

# Leverage and Productivity

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## Abstract

Financial frictions can reduce aggregate productivity, in particular when firms with high productivity cannot borrow against their earnings. This paper investigates the quantitative importance of this form of borrowing constraint using a large panel of firms in Japan. The firms are young and unlisted, precisely the firms for which credit frictions are expected to be the most severe. In this data, I find that firm leverage (asset-to-equity ratio) and firm output-to-capital ratios rise with firm productivity, both over time in a firm and across firms of the same age and cohort. I use these facts in indirect inference to estimate a standard general equilibrium model where financial frictions arise from the limited pledgeability of earnings and assets. In this model more financially constrained firms have higher output-to-capital ratios. The model matches the two facts the best when firms can pledge the equivalent of over half of their one-year-ahead earnings and one-fifth of their assets. Compared to the common assumption that firms can pledge only assets, aggregate productivity loss due to financing frictions is one-third smaller when earnings are also pledgeable to the degree seen in Japan.

Keywords: aggregate productivity, capital misallocation, financial frictions

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## Introduction

To what extent are young and unlisted firms borrowing constrained? The answer to this question matters for understanding the impact of financial frictions on aggregate productivity. Recent quantitative studies of aggregate productivity loss due to financial frictions found that frictions at the entry margin is key to generating large aggregate productivity losses. Also, the answer may be informative as an upper bound on the overall severity of financing frictions because one would expect young and unlisted firms to be affected the most by financing frictions. They have not yet accumulated retained earnings and have not been able to tap equity markets. Furthermore, many countries have programs that give financing to young firms because it is believed that they are important for aggregate growth but are financially constrained<sup>1</sup>.

One important source of financial frictions derives from firms not being able to credibly commit to fully repay loans out of their future earnings or assets. As a result, the borrowing capacity of a firm may be smaller than the financing needed for the firm to produce at its optimal scale, which could happen for a firm with low internal funds relative to its productivity. When many firms are borrowing constrained in this way, aggregate productivity can be significantly smaller than in an economy where firms can fully commit to repay. In this paper, we investigate the extent of this form of friction for a large panel of young and unlisted firms from Japan<sup>2</sup>. In particular, we study whether the borrowing constraint is due to low pledgeability of earnings or low pledgeability of assets. We also demonstrate why this distinction matters for aggregate productivity loss due to financial frictions.

More specifically, we use a standard general equilibrium model of aggre-

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<sup>1</sup>For example, expansion of financing to young firms features prominently in the current Japanese government's growth strategy (See "Japan Revitalization Plan" Cabinet Office 2014) and the SBIR grants in the U.S. give funds to young firms to stimulate innovation.

<sup>2</sup>We use Japanese data because it has a measure of inside fund that can be used for robustness checks. Moreover, policymakers in Japan have often attributed Japan's long stagnation, in part, to the stunted growth of young firms due to financial frictions (See "Japan Revitalization Plan" Cabinet Office (2014)).

gate productivity loss due to financial frictions where the extent of the frictions depends on the share of assets and one-period-ahead earnings that a lender can recover when its client-firm defaults. We depart from standard inference approaches by allowing the share pledgeable to differ for assets and earnings. When earnings are not pledgeable, the borrowing capacity of a firm is proportional of its inside fund and does not vary with the firm's productivity. On the other hand, when earnings are pledgeable, more productive firms have higher earnings and hence can borrow more than less productive firms even if they all have the same inside funds<sup>3</sup>.

We estimate these two shares by indirect inference with our micro-data. More precisely, we choose the shares so that the empirical regression coefficients of leverage<sup>4</sup> and output-to-capital ratio on productivity and inside fund matches as close as possible with that obtained from the same regression ran on data simulated from the model. Here, leverage is defined as total asset over internal equity and output-to-capital ratio is a proxy for the marginal product of capital. We choose these empirical targets for the following reason. The model has two forces governing how leverage vary with firm productivity conditional on firm's inside fund. In one case, firms are unconstrained and more productive firms have higher leverage because their optimal production scale is larger. In the second case, firms are constrained and more productive firms have higher leverage because they have higher borrowing capacity. The second force is turned on only when earnings are pledgeable. Hence we can identify the share of earnings that is pledgeable by looking at how leverage varies with productivity for constrained firms, who have higher output-to-capital ratio in the model.

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<sup>3</sup>More productive firms having higher borrowing capacity is also a feature of microfoundations where more productive firms have more to lose if they default. See, for example, Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Buera and Shin (2013) and Arellano et al. (2012).

<sup>4</sup>We measure leverage by the ratio of total assets to inside equity, which corresponds to the equity multiplier measure of leverage, as in Berk and DeMarzo (2013). A more commonly used measure of leverage in empirical corporate finance is the debt-to-asset ratio, because the focus of these studies is the choice between equity and debt. We do not distinguish between debt and outside equity because the firms in our dataset are young, unlisted, and owner-managed.

In the extreme when earnings are not pledgeable at all, leverage and output-to-capital ratio cannot *simultaneously* rise with productivity after controlling for inside funds.

In the data, we find *both* leverage and output-to-capital ratio rise strongly with firm productivity after controlling for inside equity (see Figure 1 and 2). Leverage increases by 1%, on average, for every 1% increase in firm productivity, while the output-to-capital ratio rises by 0.7%, on average, for every 1% increase in firm productivity. This pattern of leverage and output-to-capital ratio both rising with firm productivity holds within a firm over time and across firms of the same cohort, age and detailed industry group under various empirical specifications. It is also robust to several alternative empirical specifications such as using an alternative measure of inside fund and capital. The model matches these empirical elasticities the best when firms can pledge half of their one-year-ahead profits and a fifth of their assets. At these parameter values the aggregate productivity loss due to financial frictions is 11%<sup>5</sup>.

We explore the aggregate implications of our findings by comparing aggregate productivity loss due to financial frictions under our benchmark estimation with the loss when we estimate the model restricting the share of earnings that can be pledged to be zero. This restriction appears in many papers quantifying aggregate productivity loss due to financial frictions. We find aggregate productivity loss is approximately 14% in the restricted parameterization. That is, the common assumption that firms cannot borrow against earnings results in an overstatement of aggregate productivity loss by 30% relative to the loss that is consistent with our empirical findings. Hence our empirical findings suggest that assuming borrowing capacity does not depend on productivity is not innocuous: it can lead to quantitatively significant overstatement of aggregate productivity loss due to financial frictions.

There is an extensive literature studying the impact of financial frictions on

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<sup>5</sup>Aggregate productivity loss is defined as the difference between the first-best productivity and the model productivity as a percentage of the model productivity. Here, the first best is achieved when all firms are unconstrained.

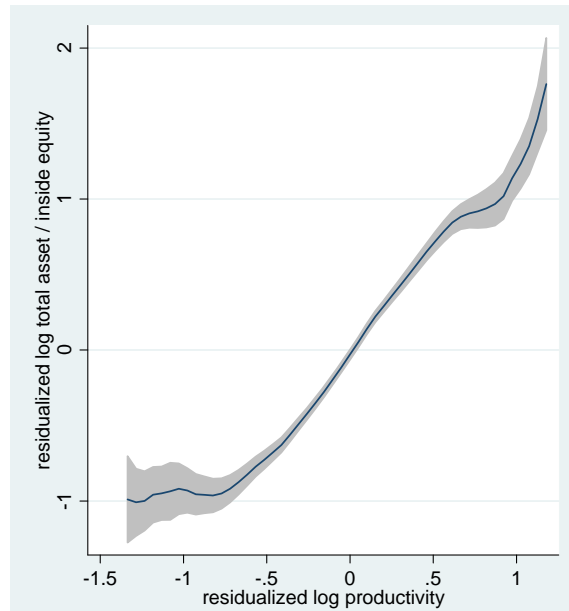


Figure 1: Firm leverage rises with firm productivity

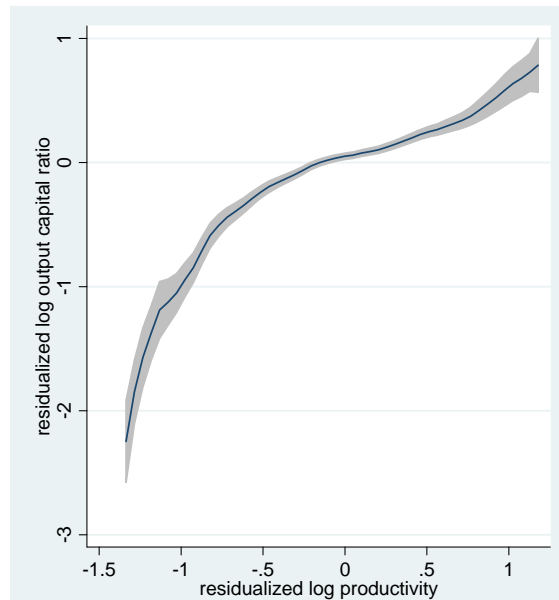


Figure 2: Firm output-capital ratio rises with firm productivity

aggregate output and productivity. However, there does not appear to be a consensus on the relationship between productivity and borrowing capacity. For example, recent articles such as Khan and Thomas (2013), Gopinath et al. (2015), Buera and Shin (2013) and Midrigan and Xu (2014) assume a firm's borrowing capacity is constant with respect to its productivity while other recent articles such as Cooley and Quadrini (2001), Arellano et al. (2012), Buera et al. (2014) and Buera et al. (2011) allow borrowing capacity to rise with productivity. Well-known theoretical papers (e.g. Kehoe and Levine (1993) and Albuquerque and Hopenhayn (2004)) that give micro-foundations to borrowing capacity are agnostic about whether borrowing capacity should rise with firm productivity. This paper contributes to the literature by providing empirical evidence consistent with borrowing capacity rising with productivity and showing the quantitative importance of this empirical pattern.

Our paper is closest in spirit to Brooks and DAVIS (2013). They use Colombia's trade liberalization in 1985 to differentiate between backward and forward looking borrowing limits. They find that the change in the age distribution of exporters after the liberalization is more consistent with forward looking borrowing limits and that aggregate gains from more trade is similar to a perfect credit market economy. We differ in that we look at aggregate productivity rather than trade gains. Also, we use firm-level data on financing, networth and productivity instead of assuming a relationship between age, cohort, and networth.

There are numerous studies of productivity and leverage for listed firms. A common finding is that smaller firms or less well performing firms have higher leverage, where leverage is measured as the debt to equity ratio (see Frank and Goyal (2008) for a survey). This appears to be driven by better firms having better access to *outside* equity<sup>6</sup>. In contrast, we find a positive correlation be-

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<sup>6</sup>For example, for listed firms in Japan, Pushner (1995) finds that leverage (debt-to-asset ratio) is negatively correlated with TFP because active institutional shareholders reduce the cost of outside equity financing while disciplining managers, leading to both a lower share of debt and higher productivity.

tween productivity and leverage. This is likely due to the firms in our dataset not having access to the outside equity market markets<sup>7</sup>. Higher productivity evidently allows firms in our sample to borrow more and have higher ratios of debt to *inside* equity. Nonetheless, our point that better firms have better access to financing is consistent with existing findings of larger firms having lower borrowing costs or better access to credit markets, e.g., Gilchrist et al. (2013), Hennessy and Whited (2007).

The paper is organized as follows. Section 1. presents a stylized model of aggregate TFP loss due to financial frictions. Section 2. lays out the inference methodology. Section 3. describes our dataset and empirical findings. Section 4. estimates the model and presents results from counterfactual exercises. Finally, Section 5. concludes. Details are provided in the Appendix.

## 1. Model

In this section we layout a stylized model of aggregate productivity loss due to financial frictions to illustrate how assumptions about the borrowing capacity affect the inference of productivity loss due to financial frictions<sup>8</sup>. In particular, we show that, for the same observed average debt ratios, aggregate productivity loss is smaller when borrowing capacity increases with firm productivity.

Consider an economy populated with a continuum of infinitely lived entrepreneurs born with wealth  $a_0$  and productivity  $z_0$  drawn from distribution  $G(a, z)$ . Each entrepreneur's productivity post birth is governed by an AR(1) process with autocorrelation parameter  $\rho$  and iid normal innovation shock with

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<sup>7</sup>This is likely to be due to the high fixed costs associated with accessing stock and bond markets. See Russ and Valderrama (2009) and Begenau and Salomao (2014) for evidence and theory.

<sup>8</sup>Here we abstract from other dimensions of entrepreneur's decisions such as heterogeneity in preferences that may lead to an overstatement of the cost of financial frictions. For example, Hurst and Pugsley (2011) find small businesses choosing to stay small due to owner's preferences and Strebulaev and Yang (2013) find family firms tend to use zero debt.

mean  $\mu_e$  and standard deviation  $\sigma_e^2$ . That is

$$\ln z' = \rho \ln z + \epsilon, \quad \epsilon \stackrel{iid}{\sim} N(\mu_e, \sigma_e^2)$$

The economy also has  $L$  measure of hand-to-mouth workers each supplying one unit of labor.

The entrepreneurs can save with financial intermediaries for a net rate of return  $r$ . The financial intermediaries in turn lend capital to entrepreneurs. We assume the financial intermediation sector is perfectly competitive. Since there are no aggregate risks, this implies that the net rate of return to capital equals the return to savings plus depreciation,  $r + \delta$ .

In each period, entrepreneurs have access to a Cobb-Douglas production technology with capital share  $\alpha$  and returns-to-scale  $\eta \in (0, 1]$ .<sup>9</sup>

$$y = f(z, k, l) = z(k^\alpha l^{1-\alpha})^\eta$$

When  $\eta = 1$ , firms have constant-returns-to-scale technology. In order for the distribution of firms to be well defined, we assume each entrepreneur can operate only one business. This assumption serves the same purpose as introducing a fixed cost of production.

The entrepreneur decides capital and labor inputs after seeing her productivity to maximize profit each period.

$$\pi(a, z) := \max_{k, l} f(z, k, l) - (r + \delta)k - wl, \quad k \leq \bar{k}(a, z)$$

$\bar{k}(a, z)$  denotes the borrowing capacity of the entrepreneur. It is the maximum amount of capital the entrepreneur can raise. Higher  $\bar{k}(a, z)/a$  means more capacity to raise external financing per unit of internal funds. The constant leverage model assumes  $\frac{\partial \bar{k}(a, z)}{\partial z}$  is constant. That is, firms with the same internal

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<sup>9</sup>The solution to the firm's problem is the same as a model where firms have constant-returns-to-scale production technology but face CES demand with elasticity of substitution  $\frac{1}{1-\eta}$ .



funds have the same borrowing capacity regardless of their productivity.

The entrepreneur's problem can be written recursively as

$$V(a, z) = \max_{a', c} u(c) + \beta \mathbb{E} [V(a', z') | z]$$

subject to

$$c + a' \leq a(1 + r) + \pi(a, z)$$

Let  $\underline{z}$  denotes the productivity level of the least productive firm among the active firms. When  $\eta < 1$ , all firms are active because the marginal return to scale is infinite at zero units of output. So  $\underline{z}$  is the lower support of the exogenous productivity distribution. When  $\eta = 1$ , the marginal return to scale is constant and only firms with marginal return to scale exceeding marginal costs produce.

There are two markets that need to clear at the equilibrium. First, capital market clearing requires aggregate capital demand equate total wealth in the economy

$$K := \int_{a, z \geq \underline{z}} k(a, z) dG(a, z) = \int_{a, z} a dG(a, z) =: A.$$

The labor market clearing condition is

$$L = \int_{a, z \geq \underline{z}} l(a, z) dG(a, z).$$

Furthermore, at the equilibrium, the evolution of wealth and productivity distribution must be consistent with the law of motion of the entrepreneurs' productivity and savings policy functions. That is

$$G(a', z') = \int_{a, z} \text{Prob}(z' | z) \mathbf{1}\{a' = \text{Savings}(a, z)\} dG(a, z)$$

**Equilibrium definition:** A stationary competitive equilibrium consists of labor demand  $l(a, z)$ , capital demand  $k(a, z)$ , productivity cutoff level  $\underline{z}$ , savings policy, interest rate and wage, wealth and productivity distribution  $G(a, z)$  such

that

1. given prices,  $l(a, z)$ ,  $k(a, z)$ ,  $\underline{z}$  and savings policy function solve the entrepreneur's problem
2. capital and labor markets clear
3.  $G(a, z)$  is consistent with the savings policy function and the law of motion of  $z$

In this economy, aggregate output equals the sum of all entrepreneur's output

$$Y = \int_a \int_z y(a, z) dG(a, z)$$

and we define aggregate productivity, TFP, as

$$Z := \frac{Y}{(K^\alpha L^{1-\alpha})^\eta}$$

When  $\eta < 1$ , aggregate productivity under the first best allocation is given by

$$Z^{fb} := [\mathbb{E}_z z^{\frac{1}{1-\eta}}]^{1-\eta}$$

where the expectation is taken with respect to the stationary distribution of productivity. When  $\eta = 1$ , the first best aggregate productivity is just the upper bound of the stationary productivity distribution. The first best TFP is characterized by a non-dependence on the wealth distribution. We define TFP loss due to financial frictions as the gap between the model TFP and the first best TFP as a percentage of the model TFP

$$\text{loss} := \frac{Z^{fb} - Z}{Z}.$$

We interpret the variation in the model TFP with exogenous parameters governing the borrowing capacity  $\bar{k}(a, z)$  as the impact of financial frictions on aggregate productivity.

## 1.1. Analytical example

Here we consider an analytical example to illustrate how the choice of borrowing capacity function  $\bar{k}(a, z)$  affects the inference of TFP loss due to financial frictions. Let us assume constant returns to scale ( $\eta = 1$ ), entrepreneurs have log utility and the borrowing capacity can be written as  $\bar{k}(a, z) = \lambda(z)a$ . As shown in Moll (2014), these assumptions yield an analytical expression of aggregate productivity where TFP is just the capital-share weighted average of all active entrepreneur's productivity. We can further decompose that expression for TFP into the following terms

$$TFP^{\frac{1}{\alpha}} = Cov_{\omega}[z^{\frac{1}{\alpha}}, \lambda(z)|z \geq \underline{z}](1 - D/K) + \mathbb{E}_{\omega}[z^{\frac{1}{\alpha}}|z \geq \underline{z}]$$

where  $\omega(z)$  denotes the wealth share of entrepreneurs with productivity  $z$  and  $D/K$  is the aggregate debt-to-capital ratio. The  $\omega(z)$  term reflects the effect of wealth accumulation by productivity types on the TFP and the  $\underline{z}$  term reflects competition for capital that changes the break even productivity. Holding these general equilibrium forces fixed, as in the case of IID productivity shocks and calibrating the model to  $D/K$  in the data, a higher covariance between leverage capacity  $\lambda(z) = \bar{k}(a, z)/a$  and productivity has a direct positive impact on TFP. That is, two model economies with the same observed aggregate financing ( $D/K$ ), same wealth distribution by productivity types and same productivity cutoff for entry, can have different aggregate productivity simply because one economy has a higher positive covariance between productivity and leverage capacity. Since the first best TFP is the same for the two economies, the economy with  $\lambda(z)$  increasing more in  $z$  will have a smaller TFP loss due to financial frictions.

The intuition for why assuming a constant  $\lambda(z)$  may lead to overstating TFP loss due to financial frictions is that it does not allow firms who are more productive relative to their wealth level to borrow more. In the constant-returns-to-scale case, all operating firms borrow up to the borrowing limit. So  $\lambda(z)a$

is the capital used by the firm. Under the efficient allocation, marginal return to capital equates. Since more productive entrepreneurs have higher marginal return to capital, efficient allocation gives them more capital regardless of their wealth level. A constant  $\lambda(z)$  prevents marginal returns to capital from equating between entrepreneurs with the same wealth level but different productivity.

## 2. Inference methodology

To get a sense of the magnitude of the overstatement of TFP loss due to financial frictions under the constant leverage model, we estimate a parsimonious model of borrowing capacity from Buera et al. (2011) that nests the constant leverage case. In this model, financial frictions arise from limited enforcement of contracts. If the entrepreneur defaults, she can keep  $1 - \phi_y$  fraction of revenue and  $1 - \phi_k$  fraction of depreciated capital but lose all her wealth  $a(1+r)$ . It is assumed that entrepreneurs can use the financial market after one period without further penalties. Borrowing capacity is the maximum capital  $\bar{k}(a, z)$  that satisfies the following incentive compatibility constraint so that there is no default in the equilibrium

$$\phi_y \max_l \{f(z, k, l) - wl\} + (1+r)a \geq (r+\delta)k + (1-\phi_k)(1-\delta)k$$

When  $\phi_y = 0$ , the borrowing limit reduces to a constant proportion of inside fund where the proportion is  $\lambda = \frac{1+r}{R+(1-\phi_k)(1-\delta)}$ . When  $\phi_y > 0$ , the borrowing capacity increases with productivity because the benefit of defaulting relative to paying the debt decreases with  $z$ .<sup>10</sup>

Existing approaches use measures of aggregate external financing such as the aggregate debt-to-GDP ratio to calibrate  $\phi_y$  and  $\phi_k$ . For example, Buera et al (2011, 2014) assume a  $\phi_y = \phi_k$  and calibrate the model to match the aggregate

<sup>10</sup>This property can be micro-founded using Albuquerque and Hopenhayn (2004) where the borrowing capacity is determined by the relative size of continuation value and outside option of the entrepreneur.

debt ratio. Other papers assume  $\phi_y = 0$ . Here, we allow  $\phi_y$  and  $\phi_k$  to vary and use micro data on leverage and output-to-capital ratio to calibrate the two parameters.

The key moments of the model that identifies  $\phi_y$  and  $\phi_k$  are the capital usage of borrowing constrained firms. For the same level of average capital usage, we infer a more positive  $\phi_y$  when capital usage of the constrained firms with the same  $a$  increases faster with  $z$ . In general, it is difficult to identify which firms are borrowing constrained. We use the model to do this. The model points to firms with higher marginal return to capital as being more constrained. Furthermore, the model equates marginal return to the average return of capital. Hence, we use the average return to capital in the data to measure the extent to which a firm is constrained. We target the elasticity of average return to productivity and wealth as well as the elasticity of leverage to productivity to estimate  $\phi_y$  and  $\phi_k$ .

### 3. Data

This section documents empirical relationships between firm borrowing and firm productivity using balance sheet information from TSR (Tokyo Shouko Research)<sup>11</sup>. TSR is the largest credit rating agency in Japan. Their data is known for its coverage and rich information of small private firms that is not found in other datasets for Japan. For example, it is used by the Japanese government whitepapers such as the White Paper on Small and Medium Enterprises. We observe the balance sheets of these firms from 2004 to 2013 as well as information on their incorporation date, legal status, detailed industry classification, listing status etc.

TSR is a credit rating agency so the unit of observation in our data is a unit that requires independent credit rating. This is suitable for our study of firm

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<sup>11</sup>Definitions of variables can be found at [Orbis glossary](#). We also provide a table with the definition of the key variables in the Appendix.

legal form	number of firms	share of firms
Corporation	791980	67.0%
Limited liability company	300391	25.4%
Sole proprietorship	55529	4.7%
Medical corporation	19204	1.6%
Partnership	5607	0.5%
Cooperative company	3740	0.3%
Agricultural collective interest company	2888	0.2%
Religious institution	621	0.1%
Association	595	0.1%
General partnership	563	0.0%
Educational foundation	495	0.0%
unknown and others	993	0.1%

Table 1: Number of firms incorporated 2001-2014 in the TSR-Orbis database by legal form at 2014

financing. Table 1 shows a breakdown of units that formed between 2001 - 2014 by Japan's legal forms observed in 2014 when the data was downloaded. We have some sole-proprietorships but the sample size is very small relative to the total number of sole-proprietors in the economy and a proxy for their inside equity is not readily available. Hence we use only the top two largest categories ("Corporations" and " Limited liability company") for our analysis.

### 3.1. Mapping model to data

Table 2 displays the data variables we use in the benchmark regressions. We do not have material costs in the data so we use industry level material shares to impute a value added measure from the operating revenue of the firm. The in-

Variable	Notation	Data item
capital	$k$	book value of capital stock (total asset)
value added	$y$	operating revenue $\times$ (1 - factor share of materials)
inside equity	$a$	<i>shihonkin</i> or shareholders fund
labor	$l$	number of employees

Table 2: Map of variables used in the empirical analysis to balance sheet items

dustry material shares come from JIP Database 2013 created by RIETI, Japanese government METI's main research branch. It has 108 sectors which we match to the TSR-Orbis database using the official JIP-ISIC concordance table. We use the average material share over 2000-2010. While there are likely to be intra-industry differences in in factor shares, using industry shares reduces the dispersion in measured productivity due to firm level measurement error in cost shares. Syverson (2004) for example uses both industry averages and and plant-level shares in his benchmark results.

We calculate firms productivity using a decreasing-returns-to-scale Cobb-Douglas production function which will be later embedded in our general equilibrium model. More specifically, productivity  $z$  of a firms is measured by

$$\ln z = \ln y - \eta\alpha \ln k - \eta(1 - \alpha) \ln l$$

where the scale parameter  $\eta = 0.85$  is taken from Midrigan and Xu (2014) and capital intensity  $\alpha$  is calculated from the JIP database in the same manner as the material shares.

### 3.2. *shihonkin* as a proxy for founders' stake

The key to our empirical strategy is controlling for  $a$ , inside equity. To this end, we use an item in Japanese firms' balance sheet called *shihonkin* as the proxy

for inside equity. *shihonkin* is also called stated/share/legal/paid-in capital.<sup>12</sup> In Japan, the legal definition of *shihonkin* is the “the amount of properties contributed by persons who become shareholders at the incorporation or share issued” (Article 445 of the Companies Act). While a straight reading of the law says that half of a firm’s initial equity is *shihonkin*, firms appear to have more flexibility in setting the level of *shihonkin*. In practice, it is seen as the founder’s own stake in the firm at the time of incorporation. For example, [J-NET 21](#), a government website providing advice to entrepreneurs setting up small businesses, advises founders that not all of the initial financing needs to be put down as *shihonkin* because the initial financing can also be entered as a loan from the company head. It observes that founders tend to register more *shihonkin* if the business requires fixed investment and operating finances while founders who only want to incorporate would put in less. Another [article](#) on the same website also says that *shihonkin* is “a measure of trust”. *shihonkin* is seen as the entrepreneurs “own skin” in the firm and is the minimum recoverable amount for creditors. Furthermore, the Ministry of Economy, Trade and Industry SME Agency website on changes in the corporate law in 2006 advice new firms to decide *shihonkin* based on financing needs and does not even mention the need to register at least half of their initial financing as *shihonkin*<sup>13</sup>.

*shihonkin* is registered at the Ministry of Justice at the time of incorporation. To incorporate a firm, the founder must provide a *shihonkin* level and show ev-

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<sup>12</sup>The institution of *shihonkin* existed in the U.S. until the late 1950’s and is still prevalent in many European countries and China.

<sup>13</sup>Existing companies have less flexibility in determining the fraction of new share issuance that goes into *shihonkin*. Articles 199 to 213 defines the process for existing firms to increase equity through share subscription. In case new stock is issued, the subscription price needs to be calculated “fairly” so not to be disadvantageous to existing shareholders. Also, when the actual price paid for the shares are less than the market value of the shares at the time of issuance, the buyer may need to pay a gift tax<sup>14</sup>. Hence, while the actual contribution can differ from the issuance value due to differences in market valuation and firm’s issuance value, firms do not appear to have a large margin of control over the total value of the equity subject to *shihonkin* registration. Nonetheless, there are firms that put more than half of the new issuance as *shihonkin*. The justification for this regulation is that general shareholders are external financiers. They are more like creditors than owners of the firms and need protection from managers diluting the value of the firm. However as our focus is young firms, we do not delve more into this aspect of *shihonkin*.



idences that deposits/physical production inputs of the declared amount is put into the firm. In the case of cash, evidence is a special bank deposit certificate. In the case of physical assets such as buildings or land, the firm needs to receive evaluation by an approved third party. Once the evidences are approved, the Ministry of Justice discloses the registered *shihonkin* in its public registry that can be accessed by anyone at the municipal registry. Changes to the *shihonkin* are updated at the Ministry of Justice. So *shihonkin* is not just a number the firm writes down but actually reflects the value of contribution by its founders.

Registering *shihonkin* carries at least two costs. First, the firms pay 0.7% tax on the amount it registers<sup>15</sup>. For example, if a firm registers ¥1000 at time of incorporation and then increases that by ¥1000 after incorporation, it pays a total of ¥14 in tax. Second, the level of registered *shihonkin* sets a lower bound on the networth needed to pay dividend<sup>16</sup>. For example if a firm has ¥3 million in *shihonkin*, it could not pay dividend to its shareholders unless its networth (total asset - debt - *shihonkin* - capital reserves). While debt and capital reserves can be adjusted easily, reduction of *shihonkin* is extremely difficult. It first requires over 2/3 (kabushiki) and 3/4 (yugen) approval rate at a meeting with over 50% of shareholders with voting rights attending. Then the firm needs to undergo a debtholder protection procedure which involves an announcement on media and government's official gazette, individually contacting each debtholder known to the firm and negotiating with disapproving debtholders. The proposed reduction only becomes effective when no debtholder vetos.

### 3.3. Data cleaning and coverage

We measure the age of a firm by the difference between the year of observation and the year of the reported incorporation date<sup>17</sup>. Table 3 gives a breakdown of

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<sup>15</sup>National Tax Agency tax schedule (in Japanese): <https://www.nta.go.jp/taxanswer/inshi/7191.htm>

<sup>16</sup>Article 446 of the Companies Act, Article 290 of the Commercial Code and Article 46 of the Yugenkaisha Act

<sup>17</sup>A few firms are observed before their incorporation date probably due to a change of legal form. We drop these observations from the data used for the analysis.

incorporation year	share of firms
before 2001	63.9%
2001	2.3%
2003	2.4%
2005	2.8%
2007	2.8%
2009	2.7%
2011	3.0%
2013	3.2%

Table 3: Breakdown of firms in the TSR-Orbis database by incorporation year

firms with known incorporation date by incorporation year (only for odd years to save space). We use only the sample that incorporated between 2001 and 2013 because we observe firms from 2004 to 2013. The cutoff at 2001 ensures we observe firms when they are young and *shihonkin* or shareholders fund are good proxies for the inside equity of the founders. For the same reason, we drop firms that were ever listed.

### 3.4. Availability of data on variables

Table 4 displays the drop in the number of firms incorporated in 2006 as we exclude firms with missing variables. For example, most of the companies are never listed companies. Of these 8690 has *shihonkin* observed at some point between 2004-2014 and 6767 has both *shihonkin* and *totalasset* observed at some point between 2004-2014.

Table 4 shows the number of firms with data at some point between 2004-2014. However, the data has an unbalanced panel structure. Table 5 counts the number of firms incorporated in 2006 with all variables observed at a particular

cumulative criteria	number of firms
has incorporate date	107441
+ is a company	94142
+ never listed	94129
+ with shihonkin	8690
+ with total asset	6767
+ with revenue	6686
+ with shareholders fund	5559
+ with employment	4906

Table 4: Number of firms incorporated in 2006 in the TSR-Orbis database by cumulative availability of variables

age. The total firm-year observation is larger than the total number of firms with all variables observed at some point between 2004-2014 because some firms were observed in multiple years. Most of our observations are after age 4. Due to this structure, we will run our estimation within age and cohort.

We check the coverage of our dataset by comparing the number of firms in our final dataset with the Japanese Census<sup>18</sup>. The Census (or Establishment and Enterprise Survey before 2009) reports the number of establishments by opening year, legal form, single versus multi-unit, branch versus main, *shihonkin* bin and employment bin etc. We define a firm in the Census as an establishment whose legal form is company (*kaisha*) and is either a single unit or a main branch. The unit of observation in the Census is an establishment with continuous economic activities at a physical location under a single management. The opening year is not the creation year or incorporation year of the firm but the year when operation began at the location under the current management. Also, it is the number of establishments operating at the time

<sup>18</sup>Details of these datasets are provided in the appendix.

age when observed	number of firms
0	711
1	1541
2	2033
3	2686
4	4189
5	4856
6	4557
7	1898

Table 5: Number of firms incorporated in 2006 with all variables available by the age at the time of observation

of the Census. For example, the firm count for the 2011 cohort is the number of single or main establishments that began operating at the surveyed location in 2011 and is operating on the Census survey date Feb 1st, 2012. It is likely that incorporation took place before operation started and some incorporated firms may not reached the operation stage or survived to the time of the Census survey.

For the available census years, Table 6 displays the Census count of non-agricultural companies that opened in that year as well as the TSR-Orbis count of companies incorporated in that year with all variables observed at some point between 2004-2014. It shows that the sample size in TSR is close to 30% of the Census counts.

Next we compare the number of firms in our dataset with the number of firms in the Census. Table 7 displays the number of firms incorporated by year against the number of newly opened single-unit or main branch establishments in the Census and the number of new registrations in the Ministry of Justice data. The number of firms in our dataset does not represent the number of

incorp year	TSR-Orbis	Census <sup>1</sup>
2001	8,995	35,114
2006	9,826	28,946
2011,2012	9,405	21,312

<sup>1</sup> single unit or main companies establishments

Table 6: Company counts. TSR-Orbis, Census

firms in the cross-section because we are counting the number of firms that appeared at least once in our dataset. For example, for 2004, this would be the number for firms that appear at least once between 2004 and 2013. It shows that the firm counts roughly matches. In some year, our firm-level data have more firms than the Ministry of Justice data. This is because our data also contains some sole-proprietors while MOJ only contains corporations.

incorp year	# of firms	# of firms* in Census 2009	# of new reg. in MOJ data
2004	91106	109780	101100
2005	96672	109909	103545
2006	107441	116764	115178
2007	99088	102316	101981
2008	101126	103061	92097
2009	95631	-	86016

Table 7: Firm counts. TSR, Census and Ministry of Justice firm registration

Next, we compare the number of firms in our dataset with all variables with the number of firms in the Census. However, Table 8 shows that there is only 10% of the firms that have all variables (revenue, asset, employment *shihonkin*).

incorp year	# of firms with all variables	# of firms	# of firms* in Census 2009
2004	8195	91106	109780
2005	8432	96672	109909
2006	9586	107441	116764
2007	8133	99088	102316
2008	6761	101126	103061
2009	5793	95631	-

Table 8: Firm with all variables. TSR, Census and Ministry of Justice firm registration

We trim our data to remove observations with negative values of *shihonkin* and total asset being smaller than *shihonkin*. Table 9 compare the number of companies in our dataset that we used for our regressions with the number of companies in the Census for the available years. The number here is for the cross-section. Our firm dataset covers more than 10% of the companies when observed at age four and less than 10% when observed at age 1.

incorp year	# of companies with all variables, age 4	# of companies with all variables, age 1	# of companies in Census <sup>1</sup>
2004	2243	800	15216
2006	4387	1575	28946
2009	1382	2572	12921

<sup>1</sup> single unit or main companies establishments

Table 9: Data: coverage of companies, observed at age 1 and 4

Figure 3 and 4 compares the *shihonkin* and worker distribution in our final sample and the 2006 Census<sup>19</sup> for newly incorporated companies<sup>20</sup>. It shows that our dataset slightly selects on larger entrants in terms of their *shihonkin* level but more than half of our observations are small firms (with *shihonkin* below 100K dollars or below 10 employees). In terms of workforce, we actually select on smaller firms. This could be because the Census includes owners and non-paid family members while TSR-Orbis does not.<sup>21</sup>

Our dataset is close to being representative along the employment and *shihonkin* margin. Ideally, we also want to show our dataset is representative of productivity, leverage and putput-to-capital ratio. Unfortunately, the Census does not cover revenue and asset until 2012. For 2012, it does not publicize the

<sup>19</sup>the Census is for all establishments because publicized data does not report a breakdown by single, branch and main establishments.

<sup>20</sup>As the Census count of newly incorporated companies include those incorporated in 2005, we use the 2005 and 2006 cohorts distributions in 2006 from TSR-Orbis to make the comparison.

<sup>21</sup>For our numerical exercise in Section 4., this selection issue may not be a problem. The economic significance of our findings increases with the standard deviation of firm productivity and the elasticity with which leverage capacity increases with productivity. Our data selects on larger entrants who are likely to have better access to financing. Hence the selection issue is likely to lead to an understatement of productivity variation and the elasticity of leverage with respect to productivity. So we believe the results of our numerical exercise is not inflated by data selection.

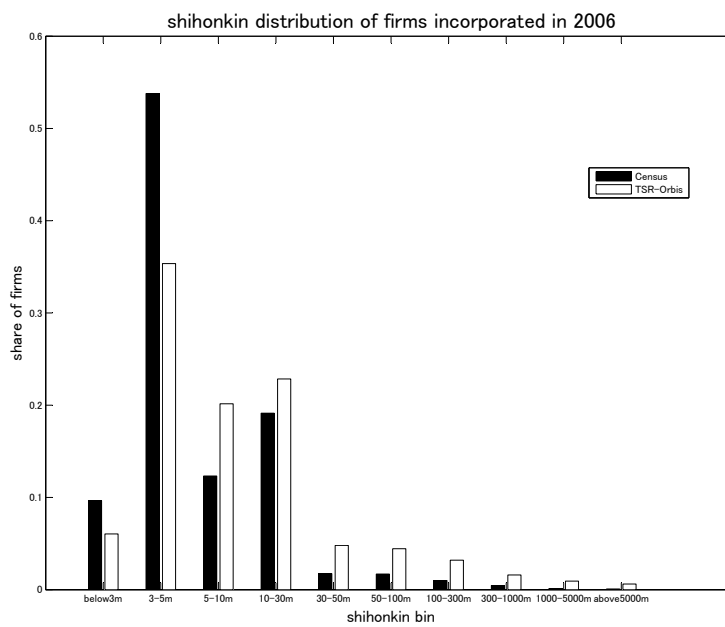


Figure 3: *shihonkin* distribution, Orbis-tsr versus Census

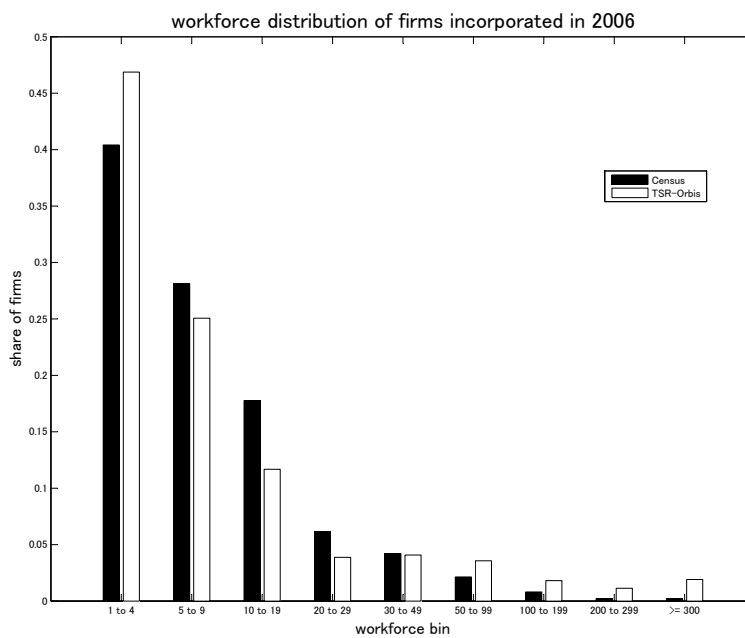


Figure 4: Workforce distribution, Orbis-tsr versus Census



revenue and capital distribution by incorporation year so we do not have a way of checking the representativeness of our data on these important dimensions.

### 3.5. Empirical patterns

Figure 5 and 6 display the main empirical patterns we will use to discipline the model. These are local linear regressions of log leverage  $\ln \frac{k}{a}$  and log output-to-capital ratio  $\ln \frac{y}{k}$  on industry fixed effects at the NAICS 6-digit level, log productivity and log inside equity. That is, the regression equations are

$$\ln \frac{y_i}{k_i} = \text{Industry FE} + \theta_1 \ln z_i + \theta_2 \ln a_i$$

$$\ln \frac{k_i}{a_i} = \text{Industry FE} + \nu_1 \ln z_i + \nu_2 \ln a_i$$

We run this regression within cohort-year. The figures display the result for the 2006 cohort in 2011, when they are five years old. We find that both firm leverage and output-to-capital ratio rise strongly with firm productivity. Furthermore, the pattern appears linear. We obtain similar results for other cohort-years

We also run an OLS regression of the above form. Column (1) in Table 10 shows the coefficient on log productivity for each dependent variable. It shows that leverage rises close to one-for-one with productivity while the elasticity of output-to-capital ratio with respect to productivity is approximately 0.7.

Columns (2) to (6) display robustness checks for the regression results. Column (2) displays the regression coefficients on log productivity after including the quadratic term  $(\ln z)^2$  in the regression. This is to control for rising leverage and output-to-capital due to a combination of constrained and unconstrained firms. The coefficients of interest do not change much. This is not surprising given that in our non-parameteric regression, the relationship appears linear.

In the third column, we use shareholders fund (total asset - total debt) instead of *shihonkin* to proxy for inside equity. In TSR-Orbis, total sharehold-

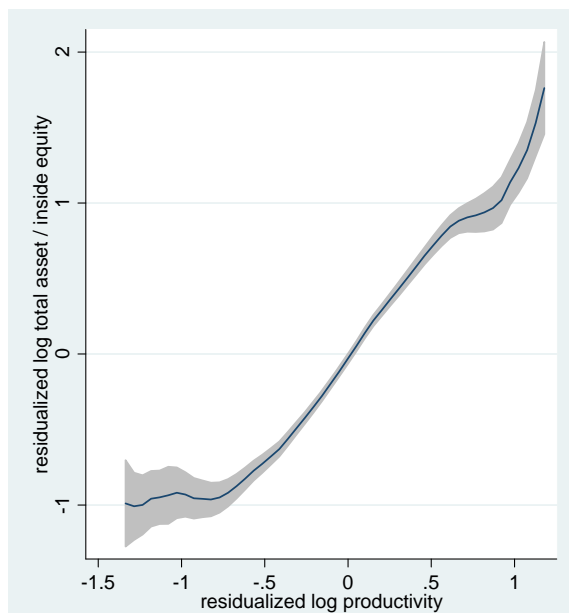


Figure 5: Firm leverage rises with firm productivity

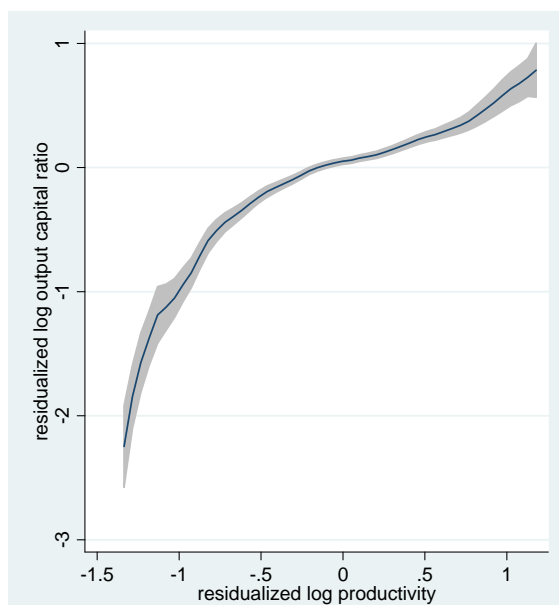


Figure 6: Firm output-capital ratio rises with firm productivity

ers fund is the sum of *shihonkin* and all other shareholders funds not linked to *shihonkin* such as reserve capital, undistributed profit, include also minority interests if any. This is to address the concern that firms may have other unobservable inside stake that positively correlate with firm productivity. For example, suppose the true inside equity held by the firm is  $a^* = \hat{a} + \epsilon$  where  $\hat{a}$  is our proxy *shihonkin*. Then our regression equation for leverage on productivity becomes

$$\ln \frac{k}{\hat{a}} = \nu_0 + \nu_1 \ln z + \ln \left( 1 + \frac{\epsilon}{\hat{a}} \right)$$

So if more productive firms have more unobserved inside equity, we would infer a positive on productivity in the leverage regression even if the true  $\nu_1$  is zero. We find that using this alternative measure of inside fund does not change our benchmark regression results.<sup>22</sup>

The fourth column uses fixed assets instead of total assets to proxy for capital. The coefficient on productivity in the leverage regression drops from 1.125 to 0.346 while the coefficient on output-to-capital ratio doubles. We do not use this as our benchmark results because the firms in our data are small young firms that do not have a lot of financial assets unrelated to production. Working capital and trade credits make up the bulk of non-fixed asset. These items should be counted towards firm borrowing used in their main production.

In column (5) we run our regression with firm fixed effects to control for unobserved cross-section variation in firms. We find that even within firms, one

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<sup>22</sup>Of course, the shareholders fund may not capture all of the unobserved inside equity. For example, Berger and Udell (1998) and Robb and Robinson (2014) find that U.S. young and startups firms have significant personal loan guarantees and collaterals such as home equity from owners that do not appear on the balance sheet. Anecdotes suggest this is also true in Japan. Hence an alternative hypothesis is that leverage capacity is constant with respect to productivity but the off balance sheet inside equity is positively correlated with productivity. We do not have data to form a strong test against this case. However, it is likely that as a firm expands with age, the off balance sheet equity share of total equity declines (e.g. Berger and Udell (1998) find older firms accumulate retained earnings). This means that if the alternative hypothesis is true, when we use shareholders fund as the proxy for inside equity, we should expect to see a higher estimate of  $\nu$  for younger firms. In contrast, as shown in Figure ??, the estimated  $\nu$  is smaller for age 1 firms<sup>23</sup>. So it is unlikely that the alternative hypothesis is driving the positive correlation between leverage and productivity.

	(1)	(2)	(3)	(4)	(5)	(6)
Dep. Var	linear	quad.	diff $a$	diff $k$	firm FE	2SLS
leverage	1.125 (0.039)	1.120 (0.050)	0.973 (0.040)	0.346 (0.051)	0.489 (0.034)	0.207 (0.048)
output- capital	0.690 (0.028)	0.598 (0.035)	0.751 (0.032)	1.316 (0.039)	1.126 (0.034)	3.205 (0.684)
$N$	5872	587a2	5872	5870	21962	5872

NAICS 6-digit industry FE. Control for log inside fund. 2SLS use employment to instrument for productivity. 2006 cohort. Age 5 except for (5). Similar results for other year-cohort.

Table 10: Regression coefficient on log productivity

percent increase in productivity is associated with 0.5% increase in leverage and slightly over 1% increase in the output-to-capital ratio.

Finally, one may be concerned that the positive relationship between leverage and output-to-capital ratio with productivity is driven by measurement error in capital and output that affects both measured productivity and the dependent variables. Assuming the employment is well measured and is uncorrelated with the measurement error in capital and output but is correlated with firm productivity, we apply 2SLS using labor as an instrument. We find that while the elasticity of leverage with respect to productivity drops to 0.2 while the elasticity of output-to-capital ratio with respect to productivity rises to 3.2.

## 4. Parameterization

In this section we first describe how we solve the model given parameters. Then, we layout our strategy for choosing the parameters. In short, we use indirect inference to choose  $\phi_k$  and  $\phi_y$  that best match the empirical relationship between firm leverage, output-to-capital ratio and firm productivity. We calibrate

the remaining parameters to firm level data and common values used in the literature.

#### 4.1. Solving the model

Given parameters, we find the stationary equilibrium by first computing all equilibrium objects given guesses of the interest rate and the wage. This involves solving the entrepreneur's dynamic programming problem and calculating the stationary joint distribution of asset and productivity that is consistent with the entrepreneur's optimal savings decision. We then apply bisection methods to find the pair of prices that satisfies the capital and labor market clearing conditions (See Appendix for details of computation methods).

The entrepreneur's production decision is straightforward to derive. All entrepreneurs choose to produce because there are no fixed costs and the marginal return to producing is infinite at zero production. Conditional on producing, the entrepreneur's factor demand can be derived using standard first-order conditions. Given a particular capital input level, the entrepreneur chooses labor so that the marginal product of labor equals the marginal cost of labor  $w$ . This yields a profit function that is concave in capital. Then, the entrepreneur chooses her capital level subject to the borrowing constraint. Without the borrowing constraint, the entrepreneur chooses the capital level which equate the marginal increase in profit with the marginal cost of capital,  $r + \delta$ . The optimal level is higher for more productive entrepreneurs because they have higher marginal returns to capital. More specifically, the unconstrained optimal capital demand increases with productivity with an elasticity of  $\frac{1}{1-\eta}$  and at this optimal level, the marginal product of capital equals  $r + \delta$ :

$$k^u(a, z) \propto z^{\frac{1}{1-\eta}} \quad \eta\alpha \frac{y^u(a, z)}{k^u(a, z)} = r + \delta.$$

The entrepreneur can choose the unconstrained optimal capital level only if it is below her borrowing capacity  $\bar{k}(a, z)$ . If it exceeds her maximum borrowing

capacity, then she will hit her borrowing limit because, below this level, profit is monotonically increasing in capital. That is, the entrepreneur's optimal capital demand is the smaller of  $k^u(a, z)$  and  $\bar{k}(a, z)$ . When the entrepreneur is constrained, her marginal product of capital exceeds the marginal cost of capital. The size of the gap, or the excess return to capital, is higher for entrepreneurs with higher productivity relative to their maximum borrowing limit. That is, the constrained entrepreneurs have capital demand and output-to-capital ratio

$$k^c(a, z) = \bar{k}(a, z) \quad \eta\alpha \frac{y^c(a, z)}{k^c(a, z)} = r + \delta + \mu(a, z) \propto \left( \frac{z}{\bar{k}(a, z)^{1-\eta}} \right)^{\frac{1}{1-(1-\alpha)\eta}}$$

In what follows, we say an entrepreneur is more financially constrained than another when her excess return  $\mu(a, z)$  is higher.

## 4.2. Parameterization

We parameterize the model by calibrating  $\phi_k$ ,  $\phi_y$  and  $\sigma$  to the firm level data and setting the remaining parameters to values commonly used in the literature. Table 11 display the values we used in our benchmark calibration. Following Midrigan and Xu (2014), we set the scale parameter of production,  $\eta$ , to be 0.85 and the capital intensity parameter,  $\alpha$ , to be 0.33. Moll (2014) which shows that aggregate productivity loss due to financial frictions at the steady state decreases with the persistence of the productivity shocks. We choose a highly persistent process that is consistent with the literature. Finally, we choose  $\sigma_e$  to match the 90/10 ratio of log productivity in our firm level data. That is, in the model, the cross-sectional distribution of  $z$  in the steady state has a log normal distribution with mean parameter  $\mu_e/(1 - \rho)$  and standard deviation parameter  $\sigma = \sigma_e/(1 - \rho^2)$ . We use the 90/10 ratio of the observed distribution of  $z$  to calibrate  $\sigma$ . We then back out the  $\sigma_e$  value that is consistent with the calibrated value of  $\sigma$ .

We use indirect inference (Smith (1993) and Gourieroux et al. (1993)) to choose  $\phi_k$ ,  $\phi_y$ . The data objects we choose to match are the regression coefficients

Parameter	Description	Value	Source
$\eta$	returns-to-scale	0.85	Midrigan and Xu (2014)
$\alpha$	capital intensity	0.33	Midrigan and Xu (2014)
$\rho$	productivity persistence	0.95	Moll (2014)
$\sigma_e$	productivity dispersion	0.627	90/10 ratio of productivity

Table 11: Pre-set parameters

of leverage, output-to-capital on moments of productivity and inside equity. Namely

$$\ln \frac{y}{k} = \beta_0 + \beta_1 \ln z + \beta_2 \ln z^2 + \beta_3 \ln a \quad (1)$$

$$\ln \frac{k}{a} = \theta_0 + \theta_1 \ln z + \theta_2 \ln z^2 + \theta_3 \ln a \quad (2)$$

We choose these data objects because they speak directly to the financial constraint and borrowing capacity in the model. Table 12 illustrates why the regression coefficients are informative of  $\phi_y$  and  $\phi_k$ . When  $\phi_k$  is very high, many firms are unconstrained and capital rises with productivity after controlling for inside fund due to the increase in capital demand with productivity. On the other hand, since many firms are unconstrained, output-to-capital ratio is flat with respect to productivity. When  $\phi_k$  is low but  $\phi_y$  is zero, most firms are financially constrained and leverage does not vary with productivity after controlling for inside equity. However, the output-to-capital ratio rise strongly with productivity because the firms with high productivity relative to their inside equity are more constrained. In order to match both rising leverage and output-to-capital ratio with firm productivity, the model needs a positive  $\phi_y$  and low  $\phi_k$ .

We choose  $\phi_y$  and  $\phi_k$  to minimize the distance between regression coefficients from the model simulated data and that from the actual data where distance is defined using weighting matrix  $\Omega$ . That is, the parameter estimates

	Leverage	Output-capital
high $\phi_k$	+	0
low $\phi_k$ , low $\phi_y$	0	+
low $\phi_k$ , high $\phi_y$	+	+

Table 12: Identification

are defined by

$$[\hat{\phi}_y, \hat{\phi}_k] := \arg \min_{\phi_y, \phi_k} ([\beta, \theta] - [\hat{\beta}, \hat{\theta}])\Omega([\beta, \theta] - [\hat{\beta}, \hat{\theta}])^T$$

Here,  $\beta$  and  $\theta$  denote the vector of regression coefficients  $[\beta_1, \beta_2, \beta_3]$  and  $[\theta_1, \theta_2, \theta_3]$  from the simulated data. The value of these depends on parameters  $\phi_y$  and  $\phi_k$ . The hatted versions denote the regression coefficients from the empirical data. For the choice of  $\Omega$ , we use equal weighting ( $\Omega$  equals the identity matrix) as the benchmark and report the results with OLS robust standard error variance-covariance matrix ( $\Omega := [X'X]^{-1}\hat{\epsilon}\hat{\epsilon}'[X'X]^{-1}$ ) as a robustness check. The latter is the efficient weighting matrix if the linear model is correctly specified<sup>24</sup>.

For the equal weighting scheme, the optimal parameters are  $\hat{\phi}_k = 0.2$ ,  $\hat{\phi}_y = 0.5$ . Under these parameters, aggregate debt-to-capital ratio is  $D/K = 0.27$ , which corresponds to an average leverage of 1.35. For the OLS weighting scheme, the resulting values are  $\hat{\phi}_k = 0.2$ ,  $\hat{\phi}_y = 0.6$  which yields an aggregate debt-to-capital ratio of  $D/K = 0.284$ . The difference between the two results is due to the OLS weighting scheme putting more weight on the curvature parameters<sup>25</sup>.

<sup>24</sup>The regression we run on the actual data also controls for industry fixed effects which is not in the simulated regressions. We construct the OLS weighting matrix by first carrying out partial regression to remove the industry fixed effects from the data regression. More specifically, we regress the dependent variable and each independent variable on industry dummies and use the residuals as dependent and independent variables. The resulting coefficients are numerically identical to running a direct regression with the industry dummies. We apply the OLS robust standard error weighting matrix formula to the regression results with the residualized variables to construct the weighting matrix for in the robustness check.

<sup>25</sup>One interesting observation is the low plegeability of asset. This could be due a large share of assets held by firms in our data are working capital, which may be difficult for lenders to



We also parameterize the model using the two common approaches in the literature: one where we set  $\phi_y = 0$  and another we set  $\phi_y = \phi_k$ . In both approaches, to make them comparable to our benchmark parameterization, we choose  $\phi_k$  to match the  $D/K$  ratio in our benchmark parameterization. Table 13 displays the chosen values in the three approaches and as well as resulting the model and data coefficients at these values. The specification that performs the worst is when  $\phi_y = 0$ . Imposing the restriction  $\phi_k = \phi_y$  brings the model closer to our empirical findings. This suggests that when only aggregate moments are available, it is better to use the  $\phi_k = \phi_y$  model instead of the  $\phi_y = 0$  model. Overall, the model fails to match the lack of curvature in the data. This suggests that alternative models of borrowing capacity may be needed.

### 4.3. TFP loss

The last lines of Tables 13 and 14 display the aggregate productivity loss for each parameterization approach. It shows that compared to the benchmark parameterization, assuming leverage capacity is constant leads to more than 30% ( $\frac{14.6-10.7}{10.7}$ ) larger productivity loss due to financial frictions. Assuming  $\phi_y = \phi_k$  leads to approximately 15% ( $\frac{11.4-10.7}{10.7}$ ) overstatement of losses. These results suggest that it is quantitatively important to model borrowing capacity rising with firm productivity.

## 5. Conclusion

Using private firm data from Japan we documented that both leverage and output-to-capital ratio increase with measured productivity in a way that is inconsistent with the decreasing returns to scale model with a common leverage capacity that is often used in quantitative studies of the impact of financial frictions on TFP. We showed that allowing firms to pledge current and future revenue

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recover. For example, for the 2006 cohort at age 5, the average share of total assets that is fixed assets is 27% while the median share is only 17%.

Regression	benchmark $\phi_y = 0.5, \phi_k = 0.2$	$\phi_y = 0$ $\phi_y = 0, \phi_k = 0.3$	$\phi_y = \phi_k$ $\phi_y = 0.3, \phi_k = 0.3$	Data (OLS s.e.)
<b>Dep var = <math>\ln \frac{k}{a}</math></b>				
$\ln z$	2.297	1.710	2.060	1.120 (0.050)
$\ln a$	-0.351	-0.318	-0.317	-0.512 (0.018)
$(\ln z)^2$	-0.852	-1.253	-1.021	-0.002 (0.013)
<b>Dep var = <math>\ln \frac{y}{k}</math></b>				
$\ln z$	1.523	1.727	1.605	0.598 (0.035)
$\ln a$	-0.226	-0.238	-0.238	-0.206 (0.010)
$(\ln z)^2$	0.297	0.437	0.356	-0.029 (0.008)
D/K	0.266	0.236	0.303	
Distance	0.516	0.574	0.520	
TFP loss	10.7%	14.6%	11.4%	

Table 13: Quality of fit of estimated key parameters versus common approaches in the literature, equal weighting

Regression	benchmark $\phi_y = 0.6, \phi_k = 0.2$	$\phi_y = 0$ $\phi_y = 0, \phi_k = 0.4$	$\phi_y = \phi_k$ $\phi_y = 0.3, \phi_k = 0.3$	Data (OLS s.e.)
<b>Dep var = <math>\ln \frac{k}{a}</math></b>				
$\ln z$	2.489	1.823	2.060	1.120 (0.050)
$\ln a$	-0.379	-0.341	-0.317	-0.512 (0.018)
$(\ln z)^2$	-0.789	-1.323	-1.021	-0.002 (0.013)
<b>Dep var = <math>\ln \frac{y}{k}</math></b>				
$\ln z$	1.456	1.688	1.605	0.598 (0.035)
$\ln a$	-0.275	-0.230	-0.238	-0.206 (0.010)
$(\ln z)^2$	-0.216	0.461	0.356	-0.029 (0.008)
D/K	0.266	0.316	0.303	
Distance	0.368	0.683	0.474	
TFP loss	10.1%	13.7%	11.4%	

Table 14: Quality of fit of estimated key parameters versus common approaches in the literature, OLS weighting

is more consistent with these empirical patterns and that ignoring this heterogeneity in leverage capacity can lead to an economically significant overstatement of the loss in TFP due to financial frictions. We leave documenting these facts for other countries and studying the implications of our findings for the impact of financial frictions on endogenous growth and business cycles for future research.

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## A Data

### A1. Definition of terms from Orbis

Can also be found at [Orbis glossary](#).

Table 15: Glossary of terms from TSR-Orbis

Balance Sheet			
BvD Code	Label	Formula	Definition
FIAS	Fixed Assets	IFAS + TFAS + OFAS	Total amount (after depreciation) of non current assets (Intangible assets + Tangible assets + Other fixed assets).
CUAS	Current Assets	STOK + DEBT + OCAS	Total amount of current assets (Stocks + Debtors + Other current assets).
TOAS	Total Assets	FIAS + CUAS	Total assets (Fixed assets + Current assets)
SHFD	Shareholders Funds	CAPI + OSFD	Total equity (Capital + Other shareholders funds)
CAPI	Capital		Issued Share capital (Authorized capital).
OSFD	Other Shareholders Funds		All Shareholders funds not linked with the Issued capital such as Reserve capital, Undistributed profit, include also Minority interests if any.
EMPL	Number of Employees		Total number of employees included in the company's payroll
Profit & Loss Account			

OPRE	Operating revenue (Turnover)		Total operating revenues (Net sales + Other operating revenues+ Stock variations). The figures do not include VAT. Local differences may occur regarding excises taxes and similar obligatory payments for specific market of tobacco and alcoholic beverage industries.
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## B Numerical procedures

In this section we first describe the numerical procedure we used to calculate the stationary equilibrium for given parameter values. Then, we describe how we estimate  $\phi_k$  and  $\phi_y$ .

### B1. Solving the model

We find the stationary equilibrium by iterating on the interest and wage rates. First, we guess a pair of interest and wage rates. Then we compute all equilibrium objects at these prices. This involves solving the entrepreneur's dynamic programming problem and calculating the stationary joint distribution of asset and productivity that is consistent with the entrepreneur's optimal savings decision. Using the resulting stationary distribution, we check whether the capital and labor markets cleared by calculating net capital and labor demand. We adjust the interest and wage rate by bisection method. That is, we increase the interest (wage) rate if net capital (labor) demand is positive and reduce it if net demand is negative. We then repeat the procedure at the new prices. We iterate on the prices until net capital and labor demand are close enough to zero.

Solving the entrepreneur's decision problem involves two parts: production decision and savings decision. Solving the production decision is straightforward. All entrepreneurs choose to produce because there are no fixed costs and the marginal return to producing is infinite at zero production. Conditional on producing, the entrepreneur's factor demand can be derived using standard first-order conditions. Given a particular capital input level, the entrepreneur chooses labor so that the marginal product of labor equals the marginal cost of labor  $w$ . This yields a profit function that is concave in capital. Then, the entrepreneur chooses her capital level subject to the borrowing constraint. Without the borrowing constraint, the entrepreneur chooses the capital level which equate the marginal increase in profit with the marginal cost of capital,  $r + \delta$ . The optimal level is higher for more productive entrepreneurs because they have higher marginal returns to capital. More specifically, the unconstrained optimal capital demand increases with productivity with an elasticity of  $\frac{1}{1-\eta}$  and at this optimal level, the marginal product of capital equals  $r + \delta$ . We check whether this capital level satisfies the borrowing constraint. If it satisfies the borrowing constraint, we choose it as the capital demand. If not, we set capital demand as the maximum level of capital that satisfies the borrowing constraint. This maximum level is calculated using the standard constrained minimization solver in Matlab.

Solving the entrepreneur's savings decision is less straightforward as there are no analytical solutions to the dynamic programming problem. We solve the problem using fitted value function iteration with linear interpolation. We choose this method because it has been shown to be a contraction mapping for a general class of income processes including the one in the current model (see Li and Stachurski (2014)). Having a globally convergent method that works over a wide range of parameter space is essential for our estimation procedure. To implement fitted value iteration, we need to discretize the state space of productivity and asset. For assets, we choose a grid that assigns more points to the lower

end of assets where there is more curvature in the value function. For productivity, we use the Rouwenhorst method which Kopecky and Suen (2010) found to be significantly more accurate than Tauchen's method for calculating aggregates. Given the entrepreneur's policy function, we need to find the stationary joint distribution of asset and productivity that is consistent with the entrepreneur's savings decisions. There exists several methods for calculating the distribution from given policy functions. We choose the forward iteration technique proposed by Young (2010) because it has been shown to be effective for solving similar models (see Haan et al. (2010)). After finding the stationary distribution, we calculate the aggregate net demand and supply of labor and capital by integrating the demand and supply functions over the stationary distribution.

## B2. Estimation

We find the optimal values  $[\hat{\phi}_y, \hat{\phi}_k]$  in the following way. First, we recover the coefficients from the actual data. Let  $n$  denote the sample size. Then, for a given pair of  $\phi_k$  and  $\phi_y$ , we solve for the stationary equilibrium using numerical methods described previously. We then simulate  $Hn$  samples of  $\{a_i, z_i, k_i, y_i\}$  from the stationary equilibrium. To simulate the sample, we following  $Hn$  households for  $T$  periods, keeping only the last period. Ergodic properties of the model ensure that when  $T$  is large, simulation from the last period well approximates simulation from the stationary distribution (Braun et al. (2012)). We set  $T$  to 500. We then run the two regressions in equations 1 and 2 on the simulated data. As shown in Gourieroux et al. (1993), this is asymptotically equivalent to simulating  $H$  samples of histories of size  $n$ . We then calculate the distance from the empirical coefficients by the aforementioned criterion. We repeat this exercise for the combinations of  $\phi_y$  and  $\phi_k$  over  $[0, 1]$  with 0.1 increments. To avoid the cluttering problem in simulated estimation methods, we use the same draws of  $z$  for every pair of  $[\phi_y, \phi_k]$ .