Discussion on “Stagnation Traps”

Jang-Ting Guo

Department of Economics
University of California, Riverside

May 15, 2015
Existence and Persistence of Stagnation Trap in a Monetary Endogenous Growth Model with Quality Ladders

⇒ Coexistence of Positive Unemployment, Low Growth, and Liquidity Trap
Objective

• Existence and Persistence of Stagnation Trap in a Monetary Endogenous Growth Model with Quality Ladders

⇒ Coexistence of Positive Unemployment, Low Growth, and Liquidity Trap

Findings

• The Key Mechanism

(1) Unemployment and Weak Aggregate Demand ⇒ Reduces Firms’ Investment in Innovation ⇒ Low Growth

(2) Low Growth ⇒ Reduces Real Interest Rate ⇒ Pushes Nominal Interest Rate to Zero

Comments

Jang-Ting Guo
Two Steady States in Baseline Model

(1) Full Employment $y^f = 1$, High Growth $g^f$, Positive Nominal Interest Rate $i^f > 0$, and Positive/Negative Inflation Rate $\pi^f \geq 1$

(2) Unemployment $y^u < 1$, Low Growth $g^u < g^f$, Zero Nominal Interest Rate $i^u = 0$, and Negative Inflation Rate $\pi^u < 1$
Two Steady States in Baseline Model

(1) Full Employment $y^f = 1$, High Growth $g^f$, Positive Nominal Interest Rate $i^f > 0$, and Positive/Negative Inflation Rate $\pi^f \geq 1$

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Two Extensions: Precautionary Savings and Time-Varying Inflation Rate

Constant or Countercyclical Subsidy to Firms’ Investment in Innovation $\Rightarrow$ Removal of Low-Growth Steady State
Two Steady States: $y^f = 1$ and $y^u < 1$

$\Rightarrow y$ Denotes the Level of Actual Output

$\Rightarrow 1 - y =$ Output Gap
Two Steady States: \( y^f = 1 \) and \( y^u < 1 \)

\[ \Rightarrow y \text{ Denotes the Level of Actual Output} \]

\[ \Rightarrow 1 - y = \text{Output Gap} \]

Figure 1 \( \Rightarrow \) Local Stability Property of Each Steady State: Saddle, Sink or Source

Possibility of Global Indeterminacy \( \Rightarrow \) Various Forms of Bifurcations
This Paper

\[
\max \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1 - \sigma}, \quad 0 < \beta < 1
\]

\[
C_t = \exp \left( \int_0^1 \ln q_{jt} c_{jt} dj \right) \quad \text{and} \quad Q_t = \exp \left( \int_0^1 \ln q_{jt} dj \right)
\]

\[
\left( \frac{c_{t+1}}{c_t} \right)^{\sigma} = \beta (1 + r_t) g_{t+1}^{1-\sigma}, \quad \text{where} \quad g_{t+1} = \frac{Q_{t+1}}{Q_t}
\]
\begin{itemize}
    \item This Paper

    \[
    \max \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1 - \sigma}, \quad 0 < \beta < 1
    \]

    \[
    C_t = \exp \left( \int_{0}^{1} \ln q_j c_j d\gamma \right) \quad \text{and} \quad Q_t = \exp \left( \int_{0}^{1} \ln q_j d\gamma \right)
    \]

    \[
    \left( \frac{c_{t+1}}{c_t} \right)^{\sigma} = \beta (1 + r_t) g_{t+1}^{1-\sigma}, \quad \text{where} \quad g_{t+1} = \frac{Q_{t+1}}{Q_t}
    \]

    \item Need $\sigma > 1$ such that

    (1) Positive Relationship between Present Consumption and Innovation Growth

    (2) Existence of Unemployment Steady State

    (3) $i^f > 0$ at Full-Employment Steady State
\end{itemize}
Alternative Specification (Footnote 14)

$$\max \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}, \quad 0 < \beta < 1$$

$$y_t = f \left( \int_0^1 q_{jt} X_{jt} dj \right) = f(Q_t)$$
Alternative Specification (Footnote 14)

\[
\max \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}, \quad 0 < \beta < 1
\]

\[y_t = f \left( \int_0^1 q_{jt} X_{jt} dj \right) = f(Q_t)\]

\[
\left( \frac{c_{t+1}}{c_t} \right) = \beta (1 + r_t)
\]

\[
\left( \frac{c_{t+1}}{c_t} \right)^\sigma = \beta (1 + r_t) g_{t+1}^{1-\sigma}, \text{ where } g_{t+1} = \frac{Q_{t+1}}{Q_t}
\]

⇒ Isomorphic Formulations Only When \( \sigma = 1 \)
Objective

This Paper

Euler: \( \left( \frac{c_{t+1}}{c_t} \right)^\sigma = \beta (1 + i_t) \bar{\pi} g_{t+1}^{1-\sigma} \)

Growth: \( 1 = \beta \left[ \left( \frac{c_t}{c_{t+1}} \right)^\sigma g_{t+1}^{1-\sigma} (\chi \frac{\gamma - 1}{\gamma} y_{t+1} + 1 - \frac{\ln g_{t+2}}{\ln \gamma}) \right] \)

When \( \sigma > 1 \Rightarrow \) Positive Relationship between \( y_{t+1} \) and \( g_{t+1} \)
This Paper

\[ \text{Euler: } \left( \frac{c_{t+1}}{c_t} \right)^\sigma = \beta \left(1 + i_t\right) \bar{\pi} g_{t+1}^{1-\sigma} \]

Growth: \[ 1 = \beta \left[ \left( \frac{c_t}{c_{t+1}} \right)^\sigma g_{t+1}^{1-\sigma} (\chi \frac{\gamma - 1}{\gamma} y_{t+1} + 1 - \frac{\ln g_{t+2}}{\ln \gamma}) \right] \]

When \( \sigma > 1 \Rightarrow \) Positive Relationship between \( y_{t+1} \) and \( g_{t+1} \)

Market Clearing: \[ c_t + \frac{\ln g_{t+1}}{\chi \ln \gamma} = y_t \]

Monetary Policy: \[ 1 + i_t = \max \left\{ (1 + \bar{i}) y_t^\phi, \ 1 \right\} \]
Alternative Specification

Period Utility: \[ \frac{c_t^{1-\sigma} - 1}{1 - \sigma} \]
**Alternative Specification**

Period Utility: \[
\frac{c_t^{1-\sigma} - 1}{1-\sigma}
\]

Final Good: \[
Y_t = A \int_0^1 (q_{jt} X_{jt})^\alpha dj, \quad A > 0, \quad 0 < \alpha < 1
\]

Demand for \(X_{jt}\): \[X_{jt} = \left(\frac{A\alpha q_{jt}^\alpha}{P_{jt}}\right)^{\frac{1}{1-\alpha}}\]
**Alternative Specification**

Period Utility: \[ c_t^{1-\sigma} - 1 \]

Final Good: \[ Y_t = A \int_0^1 (q_{jt} X_{jt})^\alpha dj, \quad A > 0, \quad 0 < \alpha < 1 \]

Demand for \( X_{jt} \): \[ X_{jt} = \left( \frac{A \alpha q_{jt}^\alpha}{P_{jt}} \right)^{\frac{1}{1-\alpha}} \]

Supply for \( X_{jt} \): \( X_{jt} = L_{jt} \), where \[ \int_0^1 L_{jt} dj + L_{jt}^{RD} + U_t = L \]

R&D Firms’ Profits: \[ \pi_{jt} = (P_{jt} - W_t) X_{jt}, \quad \frac{W_t}{W_{t-1}} = \bar{\pi} \]
Monopoly Pricing: \[ P_{jt} = \frac{W_t}{\alpha} \]

Equilibrium Quantity: \[ X_{jt} = \left( \frac{A\alpha^2 q_{jt}}{W_t} \right)^{\frac{1}{1-\alpha}} \]
Monopoly Pricing: \( P_{jt} = \frac{W_t}{\alpha} \)

Equilibrium Quantity: \( X_{jt} = \left( \frac{A\alpha^2 q_{jt}^\alpha}{W_t} \right)^{\frac{1}{1-\alpha}} \)

Aggregate Output: \( Y_t = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} W_t^{\frac{-\alpha}{1-\alpha}} Q_t, \)

where \( Q_t = \int_0^1 q_{jt}^{\frac{\alpha}{1-\alpha}} dj \)

Equilibrium Profit: \( \pi_{jt} = \alpha(1 - \alpha)q_{jt}^{\frac{\alpha}{1-\alpha}} \frac{Y_t}{Q_t} \)
Probability of Innovating = \frac{\chi L^{RD}_t}{L} = \chi \mu_t
Probability of Innovating: \( \frac{\chi L_t^{RD}}{L} = \chi \mu_t \)

Value Function: \( V_t = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \left[ \pi_{jt+1} + (1 - \chi \mu_{t+1}) V_{t+1} \right] \)

Free Entry: \( L_t^{RD} W_t = \chi \mu_t V_t \Rightarrow LW_t = \chi V_t \)

Innovation Growth: \( g_{t+1} = \frac{Q_{t+1}}{Q_t} = \chi \mu_t \gamma^{1-\alpha} \Rightarrow \frac{Y_{t+1}}{Y_t} = g_{t+1} \bar{\pi}^{\frac{-\alpha}{1-\alpha}} \)
Probability of Innovating = \( \frac{\chi L^R \mu}{L} = \chi \mu_t \)

Value Function: 
\[
V_t = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \left[ \pi j_{t+1} + (1 - \chi \mu_{t+1}) V_{t+1} \right]
\]

Free Entry: 
\[
L^R W_t = \chi \mu_t V_t \Rightarrow LW_t = \chi V_t
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Innovation Growth: 
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g_{t+1} = \frac{Q_{t+1}}{Q_t} = \chi \mu_t \gamma^{\frac{\alpha}{1-\alpha}} \Rightarrow \frac{Y_{t+1}}{Y_t} = g_{t+1} \bar{\pi}^{\frac{-\alpha}{1-\alpha}}
\]

Growth: 
\[
1 = \left( \beta \bar{\pi}^{\frac{\sigma \alpha}{1-\alpha}} \right) g_{t+1}^{-\sigma} \left[ \alpha (1 - \alpha) q_j^{\frac{\alpha}{1-\alpha}} \frac{Y_{t+1}}{LW_t Q_{t+1}} + \bar{\pi} (1 - \frac{g_{t+2}}{\gamma^{\frac{\alpha}{1-\alpha}}}) \right]
\]

When \( \sigma > 0 \) \Rightarrow Positive Relationship between \( \frac{Y_{t+1}}{Q_{t+1}} \) and \( g_{t+1} \)
Alternative Specification

Euler: \[ \left( \frac{c_{t+1}}{c_t} \right) \sigma = \beta \frac{1 + i_t}{\bar{\pi}} \]

Growth: \[ 1 = \left( \beta \bar{\pi}^{1-\alpha} \right) g_{t+1}^{-\sigma} \left[ \alpha(1 - \alpha)q_j(t+1) \frac{\chi Y_{t+1}}{LW_t Q_{t+1}} + \bar{\pi}(1 - \frac{g_{t+2}}{\gamma^{1-\alpha}}) \right] \]

When \( \sigma > 0 \) \( \Rightarrow \) Positive Relationship between \( \frac{Y_{t+1}}{Q_{t+1}} \) and \( g_{t+1} \)
Alternative Specification

Euler: \[
\left( \frac{c_{t+1}}{c_t} \right) ^\sigma = \beta \left( 1 + i_t \right)
\]

Growth: \[
1 = \left( \beta \bar{\pi}^{\frac{\alpha}{1-\alpha}} \right) g_{t+1}^{-\sigma} \left[ \alpha (1 - \alpha) q_j(t+1) \chi Y_{t+1} \right. \\
\left. LW_t Q_{t+1} + \bar{\pi} (1 - \frac{g_{t+2}}{\gamma^{1-\alpha}}) \right]
\]

When \( \sigma > 0 \Rightarrow \) Positive Relationship between \( \frac{Y_{t+1}}{Q_{t+1}} \) and \( g_{t+1} \)

Market Clearing: \( c_t = Y_t \Rightarrow \frac{c_{t+1}}{c_t} = \frac{Y_{t+1}}{Y_t} = g_{t+1} \bar{\pi}^{\frac{-\alpha}{1-\alpha}} \)

Monetary Policy: \( 1 + i_t = \max \{ (1 + \bar{i}) \frac{Y_t}{Q_t}, 1 \} \)
growth \( g \) vs. output gap \( y \)

- \( (y^u, g^u) \)
- \( (1, g^f) \)

- \( AD \)
- \( GG \)
At Unemployment Steady State

(1) Baseline $\bar{\pi} < 1 \Rightarrow$ Deflation

Extension with Precautionary Savings, but Unemployed Households Cannot Borrow or Trade Firms’ Shares
At Unemployment Steady State

(1) Baseline \( \bar{\pi} < 1 \Rightarrow \text{Deflation} \)

Extension with Precautionary Savings, but Unemployed Households Cannot Borrow or Trade Firms’ Shares

(2) Zero Nominal Interest Rate \( i_u = 0 \)

Negative Nominal Interest Rates Observed in Europe: ECB’s Deposit Rate of \(-0.2\%\), and Swiss National Bank’s Deposit Rate of \(-0.75\%\)

\[ \Rightarrow 1 + i_t = \max \left\{ (1 + \bar{\pi}) y_t^\phi, \ i \right\}, \text{ where } i < 1 \]