Discussion of “Coordinating Business Cycles”

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Conference on Multiple Equilibria and Financial Crises

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Multiple equilibria in a model of investment for productivity increase

- Fixed aggregate labor (only input)
- Each firm: if investment at cost \( c \), then constant marginal lowered from 1 to \( \alpha < 1 \).
- Profit fixed fraction of sales, \( \Rightarrow \) profit increase proportional to sales \( px: \pi_{i,j} \).
- \( \pi_{i,j}, i, j \in \{0, 1\} \) with \( i = 1 \) when firm invests, \( j = 1 \) when other firms invest.

\[
\pi_{1,0} = \left(\frac{1}{\alpha}\right)^{\sigma - 1}\pi_{0,0}, \quad \pi_{1,1} = \left(\frac{1}{\alpha}\right)^{\sigma - 1}\pi_{0,1}, \quad \pi_{0,1} = \left(\frac{1}{\alpha}\right)^{\sigma - 1}\pi_{0,0}.
\]

\[
\pi_{0,1} > \pi_{0,0} \quad \text{iff} \quad \sigma < 2.
\]

- Multiple equilibria if

\[
(\alpha^{1-\sigma} - 1)^{\pi_{0,0}} < c(1 + \rho) < (\alpha^{1-\sigma} - 1)^{\pi_{0,1}}.
\]

- With more substitution (endogenous labor and capital), the upper-bound on \( \sigma \) increases above 2.
Multiple equilibria in a model of investment for productivity increase (2)
Remarks

- Extension to growth (many equilibria).

- In the STD model, the individual decision is not investment but a “capacity utilization”. Because of the equivalence of price and production in the imperfect competition model, this is equivalent to a lower cost of production.

- Endogenous labor (and capital) in the STD model, condition $\sigma < S$ with $S > 2$. 
• Aggregate productivity parameter $\theta_t = \rho \theta_{t-1} + \epsilon_t$.

• At the end of each period $t$, agents learn $\theta_t$ perfectly (from the production).

• Global game because of the possibility of arbitrarily large jumps of $\epsilon_t$. 
Mass 1 of agents, action 0 (low) or 1 (high). $x_t$ is the mass of “high” in period $t$.

Payoff of low is 0, payoff of high is $E[\theta(x + 1) - c]$. ($c$ cost of high).

Perfect information: multiple equilibria if $c/2 < \theta_t < c$.

Imperfect information: $\theta_t - b = a(\theta_{t-1} - b) + \eta_t$, $\eta_t \sim \mathcal{N}(0, 1/q_\eta)$

agent information $s_{it} = \theta_t + \epsilon_t$, $\epsilon_t \sim \mathcal{N}(0, 1/q_\epsilon)$

Critical value $s^*$. Mass of investment $x_t(\theta_t - s^*_t) = F(\sqrt{p_\eta}(\theta_t - s^*_t))$.

Marginal $s^*$: $E[\theta_t(x_t + 1)|s^*_t] = c$.

$E[\theta_t|s^*_t] + \int \theta F(\sqrt{p_\eta}(\theta_t - s^*_t))dF_{s_t^*}(\theta) = c$.

Assume that the precision $q_\epsilon$ is arbitrarily large: $s^* \approx 2c/3$

Because the distribution is highly concentrated, most agents invest if $\theta_t > 2c/3$. 
Evolution of output

No hysteresis
Serial correlation of output comes from the fundamental

(STD)
Comparison with Guimaraes and Machado, 2014

(Std)

No hysteresis
Serial correlation of output
comes from the fundamental

\[
\text{Investment No!}
\]

(Guimaraes and Machado, 2014)
Comparison with Chamley, “Coordinating Regime Switches,” QJE 1999

No hysteresis!

Serial correlation of output comes from the fundamental

Serial correlation of output: learning the fundamental and coordination