Global Sunspots and Asset Prices in a Monetary Economy

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Financial Crises

• This paper presents a calibrated version of the Cass-Shell sunspots paper

• I argue that incomplete participation is a big deal in real world financial markets
Assumptions

• Exchange economy
  • No fundamental uncertainty
  • Incomplete markets

• Heterogeneous agents
  • Two types that differ in their discount rates
Assumptions

- Nominal assets
  - existence of multiple equilibria
- Incomplete participation
  - allows for sunspots
Main Idea

• Fundamental equilibrium is a highly persistent difference equation
• Incomplete participation due to demographic structure
• Natural market incompleteness
Main Idea

perfect foresight equilibria are solutions to a difference equation

\[ m' = F(m, b) \]  
\[ b' = G(m, b) \]

m is the equilibrium discount factor
b is the real value of government debt
Main Idea

There is one initial condition

\[ \delta_0 + \delta_1 b + \delta_2 p_k = a_{1,0} \]

where \( a_{1,0} \) are net claims by initial type 1 people

Because debt is nominal, this does not pin down an initial value of \( p_k \)
Perfect Foresight Equilibria

Figure 1: The set of perfect foresight equilibria
Three traded assets

• Government debt
  Costs \( Q^N D' \) dollars
  Pays \( D' \) dollars

• Trees
  Cost \( p_k \) apples
  Pays \( \pi\left[p_k(S') + 1\right] \)

• Arrow securities
  Cost \( Q(S') \) apples
  Pays 1 apple
Government budget constraint

\[ \frac{D'}{R^N} = D - \tau p \]

In real terms

\[ m'b' = b - \tau \]

\[ m' = \frac{p'}{pR^N} \]

\( m' \) is the pricing kernel
Households

\[ V_i[W_i] = \max_{a_i(S')} \left\{ \log C_i + \pi \beta_i E V[W_i'(S')] \right\} \]

Value function

\[ \sum_{S'} \pi m(S') W_i'(S') + C \leq a_i(S) \]

Budget constraint

\[ W_i = a_i(S) \]

Definition of wealth

\[ W_{i,0} = p_k(S) \]

Initial wealth
Solution

\[ AC_1 = a_1(S) \quad \text{Type 1 people} \]

\[ BC_2 = a_2(S) \quad \text{Type 2 people} \]

\[ A = \frac{1}{1 - \beta_1 \pi} \quad B = \frac{1}{1 - \beta_2 \pi} \]

\[ A > B \quad \text{Type 1 more patient} \]
Policy

Passive monetary policy

\[ R^N = 1.05 \]

Active fiscal policy

\[ \frac{D'}{R^N} = D - \tau p \]

\[ m'b' = b - \tau \]

Fiscal theory of the price level DOES NOT hold
No Ricardian equivalence

\[ W = p_k - T + b \]  
Aggregate wealth

\[ T = p_k \tau \]  
Tax obligation of current generation

\[ \tilde{T} \]  
Tax obligation of future generations

\[ b = T + \tilde{T} \]  
Government budget balance

BUT \[ b \neq T \]
Marginal rates of substitution

\[ C_i \] Aggregate consumption of all type \( i \) people alive today

\[ C_i^o (S') \] Aggregate consumption of all type \( i \) people who are still alive tomorrow

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Marginal rates of substitution

\[ C_i = \theta_{0,i} + \theta_{1,i} \left[ p_k (1 - \tau) + b \right] \]

\[ C_i^O (S') = \eta_{0,i} + \eta_{1,i} p'_k (S')(1 - \tau) + \eta_{2,i} b'(S') \]

These equations come from solving individual optimization problems

these are different
Marginal rates of substitution

Using the expressions for consumption

\[ m_1(b, p_k, b', p'_k) = m_2(b, p_k, b', p'_k) \]

we can equate the marginal rates of substitution state by state... and solve for \( p'_k \)

\[ p'_k = \psi(b, p_k, b') \]
Marginal rates of substitution

We can substitute this into the MRS of either person to obtain an expression for the pricing kernel

\[ \phi(b, p_k, b') = m_1 \left[ b, p_k, b', \psi(b, p_k, b') \right] \]
Equilibria

Equilibria are solutions to the equations

\[ p'_k = \psi(b, p_k, b') \]

\[ b = b'\phi(b, p_k, b') + \tau \]

\[ \delta_0 + \delta_1 b + \delta_2 p_k = a_{1,0} \]

There is no initial condition
Change of variables

Change of variables:

\[ \{p_k', b'\} \rightarrow \{b', m' = \phi(p_k, b, b')\} \]
Computing solutions

Equilibria are solutions to the equations

\[ m' - F(m, b) = 0 \]
\[ b' - G(m, b) = 0 \]
Properties of solutions

Unique feasible steady state with positive $b$
and non-negative consumption of both types

$$\bar{m} - F\left(\bar{m}, \bar{b}\right) = 0$$

$$\bar{b} - G\left(\bar{m}, \bar{b}\right) = 0$$

The steady state is a saddle
Approximate solution

Search for functions $f(\cdot)$ and $g(\cdot)$ that solve the following functional equation

\[
\begin{align*}
  f(m) - F(m, g(m)) &= 0 \\
g\left[f(m)\right] - G(m, g(m)) &= 0
\end{align*}
\]
Equilibria

A solution to this difference equation for any initial condition in \([D_1, D_2]\) is an equilibrium

\[
m' - f(m) = 0 \]

\[
b = g(m) \]

The initial price level is indeterminate
Approximate solution

• Method, Chebyshev polynomials of degree 5 with colocation.
• Solution is exact at the colocation points
• Can show that $C_1$ is increasing (and $C_2$ is decreasing) in $m$
• Pick the domain of $m[D_1,D_2]$ such that $C_1(D_1)=0$ and $C_2(D_2)=0$
## Calibration

### Table 1

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Parameter Name</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor of type 1</td>
<td>$\beta_1$</td>
<td>0.98</td>
</tr>
<tr>
<td>Discount factor of type 2</td>
<td>$\beta_2$</td>
<td>0.90</td>
</tr>
<tr>
<td>Survival probability</td>
<td>$\pi$</td>
<td>0.98</td>
</tr>
<tr>
<td>Fraction of type 1 in the population</td>
<td>$\mu_1$</td>
<td>0.5</td>
</tr>
<tr>
<td>Gross nominal interest rate</td>
<td>$R^N$</td>
<td>1.05</td>
</tr>
<tr>
<td>Primary deficit</td>
<td>$\tau$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

This calibration leads to the steady state values reported in Table 2.
Steady state values

Table 2

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Parameter Name</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium discount factor</td>
<td>$\bar{m}$</td>
<td>0.97</td>
</tr>
<tr>
<td>Equilibrium government debt</td>
<td>$\bar{b}$</td>
<td>0.69</td>
</tr>
<tr>
<td>Equilibrium asset price</td>
<td>$\bar{p}_k$</td>
<td>20.6</td>
</tr>
<tr>
<td>Return to a tree</td>
<td>$R^p_k$</td>
<td>1.03</td>
</tr>
<tr>
<td>Return to debt</td>
<td>$R^b$</td>
<td>1.03</td>
</tr>
</tbody>
</table>
Approximate solution

$f(m) - m$

$b = g(m)$
Approximate solution

This is how $p_k$ changes with $m$.

$p_k$ determines the initial wealth of a newborn.
Approximate solution

![Graph showing consumption of type 1 and type 2](image-url)

- Consumption of type 2
- Consumption of type 1
Global sunspot equilibria

\[ E[m'] - f(m) = 0 \]

\[ b = g(m) \]

Randomizations across perfect foresight equilibria are also equilibria
Global sunspot equilibria

\[ m' : B(\alpha, \beta) \]

I model \( m' \) as a linear function, \( T \), of a Beta distributed random variable

\[
T \left( \frac{\alpha}{\alpha + \beta} \right) = f(m) = E[m']
\]

There is one degree of freedom in picking \( \alpha \) and \( \beta \)
Global sunspots

• I define a function of $m$ that gives the largest possible variance for $m'$ at all points in the domain of $m$
• I chose a constant parameter, $k$, that scales this function
• High $k$ means low variance
• Low $k$ means high variance
Global sunspot equilibria

I chose $k = 2$ in my simulations
Simulating equilibria

The safe rate, inflation and the risky rate in 60 years of simulated data.

The Sharpe ratio is 0.1 for this draw.
Simulating equilibria

This graph plots the average Sharpe ratio in 6,000 draws of 60 years of data.

This is the distribution of average safe and risky rates over these 6,000 simulations.
Simulating equilibria

The distribution of Sharpe ratios has a fat left tail.
Volatility

Bursts of volatility are common
Bursts of volatility are common
Volatility

Volatility is associated with reversals in the pattern of the distribution of consumption across types.
Volatility

... this is also a period of deflation that can last for a decade or more
Conclusion

• Excess volatility and the term premium can be explained in a simple and parsimonious model

• Parameters are disciplined by demographics