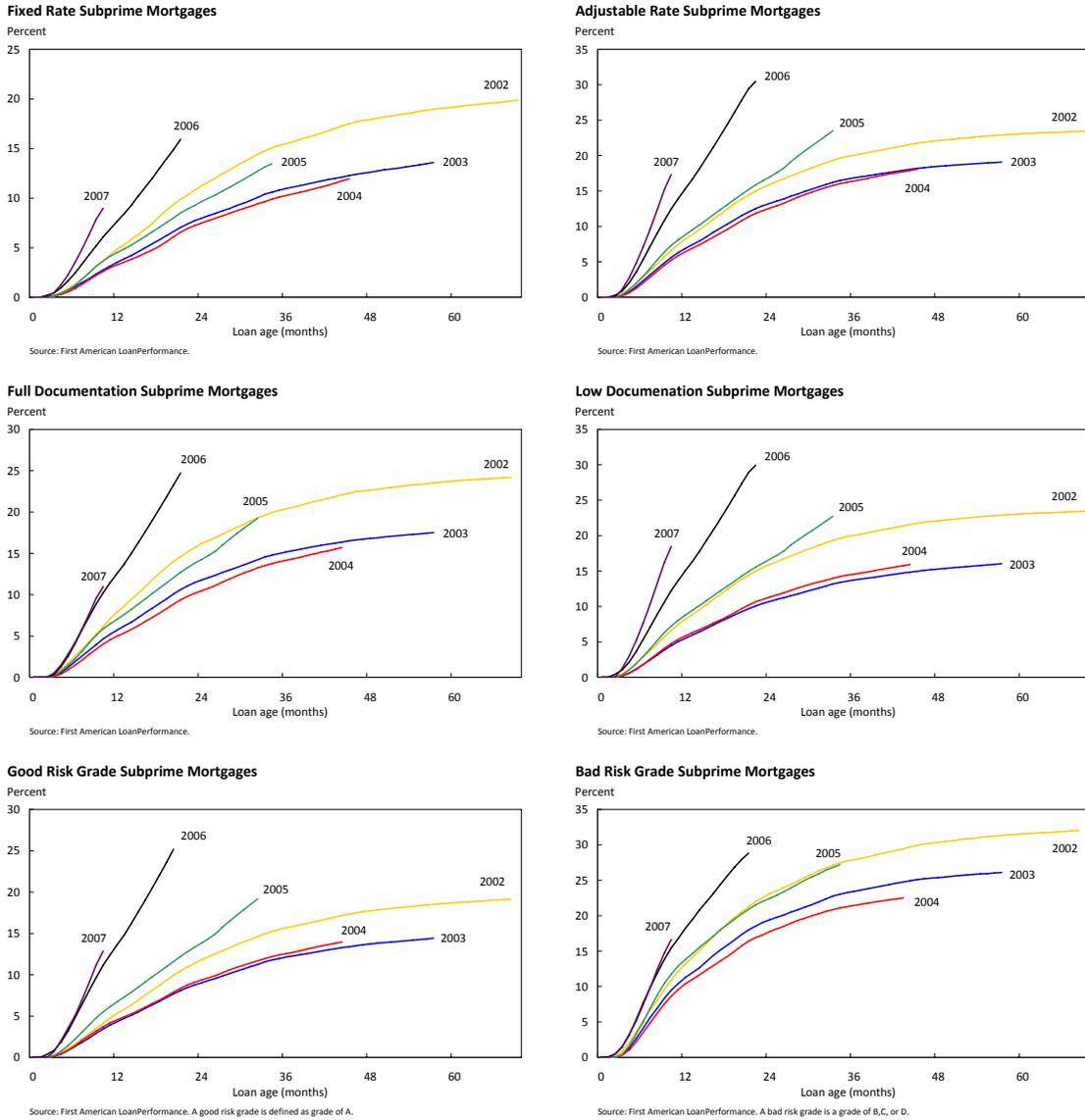
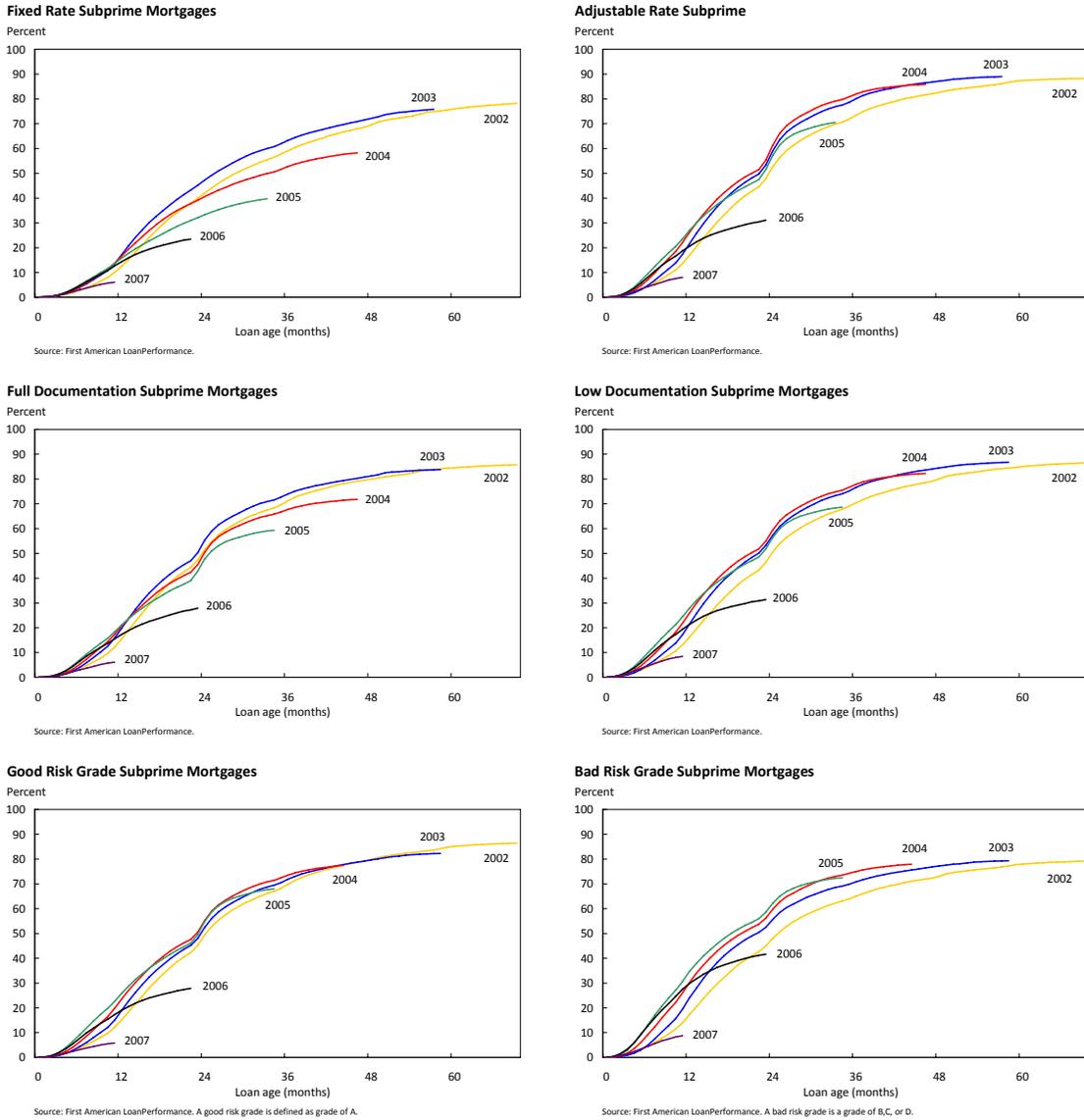


Figure 2.8: Cumulative Prepayment Rates on Different Classes of Subprime Mortgages



3 Literature Review

Explanations for the initial boom in subprime lending, and the later rise in default rates, generally focus on three factors: lax underwriting, changes in house price appreciation, and financial innovations.

Demyanyk and Van Hemert (2008) show that subprime mortgages originated between 2001 and 2006 experienced a gradual deterioration in underwriting quality, as evidenced by rising loan-to-value (LTV) and debt-to-income (DTI) ratios, and laxer documentation requirements. Dell’Ariccia, Igan and Laeven (2008) observe that this deterioration coincided with a surge in loan applications and a rapid entry of new competitors into the lending markets. Using regional breakdowns of Home Mortgage Disclosure Act (HMDA) data, they find that denial rates on mortgages fell more in areas with large increases in the volume of loan applications and the number of lending institutions. Using a different disaggregated dataset, Mian and Sufi (2008) find that credit expansions cannot be justified by improvements in economic fundamentals alone. In particular, regions with the highest denial rates in the mid 1990s experienced the largest lending booms in the post-2000 period even though they had relatively slower gains in income and employment. This branch of the literature therefore attributes the subprime boom to a growing tendency for lenders to accommodate uncreditworthy borrowers. While this behavior allowed the market to flourish, it introduced extra risk into the mortgage pool, and contributed to the eventual surge in default rates.

Changes in house price appreciation are another frequently highlighted cause of the boom and bust in the subprime market. Doms, Furlong, and Krainer (2007) estimate subprime mortgage defaults using various measures of loan risk, house prices, and macro variables such as unemployment, and find that patterns in house price appreciation are far and away the best predictor of default rates. Demyanyk and Van Hemert (2008) perform a similar empirical analysis using a mix of loan and borrower characteristics, and conclude that for both delinquencies and foreclosure rates, house prices appreciation explains the largest portion of the deterioration of post-2005 vintage loans. Gerardi, Shapiro and Willen (2007) emphasize

the impact of slowing house price appreciation on foreclosures rates in Massachusetts.

Finally, financial innovations and structural changes in the mortgage market may have misaligned agent incentives. Technological advances in credit scoring led to more accurate and consistent assessments of borrower risk (see Weicher 2007). LaCour-Little (2000) estimates that automated and computerized underwriting has resulted in savings of up to 3% of total loan values, while Davis (2001) finds savings of about \$916 per loan. These efficiency gains were a double-edged sword, however, because they encouraged mass-production of loans at the expense of due diligence and more comprehensive risk assessment.

Furthermore, advances in risk-scoring allowed an expansion of the originate-to-distribute model, where lenders took mortgages off their books by securitizing and selling them to investors. Keys et. al (2008) and Kiff and Mills (2007) argue that this practice adversely affected mortgage quality in two ways. On the lending side, it allowed mortgage originators to disperse risk to more remote parts of the financial system, and thus diminished their incentives to screen and monitor borrowers. On the securitization side, it increased the demand for low-quality loans, because fee-driven remuneration meant that securitizers profited more from processing a high volume of loans than from processing high-quality loans. Dell’Ariccia, Igan and Laeven (2008) show that regions with higher rates of subprime securitization experienced greater declines in lending standards. These results suggest that the “disintermediation” of the mortgage market fueled more irresponsible lending than would otherwise have occurred under the originate-and-hold model. Mian and Sufi (2008), Kregel (2008), and Wray (2007) put forth similar arguments.

We do not dispute the validity of these explanations. As a whole, however, they rationalize subprime developments by focusing on specific market failures. To the best of our knowledge, we are the first to examine the subprime crisis through the perspective of the *investor*, and to model the boom and breakdown of the market as rational responses to shifting investor risk perceptions.

4 The Model

4.1 The Household Problem

On top of a standard consumption model, households decide between owing and renting each period. Owned housing is purchased with a mortgage and all households own if the mortgage interest rate is sufficiently low. When the interest rates fall, a larger pool of potential borrowers exists.⁴

The key feature of the household problem is a latent variable that determines the household's position within the continuation region and the stopping times of the optimization problem.⁵ When the variable gets too high (income falls, house prices fall, the current mortgage rates rise), the household optimally defaults. When the variable gets too low (income rises, house prices increase, or alternative mortgage rates fall), the household chooses to refinance. Our model replicates the basic features of these earlier studies.

4.1.1 Renter's Problem

Every renter solves the following program:

$$V^r(A, P, R) = \max_{T, \{c_t\}_{s=0}^{T-1}} \left\{ \begin{array}{l} \sum_{s=0}^{T-1} \beta^s u(c, 0) + \\ \beta^T \max_H \{V^o(A_T, H_T, P_T, R_T)\} \end{array} \right\} \quad (1)$$

$$A_{s+1} = (1 + r_t) A_s + W_s - c - (P_T H - P_T \theta H) 1_{(s+1=T)} > 0 \quad (2)$$

$$u(c_s, 0) = \frac{(c_s^\alpha (0 + \gamma)^{1-\alpha})^{1-\sigma}}{1 - \sigma} \quad (3)$$

⁴The household problem is a multi-dimensional optimal stopping problem. The key features of this model are studied in similar problems by Grossman and Laroque (1989), Martin (2003), and Stokey (2008). In these problems, the household's state variables determine an optimal stopping time – whenever the state variable reaches certain values the household makes a discrete decision.

⁵Although the state-space itself may be multi-dimensional, the value function of the household summarizes all of the variables. In this case, the problem can always be summarized by a single random variable moving between two boundaries, which may themselves be a function of the state-space.

The parameter γ is the flow utility of rental housing. $V^r(A, P, R)$ is the value of renting when the agent holds assets A . The current price of owner-occupied housing is P , the current wage is W , and the mortgage rate faced by this household (if it purchases a house in the current period) is R . Assets evolve over the inaction region according to equation (2). The household receives a potentially time-varying interest rate on its holdings of assets, r_t , earns labor income, W_s , and consumes c_s units of the numeraire good. The final term is the cost of acquiring a house of size H . The household purchases the house at a cost PH and finances θ percent of the purchase price.

We assume all rental housing is the same size and that there is an infinite amount of rental housing available; hence, rents are zero. Each household gets a flow value of γ by consuming rental housing. This outside option ensures a ceiling on the market-clearing mortgage interest rate.

Because there are no transaction costs of moving from the rental housing to owner-occupied housing and because households often enter the rental state simply to re-optimize their house size (see below), the stopping time in the rental problem may be at zero: $T = 0$. In this scenario, $V^r(A, P, R) = V^o(A - (1 - \theta)PH, H, P, R)$, where V^o is determined from the owner's problem.

4.1.2 Owner's Problem

Each owner solves:

$$V^o(A, H, P, R) = \max_{T, \{c_s\}_{s=0}^{T-1}} E \left\{ \sum_{s=0}^{T-1} \beta^s u(c, H) + \beta^T \max_{I_p} \begin{cases} V^r(A_T, P_T, W_T, R_T) \\ -\Upsilon_p I_p - \Upsilon_d(1 - I_p) \end{cases} \right\} \quad (4)$$

$$A_{s+1} = (1 + r_t) A_s + W_s - c - (R - 1)\theta PH + I_p(P_T H - P\theta H) \quad (5)$$

$$A_{s+1} - (P_T H - P\theta H) > 0 \quad (6)$$

$$I_p = \begin{cases} 1 & \text{if } s = T \text{ and the mortgage is paid off} \\ 0 & \text{o/w} \end{cases} \quad (7)$$

The setup of the owner's problem resembles that of the rental problem. In addition to choosing the level of consumption, the household must decide each period whether to continue living in the same house and continue making mortgage payments, to default and return to the rental state, or to payoff its current mortgage and return to the rental state. If the household either pays off the mortgage or defaults, it incurs a utility cost of Υ_p or Υ_d . These costs ensure a state-dependent inaction region over which the household does not move (Martin (2003)).

We will take advantage of the fact that the household's decision rule is summarized by the evolution of A and P over the inaction region. We derive our estimating equation from the household's budget constraint. The two key stochastic terms of the asset evolution equation are wages and prices. For both of these, we believe there are idiosyncratic and aggregate elements. That is, some of the innovations to wages are agent-specific (i.e. his productivity, his labor force status) and some are aggregate (i.e. boom times versus bust times). The same is true of prices. Any individual house price contains local, regional, and national elements. We are going to subsume all of this uncertainty into a single equation

with latent variable x .

$$x_{t,j} = \begin{cases} c + x_{t-1,j} + (1 - \omega)\varepsilon_{t,j} + \omega\eta_t & \text{if } t < T \\ b(\mu_{j,t}), B(\mu_{j,t}) & \text{if } t \geq T \end{cases}$$

All idiosyncratic risk is absorbed in the term ε and all aggregate risk is absorbed in the term η . The term ω then governs the relative weight on aggregate versus idiosyncratic risk. The constant c absorbs any tendency of the wages, prices and interest rates to move systematically. The boundaries b and B are induced by the combined values of wages, prices and interest rates, which in turn induce a stopping time in the owner's problem. We estimate the parameters of this reduced-form household problem by calibrating the model to replicate data on observed defaults and prepayments.

4.2 The Mortgage Market

The remainder of the paper focuses on the subprime market from the perspective of the investor. Potential investors in subprime assets must make portfolio choices over an array of assets. We assume the key determinant of the investors' portfolio will be the perceived Sharpe Ratio – the ratio of the expected return to the expected variance of any asset. Assets with high Sharpe ratios are most desirable; assets with sufficiently low Sharpe ratios are dominated assets and are not demanded by investors.

With respect to the subprime market, investors have different estimates of the weight of aggregate risk in the household problem. Some investors view household defaults as being determined primarily by idiosyncratic risk while others believe that aggregate risk is of utmost importance. Differences in these views shape beliefs over both the mean return and variance of portfolios of subprime assets.

Over time, investors use incoming information to re-estimate the weight of aggregate risk. As their beliefs evolve, the demand for subprime assets fluctuates, and the size of the subprime investment pool changes accordingly. This feature is important for explaining the

evolution of the subprime market.

4.3 Individual Mortgages

The lender views each subprime mortgage holder as a statistical process characterized by a payment stream, a risk of prepayment, and a risk of default. In default, the payment stream goes to zero and the lender recovers one final payment – the amount recovered through a foreclosure proceeding.

The investor understands the latent process that determines defaults and prepayments by households and therefore views individual borrowers as dividend streams that follow the statistical process:

$$\delta_{j,t} = \mu_{j,t}(1 - \Gamma_{j,t}) + D_{j,t}\Gamma_{j,t} \quad (8)$$

$$\Gamma_{j,t} = \left\{ \begin{array}{l} 0 \text{ if } t < T \text{ where } T \text{ is a stopping time of } x \\ 1 \quad \text{o/w} \end{array} \right\} \quad (9)$$

$$x_{t,j} = \left\{ \begin{array}{l} c + x_{t-1,j} + (1 - \omega)\varepsilon_{t,j} + \omega\eta_t \text{ if } t < T \\ b(\mu_{j,t}), B(\mu_{j,t}) \text{ if } t \geq T \end{array} \right\} \quad (10)$$

$$D_{j,t} = \left\{ \begin{array}{l} 1 \text{ if } x_{j,t} = b(\mu_{j,t}) \quad t > M \\ F \text{ if } x_{j,t} = B(\mu_{j,t}) \end{array} \right\} \quad (11)$$

The subscript j, t denotes borrower j at time t ; hence, $\delta_{j,t}$ denotes the time t dividend on borrower j 's mortgage. The term Γ is an indicator function that determines whether or not the mortgage is alive. So long as Γ is 0, the borrower makes a mortgage payment equal to $\mu_{t,j}$, the contractual interest rate.

The mortgage dies if $\Gamma = 1$. If the mortgage is prepaid, the final return is one, $D_{j,t} = 1$, as the outstanding capital is paid off. If the mortgage defaults, the final return is equal to a terminal payment, $D_{j,t} = F$ (the foreclosure recovery), that is assumed to be less than one. The value of Γ is determined by an underlying latent stochastic process x . The mortgage

also dies after a specific number of months M . At this time, we assume the principle is paid off.

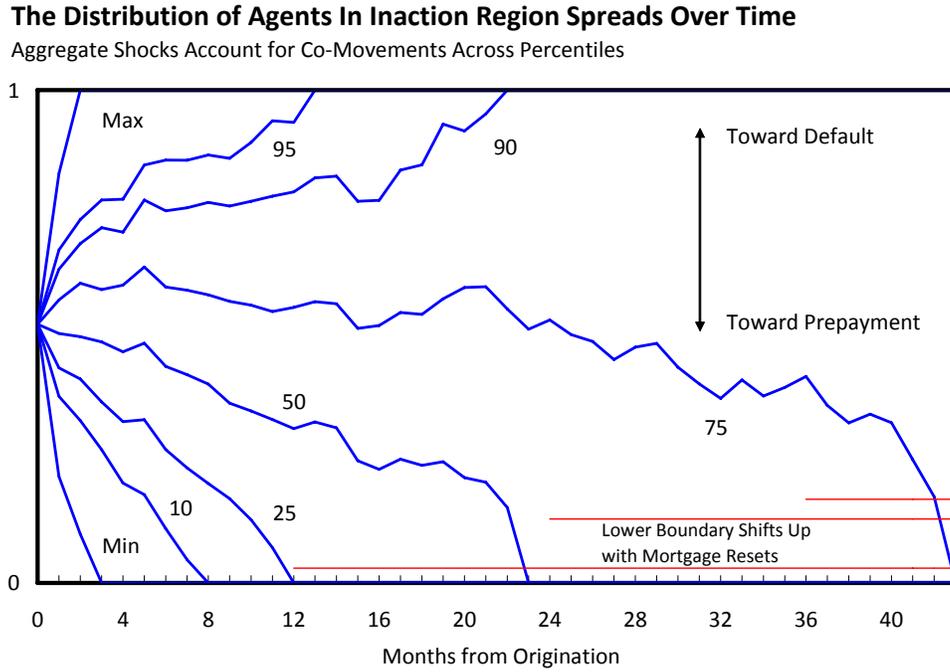
The variable x is a random walk with drift term c , and is driven by two stochastic processes. The first process, ε , is normally distributed and independent across borrowers. The second process, η , is normally distributed and is common across all borrowers. The process has lower and upper absorbing states, denoted b and B . The total variance of x is a weighted average of the two shock variables. Values of ω near zero imply default risks that are independent across agents. Values of ω near one imply highly correlated aggregate defaults. Date T is a stochastic stopping time and denotes the first date at which x hits either boundary. The probability of an individual mortgage hitting either boundary is a function of the variance of x , the drift term c , and the initial value of x .

Figure 4.1 is a visual representation of the model, and shows the distribution of mortgage quality over time. The lines in the chart show different percentiles of mortgages from origination through 45 months. The variance in the calibrated model is sufficiently high that we begin to observe both defaults and prepayments 3 months after origination. By 18 months, more than 5% of mortgages have defaulted and more than 25% of prepaid. By 24 months, these numbers rise to 10% and 50%. The common movements in the lines are induced by particularly large draws of the aggregate shock.

4.4 The value of ω

The parameter ω determines whether the majority of the variance of individual mortgage returns arise from the common shock or from idiosyncratic shocks. Figure 4.2a plots the average return of the portfolio as ω ranges from 0.3 to 0.8. The solid red line shows the expected return for a high initial value of x and the dashed black line shows results for a low initial value of x . High x 's are more likely to default than are low x 's. For both values of x , the expected return is increasing in ω for low values. However, as the weight on the aggregate shock increases, the expected return eventually begins to decrease. This

Figure 4.1: The Spread of Mortgage Quality

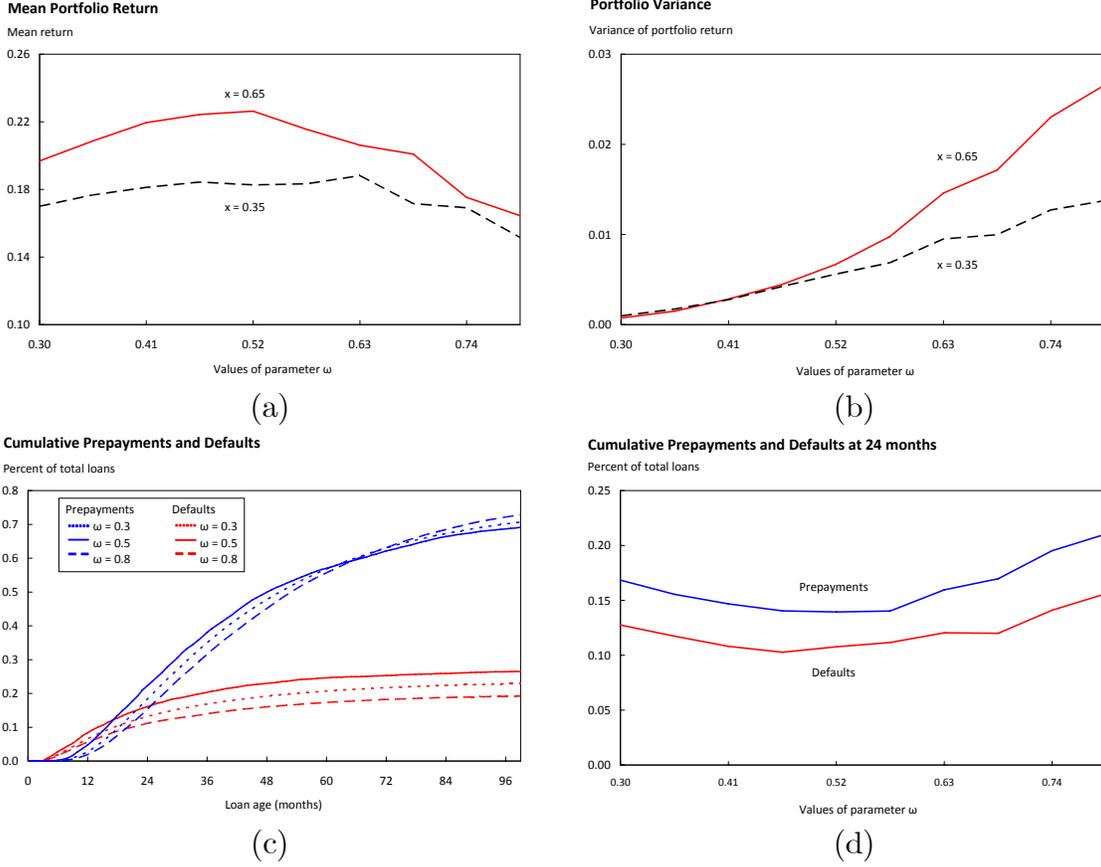


hump-shape is important.

Figure 4.2b plots the variance of the return for the same values of ω and x as in Figure 4.2a. Unlike the expected return, the variance is a monotonic increasing function of the weight on the aggregate shock. This result is not surprising. The aggregate shock is not being averaged across individual mortgages and so as we increase its weight, we expect the variance of the portfolio to increase. If the investor has knowledge of the variance of the portfolio, ω is identified. Importantly, the Sharpe Ratio of the mortgage portfolio is strictly decreasing in ω .

Figures 4.2c shows cumulative defaults and cumulative prepayments for a high value of x . These lines exhibit very little variation with changes in ω . However, Figure 4.2d shows the cross-section of cumulative defaults and cumulative prepayments at 24 months from origination. Both defaults and prepayments are u-shaped in ω . There is an isomorphic mapping between the shape of these functions and the expected return.

Figure 4.2: The Value of ω



4.5 The Investor Problem

The investor understands the latent process that drives prepayments and defaults and cares only about the Sharpe ratio of his mortgage portfolio. The investor has knowledge over the set of model parameters X :

$$X = [\mu, c, \mu_\varepsilon, \sigma_\varepsilon, \mu_\eta, \sigma_\eta, b(\mu), B(\mu), \omega]$$

with the exception of ω , the weight on aggregate risk. This parameter must be inferred from data flow.

The investor has complete knowledge of the aggregate performance of his portfolio, but cannot observe individual mortgages. Therefore, the information flow consists of the number of mortgages within the portfolio that either default or prepay period-by-period. Each

period, the investor compares the observed outcome to the probability of observing the outcome conditional on different values of ω and then updates his prior distribution of its value.

The evolution of the density function obeys the following equation (Bayes' rule):

$$f_t(\omega \mid \mathbf{d}, \mathbf{p}) = \frac{\Pr(\mathbf{d} = d \pm \tau_d, \mathbf{p} = p \pm \tau_p \mid \omega) f_{t-1}(\omega)}{\int_0^1 \Pr(\mathbf{d} = d \pm \tau_d, \mathbf{p} = p \pm \tau_p \mid \omega) f_{t-1}(\omega) d\omega}$$

where \mathbf{d} and \mathbf{p} are the observed value of cumulative defaults and prepayments each month. That is, the investor is uncertain over the parameter ω . However, he knows the data generating process and understands how changes in ω alter the probability of observing a particular level of defaults. The investor uses this information to decide on his portfolio holdings.

5 Data

Aggregate data on subprime mortgage interest rates, default rates, and prepayment rates is compiled from the First American LoanPerformance database. This proprietary database covers 85% of all securitized subprime mortgages. Between 2001 and 2006, these securitized mortgages constituted 54%, 63%, 61%, 76%, 76%, and 75% of the national subprime market, respectively.⁶ The LoanPerformance data also contains a comprehensive breakdown of aggregate loan performance by interest rate regime year of origination, FICO score, LTV ratio, and a host of other mortgage characteristics and underwriting standards.

Mortgage delinquencies in the LoanPerformance data are calculated on a Mortgage Bankers Association (MBA) basis. Under the MBA method, a loan would be considered delinquent if the payment had not been received by the end of the day immediately preceding the loans next due date (generally the last day of the month which the payment was due). For the purposes of this paper, we define defaults as loans that are seriously delinquent – those that

⁶Mortgage Market Statistics Annual (2007).

are 90 or more days overdue on their last payment or are in foreclosure. To derive our default rates, we therefore sum all loans that are 90 or more days overdue or in foreclosure and divide this sum by the total number of loans. Similarly, for prepayment rates, we calculate the fraction of total loans that terminated due to a borrower refinancing into another mortgage or selling his home. We only consider subprime loans, and exclude Alt-A loans because they fall in the nebulous area between prime and subprime. Data on issuance of subprime securities comes from Inside Mortgage Finance, and is published in Mortgage Market Statistics Annual. House price statistics are from the Office of Federal Housing Enterprise Oversight (OFHEO).

6 Calibration and Parameterization

We calibrate most of the model parameters to replicate the time series of prepayments and defaults on mortgages originated in 2003. To replicate the series, we find the average value of defaults and prepayments over 10,000 draws of the time series for x , with 10,000 mortgages in each portfolio. This number of draws is sufficient to generate a law of large numbers.

We must find the constant c , the boundaries b and B , the standard deviations for η and ε , σ_η and σ_ε , and an initial value for x such that the mortgage portfolio matches, month-by-month, the defaults on 2003 vintages. It turns out that the model is sufficiently flexible that several solutions can be found. Moreover, we can replicate the 2003 defaults assuming that the variance arises from either aggregate risk or from idiosyncratic risk. The main difference between these two calibrations is the variance of defaults over different draws of the shocks.

Calibrating the model is complicated by the mortgage resets, which occur at 12, 24, and 36 months after origination. In the data, the main impact around these reset points is an increase in the number of prepayments. We handle prepayments by shifting the boundaries

Table 6.1: Calibrated Parameters

Parameter	Symbol	Value
Discount factor	β	0.997
Relative risk aversion of investor	α	2
Initial borrower credit quality	x_0	0.525
Credit quality drift	c	-0.015
Prepayment boundary	b	0
Default boundary	B	1
Variance of idiosyncratic shock	σ_ε	0.1
Variance of aggregate shock	σ_η	0.025
Weight on aggregate shock	ω	0.8
Contractual mortgage rate	μ	.09
Recovery rate	F	0.75

inward in these months. The drift of the latent process is also allowed to change.

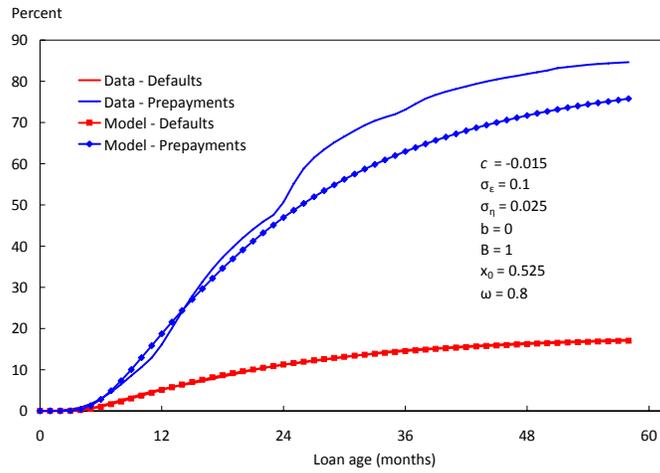
Figure 6.1 shows both the model solution for defaults and prepayments and the data for these two series. The solid lines are actual data and the marked lines are the model output. In the calibration in figure 6.1a, we do not allow any parameter changes. In particular, the boundaries of the inaction region are constant. The parameters used in the calibration are shown in each of the figures.

With constant parameters, we match default rates but not prepayment rates. To match the prepayment data, we allow the prepayment boundary to shift inwards at each of the reset dates. We choose the amount of the change to replicate the shifts in prepayments at these dates. As can be seen in Figure 6.1b, these changes allow us to come much closer to replicating the 2003 data. The fit for defaults is approximately the same and the fit for prepayments is significantly improved, although we continue to underestimate prepayments after 24 months.

Figures 6.1a and 6.1b are produced using a high value of ω . The same fit can, of course, be obtained using a very low weight on ω . In Figure 6.1c, we hold all parameters as above

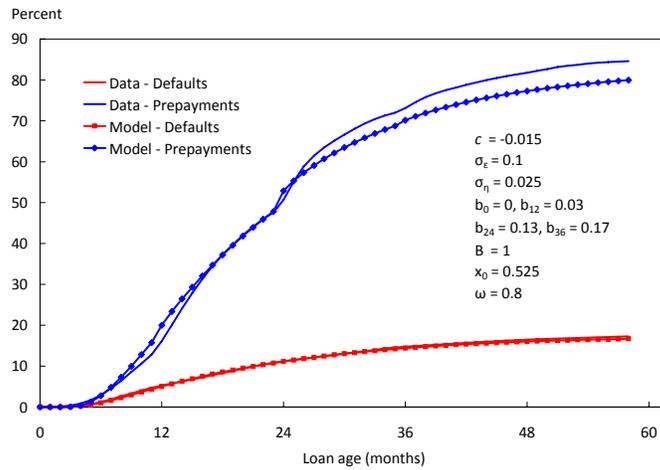
Figure 6.1: Model Simulations of 2003 Defaults and Prepayments

High Weight on Aggregate Risk: No Change in Boundaries



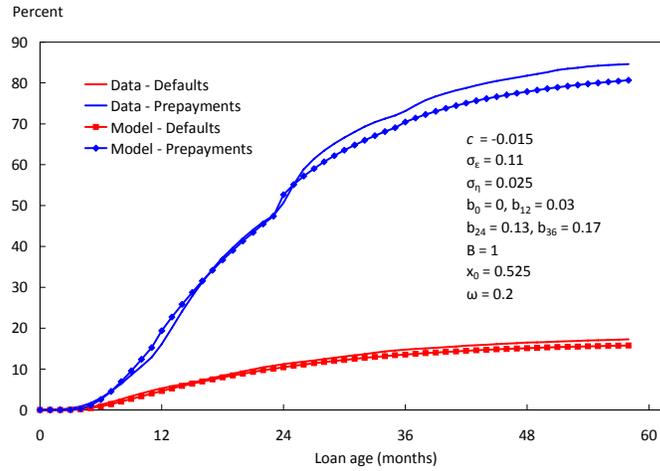
(a)

High Weight on Aggregate Risk: With Boundary Changes



(b)

Low Weight on Aggregate Risk: With Boundary Changes



(c)

except for the weight on aggregate risk and the standard deviation of the idiosyncratic shock. The similarity of the two pictures illustrates the investor's problem in the identification of ω . The weight on the aggregate shock simply cannot be gleaned from the average level of defaults even if the variance of the two shocks is known.

Only through the variance over time can the two samples be reliably identified. We compute the variance across the 10,000 realizations of the time path at 12 months. The variance across samples using a high weight on the aggregate shock is 10 times larger than the variance when the weight on the aggregate shock is small. Similarly, at any date the range of defaults and prepayments observed is an increasing function of ω . Importantly, and discussed below, the probability of observing 2006 default levels is also an increasing function of ω . When ω is set equal to 0.2, aggregate defaults at 12 months are not observed higher than 7% in our sample of 10,000 paths. Defaults above 7% occur along nearly 30% of the paths when the aggregate weight is set to 0.8.

7 Simulation: The Subprime Market 2002 to 2007

Given our calibration of the model, what does it have to say about the evolution of the subprime market between 2002 and 2007? How did investor's views on the key parameter ω evolve over this period? Why did the subprime market double in size between 2003 and 2005? Why did underwriting standards relax? Why did spreads fall at the same time? And, most importantly, why did the market collapse in 2007? In this section, we take what we have learned about the model and apply it to the experience of the subprime market over these years.

7.1 Inferring the Value of ω in 2003 and 2004

The subprime market was a relatively new phenomena in 2002. Investors knew very little about the parameters of the model, and especially little about ω , the relative weight

of aggregate risk in the problem. The main source of information at the end of 2002 was that, during the 2001 recession, aggregate default rates on subprime mortgages rose only slightly and the spread between prime and subprime defaults was quite narrow – surely an indication of small aggregate risk.

Investors therefore formed their own expectations of ω ; there existed a distribution of priors over this parameter. Investors who believed that ω was low invested in subprime assets, and those who believed ω was high avoided subprime assets. This polarization of beliefs is illustrated by the difference between the long positions taken by Countrywide Financial, an early and large subprime investor who likely believed the weight was near zero, and the short position taken by Goldman Sachs, a late and small subprime investor who must have believed the weight was substantial.

Beliefs over a low aggregate shock weight fueled the nascent subprime market, and as larger pools of mortgages emerged, more information could be inferred. Investors, both in the market and on the outside, observed the flow of aggregate defaults and aggregate prepayments by monthly vintages. They used this information to infer the weight of aggregate risk.

Independent of the investor’s priors on the weight of aggregate risk, the level of defaults and the variance of defaults across months and across vintages pushed investors towards the belief that the weight on aggregate risk was quite low. Using the parameters of the calibrated model, the probability of observing the actual 2003 levels of defaults and prepayments is close to 67% if the weight on the aggregate shock is 0.2 (a low value). If the weight on the aggregate shock is 0.8 (a high value), the probability of observing the 2003 outcome is around 29%.⁷ These contrasting probabilities cause very rapid Bayesian updating. Since the outcomes in the data were essentially the same in 2004, the probability of observing these

⁷These probabilities are computed by searching for the percentage of time paths that have 12-month outcomes consistent with the data. The lower bound of outcomes consistent with the data is the number of defaults or prepayments at 11 months and the upper bound is the number of defaults or prepayments at 13 months. The distribution of outcomes under the two weights is shown in the next section. The model with a higher weight on the aggregate shock produces a distribution of outcomes with wider support.

Table 7.1: Investors Moved Toward Low-Weight Beliefs Very Quickly

	Prior Belief	End 2003	End 2004	End 2005	End 2006
Probability	0.05	0.11	0.21	0.22	0
that ω is low	0.25	0.43	0.62	0.63	0
(0.2)	0.50	0.69	0.83	0.84	0

outcomes is also similar. In 2005, defaults were slightly higher and prepayments slightly lower, but the probabilities still slightly favored the lower weight. Investors continued to place higher probability on a low weight outcome because, by this time, their priors had shifted towards a low aggregate shock weight.

Assuming that the value of ω must be either 0.2 or 0.8, Table 7.1 shows the probability investors placed on 0.2 after each year of data.

The probability investors place on a low value of ω increased rapidly based on the realized data in 2003 and 2004. Recall that, in 2005, the number of defaults was higher than in either 2003 or 2004. A priori, one might assume that this data would lead investors to reduce their perceived probability of a low ω . This is not the case in our model; although the 2005 outcome has lower probability under either weight, it has higher probability with a low value of ω – 8.6% versus 8.0%. Therefore, even as late as the end of 2005, investors continued to increase the subjective probability placed on the low weight case.

By 2005, even an investor endowed with a very low prior of the weight on the aggregate shock placed a 63% probability by 2005. Indeed, only those investors who placed smaller than a 10% probability on the low aggregate shock weight continued to place less than a 50% probability by the end of 2005. The last column of the table shows investor beliefs by the end of 2006 assuming that all model parameters remain as in the 2003 calibrated version. We return to this column in the next section.

7.2 The Explosion of Subprime Lending: 2002-2005

The timing of our investors' change in priors on the subprime market matches the timing of the explosion in subprime lending. In figure 2.4, we showed the volume of gross issuance of Non-GSE subprime backed securities from 2001 to 2008, which we interpret as a reasonable approximation of the total amount of subprime issuance. Between 2001 and 2002, the volume of issuance was between \$15 and \$30 billion. We deduce that these are the lenders who drew high prior beliefs over a low value of ω – about 15% of the total pool of subprime investors. By the end of 2003, subprime issuance was starting to boom. In the first quarter of 2004, total issuance was near \$70 billion, and by the first quarter of 2005, it was approaching \$100 billion. The subprime market peaked, as measured by issuance, in the fourth quarter of 2006 at \$130 billion. Over the following six quarters, subprime issuance plummeted to nearly zero.

Consistent with this interpretation of the subprime market, spreads of subprime interest rates over prime interest rates (shown in Figure 2.2) were fairly constant between 2001 and 2003, but between 2003 and the beginning of 2006, fell 2.5 percentage points from 3.5% to 1.0%. According to the model, an investor who moved from a 25% probability of a low weight on the aggregate shock to an 80% weight perceived a substantial increase in the Sharpe Ratio.

7.3 The Collapse of the Subprime Market: 2006-2007

In our model, a collapse in the subprime market can only occur if all investors suddenly believe the value of ω is high. In this section, we show that the evolution of the subprime market in 2006 and 2007 pushed investors off of their low weight beliefs. Under the various assumptions on how the model produces the 2006 outcome, investors either immediately slam to a zero probability of a low weight or quickly move to a very small probability of a low weight.

We use the model to replicate default rates in 2006 in two different ways. In the first

method, we maintain our assumptions on all of the model parameters. In particular, we do not allow the variance of the shocks to change. We then compute the probability of observing the actual level of 2006 defaults under this assumption. We find that this probability is essentially zero unless the weight on the aggregate shock is high. Second, we replicate default rates in 2005, 2006, and 2007 by gradually increasing the variance and the mean of the aggregate shock. As the variance rises, the number of defaults increases and the variability of these defaults rises.

7.3.1 The Probability of Observing 2006 Defaults using 2003 Parameter Values

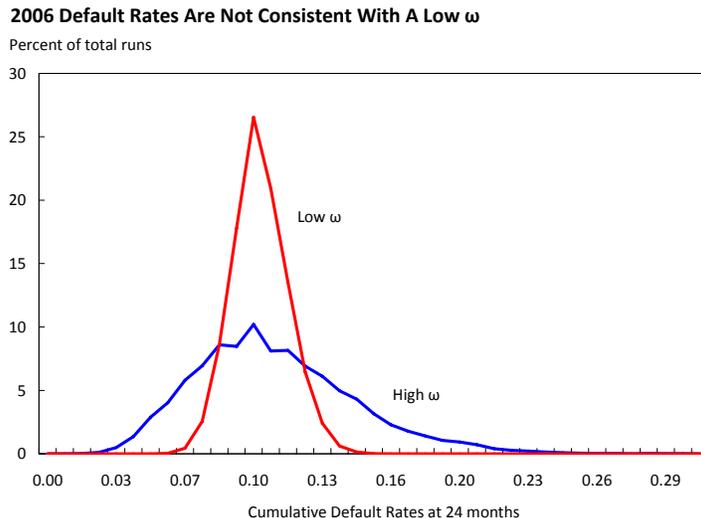
In this section, we assume that there is no change in the distribution of the shocks faced by the economy. All parameters remain as in the calibrated version above. If this assumption is maintained, then all investors observing the level of defaults in the 2006 vintage mortgages must update their priors and assume that the weight on the aggregate shock is very high.

Figure 7.1 shows the distribution of default rates 24 months after origination assuming alternatively that the weight on the aggregate shock is 0.8 and 0.2. The data is from the same simulations that produced the calibrated figures above. As such, the mean of both distributions is essentially identical. This picture makes obvious the fact that an increase in weight of aggregate risk produce outcomes that are a mean-preserving spread of the outcomes from lower weights.

Looking at the low weight line, the range of cumulative defaults is from about 6.5% to about 13%. In none of the simulated time series were defaults more than about one-half of the observed level of defaults. In contrast, the distribution implied by a high weight ranges from 2% to 28%. Only under a high weight on the aggregate shock is it possible to observe the 2006 level of defaults conditional on the distribution of both idiosyncratic and aggregate risk from the 2003 calibrated model.

Any investor who did not believe the economy had changed in any way must infer a high weight on the shock. In terms of the Bayesian updating shown above, the investor has

Figure 7.1: Distribution of 2006 Defaults Using 2003 Calibration



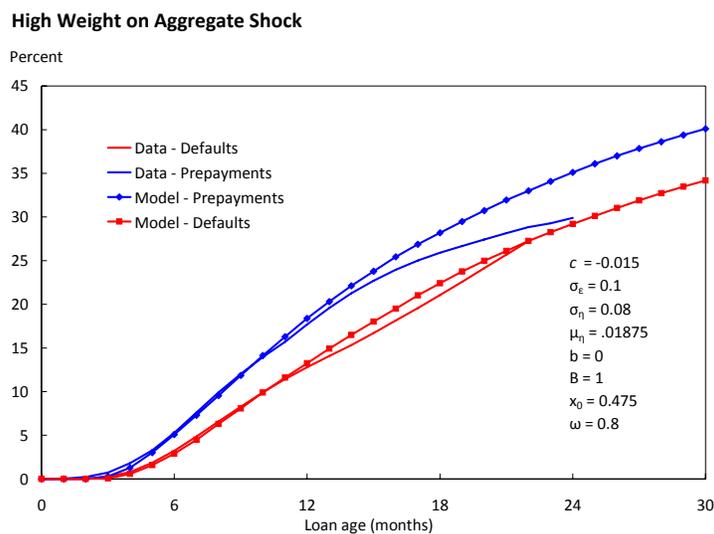
observed a zero probability event under the hypothesis that the weight is low: the updated probability that ω is a low value is zero. Therefore, by the end of 2006, these investors would no longer wish to hold new subprime assets.

However, even with a high weight on the aggregate shock, the probability of observing cumulative defaults above 26% is still very low (about 0.1% of observations). The subprime market did not end instantaneously; rather, issuance was positive for several quarters after the end of 2006. Although there may be many reasons for this gradual fall, it is possible that investors perceived a change in some of the other parameters of the model and may not have placed zero weight on low values of ω . In the next subsection, we explore what distributional changes are required to move from the 2003 default levels to the 2006 default levels, assuming each year is an average year conditional on the distribution.

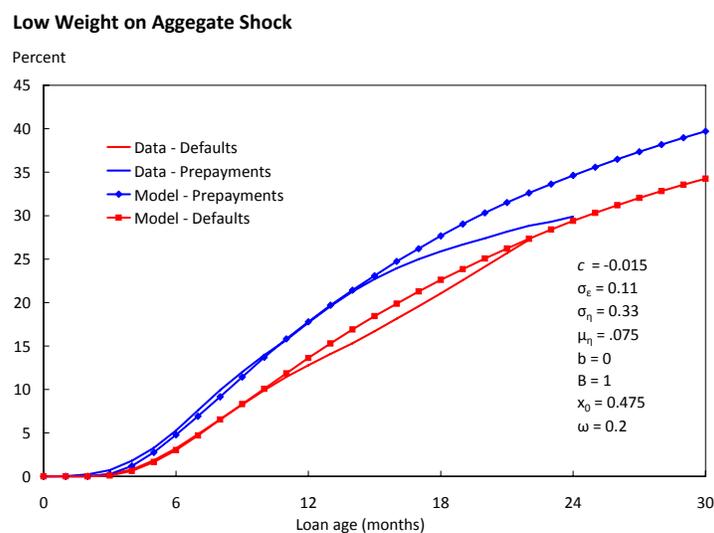
7.3.2 Increasing the Variance and Mean of the Aggregate Shock

In this section, we allow the aggregate economy to deteriorate slightly in 2005 and 2006. We model this deterioration by increasing the variance and mean of η . Figure 7.2 shows the results of this exercise. To match the 2006 data, we increased the standard deviation of the

Figure 7.2: Model Simulations of 2006 Defaults and Prepayments



(a)



(b)

aggregate shock from 0.025 to 0.08 and the mean of the shock from 0 to 0.01875.⁸ We also found it necessary to increase the initial credit quality of households by about 10%. Without this shift, we could not match the high level of early prepayments. This result foreshadows our later finding that the 2006 pattern cannot be replicated through a deterioration in credit quality.

⁸An increase in the mean of the shock is equivalent to increasing the drift term, c . An increase pushes the households toward the default boundary over time.

In figure 7.2a, we do a good job of predicting aggregate defaults; however, by 12 months from origination, we significantly over-predict the number of prepayments. We hypothesize that this may be related to the breakdown of the subprime market beginning in 2007. The breakdown likely required a subprime household to have a much larger increase in credit quality before being able to refinance. Unlike in 2003, however, we are unable to find a set of parameters that replicates this region. Moving the prepayment boundary down does not improve the fit. While the level of the curve can be matched with appropriate movements, the contour of the curve cannot.

We can also replicate the same data when the weight on the aggregate shock is low. Figure 7.2b has essentially the same properties as Figure 7.2a. However, the parameters used to generate the model are quite different. The standard deviation of the aggregate shock is four times larger than when ω was high – 0.33 as opposed to 0.08 – and the mean is also much larger – 0.075 as opposed to 0.01875.

The change in parameters is particularly striking relative to the changes needed to replicate the 2003 data. Recall that moving from a ω of 0.8 to 0.2 required a shift in the standard deviation of the idiosyncratic shock from 0.1 to 0.11, a 10% change. A shift of this magnitude is unlikely to be identifiable in macroeconomic data. In contrast, in 2006, both the mean and the variance must be shifted by a full factor of 4 in order to account for the difference. In other words, the economy must be four times worse both in terms of volatility and averages in 2006 in order to achieve the observed level of defaults and prepayments if the true ω is 0.2 rather than 0.8.

7.3.3 Deteriorating Initial Credit Quality Cannot Explain 2006

One of the more common reasons given for the high level of 2006 defaults on mortgages is a general deterioration in underwriting standards. Indeed, experts on the issue (to whom we defer) have identified a slight decrease in underwriting standards over this period, even though average FICO scores improved slightly. In this section, we use our model to examine

the implications of a deterioration in credit quality. Throughout the section, we use the same parameter values as in our 2003 calibration.

Given the derived properties of the portfolios, deteriorating credit quality, mapped into higher initial values of x in the model, will naturally lead to higher defaults. In fact, a sufficiently large deterioration will lead to any observed level of defaults. However, the deterioration has counter-factual implications for the level of prepayments. Importantly, despite the rise in foreclosures, subprime prepayment rates have remained above default rates.

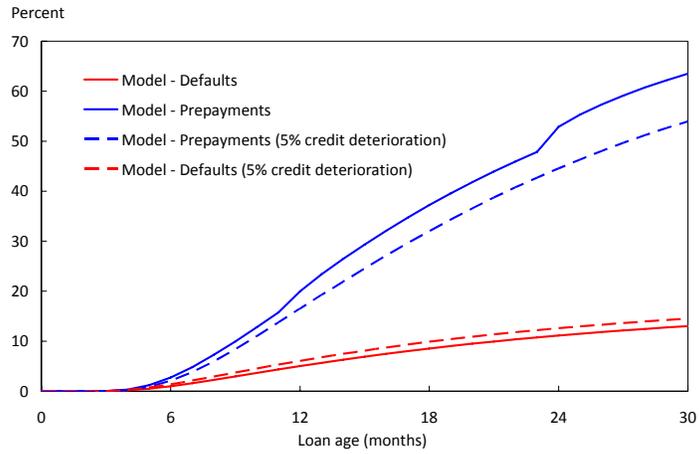
Figure 7.3a shows the results for a 5% deterioration in initial credit quality: the initial value of x is increased from 0.525 to 0.551. Default rates are higher everywhere and prepayment rates are lower everywhere under this assumption. However, a 5% deterioration does not bring the model anywhere close to matching the 2006 default rates.

In figure 7.3b, we allow the credit quality to fall by 25% relative to the base case (an x value of 0.66). In this scenario default rates, shown by the dashed blue line, approach 2006 default rates. Although 25% is much larger than actual estimates of credit deterioration, the mapping between changes in our initial credit quality and that in the data might not be exact, and so the deterioration might be reasonable.

However, we reject this as an explanation of the subprime market, because the pattern of prepayments is not consistent with the data. In this simulation, the number of prepayments is well under 50% of the observed level – about 30% of 2006 vintage subprime mortgages had prepayed by 24 months. At least in our model, one cannot move from the observed data in 2003 to the observed data in 2006 by loosening underwriting standards alone.

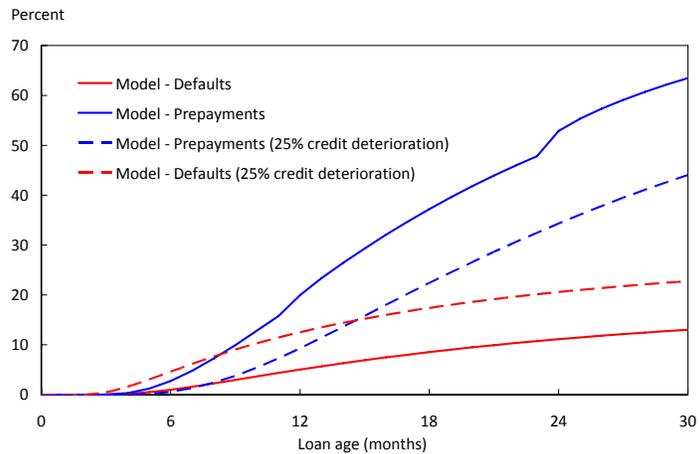
Figure 7.3: Simulating Credit Deterioration

A Five Percent Deterioration in Credit Quality



(a)

A Twenty-five Percent Deterioration in Credit Quality



(b)

8 Evaluating Policy Options

Our model provides no compelling rationale for policy intervention. Defaults, prepayments and the deterioration of the mortgage market arose without a market failure. The subprime market flourished and then collapsed for rational reasons, and if the model provides a reliable guide, the breakdown of the subprime market will prove to be permanent. Therefore, within the confines of the model there is no need for government intervention. The mortgage market should be left to its own designs.

However, if we look beyond the model, one may find reasons for intervening and alleviating the crisis. The subprime collapse may impose externalities on households outside of this specific market. In particular, the greater-than-normal number of foreclosures may create a capital overhang in the housing market. This capital overhang may not only serve as a long-term drag on economic growth, but is also likely to lower house prices and push additional households into foreclosure. Since the interests of households and firms without direct stakes in the subprime market are diffuse, these agents have difficulty coordinating. In this case, if the costs of these externalities are sufficiently high, there is a role for government intervention.

Conditional on finding intervention optimal, our model has much to say on the likely effectiveness of different policies and the channels through which these policies are likely to operate. In this section, we examine different policies and evaluate their potential impacts on subprime households.

8.1 A Moratorium on Foreclosures

This proposal mandates a moratorium on foreclosures for a fixed period of time. In terms of our model, the latent process determining household defaults crosses the boundary but the mortgage does not die. There are several interesting aspects to this problem; we examine them sequentially.

Without any modification to our model, the moratorium is effective and plays off of the empirical observation that seasoned mortgages of all classes are less likely to default than newly-issued debt. Consider the random walk nature of our latent process. Even without drift, a mortgage that would have defaulted today but is kept artificially alive for another period has a 50-50 chance of re-entering the inaction region. Furthermore, as the moratorium is extended, the probability that the process hits the prepayment boundary approaches one. Therefore, with this strict interpretation of our model, mortgage defaults are sharply reduced. This result is merely strengthened if (as we already assume) the

underlying drift of the mortgage is toward the prepayment boundary.

However, a strict interpretation of our model might not be warranted in this case. Several outside-of-the-model questions must be answered. What do households do when they hit the default boundary? What happens to the default boundary itself? Does the inherent drift of the process change once a household crosses the default boundary?

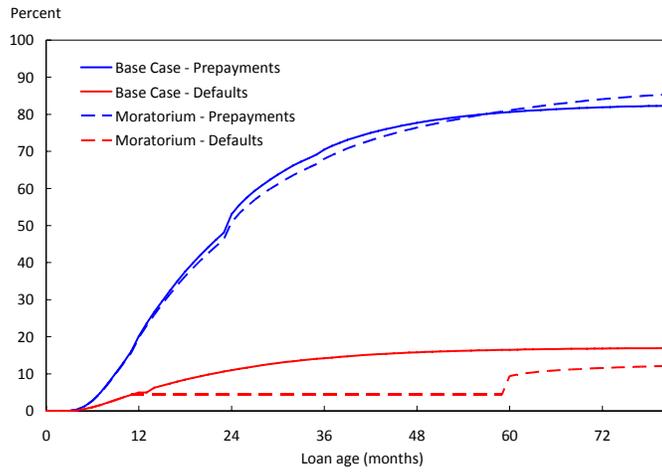
It seems likely that a majority of households entering foreclosure do so because a change in their financial condition has reduced their ability to meet their financial obligations. Therefore, we assume that any household that would have defaulted in the absence of the moratorium does not make any payments while above the default boundary. During this time period, the household accumulates debt at the rate of interest. We model the falling equity as an increase in the drift term, c – the household drifts towards the default boundary.

While the specifics of the plan matter, we can simulate one representative moratorium within the model. In the simulation, we assume that, 12 months after origination, a 48-month moratorium is put in place: no mortgages are permitted to default during this period. At the same time, we lower the default boundary by 10% to 0.9 and increase the drift from -0.015 in the baseline model to 0.01 on any mortgage that is currently above the default boundary. Figure 8.1a shows the time series of average defaults in the model using the 2003 calibration parameters. The solid red line shows the defaults as they would occur without the moratorium – the base case – and the dashed red line shows defaults under the moratorium.

The two lines are identical before the moratorium is established. From 12 to 60 months, there is no increase in defaults under the moratorium case. During this period, defaults in the base case rise from 5% to 17%. When the moratorium is lifted, the number of actual defaults jumps. Of course, the jump is not surprising since some portion of mortgages would be expected to remain over the default boundary regardless of the underlying assumptions (the latent process is a random walk). What is, perhaps, surprising is that the cumulative

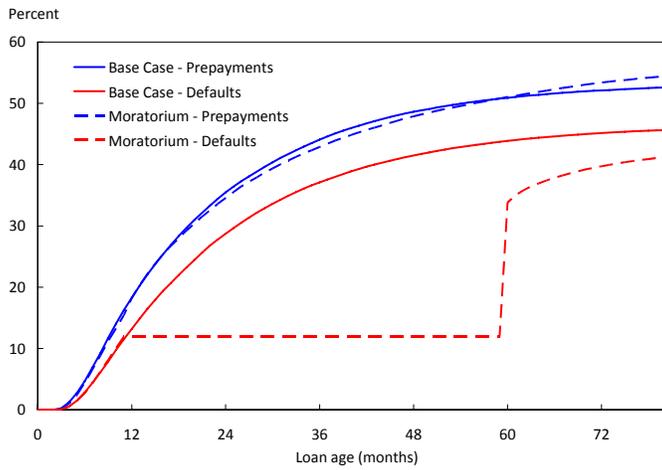
Figure 8.1: The Effects of a Foreclosure Moratorium

After the Moratorium Ends, Defaults Remain Below the Base Case



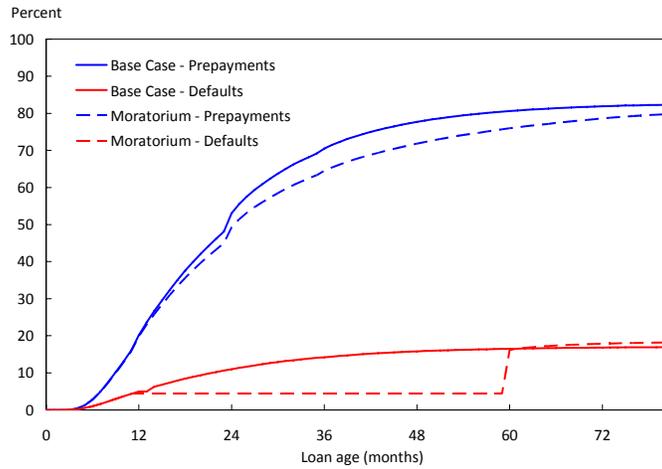
(a)

The Answer Does Not Change When Simulating 2006



(b)

Defaults After Moratorium May Be Higher Than the Base Case



(c)

number of defaults after the moratorium remains below that of the base case. The volatility of the underlying process is sufficient to push a sizable portion of mortgages from default into prepayment. Figure 8.1b shows that the effects of a moratorium are similar when we simulate 2006 defaults and prepayments.

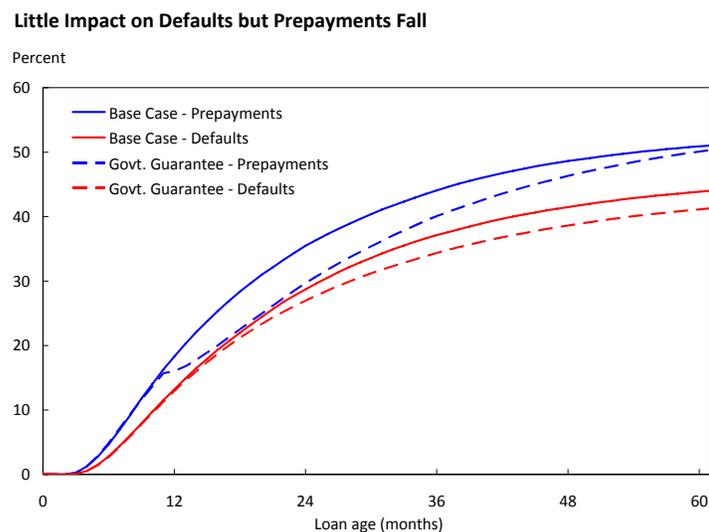
The previous result occurred because the drift on mortgages that would have defaulted does not change enough to prevent mortgages from wandering back into the inaction region and eventually into prepayment. Our choice of a 0.01 upward drift was somewhat arbitrary. The drift of a mortgage that would have defaulted might be much larger; after all, accumulated interest is rapidly adding to the debt owed and placing additional financial strains on the borrower. In Figure 8.1c, we increase the drift of the latent process until the number of defaults under the moratorium case essentially matches the number of defaults under the base case. We find that a drift of 0.05 is sufficient.

Lenders may or may not like the moratorium. Mortgages above the default line do not make mortgage payments, but mortgages continue to accrue interest. On the whole, this mortgage-market distortion would reduce the availability and increase the cost of future credit. However, in the case of the subprime market, there is no future lending to distort. Assuming lenders learned the true weight of aggregate risk, then future issuance of subprime mortgages is likely to be small regardless.

8.2 Government Guarantee of Mortgages

Under a government guarantee, the federal or state government assumes the borrower's repayment risk in return for either a restructuring of the household's mortgage or a reduction in the interest rate. Presumably, under a government guarantee, the household has a mortgage with better terms than could be secured in the absence of such a guarantee. Similar to our approach in the previous section, we view this guarantee as pushing out the prepayment boundary: households will be hesitant to pay off a mortgage with such good terms. This shift is partially offset by a slight increase in the downward drift of the

Figure 8.2: The Effects of a Government Guarantee



household's credit quality and a higher default boundary, both induced by the better terms.

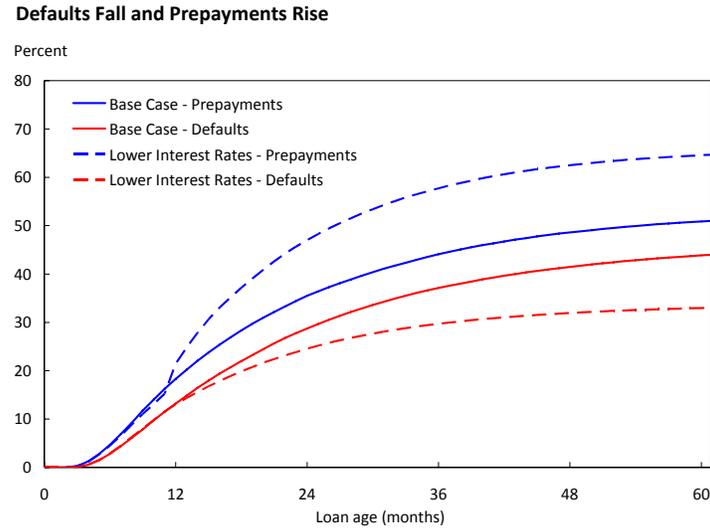
Figure 8.2 shows the effects of a guarantee on the 2006 base calibration of the model. We simulate the guarantee by pushing out the prepayment boundary to -0.2 and raising the drift from -0.015 to -0.005 .

We find that the overall performance of this mortgage portfolio is worse than without the guarantee. With the competing risk of prepayment eliminated, the risk of default rises. Of course, since government picks up the payment stream, the lenders do not mind the credit deterioration.

8.3 A Decrease in Mortgage Interest Rates

A reduction in mortgage interest rates has two effects in the model: it lowers the reset value of mortgages and lowers the interest rate on alternative rates. The first effect pushes down the prepayment boundary of the inaction region and the second raises it. Because not all subprime mortgages reset and because mortgage resets tend to be very high, we believe the second effect dominates and that a reduction in mortgage interest rates leads to a higher prepayment boundary and induces a larger proportion of households to refinance rather than

Figure 8.3: The Effects of a Decrease in Interest Rates



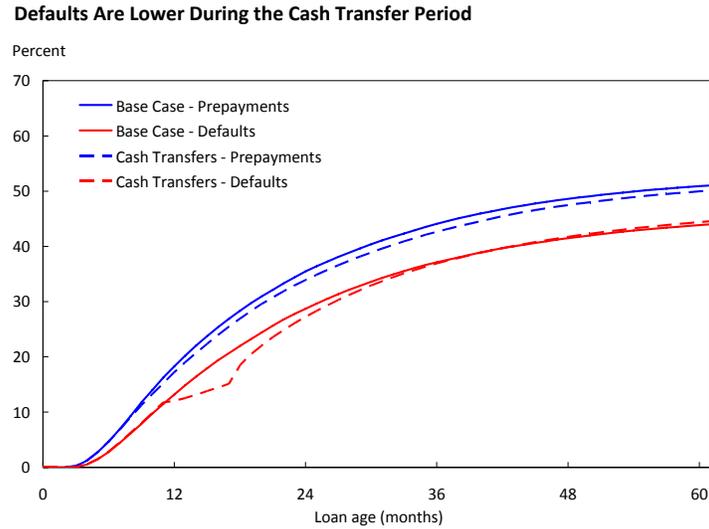
default.

In addition to the direct effect on the default and prepayment boundaries, lower real interest rates raise the value of housing, while the higher inflation outcomes lower the real value of household debt and may raise the nominal value of their income, reducing their payment burden. All of these forces work to increase the drift of the household's latent process toward the prepayment boundary, reducing the level of mortgage defaults. Figure 8.3 depicts these effects in a simulation using the 2006 base calibration of the model.

8.4 Cash Transfers to Households

In May 2008, the federal government mailed rebate checks to a large number of tax-paying U.S. households. How do these checks change the behavior of households? We do not know for sure, but we assume that any household would prefer to spend their rebate check on their mortgage rather than entering foreclosure. In the simulation depicted in figure 8.4, we assume that such a cash transfer is sufficient to postpone mortgage default by six months; this assumption is arbitrary. Any household that would have crossed the default

Figure 8.4: The Effects of a Cash Transfer



boundary in a two-month window around the time of the check delivery does not default but remains just below the boundary for the next six months. At the end of these six months, the household once again floats freely within the state space.

However, since the amount of this transfer is small relative to average mortgage payments, not all recipients are able to postpone default, and thus some households continue to hit the default boundary. Therefore, throughout this six month period, defaults continue to rise, albeit at a reduced pace. After the effect of this cash stimulus dissipates, however, default levels return to the levels seen in the base case. The transient benefits of such a policy are therefore quite limited, especially since prepayment behavior remains largely unchanged.

9 Conclusion

While we will likely never know the true reason the subprime market boomed and subsequently collapsed, this paper provides a cautionary tale for those who would overzealously attribute these outcomes to investor exuberance or household ignorance. We have shown that the boom in subprime lending may have occurred rationally given the information flow

during the early years of the boom.

In order to deem the subprime market a product of irrational exuberance, one must also assert that the initial priors over the aggregate risk weight were unfounded. Were those first agents with strong beliefs in a low aggregate shock weight acting in a manner inconsistent with the available data? While this question is beyond the scope of this paper, most studies of the housing market in the late 1990s and early 2000s indicated low or zero correlation in housing market performance across metropolitan areas. For example, Flavin and Yamashita (1998) find a block diagonal correlation matrix in house price returns across major metropolitan areas in the United States. Their finding is consistent with the idea that risk across regions – a form of idiosyncratic risk – is very low. We do not take a firm stand on the rationality of the investor’s initial beliefs, but merely point out that they may not have been too far from mainstream thinking on the topic.

If our tale is true, the future of the subprime market is in doubt. Securitization of subprime assets was not a panacea that resolved all risk in the market. Subprime assets embody a significant amount of aggregate risk. And, because the subprime crisis does not appear to have stemmed simply from deteriorating underwriting standards, there does not appear to be an easy regulatory fix.

The fate of the subprime market may also be tied to the housing market. During the early years of the subprime boom, the homeownership rate in the United States jumped from an already-elevated level of 67.5% to a record 69%; this rise coincided with the rapid increase in house prices. We do not know how much of this increase can be attributed to the surge in subprime lending, but since the beginning of the subprime market’s collapse in early 2007, homeownership rates have already fallen back near their 2000 levels. In contrast, homeownership rates did not decline in either of the last two recessions. Since renters typically consume less housing than owners, this decline may represent a sizeable decrease in housing demand. To the extent that the increase and decline is attributable to the subprime market, housing demand may have suffered a permanent negative shock.

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