The Macroeconomics of Shadow Banking

Alan Moreira[*] Alexi Savov[†]

July 2014

First draft: August 2013

Abstract

We build a macroeconomic model that centers on liquidity transformation in the financial sector. Intermediaries maximize liquidity creation by issuing securities that are money-like in normal times but become illiquid in a crash when collateral is scarce. We call this process shadow banking. A rise in uncertainty raises demand for crash-proof liquidity, forcing intermediaries to delever and substitute toward safe, collateral-intensive liabilities. Shadow banking shrinks, causing the liquidity supply to contract, discount rates and collateral premia spike, prices and investment fall. The model produces slow recoveries, collateral runs, and flight to quality and it provides a framework for analyzing unconventional monetary policy and regulatory reform proposals.

JEL Codes: E44, E52, G21.

Keywords: Macro finance, financial intermediation, liquidity transformation, shadow banking, unconventional monetary policy.

[*]Yale University School of Management, alan.moreira@yale.edu.

[†]New York University Stern School of Business and NBER, asavov@stern.nyu.edu.

We thank Markus Brunnermeier, Douglas Diamond, Gary Gorton, Valentin Haddad, Zhiguo He, Arvind Krishnamurthy, Matteo Maggiori, Andrew Metrick, Tyler Muir, Cecilia Parlatore Siritto, Thomas Philippon, and Michael Woodford for feedback. We also thank seminar participants at Yale School of Management, New York University Stern School of Business, Kellogg School of Management, Wisconsin Business School, and Princeton University, and conference participants at the Kellogg Junior Finance Conference, the NBER Monetary Economics Program Meeting, the Safe Assets and the Macroeconomy Conference at LBS, and the annual meetings of the WFA and SED.

Economic performance over the past ten years is the story of a boom, a bust, and a slow recovery. Recent work (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014*b*) shows how a scarcity of capital in the financial sector amplifies the bust. At the same time, there is an understanding that the connection between finance and macro also runs through the liabilities of financial institutions: The ups and downs of the recent cycle closely track the rise and fall of the shadow banking sector, whose liabilities are an important source of liquidity in the financial system.[1] The capital scarcity view prevalent in the macro finance literature does not offer an interpretation of shadow banking that can be used to draw this connection.

We interpret shadow banking as liquidity transformation: issuing safe liquid liabilities against risky illiquid assets. We build a dynamic macro finance model that puts this process at the center. In doing so, it joins the macroeconomic cycle to the liquidity transformation cycle in the financial sector.

Here is how it works. Households demand liquidity to insure against shocks. Intermediaries supply liquidity by tranching illiquid assets, subject to a collateral constraint due to crash risk; issuing equity is costless. In quiet times, intermediaries lever up the collateral value of their assets, expanding the quantity but also the fragility of liquidity, a shadow banking boom. Over time, investment in risky capital creates an economic boom, but it also builds up economic fragility.

A rise in uncertainty causes households to demand crash-proof, fully-collateralized liquid securities. Intermediaries delever to meet this demand. Shadow banking shuts down, contracting the liquidity supply and driving up discount rates. Asset prices fall, amplified by endogenous collateral runs (rising haircuts reinforce falling prices). Investment and economic growth also fall. A flight to quality effect pushes up the prices of safe assets, causing intermediaries to shift investment to storage-like capital. This sets up a slow recovery once uncertainty recedes. Recovery is further slowed by a "collateral decelarator" in which haircuts rise as exposure to uncertainty once again picks up.

At the heart of our model lies the distinction between liquidity and wealth. Securities

[1] From Bernanke (2013): "Shadow banking... was an important source of instability during the crisis.... Shadow banking includes vehicles for credit intermediation, maturity transformation, liquidity provision, and risk sharing. Such vehicles are typically funded on a largely short-term basis from wholesale sources. In the run-up to the crisis, the shadow banking sector involved a high degree of maturity transformation and leverage. Illiquid loans to households and businesses were securitized, and the tranches of the securitizations with the highest credit ratings were funded by very short-term debt, such as asset-backed commercial paper and repurchase agreements (repos). The short-term funding was in turn provided by institutions, such as money market funds, whose investors expected payment in full on demand and had little tolerance for risk to principal.... When investors lost confidence in the quality of the assets... they ran. Their flight created serious funding pressures throughout the financial system, threatened the solvency of many firms, and inflicted serious damage on the broader economy."

are liquid only to the extent that they are backed by sufficient collateral to make their payoffs insensitive to private information. We show that intermediaries can provide liquidity efficiently by tranching the economy's capital assets. This is the source of their economic value.

By contrast, the capital scarcity view relies on the distinction between inside and outside intermediary capital, imposing a friction on the ability of outside capital to flow in. To highlight the liquidity view, we abstract entirely from this type of friction, though the two can interact in interesting ways.

The role for liquidity in our model originates with households who experience idiosyncratic liquidity events (Diamond and Dybvig, 1983): short-lived opportunities to consume a burst of wealth at high marginal utility, which necessitates a rapid sale of securities. As a result of this urgency, liquidity-event consumption must be financed by sales of liquid securities, namely those that can be traded quickly and in large quantities at low cost.[2]

We endogenize liquidity based on the notion of low information sensitivity (Gorton and Pennacchi, 1990): A security is liquid if its expected payoff does not depend too much on private information about the state of the economy. This makes it immune to adverse selection and allows it to trade without incurring price impact or other costs.

Liquidity provision is constrained by the supply of collateral as all promises must be backed by assets (e.g. Holmström and Tirole, 1998). The efficient use of collateral, which is especially scarce in crashes, leads to the rise of shadow banking. Whereas an always-liquid, money-like security requires enough collateral to remain informationally insensitive at all times, even in a crash, a near-money security that is only liquid absent a crash uses collateral mainly when it is more abundant, making it cheaper to produce. We refer to the crash-proof liquid security as money (e.g. deposits or Treasury-backed repos) and the normal-times liquid security as shadow money (e.g. asset-backed commercial paper or private-label repos). Shadow banking is the process of issuing shadow money. It is in this sense that it embodies liquidity transformation; it allows greater liquidity creation for each dollar of available collateral.

The supply of collateral sets the liquidity possibility frontier, and its intersection with household demand pins down the equilibrium liquidity mix and security spreads. Households' willingness to hold shadow money instead of money falls as the probability of a crash rises because shadow money becomes increasingly likely to cease to be liquid when

[2] Empirical evidence for the importance of liquidity demand is found in Krishnamurthy and Vissing-Jorgensen (2012*a*,*b*), Sunderam (2013), Greenwood, Hanson and Stein (2014), and Nagel (2014). These papers show that investors accept lower returns for holding securities with money-like liquidity.

a liquidity event arrives. This sets up a tradeoff between the quantity and fragility of the liquidity supply.

Crash risk drives a wedge between the current value of an asset and its collateral value. We model crashes as compensated (mean-zero) Poisson shocks with time-varying probability that hence represents a measure of uncertainty. We micro-found the dynamics of uncertainty as the outcome of a learning process: It tends to drift down in normal times, leading to long quiet periods like the "Great Moderation". It jumps following the realization of a crash and the jump is largest from relatively low levels, a dynamic in the spirit of a "Minsky moment" (1986).[3]

When uncertainty is low as in a prolonged quiet period, households are willing to hold shadow money at only a small spread over money. This allows intermediaries to expand liquidity provision, crowding out money which makes the liquidity supply more fragile. Greater liquidity promotes saving as it allows households to shift consumption to liquidity events when it is most valuable. Greater saving results in lower funding costs for intermediaries, which get passed through as lower discount rates for their assets. Lower discount rates lead to higher prices, investment, and growth. The effect is strongest for assets with low collateral values (like riskier mortgages or commercial loans), which shifts the economy's capital mix towards greater risk. Low uncertainty thus leads to shadow banking-driven booms in liquidity transformation that spur economic booms while also building up economic fragility.[4]

In this way shadow banking increases the economy's exposure to uncertainty shocks. When such a shock arrives, household flight to crash-proof liquidity causes the spread between shadow money and money to open up. The supply of liquidity contracts sharply as intermediaries strive to meet the demand for money, which is more collateral-intensive. Shadow banking shuts down. Discount rates and collateral premia rise, asset prices and investment fall, growth turns negative. Intermediaries turn to investing only in the safest storage-like assets (like government debt or prime mortgages) at the expense of riskier but productive assets. Over time, intermediaries build a "fortress balance sheet", allowing for high levels of liquidity that is crash-proof. Yet growth remains low, leading into a slow

[3]In July 2007, two Bear Stearns hedge funds failed. We think of this episode as exemplifying the uncertainty shocks in our model. The Financial Crisis Inquiry Report (2011) refers to it as "a canary in the mineshaft," borrowing the phrase from a Wall Street insider. At the June FOMC meeting, Boston Fed President Cathy Minehan also expressed a sense of uncertainty, "While the Bear Stearns hedge fund issue may well not have legs... What happens when the bottom falls out and positions thought to be at least somewhat liquid become illiquid? Is there a potential for this to spread and become a systemic problem?" (Federal Open Market Committee, 2007).

[4]Evidence consistent with a link between liquidity transformation and economic fragility is documented in Adrian, Moench and Shin (2010); Schularick and Taylor (2012); Krishnamurthy and Vissing-Jorgensen (2012*b*); Bai, Krishnamurthy and Weymuller (2013), and Baron and Xiong (2014).

recovery. Thus it is not liquidity provision per se but liquidity transformation that enables growth.[5]

Our model produces amplification via endogenous "collateral runs" or margin spirals, episodes when movements in prices and haircuts reinforce each other (Brunnermeier and Pedersen, 2009).[6] Collateral runs are a side effect of shadow banking and the fragility it generates. At times of high liquidity transformation, an uncertainty shock not only contracts liquidity provision, increasing discount rates and reducing prices, it also increases the volatility of liquidity provision going forward. The heightened exposure to future uncertainty shocks makes discount rates more sensitive to crashes. As a result, collateral values drop faster than prices (haircuts rise), further amplifying the contraction in the supply of liquidity.

Once the economy's reliance on shadow banking ends, the haircut-price dynamics turn benign with falling haircuts decelerating a further fall in prices. While this "collateral decelerator" protects the economy from further collapse, it makes the recovery more protracted in the reverse. This asymmetry between busts and recoveries can help to reconcile the speed at which financial crises unfold with the slowness of the recoveries that follow them (Reinhart and Rogoff, 2009).

Our model also generates endogenous flight to quality effects in asset markets whenever the liquidity supply contracts sharply, as it does in a crash after a shadow banking boom.[7] The resulting rise in collateral premia makes safe, collateral-rich assets appreciate even as overall prices are falling. The negative co-movement between safe and risky assets during crashes reduces the information sensitivity of diversified asset pools, allowing intermediaries to expand the liquidity supply. This complementarity has important implications for the efficacy of a number of policy interventions.

The unprecedented actions of central banks and governments in the wake of the 2008 financial crisis have inspired a debate on unconventional monetary policy and financial stability. We analyze a subset of interventions from the perspective of the liquidity transformation role of the financial sector.

Under our framework, large-scale asset purchases (LSAP) support asset prices by relieving collateral shortages during a crisis. A mistimed intervention or talk of an early exit

[5]The tight connection between liquidity transformation and the macroeconomic cycle that our model produces is consistent with the results of Bai, Krishnamurthy and Weymuller (2013), who show that liquidity transformation in the U.S. expanded up to 2007, contracted sharply in 2008 and 2009, and has remained persistently low ever since.

[6]See Gorton and Metrick (2012) and Krishnamurthy, Nagel and Orlov (2014) for evidence that over-collateralization (haircuts) in parts of the repo market increased sharply during the financial crisis.

[7]See Krishnamurthy (2010) and McCauley and McGuire (2009) for evidence of flight to quality in U.S. Treasury and foreign exchange markets.

(i.e. "tapering"), can undo the effectiveness of LSAP through an expectations channel. Other policies like "Operation Twist" end up tightening collateral constraints by syphoning off long-duration safe assets that are complementary to risky assets in the provision of liquidity.[8]

Regulatory reform in the spirit of the "Volcker rule" or the Glass-Steagall Act, which we interpret as the mandatory segregation of safe and risky assets, can also reduce liquidity provision by preventing intermediaries from pooling these assets to take advantage of their complementarity. Stricter liquidity requirements like those proposed by the Basel III Committee raise funding costs, but are effective at reducing financial and macroeconomic volatility.

The rest of the paper is organized as follows: Section 1 reviews the literature, Section 2 sets up the model, Section 3 provides qualitative analysis, Section 4 presents results from a numerical implementation, Section 5 analyzes policy, and Section 6 concludes.

1 Related literature

Our paper draws on insights from the banking literature and applies them to macro finance. As in Diamond and Dybvig (1983), financial intermediaries add value by issuing securities that are more liquid than their assets. Following Gorton and Pennacchi (1990), we model the liquidity of a security as a function of its information sensitivity.

The macro finance literature emphasizes the scarcity of entrepreneurial or intermediary capital in amplifying and propagating fundamental shocks (for a recent survey, see Brunnermeier, Eisenbach and Sannikov, 2013). In Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999), funding costs are decreasing in entrepreneurial net worth. In Kiyotaki and Moore (1997), external funding is constrained by the collateral value of assets. Geanakoplos (2003) derives this type of constraint under differences of opinion, whereas Gorton and Ordoñez (2014) and Dang, Gorton and Holmström (2012) also base it on the notion of information sensitivity.[9] The financial crisis has shifted the focus to financial intermediaries, see He and Krishnamurthy (2012, 2013); Gertler and Kiyotaki (2010, 2013); Gârleanu and Pedersen (2011); Brunnermeier and Sannikov (2014*b*); Adrian and Boyarchenko (2012); Rampini and Viswanathan (2012); Di Tella (2012); Sannikov (2013) and Maggiori (2013). In these papers effective limits on the issuance of debt

[8]These predictions are consistent with the findings in Krishnamurthy and Vissing-Jorgensen (2013) that central bank purchases of risky assets are more effective than purchases of long-dated government debt.

[9]In a related strand of the literature, Kurlat (2013) and Bigio (2013) consider the macroeconomic effects of time-varying adverse selection more broadly.

or equity make intermediary net worth the key state variable. In our case, collateral constrains specifically liquidity provision because it is needed for maintaining low information sensitivity. Net worth drops out because equity issuance is costless.

In our model intermediary leverage is pro-cyclical. The empirical evidence on leverage varies across types of institutions. He, Khang and Krishnamurthy (2010); Adrian and Shin (2010); Ang, Gorovyy and van Inwegen (2011) and Adrian, Etula and Muir (2014) document counter-cyclical leverage for commercial banks and pro-cyclical leverage for hedge funds and broker-dealers. In our model equity displaces shadow banking-type funding, so we view its leverage dynamics as applying most closely to repo-dependent institutions such as hedge funds and broker-dealers.

Demand for safety or liquidity on the part of households or firms also plays a role in Bansal, Coleman and Lundblad (2010); Greenwood, Hanson and Stein (2014); Krishnamurthy and Vissing-Jorgensen (2012*b*); Kiyotaki and Moore (2012); Hanson et al. (2014) and Caballero and Farhi (2013). Our contribution to this literature is to develop a fully dynamic macroeconomic model, as well as to highlight the role of near-safe securities for efficient liquidity provision.

Shadow banking and securitization more broadly have also been viewed through the lenses of behavioral bias (Gennaioli, Shleifer and Vishny, 2012, 2013) and regulatory arbitrage (Acharya, Schnabl and Suarez, 2013; Harris, Opp and Opp, 2014). These perspectives emphasize excessive risk taking that arises in shadow banking. We share with Gennaioli, Shleifer and Vishny (2013) the emphasis on how beliefs about tail events drive shadow banking activity.

Finally, our analysis of the interaction between financial intermediation and monetary policy complements work by Adrian and Shin (2009); Gertler and Karadi (2011); Ashcraft, Gârleanu and Pedersen (2011); Kiyotaki and Moore (2012); Brunnermeier and Sannikov (2014*a*), and Drechsler, Savov and Schnabl (2014). Our contribution is to take the perspective of the liquidity transformation channel, showing the key role of complementarities between assets on bank balance sheets for the transmission of policy interventions.

2 Model

In this section we lay out the setup of our model. The economy evolves in continuous time $t \geq 0$. It is populated by a unit mass of households.

2.1 Technology

The available production technologies embed a tradeoff between productivity and risk: Technology A is high-productivity but risky, technology B is low-productivity but safe. We think of A as representing new, high-potential but untested investment projects, versus the storage-like safe but less productive technology B. As examples, one can think of commercial real estate loans versus prime mortgages.[10]

Let k_t^a and k_t^b denote the efficiency units of capital devoted to each technology. The rate of output is $y_t = \gamma^a k_t^a + \gamma^b k_t^b$, where $\gamma^a > \gamma^b$ reflects the productivity advantage of A capital. The total stock of each type of capital $i = a, b$ follows

$$\frac{dk_t^i}{k_t^i} = \mu \left(k_t^a + k_t^b\right) dt + \left[\phi\left(\iota_t^i\right) - \delta\right] dt - \kappa^i dN_t \quad (1)$$

$$dN_t = dJ_t - \lambda_t dt, \quad (2)$$

where μ is a constant level inflow, ι_t^i is the investment rate, $\phi(\cdot)$ is a concave investment adjustment cost function, δ is depreciation, and $\kappa^i > 0$ is risk exposure with $\kappa^a > \kappa^b$. The level inflow term serves a technical purpose.[11] To keep things simple, we assume it accrues to the total capital stock but not inside the portfolios of investors. For example, it could stand in for technological progress embodied into new vintages of capital as in Gârleanu, Panageas and Yu (2012).

The process dN_t represents productivity or simply cash-flow shocks. It consists of the crash component dJ_t, a Poisson jump process with unit size and time-varying intensity λ_t, and a compensating term that makes the shocks mean zero. The compensating term ensures that innovations to λ_t represent pure uncertainty shocks (see Brunnermeier and Sannikov (2014*b*) for a similar formulation). As we will see, households are risk neutral, so uncertainty only matters due to frictions.

We require jumps, which we call crashes, in order to create a wedge between an asset's current value and its collateral value.

[10] To motivate these examples, note that commercial real estate is strongly pro-cyclical; commercial real estate loans enjoy no government guarantees; and they offer a higher return on investment. From an ex ante perspective and for the same reasons, subprime loans can also be interpreted as type-A capital, though of course ex post their "productivity" has been called into question.

[11] This term makes the boundaries of our capital mix state variable reflecting rather than absorbing. The effect is a more gradual change in prices along the capital mix dimension. We keep μ small or zero in calibrations.

2.2 Uncertainty

We model uncertainty as the outcome of a filtering problem. This allows us to incorporate some useful features described below. It also allows us to define liquidity based on the notion of sensitivity to private information.

A latent crash intensity $\tilde{\lambda}_t$ follows a two-state continuous time Markov chain, $\tilde{\lambda}_t \in \{\lambda^L, \lambda^H\}$, with transition intensities q^L and q^H from the low and high states, respectively. Agents learn about $\tilde{\lambda}_t$ from crash realizations, and from an exogenous news signal with precision $1/\sigma$.[12] As an example of such news, one can think of the failure of the two Bear Stearns hedge funds in June 2007.

In Appendix A, we show how to compute the innovations to $\lambda_t = E_t\left[\tilde{\lambda}\right]$, which gives the filtered dynamics

$$\frac{d\lambda_t}{(\lambda^H - \lambda_t)(\lambda_t - \lambda^L)} = \left(-\frac{q^H}{\lambda^H - \lambda_t} + \frac{q^L}{\lambda_t - \lambda^L}\right) dt + \frac{1}{\sigma} dB_t + \frac{1}{\lambda_t} dN_t, \tag{3}$$

where dB_t is a standard Brownian motion that conveys the news signal.

The filtered crash intensity has three useful features. First, it drifts down absent a crash, so crashes are perceived to be less likely following a long quiet period like the Great Moderation. Second, crash realizations cause λ_t to jump up (Reinhart and Reinhart, 2010, find that half of all economies that suffer a financial crisis experience aftershocks.) Third, it jumps most from relatively low levels, a type of "Minsky moment" (1986).[13]

2.3 Households

Households have risk-neutral preferences and are subject to liquidity events in the spirit of Diamond and Dybvig (1983). Household h maximizes

$$V_t^h = \max E_t\left[\int_t^\infty e^{-\rho(s-t)} W_s \left(c_s ds + \psi C_s dN_s^h\right)\right], \tag{4}$$

where c_t is the consumption-wealth ratio outside a liquidity event. A realization $dN_t^h = 1$ signifies a liquidity event, defined as an opportunity to consume a "burst" of wealth C_t at a high marginal value $\psi > 1$.

[12] Note that the evolution dynamics (1) are specified in terms of the filtered compensated crash process dN_t. This is not essential. It means that households cannot learn from the compensating drift of productivity outside of crashes. An alternative is to write (1) in terms of the latent uncompensated crash process but then λ would affect expected productivity growth. We opt for the specification in (1) in order to have a natural frictionless benchmark where λ has no effect. Appendix A has more details on the filtering.

[13] The jump is largest near $\sqrt{\lambda^L \lambda^H}$, which is below the midpoint $\frac{1}{2}\left(\lambda^L + \lambda^H\right)$.

Liquidity events are not verifiable. This creates the potential for adverse selection in financial markets that limits the set of securities that can be used to finance liquidity-event consumption (see Section 2.4 below).

We specify the liquidity event process in a way that makes it possible for a household to experience a liquidity event during a crash. This makes demand for crash-proof liquidity increase with the likelihood of a crash as one would naturally expect. The interpretation is that crashes last more than an instant though we model them as instantaneous for tractability.

To this end, dN_t^h has two components: one for normal times and one for crashes. The normal-times component is dJ_t^h, a Poisson jump process that is independent across households, and the systematic component is the uncompensated crash process dJ_t. We require two moment conditions of dN_t^h:[14]

$$E_t\left[dN_t^h dJ_t^h\right] = he^{-\tau\lambda_t}dt \tag{5}$$

$$E_t\left[dN_t^h dJ_t\right] = h\left(1-e^{-\tau\lambda_t}\right)dt. \tag{6}$$

Equations (5)–(6) imply that $E_t\left[dN_t^h\right] = hdt$, so the intensity of liquidity events is always constant at h, but it splits up between normal times and crashes depending on λ_t. When λ_t is high, a liquidity event is more likely to coincide with a crash. Thus, it is not total liquidity demand that increases with uncertainty, but only crash-proof liquidity demand.

The parameter τ can be interpreted as the duration of a crash. When $\tau = 0$, all liquidity events occur outside of crashes, whereas when $\tau \to \infty$ they all occur during crashes.[15]

Liquidity-event consumption is bounded by $\overline{C}_t$. To generate a concave demand for liquidity, we assume $\overline{C}_t$ is i.i.d. exponential with mean $1/\eta$. For example, a household may have to make a large payment of an uncertain amount.

Importantly, liquidity-event consumption is further constrained by the household's liquid holdings l_t, which we characterize below.[16] In sum,

$$C_t \leq \min\left\{\overline{C}_t, l_t\right\}. \tag{7}$$

[14] A formulation satisfying (5)–(6) is $dN_t^h = e^{-\tau\lambda_t}dJ_t^h + \frac{h}{\lambda_t}\left(1-e^{-\tau\lambda_t}\right)dJ_t$ with the intensity of dJ_t^h set to h.

[15] More precisely, in a model with constant-intensity crashes lasting τ periods, the probability of a liquidity event coinciding with a crash is exactly $h\left(1-e^{-\tau\lambda_t}\right)dt$.

[16] We are ruling out liquidity insurance contracts, i.e. credit lines or collateral rehypothecation. Our collateral constraint limits both of these so they would not change the model significantly. The intuition is similar to Holmström and Tirole (1998). Credit lines must be secured by a household's security holdings, which can only back a limited amount of borrowing for the credit line to be informationally insensitive and hence feasible. Rehypothecation similarly amounts to a more efficient use of collateral that nevertheless remains scarce.

In a liquidity event, households must quickly sell off a substantial part of their savings. For some securities, the cost of doing so is prohibitive due to price impact or search, rendering them illiquid and effectively constraining liquidity-event consumption.

2.4 Liquidity

Following Gorton and Pennacchi (1990), we define a security as liquid if its value is sufficiently insensitive to private information about the state of the economy, in particular about $\tilde{\lambda}_t$ since it is the only source of imperfect information in the model.

Definition 1. *A security S with return process $dr_{S,t}$*

i. is liquid if

$$\frac{1}{dt}\left|\frac{E_t\left[dr_{S,t}\middle|\tilde{\lambda}_t\right]-E_t\left[dr_{S,t}\middle|\lambda_t\right]}{\tilde{\lambda}_t-\lambda_t}\right| \leq \overline{\kappa}, \tag{8}$$

where $\overline{\kappa}$ is a fixed constant; and

ii. it is crash-proof liquid if it is liquid and if $dr_{S,t}dN_t \geq 0$.

The first part of Definition 1 says that ability to profit from predicting a security's return per unit of private information, i.e. its sensitivity to private information, must not exceed a fixed bound. We provide a microfoundation for this condition in Appendix B. Low information sensitivity ensures that information acquisition is unprofitable, removing the prospect of adverse selection and allowing large trades with a wide variety of counterparties without price impact.

The second part of Definition 1 says that a security ceases to be liquid if it suffers a capital loss (i.e. default).[17] This condition extends the logic of information sensitivity to the immediate aftermath of a crash. Upon default, undercollateralized securities become junior in the capital structure, which exposes them to adverse selection. The ability to liquidate collateral might be delayed due to a "failure to deliver" or extended legal proceedings (see He and Milbradt, 2013). Lehman Brothers commercial paper provides a useful example.

[17] This drives a wedge between the amount of collateral needed to back a dollar of liquidity and the amount of liquidity households derive from seizing that collateral in a crash. If instead the seized collateral remained liquid, money is not issued in equilibrium.

2.5 Markets

We distinguish between the markets for assets, which are claims to the economy's productive capital, and securities, which are issued by intermediaries against those assets.

2.5.1 Asset markets

There is a liquid secondary market for existing capital. Households and intermediaries can both purchase capital, though households will not do so in equilibrium. Intermediaries can profit from liquidity creation by tranching capital, so their valuation cannot be below that of the households. We rule out short-selling of capital by households, which prevents synthetic creation of collateral.[18]

Each type of capital has price π_t^i for $i = a, b$ with corresponding dynamics

$$\frac{d\pi_t^i}{\pi_t^i} = \mu_{\pi,t}^i dt - \sigma_{\pi,t}^i dB_t - \kappa_{\pi,t}^i dN_t. \tag{9}$$

We solve for asset prices and their dynamics in equilibrium, and they determine investment and growth.

2.5.2 Securities markets

There are two aggregate shocks in this economy (dB_t and dN_t) and we introduce three securities that span them. We call these money, shadow money, and equity in order to convey their risk and liquidity profiles. We will show in Section 3.2 that the optimal provision of liquidity generally requires all three securities, and that the three are indeed sufficient.

We write the return processes for money, shadow money, and equity respectively as

$$dr_{m,t} = \mu_{m,t} dt \tag{10}$$

$$dr_{s,t} = \mu_{s,t} dt - \kappa_{s,t} dN_t \tag{11}$$

$$dr_{e,t} = \mu_{e,t} dt - \sigma_{e,t} dB_t - \kappa_{e,t} dN_t. \tag{12}$$

Whereas the drifts are determined by supply and demand in securities markets, the loadings are determined by the capital structure decisions of financial intermediaries. Money is fully safe, which makes it crash-proof liquid. Shadow money is safe except in a crash.

[18] In order to short-sell capital, households would have to pledge securities as collateral (as intermediaries do). The pledged securities would be unavailable for liquidity-event consumption, so households would have no more incentive to short-sell capital than intermediaries.

For it to be liquid, according to Definition 1 it must be the case that

$$\kappa_{s,t} \quad \leq \quad \overline{\kappa}. \tag{13}$$

To be liquid, shadow money must not suffer too large a loss in a crash. Otherwise, its final payoff would be too sensitive to private information about $\tilde{\lambda}_t$, exposing it to adverse selection. We will show that intermediaries optimally choose to respect this cap, and that it binds. That is they pledge $1-\overline{\kappa}$ worth of collateral per dollar of shadow money issued. Equity absorbs all normal-times risk and all remaining crash risk, making it illiquid.

2.6 Intermediaries

Intermediaries buy capital, set investment, and issue securities to maximize discounted profits. Figure 1 represents the intermediary balance sheet in relation to the rest of the economy.

Although we combine the investment and liquidity provision functions under one roof, these can easily be separated by adding entrepreneurs. We think of the intermediaries as financial institutions since they add value by tranching assets.

The environment is perfectly competitive so each intermediary takes prices as given. Moreover, in contrast to the literature, intermediaries are free to issue and repurchase equity at no cost, so current net worth is not a state variable. This allows us to solve for the intermediary's optimal capital structure in closed form given prices and their dynamics.

Intermediaries face a collateral constraint that restricts their ability to provide liquidity. Specifically, all claims must be secured by their assets.[19] We further restrict attention to securities whose payoff is nondecreasing in asset value, a natural and commonplace restriction.[20] In Section 3.2 we show that under these conditions crash-proof liquid money, liquid shadow money, and illiquid equity as in (10)–(12) implement the solution to the intermediaries' optimization problem.

Each intermediary maximizes the value of its existing equity V_t, which is discounted at the equilibrium equity rate of return:

$$\mu_{e,t} V_t dt \quad = \quad \max_{k_t^a, k_t^b, \iota_t^a, \iota_t^b, m_t, s_t} \left[\left(\gamma^a - \iota_t^a\right) k_t^a + \left(\gamma^b - \iota_t^b\right) k_t^b \right] dt + E_t \left[dV_t\right], \tag{14}$$

[19] A collateral constraint can be microfounded with lack of commitment. Payments not explicitly backed by assets are not optimal ex post as there are no reputation or distress costs of reneging on a promise.

[20] See Innes (1990), Nachman and Noe (1994), DeMarzo and Duffie (1999), and DeMarzo (2005) for uses and justifications for such a restriction.

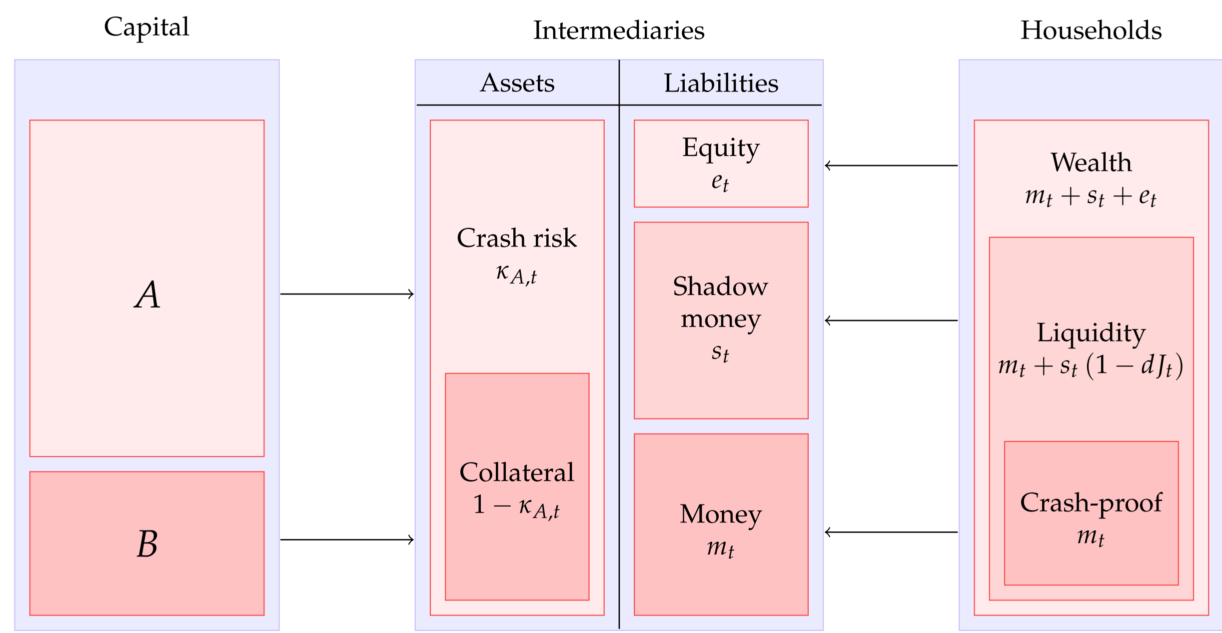


Figure 1: Balance sheet view. This figure illustrates the flow of funds in the economy. The left side represents that economy's capital stock. Intermediaries hold this capital as their assets, and use it to back securities as their liabilities. Households hold these securities as wealth, a subset of which are liquid and crash-proof liquid.

where k_t^a and k_t^b are units of capital, ι_t^a and ι_t^b are investment rates, and m_t and s_t are the shares of money and shadow money on the liabilities side of the intermediary balance sheet. Neither can be negative, $m_t, s_t \geq 0$, nor can equity $e_t = 1 - m_t - s_t$, so $m_t + s_t \leq 1$.

Let $A_t = \pi_t^a k_t^a + \pi_t^b k_t^b$ be the value of the intermediary's assets, which pins down the size of its balance sheet. Any discrepancy between A_t and the combined value of existing equity, shadow money, and money must be made up by issuing new equity. Existing equity follows

$$dV_t \quad = \quad dA_t - A_t\left(m_t dr_{m,t} + s_t dr_{s,t}\right) - \left[A_t - A_t\left(m_t + s_t\right) - V_t\right] dr_{e,t}. \tag{15}$$

The change in existing equity is equal to the change in assets net of payouts to money, shadow money, and newly-issued equity (in brackets). There is no wedge between the value of equity and the value of assets net of payouts because equity issuance is costless, as is the buying and selling of assets.

Under the collateral constraint, the intermediary must have enough assets left following a crash to cover its promises. Let $\kappa_{A,t}$ be the fraction of assets lost in a crash so that

$1-\kappa_{A,t}$ is the available asset collateral. We then have

$$m_t + s_t\left(1-\kappa_{s,t}\right) \quad \leq \quad 1-\kappa_{A,t}. \tag{16}$$

Each dollar of collateral can back one dollar of money and $1/\left(1-\kappa_{s,t}\right)$ dollars of shadow money. This makes it cheaper for the intermediary to deliver normal-times liquidity even as households prefer crash-proof liquidity, a key tradeoff.

3 Analysis

In this section we present analytical results that characterize households' demand for liquidity and intermediaries' supply of liquidity given asset prices and their dynamics. Results for the full equilibrium are in Section 4 below.

3.1 The demand for liquidity

Households use liquid securities to self-insure against liquidity events. Abundant liquidity encourages saving and drives down discount rates. As crashes become more likely, demand for money, which is crash-proof liquid rises and demand for shadow money falls.

The household problem (4) can be written as the Hamilton-Jacobi-Bellman equation

$$\rho V_t^h \quad = \quad \max_{c_t, C_t, m_t, s_t} W_t\left(c_t dt + \psi E_t\left[C_t dN_t^h\right]\right) + E_t\left[dV_t^h\right], \tag{17}$$

where m_t and s_t are portfolio holdings in money and shadow money. The budget dynamics and liquidity constraints are

$$\frac{dW_t}{W_t} \quad = \quad -c_t dt - C_t dN_t^h + dr_{e,t} + m_t\left(dr_{m,t} - dr_{e,t}\right) + s_t\left(dr_{s,t} - dr_{e,t}\right) \tag{18}$$

$$C_t \quad \leq \quad \min\left\{\overline{C}_t, l_t\right\} \tag{19}$$

$$l_t \quad = \quad m_t + s_t\left(1 - dJ_t\right). \tag{20}$$

Constraint (19) says that liquidity-event consumption must be financed by liquid balances. It always binds since $\psi > 1$. Equation (20) defines those liquid balances, capturing the fact that shadow money ceases to be liquid in a crash.

Risk neutrality gives $V_t^h = W_t$. Outside a liquidity event, consumption is elastic.[21]

[21] This is a standard feature of risk-neutral preferences and it implies that consumption can be negative. Note that even though liquidity-event consumption is lumpy, non-liquidity-event consumption is of order

The portfolio optimality conditions are

$$\mu_{e,t} - \mu_{m,t} = h(\psi - 1)\left[e^{-\tau\lambda_t} e^{-\eta(m_t+s_t)} + \left(1 - e^{-\tau\lambda_t}\right) e^{-\eta m_t}\right] \tag{21}$$

$$\mu_{s,t} - \mu_{m,t} = h(\psi - 1)\left(1 - e^{-\tau\lambda_t}\right) e^{-\eta m_t}. \tag{22}$$

The calculations are in Appendix C. Equity pays a premium over shadow money, and shadow money a premium over money. The total equity premium equals the marginal value of liquidity in all states. The premium of shadow-money over money, which we call the safety premium, equals the marginal value of liquidity in a crash. It is increasing in the level of uncertainty λ_t.

Evaluating the HJB equation at the optimum gives the total return on savings, which equals the aggregate cost of capital in equilibrium,

$$\rho - \frac{h}{\eta}(\psi - 1) + \frac{h}{\eta}(\psi - 1)\left[e^{-\tau\lambda_t} e^{-\eta(m_t+s_t)} + \left(1 - e^{-\tau\lambda_t}\right) e^{-\eta m_t}\right]. \tag{23}$$

The cost of capital is decreasing in the supply of liquidity. Greater liquidity promotes saving by raising the opportunity for consumption during a liquidity event when it is most valuable.

3.2 The supply of liquidity

The intermediary problem consists of the HJB equation (14) subject to the equity dynamics (15), the collateral constraint (16), and the non-negativity constraints $m_t, s_t, e_t \geq 0$. In this section we focus on the liabilities side of this problem.

Although there are no differences in expertise or preferences between households and intermediaries, holding capital without intermediation is not efficient in this economy. In all regions of the state-space, at least two of the three available securities are required for optimal liquidity provision, as we show below. Tranching is always efficient, and this is how intermediaries add value.

In Appendix C, we state the intermediary's problem more generally as a security design problem subject to the information sensitivity constraints imposed by Definition 1. This allows us to show rather than assume that crash-proof liquid money, liquid shadow money, and illiquid equity as in (10)–(12) are optimal. We state the end result here as follows:

dt since non-liquidity-event marginal utility is constant at one.

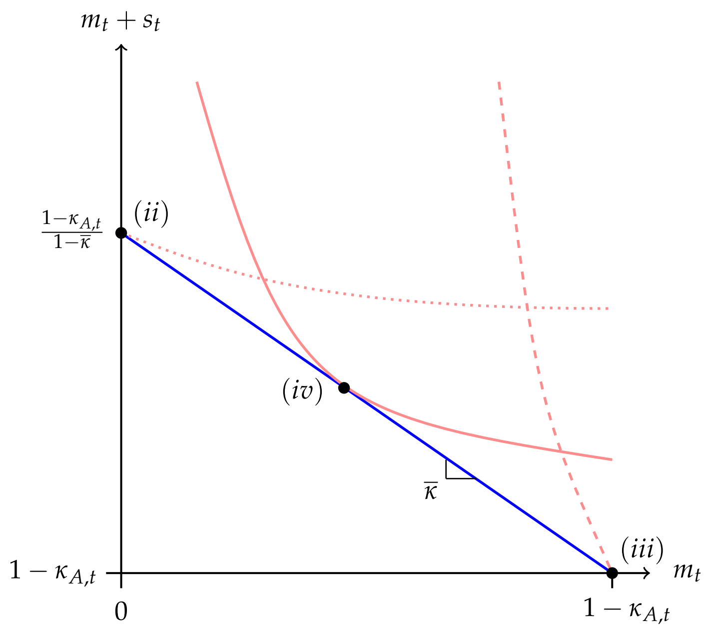


Figure 2: Optimal capital structure. This figure illustrates cases (ii)–(iv) in Proposition 1. The horizontal axis plots crash-proof liquidity m_t and the vertical axis plots total liquidity $m_t + s_t$. Note the horizontal axis crosses at $1 - \kappa_{A,t}$. The blue line represents the collateral constraint of the intermediary. The red lines represent household indifference curves.

Proposition 1. *Let $\kappa_{A,t} > 0$ and $\sigma_{A,t} > 0$ be the crash risk and normal-times risk exposures of an intermediary's balance sheet. Then the intermediary's optimal capital structure policy can be implemented with crash-proof liquid money m_t, liquid shadow money s_t (with $\kappa_{s,t} = \overline{\kappa}$), and illiquid equity $e_t = 1 - m_t - s_t$ (with $\kappa_{e,t} = 1$) as follows:*

i. $m_t = 1 - \frac{\kappa_{A,t}}{\overline{\kappa}}$ *and* $s_t = \frac{\kappa_{A,t}}{\overline{\kappa}}$ *if* $\kappa_{A,t} \leq \overline{\kappa}$ *and* $\kappa_{A,t} < \frac{\overline{\kappa}}{\eta} \log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}} \frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$;

ii. $m_t = 0$ *and* $s_t = \frac{1-\kappa_{A,t}}{1-\overline{\kappa}}$ *if* $\kappa_{A,t} > \overline{\kappa}$ *and* $\kappa_{A,t} > 1 - \frac{1-\overline{\kappa}}{\eta} \log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}} \frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$;

iii. $m_t = 1 - \kappa_{A,t}$ *and* $s_t = 0$ *if* $\lambda_t > -\frac{1}{\tau} \log(1 - \overline{\kappa})$ *and* $\kappa_{A,t} > \underline{e}$*; and*

iv. $m_t = 1 - \kappa_{A,t} - \frac{1-\overline{\kappa}}{\eta} \log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}} \frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *and* $s_t = \frac{1}{\eta} \log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}} \frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *otherwise.*

Proof. The proof is contained in Appendix C. □

Case (i) of Proposition 1 corresponds to very low asset risk $\kappa_{A,t}$ and uncertainty λ_t, allowing intermediaries to reduce equity to a minimum. Figure 2 illustrates the intuition of cases (ii)–(iv). It plots the supply of total liquidity in the form of money and shadow money issuance $m_t + s_t$ against crash-proof liquidity m_t. The blue line represents the intermediary's liquidity provision budget, which is pinned down by available collateral

$1-\kappa_{A,t}$. Changes in collateral thus shift the feasibility frontier, expanding and contracting the total supply of liquidity.

The red lines in Figure 2 depict household demand, which shifts the composition of liquidity. The indifference curves are defined over crash-proof liquidity and total liquidity, as implied by the first-order conditions (21) and (22). Their slope depends on uncertainty λ_t which determines household willingness to substitute between normal-times and crash-proof liquidity.

When λ_t is sufficiently low, we get case (*ii*) in which shadow money completely crowds out money and no crash-proof liquidity is produced. As λ_t rises, the marginal rate of substitution between normal-times and crash-proof liquidity falls so that in case (*iv*) it is equated to the marginal rate of transformation coming from the intermediary collateral constraint. Then in case (*iii*) high uncertainty pushes intermediaries to a corner solution in which shadow banking shuts down.

Proposition 1 shows that the optimal provision of liquidity generally requires shadow banking. The sole exception is case (*iii*). The rest of the time, shadow banking allows the financial sector to increase the supply of liquidity by levering up limited collateral. At the same time, the liquidity it provides is fragile: it disappears when uncertainty rises.

We also see that equity takes all normal-times risk $\sigma_{A,t}$. This is optimal because if a liquid security had a positive normal-times risk exposure, it would require additional collateral in order to remain informationally insensitive and hence liquid ex ante. Residual risk makes equity an inefficient security for providing liquidity.

As Figure 2 shows, an intermediary's ability to issue liquid securities is constrained by the collateral value of its assets (see (16)). Applying Ito's Lemma to assets A_t and taking the crash-risk component, this collateral value is

$$1-\kappa_{A,t} = \frac{\pi_t^a k_t^a}{\pi_t^a k_t^a + \pi_t^b k_t^b}\left(1-\kappa^a\right)\left(1-\kappa_{\pi,t}^a\right) + \frac{\pi_t^b k_t^b}{\pi_t^a k_t^a + \pi_t^b k_t^b}\left(1-\kappa^b\right)\left(1-\kappa_{\pi,t}^b\right). \quad (24)$$

The collateral value of assets is a value-weighted average of the collateral value of each type of asset on the intermediary's balance sheet. In turn, the collateral value of each type of asset depends on the exposure of its cash flows (κ^a and κ^b) and its price ($\kappa_{\pi,t}^a$ and $\kappa_{\pi,t}^b$) to crash risk. The model thus features both exogenous and endogenous risk. Even a cash-flow safe asset (e.g. $\kappa^b = 0$) cannot in general back money directly, as fluctuations in liquidity premia inject risk into its price ($\sigma_{\pi,t}^b, \kappa_{\pi,t}^b > 0$). The liquidity services that an asset can support thus become an important component of its value.

We turn to the intermediaries' asset choice in the following section.

3.3 Asset prices and investment

On the asset side of their balance sheet, intermediaries purchase capital and set investment. The optimality conditions of the intermediary problem with respect to capital pin down asset prices:

$$\pi_t^i = \frac{\gamma^i - \iota_t^i}{\left[\mu_{e,t} - \theta_{1,t}\left(1-\kappa^i\right)\left(1-\kappa_{\pi,t}^i\right) - \theta_{2,t}\right] - \left[\mu_{\pi,t}^i + \kappa^i \kappa_{\pi,t}^i \lambda_t + \phi\left(\iota_t^i\right) - \delta\right]} \tag{25}$$

for $i = a, b$, where $\theta_{1,t}$ and $\theta_{2,t}$ are the Lagrange multipliers on the collateral and equity non-negativity constraints. The derivation is in Appendix C.

Prices have the familiar form of current net cash flows over a discount rate minus a growth rate. The net cash flow, output minus investment, tends to be higher for A capital since $\gamma^a > \gamma^b$. The growth rate (second bracketed term in the denominator) consists of price growth, physical growth, and depreciation.

The first bracketed term in the denominator of (25) is the discount rate. It varies across the two assets as a result of their differential ability to back liquidity provision. Both assets are funded at a discount from the cost of equity, and this discount depends on their collateral values. Since A capital is riskier, $\kappa^a > \kappa^b$, and since collateral is scarce, $\theta_{1,t} > 0$, A's discount rate tends to be higher than B's. Although it has higher cash flows, A can have a lower price than B if liquidity is sufficiently scarce. As liquidity becomes more abundant and the overall cost of capital falls, the wedge in discount rates between the two types of capital shrinks.

The optimal investment policy follows standard q-theory:

$$1 = \pi_t^i \phi'\left(\iota_t^i\right) \tag{26}$$

for $i = a, b$. Since ϕ is concave, investment is increasing in asset prices. As liquidity affects discount rates and prices, it also affects investment, both overall and across assets.

Taking prices as given and setting investment accordingly, intermediaries shape the evolution of the economy's capital mix. This capital mix is slow-moving due to technological illiquidity (convex adjustment costs). It can be summarized by the A-capital share $\chi_t = k_t^a / \left(k_t^a + k_t^b\right)$, which becomes a second state variable (after λ_t). Applying Ito's

Lemma and substituting (1), the dynamics of χ_t are

$$\begin{aligned} d\chi_t &= \mu\left(1-2\chi_t\right)dt+\chi_t\left(1-\chi_t\right)\left[\phi\left(\iota_t^a\right)-\phi\left(\iota_t^b\right)+\lambda_t\left(\kappa^a-\kappa^b\right)\right]dt \\ &\quad -\chi_t\left(1-\chi_t\right)\left[\frac{\kappa^a-\kappa^b}{\chi_t\left(1-\kappa^a\right)+\left(1-\chi_t\right)\left(1-\kappa^b\right)}\right]dJ_t. \end{aligned} \tag{27}$$

In a crash, χ_t falls as more of the risky capital is wiped out. Crashes thus shift the economy's capital stock towards safety, which produces a dampening effect; uncertainty shocks absent a crash have a stronger impact on asset prices.[22]

Absent a crash, the risky capital share drifts according to relative investment in the two technologies, $\phi\left(\iota_t^a\right)-\phi\left(\iota_t^b\right)$ (the remaining terms of (27) consists of a level-inflow and crash-compensating terms). This means that in low-uncertainty states when liquidity is abundant so discount rates and collateral premia are low, the economy tends to drift towards a riskier capital mix. Conversely, the same force pushes towards retrenchment at the expense of future growth when liquidity provision contracts following a crash.

The model thus features endogenous buildups of economic fragility during booms and slow recoveries following crashes. Both are dynamic effects resulting from variation in the level and cross-sectional dispersion of discount rates induced by the expansion and contraction of liquidity provision through shadow banking.

4 Results

In this section, we present results for the full dynamic equilibrium of the model, focusing on the interaction between the macroeconomy and financial markets. We solve for prices $\pi^a\left(\lambda_t,\chi_t\right)$ and $\pi^b\left(\lambda_t,\chi_t\right)$ using projection methods, specifically Chebyshev collocation. Appendix D provides details.

We follow Brunnermeier and Sannikov (2014*b*) and pick an investment cost function that implies quadratic adjustment costs, $\phi\left(\iota_t\right)=\frac{1}{\varphi}\left(\sqrt{1+2\varphi\iota_t}-1\right)$. Our benchmark parameter values are available in Table 1. We view these as an illustration rather than a calibration.

Table 1 about here.

[22] There are several ways to remove the dampening effect: (1) introduce a capital composition shock that converts some fraction of the safe capital into risky capital when a crash hits; and (2) a crash need not destroy any capital in the aggregate, it is enough that it destroy some units of capital while benefiting others (a dispersion shock) as long as investors cannot hold fully diversified portfolios. We pursue the latter approach in Sections 4.4 (Flight to quality) and 5.2 (Operation Twist).

4.1 Macroeconomic effects

Our model links liquidity provision in financial markets to macroeconomic performance via discount rates and asset prices. Figure 3 illustrates these links under our benchmark parameters. In each plot, quantities are plotted against uncertainty λ_t at three different levels of the risky capital share χ_t.

The top two panels show the prices of the high-productivity low-collateral A capital and the low-productivity high-collateral B capital. The price of A is generally higher, reflecting its productivity advantage. However, it declines steeply with λ_t and gradually with χ_t. By contrast, the price of B is increasing in χ_t and flat in λ_t, except when χ_t is high where it is at first increasing and then decreasing in λ_t. The price of B can even exceed the price of A when uncertainty λ_t and asset risk χ_t are both high.

Figure 3 about here.

The middle panels of Figure 3 plot the supply of money and shadow money, which help to understand these price effects. When uncertainty is low, households are willing to hold shadow money to meet their liquidity needs. Intermediaries are similarly eager to supply shadow money as it allows them to lever up the collateral value of their assets, creating more liquidity and lowering their funding costs. As a result, shadow money displaces money at low levels of uncertainty and especially when the supply of collateral is low (χ_t is high). The liquidity transformation enabled by shadow banking lowers the funding cost of the productive asset, boosting its price.

The bottom left panel of Figure 3 plots the overall supply of liquidity measured as the flow rate of liquidity-event consumption. Shadow banking allows for a high level of liquidity even when the capital mix is risky as long as uncertainty is low. It is in this region (low λ_t, high χ_t that the financial sector is engaging in liquidity transformation and not just liquidity provision.

Output growth in the bottom-right panel of Figure 3 is also highest when uncertainty is low and the risky capital share is high. By increasing the supply of liquidity for a given amount of collateral, shadow banking lowers discount rates and pushes up prices, investment, and growth. Note that a highly liquid economy need not coincide with a fast-growing one: the supply of liquidity can be very high when asset risk χ_t is low but the low level of the productive A capital actually causes the economy to shrink. In other words, it is shadow banking and liquidity transformation that enables economic booms.

A rise in uncertainty sets off a contraction in liquidity and ultimately a recession. Household demand shifts abruptly from shadow money to money as only money provides crash-proof liquidity. Intermediaries cater to this demand by adjusting their liabilities. The shadow banking sector effectively shuts down within a narrow range of uncertainty. In the financial crisis of 2007 to 2008, the market for asset-backed commercial paper suffered a similarly rapid collapse (Acharya, Schnabl and Suarez, 2013).

Issuing more money requires raising equity. The result is less liquidity transformation, less liquidity, and higher discount rates. The price of A capital falls sharply while B capital becomes more valuable as the liquidity shortage increases the premium for collateral. This effect is strongest when collateral is scarce to begin with (χ_t is high) and liquidity transformation is at its peak. The reversal of investment from productive to safe capital can be interpreted as collateral mining or building a "fortress balance sheet".

Importantly, it is neither a lack of intermediary capital nor a lack of investment opportunities that causes intermediaries to stop investing in productive capital when uncertainty rises. It is instead the low level of liquidity transformation resulting from the contraction of shadow banking that initiates the downturn.

4.2 Persistence

Our model features persistence due to learning and variation in investment, which interact dynamically to produce cycles. The endogenous dynamics of collateral values near the bottom of the cycle play an important role in this process, a discussion we postpone until Section 4.3 below.

A rise in uncertainty contracts liquidity provision, which changes the economy's target capital mix so that productivity growth remains low even after uncertainty recedes, a form of slow recovery following a financial crisis. At the other end, low uncertainty promotes the accumulation of risky capital backed by shadow money issuance, which builds up fragility during booms.

To illustrate these forces, Figure 4 plots impulse response functions for the state variables uncertainty, λ_t, and the risky capital share, χ_t. The plots on the left condition on a low initial level of λ_t and those on the right on a high one, i.e. a boom versus a bust.

The top panels look at λ_t. The gray shading represents a contour plot of its conditional density, which is computed by solving the forward Kolmogorov equation in Appendix E. From the solid red lines, which represent conditional means, we see that λ_t is very persistent. From the shaded densities we see that this persistence is due to the combination of a negative drift that pulls much of the mass down and occasional jumps that send chunks of it back up. In this way uncertainty tends to be very low after a long quiet period but it can rise suddenly as in a "Minsky moment".

Figure 4 about here.

The bottom panels of Figure 4 plot the conditional means of the risky capital share χ_t starting from a low, medium, or high level under each of the two λ_t scenarios. On the left, where λ_t is low, χ_t is increasing over time except when it is very high to begin with. Low uncertainty promotes shadow banking which enables greater liquidity, reducing discount rates and collateral premia. The price of the risky asset rises and stimulates investment. Over time, this leads to a riskier capital mix.

On the right, where λ_t is high, χ_t drifts down except when it is very low from the start. When uncertainty is high, households are unwilling to hold shadow money, which makes investment in the risky asset less attractive. At the same time, demand for collateral causes investment in the safe but unproductive asset to pick up ("collateral mining"), causing χ_t to fall over time. This sets up a slow recovery once uncertainty diminishes.

Overall, Figure 4 illustrates the interaction between the model's financial and macroeconomic cycles. In the next section we show how the dynamics of collateral values contribute to this interaction.

4.3 Collateral runs

Collateral runs (or margin spirals in the language of Brunnermeier and Pedersen, 2009) are episodes during which liquidity creation requires progressively greater amounts of collateral, which is equivalent to a rise in haircuts in financial markets.[23]

To be concrete, an asset i that can back $1-\kappa$ dollars of money per dollar of value has a haircut of κ. The haircut measures the size of the drop in value that must occur before debt holders take a hit. In our model haircuts have an exogenous cash flow component κ^i and an endogenous price component κ^i_π, $i = a, b$ for an overall haircut of $1 - \left(1-\kappa^i\right)\left(1-\kappa^i_\pi\right)$. A collateral run occurs when $\kappa^i_\pi = 1 - \pi^i_+/\pi^i$ rises as π^i falls (pluses denote after-crash prices). This requires a simultaneous rise in the the level and volatility of discount rates. The higher level puts downward pressure on prices, while the higher volaitlity depresses after-crash collateral values, increasing haircuts. The resulting tightening in the collateral constraint further amplifies the initial increase in discount rates, causing a downward price spiral.

Figure 5 about here.

[23] A collateral run is distinct from a classic bank run. Agents in our model demand bigger haircuts when financial conditions become more sensitive to uncertainty shocks. They do not face a first-come-first-served constraint as in Diamond and Dybvig (1983). A bank run could arise in our model if collateral is rehypothecated among investors. We hope to explore this possibility in future research.

To demonstrate the effects of collateral runs, Figure 5 compares haircuts, capital structures, funding costs, and asset prices in economies with and without shadow banking.[24] We fix $\chi_t = 0.5$ and look across λ_t.

In a shadow-banking economy near the peak of a boom (low λ_t), an uptick in uncertainty leads to a rise in both haircuts and discount rates. Higher haircuts drive a sharp contraction in liquidity transformation, forcing a shift towards more expensive funding. As a result, prices fall further, which represents a collateral run.

In a collateral run, money yields fall while shadow money yields rise sharply. Since the economy is near its peak, shadow banking activity is very high and so overall discount rates rise. The risky A capital suffers the greatest price decline as its funding is most fragile. Although the amplification of collateral runs causes discount rates to spike above their levels in the no-shadow-banking economy, the price of A capital is always higher with shadow banking than without because prices capitalize the lower funding costs in a shadow banking-driven boom. The possibility of future booms leads to less severe downturns.

On the other side, when λ_t is high and liquidity transformation is near bottom, asset prices become less sensitive to uncertainty so a rise in λ_t actually reduces haircuts. In other words, the haircut-price dynamic reverses so that a "collateral decelerator" eventually puts a floor under asset prices. The same dynamic means, however, that haircuts initially rise as uncertainty begins to subside, which slows down the recovery of asset prices. The same mechanism that amplifies downturns also prolongs their aftermath.

4.4 Flight to quality

Our model generates flight to quality, a rise in the value of safe claims even as overall prices fall. A rise in uncertainty triggers a demand shift from shadow money to money, causing the spread between them to open up. As intermediaries absorb the excess demand for money, liquidity transformation shuts down. The premium for collateral rises, causing the safe B capital to appreciate relative to the risky A capital. When this relative price change dominates the overall rise in discount rates, the yield of money falls and the price of B capital rises.

Our benchmark parametrization produces strong flight to quality in securities markets and modest flight to quality in asset markets (see Figure 3). Intuitively, flight to quality is the result of a shortage of collateral due to a sharp contraction in liquidity trans-

[24] We implement an economy with no shadow banking by setting $\overline{\kappa} = 0$, which removes the leverage advantage of shadow money.

formation. In our benchmark model crashes actually increase the supply of collateral as a higher proportion of the safe B capital remains intact. In this section, we modify the model slightly to remove this dampening, which intensifies flight to quality effects.

Specifically, we transform the aggregate cash flow shock into a dispersion shock. Let $k^a_{i,t}$ and $k^b_{i,t}$ be the capital holdings of intermediary i and modify equation (1):

$$\frac{dk^a_{i,t}}{k^a_{i,t}} = \left[\phi\left(\iota^a_{i,t}\right) - \delta\right] dt - \kappa^a_{i,t} dN_t, \tag{28}$$

where $\kappa^a_{i,t} = \pm\kappa^a$ with probability 1/2 each and $\kappa^b_{i,t} = 0$ for simplicity. We have further simplified our benchmark specification by setting $\mu = 0$, which increases the scarcity of collateral at high levels of χ_t. We also increase η slightly to 2.8, which reduces the elasticity of substitution between money and shadow money.[25]

The dispersion shock makes A capital risky for an individual intermediary but safe in the aggregate. This keeps pledgeability low but eliminates aggregate cash flow risk, highlighting the fact that our model is about collateral rather than risk.[26]

Figure 6 about here.

Figure 6 shows the strong flight to quality in securities markets. When uncertainty λ_t rises, the yield on money falls and the spread between shadow money and money (the safety premium) opens up. The effect is strongest when liquidity transformation is initially high ($\chi_t = 0.9$) so that collateral is scarce. In this case overall liquidity falls, which causes discount rates to rise as reflected in the equity premium. These dynamics resemble developments in U.S. markets after July 2007.

Figure 6 also shows strong flight to quality in asset markets. In the bottom right panel, the price of B capital rises most in a crash when it occurs near the peak of the liquidity cycle; that is when uncertainty is low and the capital mix is risky ($\chi_t = 0.9$). U.S. long-term bonds similarly appreciated as the financial crisis unfolded in 2007–2008 (Krishnamurthy, 2010). Once uncertainty rises sufficiently (or the capital mix becomes safe enough) so that shadow banking shuts down, flight to quality disappears. Thus,

[25] The modified model is solved easily by simply altering the dynamics of χ_t in (27) to $d\chi_t = \chi_t(1-\chi_t)\left[\phi\left(\iota^a_t\right) - \phi\left(\iota^b_t\right)\right] dt$, while keeping the pricing PDEs (25) unchanged.

[26] We are implicitly assuming that intermediaries cannot diversify the dispersion shock. As a possible example, it may be difficult to distinguish ex ante which assets are likely to co-move in the rare event of a crash, (e.g. mortgages in Miami and Las Vegas). Alternatively, individual intermediaries might develop special expertise in particular markets. See also Di Tella (2012). Note that flight to quality does not require the dispersion shock modification.

flight to quality results from the acute shortage of collateral that occurs when uncertainty rises suddenly after a shadow banking boom.

Our model's learning dynamics tie together normal-times (dB) and crash (dN) flight to quality (both raise λ). The two, however, are conceptually distinct as normal-times shocks are borne entirely by equity whereas crashes affect collateral values and the supply of liquidity. In our model only crash-driven flight to quality is important ex ante because equity markets are frictionless.[27] This observation suggests that differences in the pricing of instruments that act as normal-times versus crash-risk hedges can be used to assess the importance of equity- versus collateral-based frictions.

5 Policy interventions

In the aftermath of the 2008 financial crisis, central banks around the world and the U.S. Federal Reserve in particular have resorted to a wide variety of interventions, broadly referred to as unconventional monetary policy. We consider two of these interventions, the Large Scale Asset Purchase (LSAP) program of 2008–2010 and the Maturity Extension Program also known as "Operation Twist" of 2011–2012. Under LSAP, the FED purchased large amounts of mortgage-backed securities in an effort to support their prices.[28] Under Operation Twist, the FED purchased long-dated Treasurys and sold short-dated ones with the stated goal of reducing long-term interest rates.[29]

Alongside central banks, regulators have entertained a broad range of proposals and implemented a subset of them. We look at two of these, the so-called "Volcker rule" which seeks to separate commercial banking and proprietary trading, and liquidity requirements as have been adopted by the Basel III Committee.[30]

Our aim in this section is to shed light on the interaction of these policies with the

[27] We explore the policy implications of flight to quality in Section 5.2 (Operation Twist) and Section 5.4 (Volcker rule).

[28] The press release announcing the program reads, "Spreads of rates on GSE debt and on GSE-guaranteed mortgages have widened appreciably of late. This action is being taken to reduce the cost and increase the availability of credit for the purchase of houses, which in turn should support housing markets and foster improved conditions in financial markets more generally" (Federal Open Market Committee, 2008).

[29] The program's announcement following the September 2011 FOMC meeting reads, "The Committee intends to purchase, by the end of June 2012, \$400 billion of Treasury securities with remaining maturities of 6 years to 30 years and to sell an equal amount of Treasury securities with remaining maturities of 3 years or less. This program should put downward pressure on longer-term interest rates and help make broader financial conditions more accommodative" (Federal Open Market Committee, 2011).

[30] The Basel III framework consists of two parts, the Liquidity Coverage Ratio (LCR), and the Net Stable Funding Ratio (NSFR) (Bank for International Settlements, 2010). LCR affects asset liquidity and NSFR affects liquidity provision directly. From the point of view of the model, what matters is their combined effect on liquidity transformation.

liquidity transformation channel that lies at the heart of the paper. We leave a comprehensive welfare analysis that incorporates a broader set of considerations for future work.

5.1 Asset purchases

We interpret LSAP as replacing risky A capital with safe B capital. The direct effect is to increase the amount of collateral available on intermediary balance sheets, and by extension liquidity provision, asset prices, and investment. Since ex post prices determine ex ante collateral values, LSAP also has an indirect ex ante effect.

In an LSAP, a given dollar amount of A capital is exchanged for an equal dollar amount of B capital. Any cash flow mismatch between the swapped assets is backed by lump-sump taxation of households (the government does not face a collateral constraint). We model LSAP as a one-at-a-time intervention to acknowledge the limits of this taxation power. The model is non-Ricardian because collateral impacts asset prices.

We assume LSAP takes effect with a given probability immediately following a crash (when it is most needed) and it is eventually withdrawn at a given intensity. This setup introduces a simple binary state variable that corresponds to the state of the central bank's balance sheet (empty or full). Appendix F has the details.

Figure 7 implements an LSAP that reduces the risky capital share χ_t by 0.2. It gets triggered with 50% probability after a crash and is expected to last ten years.

Figure 7 about here.

The top panels of Figure 7 show that in most of the state-space, LSAP pushes the price of risky capital up and the price of safe capital down, with the effects for both assets being strongest when the capital mix is risky ($\chi_t = 0.9$). In this region, collateral is scarce and liquidity is low. By supplying safe assets, the central bank increases the collateral value of intermediary balance sheets, allowing for greater liquidity provision. The result is both a decline in overall discount rates which tends to push all prices up, and a decline in the collateral premium, which pushes the price of risky capital up and the price of safe capital down. The net effect is positive for the risky asset and negative for the safe asset.

Interestingly, enacting a program when liquidity is abundant (when χ_t and λ_t are low) backfires and actually reduces the price of the risky asset. This is a result of the central bank's limited capacity. The economy becomes riskier because the central bank will likely be out of ammunition at the next crash.

Looking along the λ_t dimension, the price effects are strongest when uncertainty is moderately high. This is also the region where collateral runs push haircuts to their highest levels (see Section 4.3). Thus, LSAP is most effective when the shadow banking sector is maximally stressed and contracting precipitously.

The middle two panels of Figure 7 consider the ex ante effect of LSAP by comparing prices across economies with and without the possibility of an LSAP intervention. Risky capital prices are higher and safe capital prices are lower in the LSAP economy throughout the state-space, as expected. Importantly, the ex ante effects are stronger than the ex post effects in the low uncertainty region. LSAP has a stabilizing effect ex post when uncertainty is high, which means it boosts collateral values ex ante when uncertainty is low. In this way, expectation of future LSAP interventions amplifies shadow banking booms. This mechanism echoes concerns raised by Rajan (2005).

Finally, we consider the effect of an announcement that policy accommodation will be withdrawn sooner than anticipated.[31] In the lower panels of Figure 7 we show that "taper talk" produces sharp asset price movements in our model as in the data. Expectations of premature policy withdrawal can undermine the effectiveness of an asset purchase program.

Our results show that when liquidity transformation is impaired by high uncertainty, an LSAP intervention allows the central bank to support asset prices via collateral transformation.

5.2 Operation Twist

We model Operation Twist as a market intervention that reduces the duration of safe assets on intermediary balance sheets. Interestingly, in our framework this can reduce pledgeability as long-term safe capital acts as a crash-hedge due to flight to quality. To demonstrate this effect, we use the parameters from Section 4.4 on flight to quality.

We map the safe asset to government debt by assuming that the private sector cannot create it but that the government issues it by following the same policy that intermediaries do in the benchmark economy (set Tobin's q to one as in (26)). To model a change in duration within an economy, we introduce two types of government debt, zero-duration floating-rate debt and long-term fixed coupon bonds. Floating debt pays the floating rate μ_m (the yield of money) and trades at par. Long bonds pay the fixed coupon γ^b as in the baseline model. The central bank sets the shares of the two types of bonds as a policy variable. Details are in Appendix F.

In an Operation Twist intervention, the central bank buys long-term bonds and sells

[31]In the summer of 2013, discussion of policy withdrawal, or "taper talk", led to sharp corrections across asset markets.

floating rate bonds of equal dollar amounts.[32] Any cash-flow mismatch between long and short-term bonds are again backed by lump-sum taxation of households. We consider a one-off unanticipated intervention in which government debt is restructured from fully long-term to fully floating.

Figure 8 about here.

The left panel of Figure 8 shows that Operation Twist reduces the price of the risky A capital. This is due to a strong flight to quality effect that makes long-term debt appreciate in a crash (i.e. $\kappa^{b}_{\pi,t} < 0$). As a result, it acts as a hedge for the crash risk of A capital, thereby raising the aggregate supply of collateral. Floating-rate debt always trades at par, so it cannot act as a hedge. In this way, Operation Twist ends up reducing total collateral as the bottom panel of Figure 8 shows, which causes discount rates to rise and overall prices to fall. The reduction in collateral supply causes the price of long-term bonds to go up. This means that the effectiveness of Operation Twist cannot be judged by the price of long-term bonds which can go up even if the policy is counter-productive.

The case for Operation Twist is predicated on the idea that risky productive assets are exposed to duration risk just like long-term bonds, so that reducing the supply of long-term bonds might free up balance sheet capacity for risky investment. In our economy the opposite happens because duration becomes a hedge when flight to quality is strong. This makes long-term bonds complements rather than substitutes for risky investment. We return to this discussion in Section 5.4 in the context of the Volcker rule.

5.3 Liquidity requirements

A liquidity requirement limits the liquidity mismatch between a bank's assets and liabilities, and it is typically promoted as a tool to mitigate costly fire sales (see Stein, 2013). Our model features a type of fire sale in the form of a collateral run, an episode when collateral values become depressed due to rapid deleveraging in the financial sector. In this section we show that liquidity requirements are effective at mitigating this type of fire sale.

Since assets in our model are themselves illiquid, regulating liquidity mismatch here amounts to imposing an upper bound on the issuance of liquid securities:

$$m_t + s_t \leq \bar{l}, \tag{29}$$

where $\bar{l} < 1$. This constraint is always tighter than the limited liability constraint $m_t + s_t \leq 1$, so it takes its place.

[32] The merits of this type of policy are also discussed in Greenwood, Hanson and Stein (2014).

Solving the model with a liquidity requirement involves an additional case in the intermediary's optimal capital structure policy. The specifics are in Appendix F.

Figure 9 about here.

Figure 9 compares an economy with a 10% liquidity requirement ($1 - \bar{l} = 10\%$) to the benchmark case of a zero liquidity requirement. We fix $\chi_t = 0.5$ and look across λ_t.

From the top two panels, we see that the liquidity requirement leads to a drastic reduction in shadow money issuance. The reason is that the cap on liquidity provision curtails the principal advantage of shadow money, the ability to create liquidity with less collateral. In fact, money issuance rises as it is no longer crowded out by shadow money during booms.

The middle two panels plot prices, which are lower in the economy with a liquidity requirement due to lower liquidity provision. Note that this happens even at high uncertainty when the liquidity requirement is not binding. The reason is that it is expected to bind when uncertainty is low. In this way taming the boom deepens the bust.

The bottom panels look at haircuts (recall from Section 4.3 that an asset's haircut is one minus its collateral value). We see that the haircuts of both assets fall, though asset A has a much larger haircut to begin with due to greater risk.[33]

The intuition for this result is that by restraining shadow banking in booms, liquidity requirements reduce the economy's exposure to uncertainty shocks. This means that prices do not fall as fast when uncertainty rises, so collateral values remain relatively high and haircuts low.

Recall that a collateral run is an event where haircuts rise as prices fall and the two reinforce each other (Section 4.3). Figure 9 shows that liquidity requirements can indeed arrest this dynamic by slowing the rise in haircuts. This suggests they can be used to promote financial stability.

5.4 The Volcker rule

The Volcker rule seeks to prevent banks from engaging in proprietary trading due to high levels of risk. Observers have argued that the distinction between market making and proprietary trading is extremely difficult as market making typically involves holding a substantial inventory of risky assets.

In this section we map market making activity and its associated inventory accumulation into holdings of risky capital and interpret the Volcker rule as imposing a segregation

[33] By fixing $\chi_t = 0.5$, we are looking at a region with enough collateral where flight to quality does not take place.

between intermediaries that hold risky and safe assets.[34] At this level, our analysis could also refer to the reintroduction of the Glass-Steagall act, which banned affiliations between commercial banks and securities firms. Once again, our scope here is limited to the liquidity transformation channel.

The key point is that our model features a complementarity between risky and safe asset holdings whenever flight to quality effects are present. The intuition is the same as in the discussion of Operation Twist in Section 5.2: Flight to quality turns safe capital into a hedge for risky capital on intermediary balance sheets, which raises the overall collateral value of intermediary assets. Under a Volcker rule, collateral is effectively wasted as safe banks have too much and risky banks too little.

To illustrate, suppose there is flight to quality so that the value of safe capital rises in a crash (e.g. $\kappa^b = 0$, $\kappa^b_{\pi,t} < 0$). Consider a Volcker rule economy with two (types of) intermediaries, a risky-asset bank $i = a$ and a safe-asset bank $i = b$. Each bank's balance sheet must satisfy limited liability and the collateral constraint while holding only one type of capital:

$$m^i + s^i \leq \min\left\{1, \left(1 - \kappa^i_\pi\right)\left(1 - \kappa^i\right) + s^i\overline{\kappa}\right\}. \tag{30}$$

Flight to quality implies that the safe bank has excess collateral that allows it to issue 100% money, $m^b = 1$ and $s^b = 0$. The risky bank behaves as in our model, so its crash-solvency constraint binds, $m^a + s^a = (1 - \kappa^a_\pi)(1 - \kappa^a) + s^a\overline{\kappa}$. Let x be the value-weighted share of A capital. Then total liquidity under the Volcker rule and the benchmark economy is

$$m^a + s^a + m^b + s^b = x(1 - \kappa^a_\pi)(1 - \kappa^a) + (1 - x) + s^a\overline{\kappa} \tag{31}$$

$$m + s = x(1 - \kappa^a_\pi)(1 - \kappa^a) + (1 - x)\left(1 - \kappa^b_\pi\right)\left(1 - \kappa^b\right) + s\overline{\kappa}. \tag{32}$$

Comparing (31) and (32) it is clear that flight to quality, $\left(1 - \kappa^b_\pi\right)\left(1 - \kappa^b\right) > 1$, leads to lower liquidity provision in the Volcker economy. In general both normal-times and crash-proof liquidity are lower even though the Volcker rule only tightens the collateral constraint coming from crash risk. The scarcity of collateral on the risky bank's balance sheet reduces its capacity to provide normal-times liquidity with shadow money. At the same time, the excess collateral on the safe bank's balance sheet goes unused.

[34] Note that liabilities-side segregation (e.g. between issuers of money and shadow money) has no effect in our model as the ability to issue equity costlessly allows capital to flow freely across intermediaries.

6 Conclusion

We present a model that joins the macroeconomic cycle to the liquidity transformation cycle in the financial sector. The key friction is that while liquidity is valuable, it requires collateral. Since collateral is particularly scarce after large shocks (crashes), intermediaries optimally produce state-contingent liquidity in the form of securities that are money-like in normal times but cease to be liquid in a crash. This allows them to produce liquid securities from risky assets, that is to engage in liquidity transformation. We call this process shadow banking.

Shadow banking creates booms in which liquidity provision expands, discount rates fall, asset prices and investment rise, and output grows. At the same time, it builds fragility by raising the economy's exposure to uncertainty shocks. Such shocks unleash severe effects when preceded by a boom: shadow banking disappears and liquidity contracts sharply, prices of risky assets fall, amplified by collateral runs, and prices of safe assets rise through flight to quality; investment and growth fall. Recoveries are slow due to fortress balance sheets and a "collateral decelerator" under which haircuts rise as revived liquidity transformation once again restores uncertainty exposure.

In sum, shadow banking enables high levels of liquidity in good times at the expense of bad times. As it expands and contracts, shadow banking activity shifts the balance between financial stability and growth.

References

Acharya, Viral, Philipp Schnabl, and Gustavo Suarez. 2013. "Securitization without risk transfer." *Journal of Financial Economics*, 107(3): 515–536.

Adrian, Tobias, and Hyun Song Shin. 2009. "Money, Liquidity, and Monetary Policy." *The American Economic Review*, 99(2): pp. 600–605.

Adrian, Tobias, and Hyun Song Shin. 2010. "Liquidity and leverage." *Journal of Financial Intermediation*, 19(3): 418–437.

Adrian, Tobias, and Nina Boyarchenko. 2012. "Intermediary leverage cycles and financial stability." *Working paper*.

Adrian, Tobias, Emanuel Moench, and Hyun Song Shin. 2010. "Financial intermediation, asset prices, and macroeconomic dynamics." *Federal Reserve Bank of New York Staff Report*, 422.

Adrian, Tobias, Erkko Etula, and Tyler Muir. 2014. "Financial Intermediaries and the Cross-Section of Asset Returns." *The Journal of Finance, doi: 10.1111/jofi.12189*.

Ang, Andrew, Sergiy Gorovyy, and Gregory B. van Inwegen. 2011. "Hedge fund leverage." *Journal of Financial Economics*, 102(1): 102–126.

Ashcraft, Adam, Nicolae Gârleanu, and Lasse Heje Pedersen. 2011. "Two monetary tools: Interest rates and haircuts." In *NBER Macroeconomics Annual 2010, Volume 25*. 143–180. University of Chicago Press.

Bai, Jennie, Arvind Krishnamurthy, and Charles-Henri Weymuller. 2013. "Measuring the liquidity mismatch in the banking sector." *Working paper*.

Bank for International Settlements. 2010. *Basel III: International framework for liquidity risk measurement, standards and monitoring.*

Bansal, Ravi, Wilbur Coleman, and Christian Lundblad. 2010. "Endogenous Liquidity Supply." *Working paper*.

Baron, Matthew, and Wei Xiong. 2014. "Credit Expansion and Neglected Crash Risk." *Working paper*.

Bernanke, Ben S. 2013. "Monitoring the Financial System." At the 49th Annual Conference on Bank Structure and Competition sponsored by the Federal Reserve Bank of Chicago, Chicago, Illinois.

Bernanke, Ben S, and Mark Gertler. 1989. "Agency costs, net worth, and business fluctuations." *The American Economic Review*, 14–31.

Bernanke, Ben S, Mark Gertler, and Simon Gilchrist. 1999. "The financial accelerator in a quantitative business cycle framework." *Handbook of macroeconomics*, 1: 1341–1393.

Bigio, Saki. 2013. "Endogenous liquidity and the business cycle." *Working paper*.

Brunnermeier, Markus K, and Lasse Heje Pedersen. 2009. "Market liquidity and funding liquidity." *Review of Financial studies*, 22(6): 2201–2238.

Brunnermeier, Markus K, and Yuliy Sannikov. 2014*a*. "The I Theory of Money." *Working paper*.

Brunnermeier, Markus K, and Yuliy Sannikov. 2014*b*. "A macroeconomic model with a financial sector." *The American Economic Review*, 104(2): 379–421.

Brunnermeier, Markus K, Thomas Eisenbach, and Yuliy Sannikov. 2013. "Macroeconomics with Financial Frictions: A Survey." *Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society*. New York:Cambridge University Press.

Caballero, Ricardo J, and Emmanuel Farhi. 2013. "A Model of the Safe Asset Mechanism (SAM): Safety Traps and Economic Policy." *NBER working paper*.

Dang, Tri Vi, Gary Gorton, and Bengt Holmström. 2012. "Financial crises and the optimality of debt for liquidity provision." *Working paper*.

DeMarzo, Peter, and Darrell Duffie. 1999. "A liquidity-based model of security design." *Econometrica*, 67(1): 65–99.

DeMarzo, Peter M. 2005. "The pooling and tranching of securities: A model of informed intermediation." *Review of Financial Studies*, 18(1): 1–35.

Diamond, Douglas W, and Philip H Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3): pp. 401–419.

Di Tella, Sebastian. 2012. "Uncertainty shocks and balance sheet recessions." *Working paper*.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl. 2014. "A Model of Monetary Policy and Risk Premia." *Working paper*.

Federal Open Market Committee. 2007. "Minutes from the Meeting of the Federal Open Market Committee on June 27–28, 2007." *US Federal Reserve*.

Federal Open Market Committee. 2008. "Press Release of November 25, 2008." *US Federal Reserve*.

Federal Open Market Committee. 2011. "Statement following the meeting of the Federal Open Market Committee on September 11, 2011." *US Federal Reserve*.

Financial Crisis Inquiry Commission. 2011. "The Financial Crisis Inquiry Report." *US Government Printing Office*.

Gârleanu, Nicolae, and Lasse Heje Pedersen. 2011. "Margin-based Asset Pricing and Deviations from the Law of One Price." *Review of Financial Studies*.

Gârleanu, Nicolae, Stavros Panageas, and Jianfeng Yu. 2012. "Technological Growth and Asset Pricing." *The Journal of Finance*, 67(4): 1265–1292.

Geanakoplos, John. 2003. "Liquidity, Default, and Crashes: Endogenous Contracts in General Equilibrium." In *Advances in Economics and Econometrics*. Vol. 2, , ed. Mathias Dewatripont, Lars Peter Hansen and Stephen J. Turnovsky, 170–205. Cambridge University Press.

Gennaioli, Nicola, Andrei Shleifer, and Robert W. Vishny. 2012. "Neglected risks, financial innovation, and financial fragility." *Journal of Financial Economics*, 104(3): 452 – 468. Market Institutions, Financial Market Risks and Financial Crisis.

Gennaioli, Nicola, Andrei Shleifer, and Robert W. Vishny. 2013. "A Model of Shadow Banking." *The Journal of Finance*, 68(4): 1331–1363.

Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*. Vol. 3 of *Handbook of Monetary Economics*, , ed. Benjamin M. Friedman and Michael Woodford, Chapter 11, 547–599. Elsevier.

Gertler, Mark, and Nobuhiro Kiyotaki. 2013. "Banking, Liquidity and Bank Runs in an Infinite-Horizon Economy." *Working paper*.

Gertler, Mark, and Peter Karadi. 2011. "A model of unconventional monetary policy." *Journal of Monetary Economics*, 58(1): 17–34.

Gorton, Gary, and Andrew Metrick. 2012. "Securitized banking and the run on repo." *Journal of Financial Economics*, 104(3): 425–451.

Gorton, Gary, and George Pennacchi. 1990. "Financial Intermediaries and Liquidity Creation." *The Journal of Finance*, 45(1): 49–71.

Gorton, Gary, and Guillermo Ordoñez. 2014. "Collateral crises." *The American Economic Review*, 104(2): 343–378.

Greenwood, Robin, Sam Hanson, and Jeremy Stein. 2014. "A Comparative-Advantage Approach to Government Debt Maturity." *Journal of Finance, forthcoming*.

Hanson, Floyd B. 2007. *Applied stochastic processes and control for jump diffusions: Modeling, analysis, and computation.* Society for Industrial and Applied Mathematics.

Hanson, Samuel G, Andrei Shleifer, Jeremy C Stein, and Robert W Vishny. 2014. "Banks as Patient Fixed Income Investors." *Working paper*.

Harris, Milton, Christian Opp, and Marcus Opp. 2014. "Higher Capital Requirements, Safer Banks? Macroprudential Regulation in a Competitive Financial System." *Working paper*.

He, Zhigu, and Arvind Krishnamurthy. 2012. "A Model of Capital and Crises." *The Review of Economic Studies*, 79(2): 735–777.

He, Zhiguo, and Arvind Krishnamurthy. 2013. "Intermediary Asset Pricing." *American Economic Review*, 103(2): 732–770.

He, Zhiguo, and Konstantin Milbradt. 2013. "Endogenous liquidity and defaultable bonds." *Econometrica, forthcoming*.

He, Zhiguo, In Gu Khang, and Arvind Krishnamurthy. 2010. "Balance sheet adjustments during the 2008 crisis." *IMF Economic Review*, 58(1): 118–156.

Holmström, Bengt, and Jean Tirole. 1998. "Private and Public Supply of Liquidity." *Journal of Political Economy*, 106(1): 1–40.

Innes, Robert D. 1990. "Limited liability and incentive contracting with ex-ante action choices." *Journal of economic theory*, 52(1): 45–67.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *Journal of Political Economy*, 105(2).

Kiyotaki, Nobuhiro, and John Moore. 2012. "Liquidity, business cycles, and monetary policy." *Working paper*.

Krishnamurthy, Arvind. 2010. "How Debt Markets Have Malfunctioned in the Crisis." *The Journal of Economic Perspectives*, 24(1): 3–28.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2012*a*. "The aggregate demand for treasury debt." *Journal of Political Economy*, 120(2): 233–267.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2012*b*. "Short-term Debt and Financial Crises: What we can learn from US Treasury Supply." *Working paper*.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2013. "The Ins and Outs of LSAPs." *2013 Federal Reserve Bank of Kansas City Economic Policy Symposium, Jackson Hole, Wyoming*.

Krishnamurthy, Arvind, Stefan Nagel, and Dmitry Orlov. 2014. "Sizing up repo." *The Journal of Finance, doi: 10.1111/jofi.12168*.

Kurlat, Pablo. 2013. "Lemons Markets and the Transmission of Aggregate Shocks." *American Economic Review*, 103(4): 1463–1489.

Liptser, Robert S, and Albert N Shiryaev. 2001. *Statistics of Random Processes II: II. Applications.* Vol. 2, Springer.

Maggiori, Matteo. 2013. "Financial intermediation, international risk sharing, and reserve currencies." *Working paper*.

McCauley, Robert, and Patrick McGuire. 2009. "Dollar appreciation in 2008: safe haven, carry trades, dollar shortage and overhedging." *BIS Quarterly Review*.

Minsky, Hyman P. 1986. *Stabilizing an unstable economy.* Yale University Press New Haven, CT.

Nachman, David C, and Thomas H Noe. 1994. "Optimal design of securities under asymmetric information." *Review of Financial Studies*, 7(1): 1–44.

Nagel, Stefan. 2014. "The Liquidity Premium of Near-Money Assets." *Working paper*.

Rajan, Raghuram G. 2005. "Has finance made the world riskier?" *2005 Federal Reserve Bank of Kansas City Economic Policy Symposium, Jackson Hole, Wyoming*.

Rampini, Adriano, and S Viswanathan. 2012. "Financial intermediary capital." *Working paper*.

Reinhart, Carmen M, and Kenneth Rogoff. 2009. *This time is different: Eight centuries of financial folly.* Princeton University Press.

Reinhart, Carmen M, and Vincent R Reinhart. 2010. "After the fall." *Working paper*.

Sannikov, Yuliy. 2013. "Quiet Times and Financial Fragility." *Working paper*.

Schularick, Moritz, and Alan M Taylor. 2012. "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008." *American Economic Review*, 102(2): 1029–61.

Stein, Jeremy. 2013. "The Fire-Sales Problem and Securities Financing Transactions." At the Federal Reserve Bank of New York Workshop on Fire Sales as a Driver of Systemic Risk in Triparty Repo and other Secured Funding Markets, New York, 4 October 2013.

Sunderam, Adi. 2013. "Money Creation and the Shadow Banking System." *Working paper*.

Wonham, W Murray. 1965. "Some applications of stochastic differential equations to optimal nonlinear filtering." *Journal of the Society for Industrial and Applied Mathematics, Series A: Control*, 2(3): 347–369.

Table 1: Benchmark parameter values

This table contains the values for the model parameters used for producing the benchmark results of the paper.

Description	Parameter	Value
Subjective discounting parameter	ρ	0.40
Depreciation	δ	0.03
Level-inflow of capital	μ	0.01
Adjustment cost parameter	φ	5
Asset A cash flow risk	κ^a	0.4
Asset B cash flow risk	κ^b	0.0
Asset A productivity	γ^a	0.08
Asset B productivity	γ^b	0.03
Low uncertainty state	λ^L	0.1
High uncertainty state	λ^H	2.1
Low uncertainty state exit rate	q^L	0.05
High uncertainty state exit rate	q^H	0.05
Uncertainty news signal precision	$1/\sigma$	0.1
Liquid asset risk upper bound	$\overline{\kappa}$	0.7
Liquidity event duration	τ	1.032
Liquidity event intensity	h	0.1
Value of liquidity-event consumption	ψ	10
Average liquidity shock size	$1/\eta$	0.5

Figure 3: Prices, issuance, liquidity-event consumption, and growth

This figure plots capital prices, money issuance, shadow money issuance, output growth, and liquidity services using the benchmark parameter values in Table 1. Each quantity is plotted against uncertainty λ_t while holding the risky capital share χ_t fixed at a low (0.1), medium (0.5), and high (0.9) level. Growth is the expected growth rate of output $y_t = \gamma^a k_t^a + \gamma^b k_t^b$. Liquidity-event consumption is the flow rate of $E_t\left[C_t dN_t^h\right]$.

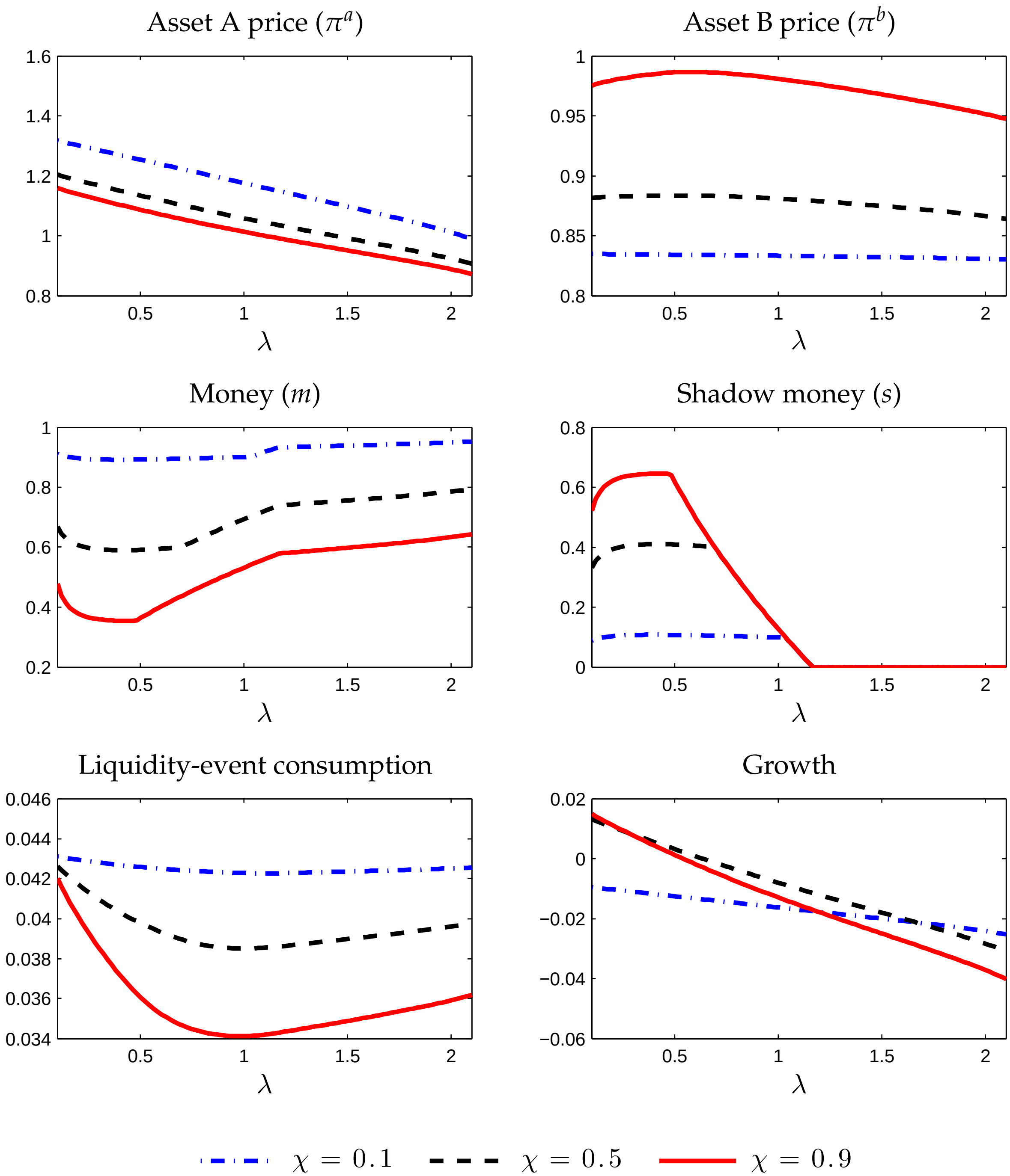


Figure 4: Persistence

The evolution of uncertainty λ_t, and the capital mix χ_t over time starting from a initial condition of low (left, $\lambda_0 = 0.6$) or high (right, $\lambda_0 = 1.6$) levels of λ_t, and low ($\chi_0 = 0.1$), medium ($\chi_0 = 0.5$), or high ($\chi_0 = 0.9$) levels of χ_t. The top two panels show the evolution of λ_t (which is independent of χ_t) with solid red lines for the conditional mean of λ_t, $E_0[\lambda_t | \lambda_0]$, and gray shading for the conditional density of λ_t. The bottom panels show the conditional means of χ_t, $E_0[\chi_t | \lambda_0, \chi_0]$. The conditional means and densities are computed using the forward Kolmogorov equation in Appendix E.

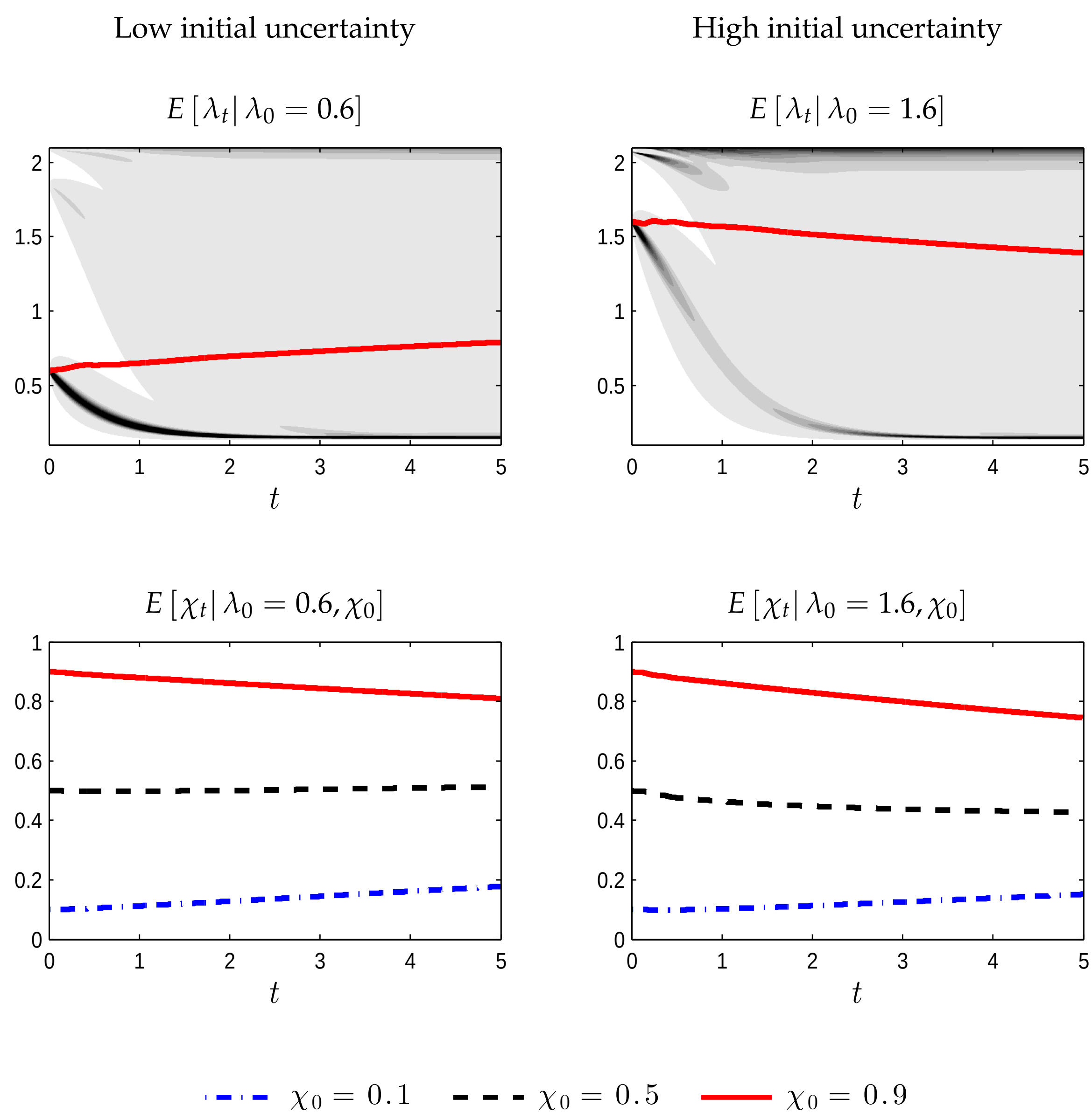


Figure 5: Collateral runs

Equilibrium haircuts, capital structure, funding costs, and prices for economies with (no markers) and without (circle markers) shadow banking. The risky capital share is fixed at $\chi_t = 0.5$. Haircuts are defined as one minus the collateral value of the asset, i.e. $1 - \left(1 - \kappa^i\right)\left(1 - \kappa^i_\pi\right)$ for $i = a, b$.

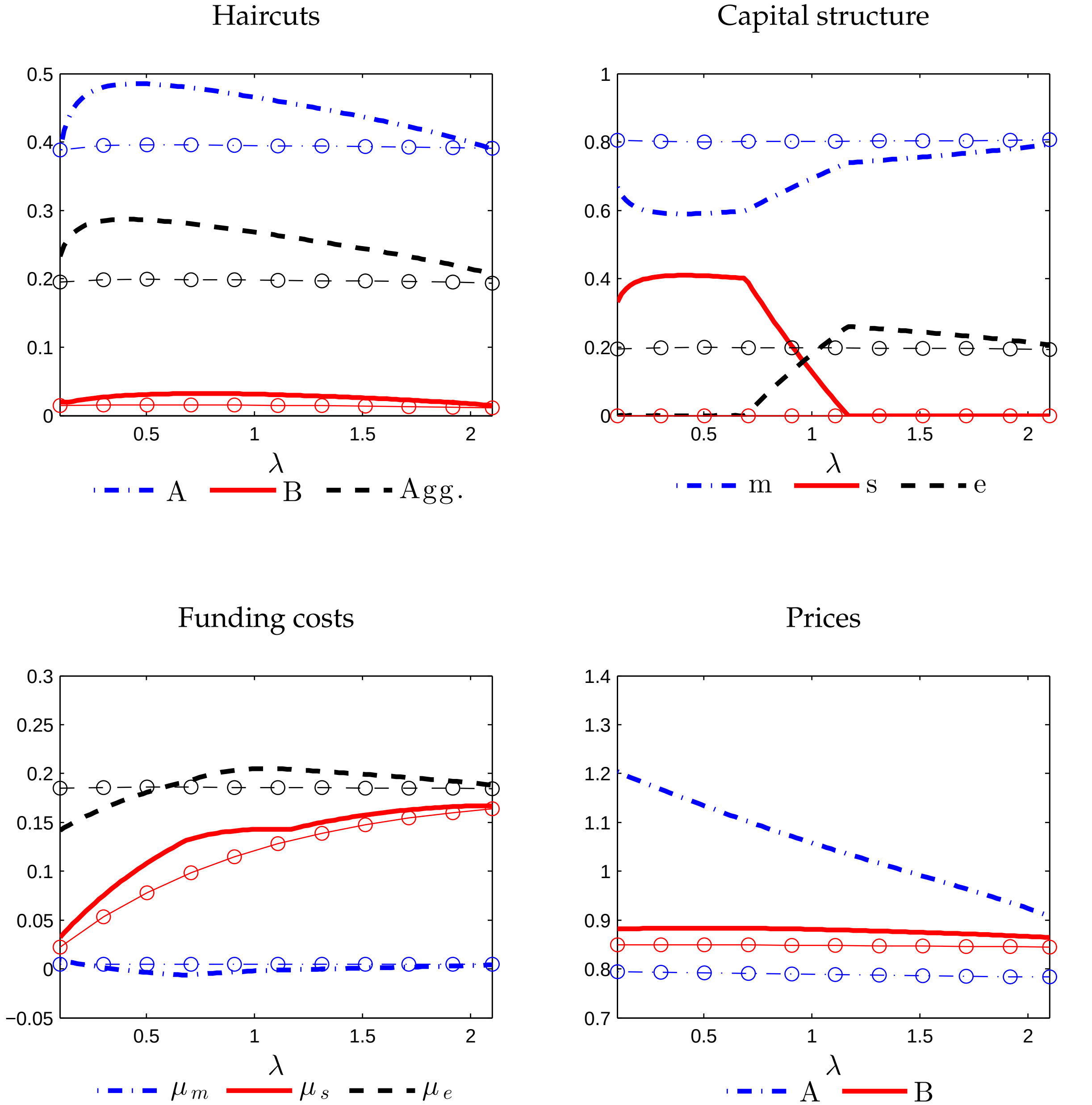


Figure 6: Flight to quality

The yield on money, the spread between shadow money and money (the safety premium), the equity premium, and the crash-return of B capital for the model with a dispersion shock $\kappa_i^a = 0.6$ (see discussion in Section 4.4), $\mu = 0$ and $\eta = 2.8$.

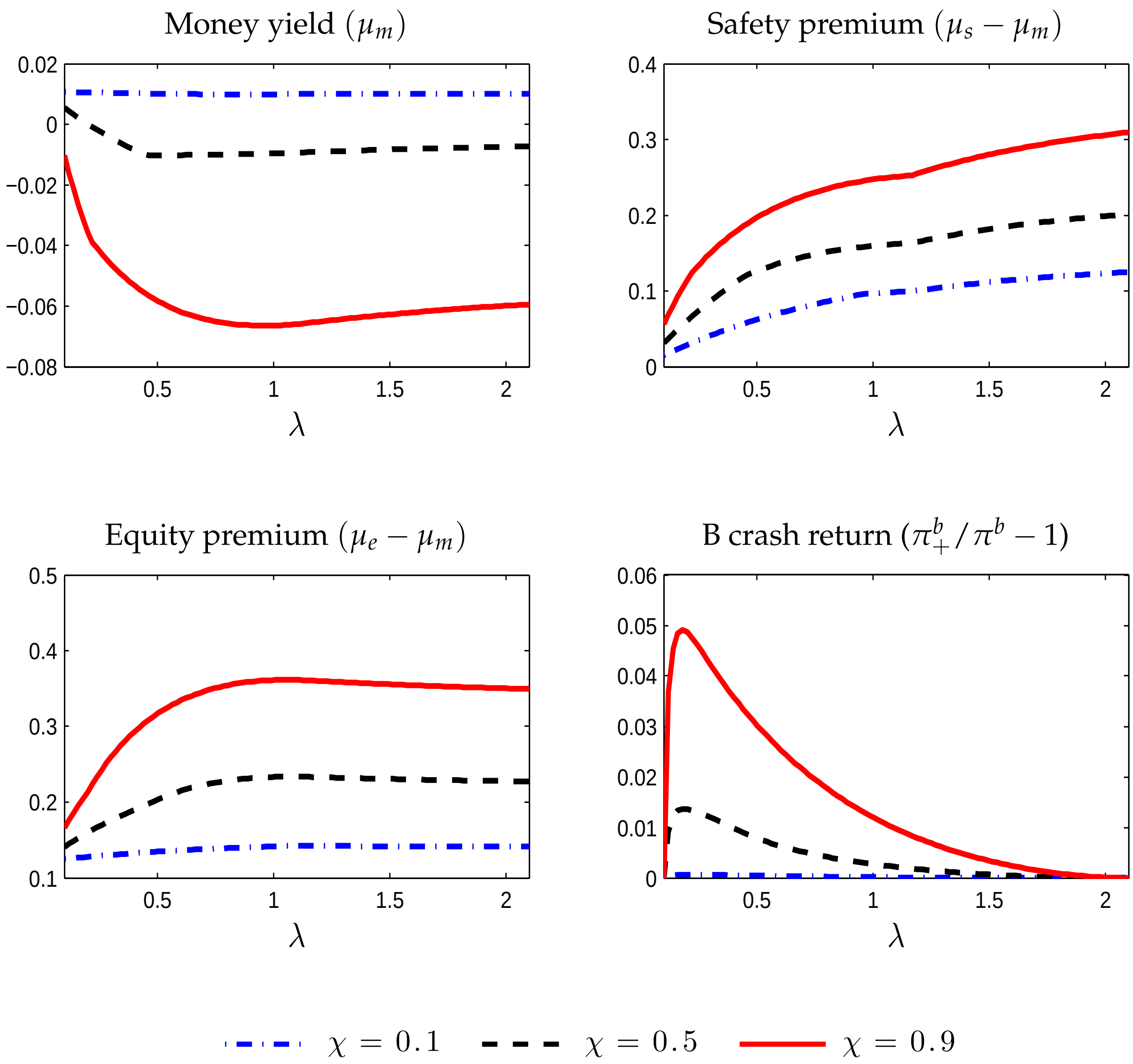


Figure 7: Asset purchases (LSAP)

In an asset purchase (LSAP) intervention, the central bank buys risky capital and sells safe capital. Here, LSAP lowers the risky capital share χ_t by 0.2, it arrives with 50% probability after a crash, and it is withdrawn at a 10% intensity rising to 100% in the last two panels. The announcement effect compares returns in crashes with and without LSAP. The ex ante effect compares prices in economies with and without the possibility of LSAP. See Appendix F for details.

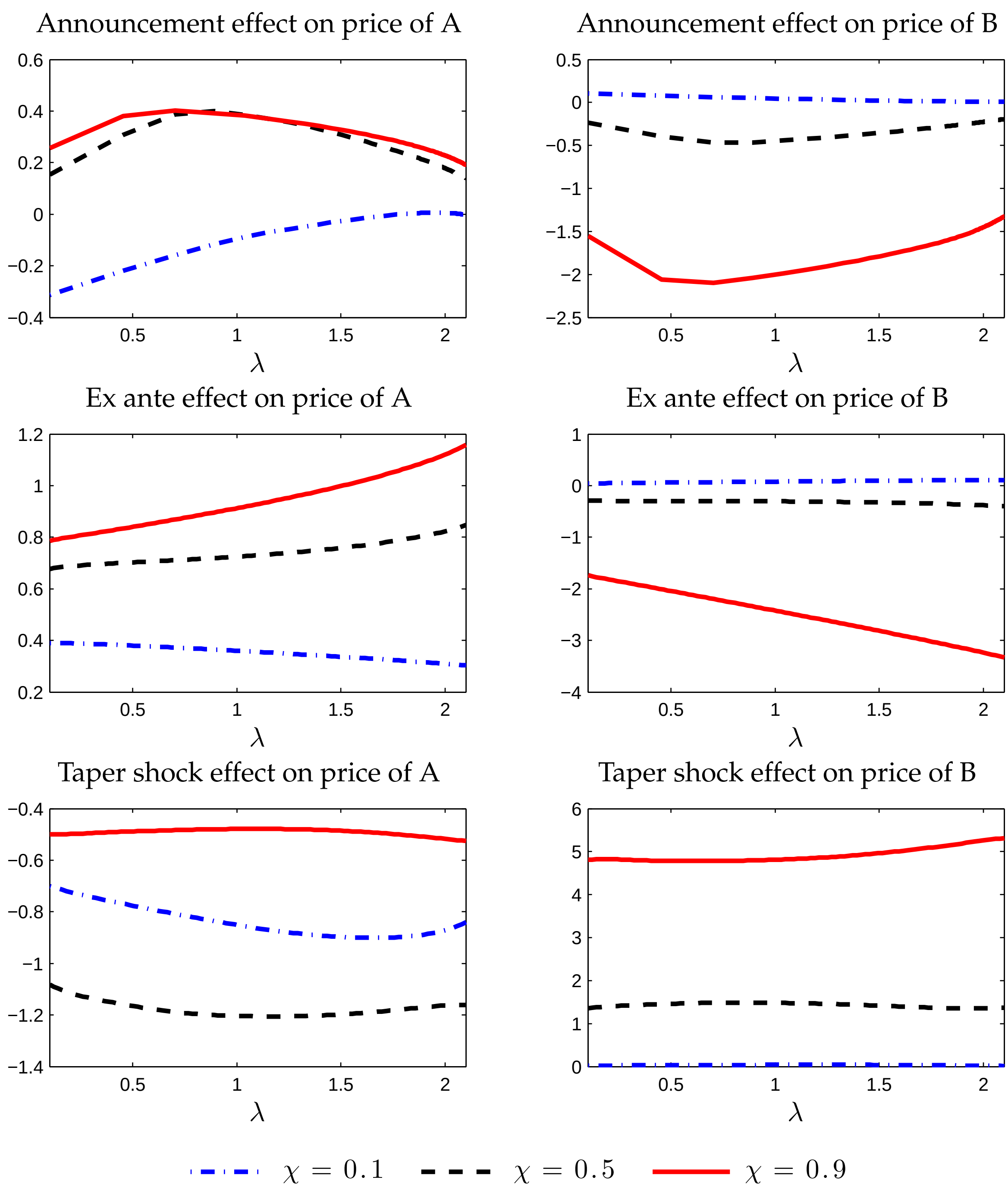


Figure 8: Operation Twist

This figure shows the effects on prices and pledgeability of changing the mix of safe government bonds from fixed-coupon (long-duration) bonds to floating-rate (zero-duration) bonds. The top left (right) plot shows the change in the price of the risky A (long-duration safe LB) capital π_t^a (π_t^{lb}), and the bottom plot shows the change in aggregate pledgeability $1-\kappa_{A,t}$. This figure uses the parameter values from Section 4.4 to generate strong flight to quality and $\mu = 0$ to keep growth invariant to the government's debt structure. Quantities on the vertical axis are in percent.

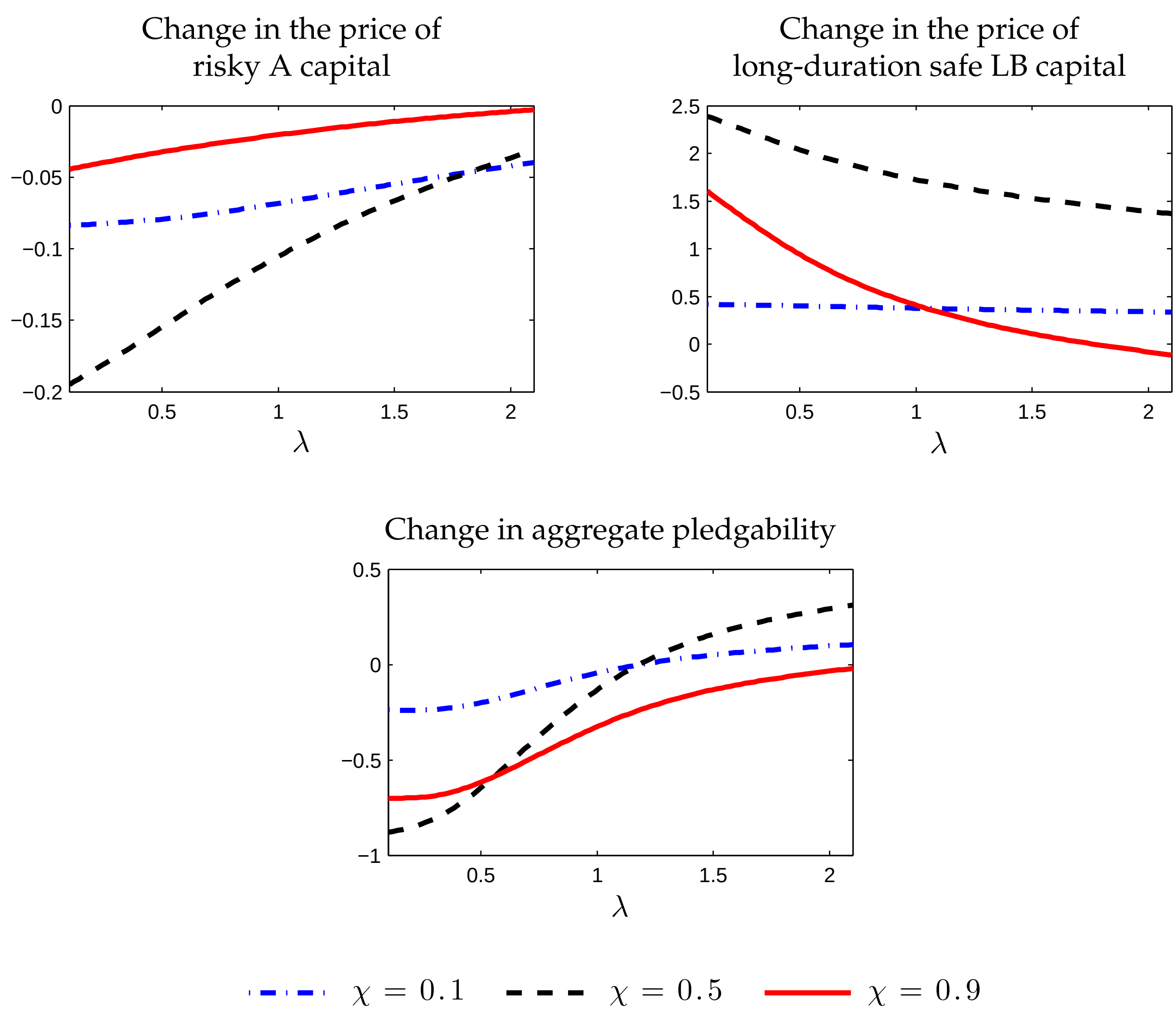
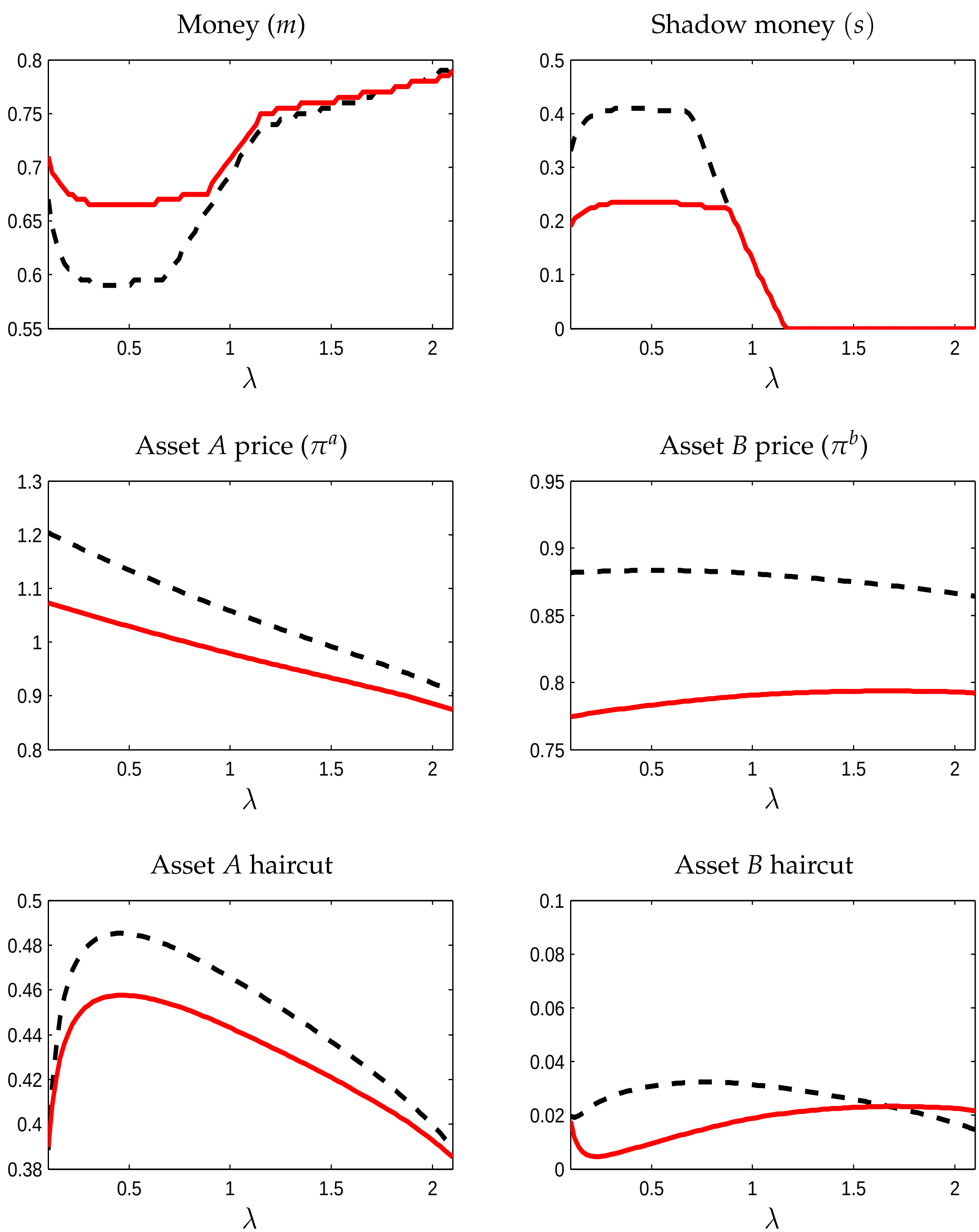


Figure 9: Liquidity requirements

This figure shows the effects of imposing a liquidity requirement on issuance, prices, and haircuts. Solid red lines plot equilibrium quantities under a liquidity requirement of $1 - \bar{l} = 10\%$. Dashed black lines are for the benchmark case of no liquidity requirement. Haircuts equal one minus collateral values, or $1 - \left(1 - \kappa^{i}\right)\left(1 - \kappa^{i}_{\pi}\right)$ for $i = a, b$. The figure fixes $\chi_t = 0.5$.



Appendix

A Filtering

Let $\mathcal{F}_t$ represent agents' information filtration. Agents form beliefs

$$\lambda_t = E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right]. \tag{A.1}$$

They learn about $\tilde{\lambda}$ from crash realizations and from uncertainty news. The solution to the filtering problem with Markov switching is analyzed in Wonham (1965) and Liptser and Shiryaev (2001).

Uncertainty news are given by the process $\left(\tilde{\lambda} - \lambda\right) dt + \sigma d\tilde{B}_t$. The crash realization process is the uncompensated Poisson process $d\tilde{J}_t$, which must coincide with the observed crash realization process dJ_t. The innovation to the household filtration $\mathcal{F}_t$ can therefore be represented as the 2×1 signal

$$d\zeta_t = \begin{bmatrix} \left(\tilde{\lambda}_t - \lambda_t\right) dt + \sigma d\tilde{B}_t \\ d\tilde{J}_t - \lambda_t dt \end{bmatrix}, \tag{A.2}$$

We seek to compute an innovations representation of the form

$$d\lambda_t = A_t dt + B_t' d\zeta_t. \tag{A.3}$$

Note that

$$\begin{aligned} d\lambda_t &= E\left[\tilde{\lambda}_{t+dt} \middle| \mathcal{F}_t, d\zeta_t\right] - E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right] && \text{(A.4)} \\ &= E\left[\tilde{\lambda}_t + d\tilde{\lambda}_t \middle| \mathcal{F}_t, d\zeta_t\right] - E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right] && \text{(A.5)} \\ &= \left[-\left(\lambda_t - \lambda^L\right) q^H + \left(\lambda^H - \lambda_t\right) q^L\right] dt + E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t, d\zeta_t\right] - E\left[\tilde{\lambda}_t \middle| \mathcal{F}_t\right]. && \text{(A.6)} \end{aligned}$$

The last line follows from the fact that the crash and news innovations are uncorrelated with the switching process for $\tilde{\lambda}$. The innovation representation is therefore the conditional mean of the population regression

$$\tilde{\lambda}_t - \lambda_t = \left[-\left(\lambda_t - \lambda^L\right) q^H + \left(\lambda^H - \lambda_t\right) q^L\right] dt + B_t' d\zeta_t + \epsilon_t. \tag{A.7}$$

The orthogonality condition for ϵ_t gives

$$\begin{aligned} B_t &= E\left[d\zeta_t d\zeta_t' \middle| \mathcal{F}_t\right]^{-1} E\left[d\zeta_t \left(\tilde{\lambda}_t - \lambda_t\right) \middle| \mathcal{F}_t\right] && \text{(A.8)} \\ &= \begin{bmatrix} \sigma^2 dt & 0 \\ 0 & \lambda_t dt \end{bmatrix}^{-1} \begin{bmatrix} \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right) dt \\ \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right) dt \end{bmatrix} && \text{(A.9)} \\ &= \begin{bmatrix} \frac{1}{\sigma^2} \\ \frac{1}{\lambda_t} \end{bmatrix} \left(\lambda^H - \lambda_t\right)\left(\lambda_t - \lambda^L\right). && \text{(A.10)} \end{aligned}$$

Therefore, we can write the dynamics of the perceived crash intensity as

$$
\begin{aligned}
d\lambda_t &= \left[-\left(\lambda_t-\lambda^L\right)q^H+\left(\lambda^H-\lambda_t\right)q^L\right]dt+\left(\lambda^H-\lambda_t\right)\left(\lambda_t-\lambda^L\right)\left[\begin{array}{cc}\frac{1}{\sigma^2} & \frac{1}{\lambda_t}\end{array}\right]d\zeta_t && \text{(A.11)}\\
&= \left[-\left(\lambda_t-\lambda^L\right)q^H+\left(\lambda^H-\lambda_t\right)q^L\right]dt+\left(\lambda^H-\lambda_t\right)\left(\lambda_t-\lambda^L\right)\left(\frac{dB_t}{\sigma}+\frac{dN_t}{\lambda_t}\right), && \text{(A.12)}
\end{aligned}
$$

where $dB_t=\frac{1}{\sigma}\left(\tilde{\lambda}_t-\lambda_t\right)dt+d\tilde{B}_t$ is a Brownian motion and $dN_t=d\tilde{J}_t-\lambda_t dt=dJ_t-\lambda_t dt$ is a compensated Poisson jump process with intensity λ_t, both adapted to $\mathcal{F}_t$. Thus,

$$
\frac{d\lambda_t}{\left(\lambda^H-\lambda_t\right)\left(\lambda_t-\lambda^L\right)} = \left(-\frac{q^H}{\lambda^H-\lambda_t}+\frac{q^L}{\lambda_t-\lambda^L}\right)dt+\frac{1}{\sigma}dB_t+\frac{1}{\lambda_t}dN_t. \qquad \text{(A.13)}
$$

This confirms (3).

B Liquidity and information sensitivity

We show how to motivate our definition of liquidity as a sufficient condition for avoiding information acquisition by arbitrageurs and the resulting adverse selection problem.

Consider an arbitrageur who runs a fund with assets under management normalized to one. For simplicity, assume that the fund is short-lived so its investment horizon has length dt. At the end of this period, the arbitrageur is compensated for outperforming a leverage-adjusted benchmark composed of the underlying assets.

We consider arbitrageurs in the equity and shadow money markets separately. This is without loss of generality as in equilibrium we will show that at any given state only one of the two markets is liquid (typically shadow money), ruling out informed arbitrage trades that exploit equity and shadow money simultaneously.

An equity fund trades money and equity, yielding a rate of return $dr_{S,t}=dr_{m,t}+w_t\left(dr_{e,t}-dr_{m,t}\right)$, where w_t is the portfolio weight in equity. On the other hand, her benchmark return is $dr_{I,t}=dr_{m,t}+\overline{w}_t\left(dr_{e,t}-dr_{m,t}\right)$, where $\overline{w}_t=E_t\left[w_t|\,\lambda_t\right]$. In order to obtain finite demand, suppose the fund faces a quadratic position cost $\frac{\alpha}{2}\left(w_t-\overline{w}_t\right)^2$. This means that an infinite position is prohibitively expensive. This could be due to some form of risk aversion or more broadly decreasing returns to scale at the trading strategy level.

Let ϑ_t be the LaGrange multiplier for the constraint $\overline{w}_t=E_t\left[w_t|\,\lambda_t\right]$. An informed arbitrageur thus maximizes

$$
\begin{aligned}
V_t^{Arb}dt &= \max_{w_t}\; E_t\left[dr_{S,t}-dr_{I,t}|\,\tilde{\lambda}\right]-\frac{\alpha}{2}\left(w_t-\overline{w}_t\right)^2dt+\vartheta_t\left(\overline{w}_t-E_t\left[w_t|\,\lambda_t\right]\right)dt && \text{(B.1)}\\
&= \max_{w_t}\; E_t\left[\left(w_t-\overline{w}_t\right)\left(dr_{e,t}-dr_{m,t}\right)|\,\tilde{\lambda}\right]-\frac{\alpha}{2}\left(w_t-\overline{w}_t\right)^2dt && \text{(B.2)}\\
&\quad +\vartheta_t\left(\overline{w}_t-E_t\left[w_t|\,\lambda_t\right]\right)dt.
\end{aligned}
$$

The optimal policy is

$$w_t = \overline{w}_t + \frac{1}{\alpha}\left[\mu_{e,t} - \mu_{m,t} - \left(\frac{1}{\sigma}\sigma_{e,t} + \kappa_{e,t}\right)\left(\tilde{\lambda}_t - \lambda_t\right) - \vartheta_t\right]. \tag{B.3}$$

Averaging over $\tilde{\lambda}_t$ gives $\vartheta_t = \mu_{e,t} - \mu_{m,t}$. Therefore,

$$w_t = \overline{w}_t - \frac{1}{\alpha}\left(\frac{1}{\sigma}\sigma_{e,t} + \kappa_{e,t}\right)\left(\tilde{\lambda}_t - \lambda_t\right). \tag{B.4}$$

The resulting maximized objective is

$$V_t^{Arb} = \frac{1}{2\alpha}\left(\frac{1}{\sigma}\sigma_{e,t} + \kappa_{e,t}\right)^2\left(\tilde{\lambda}_t - \lambda_t\right)^2. \tag{B.5}$$

The ex ante (prior to the learning decision) maximized objective is

$$E_t\left[V_t^{Arb}\middle|\,\lambda_t\right] = \frac{1}{2\alpha}\left(\frac{1}{\sigma}\sigma_{e,t} + \kappa_{e,t}\right)^2 Var_t\left(\tilde{\lambda}_t\middle|\,\lambda_t\right). \tag{B.6}$$

Suppose the cost of learning accrues at a rate $f\,Var_t\left(\tilde{\lambda}_t\middle|\,\lambda_t\right)$. Then there will be no learning provided $\frac{1}{2\alpha}\left(\frac{1}{\sigma}\sigma_{e,t} + \kappa_{e,t}\right)^2 < f$ or simply

$$\left|\frac{1}{\sigma}\sigma_{e,t} + \kappa_{e,t}\right| < \sqrt{2\alpha f}. \tag{B.7}$$

When this restriction fails to hold, arbitrageurs find it optimal to acquire information. An adverse selection problem arises, reducing market liquidity. As a result, a household experiencing a liquidity event cannot quickly unload large holdings of informationally sensitive securities without incurring substantial costs in the form of price impact or bid-ask spreads. Outside of a liquidity event, households can avoid these costs by trading more patiently or by simply buying and holding. It is with this interpretation in mind that we model the trading of equity as prohibitively expensive in a liquidity event yet costless at other times.

By analogy, shadow money remains liquid as long as

$$|\kappa_{s,t}| < \sqrt{2\alpha f}. \tag{B.8}$$

We can therefore equate $\sqrt{2\alpha f}$ with $\overline{\kappa}$ in the model. This condition ensures that households can liquidate their shadow money holdings quickly at no cost when the need arises as their trading partners can be confident that they are not dealing with privately-informed agents.

C Proofs and derivations

C.1 Household policy

The following calculation is used in the household problem:

$$
\begin{aligned}
E_t\left[C_t dN_t^h\right] &= E_t\left[\min\left\{\overline{C}_t, l_t\right\} dN_t^h\right] && \text{(C.1)}\\
&= Pr\left(dJ_t^h = 1\right) E_t\left[\min\left\{\overline{C}_t, l_t\right\} dN_t^h \middle| dJ_t^h = 1\right] && \text{(C.2)}\\
&\quad + Pr\left(dJ_t = 1\right) E_t\left[\min\left\{\overline{C}_t, l_t\right\} dN_t^h \middle| dJ_t = 1\right] &&\\
&= he^{-\tau\lambda_t}\left[\int_0^\infty \min\left\{\overline{C}_t, m_t + s_t\right\} df\left(\overline{C}_t\right)\right] dt && \text{(C.3)}\\
&\quad + h\left(1 - e^{-\tau\lambda_t}\right)\left[\int_0^\infty \min\left\{\overline{C}_t, m_t\right\} df\left(\overline{C}_t\right)\right] dt &&\\
&= \frac{h}{\eta}\left[e^{-\tau\lambda_t}\left(1 - e^{-\eta(m_t+s_t)}\right) + \left(1 - e^{-\tau\lambda_t}\right)\left(1 - e^{-\eta m_t}\right)\right] dt, &&
\end{aligned}
$$

where $f(\cdot)$ is the density of an exponentially-distributed random variable with mean $1/\eta$. Plugging into the HJB equation (17) and taking first-order conditions gives (21) and (22).

C.2 Liquidity provision policy

Let l_t^n and l_t^c be the amount of liquidity and crash-proof liquidity (as shares of total assets) that an intermediary provides. These liabilities enjoy funding cost advantages due to household demand for liquidity but they must be sufficiently collateralized to keep their information sensitivity low. We can write the liquidity provision formulation of the intermediary's problem as follows:

$$
\mu_{e,t} V_t dt = \max_{k_t^i, \iota_t^i, l_t^j, \sigma_{j,t}, \kappa_{j,t}} \left[\left(\gamma^a - \iota^a\right) k_t^a + \left(\gamma^b - \iota^b\right) k_t^b\right] dt + E_t\left[dV_t\right] \tag{C.4}
$$

for $i = a, b$ and $j = n, c$, subject to the myriad constraints

$$
\begin{aligned}
l_t^c(1 - \kappa_{c,t}) + l_t^n(1 - \kappa_{n,t}) &\leq 1 - \kappa_{A,t} && \text{(C.5)}\\
l_t^c + l_t^n &\leq 1 && \text{(C.6)}\\
l_t^j &\geq 0, \quad j = n, c && \text{(C.7)}\\
\kappa_{j,t} + \frac{\sigma_{j,t}}{\sigma} &\leq \overline{\kappa}, \quad j = n, c && \text{(C.8)}\\
\sigma_{j,t} &\geq 0, \quad j = n, c && \text{(C.9)}\\
l_t^c \sigma_{c,t} + l_t^n \sigma_{n,t} &\leq \sigma_{A,t} && \text{(C.10)}\\
l_t^c \kappa_{c,t} + l_t^n \kappa_{n,t} &\leq \kappa_{A,t} && \text{(C.11)}\\
\kappa_{c,t} &= 0 && \text{(C.12)}\\
\kappa_{n,t} &\geq 0. && \text{(C.13)}
\end{aligned}
$$

The collateral value of assets $1-\kappa_{A,t}$ is given in (24). Constraint (C.5) is the collateral constraint. Constraints (C.6)–(C.7) are non-negativity constraints. Constraint (C.8) is the information sensitivity constraint (part (i) of Definition 1). Constraints (C.9)–(C.13) say that the payouts on the liabilities of the intermediary must be nondecreasing in the value of its assets (see Section 2.6). Constraint (C.12) also says that crash-proof liquidity cannot suffer a loss (part (ii) of Definition 1).

Adapting (15) to the liquidity provision problem stated here and substituting, the intermediary HJB equation becomes

$$\begin{aligned} 0 \;=\; & \max_{k_t^i,\iota_t^i,l_t^j,\sigma_{j,t},\kappa_{j,t}} \left[\left(\gamma^a-\iota^a\right)k_t^a+\left(\gamma^b-\iota^b\right)k_t^b\right]dt+E_t\left[dA_t\right] \\ & -A_t\left[\mu_{e,t}-l_t^c\left(\mu_{e,t}-\mu_{m,t}\right)-l_t^n\left(\mu_{e,t}-\mu_{s,t}\right)\right]dt. \end{aligned} \tag{C.14}$$

for $i=a,b$ and $j=n,c$. The linearity of the intermediary problem allows us to focus on liquidity provision given only asset risk $\sigma_{A,t}$ and $\kappa_{A,t}$ (we characterize the asset choice in Section 3.3). This amounts to solving the funding cost minimization problem

$$\max_{l_t^j,\sigma_{j,t},\kappa_{j,t}} \; -\mu_{e,t}+l_t^c\left(\mu_{e,t}-\mu_{m,t}\right)+l_t^n\left(\mu_{e,t}-\mu_{s,t}\right) \tag{C.15}$$

for $j=n,c$ subject to (C.5)–(C.13) with $\sigma_{A,t}$ and $\kappa_{A,t}$ taken as given. The funding spreads come from the household optimality conditions (21) and (22) and in equilibrium they must be consistent with the amount of liquidity provided by the representative intermediary. The simple nature of this problem is due to the fact that liabilities can be re-optimized at each date and at no cost. We summarize the solution with the following proposition:

Proposition A.1. *For a given level of asset crash risk $\kappa_{A,t}$, the intermediary's optimal liquidity provision policy is characterized by $\sigma_n=\sigma_c=\kappa_c=0$, $\kappa_n=\overline{\kappa}$, and*

i. $l_t^c=1-\frac{\kappa_{A,t}}{\overline{\kappa}}$ *and* $l_t^n=\frac{\kappa_{A,t}}{\overline{\kappa}}$ *if* $\kappa_{A,t}\le\overline{\kappa}$ *and* $\kappa_{A,t}<\frac{\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$*;*

ii. $l_t^c=0$ *and* $l_t^n=\frac{1-\kappa_{A,t}}{1-\overline{\kappa}}$ *if* $\kappa_{A,t}>\overline{\kappa}$ *and* $\kappa_{A,t}>1-\frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$*;*

iii. $l_t^c=1-\kappa_{A,t}$ *and* $l_t^n=0$ *if* $\lambda_t>-\frac{1}{\tau}\log\left(1-\overline{\kappa}\right)$*; and*

iv. $l_t^c=1-\kappa_{A,t}-\frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *and* $l_t^n=\frac{1}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *otherwise.*

Proof. Dropping time subscripts and substituting $\kappa_c=0$ (from (C.12)), the Lagrangian is

$$\begin{aligned} \max_{l^j,\sigma_j,\kappa_n} \; & -\mu_e+l^c\left(\mu_e-\mu_m\right)+l^n\left(\mu_e-\mu_s\right)+\theta_1\left[1-\kappa_A-l^c-l^n(1-\kappa_n)\right] \\ & +\theta_2\left(1-l^c-l^n\right)+\theta_3 l^c+\theta_4 l^n+\theta_5\left(\overline{\kappa}-\sigma_c/\sigma\right)+\theta_6\left(\overline{\kappa}-\kappa_n-\sigma_n/\sigma\right) \\ & +\theta_7\sigma_c+\theta_8\sigma_n+\theta_9\left(\sigma_A-l^c\sigma_c-l^n\sigma_n\right)+\theta_{10}\left(\kappa_A-l^n\kappa_n\right)+\theta_{11}\kappa_n. \end{aligned} \tag{C.16}$$

for $j=n,c$. Since $\mu_m,\mu_s<\mu_e$, the intermediary wants to maximize l^n and l^c.

Consider the choice of σ_c. It is bounded below by 0 and above by $\overline{\kappa}\sigma$. It can potentially tighten liquidity provision through θ_9 (since $l^c\ge 0$). Therefore, we can set $\sigma_c=0$ without loss of generality, which gives $\theta_5=0$.

Next, consider the choice of σ_n. It can only tighten liquidity provision through θ_6 or θ_9 so we can set $\sigma_n = 0$ without loss of generality. This implies $\theta_9 = 0$.

To determine κ_n, we will first show that θ_1 always binds. Suppose θ_1 is slack. Then θ_2 must bind since if it does not l^c can be raised further (recall $\sigma_c = 0$). Thus if θ_1 is slack, θ_2 must bind. But then it is profitable to raise l^c and reduce l^n one-for-one since l^c has a greater funding advantage ($l^n > 0$ since otherwise $l^c = 1$ and θ_1 would be violated). We conclude that θ_1 binds.

Now back to κ_n: Note that since θ^1 binds, $1-\kappa_A = l^c + l^n(1-\kappa_n)$, and since $l^n + l^c \le 1$, we automatically have $l^n\kappa_n \le \kappa_A$ so θ_{10} is always satisfied and therefore redundant. This means that it is unambiguously better to raise κ_n so as to increase liquidity production. As κ_n is bounded from above by $\overline{\kappa}$, we conclude $\kappa_n = \overline{\kappa}$ (and $\theta_{11} = 0$).

Turning to l^n and l^c, we can now substitute $\sigma_c = \sigma_n = 0$, $\kappa_n = \overline{\kappa}$, $\theta_5 = \theta_9 = \theta_{11} = 0$, as well as impose complementary slackness on θ_6, θ_7 and θ_8, none of which involve l^n or l^c. We can also remove the redundant θ_{10}. The Lagrangian simplifies to

$$\begin{aligned}\max_{l^n,l^c} \quad & -\mu_e + l^c(\mu_e - \mu_m) + l^n(\mu_e - \mu_s) \\ & +\theta_1\left[1-\kappa_A - l^c - l^n(1-\overline{\kappa})\right] + \theta_2(1-l^c-l^n) + \theta_3 l^c + \theta_4 l^n.\end{aligned} \tag{C.17}$$

The optimality conditions are

$$\mu_e - \mu_m = \theta_1 + \theta_2 - \theta_3 \tag{C.18}$$
$$\mu_e - \mu_s = (1-\overline{\kappa})\theta_1 + \theta_2 - \theta_4. \tag{C.19}$$

We already know $\theta_1 > 0$. Since $\mu_e - \mu_m > \mu_e - \mu_s$, we also have $\overline{\kappa}\theta_1 > \theta_3 - \theta_4$.

Case (i): Suppose $\theta_2 > 0$ so $l^n + l^c = 1$. Since $\theta_1 > 0$, $l^n = \kappa_A/\overline{\kappa}$ and so $l^c = 1-\kappa_A/\overline{\kappa}$. This clearly requires $\kappa_A \le \overline{\kappa}$ and gives $\theta_3 = \theta_4 = 0$. Substituting for the spreads from the household problem, $\theta_1 = \frac{1}{\overline{\kappa}}h(\psi-1)\left(1-e^{-\tau\lambda}\right)e^{-\eta(1-\kappa_A/\overline{\kappa})}$ and $\theta_2 = h(\psi-1)\left[e^{-\tau\lambda}e^{-\eta} + \left(1-\frac{1}{\overline{\kappa}}\right)\left(1-e^{-\tau\lambda}\right)e^{-\eta(1-\kappa_A/\overline{\kappa})}\right]$. For θ_2 to be positive, this also requires $\kappa_A < \frac{\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$.

Case (ii): Suppose $\theta_3 > 0$ so $l^c = 0$. Then $\theta_2 = 0$ by case (i). Since $\theta_1 > 0$, $l^n = (1-\kappa_A)/(1-\overline{\kappa})$. For $\theta_2 = 0$, we must have $\kappa_A > \overline{\kappa}$. It follows that $\theta_4 = 0$. Substituting for the spreads gives $\theta_3 = h(\psi-1)\left[\frac{\overline{\kappa}}{1-\overline{\kappa}}e^{-\tau\lambda}e^{-\eta\left(\frac{1-\kappa_A}{1-\overline{\kappa}}\right)} - \left(1-e^{-\tau\lambda}\right)\right]$ and $\theta_1 = h(\psi-1)\frac{1}{1-\overline{\kappa}}e^{-\tau\lambda}e^{-\eta\left(\frac{1-\kappa_A}{1-\overline{\kappa}}\right)}$. Since $\theta_3 > 0$, $\kappa_A > 1 - \frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$.

Case (iii): Suppose $\theta_4 > 0$ so $l^n = 0$. Then $\theta_2 = 0$ by case (i). Since $\theta_1 > 0$, $l^c = 1-\kappa_A$ and so $\theta_2 = \theta_3 = 0$. Substituting for the spreads, $\theta_1 = h(\psi-1)e^{-\eta(1-\kappa_A)}$ and $\theta_4 = h(\psi-1)\left[(1-\overline{\kappa}) - e^{-\tau\lambda}\right]e^{-\eta(1-\kappa_A)}$. This case thus requires $\lambda > -\frac{1}{\tau}\log(1-\overline{\kappa})$ (so that $\theta_4 > 0$).

Case (iv): Suppose $\theta_2 = \theta_3 = \theta_4 = 0$. Since $\theta_1 > 0$, $l^n = 1-\kappa_A + l^n\overline{\kappa} - l^c$. Substituting for the spreads and solving, $l^n = \frac{1}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$, $l^c = 1-\kappa_A - \frac{1-\overline{\kappa}}{\eta}\log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}}\frac{e^{-\tau\lambda}}{1-e^{-\tau\lambda}}\right)$, and $\theta_1 = h(\psi-1)e^{-\eta(1-\kappa_A)}\left(\frac{e^{-\tau\lambda}}{1-\overline{\kappa}}\right)^{1-\overline{\kappa}}\left(\frac{1-e^{-\tau\lambda}}{\overline{\kappa}}\right)^{\overline{\kappa}}$.

□

We can now show that the optimal liquidity provision policy characterized in Proposition A.1 can be implemented with crash-proof liquid money, liquid shadow money, and illiquid equity as stated in Proposition 1 in the text:

Proof of Proposition 1. We already showed that $\kappa_{s,t} = \overline{\kappa}$ is optimal for delivering normal-times liquidity.

Case (i): The optimal capital structure can be implemented with $m_t = 1 - \frac{\kappa_{A,t}}{\overline{\kappa}}$ and $s_t = \frac{\kappa_{A,t}}{\overline{\kappa}}$. This case technically requires an infinite $\sigma_{e,t}$ but in practice it can be implemented with an arbitrarily small amount of equity and finite $\sigma_{e,t}$ with no impact on the equilibrium. Equity is illiquid.

Case (ii): The optimal capital structure can be implemented with $m_t = 0$ and $s_t = \frac{1-\kappa_{A,t}}{1-\overline{\kappa}}$. Equity is $e_t = \frac{\kappa_{A,t}-\overline{\kappa}}{1-\overline{\kappa}}$ with $\kappa_{e,t} = 1$ so equity is illiquid.

Case (iii): The optimal liquidity supply can be implemented with $m_t = 1 - \kappa_{A,t}$ and $s_t = 0$. Equity is $1 - m_t - s_t = \kappa_{A,t}$ and is wiped out in a crash ($\kappa_{e,t} = 1$) so equity is illiquid.

Case (iv): The optimal capital structure can be implemented with $m_t = (1 - \kappa_{A,t}) - \frac{1-\overline{\kappa}}{\eta} \log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}} \frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ and $s_t = \frac{1}{\eta} \log\left(\frac{\overline{\kappa}}{1-\overline{\kappa}} \frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$. Equity is again wiped out in a crash, $\kappa_{e,t} = 1$ so it is illiquid. □

C.3 Asset-side policy

From (14) and (15), applying Ito's Lemma to total assets and dropping time subscripts, the asset-side intermediary problem is

$$\begin{aligned} 0 \;=\; \max_{k^a,k^b,\iota^a,\iota^b} \; & \left(\gamma^a - \iota^a\right)k^a + \left(\gamma^b - \iota^b\right)k^b + \pi^a k^a \left[\mu_\pi^a + \phi\left(\iota^a\right) - \delta + \kappa^a \kappa_\pi^a \lambda\right] \\ & + \pi^b k^b \left[\mu_\pi^b + \phi\left(\iota^b\right) - \delta + \kappa^b \kappa_\pi^b \lambda\right] - A\left[\mu_e - \theta_1\left(1-\kappa_A\right) - \theta_2\right], \end{aligned} \tag{C.20}$$

where θ_1 is the multiplier on the collateral constraint and θ_2 is the multiplier on the equity capital requirement (both are scaled by assets). We have already substituted the liabilities-side optimality conditions (C.18) and (C.19). The multipliers θ_1 and θ_2 are determined in equilibrium by aggregate liquidity provision and collateral values, their formulas are included in the proof of Proposition A.1.

Taking derivatives of (C.20) with respect to ι^a and ι^b gives (26). Substituting for κ_A using (24) and differentiating with respect to k^a and k^b gives (25).

D Numerical solution

Using the adjustment cost function $\phi(\iota) = \frac{1}{\varphi}\left(\sqrt{1+2\varphi\iota} - 1\right)$, the optimality conditions for capital (25) become

$$\mu_e - \theta_1\left(1-\kappa_\pi^i\right)\left(1-\kappa^i\right) - \theta_2 \;=\; \frac{\gamma^i + \frac{1}{2\varphi}}{\pi^i} + \frac{\pi^i}{2\varphi} + \mu_\pi^i - \frac{1}{\varphi} - \delta + \kappa^i\kappa_\pi^i\lambda \tag{D.1}$$

for $i = a, b$. The multipliers are provided in the proof of Proposition A.1. Substituting for the investment cost function and the investment optimality conditions into (27), the dynamics of χ are

$$\begin{aligned} d\chi &= \mu(1-2\chi)\,dt + \chi(1-\chi)\left[\frac{\pi^a - \pi^b}{\varphi} + \lambda\left(\kappa^a - \kappa^b\right)\right] dt \qquad \text{(D.2)} \\ &\quad -\chi(1-\chi)\left[\frac{\kappa^a - \kappa^b}{\chi(1-\kappa^a) + (1-\chi)\left(1-\kappa^b\right)}\right] dJ. \end{aligned}$$

To get the dynamics of prices, apply Ito's Lemma:

$$\begin{aligned} \frac{d\pi^i}{\pi^i} &= \left[\frac{\pi^i_\lambda}{\pi^i}\left(\lambda - \lambda^L\right)\left(\lambda^H - \lambda\right)\left(-\frac{q^H}{\lambda^H - \lambda} + \frac{q^L}{\lambda - \lambda^L} - 1\right)\right. \qquad \text{(D.3)} \\ &\quad + \frac{1}{2}\frac{\pi^i_{\lambda\lambda}}{\pi^i}\left(\lambda - \lambda^L\right)^2\left(\lambda^H - \lambda\right)^2\frac{1}{\sigma^2} + \frac{\pi^i_\chi}{\pi^i}\chi(1-\chi)\left(\mu\left(\frac{1}{\chi} - \frac{1}{1-\chi}\right)\right. \\ &\quad \left.\left. + \frac{\pi^a - \pi^b}{\varphi} + \lambda\left(\kappa^a - \kappa^b\right)\right)\right] dt + \frac{\pi^i_\lambda}{\pi^i}\left(\lambda - \lambda^L\right)\left(\lambda^H - \lambda\right)\frac{1}{\sigma}dB \\ &\quad - \left[1 - \frac{\pi^i\left(\lambda + \frac{1}{\lambda}\left(\lambda - \lambda^L\right)\left(\lambda^H - \lambda\right), \chi - \frac{\chi(1-\chi)\left(\kappa^a - \kappa^b\right)}{\chi(1-\kappa^a) + (1-\chi)\left(1-\kappa^b\right)}\right)}{\pi^i}\right] dJ \end{aligned}$$

for $i = a, b$. We solve for π^a and π^b using Chebyshev collocation.

E Conditional state density

Let $f_t(\lambda, \chi)$ be the joint density of λ and χ at t given initial density $f_0(\lambda, \chi)$. We calculate f by solving the associated forward Kolmogorov equation (Hanson, 2007, Theorem 7.7):

$$\begin{aligned} \frac{\partial f_t}{\partial t} &= -\frac{\partial}{\partial \lambda}\left(\mu_\lambda f_t\right) - \frac{\partial}{\partial \chi}\left(\mu_\chi f_t\right) + \frac{1}{2}\frac{\partial^2}{\partial \lambda^2}\left(\sigma^2_\lambda f_t\right) \qquad \text{(E.1)} \\ &\quad + \left[\lambda^- f_t\left(\lambda^-, \chi^-\right)\left|\frac{\partial \lambda^-}{\partial \lambda}\right|\left|\frac{\partial \chi^-}{\partial \chi}\right| - \lambda f_t(\lambda, \chi)\right]. \end{aligned}$$

We have

$$\frac{\partial}{\partial\lambda}\left(\mu_\lambda f_t\right) = \left[2\lambda-\left(q^H+q^L\right)-\left(\lambda^H+\lambda^L\right)\right]f_t + \left[-q^H\left(\lambda-\lambda^L\right)+q^L\left(\lambda^H-\lambda\right)-\left(\lambda-\lambda^L\right)\left(\lambda^H-\lambda\right)\right]\frac{\partial f_t}{\partial\lambda} \tag{E.2}$$

$$\frac{\partial}{\partial\chi}\left(\mu_\chi f_t\right) = \left[\frac{1}{\varphi}\left(\pi^a-\pi^b\right)+\lambda\left(\kappa^a-\kappa^b\right)\right]\left[(1-2\chi)f_t+\chi(1-\chi)\frac{\partial f_t}{\partial\chi}\right] + \mu\left[(1-2\chi)\frac{\partial f_t}{\partial\chi}-2f_t\right]+\frac{1}{\varphi}\left(\pi^a_\chi-\pi^b_\chi\right)\chi(1-\chi)f_t \tag{E.3}$$

$$\frac{\partial^2}{\partial\lambda^2}\left(\sigma_\lambda^2 f_t\right) = 2\left[\left(\frac{\partial\sigma_\lambda}{\partial\lambda}\right)^2+\sigma_\lambda\left(\frac{\partial^2\sigma_\lambda}{\partial\lambda^2}\right)\right]f_t+4\sigma_\lambda\left(\frac{\partial\sigma_\lambda}{\partial\lambda}\right)\frac{\partial f_t}{\partial\lambda}+\sigma_\lambda^2\frac{\partial^2 f_t}{\partial\lambda^2}, \tag{E.4}$$

where $\sigma_\lambda=\left(\lambda-\lambda^L\right)\left(\lambda^H-\lambda\right)\frac{1}{\sigma}$, $\frac{\partial\sigma_\lambda}{\partial\lambda}=\left(\lambda^L+\lambda^H-2\lambda\right)\frac{1}{\sigma}$, and $\frac{\partial^2\sigma_\lambda}{\partial\lambda^2}=-\frac{2}{\sigma}$. The pre-jump state values λ^- and χ^- are given by

$$\lambda^- = \frac{\lambda^L\lambda^H}{\lambda^L+\lambda^H-\lambda} \tag{E.5}$$

$$\chi^- = \frac{1}{1+\left(\frac{1-\kappa^a}{1-\kappa^b}\right)\left(\frac{1-\chi}{\chi}\right)} \tag{E.6}$$

with $\frac{\partial\lambda^-}{\partial\lambda}=\frac{\lambda^L\lambda^H}{\left(\lambda^L+\lambda^H-\lambda\right)^2}$ and $\frac{\partial\chi^-}{\partial\chi}=\frac{\left(\frac{1-\kappa^a}{1-\kappa^b}\right)\left(\frac{1}{\chi^2}\right)}{\left[1+\left(\frac{1-\kappa^a}{1-\kappa^b}\right)\left(\frac{1-\chi}{\chi}\right)\right]^2}$. We solve the Kolmogorov equation with Chebyshev collocation as in the rest of the paper.

F Implementation of policy interventions

F.1 Large Scale Asset Purchases

Let $\zeta_t\in\{\zeta_E,\zeta_F\}$ denote the state of the central bank's balance sheet which is either "empty" ($\zeta_t=\zeta_E$) or "full" ($\zeta_t=\zeta_F$). An empty balance sheet is filled with probability $\beta_{LSAP}(\lambda_t,\chi_t)$ immediately after a crash in which case it becomes full. The size of the program is given as a state-contingent fraction $\alpha(\lambda_t,\chi_t)$ of the outstanding supply of A capital. A full balance sheet is emptied ($\zeta_t=\zeta_E$) with intensity $\beta_{TAPER}(\lambda_t,\chi_t)$.

We nest a permanent and transitory LSAP programs with $\beta_{TAPER}\left(\lambda_t,\chi_t\right)=0$ under a permanent policy and $\beta_{TAPER}\left(\lambda_t,\chi_t\right)>0$ under a transitory policy. We can express prices as $\pi^i_t=\pi^i\left(\lambda_t,\chi_t,\zeta_t\right)$ for $i=a,b$. The drift and after-crash value of prices when $\zeta_t=\zeta_E$ are

$$\mu^i_{\pi,\zeta_E} = \mu^i_{\pi,\zeta_0}-\lambda\beta_{LSAP}\left(\kappa^i_{\pi,LSAP}-\kappa^i_\pi\right) \tag{F.1}$$

$$1-\kappa^i_{\pi,\zeta_E} = \min\left\{1-\kappa^i_{\pi,LSAP},1-\kappa^i_\pi\right\}, \tag{F.2}$$

for $i = a, b$ where μ^i_{π,ζ_0} is given in (D.3), $1 - \kappa^i_{\pi,LSAP} = \frac{\pi^i(\lambda_+,\chi_+-\alpha,\zeta_F)}{\pi(\lambda,\chi,\zeta_E)}$ (LSAP shifts χ^+ to $\chi^+ - \alpha$), and $1 - \kappa^i_\pi = \frac{\pi(\lambda_+,\chi_+,\zeta_E)}{\pi(\lambda,\chi,\zeta_E)}$. These modified dynamics enter into (25). Under $\zeta_t = \zeta_F$, we have

$$\mu^i_{\pi,\zeta_F} = \mu^i_{\pi,\zeta_0} - \beta_{TAPER}\kappa^i_{\pi,TAPER}. \tag{F.3}$$

for $i = a, b$ with $1 - \kappa_{\pi,TAPER} = \frac{\pi(\lambda,\chi+\alpha,\zeta_E)}{\pi^i(\lambda,\chi,\zeta_F)}$. To keep things simple, we do not impose collateral constraints with respect to the reversal shock. This has the effect of understating the impact of policy withdrawal. In addition, the policy's entry and exit are generally not of equal size as the economy drifts in the meantime. In this case one can think of the central bank's balance sheet as retaining a residual position.

We measure the announcement effect of an LSAP program as the difference in crash returns with and without the intervention, $\kappa^i_\pi - \kappa^i_{\pi,LSAP}$. A policy reversal shock is measured analogously as $\frac{\pi^i(\lambda,\chi-\alpha,\zeta_E)}{\pi^i(\lambda,\chi,\zeta_F)} - 1$. We measure the effect of an unanticipated tapering shock that raises β_{TAPER} as $\frac{\pi^i(\lambda,\chi,\zeta_F|\beta^H_{TAPER})}{\pi^i(\lambda,\chi,\zeta_F|\beta^L_{TAPER})} - 1$.

In the case of a permanent intervention, the economy at $\zeta_t = \zeta_F$ corresponds to the benchmark economy and we can solve backwards to obtain prices under ζ_E. When interventions are transitory, there is two-way flow between ζ_E and ζ_F and we solve for prices under the two regimes simultaneously.

F.2 Operation Twist

We map the safe capital k^b_t to government debt by assuming that the private sector cannot create it but that the government issues it at the same rate as in the baseline model.[35] To model a change in duration within a given economy, we split type k^b capital into two pieces: zero-duration floating-rate debt k^{fb} and long-duration safe bonds k^{lb} (so $k^b_t = k^{fb}_t + k^{lb}_t$). Floating debt pays the floating rate μ_m (the yield of money) and trades at par in equilibrium. Long bonds pay the fixed coupon γ^b as in the baseline model. We avoid introducing an additional state variable by assuming that the central bank sets the relative shares $\alpha_t = k^{fb}_t / k^b_t$ and $1 - \alpha_t = k^{lb}_t / k^b_t$ as a policy variable.

In an Operation Twist intervention, the central bank buys long-term bonds and sells floating rate bonds of equal dollar amounts at post-announcement prices. This changes their relative shares α_t and $1 - \alpha_t$. Operation Twist thus changes the composition of government debt α_t, while keeping its value constant. The change in the quantity of government debt is characterized as follows.

Let x_- and x_+ denote the pre- and post-intervention values of a given quantity x. For example π^{bl}_+ is the post-intervention price of the long-term bond. Assuming that the government trades at post-announcement prices and that the intervention does not change

[35] Specifically, private investment ι^b_t is restricted to zero in the pricing equation (25). Existing long bonds depreciate on the balance sheet but the government controls their aggregate supply by setting issuance to $dk^{lb}_t = \left[\phi\left(\iota^{lb}_t\right) - \delta\right] dt$ absent a policy shock with $\pi^{lb}_t \phi'\left(\iota^{lb}_t\right) = 1$ and similarly for dk^{fb}_t.

the overall value of government liabilities, the pre- and post- quantities must respect

$$k_{-}^{b}\left[\alpha_{-}\pi_{+}^{fb}+\left(1-\alpha_{-}\right)\pi_{+}^{lb}\right] \quad = \quad k_{+}^{b}\left[\alpha_{+}\pi_{+}^{fb}+\left(1-\alpha_{+}\right)\pi_{+}^{lb}\right]. \tag{F.4}$$

We solve for net new issuance $\frac{k_{+}^{b}}{k_{-}^{b}}$ as a function of equilibrium prices and the change in the debt maturity mix $\alpha_{+}-\alpha_{-}$.

F.3 Liquidity requirements

The solution to the intermediary capital structure problem in the presence of a liquidity requirement is characterized by

Proposition F.1. *Let $\bar{l}$ be the upper bound on liquidity creation. The intermediary's optimal liquidity provision policy is implemented by*

i. $m_t=\bar{l}$ *and* $s_t=0$ *if* $\kappa_{A,t}\leq 1-\bar{l}$;

ii. $m_t=\bar{l}-\frac{\kappa_{A,t}-(1-\bar{l})}{\bar{\kappa}}$ *and* $s_t=\frac{\kappa_{A,t}-(1-\bar{l})}{\bar{\kappa}}$ *if* $1-\bar{l}\leq\kappa_{A,t}\leq 1-\bar{l}+\bar{\kappa}\bar{l}$ *and* $\kappa_{A,t}<1-\bar{l}+\frac{\bar{\kappa}}{\eta}\log\left(\frac{\bar{\kappa}}{1-\bar{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$;

iii. $m_t=0$ *and* $s_t=\frac{1-\kappa_{A,t}}{1-\bar{\kappa}}$ *if* $\kappa_{A,t}>1-\bar{l}+\bar{\kappa}\bar{l}$ *and* $\kappa_{A,t}>1-\frac{1-\bar{\kappa}}{\eta}\log\left(\frac{\bar{\kappa}}{1-\bar{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$;

iv. $m_t=1-\kappa_{A,t}$ *and* $s_t=0$ *if* $\lambda_t>-\frac{1}{\tau}\log\left(1-\bar{\kappa}\right)$ *and* $\kappa_{A,t}\geq 1-\bar{l}$;

v. $m_t=\left(1-\kappa_{A,t}\right)-\frac{1-\bar{\kappa}}{\eta}\log\left(\frac{\bar{\kappa}}{1-\bar{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *and* $s_t=\frac{1}{\eta}\log\left(\frac{\bar{\kappa}}{1-\bar{\kappa}}\frac{e^{-\tau\lambda_t}}{1-e^{-\tau\lambda_t}}\right)$ *otherwise.*

Proof of Proposition F.1. The proof follows the recipe for Propositions A.1 and 1, replacing the limited liability constraint with the liquidity requirement. The only new case is case (i) which arises when total risk is so low that the solution in case (ii) would violate the liquidity requirement. □