Aggregation Level in Stress Testing Models*

Galina Hale†
John Krainer‡
Erin McCarthy§

June 9, 2015

Abstract

We explore the question of optimal aggregation level for stress testing models when the design of the stress test is specified in terms of aggregate macroeconomic variables, but the underlying performance data is available at a loan level. Using standard model performance measures, we ask whether it is better to formulate models at a disaggregated level ("bottom up") and then aggregate the predictions in order to obtain portfolio loss values? Or is it better to use aggregated models ("top down") at the portfolio level. We study this question for a large portfolio of home equity lines of credit. We conduct model comparisons of loan-level default probability models, county-level models, aggregate portfolio-level models, and hybrid approaches based on portfolio segments such as debt-to-income (DTI) ratios, loan-to-value (LTV) ratios, and FICO risk scores. For each of these aggregation levels we choose the model that fits the data best in terms of in-sample and out-of-sample performance, and then compare winning models across all approaches. We document three main results. First, all the models considered here are capable of fitting our data when given the benefit of the using the whole sample period for estimation. Second, in out-of-sample exercises, large performance differences emerge between the different models. Loan-level models appear to be particularly unreliable in the out-of-sample exercises, apparently due to their lower sensitivity to the macro risk factors in the stress scenario. We find that the average forecast performance is best for portfolio and county-level models. However, for portfolio level, small perturbations in model specification may result in large forecast errors, while country-level models tend to be very robust. Third, we find that more aggregated models tend to produce more conservative forecasts than county-level or loan-level models.

Keywords: bank stress testing, forecasting, portfolio granularity, probability of default, mortgage, home equity

*The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

†galina.b.hale@sf.frb.org

‡john.krainer@sf.frb.org

§erin.mccarthy@sf.frb.org

1 Introduction

Under the Dodd-Frank Act, the Federal Reserve is required to conduct annual stress tests of the systemically-important U.S. banking institutions.[1] Unlike the traditional stress testing procedures which try to measure the severity of losses from an empirical loss distribution, the centerpiece of the supervisory stress tests is a specific set of scenarios for the aggregate economy. A scenario consists of explicit paths for variables such as interest rates, asset prices, unemployment, inflation, and GDP growth. The scenarios are not necessarily considered likely: no probabilities are attached to individual scenarios. However, the scenarios are meant to be coherent in the sense that, even though some variables, such as unemployment, may move to extreme values, other variables in the scenario, such as credit spreads, comove with these extreme changes in historically consistent ways.

Given the distinct structure of supervisory stress tests, our research question centers on which risk modeling and forecasting approaches may prove to be most useful for the task at hand. Specifically, if the inputs to the stress test are given at a certain level of aggregation, what then does this imply about the appropriate level of aggregation of underlying loan portfolio variables in the risk modeling? In this paper we investigate whether different levels of portfolio aggregation yield different degrees of forecasting error and stability, and sensitivity to macroeconomic variables.[2] Our application is to default forecasting for a portfolio of home equity lines and loans observed over the 2002-2013 period. We consider "bottom-up" loan-level models, where we incorporate very detailed information on loan characteristics as well as local and aggregate economic variables, "top-down" time-series models at the portfolio level, and hybrid approaches, where we aggregate the data into buckets by deciles of the risk factors or by county. The goal of the paper is to provide an out-of-

[1]The Federal Reserve has conducted several distinct rounds of stress testing since the financial crisis in 20087. Both the Supervisory Capital Assessment Program (SCAP) of 2009, and the Comprehensive Capital Analysis and Review (CCAR) are very similar in terms of format. The principal change in supervisory stress testing over the past several years has been the increase in number of institutions included in the exercise

[2]By forecast stability we mean low sensitivity of estimates to changes in data sample or small perturbations of the model specification.

sample forecast evaluation of these models and assess which levels of aggregation appear to work best in terms of a mean-squared error (MSE) of forecast as well as in terms of predicting scenario outcomes that are both plausible and conservative.

In our context of predicting the probability of default on a home equity line or loan, a number of theoretical considerations weigh on the choice of which level to aggregate the data to. At one extreme, a top-down approach would have us use fairly simple specifications to capture the time-series dynamics of default rates. This level of aggregation also fits with the loss function of a bank regulator, which would emphasize default rates or losses at an aggregate or firm level rather than at the individual loan level. The disadvantage of using highly aggregated data, or course, is that these models are almost surely misspecified. These models will perform poorly if the composition of loans is changing over time.[3] There is also a risk that the aggregation process introduces so-called aggregation bias, where parameters estimated at a macro-level deviate from the true underlying micro parameters.[4]

The loan-level model alternatives also present challenges. One of the main obstacles to estimating micro-level models has long been the scarcity of reliable loan-level data. This is much less of a problem in the current day, given the recent improvements in data collection in the banking and financial sector. However, many of the risk factors that enter into a loan-level default model are actually market-level proxies of the individual borrower's risk factor. For example, housing values (i.e., the value of the collateral for the loans) are not updated regularly at the individual borrower level. In our analysis we update the loan-to-value ratio (LTV) using a county or zip code house price index. This introduces measurement error into the estimation, which may be nonrandom.

[3] Frame, Gerardi, and Willen (2015) show how this changing loan composition led to large errors in the OFHEO loan-level default model used to stress the GSE's exposure to mortgage default risk.

[4] Going back to Theil (1954), linear models that are perfectly specified at the micro-level were known to be susceptible to aggregation bias. Grunfeld and Griliches (1960) showed that once this assumption of a perfectly specified micro model is relaxed, then aggregation could produce some potential gains in the form of aggregating out specification or other types of measurement error. Also, Granger (1980) shows that time series constructed from aggregated series can have substantially different persistence properties than is present in the underlying disaggregated data.

We encounter the same measurement problem with the unemployment rate, which we proxy for with a county-wide unemployment rate. The home-owning and home equity borrowing population may be quite different from the population in a county most exposed to unemployment shocks. Indeed, for the case of unemployment, there is a further complication. Ideally, we would have a variable telling us whether the borrower him or herself is unemployed.[5] But what in fact we have is a population average probability that the borrower is unemployed (see also Gyourko and Tracy (2013)). All of these concerns would tend to lead to an attenuation bias, or a propensity to find a weaker estimated relationship between two variables of interest than is in fact present. This bias seems particularly worrisome given the design of the Federal Reserve's stress tests which are cast in terms of exactly these variables where we have measurement difficulties.

In order to evaluate different levels of aggregation with respect to the CCAR usefulness, we need to choose an empirical specification for each. It turns out that the set of model specifications with good fit are different for different aggregation levels. For this reason we proceed in three steps: first we screen a very large number of specifications that include all potential risk drivers at various lags, as well as their interactions, for statistical significance, intuitive sign, and in-sample fit. This is done using some judgement (e.g., house prices should enter into any model of home equity default), as well R^2 and information criteria. Then we focus on a smaller number of reasonable specifications that pass the screening test. For each of these specifications, we estimate regressions using data ending in each of 12 months from June 2008 through July 2009. In each case, we construct the forecasted default frequency for the following 9 quarters, in the spirit of CCAR exercises, and compute MSEs as well as measures of how conservative the forecast is.

We find that across all these specifications, county-level regressions tend to have lower forecast errors, produce reasonably conservative results, and, most importantly, are quite stable across specification and forecast windows. Loan-level regressions tend to have the highest forecast errors

[5] There is evidence that borrowers are not completely strategic in their default behavior and require a "double trigger" of house price declines and unemployment (Gerardi, Herkenhoff, Ohanian, and Willen (2013)).

and the least conservative predictions, while aggregate regressions perform well on average but are not very robust to specification changes. Models aggregated by risk factor deciles also perform quite well and are relatively stable across specifications. They are, however, inferior to county-level regressions in terms of the forecast error. Our overall conclusion is that neither loan-level not top-down aggregate models are best for CCAR purposes. It appears the best approach is to aggregate the data to some extent — most meaningfully, to the level at which macroeconomic variables used in scenarios are available.

The paper is organized as follows. In section 2 we demonstrate that econometric theory does not provide a clear guide as to which level of aggregation will result in the lowest forecasting error. We illustrate the way that disaggregated models (i.e., loan-level risk models in our application) may suffer from measurement error, while the most aggregated top-down risk models may suffer from aggregation bias. In section 3 we describe the home equity data set we use, detail the specifics of our forecasting exercise, and present the results. Section 4 concludes.

2 Econometric framework

Our goal is to predict default rate $y \in [0, 1]$ on the entire portfolio given macroeconomic scenarios. The macroeconomic variables x do not vary by loan in portfolio, although some macroeconomic variables might vary by geographical segments of portfolio. For simplicity of notation, suppose we are only predicting one period forward, that is predicting y_{T+1} given x_{T+1} and observed history of y's and x's up to period T. Suppose the data generating process (DGP) is such that

$$y_t = X'\beta + \varepsilon,$$

where y is a vector of observed default rates (or, in case of individual loans, default indicators) over time, X is a matrix of observed covariates, including constant term, unobserved disturbance ε is distributed $N(0, \sigma^2)$. We can use linear regression to estimate b, the estimator for β, and $\hat{\sigma}$, the estimator for σ.

Suppose y and X are observed at individual loan level, and there are N loans observed for T time periods. Therefore, we have a choice of whether to estimate b and $\hat{\sigma}$ on individual loan data (using, for example, linear probability regression), on average values of y and X for sub-portfolios of any type (using pooled or fixed effects panel regression), or on overall portfolio averages (using time series regression). Given that our goal is to predict aggregate y, we want to determine which method is preferable.

Regardless of the regression estimated, the forecast can be constructed by substituting b for β in the DGP equation above. For now, let us assume that regardless of the aggregation level, we can obtain an unbiased estimate of β, therefore aggregation level will not affected expected forecast mean.

In case of unbiased estimates, therefore, we are only concerned with the precision of our forecast. Assume that all the individual observations are i.i.d. Let's denote y_L and X_L the observables measured at loan level, y_P and X_P those at portfolio level, and y_B and X_B those at sub-portfolio, or bucket, level. Portfolio and sub-portfolio variables can be expressed as averages of loan-level data:

$$y_P = \frac{1}{N} S_N' y_L, \;\; X_P = \frac{1}{N} S_N' X_L,$$

where S_N is an $(NT \times T)$ summation matrix such that each element of y_P and each row of X_P are sums of elements in a given time t.[6] Similarly,

$$y_B = \frac{1}{J} S_J' y_L, \;\; X_B = \frac{1}{J} S_J' X_L,$$

[6] $S_N = I_T \otimes 1_N$, where I_T is a $(T \times T)$ identity matrix and 1_N is a vector of N ones.

where $1 < J < N$ is the number of sub-portfolios, S_J is an $(NT \times JT)$ summation matrix such that each element of y_B and each row of X_B is the sum for a given t of all the elements of subportfolio j.[7]

One can show that differences in Brier score for predicting y_{T+1} using regressions with different level of aggregation will be determined by differences in estimated variance of the disturbance $\hat{\sigma}$, the number of loans and sub-portfolios, and differences in inverse sum of squared covariates. If the observations are i.i.d., different aggregation levels will give the same results in the limit. However, in finite samples, even if observations are drawn from i.i.d. distributions, there will be differences in forecast errors, depending on a sample. They will generally be larger the more aggregated the regression sample is.

There are two main reasons, however, to believe that the observations in the analysis are not i.i.d. and therefore the estimates of β are not necessarily unbiased: namely, measurement error and aggregation bias, to which we now turn.

2.1 Individual-level measurement error

One issue that arises in loan-level analysis is that macroeconomic variables are not measured at a loan level. For example, while a borrower's unemployment status or home price have a direct effect on his or her probability to default on the home equity loan, it is common to proxy for these variables with state or MSA-level unemployment rate and home price index, which introduces measurement error problem in loan-level regressions. With sufficient number of observations per state or MSA, these individual errors cancel out when computing averages for the state or MSA-level regressions, so the problem is specific to loan-level regressions.

Formally, let's define observables Z, a subset of X, that is only observable at aggregation level

[7] $S_J = I_T \otimes I_J \otimes 1_{N_J}$ in a special case of all subportfolios having the same number of observations, so that $J * N_J = N$.

of sub-portfolios. Thus, for loan i in subportfolio j and time t, the true covariate Z_{ijt} is

$$Z_{ijt} = \overline{Z}_{jt} + \zeta_{ijt},$$

where ζ is unobserved and is distributed with mean zero and variance ς^2. When $\overline{Z}_{jt}$ is used instead of unobserved Z_{ijt} in the regressions, they suffer from an omitted variable bias, due to correlation between the regressor $\overline{Z}_{jt}$ and the error term, which now is $\varepsilon_{ijt} + \zeta_{ijt}$. Thus, the estimator b_L is no longer unbiased. To see this, denote as $\tilde{X}$ the subset of regressors in X that is not Z combined with observable $\overline{Z}$. The unbiased estimators would be produced by the regression

$$y = \tilde{X}'b + \zeta'c + e.$$

Since ζ is unobserved, we estimate instead the regression

$$y = \tilde{X}'\tilde{b} + \tilde{e}.$$

We can show that

$$E(\tilde{b}) = \beta + (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\zeta c.$$

Given that in general, $\overline{Z}$ and ζ are correlated, $\tilde{X}$ and ζ are correlated, and therefore $\tilde{b}$ will not be an unbiased estimator of β. If the correlation is positive and $c > 0$ or if correlation is negative and $c < 0$, $E(\tilde{b}) > \beta$, otherwise, $E(\tilde{b}) < \beta$. Since c and ζ are not observed, it is generally not possible to evaluate on pure econometric basis whether the bias is positive or negative. Note that an attenuation bias would be particularly harmful if the estimates are used for scenario analysis, because lower coefficients on macroeconomic variables would lead to an underestimate of the stress scenario losses.

Moreover, one can show that

$$E(\tilde{e}'\tilde{e}) - E(e'e) = \hat{\sigma_L}^2 * k_2 + c'\zeta'\zeta c,$$

that is, the sum of squared errors and therefore the forecast error will always be larger in the presence of measurement error.

Given that we assumed that the measurement error is zero on average for each j, the level of aggregation at which Z is observed, such a problem will only arise for loan-level regressions, not for portfolio, or j-level regressions.

2.2 Aggregation bias

The measurement error problem has to be weighed against the aggregation bias problem. In the prior discussion we assumed that x_{ijt} and e_{ijt} are uncorrelated across individual loans i. This assumption is necessary to obtain b_P and b_B that are unbiased estimates of β. In practice, this is unlikely to be the case. If $\forall t\, E(x_{it}x_{jt}) \neq 0$, $E(e_{it}e_{jt}) \neq 0$, for $i \neq j$, aggregate regression estimates will not be unbiased. More specifically,

$$b_P = (X_P'X_P)^{-1}X_P'y_P = (X_L'U_NX_L)^{-1}X_L'U_Ny_L = (X_L'X_L{+}X'(U_N{-}I)X_L)^{-1}(X_L'y_L{+}X'(U_N{-}I)y_L),$$

where $U_N = S_NS_N'$, the $(NT \times NT)$ block-diagonal matrix of T $(N \times N)$ matrices of ones in the diagonal and zeros elsewhere. For this exercise we assume that the loan-level estimate is unbiased:

$$b_L = (X_L'X_L)^{-1}X_L'y_L, \quad E(b_L) = \beta.$$

If there is no within-time cross-individual correlation in x and e, the cross-product terms will be zero in expectations, that is $U_N = I$, and therefore $E(b_P) = E(b_L) = \beta$, otherwise $E(b_P) \neq E(b_L)$.[8] The same problem arises for a sub-portfolio level regressions. However, given that fewer cross-product terms appear in sub-portfolio level regressions, the problem is smaller the less aggregated the variables are.

To summarize, there is no sure way to tell what level of aggregation is going to produce the best forecast — both in terms of bias and in terms of forecast precision.[9] We have illustrated that, depending on the structure of the data, forecast accuracy can be better or worse for more aggregated data. We have also given examples in which measurement error bias is likely to arise in individual loan level regressions, while aggregation bias is likely in the aggregate regressions. Thus, what level of aggregation is the best for predicting aggregate outcomes remains an empirical question and the answer depends on the specific data set being analyzed. In the rest of the paper we present an empirical exercise for HELOCs, in which we compare out-of-sample performance of models estimated at different levels of aggregation. The optimal level of aggregation, however, may vary for different types of loans and sample characteristics.

3 Data and results

In this section we present our exercise, in which we compare the performance of default probability models evaluated at different level of aggregation. For this exercise we use a large data base of home equity loans and lines of credit, which we now describe.

[8] Note, however, that the standard estimate of the variance of b_L will not be unbiased, and the cluster-robust standard errors will need to be computed. See, for example, Arellano (1987).

[9] This result is also demonstrated, for in-sample fit, in Pesaran, Pierse, and Kumar (1989).

3.1 Data description

The data set is constructed from a five-percent sample of loans from the CoreLogic LP Home Equity Database. We keep home equity lines of credit with adjustable rates that are in the second lien position.[10] Our resulting sample is at the monthly frequency with 20,757,776 total observations and 454,724 unique loans ranging from 2002 to present; with the bulk of the loans found between 2005 until now. Delinquency is defined as the event of reaching 90-days past due. Once this event takes place the loan history is terminated, meaning that we abstract away from cures and the actual transition from default to foreclosure to loss. Thus, our measure of the delinquency or default rate is the transition rate from current into default rather than the stock of all outstanding loans in default.

All the specifications explored in our different risk models contain a grouping of observable economic factors. The main economic risk factors used in the analysis include the trailing 12-month house price appreciation from the CoreLogic monthly house price indexes (HPI). Whenever possible, we use the zip code level HPI. When this level of precision is not possible we revert back to the county-level HPI. If the loan is situated in a county where CoreLogic has no coverage at all, we drop the history from the sample. We also use the county-level unemployment rate (BLS) and a year-over-year real GDP growth series constructed from the monthly estimates provided by Macroeconomic Advisors. All of these variables speak to the general economic conditions in the borrower's local market. Moreover, aggregated versions of all of these variables are used in the CCAR stress tests.

In general we use a parsimonious approach to model selection on the grounds that these specifications tend to do better out-of-sample. As we proceed to lower levels of aggregation, however, we include increasingly more variables as a way of giving the disaggregated models the fullest oppor-

[10] We dropped the fixed-rate lines on the grounds that pre-payments and payoff behavior differ substantially across these two populations.

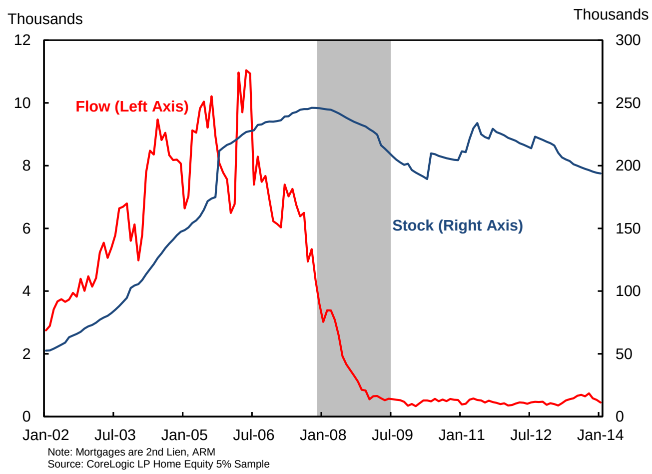
tunity to exploit the rich data set at our disposal. Thus, we include commonly used variables such as the FICO risk or credit score (FICO), the borrower debt-to-income ratio at origination (DTI), and the reported loan-to-value ratio at origination(LTV). In the loan-level model we also consider a host of other variables that speak to underwriting standards and other factors that might be correlated with unobservable borrower characteristics, such as loan age and the spread of the loan rate over the reference interest rate. We also have the ability to create an imputed current LTV by updating the loan-specific LTV at origination by its respective house price appreciation. Note that the LTV, FICO scores, and DTI have all been winsorized at the 1st and 99th percentiles of the raw empirical distribution. The summary statistics for the variables used in the risk models are in Table 1.

Table 1: Summary Statistics

	Mean	Standard Deviation	25th Percentile	75th Percentile
Loan-to-Value at origination	37.765	26.073	15.57	60.11
FICO at origination	739.451	51.109	703	779
Debt-to-Income at origination	35.667	14.527	26.8	44.7
Unemployment Rate, County	6.961	2.884	4.7	8.9
GDP growth, yearly	1.614	2.072	1.285	2.856
House Price Depreciation, yearly	-0.118	12.397	-7.669	6.892
Loan Origination, log	11.044	1.008	10.463	11.575
Margin Rate	0.548	0.995	0	.875
Current Interest Rate	5.664	2.310	3.74	7.625
Loan length, months	222.136	154.819	0	360
Total amount drawn, monthly	23028.73	57725.14	0	17769.74
Observations	20757776			

There have been some significant changes in the supply of home equity loans and lines of credit over the course of our sample period. As can be seen from Figure 1 there was a steady increase in new loan origination through the housing boom years. New originations abruptly dried up once house prices leveled off and began falling in 2006. Essentially all of our loans were originated during

Figure 1: Flow and Stock of Loans in Sample



the 2002-2006 period. We did not include any loans that were originated prior to 2002–the starting point for our loan observations–for fear of introducing survivorship bias.

3.2 Forecasting exercise

We start the exercise at the highest level of aggregation: the national level — which we also refer to as "portfolio", or "aggregate" level. We estimate variations of the model,

$$y_t = \theta_0 + X_t\beta + \varepsilon_t, \tag{1}$$

where $y_t = \frac{1}{N}\sum_{i=1}^{N} y_{it}$ is the aggregate default rate from our sample of N loans in month t , the matrix X_t contains the averages of loan-level and macroeconomic risk covariates, and lags of some of the covariates, ε_t is an error term. The model in equation 1 is truly aggregated in the sense that both left and right-hand side variables are constructed as averages from the underlying loan-level data. The regressions are estimated using OLS.[11]

We search for the best fitting model through a large number of specifications based on all combinations of variables in Table 1. We are able to narrow this large set of candidate models down to 25 models that are judged to have reasonable in-sample fit and with intuitive coefficients. For each of these specifications, we estimate the models on a series of 12 rolling samples. That is, starting with a sample ranging from January 2002 to July 2008, we estimate each model and then construct out-of-sample predictions from July 2008 for the next 9 quarters. We then repeat the exercise on an estimation sample ranging from January 2002 to August 2008, and then perform a 9-quarter ahead forecast. We proceed in this fashion so that the estimation sample gradually increases in size. In our longest subsample, we estimate the model up through June 2009. This set of rolling windows allows us to see how the model performs as it gradually learns about the dynamics of home equity defaults during the crisis. We then select the specification that performs well out-of-sample, on average, in terms of the mean squared error of forecast, does not tend to underpredict default probabilities, includes macroeconomic variables that appear in the scenarios, and has intuitive coefficients. We refer to this as a winning specification and report all 12 regressions for this specification in Table 5 in the Appendix. We also retain information on the performance of the rest of 28 specifications.

The next step in the disaggregation process is to break the delinquency rate down into subportfolios. We consider four different schemes for disaggregation: by DTI decile, LTV decile, FICO score decile, and by county.[12] The estimated model is now a fixed-effect panel model,

$$y_{jt} = \theta_j + X_{jt}\beta + \eta_{jt}, \tag{2}$$

where the θ_j is a subportfolio-specific fixed-effect, $y_{jt} = \frac{1}{J}\sum_{i=1}^{N_J} I(i \in j) y_{it}$ is the average default rate for all loans in sub-portfolio j in month t, X_{jt} is the set of average values of covariates for each

[11] The results do not change materially when we estimate equation (1) using a tobit specification.

[12] The top 25 percent of counties in the stock of loans as of 2005 comprise the county data set. This helps to improve the fit of the model by eliminating noise from counties with too few observations.

sub-portfolio, and η_t is the error term.

The subportfolio approach preserves some of the potential for aggregating out the measurement error problem, while also offering flexibility to introduce more portfolio-specific information to the regressions. In the disaggregated models we make predictions of the disaggregated delinquency rates and then aggregate these predictions to compare to the aggregate outcomes. That is, when we forecast default at times $t = 1, ..., T$ for subportfolio j, the MSE that we use for forecast evaluation is not the average difference between predicted and actual subportfolio default rates. Rather, it is the difference between the average aggregate default prediction and the aggregated portfolio default rate,

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(y_t - \frac{1}{J}\sum_{j=1}^{J}\hat{y}_{jt}\right)^2. \tag{3}$$

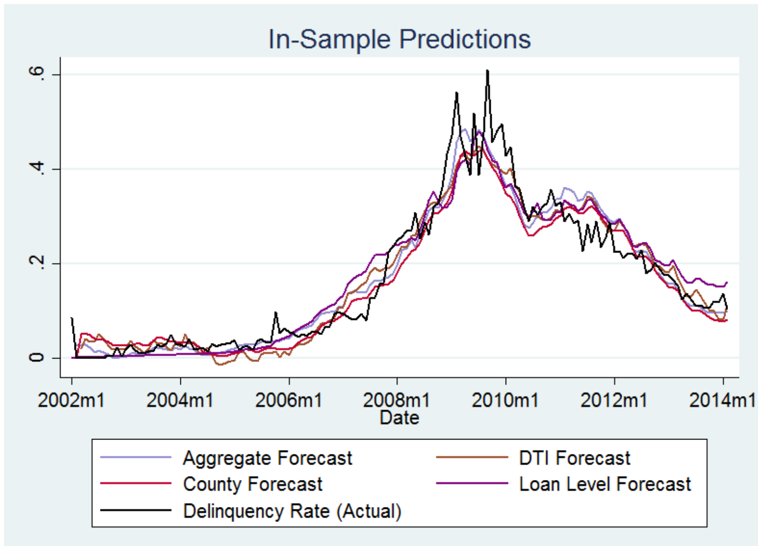
We feel that this approach more closely mimics what a bank would do when confronted with a problem of predicting total portfolio defaults or losses. If the object of interest is the portfolio default rate or loss rate, then the appropriate measure of out-of-sample fit is one where forecast error at the portfolio level is small.

For these subportfolio models we conduct the same procedure as for aggregate model in terms of specification selection. After extensive pre-testing we end up with 22 reasonable specifications for each type of aggregation: by DTI, LTV, FICO deciles, and by county. Out of these reasonable specifications, we select winning models in the same way as we do for aggregate model, and retain information of performance of other models.

Finally, we consider fully disaggregated loan-level models. These models are estimated as logit regressions,

$$D_{it} = \alpha + X_{it}\beta + \nu_{it}, \tag{4}$$

Figure 2: In-sample Model Predictions



where i is the index for individual loans or borrowers, D_{it} is a 0-1 indicator of whether loan i defaulted in month t, and ν_{it} is the error term. Some variables in the vector of covariates X_{it} , such as unemployment rate and home price depreciation, do not vary by loan but are repeated for all loans in the same county in the same month. We cluster standard errors by county to account for resulting correlation in errors.

We select 18 reasonable models among all specification permutations, and evaluate their forecasting performance over 12 rolling regressions ending in July 2008 through June 2009. As with the subportfolio models, the MSE is calculated as the average deviation of the aggregated default predictions compared to the aggregate default rate,

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(y_t - \frac{1}{N}\sum_{i=1}^{N}\hat{y}_{it}\right)^2 . \tag{5}$$

The winning models for each aggregation approach are reported in Table 2. In order to not overwhelm the reader with the results, we only report the regressions that are estimated through

January 2009, the middle of our rolling window set.[13] We can see that the best specification varies with the aggregation approach, but the effects of included variables are mostly stable across specifications. We find, as one would expect, that defaults on home equity loans are more likely when unemployment is higher, or when home prices depreciate. The combination of these factors seems to lead to an additional increase in default rates. We also consistently find that higher debt-to-income ratios and lower FICO scores of the borrowers are associated with higher default rates. All aggregate regressions tend to have good in-sample fit in terms of the R^2.

[13] All 12 regressions for each model are reported in the Appendix.

Table 2: Best Models: Regression through January 2009

	Aggregate	DTI	LTV	FICO	County	Loan Level
HPD, lag(1)	-0.0025	-0.00244	-0.01284***	0.01020	-0.00527***	0.0489***
	(0.0039)	(0.00358)	(0.00321)	(0.00585)	(0.00198)	(0.0018)
UR, lag(1)	0.0330***	0.02384***	0.00588	0.02514***	0.02091***	0.0651***
	(0.0069)	(0.00389)	(0.00859)	(0.00515)	(0.00439)	(0.0150)
HPD*UR, lag(1)	0.0020***		0.0041***		0.0016***	
	(0.0007)		(0.0008)		(0.0004)	
HPD, lag(2)		0.0077**		-0.0041	0.0058***	
		(0.0034)		(0.0051)	(0.0021)	
UR, lag(2)		0.0187***		0.0222***	0.0049	
		(0.0056)		(0.0067)	(0.0045)	
HPD*UR, lag(2)					-0.0003	
					(0.0004)	
DTI, lag(1)	0.0109***		0.00011		0.00428***	-0.0031***
	(0.0011)		(0.00207)		(0.00054)	(0.0010)
FICO, lag(1)		0.0030		-0.02594**		spline
		(0.0024)		(0.00893)		
LTV, lag(1)	0.0049**	-0.00104				spline (imputed)
	(0.0020)	(0.00180)				
Loan Amount, lag(1)		0.2466***		0.35825***		
		(0.0457)		(0.10421)		
Constant	-0.5959***					-8.4449***
	(0.1007)					(1.6041)
Additional Loan Characteristics	No	No	No	No	No	Yes
Fixed Effects	No	Yes	Yes	Yes	Yes	No
Observations	83	820	830	820	24854	6208287
Adjusted R^2	0.9300	0.8506	0.6387	0.5826	0.3655	0.1961
MSE	0.0046	0.0098	0.0130	0.0083	0.0041	0.0617
Loss	0.9766	0.9493	1.054	0.9624	1.0293	1.2754

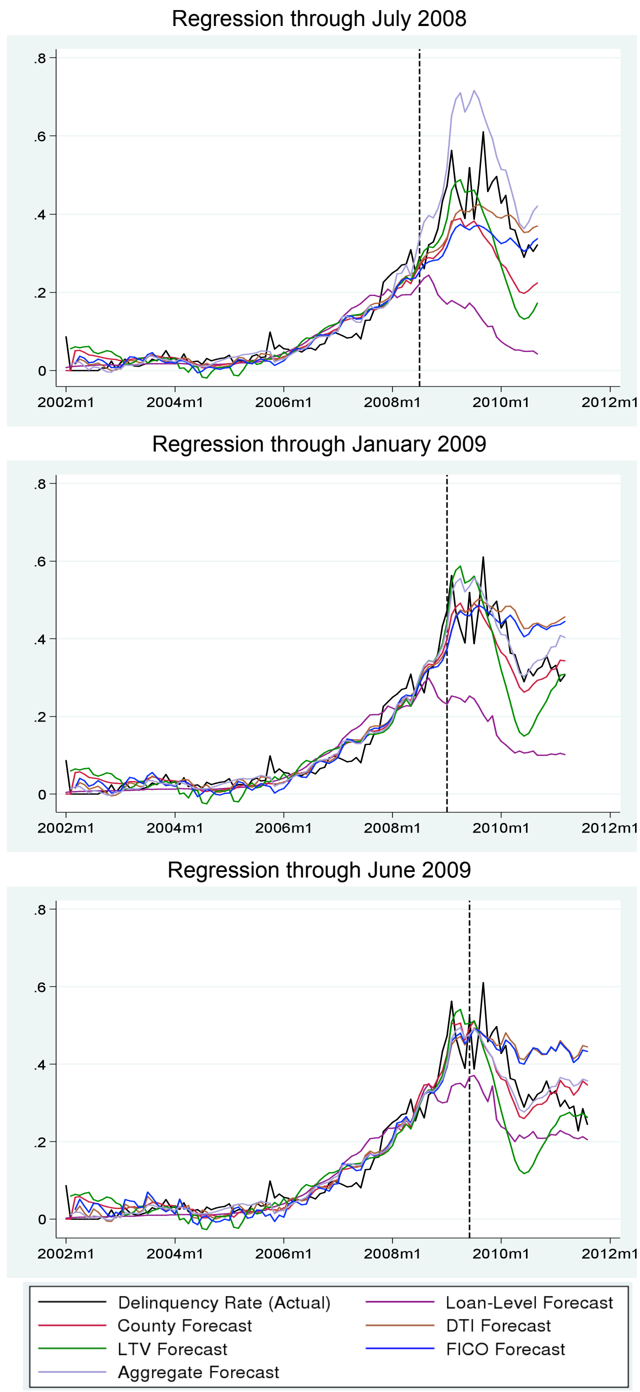
Notes: Standard errors in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Robust standard errors.

Before turning to the out-of-sample results we first examine the in-sample performance of our models when estimated over the full sample of data. These in-sample fits can be found in Figure 2. The models profiled here are actually selected on the basis of out-of-sample performance (to be discussed below). However, it is useful to demonstrate from the beginning that all three aggregation levels demonstrate very similar capability of fitting the data in-sample. All four models (loan level, county level, segment level, and aggregate) can roughly match the timing of the turnaround. None of the four models is quite able to match the peak in defaults observed. Thus, from a pure in-sample performance perspective where all the data are used in estimation, there is little a priori reason to prefer one aggregation level over another.

With this starting point we can proceed to the out-of-sample comparisons. Figure 3 shows the performance of the out-of-sample forecast of our winning models. The top panel shows the forecasts of the regressions with the estimation window through July 2008, the middle panel through January 2009 (regressions reported in Table 2), the bottom panel through June 2009. We can compare the forecasts resulting from each approach with the data. We find that the loan-level forecast consistently under-predicts loss frequency in the aggregate, while aggregate forecasts over-predicts default frequency in the beginning of our rolling window. We also observe that aggregation by LTV buckets produces quite poor results, while aggregation by DTI and FICO buckets produce good results that are similar to each other. DTI and FICO approaches tend to over-predict default frequencies in the second half of our rolling forecast window. County-level aggregation models produce forecasts that are quite accurate and stable, with the exception of the second half of the forecast horizon in the regressions that end before the crisis.

We can formalize these observations by comparing MSEs of all reasonable models across all 12 rolling forecast windows for each of aggregation approach. Table 3 presents summary statistics for all resulting MSEs, by aggregation approach, as well as average MSE for each winning model across 12 rolling forecast windows. We find that on average, county-level models have the smallest forecast

Figure 3: Out-of-Sample Forecast



errors, which also don't vary much across specifications. This is consistent with our expectations, because macroeconomic information that enters the regressions varies by country and is therefore fully explored in these regressions, while not generating measurement errors as in loan-level regressions. While the MSE of the winning model of the portfolio-level approach is smaller than that of the county-level, we can see that there is high variation in the quality of forecast of the aggregate model resulting from small perturbations in model specification.

Table 3: MSE Summary: Based on 9 quarter forecast

	Aggregate	FICO	LTV	DTI	County	Loan Level
Mean	0.0325	0.0407	0.0482	0.0362	0.0158	0.0845
Standard Deviation	0.0478	0.0743	0.0664	0.0615	0.0141	0.0130
Min.	0.0035	0.0037	0.0039	0.0052	0.0031	0.0180
Max.	0.2683	0.6804	0.4430	0.4976	0.0709	0.1153
Winning Model (Mean)	0.0064	0.0131	0.0158	0.0129	0.0080	0.0590

In the stress testing exercise, however, forecast accuracy is the exclusive goal. Given model uncertainty, it is also important that the errors of forecast are more likely to be on the conservative side. Thus, we construct a "conservative loss" measure

$$CL = \frac{1}{T}\sum_{t=1}^{T} \exp(y_t - \hat{y}_t),$$

where $\hat{y}_t$ for disaggregated models is computed as average forecasts. This measure is equal to 1 if there is no forecast error, is below 1 if the error is on the side of over-predicting default frequency, and is above 1 if the model is under-predicting defaults. Summary statistics for this loss measure are presented for all reasonable models across all 12 forecast windows for each of our aggregation approaches in Table 4. We find that aggregate model produces more conservative forecasts on average, as we saw in Figure 1, but that loss measure varies substantially across specifications. Loan-level model is very consistent in under-predicting losses. The sub-portfolio models all have similar loss measures on average with county-level loss measures being the most stable across regression specifications.

Table 4: Loss Summary

	Aggregate	FICO	LTV	DTI	County	Loan Level
Mean	0.9655	1.0089	1.0502	1.0665	1.0541	1.3178
Standard Deviation	0.1598	0.1788	0.1976	0.1837	0.1015	0.0355
Min.	0.6709	0.4934	0.5385	0.6214	0.8463	1.1288
Max.	1.6425	1.7084	1.9065	1.9821	1.2901	1.3978
Winning Model (Mean)	0.9837	0.9825	1.0806	0.9632	1.0478	1.2553

4 Conclusion

In this paper we compare risk models with different levels of aggregation: from loan-level to aggregate portfolio-level models. We consider hybrid approaches where we model default probabilities for different segments of a portfolio, such as buckets of debt-to-income ratios, loan-to-value ratios, FICO risk scores, and with loans aggregated by county. We conduct our tests on a large portfolio of home equity loans and lines of credit.

In our sample of home equity lines and loans, neither loan-level models nor portfolio-level models are ideal for the specific exercise of regulatory stress testing we have in mind. In the CCAR stress testing exercises, scenario drivers are supplied at national level, with some variables disaggregated by private data vendors by geographical regions such as state, metropolitan area, or county. Default and loss data, however, are frequently available for the banking institution at loan level. The question that arises is whether it is better to aggregate data first and then estimate the risk model, to estimate loan level model and then to aggregate projections, or to estimate some intermediate level model. We demonstrate that loan-level models are subject to measurement errors that arise from the explanatory variables that are not available at the loan level, while aggregate models are subject to aggregation bias. In our empirical exercise we find that this tension is best resolved at the intermediate level of aggregation. In particular, county-level regressions, where macroeconomic variables at county level are used, appear to perform best for the purpose of stress testing. Other hybrid approaches also perform better than either loan-level model or aggregate model.

We measure model performance using model selection criteria appropriate for the stress testing exercise. The MSE criterion puts equal weight on positive and negative forecast errors. Policymakers and bank supervisors, however, are often thought to have preferences that put more weight on downside risks than upside risks. For this reason, we also employ a "conservative loss" measure which punishes model underpredictions. In this context, the loan-level models appear to perform particularly poorly, given their persistent underprediction of home equity default rates. While aggregate models are quite conservative on average, their predictions are not robust to model specification and can at times produce very low default rates.

To be clear, our goal is not to recommend one specific approach to risk modeling. The purpose of our exercise is to illustrate, using an example of home equity lines and loans, that aggregation level matters. In some cases, intermediate levels of aggregation might be a best approach to modeling default probabilities or loss rates on banks' loan portfolios. We also provide an econometric argument that shows why this might be the case. Our hope is that researchers, regulators, and practitioners alike devote due attention to the implications of the aggregation level of models used for stress testing.

References

Arellano, M. (1987): "Computing Robust Standard Errors for Within-Group Estimators," *Oxford Bulletin of Economics and Statistics*, 49, 431–434.

Chinchalkar, S., and R. Stein (2010): "Comparing Loan-Level and Pool-Level Mortgage Portfolio Analysis," Moody's Research Labs working paper.

Covas, F., B. Rump, and E. Zakrajsek (2014, forthcoming): "Stress-Testing U.S. Bank Holding Companies: A Dynamic Panel Quantile Regression Approach," *International Journal of Forecasting*.

Deng, Y., J. M. Quigley, and R. VanOrder (2000): "Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options," *Econometrica*, 2, 275–307.

Duffie, D., and D. Lando (2001): "Term Structures of Credit Spreads with Incomplete Accounting Information," *Econometrica*, 69, 633–664.

Frame, W. S., K. Gerardi, and P. Willen (2015): "The Failure of Supervisory Stress Testing: Fannie Mae, Freddie Mac, and OFHEO," Federal Reserve Bank of Atlanta working paper.

Gerardi, K., K. Herkenhoff, L. Ohanian, and P. Willen (2013): "Unemployment, Negative Equity, and Strategic Default," Federal Reserve Bank of Atlanta Working Paper.

Granger, C. (1980): "Long memory relationships and the aggregation of dynamic models," *Journal of Econometrics*, 14, 227–238.

Grunfeld, Y., and Z. Griliches (1960): "Is Aggregation Necessarily Bad?," *Review of Economics and Statistics*, 42(1), 1–13.

Gyourko, J., and J. Tracy (2013): "Unemployment and Unobserved Credit Risk in the FHA Single Family Mortgage Insurance Fund," NBER Working Paper No 18880.

Pesaran, M., R. Pierse, and M. Kumar (1989): “Econometric Analysis of Aggregation in the Context of Linear Prediction Models,” *Econometrica*, 57(4), 861–888.

Theil, H. (1954): *Linear Aggregation of Economic Relations*. Amsterdam.

5 Appendix

Table 5: Portfolio Level

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
DTI, lag(1)	0.0118***	0.0113***	0.0106***	0.0106***	0.0107***	0.0108***	0.0109***	0.0110***	0.0109***	0.0110***	0.0112***	0.0112***
	(0.0013)	(0.0012)	(0.0012)	(0.0012)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0012)	(0.0012)
HPD, lag(1)	-0.0084	-0.0044	0.0015	0.0013	0.0009	-0.0000	-0.0025	-0.0040	-0.0048**	-0.0020	0.0002	0.0011
	(0.0064)	(0.0055)	(0.0053)	(0.0043)	(0.0040)	(0.0038)	(0.0039)	(0.0032)	(0.0023)	(0.0029)	(0.0028)	(0.0026)
UR, lag(1)	0.0447***	0.0374***	0.0275***	0.0278***	0.0284***	0.0298***	0.0330***	0.0347***	0.0354***	0.0325***	0.0296***	0.0271***
	(0.0114)	(0.0097)	(0.0095)	(0.0079)	(0.0074)	(0.0070)	(0.0069)	(0.0061)	(0.0055)	(0.0059)	(0.0061)	(0.0061)
HPD*UR, lag(1)	0.0032**	0.0024**	0.0012	0.0012	0.0013*	0.0015**	0.0020***	0.0023***	0.0024***	0.0019***	0.0015***	0.0014***
	(0.0013)	(0.0011)	(0.0010)	(0.0008)	(0.0007)	(0.0007)	(0.0007)	(0.0006)	(0.0004)	(0.0005)	(0.0005)	(0.0005)
LTV, lag(1)	0.0058***	0.0052**	0.0044**	0.0045**	0.0045**	0.0047**	0.0049**	0.0051**	0.0051**	0.0051**	0.0053**	0.0056**
	(0.0021)	(0.0021)	(0.0021)	(0.0021)	(0.0021)	(0.0021)	(0.0020)	(0.0020)	(0.0020)	(0.0021)	(0.0022)	(0.0023)
Observations	77	78	79	80	81	82	83	84	85	86	87	88
Adjusted R^2	0.8876	0.8961	0.8947	0.9060	0.9153	0.9237	0.9300	0.9391	0.9500	0.9489	0.9447	0.9425
MSE	0.0196	0.0069	0.0045	0.0043	0.0039	0.0037	0.0046	0.00671	0.0080	0.0056	0.0044	0.0045
Loss	0.8948	0.9513	1.0413	1.0382	1.0312	1.0134	0.9766	0.9542	0.9433	0.9665	0.989	1.004

Notes: Robust standard errors in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$.
Variables are based on the mean value, by date.

Table 6: DTI Level

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
HPD, lag(1)	0.0019	0.0011	0.0020	0.0013	0.0008	-0.0007	-0.0024	0.0005	0.0038	0.0038	0.0036	0.0053
	(0.0032)	(0.0035)	(0.0035)	(0.0035)	(0.0035)	(0.0034)	(0.0036)	(0.0039)	(0.0036)	(0.0036)	(0.0036)	(0.0035)
UR, lag(1)	0.0173***	0.0187***	0.0162***	0.0169***	0.0169***	0.0186***	0.0238***	0.0322***	0.0441***	0.0427***	0.0434***	0.0488***
	(0.0035)	(0.0039)	(0.0034)	(0.0035)	(0.0035)	(0.0037)	(0.0039)	(0.0047)	(0.0069)	(0.0069)	(0.0070)	(0.0071)
HPD, lag(2)	0.0034	0.0043	0.0034	0.0041	0.0046	0.0060	0.0077**	0.0046	0.0009	0.0012	0.0017	0.0002
	(0.0030)	(0.0034)	(0.0034)	(0.0033)	(0.0034)	(0.0033)	(0.0034)	(0.0037)	(0.0034)	(0.0034)	(0.0034)	(0.0034)
UR, lag(2)	0.0139**	0.0138**	0.0126**	0.0140**	0.0160**	0.0172**	0.0187***	0.0191***	0.0167**	0.0116*	0.0031	-0.0074*
	(0.0053)	(0.0054)	(0.0054)	(0.0054)	(0.0053)	(0.0054)	(0.0056)	(0.0058)	(0.0058)	(0.0054)	(0.0043)	(0.0036)
FICO, lag(1)	0.0028	0.0028	0.0028	0.0028	0.0029	0.0030	0.0030	0.0032	0.0034	0.0038	0.0040	0.0042*
	(0.0021)	(0.0021)	(0.0021)	(0.0022)	(0.0022)	(0.0023)	(0.0024)	(0.0026)	(0.0025)	(0.0024)	(0.0023)	(0.0022)
LTV, lag(1)	-0.0006	-0.0007	-0.0006	-0.0007	-0.0007	-0.0008	-0.0010	-0.0012	-0.0013	-0.0009	-0.0005	-0.0002
	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0018)	(0.0019)	(0.0019)	(0.0018)	(0.0018)	(0.0017)
Loan Amount, lag(1)	0.2087***	0.2141***	0.2003***	0.2067***	0.2117***	0.2226***	0.2466***	0.2631***	0.2808***	0.2529***	0.2265***	0.2039***
	(0.0345)	(0.0368)	(0.0357)	(0.0375)	(0.0387)	(0.0425)	(0.0457)	(0.0519)	(0.0551)	(0.0506)	(0.0443)	(0.0402)
Observations	760	770	780	790	800	810	820	830	840	850	860	870
Adjusted R^2	0.7760	0.7928	0.8030	0.8187	0.8321	0.8412	0.8506	0.8574	0.8663	0.8719	0.8741	0.8738
MSE	0.0062	0.0058	0.0070	0.0063	0.0060	0.0064	0.0098	0.0187	0.0333	0.0256	0.0167	0.0129
Loss	1.0374	1.0270	1.0492	1.0336	1.0195	0.9958	0.9493	0.8999	0.8497	0.8705	0.9023	0.9240
Fixed Effects	Y											

Notes: Robust standard errors in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$.
Variables are based on the mean value, by date.

Table 7: LTV Level

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
HPD, lag(1)	-0.0073*	-0.0083***	-0.0061**	-0.0078**	-0.0086**	-0.0102***	-0.0128***	-0.0143***	-0.0172***	-0.0144***	-0.0119***	-0.0111***
	(0.0037)	(0.0024)	(0.0019)	(0.0024)	(0.0029)	(0.0029)	(0.0032)	(0.0034)	(0.0025)	(0.0023)	(0.0017)	(0.0015)
UR, lag(1)	0.0015	0.0024	-0.0018	0.0004	0.0010	0.0029	0.0059	0.0070	0.0097	0.0057	0.0020	-0.0008
	(0.0159)	(0.0117)	(0.0103)	(0.0101)	(0.0092)	(0.0092)	(0.0086)	(0.0077)	(0.0110)	(0.0109)	(0.0102)	(0.0098)
HPD*UR, lag(1)	0.0030**	0.0032***	0.0027***	0.0031***	0.0032***	0.0036***	0.0041***	0.0044***	0.0049***	0.0044***	0.0039***	0.0038***
	(0.0009)	(0.0007)	(0.0005)	(0.0006)	(0.0007)	(0.0007)	(0.0008)	(0.0008)	(0.0008)	(0.0007)	(0.0006)	(0.0006)
DTI, lag(1)	0.0008	0.0007	0.0006	0.0005	0.0004	0.0003	0.0001	-0.0001	-0.0001	-0.0001	-0.0002	-0.0004
	(0.0018)	(0.0018)	(0.0018)	(0.0019)	(0.0019)	(0.0020)	(0.0021)	(0.0021)	(0.0023)	(0.0024)	(0.0024)	(0.0025)
Observations	770	780	790	800	810	820	830	840	850	860	870	880
Adjusted R^2	0.5324	0.5517	0.5660	0.5828	0.5996	0.6159	0.6387	0.6614	0.6590	0.6615	0.6641	0.6609
MSE	0.0159	0.0152	0.0226	0.0187	0.01788	0.0152	0.01305	0.0131	0.0142	0.0133018	0.0144	0.0166
Loss	1.0957	1.0894	1.1362	1.1154	1.1101	1.0884	1.0543	1.0387	1.0110	1.045	1.0799	1.1025
Fixed Effects	Y											

Notes: Robust standard errors in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$.
Variables are based on the mean value, by date.

Table 8: FICO Level

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
HPD, lag(1)	0.0148**	0.0135*	0.0141**	0.0132*	0.0128*	0.0115*	0.0102	0.0139*	0.0173*	0.0166*	0.0155*	0.0166*
	(0.0060)	(0.0061)	(0.0062)	(0.0060)	(0.0060)	(0.0059)	(0.0059)	(0.0066)	(0.0079)	(0.0079)	(0.0078)	(0.0078)
UR, lag(1)	0.0159***	0.0184***	0.0170***	0.0180***	0.0175***	0.0196***	0.0251***	0.0342***	0.0477***	0.0464***	0.0468***	0.0516***
	(0.0036)	(0.0041)	(0.0033)	(0.0034)	(0.0034)	(0.0038)	(0.0051)	(0.0069)	(0.0114)	(0.0110)	(0.0112)	(0.0122)
HPD, lag(2)	-0.0092	-0.0077	-0.0083	-0.0074	-0.0068	-0.0054	-0.0041	-0.0079	-0.0117	-0.0106	-0.0092	-0.0102
	(0.0052)	(0.0054)	(0.0054)	(0.0052)	(0.0051)	(0.0051)	(0.0051)	(0.0057)	(0.0070)	(0.0069)	(0.0068)	(0.0067)
UR, lag(2)	0.0151*	0.0145*	0.0135*	0.0158**	0.0187**	0.0204**	0.0222***	0.0226***	0.0190**	0.0147**	0.0068	-0.0025
	(0.0067)	(0.0068)	(0.0066)	(0.0064)	(0.0061)	(0.0064)	(0.0067)	(0.0069)	(0.0066)	(0.0055)	(0.0054)	(0.0046)
FICO, lag(1)	-0.0211**	-0.0218**	-0.0220**	-0.0228**	-0.0235**	-0.0246**	-0.0259**	-0.0274**	-0.0290**	-0.0293**	-0.0294**	-0.0293**
	(0.0076)	(0.0077)	(0.0077)	(0.0080)	(0.0082)	(0.0084)	(0.0089)	(0.0092)	(0.0099)	(0.0098)	(0.0096)	(0.0095)
Loan Amount, lag(1)	0.2902**	0.3000**	0.2883**	0.3019***	0.3112***	0.3278***	0.3582***	0.3848***	0.4115***	0.3902***	0.3615***	0.3384***
	(0.0919)	(0.0951)	(0.0889)	(0.0918)	(0.0929)	(0.0959)	(0.1042)	(0.1089)	(0.1190)	(0.1102)	(0.1023)	(0.0975)
Observations	760	770	780	790	800	810	820	830	840	850	860	870
Adjusted R^2	0.5061	0.5204	0.5345	0.5479	0.5619	0.5755	0.5826	0.5983	0.6050	0.6191	0.6263	0.6307
MSE	0.0109	0.0089	0.0105	0.0082	0.0070	0.0064	0.0083	0.0163	0.0295	0.0235	0.0153	0.0121
Loss	1.0817	1.0652	1.0805	1.0581	1.0407	1.0125	0.9624	0.9109	0.8606	0.8783	0.9093	0.9294
Fixed Effects	Y											

Notes: Robust standard errors in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$.
Variables are based on the mean value, by date.

Table 9: County Level

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
HPD, lag(1)	-0.0016	-0.0038*	-0.0031	-0.0040*	-0.0039*	-0.0041**	-0.0053***	-0.0068***	-0.0089***	-0.0102***	-0.0122***	-0.0139***
	(0.0023)	(0.0023)	(0.0022)	(0.0022)	(0.0022)	(0.0020)	(0.0020)	(0.0022)	(0.0025)	(0.0024)	(0.0024)	(0.0025)
UR, lag(1)	0.0159***	0.0198***	0.0178***	0.0194***	0.0190***	0.0189***	0.0209***	0.0257***	0.0292***	0.0268***	0.0276***	0.0302***
	(0.0053)	(0.0054)	(0.0047)	(0.0048)	(0.0048)	(0.0046)	(0.0044)	(0.0053)	(0.0054)	(0.0054)	(0.0053)	(0.0053)
HPD*UR, lag(1)	0.0014***	0.0017***	0.0016***	0.0017***	0.0016***	0.0015***	0.0016***	0.0020***	0.0023***	0.0022***	0.0025***	0.0028***
	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)
HPD, lag(2)	0.0035	0.0055**	0.0050**	0.0056**	0.0052**	0.0050**	0.0058***	0.0070***	0.0085***	0.0103***	0.0129***	0.0149***
	(0.0023)	(0.0022)	(0.0023)	(0.0022)	(0.0022)	(0.0021)	(0.0021)	(0.0023)	(0.0027)	(0.0026)	(0.0026)	(0.0025)
UR, lag(2)	-0.0012	-0.0037	-0.0020	-0.0015	0.0009	0.0034	0.0049	0.0038	0.0039	0.0041	-0.0001	-0.0048
	(0.0046)	(0.0049)	(0.0052)	(0.0050)	(0.0044)	(0.0043)	(0.0045)	(0.0052)	(0.0051)	(0.0047)	(0.0043)	(0.0036)
HPD*UR, lag(2)	-0.0005	-0.0008*	-0.0007*	-0.0007*	-0.0005	-0.0004	-0.0003	-0.0006	-0.0008	-0.0008	-0.0012**	-0.0015***
	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)
DTI, lag(1)	0.0036***	0.0037***	0.0037***	0.0038***	0.0039***	0.0040***	0.0043***	0.0045***	0.0047***	0.0047***	0.0044***	0.0043***
	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0006)	(0.0006)	(0.0006)	(0.0006)	(0.0006)
Observations	23018	23324	23630	23936	24242	24548	24854	25160	25466	25772	26078	26384
Adjusted R^2	0.2310	0.2546	0.2672	0.2904	0.3119	0.3356	0.3655	0.3945	0.4289	0.4460	0.4554	0.4620
MSE	0.01549	0.0132	0.0145	0.0116	0.00893	0.0066	0.0041	0.0036	0.0044	0.0045	0.0041	0.0046
Loss	1.1141	1.103	1.114	1.0980	1.0815	1.0613	1.0294	1.0020	0.9741	0.9806	1.0020	1.0142

Notes: Standard errors in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Standard errors clustered by county. Fixed Effects included. Variables are based on the mean value, by date.

Table 10: Loan Level

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
delinquent												
HPD, lag(1)	0.0550***	0.0543***	0.0536***	0.0528***	0.0523***	0.0512***	0.0489***	0.0467***	0.0447***	0.0441***	0.0434***	0.0429***
	(0.0020)	(0.0019)	(0.0019)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0019)	(0.0020)	(0.0020)	(0.0020)
UR, lag(1)	0.0453***	0.0467***	0.0432***	0.0478***	0.0517***	0.0559***	0.0651***	0.0768***	0.0879***	0.0875***	0.0901***	0.0917***
	(0.0171)	(0.0167)	(0.0155)	(0.0149)	(0.0148)	(0.0148)	(0.0150)	(0.0150)	(0.0152)	(0.0147)	(0.0144)	(0.0142)
win	0.0217	0.0321	0.0491	0.0547	0.0385	0.0242	-0.0105	-0.0165	-0.0292	-0.0385	-0.0511	-0.0615
	(0.0797)	(0.0809)	(0.0765)	(0.0754)	(0.0731)	(0.0726)	(0.0686)	(0.0679)	(0.0684)	(0.0672)	(0.0687)	(0.0677)
Appraisal (mil.)	0.0004***	0.0004***	0.0005***	0.0005***	0.0005***	0.0005***	0.0005***	0.0005***	0.0005***	0.0005***	0.0005***	0.0005***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Full Doc. (D=1)	-0.3492***	-0.3728***	-0.3832***	-0.4018***	-0.4213***	-0.4504***	-0.4504***	-0.4534***	-0.4662***	-0.4657***	-0.4695***	-0.4642***
	(0.0515)	(0.0513)	(0.0518)	(0.0512)	(0.0509)	(0.0510)	(0.0508)	(0.0511)	(0.0519)	(0.0511)	(0.0501)	(0.0511)
Interest Only(D=1)	-1.8574***	-1.9012***	-1.9316***	-1.9770***	-2.0378***	-2.0991***	-2.1729***	-2.2478***	-2.3187***	-2.3602***	-2.3885***	-2.4405***
	(0.0721)	(0.0752)	(0.0763)	(0.0775)	(0.0792)	(0.0797)	(0.0802)	(0.0828)	(0.0851)	(0.0842)	(0.0801)	(0.0821)
Margin Rate	-0.0104	-0.0154	-0.0166	-0.0138	-0.0080	0.0018	0.0107	0.0168	0.0149	0.0147	0.0154	0.0117
	(0.0153)	(0.0158)	(0.0156)	(0.0152)	(0.0149)	(0.0147)	(0.0152)	(0.0150)	(0.0150)	(0.0150)	(0.0148)	(0.0148)
Loan Amt	0.2441***	0.2423***	0.2450***	0.2442***	0.2456***	0.2420***	0.2384***	0.2339***	0.2400***	0.2384***	0.2374***	0.2375***
	(0.0215)	(0.0191)	(0.0175)	(0.0175)	(0.0166)	(0.0160)	(0.0157)	(0.0145)	(0.0135)	(0.0124)	(0.0116)	(0.0112)
Purchase(D=1)	0.2603***	0.3417***	0.3652***	0.4032***	0.4085***	0.4225***	0.4166***	0.4536***	0.4847***	0.4674***	0.4763***	0.5016***
	(0.0653)	(0.0632)	(0.0620)	(0.0603)	(0.0563)	(0.0495)	(0.0488)	(0.0491)	(0.0463)	(0.0454)	(0.0466)	(0.0448)
Loan Term	-0.0019***	-0.0021***	-0.0021***	-0.0022***	-0.0023***	-0.0024***	-0.0025***	-0.0026***	-0.0028***	-0.0029***	-0.0030***	-0.0031***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
Current IR	0.2514***	0.2472***	0.2502***	0.2478***	0.2455***	0.2366***	0.2197***	0.2049***	0.1976***	0.1931***	0.1928***	0.1908***
	(0.0090)	(0.0092)	(0.0092)	(0.0094)	(0.0089)	(0.0088)	(0.0089)	(0.0090)	(0.0091)	(0.0092)	(0.0093)	(0.0093)
DTI	-0.0023**	-0.0026**	-0.0029**	-0.0029***	-0.0031***	-0.0031***	-0.0031***	-0.0029***	-0.0027***	-0.0026***	-0.0027***	-0.0026***
	(0.0012)	(0.0012)	(0.0012)	(0.0011)	(0.0011)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0009)	(0.0009)	(0.0009)
Committed	0.6307***	0.6370***	0.6449***	0.6481***	0.6565***	0.6619***	0.6605***	0.6654***	0.6697***	0.6740***	0.6760***	0.6788***
	(0.0179)	(0.0177)	(0.0174)	(0.0172)	(0.0167)	(0.0165)	(0.0162)	(0.0169)	(0.0165)	(0.0155)	(0.0156)	(0.0151)
Constant	-9.0459***	-8.8904***	-8.9194***	-8.8802***	-8.4734***	-8.8679***	-8.4449***	-8.8562***	-9.2848***	-9.2778***	-9.3871***	-9.4173***
	(1.6328)	(1.6499)	(1.5915)	(1.5381)	(1.5071)	(1.5370)	(1.6041)	(1.5397)	(1.5333)	(1.4969)	(1.4896)	(1.4465)
Observations	5438690	5568757	5698048	5826645	5954505	6081629	6208287	6333879	6458268	6581310	6702895	6818612
Pseudo R^2	0.1744	0.1785	0.1805	0.1843	0.1887	0.1919	0.1961	0.2002	0.2041	0.2063	0.2079	0.2100
MSE	0.0855	0.0842	0.0900	0.0882	0.0858	0.0779	0.0617	0.0426	0.0290	0.0246	0.0201	0.01804
Loss	1.3202	1.3200	1.3381	1.3370	1.334	1.316	1.2754	1.2212	1.1759	1.1577	1.1390	1.1288

Notes: County clustered standard errors in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$.
Vintage Fixed Effects, FICO/LTV/age splines