SLOW MOVING DEBT CRISSES

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SLOW MOVING CRISES

- Sovereign crises without immediate liquidity concern
  - unexpected sharp increase in spreads
  - treasury auctions keep going ok
  - gradual, but faster accumulation of debt despite efforts at fiscal adjustment
  - investors worry about medium-run debt dynamics
- Recent example: Italy
ITALY: 10YR BOND YIELDS

Draghi's "Whatever it takes"

Something happens here
ITALY: GOVERNMENT BUDGET

IMF projections

Net surplus
Primary surplus
Interest payments

-100 -50 0 50 100 150
THIS PAPER

- Dynamic model of multiple equilibria with a fiscal rule
- Characterize maximum debt and crisis region
- What properties of fiscal rule help prevent crises?
- Also: timing/commitment issues and multiplicity
CONNECTIONS

- Role of expectations: Calvo (1988)
- Related: Navarro, Nicolini, Teles (2014)
- Monetary/fiscal issues: Corsetti-Dedola (2014)
OUTLINE

- Recursive derivation of debt capacity with short-term debt
- Applications: stationary model
- Microfoundations
SHORT-TERM DEBT

- **Time** \( t = 1, \ldots, T \)
- **Fiscal rule** \( F(s_t|s_{t-1}, b_t) \)
- **Zero recovery after default**
- **Budget constraint**

\[
q_t b_{t+1} + s_t = b_t
\]
SOLVING BACKWARDS
SOLVING BACKWARDS

repay if

$s_T \geq b_T$
SOLVING BACKWARDS

repay if

$ s_T \geq b_T $

$ Q_{T-1}(b_T, s_{T-1}) = $ 

$ \beta \Pr (s_T \geq b_T | s_{T-1}, b_T) $
SOLVING BACKWARDS

\[ Q_{T-1}(b_T, s_{T-1}) = \beta \Pr (s_T \geq b_T | s_{T-1}, b_T) \]

\[ m_{T-1}(s_{T-1}) = \max_{b'} Q_{T-1}(b', s_{T-1}) b' \]

repay if

\[ s_T \geq b_T \]
SOLVING BACKWARDS

repay if

\[ b_{T-1} - s_{T-1} \leq m_{T-1}(s_{T-1}) \]

repay if

\[ s_T \geq b_T \]

\[
Q_{T-1}(b_T, s_{T-1}) = \\
\beta \Pr(s_T \geq b_T | s_{T-1}, b_T)
\]

\[
m_{T-1}(s_{T-1}) = \\
\max_{b'} Q_{T-1}(b', s_{T-1})b'
\]
SOLVING BACKWARDS

\[ Q_{T-2}(b_{T-1}, s_{T-2}) = \beta \Pr(s_{T-1} \geq b_{T-1} - m_{T-1}(s_{T-1})|s_{T-2}, b_{T-1}) \]

\[ Q_{T-1}(b_T, s_{T-1}) = \beta \Pr(s_T \geq b_T|s_{T-1}, b_T) \]

\[ m_{T-1}(s_{T-1}) = \max_{b'} Q_{T-1}(b', s_{T-1})b' \]

repay if \[ b_{T-1} - s_{T-1} \leq m_{T-1}(s_{T-1}) \]

repay if \[ s_T \geq b_T \]
SOLVING BACKWARDS

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SOLVING BACKWARDS

repay if

\[ b_{T-2} - s_{T-2} \leq m_{T-2}(s_{T-2}) \]

repay if

\[ b_{T-1} - s_{T-1} \leq m_{T-1}(s_{T-1}) \]

repay if

\[ s_T \geq b_T \]

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\[ Q_{T-2}(b_{T-1}, s_{T-2}) = \]
\[ \beta \Pr(s_{T-1} \geq b_{T-1} - m_{T-1}(s_{T-1}) | s_{T-2}, b_{T-1}) \]

\[ m_{T-2}(s_{T-2}) = \]
\[ \max_{b'} Q_{T-2}(b', s^{T-2}) b' \]

\[ Q_{T-1}(b_{T}, s_{T-1}) = \]
\[ \beta \Pr(s_T \geq b_T | s_{T-1}, b_T) \]

\[ m_{T-1}(s_{T-1}) = \]
\[ \max_{b'} Q_{T-1}(b', s_{T-1}) b' \]
SOLVING BACKWARDS

- Result: Maximal debt and price schedules uniquely defined
- Multiple equilibria?
  - Yes
- $Q_t(b_{t+1}, s_t)b_{t+1}$ not monotone
- Laffer curve
Laffer Curve
A STATIONARY EXAMPLE

- Continuous time
- With Poisson probability $\lambda$ uncertainty is realized
- At that point surplus $S$ drawn from CDF $F(S)$
- If default, recover fraction of surplus
- Price at the Poisson event is

$$\Psi(b) = 1 - F(b) + \phi \frac{1}{b} \int_{S}^{b} S dF(s)$$
ODE

- Fiscal rule, increasing, *bounded above*

  \[ s = h(b) \]

- Budget constraint

  \[ q(b + \delta b) + s = \kappa b \]

- Pricing condition

  \[ rq = \kappa - \delta q + \lambda(\Psi(b) - q) + \dot{q} \]

- ODEs in \( b, q \)
TERMINAL CONDITIONS

- An equilibrium satisfies the ODE and a terminal condition:
  - Possibility 1: $b$ and $q$ converge to a steady state
  - Possibility 2:
    \[ b \to \infty, \quad q \to 0 \]
  - Possibility 2 leads to default in finite time and constant debt value for $b$ large enough
MULTIPLE STEADY STATES

\[ \dot{q} = 0 \]

\[ \dot{b} = 0 \]
MULTIPLE EQUILIBRIA

Figure 7: Phase diagrams with fiscal rule.

Dynamic Stability.

Next we provide a necessary and sufficient condition for saddle-path stability of a steady state.

Lemma 1. A steady state with positive debt is locally saddle-path stable if and only if the $\dot{b} = 0$ locus is downward sloping and steeper than the $\dot{q} = 0$ locus, or equivalently,

$$h_0(b) > \frac{dq}{dr} + lY_0(b).$$

As mentioned above, in a steady state with no default risk $q = 1$ and $lY_0(b) = 0$, so that condition (16) reduces to $h_0(b) > r$. This ensures that the total secondary surplus (primary minus the cost of servicing the debt) is increasing in $b$, a standard stability condition for fiscal rules in the literature on fiscal and monetary policy (Leeper, 1991). This condition is stronger than condition (15) precisely because it includes the induced change in bond prices. Condition (15) ensures the $\dot{b} = 0$ locus is decreasing; whereas condition (16) ensures it is decreasing and steeper than the $\dot{q} = 0$ locus.

Presumably, saddle-path stability is a desirable feature. However, given the model’s non-linearities, having a saddle-path stable equilibrium is not enough to rule out multiple steady states or multiple equilibria. Indeed, multiple steady states are ensured if any steady state is saddle path stable.

Proposition 6. If there is a saddle-path stable steady state with positive debt, then there is another steady state with higher debt that is not stable.

The result follows from the fact that the surplus is bounded above by $\bar{s} > 0$, implying that the fiscal policy rule cannot be responsive at high debt levels. Indeed, for high debt levels...
STABILITY

- With no default risk ODE boils down to
  \[ \dot{b} = rb - h(b) \]

- Stability condition (Leeper, 1991)
  \[ h'(b) > r \]

- Increase surplus faster than debt service
STABILITY

- Steady state saddle path stable if
  \[ h'(b) > \kappa - \delta q - \frac{\delta \lambda}{r + \delta + \lambda} \Psi'(b)b \]

- This is stronger than
  \[ h'(b) > r \]

- **Result:** If \( h \) function bounded and there is a stable s.s., there must also be another s.s. with higher debt
A SLOW MOVING CRISIS

spread

primary surplus

debt stock
SUMMING UP

- Conditions for “sustainability” are tighter than under risk-free debt
- Even if sustainability condition satisfied, basin of attraction is not necessarily safe
- Equilibrium is eventually unique
Figure 7: Regions with unique equilibrium (red), three equilibria (pink) or immediate default (yellow).

For the liquidity crisis, when this probability is not zero, the interest rate rises and the government makes an effort to reduce debt to a safe level that excludes investor runs and lowers the interest rate. Thus, high interest rates in liquidity crisis models may be present even with a decreasing path for debt.

In contrast, in our model, debt rises along the bad, high interest equilibrium path. Indeed, the rising path for debt and higher interest rates are intimately related, the one implying the other.

Another interesting distinction is that the multiplicity from liquidity crises is broader and more pervasive than the multiplicity due to slow moving crises. In the example above, we found three equilibrium interest rates. However, only two of these can be considered part of a stable equilibrium. In contrast, liquidity crises open the door to a continuum of sunspot equilibria, indexed by the constant arrival probability of the run.

Conesa and Kehoe (2012) extend liquidity crisis models to include uncertainty in income and find that debt may be increasing in some cases. Nevertheless, high interest rates are driven by the sunspot probability of a run, not by the accumulation of debt.
5.1 A Game with No Commitment

Consider a two-period model in which the government's objective function is to maximize $u(s) + bV_b$, where $s$ is current primary surplus and $b_0$ is the stock of bonds issued in the first period, to be repaid in the second period. Notice that we could interpret $u$ as the payoff resulting from a full specification of the benefits of public expenditure and the costs of taxation and $V$ as a value function in a stationary, optimizing model with an infinite horizon. The government has a stock of bonds $b$ inherited from the past that it needs to repay at the beginning of the first period, so the budget constraint is $b = q b_0$.

Investors are risk neutral and their expected repayment per bond issued is given by the function $b G(b_0)$, which includes the recovery value after default tomorrow.
MICROFOUNDTIONS

- Goal
  - write down a “game”
  - government chooses debt...
  - ... but cannot commit to not go back
  - solve it and show “Calvo outcome”
MODEL

- Three periods
- Bonds only pay in 3
- Objective of borrower is \( U(c_0, c_1, c_2) \)
- Issue bonds at \( t=0 \) and \( t=1 \): \( c_0 = q_0 b_0 \quad c_1 = q_1 (b_1 - b_0) \)
- Repayment at \( t=2 \) depends on bonds issued and shock
MULTIPLICITY AT T = 1

- Best response
  \[ B_1(b_0, q_0) \]

- Rational expectations
  \[ q_0 = 1 - F(B_1(b_0, q_0)) \]

- Multiplicity possible if preferences non-separable: low resources raised in 0 increase incentive to borrow at 1
DO WE GET THERE?
FINAL REMARKS

- Slow Moving Crises
  - dynamic Calvo
  - different from liquidity crisis a la Cole-Kehoe

- Tipping points and tipping regions
- Local/global properties of fiscal rule