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5 Policy Rules for Inflation Targeting

Glenn D. Rudebusch and Lars E. O. Svensson

5.1 Introduction

In this paper, we use a small empirical model of the U.S. economy to examine the performance of policy rules that are consistent with a monetary policy regime of inflation targeting. In the real world, explicit inflation targeting is currently pursued in New Zealand, Canada, the United Kingdom, Sweden, Australia, and the Czech Republic. Inflation targeting in these countries is characterized by (1) a publicly announced numerical inflation target (either in the form of a target range, a point target, or a point target with a tolerance interval), (2) a framework for policy decisions that involves comparing an inflation forecast to the announced target, thus providing an “inflation-forecast targeting” regime for policy, where the forecast serves as an intermediate target (cf. Haldane 1998; King 1994; Svensson 1997a), and (3) a higher than average degree of transparency and accountability.¹

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1. The rapidly growing literature on inflation targeting includes the conference volumes Leiderman and Svensson (1995), Haldane (1995), Federal Reserve Bank of Kansas City (1996), and Lowe (1997). See also the survey by Bernanke and Mishkin (1997).

We model an inflation-targeting policy regime using loss functions over policy goals. In our loss functions, inflation targeting always involves an attempt to minimize deviations of inflation from the explicit inflation target. In addition, however, our inflation-targeting loss functions also allow concerns about real output (or more precisely about the variability of output because the natural rate hypothesis is assumed). That is, we would argue there is no necessary connection between the specification of the loss function (other than that inflation variability must enter with a nonnegligible weight) and the specification of an inflation-targeting policy regime.² For support of this view, see, for example, the recent discussion by Fischer (1996), King (1996), Taylor (1996), and Svensson (1996) in Federal Reserve Bank of Kansas City (1996).³ Thus we interpret inflation targeting as consistent with a conventional quadratic loss function, where in addition to the variability of inflation around the inflation target there is some weight on the variability of the output gap.⁴

In examining policy rules that are consistent with inflation targeting, we consider two broad classes of rules: instrument rules and targeting rules. An explicit instrument rule expresses the monetary policy instrument as an explicit function of available information. We examine both optimal unrestricted instrument rules (a tradition that goes back at least to Taylor 1979; recent contributions include Blake and Westaway 1996) as well as optimal simple or restricted instrument rules, which involve only a few parameters or arguments (e.g., current inflation and output as in Taylor's 1993 rule). However, no central bank, whether inflation targeting or not, follows an explicit instrument rule (unrestricted or simple). Every central bank uses more information than the simple rules are based on, and no central bank would voluntarily restrict itself to react mechanically in a prescribed way to new information. The role of unrestricted or simple explicit instrument rules is at best to provide a baseline and comparison to the policy actually followed.

A targeting rule may be closer to the actual decision framework under inflation targeting. It is represented by the assignment of a loss function over devia-

2. One may argue, though, that the high degree of transparency and accountability serves to increase the commitment to minimizing the loss function, and to ensure that any concern about the real economy is consistent with the natural rate hypotheses and therefore reduces, or eliminates, any inflation bias.

3. As discussed in Svensson (forthcoming b), concerns about the stability of the real economy, model uncertainty, and interest rate smoothing all have similar effects under inflation targeting, namely, a more gradualist policy. Thus, if inflation is away from the inflation target, it is brought back to target more gradually (under "flexible" rather than "strict" inflation targeting, the inflation forecast hits the target at a horizon that is longer than the shortest possible). Svensson (1997b) argues that all inflation-targeting central banks in practice behave in this way, possibly with differing weights on the different reasons for doing so.

4. Because inflation-targeting central banks, like other central banks, also seem to smooth interest rates, our loss function also includes some weight on the variability of interest rate changes.

tions of a goal variable from a target level, or deviations of an intermediate target variable from an intermediate target level (cf. Rogoff 1985; Walsh 1998; Svensson 1997a, forthcoming b). A targeting rule, combined with a particular model, is only an *implicit* instrument rule; typically, the equivalent of a first-order condition has to be solved in order to find the corresponding *explicit* instrument rule. (For an intermediate target variable that the central bank has complete control over, the first-order condition is trivial: equality between the intermediate target variable and the target level.) As an example, note that one interpretation of “inflation-forecast targeting” is that the policy instrument is adjusted such that a conditional inflation forecast (the intermediate target variable) hits the inflation target at an appropriate horizon. Combined with a particular model, the instrument then becomes an implicit function of current information; when the corresponding system of equations is solved for the instrument, the explicit instrument rule results. We shall examine several such targeting rules below.

Our analysis proceeds as follows. Section 5.2 presents the empirical model we use, which is a simple two-equation model of U.S. output and inflation, somewhat similar to the theoretical model in Svensson (1997a). The model captures some realistic dynamics (e.g., monetary policy actions affect output before inflation) in a very simple but tractable form. Section 5.3 first attempts to reduce the confusion caused by the literature’s use of two different meanings of “targeting” and then presents the different instrument and targeting rules we examine. Section 5.4 reports our results, with focus on output and inflation variability under a large set of various policy rules. We find that some simple instrument and targeting rules involving inflation forecasts do remarkably well in minimizing the loss function (relative to the optimal rule). Other policy rules, some of which are frequently used in the literature as representing inflation targeting, do less well. Finally, section 5.5 concludes.

5.2 An Empirical Model of U.S. Output and Inflation

5.2.1 Motivation

Our choice of an empirical model of output and inflation is motivated by three considerations. First, we choose a simple linear model (as well as quadratic preferences below), so our analysis will be tractable and our results transparent. Our model consists of an aggregate supply equation (or “Phillips curve”) that relates inflation to an output gap and an aggregate demand equation (or “IS curve”) that relates output to a short-term interest rate. Obviously, our model glosses over many important and contentious features of the monetary transmission mechanism. Still, we feel that the model has enough richness—for example, in dynamics—to be of interest, especially when judged relative to some of the models used in previous theoretical discussions.

Second, our model captures the spirit of many practical policy-oriented macroeconometric models. Some (e.g., McCallum 1988) have argued that because there is no academic consensus on the structure of the economy, any proposed monetary policy rule should perform well in a variety of models. We are completely sympathetic to this argument. We believe that robustness to plausible model variation is a crucial issue and one that this conference volume, taken as a whole, should provide some insight into. However, we also believe that monetary policy analysis will be most convincing to central bankers (who are, of course, among the most important ultimate consumers of this research) if it is conducted using models that are similar in structure to the ones actually employed by central bankers. Thus, for example, at this stage of analysis, we focus our attention on a model that (1) uses a short-term interest rate as the policy instrument with no direct role for monetary aggregates, (2) is specified in terms of output gaps from trend instead of output growth rates, and (3) includes a Phillips curve with adaptive or autoregressive expectations that is consistent with the natural rate hypothesis. Such a structure is typical of many central bank policy models (including, e.g., the 11 models described in the central bank model comparison project for the Bank for International Settlements 1995), and because our empirical analysis uses U.S. data, we will be keen to match the properties of the Federal Reserve's venerable MPS macroeconometric model.⁵ Of course, the appropriate way to model expectations for policy analysis remains particularly contentious (see, e.g., the early discussion by Lucas 1976 and Sims 1982). We are persuaded that the importance of the Lucas critique is in large measure an empirical issue as in, for example, Oliner, Rudebusch, and Sichel (1996). In this regard, Fuhrer (1997) tests an autoregressive Phillips curve like ours against a forward-looking version and cannot reject it. Moreover, many policymakers appear more comfortable with the backward-looking version, including Federal Reserve Governor Meyer (1997) and former vice-chairman Alan Blinder (1998). Finally, in this regard, it should be noted that our backward-looking expectations may be particularly appropriate during the introduction of a new rule for inflation targeting. As stressed by Taylor (1993) and Bomfim and Rudebusch (1997), rational expectations may be unrealistic during the transition period when learning about the new policy rule is taking place.

Our third consideration in model selection is empirical fit to the data. To judge whether our model is able to reproduce the salient features of the data, we compare its fit and dynamics to an unrestricted vector autoregression

5. In 1996, the FRB/US model replaced the MPS model as the Federal Reserve Board's main quarterly macroeconometric model. The major innovation of this model is its ability to explicitly model various types of expectations including model-consistent ones (see Brayton and Tinsley 1996). Still, across a range of expectations processes, the properties of the new model are broadly similar to those of our model. E.g., the FRB/US model exhibits an output sacrifice ratio of between 2 and 5, which, as noted below, brackets our model's sacrifice ratio of about 3.

(VAR). VARs have become a very popular tool recently for describing the dynamics of monetary transmission, and they are a natural benchmark for model evaluation. Indeed, if one dislikes the structural interpretation that we attach to our model, one can simply consider it a reduced-form VAR and so our analysis is similar in spirit to Feldstein and Stock (1994) or Cecchetti (1995).

5.2.2 Model Estimates

The two equations of our model are

$$(1) \quad \pi_{t+1} = \alpha_{\pi 1} \pi_t + \alpha_{\pi 2} \pi_{t-1} + \alpha_{\pi 3} \pi_{t-2} + \alpha_{\pi 4} \pi_{t-3} + \alpha_y y_t + \varepsilon_{t+1},$$

$$(2) \quad y_{t+1} = \beta_{y1} y_t + \beta_{y2} y_{t-1} - \beta_r (\bar{i}_t - \bar{\pi}_t) + \eta_{t+1},$$

where π_t is quarterly inflation in the GDP chain-weighted price index (p_t) in percent at an annual rate, that is, $400(\ln p_t - \ln p_{t-1})$; $\bar{\pi}_t$ is four-quarter inflation in the GDP chain-weighted price index, that is, $(1/4) \sum_{j=0}^3 \pi_{t-j}$; i_t is the quarterly average federal funds rate in percent at an annual rate; \bar{i}_t is the four-quarter average federal funds rate, that is, $(1/4) \sum_{j=0}^3 i_{t-j}$; y_t is the percentage gap between actual real GDP (q_t) and potential GDP (q_t^*), that is, $100(q_t - q_t^*)/q_t^*$. These five variables were de-measured prior to estimation, so no constants appear in the equations.

The first equation relates inflation to a lagged output gap and to lags of inflation.⁶ The lags of inflation are an autoregressive or adaptive representation of inflation expectations, which is consistent with the form of the Phillips curve in the MPS model described in Brayton and Mauskopf (1987). In our empirical analysis below, we will not reject the hypothesis that the coefficients of the four inflation lags sum to one; thus we will use an accelerationist form of the Phillips curve, which implies a long-run vertical Phillips curve. The second equation relates the output gap to its own lags and to the difference between the average funds rate and average inflation over the previous four quarters—an approximate ex post real rate. The third term is a simple representation of the monetary transmission mechanism, which, in the view of many central banks, likely involves nominal interest rates (e.g., mortgage rates), ex ante real short and long rates, exchange rates, and possibly direct credit quantities as well. Equation (2) appears to be a workable approximation of these various intermediate transmission mechanisms.

The estimated equations, using the sample period 1961:1–96:2, are shown below. (Coefficient standard errors are given in parentheses, and the standard error of the residuals and Durbin-Watson statistics also are reported.)

6. Our series on the output gap is essentially identical to those that have been used in a variety of Federal Reserve and other government studies including, e.g., Congressional Budget Office (1995) and Hallman, Porter, and Small (1991). Our estimation results were little changed by using a flexible trend for potential output such as a quadratic trend.

$$\pi_{t+1} = .70\pi_t - .10\pi_{t-1} + .28\pi_{t-2} + .12\pi_{t-3} + .14y_t + \varepsilon_{t+1},$$

(.08) (.10) (.10) (.08) (.03)

$$SE = 1.009, \quad DW = 1.99,$$

$$y_{t+1} = 1.16y_t - .25y_{t-1} - .10(\bar{i}_t - \bar{\pi}_t) + \eta_{t+1},$$

(.08) (.08) (.03)

$$SE = 0.819, \quad DW = 2.05.$$

The equations were estimated individually by ordinary least squares.⁷ The hypothesis that the sum of the lag coefficients of inflation equals one had a p -value of .42, so this restriction was imposed in estimation.⁸

The subsample stability of our estimated equations is an important condition for drawing inferences from our model—whether it is given a structural or reduced-form (VAR) interpretation. In particular, because ours is a backward-looking model, the Lucas critique may apply with particular force. The historical empirical importance of this critique can be gauged by econometric stability tests (again, see Oliner et al. 1996). Our estimated equations appear to easily pass these tests. For example, consider a stability test from Andrews (1993): the *maximum* value of the likelihood ratio test statistic for structural stability over all possible breakpoints in the middle 70 percent of the sample. For our estimated inflation equation, the maximum likelihood ratio test statistic is 9.77 (in 1972:3), while the 10 percent critical value is 14.31 (from table 1 in Andrews 1993). Similarly, for the output equation, the maximum statistic is 7.87 (in 1982:4), while the 10 percent critical value is 12.27.

5.2.3 Comparison to Other Empirical Estimates

It is useful to compare our model with other empirical estimates in order to gauge its plausibility and its conformity to central bank models. From the perspective of monetary policy, two features are of particular interest: (1) the sensitivity of real activity to movements in the policy instrument and (2) the responsiveness of inflation to slack in the economy. Table 5.1 provides some evidence on both of these issues with a comparison of simulations from our model (1)–(2) and the MPS model, which was used regularly in the Federal Reserve's forecasting process for over 25 years. The experiment considered (as outlined in Smets 1995 and Mauskopf 1995) assumes that the Federal Reserve raises the federal funds rate by 1 percentage point for two years and then returns the funds rate to its original level thereafter. Table 5.1 reports for output and inflation the average difference between this simulation and a constant

7. Almost identical parameter estimates were obtained by the seemingly unrelated regressions and by system maximum likelihood methods because the cross-correlation of the errors is essentially zero.

8. This p -value is obtained from the usual F -statistic. Of course, nonstandard near-unit distributions may apply (see Rudebusch 1992), but these are likely to push the p -value even higher.

Table 5.1 Model Responses to a Funds Rate Increase (annual average difference from baseline in percentage points)

	Years after Funds Rate Increase		
	1	2	3
Output gap			
MPS ^a	-.07	-.45	-.99
Our model	-.07	-.41	-.66
Inflation			
MPS ^a	-.00	-.03	-.26
Our model	-.00	-.08	-.25

^aFrom table II.1 in Mauskopf (1995).

funds rate alternative in each of the first three years after the funds rate increase. The responses of the MPS model and our model to this temporary tightening of monetary policy are quite similar. In both models, output averages almost 0.5 percentage points lower in year 2 and between two-thirds and 1 percentage point lower in year 3, while inflation falls by about a quarter of a percentage point by year 3. Both models require about 3.3 years of a 1 percentage point output gap in order to induce a 1 percentage point change in the inflation rate—that is, they exhibit an output sacrifice ratio of just over 3.⁹ Most important, the magnitude of the link between the funds rate and inflation, which will be crucial for our inflation-targeting analysis, is essentially the same across the two models.¹⁰

Finally, it is also useful to compare the fit and impulse responses of our model to those of a VAR. While one may be deeply skeptical of the use of VARs for certain structural investigations (see Rudebusch 1998a), they can provide simple atheoretical summaries of the general dynamics of the data and thus can provide a useful benchmark for the overall fit of a model. Our model can be viewed as two restricted equations from a trivariate VAR with four lags. The VAR output equation regresses the gap on four lags of π , y , and i . The VAR inflation equation regresses inflation on the same lags as well as the contemporaneous value of the gap.¹¹ Table 5.2 compares the Schwarz and Akaike information criteria (SIC and AIC, respectively) for each VAR equation with those of our structural model. These two model selection criteria, which are functions of the residual sum of squares, are differentiated by their degrees-of-

9. For comparison, with a rough back-of-the-envelope calculation Ball (1994) reports an output sacrifice ratio for the United States of 2.4.

10. Our model estimates appear comparable to other recent small empirical structural models of the United States, including Fuhrer and Moore (1995), Clark, Laxton, and Rose (1996), and Fair and Howrey (1996). This is true even though the models use different interest rates in the IS curve: Fuhrer and Moore use an ex ante real long rate, Clark et al. use an ex ante real short rate, and Fair and Howrey use a nominal short rate. In fact, over the postwar historical sample, the four measured rates used appear to have moved together fairly closely.

11. Thus our VAR has a Cholesky factorization with a causal order of output, inflation, and, finally, the funds rate.

Table 5.2 Model Selection Criteria

	SIC	AIC
Inflation equation		
VAR	736.8	698.8
Our model	705.0	690.3
Output equation		
VAR	652.2	617.1
Our model	639.7	630.9

Note: SIC = Schwarz information criterion; AIC = Akaike information criterion.

freedom penalty for the number of parameters estimated. As shown in table 5.2, the structural model's inflation equation is favored over the VAR's inflation equation by both the SIC and the AIC. For the output equation, there is a split decision. The SIC, which more heavily penalizes extra parameters, favors the structural model, while the AIC favors the VAR. Overall, the information criteria do not appear to view our structural model restrictions unfavorably.

As a final comparison of our structural model to the VAR, figure 5.1 shows their responses to various shocks. This exercise completes the VAR with the usual VAR funds rate equation that regresses the funds rate on four lags of the three variables as well as contemporaneous values of the output gap and inflation. This VAR funds rate equation—with its interpretation as a Federal Reserve reaction function—is also added as a third equation to our model. The impulse responses of this structural system are shown as solid lines in figure 5.1, while the usual VAR impulse responses are shown as long-dashed lines along with their 95 percent confidence intervals as short-dashed lines. Because the funds rate reaction function equation is identical across the two systems, any differences in dynamics are attributable to the structural model restrictions on the output and inflation equations.

Figure 5.1 suggests that these restrictions do not greatly alter the dynamics of the model relative to an unrestricted VAR. In response to a positive funds rate shock, output and inflation decline in a similar manner in each system.¹² Also, a positive output shock persists over time and boosts inflation in a like fashion in both models. Only for an inflation shock (the left-hand column of fig. 5.1) do our model's responses edge outside the VAR's confidence intervals. This discrepancy reflects our model's output sensitivity to the real interest rate, which falls after an inflation shock because the VAR funds rate reaction function has such an extremely weak interest rate response to inflation. The implausibility of such VAR reaction functions, which mix several decades of very different Federal Reserve behavior, is highlighted in Rudebusch (1998a) and Judd and Rudebusch (1998). As shown below, with more plausible reaction

12. There is a modest, insignificant "price puzzle" exhibited by the VAR but not the structural model.

functions where the Fed raises the funds rate by more than inflation shock (so the real rate rises, as in the Taylor rule), output will fall following an inflation shock in the structural model.

5.3 Monetary Policy Rules

5.3.1 Instrument Rules and Targeting Rules

As noted in our introduction, by an (explicit) *instrument rule*, we mean that the monetary policy instrument is expressed as an explicit function of available information. Classic examples of instrument rules are the McCallum (1988) rule for the monetary base and the Taylor (1993) rule for the federal funds rate. By a *targeting rule*, we mean that the central bank is assigned to minimize a loss function that is increasing in the deviation between a target variable and the target level for this variable. The targeting rule will, as we shall see, imply an *implicit* instrument rule.

In the literature, the expression “targeting variable x_t ,” or “having a target level x^* for variable x_t ,” has two meanings. According to the first meaning, the expression above is used in the sense of “*setting a target* for variable x .”¹³ Thus “having a target” means “*using all relevant available information* to bring the target variable in line with the target,” or more precisely to minimize some loss function over expected future deviations of the target variable from the target level, for instance, the quadratic loss function

$$\min_{i_t} E_t \sum_{\tau=0}^{\infty} \delta^\tau (x_{t+\tau} - x^*)^2,$$

where δ , $0 < \delta < 1$, is a discount factor and E_t denotes the expectations operator conditional on information available in period t . We will use “targeting” according to this first meaning, following, for instance, Rogoff (1985), Walsh (1998), and Svensson (1997a, forthcoming b).

According to the second meaning, “targeting” and “targets” imply a particular *information restriction* for the instrument rule, namely, that the instrument must only depend on the gap between the target variable and the target level (and lags of this gap or lags of itself, or both).¹⁴ Thus the instrument rule is typically restricted to be

$$A(L)i_t = B(L)(x_t - x^*),$$

13. This is in line with *Webster's Ninth New Collegiate Dictionary*: **target** vt (1837) **1**: to make a target of; *esp*: to set as a goal **2**: to direct or use toward a target.

14. See, e.g., Judd and Motley (1992), McCallum (1997), and Bernanke and Woodford (1997). Bernanke and Woodford's criticism of Svensson's (1997a) use of the term “inflation-forecast targeting” seems to take the second meaning of “targeting” for granted and disregard the first meaning (which indeed is the one used in Svensson 1997a).

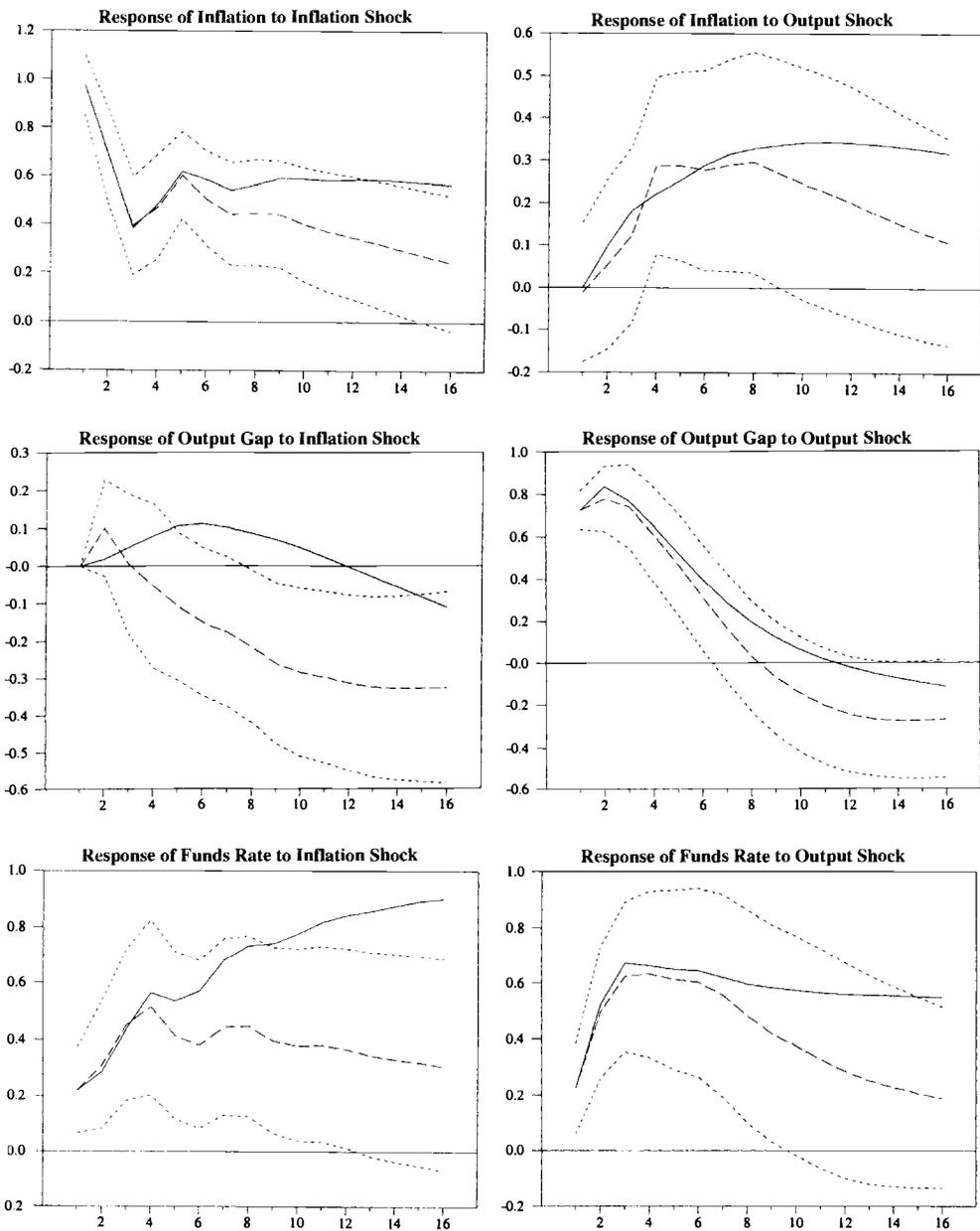
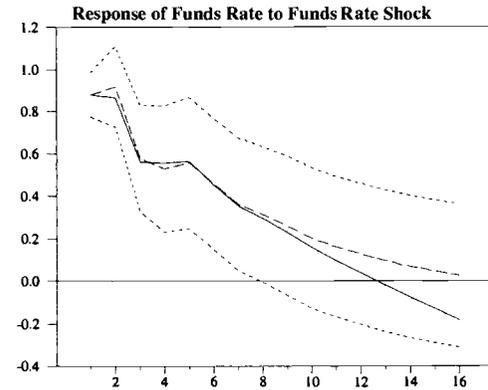
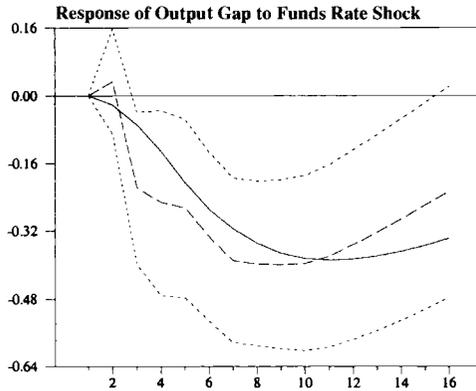
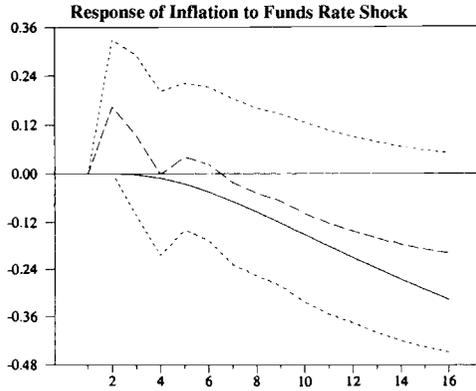


Fig. 5.1 VAR and structural model impulse responses

Note: Solid, impulse responses of the structural model (amended with the VAR interest rate equation). Long dashes, impulse responses of the VAR, with 95 percent confidence intervals (short dashes).



where $A(L)$ and $B(L)$ are polynomials in the lag operator L . To convey the second meaning, “responding only to $x_t - x^*$ ” seems more precise. Note that “inflation targeting” according to this second meaning, but *not* according to the first meaning, might correspond to an instrument rule like

$$i_t = hi_{t-1} + \phi(\pi_t - \pi^*).$$

This instrument rule turns out to perform much worse than other instrument rules. Note also that “inflation-forecast targeting” according to the second meaning (as in, e.g., Haldane 1997), but generally *not* according to the first meaning, might be an instrument rule like

$$i_t = hi_{t-1} + \phi(\pi_{t+T|t} - \pi^*),$$

where $\pi_{t+T|t}$ denotes some conditional inflation forecast of inflation T quarters ahead (more on this below).

A targeting rule for a *goal* variable is hence equivalent to having an objective for this variable. Examples of such rules are “annual inflation shall fall within the interval 1–3 percent per year on average at least three years out of four” and “minimize the expected value of a discounted sum of future weighted squared deviations of annual inflation from 2 percent per year and squared output gaps.” We shall assume an objective of the latter kind.

Similarly, a targeting rule for an *intermediate target* variable is equivalent to having a loss function for this intermediate target variable (an intermediate loss function), where the target level sometimes is not constant but depends on current information. The targeting rule can also be expressed as an equation that the target variable shall fulfill, for instance that the target level for the intermediate target is an explicit function of available information. The equation for the intermediate target variable may be interpreted as a first-order condition of an explicit or implicit loss function for the goal variable (see Svensson 1997a, forthcoming b, for examples). Thus a targeting rule in the end expresses the intermediate target level as a function of current information. Examples of intermediate target rules are “minimize the expected future deviation of M3 growth from the sum of a given inflation target, a forecast of potential output growth, and a velocity trend,” “keep the exchange rate within a ± 2.25 percent band around a given central parity,” and “adjust the instrument such that the forecast for inflation four to eight quarters ahead, conditional on the current state of the economy and on holding the instrument at constant level for the next eight quarters, is 2 percent per year.” We shall consider some targeting rules of this last kind.

A targeting rule in a given model implies a particular instrument rule, but this instrument rule is *implicit* rather than explicit. That is, the targeting rule has to be solved for the instrument rule in order to express it as a function of current information.

5.3.2 The Model

Let the model be given by equations (1) and (2), and let ϵ_t and η_t be i.i.d. zero-mean disturbances with variances σ_ϵ^2 and σ_η^2 and covariance $\sigma_{\epsilon\eta}$. The coefficients of the lagged inflation terms in equation (1) are restricted to sum to one,

$$\sum_{j=1}^4 \alpha_{\pi_j} = 1.$$

In our analysis, we will interpret “inflation targeting” as having a loss function for monetary policy where deviations of inflation from an explicit inflation target are always given some weight, but not necessarily all the weight. In particular, for a discount factor δ , $0 < \delta < 1$, we consider the intertemporal loss function in quarter t ,

$$(3) \quad E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau},$$

where the period loss function is

$$(4) \quad L_t = \bar{\pi}_t^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2$$

(π_t and $\bar{\pi}_t$ are now interpreted as the deviation from a constant given inflation target) and $\lambda \geq 0$ and $\nu \geq 0$ are the weights on output stabilization and interest rate smoothing, respectively.¹⁵ We will refer to the variables $\bar{\pi}_t$, y_t , and $i_t - i_{t-1}$ as the goal variables. As defined in Svensson (forthcoming b), “strict” inflation targeting refers to the situation where only inflation enters the loss function ($\lambda = \nu = 0$), while “flexible” inflation targeting allows other goal variables (nonzero λ or ν).

When $\delta \rightarrow 1$, the sum in equation (3) becomes unbounded. It consists of two components, however: one corresponding to the deterministic optimization problem when all shocks are zero and one proportional to the variances of the shocks. The former component converges for $\delta = 1$ (because the terms approach zero quickly enough), and the decision problem is actually well defined also for that case. For $\delta \rightarrow 1$, the value of the intertemporal loss function approaches the infinite sum of unconditional means of the period loss function, $E[L_t]$. Then the scaled loss function $(1 - \delta)E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}$ approaches the unconditional mean $E[L_t]$. It follows that we can also define the optimization problem for $\delta = 1$ and then interpret the intertemporal loss function as the unconditional mean of the period loss function, which equals the weighted sum of the unconditional variances of the goal variables,

$$(5) \quad E[L_t] = \text{var}[\bar{\pi}_t] + \lambda \text{var}[y_t] + \nu \text{var}[i_t - i_{t-1}].$$

15. Then i_t can be interpreted as the deviation of the federal funds rate from the sum of the inflation target and the natural real interest rate (the unconditional mean of the real interest rate).

We shall use equation (5) as our standard loss function, hence assuming the limiting case $\delta = 1$.

5.3.3 State-Space Representation

The model (1)–(2) has a convenient state-space representation,

$$(6) \quad X_{t+1} = AX_t + Bi_t + v_{t+1}.$$

The 9×1 vector X_t of state variables, the 9×9 matrix A , the 9×1 column vector B , and the 9×1 column disturbance vector v_t are given by

$$X_t = \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ y_t \\ y_{t-1} \\ i_{t-1} \\ i_{t-2} \\ i_{t-3} \end{bmatrix}, \quad A = \begin{bmatrix} \sum_{j=1}^4 \alpha_{\pi j} e_j + \alpha_y e_5 & & & & & & & & & \\ & e_1 & & & & & & & & \\ & & e_2 & & & & & & & \\ & & & e_3 & & & & & & \\ \beta_r e_{1,4} + \beta_{y1} e_5 + \beta_{y2} e_6 - \beta_r e_{7,9} & & & & & & & & & \\ & & & & e_5 & & & & & \\ & & & & & e_6 & & & & \\ & & & & & & e_7 & & & \\ & & & & & & & e_8 & & \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\beta_r/4 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad v_t = \begin{bmatrix} \varepsilon_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \eta_t \\ 0 \\ 0 \end{bmatrix},$$

where e_j ($j = 0, 1, \dots, 9$) denotes a 1×9 row vector, for $j = 0$ with all elements equal to zero, for $j = 1, \dots, 9$ with element j equal to unity and all other elements equal to zero, and where $e_{j:k}$ ($j < k$) denotes a 1×9 row vector with elements $j, j + 1, \dots, k$ equal to $1/4$ and all other elements equal to zero.

Furthermore, it is convenient to define the 3×1 vector Y_t of goal variables. It fulfills

$$(7) \quad Y_t = C_X X_t + C_i i_t,$$

where the vector Y_t , the 3×9 matrix C_X , and the 3×1 column vector C_i are given by

$$Y_t = \begin{bmatrix} \bar{\pi}_t \\ y_t \\ i_t - i_{t-1} \end{bmatrix}, \quad C_X = \begin{bmatrix} e_{1,4} \\ e_5 \\ -e_7 \end{bmatrix}, \quad C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Then the period loss function can be written

$$(8) \quad L_t = Y_t' K Y_t,$$

where the 3×3 matrix K has the diagonal $(1, \lambda, \nu)$ and all its off-diagonal elements are equal to zero.

5.3.4 Linear Feedback Instrument Rules

We will consider the class of linear feedback instruments rules, that is, rules of the form

$$(9) \quad i_t = fX_t,$$

where f is a 1×9 row vector. This class of rules includes the optimal instrument rule (see below).

For any given instrument rule of the form (9), the dynamics of the model follows

$$\begin{aligned} X_{t+1} &= MX_t + v_{t+1}, \\ Y_t &= CX_t, \end{aligned}$$

where the matrices M and C are given by

$$(10) \quad M = A + Bf,$$

$$(11) \quad C = C_x + C_y f.$$

For any given rule f that results in finite unconditional variances of the goal variables, the unconditional loss (5) fulfills¹⁶

$$(12) \quad E[L_t] = E[Y_t' K Y_t] = \text{trace}(K \Sigma_{yy}),$$

where Σ_{yy} is the unconditional covariance matrix of the goal variables (see the appendix).

5.3.5 The Optimal Instrument Rule

With equations (6) and (8), the problem is written in a form convenient for the standard stochastic linear regulator problem (cf. Chow 1970, Sargent 1987). Minimizing expression (3) in each quarter, subject to equation (6) and the current state of the economy, X_t , results in a linear feedback rule for the instrument of the form (9). In the limit when $\delta = 1$, the optimal rule converges to the one minimizing expression (5). The expression for the optimal instrument rule is given in the appendix.¹⁷

5.3.6 Inflation Forecasts

Given the lags in the monetary transmission mechanism, inflation-targeting central banks focus on inflation forecasts. Indeed, several of these banks have started to publish inflation reports that are completely devoted to describing the recent history and future prospects for inflation. The actual inflation forecasts that have been reported have fallen into two broad categories depending

16. The trace of a matrix A , $\text{trace}(A)$, is the sum of the diagonal elements of A .

17. Since there are no forward-looking variables, we need not distinguish between the commitment and discretion solutions because they are the same.

on how monetary policy is projected forward: constant-interest-rate inflation forecasts and rule-consistent inflation forecasts.

Constant-Interest-Rate Inflation Forecasts

Inflation-targeting central banks often refer to, and report, inflation forecasts conditional on a given constant interest rate. We will call such forecasts “constant-interest-rate inflation forecasts.” Such inflation forecasts are frequently used in the following way. If a constant-interest-rate inflation forecast for the current interest rate is above (below) target for a given horizon, monetary policy has to be tightened (eased) and the interest rate increased (decreased). If the inflation forecast is on target, the current interest rate setting is deemed appropriate (see, e.g., Mayes and Riches 1996; Svensson 1997a). Such forecasts, based on a fixed nominal rate, may seem overly simplistic,¹⁸ but they have been widely used at central banks, perhaps most notably at the Bank of England, where (before operational independence in 1997) the Bank produced such forecasts because it could not presuppose policy changes by the government.¹⁹

In an attempt to represent this, it is convenient to define the “ T -quarter-ahead constant-interest-rate inflation forecast.” By this we mean a forecast of four-quarter inflation $T \geq 2$ quarters ahead, conditional on a given constant current and future interest rate (and on the current state variables X_t). Denote this conditional forecast by $\bar{\pi}_{t+T|t}(i)$, for the given constant current and future interest rate i . It is given by

$$(13) \quad \bar{\pi}_{t+T|t}(i) \equiv e_{1,4} \tilde{M}^{T-1} (AX_t + Bi),$$

where \tilde{M} is a 9×9 matrix given by

$$(14) \quad \tilde{M} = A + Be_7$$

(we note that $e_7 X_{t+1} = i_t$).

Consider also the T -quarter-ahead constant-interest-rate inflation forecast in quarter t , when the interest rate is held constant at a level equal to that of the previous quarter, i_{t-1} . This conditional inflation forecast, the “ T -quarter-ahead unchanged-interest-rate inflation forecast,” $\bar{\pi}_{t+T|t}(i_{t-1})$, fulfills

$$(15) \quad \begin{aligned} \bar{\pi}_{t+T|t}(i_{t-1}) &\equiv e_{1,4} \tilde{M}^{T-1} (AX_t + Bi_{t-1}) \\ &= e_{1,4} \tilde{M}^{T-1} (AX_t + Be_7 X_t) \\ &= e_{1,4} \tilde{M}^T X_t. \end{aligned}$$

18. Indeed, given a long enough forecast horizon, the forecasted inflation path will normally be explosive.

19. However, even after operational independence, the Bank’s forecasts have assumed unchanged short-term interest rates (see Britton, Fisher, and Whitley 1998). Similarly, it is our

Rule-Consistent Inflation Forecasts

There are of course many other assumptions that one could make about monetary policy in order to produce inflation forecasts. For example, one could condition on a constant *real* interest rate, or one could set the rate in each future period according to a given reaction function for policy. Recently, the Reserve Bank of New Zealand (1997) has moved beyond constant-interest-rate forecasts and started to report official inflation forecasts conditional on a particular reaction function. (This results in inflation forecasts always returning to the target.) Below, we shall also consider a rule that employs such forecasts.

5.3.7 Simple Instrument Rules

By a simple instrument rule we mean an instrument rule of the form (9), where the vector f is restricted in some way. We will distinguish no fewer than nine types of simple instrument rules by characterizing them in terms of three forms and three arguments.²⁰

Three Forms

We consider three forms: smoothing, level, and difference; the latter two are special cases of the first form. The *smoothing* form, denoted S, is given by

$$(16) \quad \begin{aligned} i_t &= hi_{t-1} + gX_t, \\ f &= he_\gamma + g, \end{aligned}$$

where h is a coefficient and g is a 1×9 row vector of response coefficients. When the coefficient h fulfills $0 < h \leq 1$, this form of instrument rule is characterized by “partial adjustment,” or “smoothing,” of the instrument. The larger the coefficient h , the more smoothing (the more partial the adjustment).

Recall that i_t is the deviation from the average nominal interest rate, which in our model equals the sum of the inflation target (the average inflation rate) and the natural real interest rate (the average real interest rate). If we, temporarily in this paragraph, let all variables denote absolute levels and denote the average level of variable x_t by x^0 , we can write form (16) as

$$(17) \quad \begin{aligned} i_t &= hi_{t-1} + (1 - h)i^0 + g(X_t - X^0) \\ &= hi_{t-1} + (1 - h)(r^0 + \pi^*) + g(X_t - X^0) \\ &= hi_{t-1} + (1 - h)(r^0 + \bar{\pi}_t) + \tilde{g}(X_t - X^0), \end{aligned}$$

impression that internal staff forecasts at the Federal Reserve Board are often conditioned on a constant federal funds path. Thus constant-interest-rate forecasts may have some general advantages—perhaps, in ease of communication, as noted by Rudebusch (1995).

20. The theory and practice of simple policy rules is examined in Currie and Levine (1984).

where $\tilde{g} \equiv g - (1 - h)e_{1:4}$ and we have used $i^0 = r^0 + \pi^*$. Thus form (16) is equivalent to (17), which is a frequent way of writing instrument rules.²¹

The *level* form, denoted L, is the special case of the autoregressive form when $h = 0$, whereas the *difference* form, denoted D, is the special case when $h = 1$.²²

Three Arguments (Restrictions on g)

We consider three combinations of arguments (variables that the instrument responds to). That is, we consider three different restrictions on the vector g of response coefficients. First, we consider a response to $\bar{\pi}_t$ and y_t , denoted $(\bar{\pi}_t, y_t)$, which implies²³

$$gX_t = g_\pi \bar{\pi}_t + g_y y_t,$$

$$g = g_\pi e_{1:4} + g_y e_5,$$

where g_π and g_y are the two response coefficients. Second, we consider a response to the T -quarter-ahead unchanged-interest-rate inflation forecast only, denoted $\bar{\pi}_{t+T|t}(i_{t-1})$. This implies

$$gX_t = g_\pi \bar{\pi}_{t+T|t}(i_{t-1}),$$

$$g = g_\pi e_{1:4} \tilde{M}^T,$$

where we have used equation (15). Finally, we consider a response to both the T -quarter-ahead unchanged-interest-rate inflation forecast and the output gap, denoted $(\bar{\pi}_{t+T|t}(i_{t-1}), y_t)$, which implies

$$gX_t = g_\pi \bar{\pi}_{t+T|t}(i_{t-1}) + g_y y_t,$$

$$g = g_\pi e_{1:4} \tilde{M}^T + g_y e_5.$$

A particular instrument rule is denoted Ta , with the type $T = S, L$, or D , and the argument $a = (\bar{\pi}_t, y_t)$, $\bar{\pi}_{t+T|t}(i_{t-1})$, or $(\bar{\pi}_{t+T|t}(i_{t-1}), y_t)$. By a *Taylor-type rule* we mean a simple instrument rule of the form $L(\bar{\pi}_t, y_t)$,

$$i_t = g_\pi \bar{\pi}_t + g_y y_t.$$

21. Clarida, Galí, and Gertler (1997, 1998) model interest rate smoothing as

$$i_t = hi_{t-1} + (1 - h)(\bar{\pi}_t + \tilde{g}X_t),$$

which is obviously consistent with eq. (17) (as long as $h \neq 1$) since we can identify $(1 - h)\tilde{g}$ above with \tilde{g} in eq. (17).

22. Note that since $i_{t-1} = X_{t-1} = e_1 X_{t-1}$, we can always write $i_t = fX_t$ as $i_t = i_{t-1} + (f - e_1)X_t$. Thus, unless g_y is restricted to fulfill $g_y = 0$, the difference form does not imply any restriction.

23. Note that responding to $\bar{\pi}_t$ means responding to the discrepancy between inflation and the inflation target, since $\bar{\pi}_t$ is the deviation from the mean, and the mean coincides with the inflation target, since there is no inflation bias in our model.

The classic *Taylor rule* (Taylor 1993) is a Taylor-type rule with $g_\pi = 1.5$ and $g_y = 0.5$.²⁴

We do not include the case of a response to only $\bar{\pi}_t$, $gX_t = g_\pi \bar{\pi}_t$, since it consistently performed very badly.

An Information Lag

McCallum has in several papers, for instance McCallum (1997), argued that it is more realistic from an information point of view to restrict the instrument in quarter t to depend on the state variables in quarter $t - 1$,

$$i_t = fX_{t-1}.$$

On the other hand, it can be argued that the central bank has much more information about the current state in the economy than captured by the few state variables in the model. Then, assuming that the state variables in quarter t are known in quarter t is an implicit way of acknowledging this extra information.²⁵ This is the main reason why our baseline case has the instrument depending on the state variables in the same quarter.

For comparability with results of other authors, we nevertheless would like to be able to restrict the instrument to depend on state variables one quarter earlier. Thus we consider the case when there is response to $\bar{\pi}_{t-1}$ and y_{t-1} , denoted $(\bar{\pi}_{t-1}, y_{t-1})$, with and without interest rate smoothing,

$$(18) \quad i_t = hi_{t-1} + g_\pi \bar{\pi}_{t-1} + g_y y_{t-1}.$$

This requires some technical modifications in our state-space setup, which are detailed in the appendix.

An Instrument Rule with Response to a Rule-Consistent Inflation Forecast

Consider the following rule:

$$(19) \quad i_t = hi_{t-1} + \phi \pi_{t+T|t},$$

where $\phi > 0$ and $\pi_{t+T|t}$ ($T \geq 2$) is the rational expectation of π_{t+T} , conditional on X_t , equation (6), and equation (19). Thus $\pi_{t+T|t}$ is a rule-consistent inflation forecast as described above, although in this case the rule being conditioned on includes the forecast. This rule, where the instrument responds to a rule-consistent inflation forecast, is not an explicit instrument rule because it does not express the instrument as an explicit function of current information (or, in the context of our model, of predetermined variables). It is not a targeting rule,

24. See McCallum (1988), Bryant, Hooper, and Mann (1993), Judd and Motley (1992), and Henderson and McKibbin (1993) for other examples of explicit instrument rules.

25. In fact, obtaining a good description of the real-time information set of policymakers is a complicated assignment (see Rudebusch 1998a). E.g., simply lagging variables ignores data revisions (see Diebold and Rudebusch 1991).

in the sense we have used the term, since it is not explicitly related to some loss function. Nor does it express an intermediate target level as a function of current information. The rule is an *equilibrium condition* because the right-hand side of equation (19) is endogenous and depends on the rule itself. Hence, it is an *implicit* instrument rule. The self-referential, rational expectations nature of the rule complicates its analytical derivation in terms of an explicit instrument rule.²⁶ However, the rule remains a simple instrument rule similar in form to the $S(\bar{\pi}_{t+T|t}(i_{t-1}))$ rule described above, only the instrument responds to an endogenous variable rather than a predetermined one. We consequently denote the rule in equation (19) by $S(\pi_{t+T|t})$.

Like the $S(\bar{\pi}_{t+T|t}(i_{t-1}))$ rule, the $S(\pi_{t+T|t})$ rule has considerable intuitive appeal, inasmuch as it implies that if new information makes the inflation forecast at the horizon T increase, the interest rate should be increased, and vice versa. Even better, however, the $S(\pi_{t+T|t})$ rule uses an inflation forecast that can be conditional on a nonconstant interest rate path. The $S(\pi_{t+T|t})$ rule is similar to the reaction function used in the Bank of Canada's Quarterly Projection Model (see, e.g., Colletti et al. 1996) and the Reserve Bank of New Zealand's Forecasting and Policy System (see Black et al. 1997), and identical to the rule considered by Batini and Haldane in chapter 4 of this volume.²⁷ Indeed, this rule appears to be a frequent reference rule among inflation-targeting central banks. It is (when $h = 1$) what Haldane (1997) calls "the generic form of the feedback rule under an inflation target," which "encapsulates quite neatly the operational practice of most inflation targeters."

Nevertheless, the $S(\pi_{t+T|t})$ rule is not derived as a first-order condition of some loss function corresponding to inflation targeting.²⁸ The question then arises: How efficient is this rule in achieving an inflation target? This question is particularly relevant because of its use in the inflation projections by two prominent inflation-targeting central banks, and because of its intuitive appeal to many as representing generic inflation targeting. Consequently, we examine the performance of this rule within the framework of our model.

26. In equilibrium, the rational expectations inflation forecast becomes an endogenous linear function of the state variables (where the coefficients depend on the parameters T , ϕ , and h), which by eq. (19) results in eq. (9). For $T = 2$, the explicit instrument rule is easy to derive. For $T \geq 3$, the derivation is more complex. The details are provided in the appendix.

27. It is also used in Black, Macklem, and Rose (1997).

28. Because the rule is not derived as a first-order condition, its precise form is not obvious. As alternatives to eq. (19) one can consider

$$i_t = hi_{t-1} + (1 - h)\bar{\pi}_t + \phi \pi_{t+T|t},$$

or even

$$i_t = hi_{t-1} + g_\pi \bar{\pi}_t + \phi \pi_{t+T|t},$$

where g_π is unrestricted.

Optimal Simple Instrument Rules

In order to find the optimal simple instrument rule for a given type of rule and with a given combination of arguments, we optimize equation (5) over g , h , and ϕ taking the corresponding restrictions into account.

5.3.8 Targeting Rules

The Optimal Targeting Rule

Above we have noted the existence of an optimal instrument rule. Of course, the corresponding minimization problem defines an optimal targeting rule as well. Here, however, we show that the first-order condition for an optimum can be interpreted as an optimal intermediate targeting rule.

Consider the first-order condition for minimizing expressions (3) and (8) subject to (6) and (7),

$$\begin{aligned}
 (20) \quad 0 &= \sum_{\tau=0}^{\infty} \frac{\partial Y'_{t+\tau|t}}{\partial i_t} K Y_{t+\tau|t} \\
 &= C'_t K Y_t + \sum_{\tau=1}^{\infty} B'(A^{\tau-1})' C'_X K Y_{t+\tau|t},
 \end{aligned}$$

where we have used that

$$\frac{\partial Y_t}{\partial i_t} = C_t, \quad \frac{\partial Y_{t+\tau|t}}{\partial i_t} = C_X \frac{\partial X_{t+\tau|t}}{\partial i_t} = C_X A^{\tau-1} B, \quad \tau = 1, 2, \dots,$$

and let the discount factor fulfill $\delta = 1$. This is a linear relation between the current and conditionally forecasted future goal variables, $Y_{t+\tau|t}$, $\tau = 0, 1, 2, \dots$, conditional on the current instrument and the future policy. The task of the monetary authority can be described as setting an instrument in the current quarter so as to achieve the relation (20). This relation can then be interpreted as an intermediate target path for the forecast of future goal variables. That is, the forecasts of future goal variables are considered intermediate target variables. Then the task of the monetary authority is to choose, conditional on the current state variable X_t , a current instrument i_t and a plan $i_{t+\tau|t}$ ($\tau = 1, 2, \dots$) for future instruments, such that the resulting conditional forecasts of future goal variables $Y_{t+\tau|t}$ fulfill the intermediate target (20), where

$$\begin{aligned}
 Y_t &= C_X X_t + C_t i_t, \\
 Y_{t+\tau|t} &= C_X X_{t+\tau|t} + C_t i_{t+\tau|t} \\
 &= C_X A^\tau X_t + \sum_{j=0}^{\tau-1} C_X A^{\tau-1-j} B i_{t+j|t} + C_t i_{t+\tau|t},
 \end{aligned}$$

where $\tau = 1, 2, \dots$ and we have used that

$$\begin{aligned} X_{t+\tau+1|t} &= AX_{t+\tau|t} + Bi_{t+\tau|t} \\ &= A^{\tau+1}X_t + \sum_{j=0}^{\tau} A^{\tau-j}Bi_{t+j|t}. \end{aligned}$$

We note that the $Y_{t+\tau|t}$ ($\tau = 0, 1, 2, \dots$) that fulfill equation (20) can be seen as impulse responses of the goal variables for the optimal solution, for impulses that put the economy at its initial state. We can now imagine a governor or a board of governors pondering over a set of alternative current and future instrument settings and alternative forecasts for the goal variables that have been provided for consideration by the central bank staff, in order to decide on the current instrument setting. When the governor or board of governors ends up selecting one instrument path and corresponding goal variable forecasts that they believe are best, their behavior (if rational) can be seen as implicitly selecting forecasts that fulfill equation (20) for some implicit weight matrix K in their loss function.

In general, equation (20) involves a relation between all the goal variables. The case when inflation and the output gap are the only goal variables is examined in Svensson (1997a, forthcoming b). Since, by the Phillips curve (1), the forecast of output can be written as a linear function of the forecast of inflation, this linear function can then be substituted for the output forecast in equation (20), which results in a relation for the forecast of future inflation only. That relation can be interpreted as an intermediate target for the inflation forecast. In the special case examined in Svensson (1997a, forthcoming b), these relations for the inflation forecast are both simple and optimal. In the general case these relations need not be optimal. Here we will examine them as potential simple targeting rules, called inflation-forecast-targeting rules.

Simple Targeting Rules

Consider targeting rules for the T -quarter-ahead constant-interest-rate inflation forecast. These rules imply implicit instrument rules that are normally not "simple," since they normally depend on most state variables. We will consider four kinds of simple targeting rules, namely, strict and flexible inflation-forecast targeting, with and without smoothing.

In Svensson (1997a), the following first-order condition for the inflation forecast is derived, for the case of flexible inflation targeting with some non-negative weight on output stabilization, $\lambda \geq 0$, but zero weight on interest rate smoothing, $\nu = 0$,

$$\pi_{t+2|t}(i_t) - \pi^* = c(\lambda)(\pi_{t+1|t} - \pi^*).$$

In the model in Svensson (1997a), $\pi_{t+1|t}$ is predetermined, $\pi_{t+2|t}(i_t)$ is the inflation forecast for the earliest horizon that can be affected, and $c(\lambda)$ is an increasing function of λ , fulfilling $0 \leq c(\lambda) < 1$, $c(0) = 0$, and $c(\lambda) \rightarrow 1$ for $\lambda \rightarrow \infty$.

In the present model, we can consider a generalization of this framework,

$$(21) \quad \bar{\pi}_{t+T|t}(i_t) = c \bar{\pi}_{t+1|t},$$

where c and T fulfill $0 \leq c < 1$ and $T \geq 2$. This we refer to as *flexible T-quarter-ahead inflation-forecast targeting*, denoted FIFT(T).

The expression (21) denotes a targeting rule, where the corresponding instrument rule is *implicit*. In order to solve for the instrument rule, we use equation (13) to write equation (21) as

$$e_{1:4} \tilde{M}^{T-1} (AX_t + Bi_t) = ce_{1:4} AX_t.$$

Then the implicit instrument rule can be written

$$i_t = g(c, T) X_t,$$

where the row vector $g(c, T)$ is a function of c and T given by

$$(22) \quad g(c, T) \equiv \frac{e_{1:4}(cI - \tilde{M}^{T-1})A}{e_{1:4} \tilde{M}^{T-1} B},$$

where I is the 9×9 identity matrix (note that $e_{1:4} \tilde{M}^{T-1} B$ is a scalar and $e_{1:4}(cI - \tilde{M}^{T-1})A$ is a 1×9 row vector).

Strict T-quarter-ahead inflation-forecast targeting, denoted SIFT(T), is the special case of equation (21) when $c = 0$,

$$(23) \quad \bar{\pi}_{t+T|t}(i_t) = 0.$$

The corresponding implicit instrument rule is

$$(24) \quad i_t = g(0, T) X_t,$$

where

$$(25) \quad g(0, T) \equiv -\frac{e_{1:4} \tilde{M}^{T-1} A}{e_{1:4} \tilde{M}^{T-1} B}$$

Note that the numerator in equation (25) equals the constant-interest-rate inflation forecast corresponding to a zero interest rate, $\bar{\pi}_{t+T|t}(0)$. The denominator, $e_{1:4} \tilde{M}^{T-1} B$, is the constant-interest-rate policy multiplier for the four-quarter inflation T quarters ahead, since by equation (13)

$$(26) \quad \frac{\partial \bar{\pi}_{t+T|t}(i)}{\partial i} = e_{1:4} \tilde{M}^{T-1} B.$$

Hence, very intuitively the instrument rule corresponding to strict inflation-forecast targeting can be written as

$$i_t = -\frac{\bar{\pi}_{t+T|t}(0)}{\partial \bar{\pi}_{t+T|t}(i) / \partial i_t},$$

the negative of the *zero-interest-rate* inflation forecast divided by the constant-interest-rate policy multiplier.

We can equivalently write this instrument rule in terms of *changes* in the interest rate. By equation (13) we have

$$\bar{\pi}_{t+T|t}(i_t) - \bar{\pi}_{t+T|t}(i_{t-1}) = e_{1,4} \tilde{M}^{T-1} B (i_t - i_{t-1}).$$

By equation (23) we can write

$$i_t - i_{t-1} = - \frac{\bar{\pi}_{t+T|t}(i_{t-1})}{e_{1,4} \tilde{M}^{T-1} B} = - \frac{e_{1,4} \tilde{M}^T X_t}{e_{1,4} \tilde{M}^{T-1} B} = (f(0, T) - e_7) X_t.$$

Very intuitively, the interest rate adjustment equals the negative of the *unchanged-interest-rate* inflation forecast for the unchanged interest rate divided by the constant-interest-rate policy multiplier.

Note that strict inflation-forecast targeting implies that the inflation forecast conditional on the future instrument rule (24), rather than conditional on a constant interest rate, deviates from zero,

$$E_t \bar{\pi}_{t+T} \neq 0,$$

and in practice reaches zero later than T quarters ahead. This is apparent from the impulse responses for $\bar{\pi}_{t+\tau|t}$ under strict inflation-forecast targeting.

Note that strict T_1 -quarter inflation-forecast targeting may be approximately equal to flexible T_2 -quarter flexible inflation-forecast targeting, when the horizon for strict inflation targeting exceeds that of flexible inflation targeting, $T_1 > T_2$.

The above targeting rules can be considered under *smoothing* (partial adjustment) of the interest rate,

$$\begin{aligned} i_t &= h i_{t-1} + (1 - h) g(c, T) X_t, \\ f &= h e_7 + (1 - h) g(c, T), \end{aligned}$$

where it may be reasonable to restrict the smoothing coefficient h to fulfill $0 \leq h < 1$. Note that under smoothing, h is not generally the “net” coefficient on i_{t-1} , since $g_7(c, T)$ is generally not zero. These targeting rules under smoothing are denoted FIFTS(T) and SIFTS(T), respectively.

The *optimal* inflation-forecast-targeting rules are found by minimizing the loss function (5) over the parameters c , h , and T , taking into account the restrictions on these and that $T \geq 2$ is an integer. For instance, under strict inflation targeting without smoothing, we have $c = h = 0$, and the only free parameter is T .

Table 5.3 Results on Volatility and Loss with Various Rules ($\lambda = 1, \nu = 0.5$)

Rule	Std[$\bar{\pi}_t$]	Std[y_t]	Std[$i_t - i_{t-1}$]	Loss	Rank
Optimal	2.15	2.24	1.68	11.08	1
L($\bar{\pi}_t, y_t$) $g_\pi = 2.72, g_y = 1.57$	2.18	2.24	1.74	11.27	5
L($\bar{\pi}_{t+T_0}(i_{t-1})$) $T = 8; g_\pi = 2.55$	2.42	2.27	2.07	13.15	18
L($\bar{\pi}_{t+T_0}(i_{t-1}), y_t$) $T = 8; g_\pi = 2.53, g_y = 0.29$	2.44	2.15	2.20	13.01	17
S($\bar{\pi}_t, y_t$) $g_\pi = 2.37, g_y = 1.44, h = 0.14$	2.18	2.25	1.68	11.23	4
S($\bar{\pi}_{t+T_0}(i_{t-1})$) $T = 8; g_\pi = 1.89, h = 0.46$	2.15	2.47	1.53	11.89	12
S($\bar{\pi}_{t+T_0}(i_{t-1}), y_t$) $T = 8; g_\pi = 1.54, g_y = 0.45, h = 0.60$	2.15	2.25	1.68	11.09	2
S(π_{t+T_0}) $T = 8; \phi = 2.62, h = 0.32$	2.15	2.45	1.53	11.77	11
$T = 12; \phi = 3.65, h = 0.38$	2.13	2.41	1.55	11.58	10
$T = 16; \phi = 5.52, h = 0.41$	2.13	2.40	1.57	11.51	7
SIFT(T) $T = 8$	1.40	2.84	7.44	37.65	22
$T = 12$	1.81	2.44	3.15	14.17	19
$T = 16$	2.21	2.27	2.03	12.05	13
FIFT(T) $T = 8; c = 0.72$	2.24	1.82	5.31	22.41	21
$T = 12; c = 0.39$	2.17	2.11	2.72	12.86	16
$T = 16; c = 0.01$	2.22	2.26	2.02	12.05	13
SIFTS(T) $T = 8; h = 0.59$	1.51	3.39	3.88	21.29	20
$T = 12; h = 0.45$	1.87	2.60	1.94	12.16	15
$T = 16; h = 0.31$	2.24	2.34	1.47	11.57	8
FIFTS(T) $T = 8; c = 0.66, h = 0.71$	2.15	2.26	1.86	11.42	6
$T = 12; c = 0.35, h = 0.47$	2.18	2.28	1.59	11.17	3
$T = 16; c = 0.00, h = 0.31$	2.24	2.34	1.47	11.57	8

5.4 Results

5.4.1 Optimized Rules

In this subsection, we consider the performance of various rules for several illustrative cases of different preferences over goal variables. The rules we consider have been optimized in terms of their parameter settings for the given preferences and the given form of the rule assumed.

Tables 5.3 through 5.7 provide results for five different sets of preferences over goals. In each table, the volatility of the goal variables (measured as the unconditional standard deviations), the minimized loss, and the relative ranking in terms of loss are shown for 22 different rules. Loss is calculated under

Table 5.4 Results on Volatility and Loss with Various Rules ($\lambda = 0.2, \nu = 0.5$)

Rule	Std[π_t]	Std[y_t]	Std[$i_t - i_{t-1}$]	Loss	Rank
Optimal	1.97	2.64	1.55	6.47	1
L($\bar{\pi}_t, y_t$) $g_\pi = 3.17, g_y = 1.22$	2.00	2.61	1.65	6.71	10
L($\bar{\pi}_{t+T_t}(i_{t-1})$) $T = 8; g_\pi = 2.65$	2.37	2.28	2.17	9.00	17
L($\bar{\pi}_{t+T_t}(i_{t-1}), y_t$) $T = 8; g_\pi = 2.69, g_y = -0.25$	2.36	2.41	2.10	8.92	16
S($\bar{\pi}_t, y_t$) $g_\pi = 2.34, g_y = 1.03, h = 0.30$	2.00	2.64	1.56	6.60	9
S($\bar{\pi}_{t+T_t}(i_{t-1})$) $T = 8; g_\pi = 1.63, h = 0.69$	1.97	2.75	1.53	6.58	8
S($\bar{\pi}_{t+T_t}(i_{t-1}), y_t$) $T = 8; g_\pi = 1.42, g_y = 0.16, h = 0.74$	1.97	2.64	1.55	6.48	2
S(π_{t+T_t}) $T = 8; \phi = 2.35, h = 0.62$	1.97	2.73	1.53	6.55	7
$T = 12; \phi = 3.86, h = 0.71$	1.97	2.69	1.54	6.50	5
$T = 16; \phi = 8.33, h = 0.47$	1.97	2.68	1.54	6.49	4
SIFT(T) $T = 8$	1.40	2.84	7.44	31.21	22
$T = 12$	1.81	2.44	3.15	9.42	19
$T = 16$	2.21	2.27	2.03	7.95	14
FIFT(T) $T = 8; c = 0.69$	2.13	1.87	5.38	19.68	21
$T = 12; c = 0.24$	1.99	2.24	2.88	9.10	18
$T = 16; c = 0.00$	2.21	2.27	2.03	7.95	14
SIFTS(T) $T = 8; h = 0.71$	1.62	3.84	3.34	11.16	20
$T = 12; h = 0.60$	1.93	2.74	1.60	6.51	6
$T = 16; h = 0.45$	2.28	2.39	1.25	7.11	12
FIFTS(T) $T = 8; c = 0.53, h = 0.79$	1.98	2.67	1.67	6.74	11
$T = 12; c = 0.08, h = 0.60$	1.98	2.65	1.52	6.48	2
$T = 16; c = 0.00, h = 0.45$	2.28	2.39	1.25	7.11	12

the assumption that output and inflation variability are equally distasteful ($\lambda = 1$) in table 5.3 and that output variability is much less costly ($\lambda = 0.2$) in table 5.4 and much more costly ($\lambda = 5$) in table 5.5. Variability of nominal interest rate changes are also costly in these three tables ($\nu = 0.5$).²⁹ Variation in the costs of variability of interest rate changes are considered in tables 5.6 ($\nu = 0.1$) and 5.7 ($\nu = 1.0$) (both assuming $\lambda = 1$). The preferences in table 5.3 imply a concern not only about inflation stabilization but also about output stabilization and interest rate smoothing, which we believe is realistic for many central banks, also inflation-targeting ones. Comparison with tables 5.4

29. Such costs are suggested, in part, by the concern central banks display for financial market fragility (see, e.g., Rudebusch 1995).

Table 5.5 Results on Volatility and Loss with Various Rules ($\lambda = 5, \nu = 0.5$)

Rule	Std[$\bar{\pi}_t$]	Std[y_t]	Std[$i_t - i_{t-1}$]	Loss	Rank
Optimal	2.65	1.86	2.36	26.99	1
$L(\bar{\pi}_t, y_t)$ $g_\pi = 2.15, g_y = 2.17$	2.69	1.89	2.16	27.46	4
$L(\bar{\pi}_{t+7q}(i_{t-1}))$ $T = 8; g_\pi = 2.13$	2.69	2.22	1.70	33.32	17
$L(\bar{\pi}_{t+7q}(i_{t-1}), y_t)$ $T = 8; g_\pi = 2.24, g_y = 1.40$	2.80	1.83	2.69	28.17	7
$S(\bar{\pi}_t, y_t)$ $g_\pi = 1.26, g_y = 2.35, h = -0.11$	2.68	1.88	2.27	27.39	3
$S(\bar{\pi}_{t+7q}(i_{t-1}))$ $T = 8; g_\pi = 2.03, h = 0.06$	2.67	2.23	1.59	33.29	16
$S(\bar{\pi}_{t+7q}(i_{t-1}), y_t)$ $T = 8; g_\pi = 1.78, g_y = 1.27, h = 0.31$	2.65	1.87	2.31	27.15	2
$S(\pi_{t+7q})$ $T = 8; \phi = 2.62, h = -0.15$	2.65	2.21	1.62	32.81	15
$T = 12; \phi = 3.16, h = -0.11$	2.61	2.19	1.65	32.06	12
$T = 16; \phi = 3.91, h = -0.09$	2.59	2.18	1.67	31.78	11
SIFT(T)					
$T = 8$	1.40	2.84	7.44	69.88	22
$T = 12$	1.81	2.44	3.15	37.89	20
$T = 16$	2.21	2.27	2.03	32.59	13
FIFT(T)					
$T = 8; c = 0.81$	2.64	1.70	5.15	34.71	18
$T = 12; c = 0.64$	2.70	1.91	2.49	28.61	8
$T = 16; c = 0.44$	2.79	2.02	1.78	29.86	10
SIFTS(T)					
$T = 8; h = 0.35$	1.44	3.04	5.07	61.07	21
$T = 12; h = 0.15$	1.82	2.47	2.69	37.50	19
$T = 16; h = 0.02$	2.21	2.27	1.99	32.59	13
FIFTS(T)					
$T = 8; c = 0.80, h = 0.52$	2.63	1.87	2.52	27.48	5
$T = 12; c = 0.64, h = 0.22$	2.71	1.95	1.94	28.15	6
$T = 16; c = 0.44, h = 0.03$	2.79	2.03	1.71	29.85	9

through 5.7 allows us to note the consequences of relatively more or less emphasis on output stabilization and interest rate smoothing.

The first rule at the top of each table is the unrestricted optimal control rule—the obvious benchmark. The optimal rule in table 5.3 produces volatility results not too far from our historical sample results, which are $\text{Std}[\bar{\pi}_t] = 2.33$, $\text{Std}[y_t] = 2.80$, and $\text{Std}[i_t - i_{t-1}] = 1.09$. The next four rows consider level rules with current inflation and output, $L(\bar{\pi}_t, y_t)$, future inflation $L(\bar{\pi}_{t+8q}(i_{t-1}))$, and future inflation and current output $L(\bar{\pi}_{t+8q}(i_{t-1}), y_t)$ as arguments (where the forecasts are the 8-quarter-ahead “unchanged-interest-rate” four-quarter inflation forecast). The next three rows consider smoothing instrument rules with the same arguments. The following three rows are for the interest-

Table 5.6 Results on Volatility and Loss with Various Rules ($\lambda = 1, \nu = 0.1$)

Rule	Std[$\bar{\pi}_t$]	Std[y_t]	Std[$i_t - i_{t-1}$]	Loss	Rank
Optimal	1.96	2.12	3.02	9.25	1
L($\bar{\pi}_t, y_t$) $g_\pi = 3.43, g_y = 2.50$	2.01	2.18	2.71	9.51	5
L($\bar{\pi}_{t+7q}(i_{t-1})$) $T = 8; g_\pi = 3.46$	2.11	2.35	2.99	10.86	20
L($\bar{\pi}_{t+7q}(i_{t-1}), y_t$) $T = 8; g_\pi = 3.41, g_y = 1.00$	2.18	2.05	3.53	10.18	11
S($\bar{\pi}_t, y_t$) $g_\pi = 2.80, g_y = 2.80, h = -0.16$	2.00	2.15	2.90	9.46	4
S($\bar{\pi}_{t+7q}(i_{t-1})$) $T = 8; g_\pi = 3.15, h = 0.31$	1.94	2.47	2.47	10.51	18
S($\bar{\pi}_{t+7q}(i_{t-1}), y_t$) $T = 8; g_\pi = 2.79, g_y = 1.06, h = 0.47$	1.96	2.14	2.98	9.29	2
S(π_{t+7q}) $T = 8; \phi = 5.01, h = -0.01$	1.94	2.45	2.49	10.37	13
$T = 12; \phi = 7.99, h = 0.06$	1.92	2.41	2.55	10.13	9
$T = 16; \phi = 13.66, h = 0.09$	1.91	2.39	2.58	10.04	8
SIFT(T) $T = 8$	1.40	2.84	7.44	15.54	22
$T = 12$	1.81	2.44	3.15	10.19	12
$T = 16$	2.21	2.27	2.03	10.41	14
FIFT(T) $T = 8; c = 0.61$	1.95	1.97	5.54	10.75	19
$T = 12; c = 0.27$	2.02	2.21	2.84	9.78	7
$T = 16; c = 0.00$	2.21	2.27	2.03	10.41	14
SIFTS(T) $T = 8; h = 0.34$	1.43	3.03	5.14	13.86	21
$T = 12; h = 0.11$	1.82	2.46	2.80	10.15	10
$T = 16; h = 0.06$	2.20	2.26	2.15	10.41	14
FIFTS(T) $T = 8; c = 0.60, h = 0.45$	1.95	2.13	3.08	9.30	3
$T = 12; c = 0.27, h = 0.13$	2.03	2.23	2.46	9.73	6
$T = 16; c = 0.00, h = -0.06$	2.20	2.26	2.15	10.41	14

rate-smoothing rule $S(\pi_{t+7q})$, using the 8-, 12-, and 16-quarter-ahead rule-consistent quarterly inflation forecasts. The final twelve rows of each table present various implicit inflation-forecast-targeting rules at horizons of 8, 12, and 16 quarters. For all of the rules (except the optimal one), the relevant optimal rule parameters are given in the tables as well.

These tables suggest several conclusions: First, simple instrument rules appear to be able to perform quite well in our model. Consistently across the tables, the top-performing rule is the $S(\bar{\pi}_{t+8q}(i_{t-1}), y_t)$ one, which reacts to the constant-interest-rate inflation forecast and the current output gap. Indeed, these simple “forward-looking” Taylor-type rules are always extremely close to matching the optimal rule in terms of overall loss. This result is somewhat

Table 5.7 Results on Volatility and Loss with Various Rules ($\lambda = 1, \nu = 1$)

Rule	Std[$\bar{\pi}_t$]	Std[y_t]	Std[$i_t - i_{t-1}$]	Loss	Rank
Optimal	2.27	2.29	1.33	12.17	1
L($\bar{\pi}_t, y_t$) $g_\pi = 2.44, g_y = 1.23$	2.29	2.28	1.42	12.49	8
L($\bar{\pi}_{t+\tau_i}(i_{t-1})$) $T = 8; g_\pi = 2.24$	2.60	2.24	1.79	14.99	17
L($\bar{\pi}_{t+\tau_i}(i_{t-1}), y_t$) $T = 8; g_\pi = 2.23, g_y = 0.07$	2.61	2.20	1.82	14.97	16
S($\bar{\pi}_t, y_t$) $g_\pi = 1.12, g_y = 1.04, h = 0.27$	2.29	2.30	1.34	12.33	4
S($\bar{\pi}_{t+\tau_i}(i_{t-1})$) $T = 8; g_\pi = 1.47, h = 0.54$	2.27	2.47	1.24	12.82	12
S($\bar{\pi}_{t+\tau_i}(i_{t-1}), y_t$) $T = 8; g_\pi = 1.18, g_y = 0.30, h = 0.65$	2.27	2.30	1.34	12.18	2
S($\pi_{t+\tau_i}$) $T = 8; \phi = 1.92, h = 0.45$	2.26	2.45	1.25	12.71	10
$T = 12; \phi = 2.52, h = 0.50$	2.25	2.42	1.26	12.54	9
$T = 16; \phi = 3.63, h = 0.53$	2.25	2.41	1.27	12.48	7
SIFT(T) $T = 8$	1.40	2.84	7.44	65.29	22
$T = 12$	1.81	2.44	3.15	19.13	19
$T = 16$	2.21	2.27	2.03	14.11	15
FIFT(T) $T = 8; c = 0.77$	2.45	1.75	5.21	36.19	21
$T = 12; c = 0.47$	2.30	2.04	2.64	16.43	18
$T = 16; c = 0.12$	2.32	2.20	1.95	14.02	14
SIFTS(T) $T = 8; h = 0.66$	1.56	3.62	3.54	28.08	20
$T = 12; h = 0.56$	1.91	2.70	1.68	13.77	13
$T = 16; h = 0.45$	2.28	2.39	1.25	12.47	6
FIFTS(T) $T = 8; c = 0.69, h = 0.79$	2.28	2.33	1.47	12.77	11
$T = 12; c = 0.40, h = 0.59$	2.27	2.31	1.31	12.19	3
$T = 16; c = 0.05, h = 0.45$	2.32	2.36	1.23	12.46	5

surprising given that the inflation forecast incorporated into these rules is simply a single 8-quarter-ahead inflation projection conditioned on an unchanged interest rate path.

Perhaps even more surprising, the current inflation and output Taylor-type rules—L($\bar{\pi}_t, y_t$) and S($\bar{\pi}_t, y_t$)—are nearly as good. Particularly, in table 5.3 (with $\lambda = 1$), these rules perform with output and inflation gap variances that are similar to those of the optimal rule. In order to understand the exceptional performance of these rules, it is instructive to compare the coefficients of these simple rules to those of the optimal rule. The optimal rule in table 5.3 (the optimal rules from the other tables have broadly similar parameterizations) has the form

$$i_t = .88\pi_t + .30\pi_{t-1} + .38\pi_{t-2} + .13\pi_{t-3} + 1.30y_t - .33y_{t-1} + .47i_{t-1} \\ - .06i_{t-2} - .03i_{t-3}.$$

The $L(\bar{\pi}, y_t)$ rule, for example, comes close to matching this by setting the first four parameters all equal to 0.68 (i.e., $g_\pi/4$), the y_t parameter equal to 1.57, and the other parameters equal to zero. Because the Taylor rule has received so much attention, it is also interesting to note that across all of the tables the parameters for our $L(\bar{\pi}, y_t)$ Taylor-type rules are fairly high. Instead of the original Taylor rule parameters of 1.5 on inflation (g_π) and 0.5 on output (g_y), our optimal $L(\bar{\pi}, y_t)$ rules sets these parameters above 2 and 1, respectively, in all of the tables.³⁰

Second, in distinct contrast to the simple rules that include contemporaneous output gaps, the simple instrument rules that respond only to inflation forecasts do quite poorly—even when the weight on output stabilization is small, as in table 5.4. Of course, the optimal rule does include large coefficients on output, but presumably these reflect in large part the inflation-forecasting properties of output (especially for low λ). However, the simple instrument rules $L(\bar{\pi}_{t+8|t}(i_{t-1}))$ and $S(\bar{\pi}_{t+8|t}(i_{t-1}))$ that incorporate only future inflation do not fare very well. One might conjecture that these rules do poorly because of the mechanical nature of the forecasts used, which are simple projections assuming a constant nominal funds rate. However, the $S(\pi_{t+8|t})$ rule, which conditions the inflation forecast on a time-varying, rule-consistent interest rate path, does little better than the $S(\bar{\pi}_{t+8|t}(i_{t-1}))$ rule. More likely, the restricted fashion in which the inflation forecasts enter the rule—the instrument responds only to the deviation between the forecast and the inflation target—is to blame. This illustrates what was emphasized in subsection 5.3.7, namely, that these rules are not first-order conditions to our loss function. However, note that these rules do better for a smaller λ (table 5.4) and worse for a larger λ (table 5.5). This indicates that they are closer to a first-order condition of a loss function that only involves inflation stabilization and interest rate smoothing.³¹

Third, the inflation-forecast-targeting rules perform quite well given enough flexibility and interest-rate-smoothing ability. The FIFTS rule (flexible inflation-forecast targeting with smoothing) is essentially able to match the performance of the $S(\bar{\pi}_{t+8|t}(i_{t-1}), y_t)$ rule—and hence the optimal rule—in all cases except when there is a very high weight on output stabilization (table 5.5). Across all of the tables, the best inflation-forecast horizon to use with this rule is usually 12 quarters but sometimes 8 quarters. The IFT rules without

30. Ball (1997), in a simple, calibrated theoretical model similar to our own, argues that the optimal Taylor-type rule should have higher coefficients than the original Taylor rule. However, Ball also argues that in the optimal rule the output parameter should be larger than the inflation parameter, which is generally contrary to our results.

31. The length of the forecast horizon (T) in the $S(\pi_{t+T|t})$ rule makes only a modest contribution. I.e., the targeting horizon trade-off discussed in Haldane (1997) is relatively modest in our model with this rule.

interest rate smoothing are heavily penalized by the cost of large changes in the nominal interest rate instrument. Note that this is true even in table 5.6 when the cost of variability of interest rate change is quite low.

To augment the tables, figure 5.2 shows the trade-offs between inflation variability and output gap variability that result for varying the weight on output stabilization (λ) from 0 to 10 and assuming $\nu = 0.5$.³² The trade-off resulting from the optimal rule is shown as a solid line. For increasing λ , the optimal rule corresponds to points further southeast on the curve. The dashed lines correspond to the smoothing rules $S(\bar{\pi}_t, y_t)$, $S(\bar{\pi}_{t+8|t}(i_{t-1}))$, $S(\pi_{t+8|t})$, and $S(\bar{\pi}_{t+8|t}(i_{t-1}), y_t)$. Only the last of these is consistently close to the optimal rule. Note that $S(\pi_{t+8|t})$ is close to the optimal rule for small λ .

Also, the triangle shows the sample (1960:1–96:2) standard deviations of inflation and the output gap. The circle shows the standard deviations that result from an estimated Taylor-type rule for the sample 1985:1–96:2 (with $g_\pi = 1.76$ and $g_y = 0.74$). The square shows the standard deviations that result from the Taylor rule (with $g_\pi = 1.5$ and $g_y = 0.5$).

The trade-offs from flexible inflation-forecast targeting with smoothing (FIFTS) at 8-, 12-, and 16-quarter horizons are shown as the dashed-dotted lines. For $T = 8$ quarters, the trade-off is consistently close to that of the optimal rule.

The trade-offs from flexible inflation-forecast targeting without smoothing (FIFT) are shown as the dotted lines. A shorter horizon T is associated more with less output variability than with less inflation variability (cf. table 5.3).

Finally, figures 5.3 and 5.4 give the dynamic impulse responses of the model under various optimal simple smoothing rules and targeting rules, respectively. All of the rules have broadly similar features, especially a large, quick interest rate rise in response to a positive inflation or output shock.³³ There are, however, some subtle but telling differences among the rules. In figure 5.3, the $S(\pi_{t+8|t})$ rule, which considers only the inflation forecast, has the mildest response to an output shock, which allows inflation (through the Phillips curve) to get a bit more out of control and requires a slightly longer slowdown in output to compensate. In figure 5.4, the inflation-targeting rules without smoothing show large initial interest rate spikes in response to the shocks. With smoothing, however, the FIFTS rule is able to mimic the hump-shaped pattern of interest rates of the smoothing instrument rules.

5.4.2 Common Conference Rules

In this subsection, we consider the five rules that are to be common across all of the investigations at this conference. These rules and our results on volatility and loss (assuming $\lambda = 1$ and $\nu = 0.5$) are summarized in table 5.8. The

32. Although plots of such trade-offs are common in the literature, they sweep interest-rate-smoothing considerations under the rug, so we have some preference for the tabular results.

33. Note the great contrast between figs. 5.3 and 5.4 and the left two columns of fig. 5.1. Again, the poor results in fig. 5.1 can be traced to the misspecification of the VAR interest rate equation.

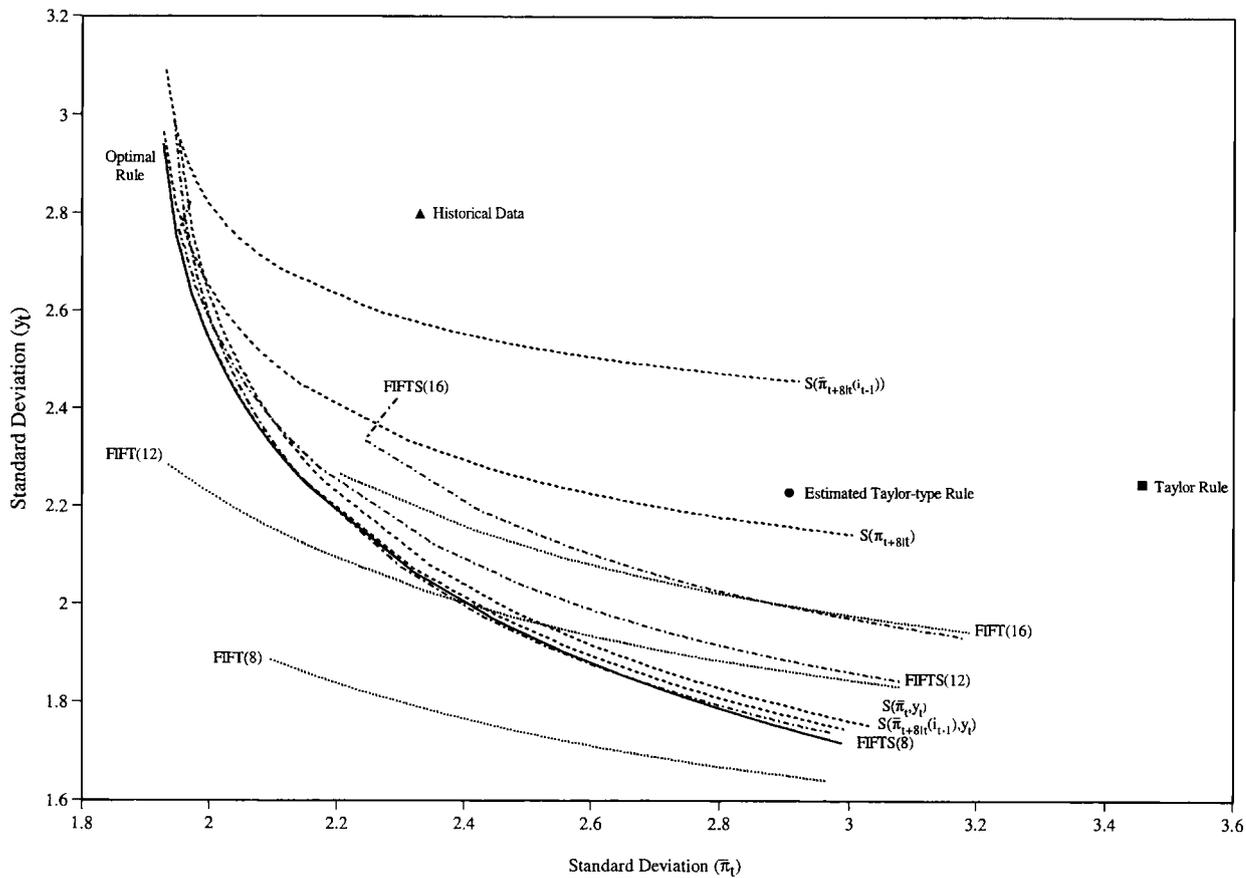


Fig. 5.2 Policy rule frontiers

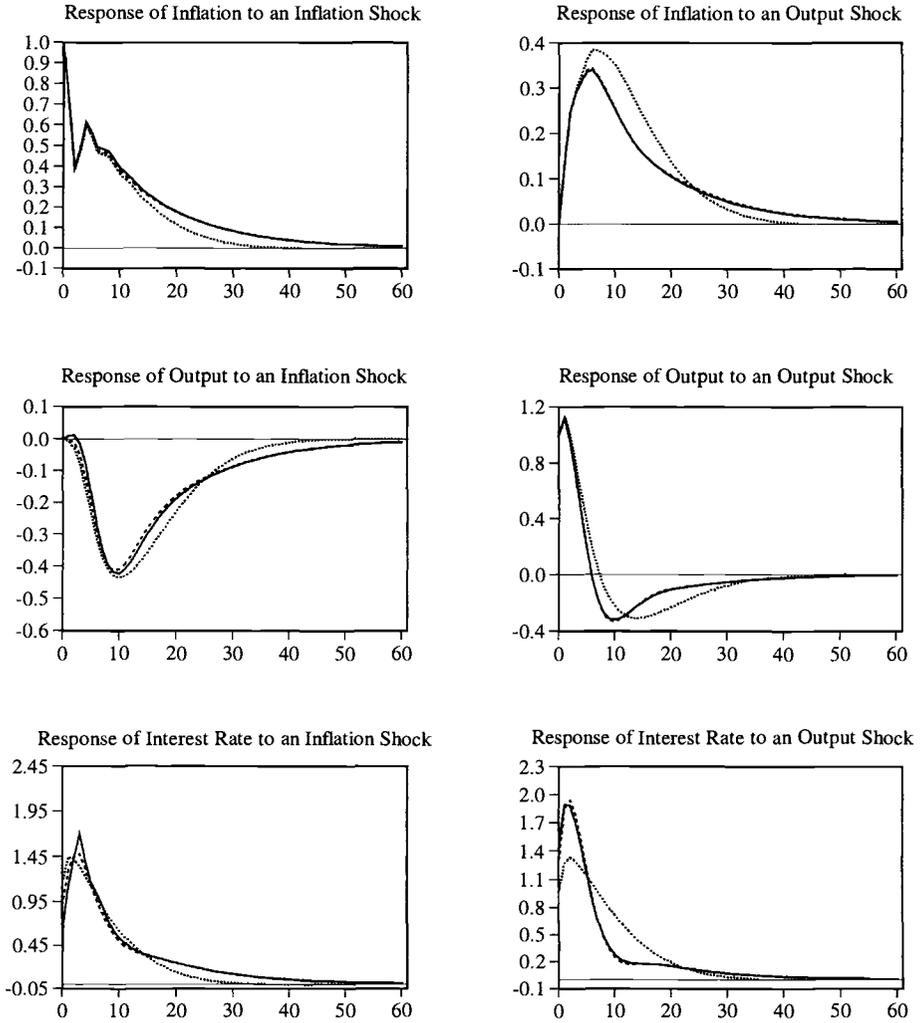


Fig. 5.3 Impulse responses for smoothing rules ($\lambda = 1, \nu = 0.5$)

Note: Solid, $S(\bar{\pi}_t, y_t)$; dashes, $S(\bar{\pi}_{t+8it}(i_{t-1}), y_t)$; dots, $S(\bar{\pi}_{t+8it})$.

results with lagged information, which are shown in the lower half of table 5.8, are qualitatively the same as those with contemporaneous information, so we concentrate on the latter.

First, consider the two level rules (in our terminology) that are common. Rules III(1) and IV(1) have much weaker inflation and output response coefficients than our optimal $L(\bar{\pi}_t, y_t)$ rule (in table 5.3), and inflation variability under the common rules is much larger than with the optimal ones, while out-

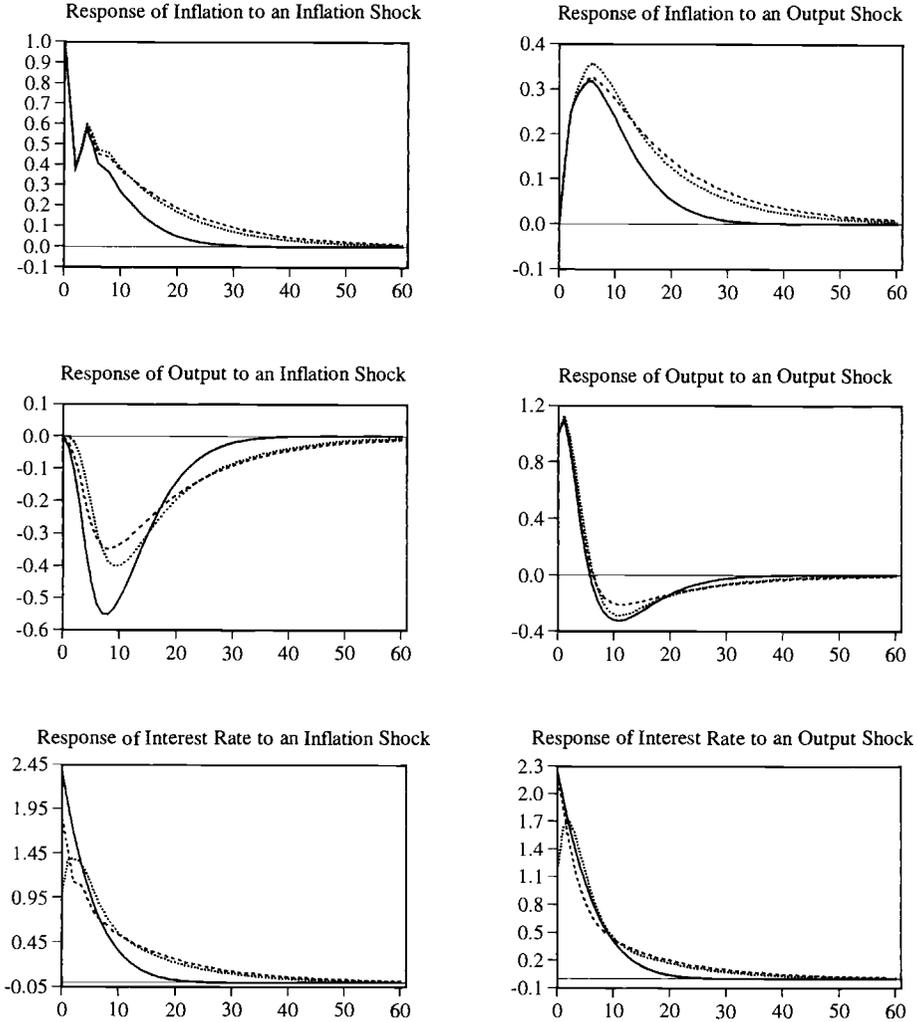


Fig. 5.4 Impulse responses for inflation-targeting rules ($\lambda = 1, \nu = 0.5$)

Note: Solid, SIFT (12); dashes, FIFT (12); dots, FIFTS (12).

put variability is slightly lower and variability of interest rate changes is about the same. The parameters of the common conference rules could only be optimal for a very large λ (much greater than 10).

Second, the set of common conference rules included two difference rules and one smoothing rule with $h = 1.3$. None of these rules provided dynamically stable solutions in our model. Note that the optimal value of h for rule $S(\bar{\pi}_t, y_t)$ equals 0.14 in table 5.3 and is hence not close to one. The optimal

Table 5.8 Results for Conference Rules ($\lambda = 1, \nu = 0.5$)

Rule	Std[$\bar{\pi}_t$]	Std[y_t]	Std[$i_t - i_{t-1}$]	Std[i_t]	Loss
<i>With contemporaneous information; $i_t = hi_{t-1} + g_\pi \bar{\pi}_t + g_y y_t$</i>					
Rule I(1); D($\bar{\pi}_t, y_t$) $g_\pi = 3.00, g_y = 0.80, h = 1.00$					Dynamically unstable
Rule II(1); D($\bar{\pi}_t, y_t$) $g_\pi = 1.20, g_y = 1.00, h = 1.00$					Dynamically unstable
Rule III(1); L($\bar{\pi}_t, y_t$) $g_\pi = 1.50, g_y = 0.50, h = 0.00$	3.46	2.25	0.71	4.94	17.25
Rule IV(1); L($\bar{\pi}_t, y_t$) $g_\pi = 1.50, g_y = 1.00, h = 0.00$	3.52	1.98	1.03	4.97	16.86
Rule V(1); S($\bar{\pi}_t, y_t$) $g_\pi = 1.20, g_y = 0.06, h = 1.30$					Dynamically unstable
Optimal L($\bar{\pi}_t, y_t$) $g_\pi = 2.72, g_y = 1.57, h = 0.00$	2.18	2.24	1.74	5.11	11.27
Optimal D($\bar{\pi}_t, y_t$) $g_\pi = 0.07, g_y = 0.27, h = 1.00$	3.85	3.80	1.07	7.80	30.42
<i>With lagged information; $i_t = hi_{t-1} + g_\pi \bar{\pi}_{t-1} + g_y y_{t-1}$</i>					
Rule I(2); D($\bar{\pi}_{t-1}, y_{t-1}$) $g_\pi = 3.00, g_y = 0.80, h = 1.00$					Dynamically unstable
Rule II(2); D($\bar{\pi}_{t-1}, y_{t-1}$) $g_\pi = 1.20, g_y = 1.00, h = 1.00$					Dynamically unstable
Rule III(2); L($\bar{\pi}_{t-1}, y_{t-1}$) $g_\pi = 1.50, g_y = 0.50, h = 0.00$	3.62	2.40	0.72	5.20	19.07
Rule IV(2); L($\bar{\pi}_{t-1}, y_{t-1}$) $g_\pi = 1.50, g_y = 1.00, h = 0.00$	3.63	2.14	1.04	5.19	18.29
Rule V(2); S($\bar{\pi}_{t-1}, y_{t-1}$) $g_\pi = 1.20, g_y = 0.06, h = 1.30$					Dynamically unstable
Optimal L($\bar{\pi}_{t-1}, y_{t-1}$) $g_\pi = 2.50, g_y = 1.50, h = 0.00$	2.38	2.44	1.69	5.42	13.03
Optimal D($\bar{\pi}_{t-1}, y_{t-1}$) $g_\pi = 0.04, g_y = 0.21, h = 1.00$	4.96	4.21	0.87	8.58	42.75

difference rule D($\bar{\pi}_t, y_t$) that is shown in table 5.8 requires very low coefficients in order to ensure stability. Even so its performance is quite poor.³⁴

5.4.3 A Nonnegative Nominal Interest Rate Constraint

In this subsection, we consider the occurrence of negative nominal interest rates. Negative nominal interest rates, although highly implausible in practice, are almost never excluded in policy rule analyses and our study is no exception. As noted in section 5.2, our model has many much-debated simplifications; however, one of its least debated approximations is its completely linear nature with its symmetry with respect to zero for all quantities including nominal

34. In rational expectations models, difference rules appear to perform much better, e.g., Fuhrer and Moore (1995) and Williams (1997).

interest rates. Indeed, it is straightforward to calculate the unconditional probability of obtaining a negative nominal funds rate for any given rule. For example, assuming an inflation target of 2 percent and an equilibrium real funds rate of 2.5 percent (which is obtained from the estimated constant term in the IS curve regression without de-meaned data), most of the optimized rules in table 5.3 give about a 20 percent probability of a negative interest rate. Clearly, these rules assume that nominal interest rates would be negative a nonnegligible proportion of the time.

Still, for policy rule analysis, we view the simple imposition of an interest rate nonnegativity constraint as unsatisfactory in several respects. Technically, such a nonlinear constraint renders our analytical methods difficult if not infeasible, though simulation methods are available; see Fuhrer and Madigan (1997) and Fair and Howrey (1996). More important, however, such a constraint, by limiting the degree to which the central bank can conduct expansionary monetary policy at low inflation rates, almost ensures dynamic instability in an otherwise linear model.³⁵ We do not view such instability as plausible. We think that there are always mechanisms by which the central bank can stimulate the economy even if short-term rates are near zero. Expansionary monetary policy could always be conducted by the injection of reserves through purchases of Treasury securities at all maturities (flattening the *entire* yield curve), or purchases of foreign exchange (unsterilized intervention), or even purchases (or financing) of corporate debentures and equity.³⁶ That is, our model, although not strictly true, may give a fairly accurate picture of the potential power of central banks. However, it must be admitted that there is little empirical basis for judging the performance of very low inflation economies in our sample.

5.5 Conclusions

An early working title of this paper was “Practical Inflation Targeting,” by which we meant an exploration of plausible policy rules using a model of a form common at central banks. In this spirit, our examination of policy rules has been in part descriptive, and closely linked to what inflation-targeting central banks actually seem to be doing, as well as partly prescriptive, involving sifting and judging among various rules. From the latter perspective, our results suggest that certain simple forward-looking rules are able to perform quite well.

Of course, our prescriptive results about particular simple rules are conditional on our particular model, and there is much room for extensions and improvements. Questions regarding parameter uncertainty and structural stability are crucial before the results can be taken too seriously; however, judging

35. Intuitively, with an estimated equilibrium real funds rate of 2.5 percent, if inflation ever falls to, say, -3 percent, then with a zero nominal funds rate, the real funds rate is still restrictive, so the output gap decreases and inflation falls even more.

36. See the related discussion in Lebow (1993).

from the results of this conference (and the analysis of Rudebusch 1998b), questions about model uncertainty are likely an order of magnitude larger. Plausible model variation may strengthen our conclusions. For example, our model is backward looking and has no explicit role for expectations and no “credibility effect” in the Phillips curve. An expectations channel for monetary policy through the Phillips curve would most likely make inflation easier to control and more self-stabilizing under inflation targeting.³⁷ In this sense, relative to some of the other papers at this conference, we are stacking the cards against inflation targeting. Nonetheless, there can be no substitute to actually investigating the robustness of our results across model specifications.

However, we would like to emphasize that a forward-looking decision framework for inflation targeting can exhibit robustness to model variation. For example, as mentioned above, one implementation of inflation-forecast targeting is to choose from the set of conditional inflation forecasts (each based on a particular path for the instrument) the one that is most consistent with the inflation target—that is, approaches the inflation target at an appropriate rate, hits the inflation target at an appropriate horizon, and, more generally, minimizes the loss function—and then follow the corresponding instrument path. The construction of conditional forecasts of course depends on the model used, but the procedure itself is robust to known model variation.³⁸ Put differently, targeting rules allow the coefficients of the implied instrument rules to change with structural shifts in the model. It is this decision framework that we have tried to capture in the optimal targeting rule and in the simple inflation-forecast-targeting rules in subsection 5.3.8. In contrast, any given optimal explicit instrument rule depends on the precise model assumed and may be rather imperfect for a different model; any given reasonably robust explicit instrument rule may still be rather imperfect for a specific model.

Appendix

Unconditional Variances

The covariance matrix Σ_{yy} for the goal variables is given by

$$(A1) \quad \Sigma_{yy} \equiv E[Y_t Y_t'] = C \Sigma_{xx} C',$$

where Σ_{xx} is the unconditional covariance matrix of the state variables. The latter fulfills the matrix equation

37. The analysis in Svensson (forthcoming c) of inflation targeting in an open economy with forward-looking aggregate demand and supply confirms this.

38. In a forward-looking model, constructing conditional inflation forecasts for arbitrary instrument paths implies some problems that are not present in a backward-looking model. Svensson (forthcoming a) provides a solution.

$$(A2) \quad \Sigma_{xx} \equiv E[X_t X_t'] = M \Sigma_{xx} M' + \Sigma_{vv}.$$

We can use the relations $\text{vec}(A + B) = \text{vec}(A) + \text{vec}(B)$ and $\text{vec}(ABC) = (C' \otimes A) \text{vec}(B)$ on equation (A2) (where $\text{vec}(A)$ denotes the vector of stacked column vectors of the matrix A and \otimes denotes the Kronecker product), which results in

$$\begin{aligned} \text{vec}(\Sigma_{xx}) &= \text{vec}(M \Sigma_{xx} M') + \text{vec}(\Sigma_{vv}) \\ &= (M \otimes M) \text{vec}(\Sigma_{xx}) + \text{vec}(\Sigma_{vv}). \end{aligned}$$

Solving for $\text{vec}(\Sigma_{xx})$ we get

$$(A3) \quad \text{vec}(\Sigma_{xx}) = [I - (M \otimes M)]^{-1} \text{vec}(\Sigma_{vv}).$$

The Optimal Instrument Rule

The optimal instrument rule is the vector f in equation (9) that fulfills

$$f = -(R + \delta B'VB)^{-1}(U' + \beta B'VA),$$

where the 9×9 matrix V fulfills the Riccati equation

$$V = Q + Uf + f'U' + f'Rf + \delta M'VM,$$

where M is the transition matrix given by equation (10) and Q , U , and R are given by

$$Q = C_x' K C_x, \quad U = C_x' K C_i, \quad R = C_i' K C_i.$$

Furthermore, the optimal value of expression (3) is

$$(A4) \quad X_t' V X_t + \frac{\delta}{1 - \delta} \text{trace}(V \Sigma_{vv}),$$

where $\Sigma_{vv} = E[v_t v_t']$ is the covariance matrix of the disturbance vector.

For $\delta = 1$ the optimal value of equation (5) is

$$(A5) \quad E[L_t] = \text{trace}(V \Sigma_{vv}).$$

An Information Lag

With our state-space setup, the information lag in equation (18) requires inserting π_{t-4} as a tenth state variable and forming the extended 1×10 state-variable vector

$$\tilde{X}_t = \begin{bmatrix} X_t \\ \pi_{t-4} \end{bmatrix}.$$

Then the restriction can be written

$$\begin{aligned}\tilde{g}\tilde{X}_t &= g_\pi\bar{\pi}_{t-1} + g_y y_{t-1}, \\ \tilde{g} &= g_\pi(\tilde{e}_{2,4} + \frac{1}{4}\tilde{e}_{10}) + g_y\tilde{e}_6, \\ \tilde{f} &= h\tilde{e}_7 + \tilde{g}, \\ i_t &= \tilde{f}\tilde{X}_t,\end{aligned}$$

where \tilde{g} and \tilde{f} are 1×10 row vectors and \tilde{e}_j and $\tilde{e}_{j,k}$ are defined as e_j and $e_{j,k}$, except that they are 10×1 vectors.

An Instrument Rule That Responds to a Rule-Consistent Inflation Forecast

Suppose $T \geq 3$ (we deal with $T = 2$ below.) Then we have to write the model in state-space form with forward-looking variables. We first note that, since in our model the first element in B is zero, the first equation in (6) is

$$(A6) \quad \pi_{t+1} = A_1 X_t + v_{1,t+1},$$

where A_1 is the row vector $(a_{1k})_{k=1}^n$. Then π_{t+1} and $\pi_{t+1|t} = A_1 X_t$ are predetermined. In order to write the system in state-space form, we now define the $(T-2) \times 1$ column vector of forward-looking variables, $x_t = (x_{lt})_{l=1}^{T-2}$, where

$$(A7) \quad x_{lt} \equiv \pi_{t+l|t}$$

for $l = 1, \dots, T-2$. Observe that for $l = 1, \dots, T-3$, by the law of iterated expectations,

$$(A8) \quad x_{l,t+1|t} = x_{l+1,t},$$

whereas for $l = T-2$ we have

$$(A9) \quad x_{T-2,t+1|t} = \pi_{t+T|t}.$$

Equation (A8) gives us $T-3$ equations for the first $T-3$ forward-looking variables x_{lt} , $l = 1, \dots, T-3$. We also need an equation for $x_{T-2,t}$. Lead equation (A6) by one period, and take expectations in period t ,

$$(A10) \quad \begin{aligned}x_{1t} \equiv \pi_{t+2|t} &= A_1 X_{t+1|t} = A_1 [A X_t + B(hi_{t-1} - \phi \pi_{t+T|t})] \\ &= A_1 (\tilde{A} X_t + \phi B x_{T-2,t+1|t}),\end{aligned}$$

where

$$(A11) \quad \tilde{A} = A + hB e_7,$$

where we have used equations (6), (19), and (A9). Solve for $x_{T-2,t+1|t}$,

$$(A12) \quad x_{T-2,t+1|t} = -\frac{1}{\phi A_1 B} A_1 \tilde{A} X_t + \frac{1}{\phi A_1 B} x_{1t},$$

which gives us the remaining equation (note that $A_1 B$ is a scalar).

Thus equations (A8) and (A12) give us $T - 2$ equations for the $T - 2$ forward-looking variables. With regard to the predetermined variables, we use equations (6), (19), (A9), (A11), and (A12) to write,

$$(A13) \quad \begin{aligned} X_{t+1} &= \tilde{A} X_t + \phi B x_{T-2,t+1|t} \\ &= \tilde{A} X_t + \phi B \left(-\frac{1}{\phi A_1 B} A_1 \tilde{A} X_t + \frac{1}{\phi A_1 B} x_{1t} \right) \\ &= \left(I - \frac{1}{A_1 B} B A_1 \right) \tilde{A} X_t + \frac{1}{A_1 B} B x_{1t}. \end{aligned}$$

By combining equations (A13), (A8), and (A12), we can now write the system in state-space form,

$$(A14) \quad \begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = D \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} v_{t+1} \\ 0 \end{bmatrix},$$

where the $(n + T - 2) \times (n + T - 2)$ matrix D is given by

$$D = \begin{bmatrix} \left(I - \frac{1}{A_1 B} B A_1 \right) \tilde{A} & \frac{1}{A_1 B} B u_{n+1} \\ D_{21} & D_{22} \end{bmatrix},$$

where u_k , $k = 1, \dots, n + T - 2$, is a $1 \times (n + T - 2)$ row vector with element k equal to unity and all other elements equal to zero and where the $(T - 2) \times n$ matrix D_{21} and the $(T - 2) \times (T - 2)$ matrix D_{22} are given by

$$D_{21} = \begin{bmatrix} 0_{(T-3) \times n} \\ -\frac{1}{\phi A_1 B} A_1 \tilde{A} \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0_{(T-3) \times 1} & I_{T-3} \\ \frac{1}{\phi A_1 B} u_{n+1} & \end{bmatrix},$$

where $0_{k \times m}$ is a $k \times m$ matrix of zeros and I_m is an $m \times m$ identity matrix.

The system (A14) can then be solved with the help of known algorithms, for instance the one in Klein (1997). The solution results in a $(T - 2) \times n$ matrix H , expressing the forward-looking variables as a linear function of the state-variables,

$$(A15) \quad x_t = H X_t.$$

The dynamics of the predetermined variable are then given by

$$(A16) \quad X_{t+1} = (D_{11} + D_{12}H)X_t + v_{t+1},$$

where D_{11} and D_{12} are the obvious submatrices of D . It furthermore follows that

$$x_{t+1|t} = D_{21}X_t + D_{22}x_t = (D_{21} + D_{22}H)X_t.$$

From equations (19) and (A9) follows that the equilibrium instrument rule can be written

$$\begin{aligned} i_t &= fX_t, \\ f &= he_7 + \phi u_{n+T-2}(D_{21} + D_{22}H). \end{aligned}$$

Then we can use f in equations (10) and (11) and proceed as in the other cases. The matrix M in equation (10) will of course equal the matrix $D_{11} + D_{12}H$ in equation (A16).

For $T = 2$, by equations (19) and (A10), we directly get

$$\begin{aligned} \pi_{t+2|t} &= A_1(\tilde{A}X_t + \phi B\pi_{t+2|t}) \\ &= \frac{1}{1 - \phi A_1 B} A_1 \tilde{A}X_t; \end{aligned}$$

hence,

$$\begin{aligned} i_t &= hi_{t-1} + \frac{\phi}{1 - \phi A_1 B} A_1 \tilde{A}X_t, \\ f &= he_7 + \frac{\phi}{1 - \phi A_1 B} A_1 \tilde{A}. \end{aligned}$$

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Comment Frederic S. Mishkin

It is a pleasure to comment on this excellent paper by Glenn Rudebusch and Lars Svensson. In doing so, I first want to clarify what the paper is really about because its title might limit its audience, particularly among central bankers. Second, I will highlight some nice features of the analysis. Finally, I will discuss some of the major results in the paper and suggest why they are so important to practicing policymakers and then will make some concluding remarks.

What the Paper Is Really About

The paper is clearly targeted at central bankers, who, as Rudebusch and Svensson put it, “are among the most important ultimate consumers of this research.” However, the paper’s title, “Policy Rules for Inflation Targeting,” suggests that the paper might only be of interest to central banks that are engaged in, or are contemplating engaging in, inflation targeting. This characterization of the paper would be incorrect because it is just as useful to central banks that have no intentions of engaging in inflation targeting as to those that do.

The authors state that they interpret inflation targeting as “implying a conventional quadratic loss function, where in addition to the variability of inflation around the inflation target there is some weight on the variability of the output gap.” Although I agree with them that inflation targeting as practiced by central banks does display a concern for output variability as well as inflation variability in the loss function (e.g., see the case studies in Mishkin and Posen 1997 and Bernanke et al. 1999), I believe that this is also true for almost any central banker whom I encountered when I was among their ranks.

Inflation targeting involves (1) an institutional commitment to inflation control as the primary goal of monetary policy, (2) a publicly announced explicit inflation goal, with a focus on inflation forecasts using all available information to guide policy rather than one specific intermediate target such as a monetary aggregate, (3) a stress on transparency and communication with the public about the strategy of monetary policy, and (4) accountability of the central bank for achieving its stated inflation goals (see, e.g., King 1994; Leiderman and Svensson 1995; Bernanke and Mishkin 1997; Bernanke et al. 1999). Although many countries have adopted inflation targeting, such as New Zealand, Canada, the United Kingdom, Sweden, Spain, and Australia, other countries such as the United States have not. Nevertheless, because non-inflation targeters also care about inflation variability in their loss functions (as well as the other elements in the loss function used in this paper), the results in the paper are highly relevant to how they should conduct monetary policy. The title of

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the paper should not deter those who are not advocates of inflation targeting from reading this paper with great interest.

The second semantic problem with the title of this paper is that it refers to rules. In the dichotomy between rules and discretion that comes out of the rules-versus-discretion debate, a rule is seen as a precise, written description of how policy is to be conducted that helps to reduce the time-inconsistency problem. The classic example is Milton Friedman's constant money growth rate rule, in which a specific monetary aggregate is stipulated to grow at the same rate every year. Central bankers, in general, are very hostile to these types of rules because they stress the inability of written rules to deal with unforeseen shocks or changes in the structure of the economy and thus see the need for some discretion. Thus central bankers may be inclined to ignore this paper because of their hostility to rules. This would also be a mistake.

The authors clearly do not advocate that the rules they study in this paper be written down and followed slavishly by central banks. They emphasize that the optimal rules they derive are optimal only in the context of the specific model they study, and they acknowledge that there is substantial uncertainty about what is the appropriate model of the economy. They also acknowledge the possibility of a Lucas critique of their analysis in which adherence to their rules might affect expectations and hence the estimated parameters of their model, thus making the simulation results with their rules somewhat suspect. Thus the paper does not intend to suggest that central bankers should announce a rule of the type they study and then be obliged to follow it. Rather they see the analysis in the paper as providing guidance to central banks as to how they should conduct monetary policy and react to new information if they have sensible objectives. For example, one strong implication of the analysis is that good monetary policy should always react to increases in either the output gap or the deviation of inflation from the target level by increasing short-term real interest rates. This is an important prescription for monetary policy and one that has not always been followed in the past (Clarida, Galí, and Gertler 1998). It applies even if a central bank is exercising some discretion and is unwilling to commit to an explicit rule.

The basic point I am making here applies to all the papers in this conference volume. They focus on explicit rules because this is the only way we can scientifically analyze different approaches to the conduct of monetary policy. However, even if the monetary authorities are exercising some discretion, which is the case not only for non-inflation targeters but also for those engaging in inflation targeting,¹ they still need the guidance supplied by the analysis in these papers as to how the setting of policy instruments should respond to information as it comes in to the central bank.

1. As is made clear in Bernanke and Mishkin (1997), Mishkin and Posen (1997), and Bernanke et al. (1999), inflation targeting is a framework, not a rule, in which there is discretion, but the discretion is constrained by the transparency and accountability of the inflation-targeting framework.

Nice Features of the Analysis

The paper conducts its analysis of policy rules in the context of a small, simple macromodel. The use of such a simple model is necessary for tractability in the analysis, and I find the model to be very sensible. (I am clearly somewhat biased here because the model used in this paper is very similar to the one Arturo Estrella and I use in chap. 9 of this volume.) Some might criticize the model because of both its simplicity and the fact that implicitly it assumes that expectations formation is backward looking. One very nice feature of the paper is subsection 5.2.3, which spends some time justifying the use of this model by showing that it fits the data well, has sensible implications for sacrifice ratios, and has reasonable dynamics. In addition, this subsection shows that the authors' small model captures the key essentials of more complex models that monetary policymakers actually use, particularly those at the Board of Governors of the Federal Reserve System. Although not everyone would agree with their model, Rudebusch and Svensson's careful discussion of its key features shows that the analysis in their paper is highly relevant to policymakers who may have a more complicated view of the world.

Another nice feature of this paper and many others in the volume is that they allow for an interest-rate-smoothing objective of the monetary authorities in discussing policy rules by including interest rate variability in the loss function, something that has rarely been done in previous literature. From my experience, I can tell you that central bankers are indeed very concerned about interest rate smoothing, and by focusing on it, the papers in this volume will be more relevant to these monetary policymakers.

Why is it that interest rate stability and smoothing is of such concern to central bankers? I see two reasons. First is that central bankers are very averse to reversing course frequently on interest rates because they are concerned that it may reduce public confidence in central bank competence. When central banks that have recently been raising interest rates suddenly lower them (or vice versa), it may look like an admission of a previous policy mistake. Because central bankers, like most of us, do not like to admit publicly that they have been wrong, it is natural that they should want to avoid quick interest rate reversals. They can avoid this by moving interest rates in short steps in the same direction over a period of time, rather than moving interest rates by a large amount. The resulting interest rate smoothing is exactly what you would get if interest rate variability is penalized in the policymakers' objective function, and this is why it is so sensible to include interest rate variability in the loss function.

The second reason for worrying about interest rate variability in the loss function is that central bankers are concerned not only about inflation and output variability but also about financial stability. Indeed, on a day-to-day basis central bankers probably spend more time concerning themselves with financial stability objectives than with price stability, although you wouldn't always

know this from their speeches. Interest rate instability can be a source of financial fragility because rises in interest rates can directly hurt the balance sheets of banks that engage in the asset transformation activity of borrowing short and lending long. Also, high interest rates can directly hurt business firms' cash flows, which also causes a deterioration in their balance sheets. The deterioration in both banks' and nonbank firms balance sheets can decrease financial stability and make moral hazard and adverse selection problems more severe in credit markets, thereby making it harder for financial markets to achieve their intended purpose of getting funds to people and firms with productive investment opportunities (e.g., see Mishkin 1997).² Thus interest rate variability can also harm the economy and rightfully belongs in the policymakers' loss function.

Because there are good reasons to include interest rate variability in the policymakers' loss function and because central bankers clearly worry about interest rate smoothing, analyzing the effects on optimal policy of interest-rate-smoothing objectives makes the results in this and the other papers in the conference that much more relevant and interesting to their targeted audience in the central banking community.

Major Results and Their Importance to Policymakers

One important finding of the paper is that rules that respond solely to inflation, particularly current inflation rather than forecasts of future inflation, do very badly, even if the objective of the central bank focuses solely on inflation stabilization and is not concerned with output stabilization. Rules that include a response to output gaps, such as Taylor-type rules do far better. In the context of the authors' macromodel, the output gap contains information about future inflation, and so this conclusion can be restated as saying that monetary policy should be very preemptive.

Some economists have argued that as long as the Federal Reserve strongly tightens monetary policy when inflation actually rises in order to squash it, inflation will be unlikely to appear because of the Fed's credibility and monetary policy will be successful at stabilizing both output and inflation. In other words, monetary policy can take a Bunker Hill stance and wait until it sees the whites of inflation's eyes before reacting. The Rudebusch-Svensson results suggest that a nonpreemptive monetary policy of this type will not be successful and will lead to much poorer monetary policy outcomes.

Rudebusch and Svensson's results on the need for preemptive monetary policy rely heavily on the feature of their Phillips-curve-type equation that inflation expectations are backward looking. In macromodels in which inflation expectations are forward looking, for example in several of the papers in this volume, "whites of their eyes" monetary policy, in which monetary policy re-

2. Mishkin (1997) also suggests that there are direct effects of interest rate increases on adverse selection because higher interest rates lead to more adverse selection in credit markets.

acts almost entirely to current inflation and hardly at all to output gaps, works quite well. The intuition is straightforward. Because expectations are forward looking, a strong reaction to inflation only when it appears is nevertheless taken into account in price-setting behavior and so inflation is far less likely rear its ugly head. With backward-looking expectations, monetary policy must be preemptive in order to head off inflation ahead of time because inflation expectations are slow to change. This and other papers in this volume therefore suggest a very important conclusion that at first sounds counterintuitive but is actually quite intuitive once you think about it: the more backward looking are inflation expectations, the more preemptive monetary policy needs to be.

Although I believe that inflation expectations reflect some element of forward-looking behavior, credibility for central banks is hard to come by and takes a long time to develop. Thus inflation expectations are likely to be more backward looking than forward looking. Indeed, this is what the evidence in papers such as Fuhler (1997) suggests—backward-looking models seem to fit the data better—and this is one reason why a model like that used by Rudebusch and Svensson is taken more seriously by policymakers than models that rely on forward-looking expectations. Thus I lean toward Rudebusch and Svensson's view that their macromodel is more realistic than ones relying on forward-looking expectations, and so successful monetary policy must have a strong preemptive component.

Another striking result in this paper is that simple rules of the Taylor type perform quite well and are not far off in their performance from the optimal rule. Part of the reason might stem from the simplicity of the authors' macromodel, but I suspect that another factor is at work. If the monetary authority has the right basic approach to monetary policy—that is, it raises nominal interest rates by more than any increase in actual or expected inflation, so that inflation never spins out of control—then monetary policy will do pretty well. (Symmetrically, policy must lower nominal interest rates by more than any decline in inflation when it occurs.) This is also the implication of John Taylor's paper (chap. 7 of this volume), which suggests that the episodes in which monetary policy made its biggest mistakes occurred when it did not follow the basic prescription outlined above.

The conclusions from the Rudebusch-Svensson and Taylor papers are thus encouraging. Even if monetary policymakers don't get things perfect, they can still do pretty well by making sure that they do not make the mistakes of the past (see Clarida et al. 1998) by thinking that they have tightened monetary policy enough when they raise interest rates in the face of inflation increases, but by an amount less than the inflation increase.

Another important conclusion from this paper is that central banks and particularly inflation targeters should not be what Mervyn King calls "inflation nutters," that is, fanatics on controlling inflation at any cost. Central bank rhetoric about monetary policy often focuses almost solely on price stability, but as Taylor (1993) has emphasized there is a trade-off between inflation variabil-

ity and output variability. Taking account of this trade-off does have important implications for monetary policy and indicates that inflation fanaticism is unwise. Two basic results in Rudebusch and Svensson's paper support this position. First, putting some weight on output gaps as well as on deviations from inflation in formulating monetary policy leads to a substantially lower value of the loss function. Second, policy settings that lead to a more gradual approach to the target level of inflation also produce lower values of the loss function. These results imply that even if the primary goal of monetary policy is price (inflation) stability, it is important for central banks to be flexible.

The need for flexibility is recognized by central banks, even those that have explicitly committed to inflation targeting. As discussed in Mishkin and Posen (1997) and in our book Bernanke et al. (1999), both the Bundesbank and inflation-targeting central banks in their disinflationary phases have lowered their inflation targets gradually toward their long-run goals. Furthermore, these central banks have expressed their concerns about the trade-off between inflation variability and output variability. Thus central banks with a strong commitment to fighting inflation cannot be characterized as "inflation nutters," and the findings in this paper provide a further rationale for the degree of flexibility that they have been exercising.

Concluding Remarks

Despite the title of the paper, which might limit its audience, this paper should be required reading for central bankers. Although it makes use of a simple macromodel, which not everyone would accept, it provides some basic insights that I believe would continue to hold up in more complex frameworks. Most important, this paper suggests that if central banks focus on the right basic strategy, they do not have to be perfect to do quite well.

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Comment James H. Stock

Glenn Rudebusch and Lars Svensson have provided a clear and interesting treatment of a large number of policy rules within a bivariate vector autoregression (VAR). They model the interest rate (the federal funds rate) as an exogenous variable under the perfect control of the Fed. Changes in the interest rate affect the deviation of real output from potential, which in turn affects inflation through an output-based Phillips curve. Control rules are evaluated in terms of their expected loss, which is a function of the variances of inflation, potential GDP, and the interest rate.

Their paper is clearly and precisely written and the results are well presented. Their discussion of loss functions and targets is lucid and compelling. The modeling decisions they made are sensible and permit the evaluation of a large number of rules. In future work along these lines, it would be of interest to consider a larger VAR that includes an additional interest rate (so that the Fed is not implicitly given control over the entire term structure in the simulations). Similarly, most methods for constructing potential GDP are questionable, and theirs is no exception. The pitfalls of estimating potential GDP could be sidestepped by specifying the Phillips curve in terms of unemployment rather than potential GDP. It would be useful to see whether their findings, particularly the importance of large coefficients in Taylor-type rules, hold up under these extensions. These comments are relatively minor, however, and in general their paper constitutes an excellent contribution to the literature on monetary policy rules.

Because Rudebusch and Svensson's paper is so clean and self-contained, in the remainder of these comments I will turn to the broader question that is one of the motivations for this conference, the construction and evaluation of control rules in the presence of model uncertainty. A policy rule that performs well under reasonable perturbations of a model, or under different plausible models, is robust to that model uncertainty. Although policy robustness is an underlying theme of this conference, it is important to emphasize two limitations of the robustness results reported in this volume.

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First, the “conference rules” have been evaluated by various authors using their estimated models, but each of the estimated models contains considerable model uncertainty arising from the estimation of the model parameters. It is possible that a rule is robust across point estimates of models, which might be similar in important dynamic respects (after all, the models are estimated using the same data), but that the rule is not robust to 1 standard error changes in the parameters of the models. Robustness to sampling uncertainty needs to be investigated more carefully before any conclusions can be drawn about the robustness of the policy rules considered in this volume (I return to this point below).

Second, a theme of several papers is that inflation-forecast-targeting rules (in which the monetary authority adjusts the interest rate to move an inflation forecast toward a target) perform well in many of the models considered here. However, this conclusion is drawn by evaluating the performance of the inflation-targeting rule using the same model that is used to compute the inflation forecast. In contrast, the essence of policy robustness is whether a specific quantitative rule performs well under a model *other than that used to develop the policy*. Rudebusch and Svensson find that inflation-forecast-targeting rules, based on conditional inflation forecasts produced by their models, work well when evaluated using their model. The proper check of robustness, however, is whether inflation-forecast-targeting rules based on, say, the Rudebusch-Svensson model forecasts work well when the true model is something else.

To illustrate this point, suppose the Fed hires Rudebusch and Svensson to make their conditional forecasts: the Fed provides them a trial value of the interest rate, Rudebusch and Svensson compute inflation forecasts, and they iterate until the inflation forecasts from their model satisfy the Fed policymakers. Now suppose, however, that Rudebusch and Svensson’s research assistant mistakenly feeds the conditional U.S. interest rate into a model of the Swedish economy rather than their U.S. model, so that Rudebusch and Svensson report back Swedish rather than U.S. inflation. One would expect this inflation-forecast-targeting rule, thus implemented, to produce outcomes for the U.S. that are badly wrong: the model used to generate the inflation forecasts differs sharply from the true model. While one would hope that such a gross mistake would not happen in practice, the essential point is that evaluating the robustness of inflation-targeting rules requires the evaluation of the model’s conditional forecasts when that model is false. I know of no research on monetary policy rules that undertakes that evaluation.

The remainder of these comments take up this problem in the form of parametric model uncertainty, by which I mean uncertainty that can be summarized as uncertainty about the value of a finite-dimensional parameter. This complements Sargent’s comments on Ball’s paper (chap. 3 of this volume), in which Sargent considers the case of uncertainty that is nonparametric in the sense that the uncertainty can be formalized as over elements of an infinite-dimensional space. In particular, I will consider two approaches to parametric

uncertainty. The first is a Bayesian approach that grows out of Brainard's (1967) early work on parameter uncertainty. I will argue that while this approach is appealing from the perspective of decision theory, and while it can yield intuitive results, in practice it places informational demands on policymakers that are wholly unrealistic and therefore fails to provide a useful framework for constructing practical policies. In its place, I propose using minimax methods to construct optimal robust policies and implement these methods quantitatively in the Rudebusch-Svensson model.

Bayesian Approaches to Model Uncertainty

The Bayesian decision analytic approach to control under parametric uncertainty posits a loss function that is a function of future macroeconomic variables. The decision maker is assumed to have priors over all parameters in the model. Optimal policy is then solved by finding the policy that minimizes the expected loss, integrating over the parameters with respect to the prior density. This is conventionally done in the context of a single model. However, in this volume several distinct models are presented, so it is of interest to consider the result of this procedure when there is uncertainty over the class of models as well. In particular, consider two stylized single-equation models of inflation:

$$(1) \quad \pi_t = \beta x_{t-1} + \varepsilon_t,$$

$$(2) \quad \pi_t = \alpha \pi_{t-1} + \gamma x_{t-1} + \eta_t,$$

where π_t is inflation and x_t is the control variable. Evidently, the two models differ only in whether lagged inflation has an effect on future inflation. Suppose that the decision maker has Gaussian priors over β in the first model, so that $\beta \sim N(\bar{\beta}, \sigma_\beta^2)$. For the second model, the decision maker has the priors $\gamma \sim N(\bar{\gamma}, \sigma_\gamma^2)$.

Suppose the decision maker has quadratic loss, $(\pi_t - \pi^*)^2$, where π^* is the target rate of inflation. If the decision maker were sure that model (1) is correct, then the optimal policy would be

$$(3) \quad x_{t-1}^{*,1} = [\bar{\beta}/(\bar{\beta}^2 + \sigma_\beta^2)]\pi^*.$$

On the other hand, if the decision maker were sure that model (2) is correct, the optimal policy would be

$$(4) \quad x_{t-1}^{*,2} = [\bar{\gamma}/(\bar{\gamma}^2 + \sigma_\gamma^2)](\pi^* - \alpha \pi_{t-1}).$$

Now suppose that the decision maker does not know which model is correct but is sure that one of them is; he or she assigns prior probability λ to the event that model (1) is the true model. In this case, the optimal policy is

$$(5) \quad x_{t-1}^* = \lambda x_{t-1}^{*,1} + (1 - \lambda)x_{t-1}^{*,2}.$$

The noteworthy feature of this result here is that when there is uncertainty over classes of models rather than just (smooth) uncertainty over the parameters in a model, the optimal policy is a linear combination of the two optimal policies in the individual models. At least in this simple example, then, one could imagine giving a board of policymakers the optimal policies resulting from the individual models and letting each policymaker compute his or her individual weighted average of these model-based policies, based on each individual's views of how likely a particular model is to be correct.

Although this result has intuitive appeal, there are reasons to doubt that its simple lessons can be made general enough to be useful for practical policy making. First, on a technical level, dynamic models with learning imply very different rules, in which there can be experimentation to learn about the parameters of the model (cf. Wieland 1996, 1998). It is not clear how this would generalize to the multimodel setting.

Second and more fundamentally, the calculations here require an unrealistic amount of information. Key to these calculations are the existence of prior distributions, which for nonlinear models need to be joint priors over all the parameters. While it is plausible that policymakers might have opinions about the value of the NAIRU or the slope of the Phillips curve, it is not plausible that they would have opinions about, say, the covariance between α_{π_3} and β_{y_2} in equations (1) and (2) in Rudebusch and Svensson's paper. Indeed, there has been great debate about how to construct priors for large autoregressive roots in univariate autoregressive models (see, e.g., the special issue of *Econometric Theory*, August/October 1994); I believe that a fair summary of this debate is that various Bayesians have agreed to disagree over how to construct their priors. If experts cannot construct priors for univariate autoregressions, it is entirely unrealistic for noneconometrician policymakers to construct priors for multiequation nonlinear dynamic models. Unfortunately, such priors are a necessity for the foregoing calculations, so the conventional decision analytic approach does not seem to be a promising direction for developing practical policy rules that address model uncertainty. It is therefore useful to explore an alternative approach based on minimax approaches to model uncertainty.

Minimax Approaches to Model Uncertainty

An alternative approach is to evaluate policies by their worst-case performance across the various models under consideration. The best policy from this perspective is the minimax policy that has the lowest maximum risk. Because Rudebusch and Svensson do not consider parameter uncertainty, as an illustration I will consider the effect of parameter uncertainty on policy choice using the Rudebusch-Svensson model.

Specifically, I consider their model (1)–(2), with their point estimates, and focus on the effects of uncertainty in two of the parameters, α_y and β_r . These are the two most interesting parameters of their model from an economic perspective: α_y is the slope of the (potential GDP) Phillips curve, and β_r is the impact effect on the GDP gap of a change in the interest rate.

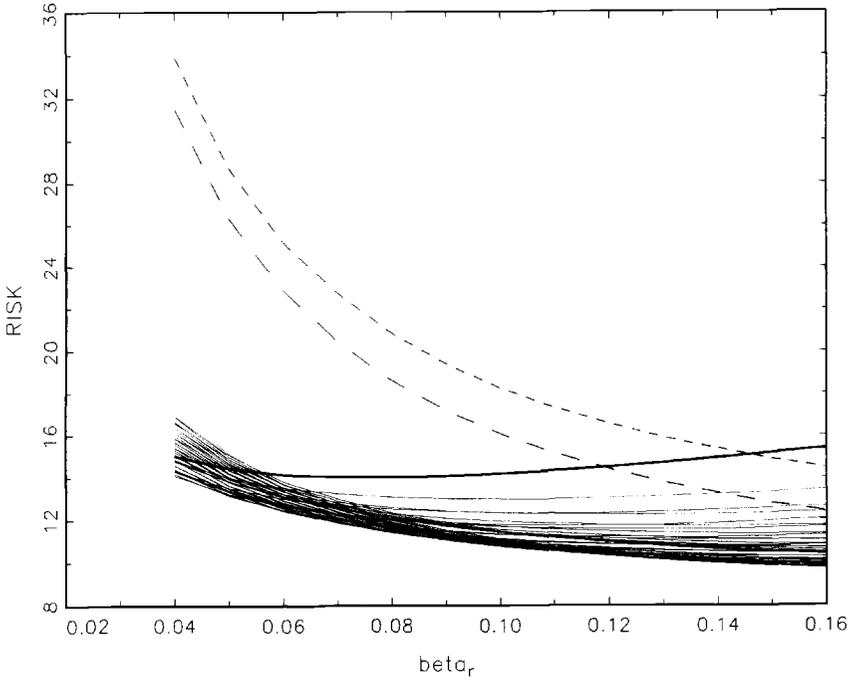


Fig. 5C.1 Risk functions for various policy rules, evaluated in the Rudebusch-Svensson model for $0.04 \leq \beta_r \leq 0.16$ and $\alpha_y = 0.20$

The loss function considered here is the one of the Rudebusch-Svensson loss functions,

$$(6) \quad \text{Loss} = \text{var}(\pi_t) + \text{var}(y_t) + \frac{1}{2} \text{var}(\Delta i_t)$$

in their notation. To capture model uncertainty, values of parameters α_y and β_r within 2 standard errors of their point estimates were considered; that is, the parameters were varied in the ranges $0.08 \leq \alpha_y \leq 0.20$ and $0.04 \leq \beta_r \leq 0.16$.

The policy rules considered here are two-parameter modified Taylor rules of the type considered by Rudebusch and Svensson, specifically,

$$(7) \quad i_t = g_\pi \bar{\pi}_t + g_y y_t.$$

Three types of policy rules were considered: the Taylor rule ($g_\pi = 1.5, g_y = 0.5$) and a modified Taylor rule with somewhat more response to output fluctuations ($g_\pi = 1.5, g_y = 1.0$); model-specific optimal rules of the type (7), in which the parameters are optimal for particular values of α_y and β_r ; and the minimax rule that minimizes expected loss over all parameter values.

Slices of the risk function surface are presented in figure 5C.1 for these various policy rules; the slices present risk as a function of β_r for $\alpha_y = 0.20$. The upper lines are the risks of the two conference rules, the Taylor rule (*short*

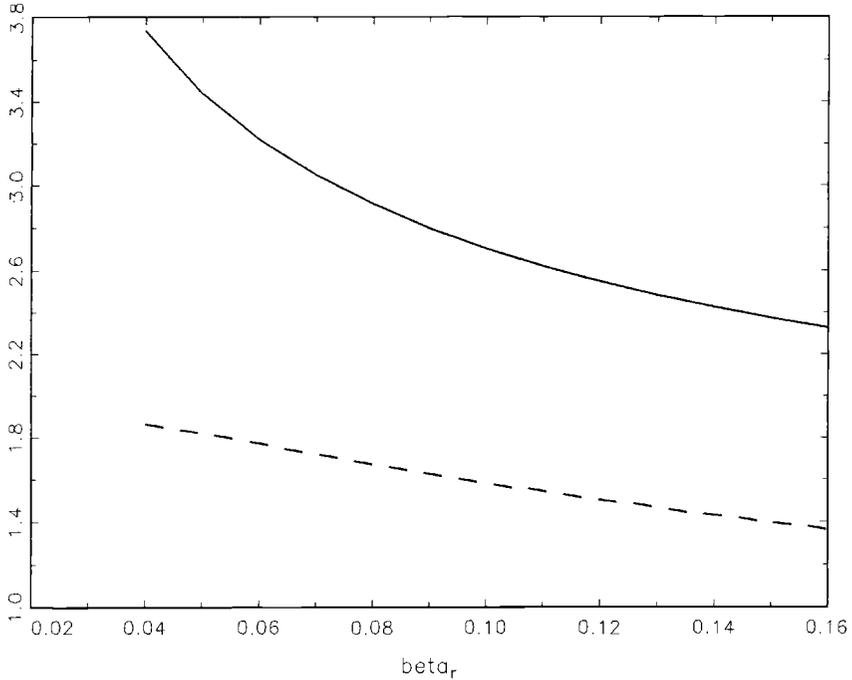


Fig. 5C.2 Optimal parameter values for Taylor-type rules in the Rudebusch-Svensson model for $0.04 \leq \beta_r \leq 0.16$ and $\alpha_y = 0.14$

dashes) and the rule with $g_\pi = 1.5$ and $g_y = 1.0$ (long dashes). Each of the light solid lines is the risk function for a policy that is optimal for a particular value of (α_y, β_r) ; the lower envelope of these dotted lines constitutes an envelope of the lowest possible risk, across these parameter values. The heavy solid line is the risk of the policy that is minimax over $0.08 \leq \alpha_y \leq 0.20$ and $0.04 \leq \beta_r \leq 0.16$. (The model-optimal and minimax policies were computed by a simulated annealing algorithm with 1,000 random trials.)

Several observations are apparent. First, the Taylor rule has very large maximum risk. The risk is greatest when β_r is lowest. When monetary policy has little effect (β_r is small), the Taylor rule produces movements in interest rates that are too small to stabilize output and inflation as quantified by the loss function (6). It turns out that the minimax rule has a risk function that is tangent to the risk envelope, with the point of tangency corresponding to the model in which monetary policy has the smallest direct impact on the GDP gap and the Phillips curve is flattest ($\beta_r = 0.04$ and $\alpha_y = 0.08$). In the Rudebusch-Svensson model, this corresponds to the case in which monetary policy is least effective. Here the minimax policy is obtained by producing the optimal rule in the least favorable case for monetary control of inflation and output.

The model-specific optimal parameter values are plotted in figure 5C.2 for

$\alpha_y = 0.14$. Evidently, when the impact effect of monetary policy is small, the optimal response of monetary policy to inflation (*solid line*) and the output gap (*dashed line*) is large. This is the case for the minimax policy, in which $g_\pi = 3.86$ and $g_y = 1.48$. The minimax risk across all models for this policy is 15.61. For the Rudebusch-Svensson model with this parametric uncertainty, the minimax-optimal Taylor-type rule exhibits very strong reactions to inflation and the output gap to guard against the possibility that the true response of the economy to monetary policy is weak.

These results are only illustrative, but they indicate that quite different conclusions can be reached once we admit that there is parameter uncertainty in our models. In the Rudebusch-Svensson model, recognizing parameter uncertainty leads to “conservative” policies that exhibit more aggressive responses than are optimal for the point estimates of the model. It would be interesting to see this sort of analysis undertaken for some of the other models presented in this volume.

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Discussion Summary

Arturo Estrella asked whether the good performance of smoothing rules in the paper is related to the fact that the IS curve depends on the difference between the short-term nominal interest rate and recent inflation. Changes in the nominal rate reflected in the smoothing rules could be proxying for the difference between the nominal rate and recent inflation. *Svensson* replied that the reason for the bad performance of difference rules was not clear. There is a tendency to get an eigenvalue equal to one or above because the coefficients sum to one in the Phillips curve.

Andrew Haldane noted that most inflation-targeting countries seem to be small open economies. It would therefore be interesting to see how the results of the paper change in an open economy setting. *Svensson's* work and the *Batini and Haldane* paper presented at the conference suggest that the change in results would be substantial. Consider the example of simple rules. The two simple rules that perform well in the paper are the Taylor rule and a constant-interest-rate inflation forecast rule. In a model with only two equations, aggregate demand and aggregate supply, these rules, which condition on just two

variables, come not surprisingly close to being fully optimal. In a setting with an important role for exchange rates, Svensson's work on inflation targeting in small open economies indicates that the Taylor rule might not do very well. The second rule holds interest rates constant, which is not admissible with a no-arbitrage condition in a forward-looking open economy setting. Regarding the latter point, *Rudebusch* replied that one of the reasons for the paper to look at constant-interest-rate inflation forecast rules is that inflation-targeting central banks, such as the Bank of England, produce these forecasts in their inflation reports. Therefore, these forecasts seem to be of interest for policy. *Svensson* agreed that simple rules work well because the model is simple enough for inflation and output to be sufficient statistics. With more variables, for example, fiscal policy, simple rules would work less well.

Volker Wieland noted that in the presence of uncertainty about multiplicative parameters, such as the effectiveness of monetary policy, in a linear model, the optimal rule exhibits a more cautious policy response. However, additive uncertainty, such as uncertainty about the natural rate, does not matter in a linear-quadratic framework. In a nonlinear model, additive uncertainty begins to matter. Nonlinearities could, for example, be in the preferences or in the constraints, such as zero-bound constraints on nominal interest rates or nonlinear Phillips curves. *John Williams* mentioned that in his own work on parameter uncertainty using the U.S. model (*Williams* 1997), the value of the loss function and the implicit optimal rule vary greatly with the parameter governing the slope of the Phillips curve. While this parameter is thus a key parameter for monetary policy, it is unfortunately also the least precisely estimated parameter of the model.

Frederic Mishkin made two points justifying rules based on constant-interest-rate inflation forecasts in the context of a closed economy. First, these rules help central banks communicate with the public. Second, these rules help guide discussions about monetary policy in central bank meetings. *Svensson* illustrated these points by noting that in the case of a strict inflation-forecast-targeting rule, the warranted change in interest rates can be expressed as the difference between the unchanged-interest-rate inflation forecast and the inflation target, divided by the policy multiplier, which is easy to communicate. In practice, inflation reports show such constant-interest-rate inflation forecasts.

William Poole stressed that to understand rules for the federal funds rate, it is essential to have two interest rates in the model because of the following reasoning. One of the attractive features of money growth rules is that the economy has a built-in stabilizing mechanism: with constant money growth, shocks to aggregate demand change interest rates, thus keeping the economy from "running off." Something similar happened in recent years with the Federal Reserve's federal funds rate targeting: long interest rates have changed in response to anticipated future federal funds rate moves, even when the Federal Reserve did not change the federal funds rate much. So the fact that bond markets are forward looking is a built-in stabilizing mechanism.

Ben McCallum approved of the emphasis on terminology in the paper. The distinction between targeting a variable and responding to a variable warrants consideration. However, the notion of inflation targeting is odd in the context of high λ -values, that is, when the weight on output variability is much higher than the weight on inflation. *Svensson* agreed with *McCallum* that to use the term “inflation targeting,” the weight on inflation should be significant.

Robert King remarked that the term “interest rate smoothing” is usually used to denote inertia in the level of interest rates, represented by a large response coefficient on the lagged interest rate and small coefficients on contemporaneous output and inflation. From both the *Rotemberg and Woodford* and the *Batini and Haldane* papers it seems as if forward-looking models could rationalize that pattern of response. How does such a rule perform in the *Rudebusch-Svensson* model? *Rudebusch* replied that these rules are not desirable in their model. Moreover, *Rudebusch* disagreed with *King*’s characterization of interest rate smoothing. Whether a rule smooths the interest rate depends on how persistent the arguments of the rule are. *John Taylor*’s original rule has small response coefficients with no lagged term, and yet, it produces a path for the funds rate that is as smooth as the historical series. In a model with persistence in the output gap and inflation, it is not clear whether a large coefficient on the lagged interest rate is needed to smooth the funds rate.

Ben Friedman noted that the point about *Brainard*-type uncertainty rules driving the policymaker toward more conservatism depends not only on the model but also on the policy rule and the policy instrument. In the context of this discussion, the policy instrument is the interest rate, and therefore conservatism presumably means less variation in interest rates. In a world with money demand shocks, conservative policy thus leads to higher variability in monetary quantities. However, with a policy rule based on the monetary base, conservatism means that the money base grows more closely along a fixed growth path, which, for the same reasons but now played in reverse, means more interest rate volatility. *Friedman* asked whether this tension is handled conceptually in the approach presented in *Stock*’s comment. *Stock* replied that in the example used in his discussion, the policy rule was based on interest rates. In a comparison of different instruments, it is not obvious that the optimal combination rule is going to be spanned by the submodels. *Stock* also remarked that conservatism does not necessarily mean gradualism. In his simulations, the *Taylor* rule was the most conservative rule in the sense that the response coefficients were smallest. However, the *Taylor* rule generated large losses and was far away from a minimax or optimal solution. *Bob Hall* remarked that the same question arises in prison sentences because of the unknown deterrent effect. Is it conservative to give felons short sentences?

Tom Sargent questioned the conclusion drawn in *Stock*’s comment regarding the averaging of rules. If the analysis suggested by *Stock* is pursued with the model at hand, a dynamic model, the posterior over models becomes part of the state of the control problem, such as in *Volker Wieland*’s thesis, implying

that decision rules in this problem become functions of this distribution. Furthermore, the control problem is going to unleash an experimentation motive. If a decision maker is confronted with more than one possible model and a prior over those models, he wants to manipulate the data to learn more. The minimax caution characterization is a static problem, which will not survive the dynamics.

Reference

Williams, John C. 1997. Simple rules for monetary policy. Washington, D.C.: Board of Governors of the Federal Reserve System. Manuscript.