Risky Mortgages, Bank Leverage and Credit Policy *

Francesco Ferrante †
Federal Reserve Board

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Abstract

A key ingredient of the Great Recession was the high exposure of leveraged financial intermediaries to risky mortgages. To capture this feature of the financial crisis, I develop a DSGE model with balance-sheet constrained banks financing both mortgages and productive capital. Mortgages are provided to agents facing idiosyncratic housing depreciation risk, implying an endogenous default decision depending on borrowers leverage and house prices. The interaction between bank leverage constraint, house prices and defaults generates novel amplification mechanisms allowing a mortgage crisis to severely affect the real economy. I study the quantitative implications of these new channels by considering two different shocks linked to the housing market: an increase in the variance of housing risk and a deterioration of mortgages collateral value for bank funding. Both shocks are able to produce co-movements in house prices, business investments and output. In addition, I show how policy interventions similar to the ones implemented by the Federal Reserve can reduce the effects of a crisis.

Keywords: Financial frictions, Housing, Mortgages, Banking, Unconventional Monetary Policy

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†Federal Reserve Board, francesco.ferrante@frb.gov

1 Introduction

Looking back at the recession of 2007-2009, a highly leveraged financial sector is commonly identified as the element that allowed a disruption in the housing market to evolve into a global economic slowdown. In particular, the high exposure of U.S. banks to mortgage-backed-securities, whose value was closely related to house prices, made them particularly vulnerable to the turmoil in the subprime market, and reduced their ability to provide loans to other sectors of the economy.

In this paper I try to model these events in a DSGE framework characterized by financial intermediaries facing an endogenous leverage constraint and lending to two non-financial sectors: firms needing to finance the purchase of capital and "impatient" households requiring funds to purchase a house. The aim of this work is to study how a financial industry with balance sheet constraints might present, in addition to the traditional “financial accelerator", also important propagation channels linked to the mortgage market, allowing for shocks originating in the housing market to seriously affect other productive sectors.

A relevant feature of this framework is that I model the funding problem for homeowners by using mortgages with endogenous default. In particular, this is done by assuming that houses are subject to idiosyncratic depreciation risk, implying a default decision that will depend on borrowers leverage and on house prices. In addition, the specification of the default procedure allows for simple aggregation making the problem more tractable in a DSGE model.

In this paper a drop in the value of houses will have direct implications for the real sector through three novel channels.

First of all there is a "spillover-channel". Lower house prices imply higher default rates, causing losses for financial intermediaries. As their net worth is eroded, leveraged banks will experience a tighter borrowing constraint that will force them to deleverage by selling assets. As a result, financial intermediaries will also decrease their demand for business loans, implying a rise in the spread they charge on these assets and a drop in the price of capital. This generates a comovement between house prices, business investments and output, a feature that has been documented empirically and that characterized the recent recession[1].

A second mechanism is a "default-channel". In fact, a tightening in their borrowing constraint will cause a decrease in banks demand for mortgages and an increase in the interest rate faced by borrowing households. This will reduce households demand for houses, depressing house prices further and increasing defaults even more. Furthermore, as defaults increase, banks will repossess more houses so that their balance sheet will be more exposed to the decline in house prices. As a consequence, banks will experience an additional deterioration of their net worth and a tightening of their leverage constraint, reinforcing the initial shock.

Finally, a decline in house prices will also play a role through a "demand-channel". In fact, borrowing households demand for consumption will be proportional to their wealth, that is affected by the value of their house. Therefore, lower house prices will imply lower demand for the final

[1] See, for example, Liu, Wang and Zha (2010) and Mian and Sufi (2009b)

good, putting downward pressure on output and wages, especially if nominal rigitidies are present.

As a result, the negative feedback loop characteristic of the financial accelerator, in this framework is enriched with the relationship between house prices, bank funding conditions and endogenous defaults.

The quantitative implications of these amplification channels are studied by considering two shocks related to the mortgage market.

The first one is a "housing risk shock", modeled as an increase in the variance of the idiosyncratic depreciation shock. The idea behind this experiment is to simulate an initial disturbance increasing the default rate on mortgages, similarly to what happened in the subprime market at the beginning of the Great Recession. The increase in defaults will interact with all the three channels described above, producing a more severe downturn compared to a model with no financially constrained intermediaries.

Another useful experiment, that I conduct in this framework, is linked to the deterioration of the collateral value of mortgages for financial intermediaries, that can be thought of as replicating the collapse of the market for mortgage-backed-securities (MBS), that represented an important external credit channel for banks. Such shock directly tightens banks leverage constraint, causing fire sales and generating a crisis through the same channels affected by a housing risk shock. It is important to stress that both these shocks, unlike a capital quality shock or a productivity shock, would not have a real impact in a frictionless setup.

In the last part of the paper, I show how this model provides a natural laboratory to evaluate the effects of large scale asset purchases performed by the Federal Reserve, with a special focus on mortgage-backed securities. In fact this paper shows how the direct intermediation of housing-related assets, provided by a central bank, can effectively reduce the consequences of a recession stemming from a turmoil in the housing market or in the MBS market.

As regards the related literature, this paper builds on the framework of Gertler and Karadi (2011), that first modeled constrained banks in a monetary DSGE model, and extends their work by introducing housing and defaultable mortgages. Compared to their "capital quality shock", the exogenous disturbances considered in this model allow to present a more realistic characterization of the shocks initiating the financial crisis, since they originate in the housing sector.

Another paper providing a DSGE model for the relationship between housing and the financial sector over the recent crisis is Iacoviello (2014). In a different setup, the author models financial intermediaries lending both to entrepreneurs and to households and studies the effects of a balance-sheet shock that affects negatively the banks but positively the borrowing households. The author identifies in this redistributive shock the driving channel of macroeconomic fluctuations during the crisis. However Iacoviello (2014) does not model defaultable mortgages and focuses on exogenous regulatory constraints on bank capital.

Jeske et al. (2013), model mortgage defaults in a macroeconomic framework similar to this paper, in order to study the welfare implications of the bailout guarantees provided by the Government Sponsored Enterprises. However in their work financial intermediaries are unconstrained and their net worth does not play a role in the aggregate economy.

Abstracting from financial intermediation, a related strand of literature is the one studying the effects of shocks linked to housing in models *á* la Kiyotaki and Moore (1997). The first example is Iacoviello (2005), in which also nominal contracts are present, followed by Iacoviello and Neri (2010) that introduce a multi-sector structure and a richer set of shocks. Liu, Wang and Zha (2010) analyze the empirical relevance of credit constraints in a model with costly contract enforcement in which houses are used as collateral for loans. They claim that a necessary condition for credit constraints to play a role in business-cycle fluctuations is to have a mechanism producing comovements between house prices and real investments. In their paper this correlation is obtained through a preference shock combined with the fact that houses also serve as collateral for credit-constrained entrepreneurs. My model can be interpreted as an alternative way to build this mechanism, without relying on treating residential land and commercial land as the same good. As a result, the role of financial intermediaries is exactly to link house prices and the funding available for final good producers.

Finally, among the papers introducing housing in incomplete markets models with heterogeneous agents, two relevant works are Favilukis, Ludvigson and Van Nieuwerburg (2011) and Kiyotaki, Michaelides and Nikolov (2008), both studying the implications of financial liberalization in a framework without banks but with two productive sectors and housing as a collateral for household finance.

The rest of the paper is organized as follows: Section 2 presents the model and the problems of the different agents. Section 3 contains the quantitative excercises performed in order to simulate a crisis. Section 4 concludes.

2 The Model

The model is based on Gertler and Karadi (2011). To their framework I add a second set of "impatient-agents", that obtain utility from housing services and purchase risky houses. The only way for them to borrow is by issuing defaultable mortgages collateralized by their house. Such mortgages are financed by banks that also invest in capital. These financial intermediaries face an agency problem when raising funds from patient households and this will imply an endogenous leverage constraint.

2.1 Patient Households

There is a continuum of patient households[2] , that consume, save in deposits or government debt, and provide labor. As in Gertler and Karadi (2011), I assume that a fraction g of these agents are

[2] I will refer to patient households also as lenders or depositors.

"workers", whereas a fraction $(1-g)$ are "bankers". Workers provide labor to the consumption-good sector and return the wage to their household. Bankers manage a financial intermediary that returns its profits back to the family at the end of every period. In order to avoid that bankers save their way out the financial constraint, I assume that with probability $1-\sigma$ they exit the financial sector and become workers; at the same time a fraction $(1-g)(1-\sigma)$ of workers replaces them, and keeps the proportion of types unchanged. New bankers will be endowed with some start-up funds, that I will explain in detail later. Bankers are the only agents that are able to lend funds to goods producers and impatient households. Whithin the household there is perfect consumption insurance. As a result each patient household effectively owns a bank, but I assume that it invests in the deposits of an intermediary it does not own.

Whenever confusion is possible I will use hatted variables to refer to patient households as opposed to impatient ones. Patient households gain utility from consumption $\hat{C}_t$, and have disutility from labor $\hat{N}_t$, according to the following preference structure[3]

$$\max \; E_t \sum_{i=0}^{\infty} \hat{\beta}^i \left[log(\hat{C}_{t+i}) - \chi \frac{\hat{N}_{t+i}^{\gamma_n+1}}{\gamma_n+1} \right] \tag{1}$$

where their discount factor $\hat{\beta}$ is larger than the one for impatient agents, β.

In addition, they can save by using one-period debt issued either by financial intermediaries (deposits) or by the government. In equilibrium both securities will be risk-free, so that we can refer to them as D_t. As a result, households maximize their discounted utility, by choosing $\hat{C}_t, \hat{N}_t$ and D_t subject to the following budget constraint

$$\hat{C}_t = \hat{w}_t \hat{N}_t + \Pi_t - D_t + R_t D_{t-1} \tag{2}$$

where $\hat{w}_t$ is the wage paid to patient agents, R_t is the risk free rate and Π_t are profits from the ownership of banks and capital producing firms.

If we define $\Lambda_{t,t+1} = \hat{C}_t / \hat{C}_{t+1}$, we obtain the following first order conditions for labor and deposits

$$\chi \hat{N}_t^{\gamma_n} = \hat{w}_t / \hat{C}_t \tag{3}$$

$$1 = E_t \hat{\beta} \Lambda_{t,t+1} R_{t+1} \tag{4}$$

2.2 Impatient Households

Impatient households[4] discount the future with discount factor[5] $\beta < \hat{\beta}$ and derive utility from consumption c_t and housing services x_t, that can be obtained by renting a house at price r_t. They

[3] For simplicity I assume that patient households don't obtain utility from housing services. This assumption is made to isolate the relationship between house prices and the choice variables of indebted households. However, it would be possible to include housing also in the utility of lenders, by appropriately adjusting the functioning of the rental market.

[4] I will refer to impatient households also as "borrowers" or "homeowners".

[5] This guarantees that in the steady state of the model they are willing to borrow by issuing mortgages.

supply one unit of labor inelastically, for which they receive a wage w_t.

Borrowers have access to two types of assets, a one period mortgage m_t and houses h_t. A house that is purchased today at a price q_t^h produces one unit of housing services next period, that can be sold for r_{t+1}[6]. For both assets I assume that short-selling is not possible.

Houses are subject to idiosyncratic depreciation shocks ξ_t, so that in period t, after having rented the house, the owner is left with $\xi_t h_{t-1}$ units of housing. In particular, ξ_t follows a cdf $F(\xi_t, \lambda_t)$ where λ_t is an exogenous disturbance, following an AR1 process, that we define as "housing risk", affecting the variance of the distribution, but not the mean. In particular $E_t(\xi_t) = 1$ for any λ_t, so that houses are in aggregate fixed supply $\bar{H}$.

The only way for impatient households to borrow is to use a one-period defaultable mortgage m_t. After observing the realization of their idiosyncratic shock ξ_t, borrowers can decide to default on their outstanding debt m_{t-1} at the only cost of losing their collateral. There is no other cost for defaulting households, and they can immediately purchase new housing with their available wealth. Such assumption implies that borrowers will default whenever the value of their house is lower than the face value of their mortgage, that is if $\xi_t q_t^h h_{t-1} < m_{t-1}$. Such specification of the default decision is similar to the one used in Jeske et al. (2013).

As a result, the defaulting borrowers will be all those with an idiosyncratic housing shock below a certain threshold $\bar{\xi}_t$, given by

$$\bar{\xi}_t(\eta_{t-1}) = \frac{m_{t-1}}{q_t^h h_{t-1}} = \frac{\eta_{t-1}}{q_t^h} \tag{5}$$

where $\eta_t = \frac{m_t}{h_t}$ represents the impatient household's leverage.

As I will show in the following sections, this simple characterization of the default decision will imply that the only individual variable affecting the price of the mortgage Q_t will be η_t, so that in the household problem we can use $Q_t(\eta_t)$.

2.3 Recursive Formulation of the Impatient Agent Problem

I assume that the borrower's utility function is given by[7]

$$U(c_t, x_t) = \rho_t \log c_t + (1 - \rho_t) \log x_t$$

where ρ_t represents a housing preference shock following an AR1 process.

It is useful to separate the problem of the impatient household between a static decision on the expenditures allocation between consumption and housing services, and a dynamic consumption-saving decision. In particular, if we define $\tilde{c}_t$ as the total expenditures in consumption and housing services, then we can write the static problem as

$$u(\tilde{c}_t, r_t) = \max U(c_t, x_t) \text{ s.t.}$$

[6] In order to simplify aggregation I consider two distinct markets for housing services and houses.

[7] The log specification simplifies aggregation. However aggregation would still be possible with CRRA utility, with more complex policy functions.

$$c_t + r_t x_t = \tilde{c}_t$$

Given the log-utility it is easy to show that[8]

$$u(\tilde{c}_t, r_t) = \log(\tilde{c}_t) + \Theta(\rho_t, r_t)$$

and in addition

$$c_t = \rho_t \tilde{c}_t \tag{6}$$

$$r_t x_t = (1 - \rho_t)\, \tilde{c}_t \tag{7}$$

Define ω_t as the financial wealth for the borrower in period t after the default decision has taken place. This represents the indivitual state variable and it includes the income from renting the house he owns and, if he has not defaulted the difference between the value of the house and the value of the mortgage[9], so that $\omega_t = \max\left\{h_{t-1}\left[\left(q_t^h \xi_t + r_t\right) - \eta_{t-1}\right], h_{t-1} r_t\right\}$.

The problem of the borrower will then be to choose total expenditures $\tilde{c}_t$, houses h_t and leverage η_t in order to solve

$$V_t(\omega_t) = \max_{c_t, h_t, \eta_t} \left\{u(\tilde{c}_t, r_t) + \beta E_t V_{t+1}(\omega_{t+1})\right\}$$

$$\tilde{c}_t + h_t\left[q_t^h - Q_t(\eta_t)\,\eta_t\right] \leq \omega_t + w_t \tag{8}$$

$$\omega_{t+1} = \begin{cases} h_t\left[\left(q_{t+1}^h \xi_{t+1} + r_{t+1}\right) - \eta_t\right] & \text{if} \quad \xi_{t+1} \geq \bar{\xi}_{t+1}(\eta_t) \\ h_t r_{t+1} & \text{if } \xi_{t+1} < \bar{\xi}_{t+1}(\eta_t) \end{cases} \tag{9}$$

Equation (8) represents the budget constraint, where $\left[q_t^h - Q_t(\eta_t)\,\eta_t\right]$ is the down-payment needed to purchase a house that is financed with a mortgage equal to a fraction η_t of the housing good.

Equation (9) is the evolution of financial wealth, that depends on whether default occurs or not. As mentioned in the previous section, the default threshold $\bar{\xi}_t$ can be written as a function of last period's leverage η_{t-1}. It is important to notice that the borrower internalizes how its leverage choice affects his default probability next period, and hence the interest rate that the lender will charge on the mortgage, $1/Q_t(\eta_t)$.

2.3.1 Characterization of the Impatient Agent Problem

In order to obtain a solution that allows for easy aggregation among borrowers, it is useful to rewrite the problem in terms of "labor claims" l_t. Each claim entitles to the future stream of wages w_t, w_{t+1}, w_{t+2} ... , and is valued at price p_t, that represents the present discounted value of future borrower's wages.

We can then rewrite the maximization in terms of a new state variable, the "effective wealth"[10] $a_t = \omega_t + l_t(w_t + p_t)$. In addition, to simplify the consumption saving decision, we can define

[8] The formula for $\Theta(\rho_t, r_t)$ can be found in the appendix.

[9] Given that labor is supplied inelastically, the labor income only depends on the aggregate variable w_t.

[10] Such terminology and approach is also used in Angeletos (2007), where the author shows that linear aggregation is possible also in a framework with idiosyncratic investment risk.

savings as $s_t = h_t \left[q_t^h - Q_t(\eta_t)\eta_t\right] + l_t p_t$. Finally, the portfolio decision can be written in terms of the share of savings that is allocated to labor claims $\varphi_t = l_t p_t / s_t$ and the one that is used for housing $(1-\varphi_t) = h_t \left[q_t^h - Q_t(\eta_t)\eta_t\right] / s_t$.

At this point we can write the borrower's problem as

$$V_t(a_t) = \max_{c_t, s_t, \varphi_t, \eta_t} \{\log(\tilde{c}_t) + \Theta(\rho_t, r_t) + \beta E_t V_{t+1}(a_{t+1})\} \quad \text{s.t.}$$

$$\tilde{c}_t + s_t = a_t \tag{10}$$

$$a_{t+1} = s_t R_{t+1}^s(\eta_t, \varphi_t, \xi_{t+1}) \tag{11}$$

$$R_t^s(\eta_{t-1}, \varphi_{t-1}, \xi_t) = \left[\varphi_{t-1} R_t^l + (1-\varphi_{t-1}) R_t^h(\eta_{t-1}, \xi_t)\right] \tag{12}$$

$$R_t^l = \frac{w_t + p_t}{p_{t-1}} \tag{13}$$

$$R_t^h(\eta_{t-1}, \xi_t) = \frac{\max\left(r_t, \left(q_t^h \xi_t + r_t\right) - \eta_{t-1}\right)}{\left[q_{t-1}^h - Q_{t-1}(\eta_{t-1})\eta_{t-1}\right]} \tag{14}$$

Equation (10) is simply a rewriting of the budget constraint. Equation (11) is the evolution of effective wealth, expressed in terms of the rate of return on savings R_t^s. Such return is simply a weighted average of the return on housing R_t^h and the return on labor claims R_t^l, where the weights are given by the portfolio shares φ_t and $(1-\varphi_t)$ as written in equation (12). Equation (13) defines the return on labor whereas equation (14) defines the one on housing. In particular, in case of default the latter is going to be given only by r_t , otherwise it also includes the difference between the residual value of the house and the face value of the mortgage.

The non-standard features of the impatient agent's problem are the possibility of default and the fact that he internalizes how his leverage decision affects the price of his debt. However, given the simple characterization of default, that does not require to keep track of the default history, this problem is going to have a simple solution as described in the following proposition[11].

Proposition 1 *Given prices, the borrower's optimal choices for consumption, housing services, housing, and mortgage debt are linear in effective wealth:*

$$c_t = \rho_t (1-\beta) a_t \tag{15}$$

$$r_t x_t = (1-\rho_t)(1-\beta) a_t \tag{16}$$

$$h_t = \frac{(1-\varphi_t)}{\left[q_t^h - Q_t(\eta_t)\eta_t\right]} \beta a_t \tag{17}$$

$$m_t = \eta_t h_t \tag{18}$$

[11] See proof in the appendix.

where η_t and φ_t are determined by

$$\frac{d\left[Q_t\left(\eta_t\right)\eta_t\right]}{d\eta_t} = E_t\left\{\frac{1}{R^s_{t+1}\left(\eta_t,\varphi_t,\xi_{t+1}\right)}1\left\{\xi_{t+1} > \frac{\eta_t}{q^h_{t+1}}\right\}\right\} \tag{19}$$

$$E_t\left\{\frac{\left(R^l_{t+1} - R^h_{t+1}\left(\eta_t,\xi_{t+1}\right)\right)}{R^s_{t+1}\left(\eta_t,\varphi_t,\xi_{t+1}\right)}\right\} = 0 \tag{20}$$

and the evolution of effective wealth follows

$$a_{t+1} = \beta a_t R^s_{t+1}\left(\eta_t,\varphi_t,\xi_{t+1}\right) \tag{21}$$

The policy functions for consumption and housing services expenditures, eq (15) and (16), simply follow from the fact that given log-utility, consumption expenditures $\tilde{c}_t$ will be a constant fraction $(1-\beta)$ of wealth[12]. Combining this with eq. (6) and (7) delivers the two equations (15) and (16).

The leverage decision of the impatient agent is described by equation (19). The left-hand side represents the benefits of issuing a mortgage equal to a fraction η_t of the housing that the borrower is purchasing. In particular this quantity can be rewritten as

$$\frac{d\left[Q_t\left(\eta_t\right)\eta_t\right]}{d\eta_t} = Q_t\left(\eta_t\right) + \eta_t Q'_t\left(\eta_t\right)$$

where the first term represents the amount received per unit of mortgage, whereas the second term takes into account how a marginal increase in η_t will affect the pricing of the mortgage. As I will explain in the following section $Q'_t\left(\eta_t\right) < 0$, due to the fact that a higher leverage increases the probability of default. The right-hand side of equation (19) represents the expected mortgage costs next period, that are given by the repayment of the face value of debt, but only in the non-default states.

The portfolio decision related to φ_t will be determined by a standard indifference condition in eq (20), that equates the expected discounted return on houses and labor.

An important result is that the system of equations ((19) and (20)) determining η_t and φ_t only depends on aggregate variables. This implies that these two variables will be the same for every impatient household, so that all borrowers will have the same leverage and consequently only one type of risky mortgage will be traded in every period.

Given η_t and φ_t, equations (17) and (18), simply follow from the budget constraint and the definition of η_t and φ_t. Finally, equation (21) is obtained from (11).

As I will show in the next section, the linearity of the policy functions together with the fact that φ_t and η_t only depend on aggregate variables will allow for a simple aggregation of the choices

[12] The linearity of the policy functions would still be present as long as we focus on homotetic utility functions with homogeneous budget constraints. CRRA utility would satisfy this requirement, but they would imply a time varying saving rate instead of a constant one.

of impatient households, without having to keep track of the wealth distribution of this type of agents.

2.4 Aggregation for Impatient Agents

If we define H_t and L_t as the aggregate amount of houses and labor claims for impatient agents, we can write the evolution of their aggregate net worth NW_t^{imp} as

$$NW_t^{imp} = H_{t-1}\left\{r_t + q_t^h \int_{\bar{\xi}_t(\eta_{t-1})}^{\infty} \xi_t f(\xi_t, \lambda_t)\, d\xi - \left(1 - F\left(\bar{\xi}_t(\eta_{t-1}), \lambda_t\right)\right)\eta_{t-1}\right\} + L_{t-1}(w_t + p_t) \tag{22}$$

$$= \bar{H}\left\{r_t + q_t^h \int_{\bar{\xi}_t(\eta_{t-1})}^{\infty} \xi_t f(\xi_t, \lambda_t)\, d\xi - \left(1 - F\left(\bar{\xi}_t(\eta_{t-1}), \lambda_t\right)\right)\eta_{t-1}\right\} + (w_t + p_t) \tag{23}$$

where I have used the result that η_t is the same for all borrowers and the market clearing results $H_t = \bar{H}$ and $L_t = 1$. Therefore, in addition to the value of rents and labor claims, the aggregate wealth for impatient agents will be increasing in the value of the houses of non-defaulting agents and decreasing in their outstanding debt.

In addition, the linearity of the policy functions implies that the borrowers' aggregate demand for consumption goods C_t, housing services X_t and houses will follow

$$C_t = \rho_t(1-\beta)\, NW_t^{imp} \tag{24}$$

$$r_t X_t = (1-\rho_t)(1-\beta)\, NW_t^{imp} \tag{25}$$

$$H_t = \frac{(1-\varphi_t)}{\left[q_t^h - Q_t(\eta_t)\,\eta_t\right]}\beta NW_t^{imp} = \bar{H} \tag{26}$$

Equation (24) together with (22) shows how the consumption of impatient agents is affected by the value of houses, since it can be shown that NW_t^{imp} is increasing in q_t^h. In addition, from equation (26) we can see that the aggregate demand for housing is linear in net worth and increasing in the amount of dollars raised from mortgages per unit of housing, $Q_t(\eta_t)\,\eta_t$.

2.5 The Banker's Problem

The role of banks is to transfer funds from patient households to intermediate goods producers to finance capital purchases and to impatient households to finance house purchases. I will refer to the first type of assets as loans, z_t and to the second one as mortgages m_t [13].

As in Gertler and Karadi (2011), I assume that there is no friction between bankers and non-financial firms, so that goods producers can issue a state contingent security, that can be thought

[13] With a slight abuse of terminology I will refer to m_t also as mortgage-backed-securities (MBS).

of as equity[14], whose price will be equal to the price of capital q_t^k, and providing a return R_t^k.

On the other hand, as described above, the relationship between banks and homeowners is characterized by defaultable debt. In particular, each bank can potentially invest in a continuum of mortgages, each indexed by the leverage of the borrowing household, $m_t(\eta_t)$, and for which the banker will pay a price $Q_t(\eta_t)$.

Each bank finances itself with retained earnings n_t, and by issuing risk-free deposits d_t to patient households. As a result, we can write the budget constraint for a bank as

$$q_t^k z_t + \int Q_t(\eta_t) m_t(\eta_t) d\eta_t = n_t + d_t$$

The expected return per unit of a mortgage with leverage η_t will be

$$E_t R_{t+1}^m(\eta_t) = E_t \frac{\left\{ \left[1 - F\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right)\right] + \gamma \frac{q_{t+1}^h}{\eta_t} \int_0^{\bar{\xi}_{t+1}(\eta_t)} \xi_{t+1} dF\left(\xi_{t+1}, \lambda_{t+1}\right) \right\}}{Q_t(\eta_t)} \tag{27}$$

$$= E_t \frac{\Psi_{t+1}\left(\eta_t, \lambda_{t+1}, \xi_{t+1}\right)}{Q_t(\eta_t)} \tag{28}$$

Equation (27) is important to understand the expected payoff of a bank financing a mortgage. With probability $1 - F\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right)$ the debt is repaid, and the bank receives the face value of the mortgage. Otherwise, when $\xi_{t+1} < \eta_t / q_{t+1}^h$, the household defaults and walks away and the bank can repossess an amount of housing whose value before depreciation is $q_{t+1}^h h_t = q_{t+1}^h m_t / \eta_t$. In addition, I assume that there are also default costs that are equal to a fraction $(1-\gamma)$ of the value of the house, that is lost in the foreclosure process.

We can then characterize the evolution of the net-worth of an individual bank as

$$n_{t+1} = q_t^k z_t R_{t+1}^k + \int \left\{ Q_t(\eta_t) m_t(\eta_t) R_{t+1}^m(\eta_t) \right\} d\eta_t - R_{t+1} d_t \tag{29}$$

$$= q_t^k z_t \left(R_{t+1}^k - R_{t+1}\right) + \int Q_t(\eta_t) m_t(\eta_t) \left[R_{t+1}^m(\eta_t) - R_{t+1}\right] d\eta_t + n_t R_{t+1} \tag{30}$$

As long as the banker makes an expected return on his assets greater or equal than R_{t+1}, he will choose z_t, m_t and d_t in order to maximize the accumulated value of his net-worth before it has to exit and become a worker. Hence, his value function at the end of time t (before knowing the realization of the exit random variable) is given by

$$V_t = E_t \sum_{i=0}^{\infty} (1-\sigma) \sigma^i \hat{\beta}^{i+1} \Lambda_{t,t+1+i} n_{t+1+i} \tag{31}$$

where σ is the probability of staying in the market. As I described above, banks are owned by

[14] At the cost of additional complexity it would be possible to model also defaultable loans to non-financial firms, by assuming some idiosyncratic disturbance to the firm return and a default decision similar to the one of impatient households. For an example see Navarro (2014).

patient households, and for this reason their stochastic discount factor enters the value function in (31). In addition, as in Gertler and Karadi (2011), I introduce an agency problem between the bank and the depositors in order to limit the amount of risky assets that the financial sector can hold and generate accordingly a gap between the rate of returns on assets and liabilities. In particular I assume that after raising deposits, the banker can default and divert a fraction θ_t^k of his loans and θ_t^m of his mortgages, back to his own household. If the banker does so, depositors can force him to bankruptcy and consequently to leave the banking sector forever, while recovering the remaining fractions of the assets.

As a result, the banker's problem entails the following incentive constraint, needed for the households to provide deposits to the bank

$$V_t \geq \theta_t^m \left[\int Q_t(\eta_t) \, m_t(\eta_t) \, d\eta_t \right] + \theta_t^k q_t^k z_t \tag{32}$$

Such constraint guarantees that the value from continuing operating the bank, the left-hand side, is larger than the value of "running-away" with the diverted assets. In addition I assume that both θ_t^j for $j = k, m$, are subject to exogenous shocks according to

$$\log \theta_t^j = (1 - \rho_{\theta^j}) \log \theta_{ss}^j + \rho_{\theta^j} \log \theta_{t-1}^j + \varepsilon_{\theta^j t} \quad j = k, m$$

The idea is that such shocks should capture changes in the tightness of credit markets that are not related to fundamental shocks. In particular $\varepsilon_{\theta^m t}$ and $\varepsilon_{\theta^k t}$ are shocks specifing to the financing of mortgages or firm loans. In the numerical experiments I will focus on a shock affecting θ_t^m as a stylized way to capture the collapse of the market for mortgage-backed securities and securitization.

We can write the banker's value function recursively as follows

$$V_t(n_t) = \max_{k_t, \{m_t(\eta_t)\}_{\eta_t}} E_t \hat{\beta} \Lambda_{t,t+1} \left\{ (1 - \sigma) n_{t+1} + \sigma V_{t+1}(n_{t+1}) \right\}$$

where the maximization is subject to (32) and (29).

It can be showed that the value function for the banker is linear in net-worth, and can be rewritten as[15] $V_t(n_t) = \nu_t n_t$. If we define μ_t as the multiplier on the incentive constraint, the implied first order conditions for z_t and m_t are

$$E_t \hat{\beta} \Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1}^k - R_{t+1} \right) = \mu_t \theta_t^k$$

$$E_t \hat{\beta} \Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1}^m(\eta_t) - R_{t+1} \right) = \mu_t \theta_t^m \quad \forall \eta_t$$

where $\Omega_t = \{(1 - \sigma) + \sigma \nu_t\}$ represents the adjusted marginal value of net-worth. As a result, if the constraint does not bind, ($\mu_t = 0, \Omega_t = 1$), the expected discounted return on both bank assets should be equal to the risk-free rate. However, when the constraint binds loans and MBS will imply

[15] See the appendix for a detailed solution of the problem of the financial intermediary

an excess return on the risk-free rate.

In addition, the equations above imply the following no-arbitrage relationship

$$E_t\hat{\beta}\Lambda_{t,t+1}\Omega_{t+1}\left(R^m_{t+1}-R_{t+1}\right)=\frac{\theta^m_t}{\theta^k_t}E_t\hat{\beta}\Lambda_{t,t+1}\Omega_{t+1}\left(R^k_{t+1}-R_{t+1}\right) \tag{33}$$

Such equation establishes a first linkage between the expected returns on capital and houses, that is also going to depend on the tighteness of the leverage constraint, as measured by μ_t. In particular, in steady state, if $\theta^m<\theta^k$, the excess return on MBS will be lower than the one on loans to the productive sector.

Given the linear form of the value function, it can be shown that when the constraint is binding, the following endogenous constraint on bank's adjusted leverage is going to be in place

$$\left[q^k_t z_t+\frac{\theta^m_t}{\theta^k_t}\int Q_t\left(\eta_t\right)m_t\left(\eta_t\right)d\eta_t\right]\leq\phi_t n_t \tag{34}$$

where

$$\phi_t=\frac{E_t\hat{\beta}\tilde{\Lambda}_{t,t+1}R_{t+1}}{\theta^k_t-E_t\hat{\beta}\tilde{\Lambda}_{t,t+1}\left(R^k_{t+1}-R_{t+1}\right)} \tag{35}$$

where $\tilde{\Lambda}_{t,t+1}=\Lambda_{t,t+1}\Omega_{t+1}$. The constraint in (34) sets the value of the bank portfolio at a point in which the incentive constraint is exactly satisfied. In particular, if $\theta^m_t<\theta^k_t$, this implies a slacker limit on the bank's investment in mortgages. Also, the maximum leverage ratio will be inversely related to θ^k_t and positively related to the spread in expected returns. Equation (34) is at the heart of the standard bank financial accelerator, by linking banks asset demand to their net-worth.

In addition, we can rewrite equation (33) in order to obtain the mortgage pricing equation that impatient agents will internalize when choosing their optimal leverage, that is

$$\begin{aligned}Q_t\left(\eta_t\right) &= \frac{E_t\hat{\beta}\tilde{\Lambda}_{t,t+1}\left\{\left[1-F\left(\bar{\xi}_{t+1}(\eta_t),\lambda_{t+1}\right)\right]+\gamma\frac{q_{t+1}}{\eta_t}\int_0^{\bar{\xi}_{t+1}(\eta_t)}\xi_{t+1}dF\left(\xi_{t+1},\lambda_{t+1}\right)\right\}}{E_t\hat{\beta}\tilde{\Lambda}_{t,t+1}R_{t+1}+\theta^m_t\mu_t} && (36)\\ &= \frac{E_t\hat{\beta}\tilde{\Lambda}_{t,t+1}\Psi^m_{t+1}\left(\eta_t,\xi_{t+1}\right)}{E_t\hat{\beta}\tilde{\Lambda}_{t,t+1}R_{t+1}+\theta^m_t\mu_t}=E_t\tilde{\Omega}_{t+1}\Psi^m_{t+1}\left(\eta_t,\xi_{t+1}\right) && (37)\end{aligned}$$

This relationship will be crucial for the additional amplification mechanism present in this paper. In fact, $\tilde{\Omega}_{t+1}$ is the stochastic discount factor that bankers use to price risky mortgages. During a crisis, the incentive constraint on financial intermediaries will become tighter. As a result μ_t will increase, putting downward pressure on $Q_t\left(\eta_t\right)$ and increasing the spread on mortgages. This will reduce borrowers demand for housing, depressing q^h_t and increasing defaults, negatively affecting banks net worth and causing real costs for the economy. This will imply a further tightening of the incentive constraint, reinforcing the initial shock. As a result, the negative feedback loop characteristic of the financial accelerator, in this framework is enriched with the relationship between house prices, endogenous defaults and banks foreclosures.

Equation (36) also shows how the costly default of mortgages introduces an additional spread between the cost of funding for banks and the one for impatient households. In fact, since the term in parenthesis in eq. (36) is smaller than one, this implies that

$$E_t \tilde{\Omega}_{t+1} \frac{1}{Q_t(\eta_t)} > 1 = E_t \tilde{\Omega}_{t+1} R^m_{t+1} \tag{38}$$

where R^m_{t+1} can be interpreted as the required rate of return for bankers. Therefore the price of a mortgage will include an additional default-premium, that compensates financial intermediaries for the possibility of default.

In addition, we can use (36) to compute the derivative of the mortgage price with respect to leverage. In particular, we obtain

$$\frac{dQ_t(\eta_t)}{d\eta_t} = -E_t \tilde{\Omega}_{t+1} \frac{1}{\eta_t} \left\{ f\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right) \frac{\eta_t}{q^h_{t+1}} (1-\gamma) + \gamma \frac{q^h_{t+1}}{\eta_t} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF\left(\xi_{t+1}, \lambda_{t+1}\right) \right\} < 0 \tag{39}$$

The negative relationship between mortgate prices and leverage is intuitive, since a higher leverage implies a higher probability of default. Furthermore it can be showed that

$$\frac{d\left[Q_t(\eta_t)\eta_t\right]}{d\eta_t} = Q_t(\eta_t) + \eta_t Q'_t(\eta_t) = E_t \tilde{\Omega}_{t,t+1} \left\{ \left[1 - F\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right)\right] - (1-\gamma) f\left(\bar{\xi}_{t+1}(\eta_t), \lambda_{t+1}\right) \frac{\eta_t}{q^h_{t+1}} \right\} \tag{40}$$

a quantity that is needed to determine the optimal η_t in (19).

Finally, it has to be noted that if the constraint does not bind then $\mu_t = 0$, $\tilde{\Lambda}_{t,t+1} = \Lambda_{t,t+1}$ and $E_t \hat{\beta} \tilde{\Lambda}_{t,t+1} R_{t+1} = 1$ so that

$$Q_t(\eta_t) = E_t \tilde{\Omega}_{t,t+1} \Psi^m_{t+1}\left(\eta_t, \xi_{t+1}\right) = E_t \beta \Lambda_{t,t+1} \Psi^m_{t+1}\left(\eta_t, \xi_{t+1}\right) \tag{41}$$

When the incentive problem does not play a role banks will be just a veil and the mortgages will be priced with the stochastic discount factor of patient households. Equation (41) will be used instead of eq. (36) to simulate the model without financially constrained banks, and to evaluate the amplification that ensues from the bankers' agency problem.

2.6 Aggregation in the Banking Sector

Given the linearity of the incentive constraint in (34), the fact that ϕ_t only depends on aggregate quantities, and that in equilibrium all mortgages will have the same leverage, we can obtain the following aggregate version of the constraint on the bank portolio

$$\left[q^k_t Z_t + \frac{\theta^m_t}{\theta^k_t} Q_t(\eta_t) M^b_t \right] \leq \phi_t NW^b_t \tag{42}$$

where M^b_t and Z_t represent banks aggregate holding of mortgages and loans, whereas NW^b_t is the aggregate net worth of the financial system. Importantly, such relationship relates the demand for asset by intermediaries to the aggregate level of net worth, so that any shock negatively affecting this variable will put downward pressure on Q_t and q_t^K.

The evolution of aggregate net-worth will be given by the wealth of the surviving bankers plus a transfer that the household will provide to the new bankers, equal to a fraction $\varpi/(1-\sigma)$ of the value of the assets of exiting bankers

$$NW_t^b = \sigma[R_t^m Q_{t-1} M_{t-1}^b + R_{t+1}^k q_t^k Z_t - R_{t+1} D_t] + NW_t^e \tag{43}$$

where

$$NW_t^e = \varpi(Q_t M_{t-1}^b + q_t^k Z_{t-1}) \tag{44}$$

From equation (43) we see how any shock affecting the realized return of the two types of assets will directly impact aggregate net-worth. This effect will be larger for the asset representing a larger share of the aggregate portfolio.

2.7 Intermediate Goods Producers

Consumption good producers are competitive and produce output to be sold to retailers at the real price P_t^m. They operate a standard Cobb-Douglas technology using capital and the combined labor of patient and impatient households

$$Y_t = A_t K_{t-1}^{\alpha} \left(N_t^{\mu} \hat{N}_t^{1-\mu} \right)^{1-\alpha}$$

where A_t represents an aggregate productivity shock. As in Iacoviello and Neri (2010) I assume complementarity between the labor of the two types of agents, so that the parameter μ represents the labor income share accruing to the impatient household.

The first order conditions with respect to labor will imply

$$\hat{w}_t = A_t \left(1-\alpha\right) \left(1-\mu\right) \frac{P_t^m Y_t}{\hat{N}_t} \tag{45}$$

$$w_t = A_t \left(1-\alpha\right) \mu \frac{P_t^m Y_t}{N_t} \tag{46}$$

where P_t^m is the real price of intermediate goods.

The firm has no initial endowment, and needs to fund the purchase of capital by issuing state contingent debt claims Z_t equal to the amount of new capital acquired K_t. By no-arbitrage these claims will have a price equal to the price of capital q_t^K. In particular, given that the firm will make zero profits state by state, we have that the one period return on capital, obtained by the bank, will be given by:

$$R_{t+1}^K = \frac{P_{t+1}^m \alpha Y_{t+1}/K_{t-1} + (1-\delta_K) q_{t+1}^k}{q_t^k} \tag{47}$$

where δ_k is the depreciation rate of capital.

2.8 Capital Producers

Capital good producers create new capital by combining final good input I_t with aggregate capital K_{t-1} according to the technology $\Phi\left(\frac{I_t}{K_{t-1}}\right)K_{t-1}$. They operate competitively and sell the new capital at the price q_t^k. Their problem will be given by

$$\max_{I_t,}\left[q_t^k\Phi\left(\frac{I_t}{K_{t-1}}\right)K_{t-1}-I_t\right]$$

As a result the price of capital will satisfy

$$q_t^k=\left[\Phi'\left(\frac{I_t}{K_{t-1}}\right)\right]^{-1}$$

Following Bocola (2014) I use $\Phi(x)=a_1x^{1-\gamma_i}+a_2$ where γ_i will measure the elasticity of the price with respect to investments and a_1 and a_2 are normalizing parameters to have a steady state price of unity.

2.9 Retail Goods Producers

The final output Y_t is a CES composite of a continuum of varieties produced by retail firms, owned by patient households, that employ intermediate output as input. The final good composite is

$$Y_t=\left[\int_0^1 Y_t(z)^{(\varepsilon-1)/\varepsilon}\,dz\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{48}$$

where $Y_t(z)$ is the output produced by retailer f. Each retailer (f) faces the demand function

$$Y_t(z)=\left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon}Y_t \tag{49}$$

where the aggregate price level P_t is given by

$$P_t=\left[\int (P_t(z))^{1-\varepsilon}\,dz\right]^{\frac{1}{1-\varepsilon}} \tag{50}$$

In addition, I introduce nominal rigidities by assuming that each period a firm is able to adjust its prices only with probability $(1-\xi)$

As a result, the problem for the firm-setting firm is to select P_t^* to maximixe

$$E_t\sum_{i=0}^{\infty}\zeta^i\hat{\beta}^i\Lambda_{t,t+1}\left[\frac{P_t^*}{P_{t+i}}-P_{t+i}^m\right]Y_{t+i}^*(z) \tag{51}$$

the first order conditions will be given by

$$E_t \sum_{i=0}^{\infty} \zeta^i \hat{\beta}^i \Lambda_{t,t+1} \left[\frac{P_t^*}{P_{t+i}} - \frac{\varepsilon}{\varepsilon - 1} P_{t+i}^m \right] Y_{t+i}^*(z) \tag{52}$$

Finally, aggregating over (50) we obtain the following evolution for P_t

$$P_t = \left[(1-\zeta) \left(P_t^*\right)^{(1-\varepsilon)} + \zeta \left(P_{t-1}\right)^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}$$

2.10 Market Clearing and Resource Constraint

The equilibrium in the capital market requires that the value of the loans held by financial intermediaries equals the value of capital in place at time t

$$Z_t = K_t$$

The equilibrium in the labor market for impatient households implies that the aggregate amount of labor and labor claims will be

$$L_t = N_t = 1 \tag{53}$$

In addition the equlibrium in the housing market and in the housing services market will be given by

$$X_t = H_t = \bar{H} \tag{54}$$

The evolution of aggregate capital will be given by

$$K_t = (1-\delta) K_{t-1} + \Phi \left(\frac{I_t}{K_{t-1}} \right) K_{t-1}$$

Finally, the aggregate resource constraint is

$$Y_t = C_t + \hat{C}_t + I_t + (1-\gamma) q_t^h H_{t-1} \int_0^{\bar{\xi}_t} \xi_t dF\left(\xi_t, \lambda_t\right) \tag{55}$$

where the last term represents the default costs. Also it is convenient to devine output net of default costs as

$$\bar{Y}_t = Y_t - (1-\gamma) q_t^h H_{t-1} \int_0^{\bar{\xi}_t} \xi_t dF\left(\xi_t, \lambda_t\right)$$

and aggregate consumption as

$$\bar{C}_t = C_t + \hat{C}_t$$

2.11 Monetary Policy

Monetary policy is characterized by the following Taylor rule

$$(i_t) = (i_{t-1})^{(\rho_i)} \left[(i_{ss}) (\pi_t)^{\kappa_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\kappa_Y} \right]^{(1-\rho_i)} \left(\varepsilon_t^i \right) \tag{56}$$

where ρ_i is a smoothing parameter on interest rates, ε_t is a monetary policy shock and the gross nominal rate i_t is given by the Fisher equation

$$i_t = R_{t+1} E_t \pi_{t+1} \tag{57}$$

2.12 Credit Policy

As part of the set of unconventional monetary policy tools employed by the Federal Reserve, the purchase of mortgage-backed securities and agency debt was the largest program put in place. The asset purchase started in January 2009 and the stock of MBS held by the Federal Reserve topped \$1.1 trn by mid-2010. The main aim of this program was to reduce mortgage interest rates on the primary and secondary market, in order to "support housing markets and foster improved conditions in financial markets more generally"[16].

In order to capture the effects of such policy in this model, I assume that the central bank is able to indirectly intermediate mortgage-related securities M_t^g, by purchasing them from the financial sector. In particular, I assume that the Fed buys its mortgages from financial intermediaries, right after they are originated, at the origination price $Q_t(\eta_t)$. Importantly, at the time of origination, these mortgages are not subject to the agency problem between depositors and bankers.

Therefore, the aggregate amount of MBS will be given by

$$M_t = M_t^b + M_t^g \tag{58}$$

As in Gertler and Karadi (2011), I assume that the central bank can finance this credit policy by issuing risk-free government debt to patient households, not subject to any agency problem. However this type of intermediation entails efficiency costs $\tau Q_t M_t^g$, that can capture, for example, the disadvantage that the Fed has in selecting the best securities to fund.

To characterize this credit policy I consider a central bank intermediating a fraction Ψ_t^M of total assets, that is

$$M_t^g = \Psi_t^M M_t \tag{59}$$

As a result, given the credit policy the total amount of mortgages finance at time t will be

$$M_t = \frac{M_t^b}{1 - \Psi_t^M} \tag{60}$$

[16] Federal Reserve press releas from November 25, 2008.

so that we see how an increase in Ψ_t^M will imply that the constraint on intermediaries leverage will have a smaller impact on the aggregate amount of mortgages intermediated.

In order to model Ψ_t^M, I assume that the Fed intervenes when the spread $\left(E_t R_{t+1}^m - R_{t+1}\right)$ rises. This spread is linked to how easily financial intermediaries can fund mortgages, and I will refer to it as MBS spread. During a financial crisis, when banks net worth is low and financial frictions are tighter this spread will be higher.

In particular, I consider the following policy rule

$$\Psi_t^M = \begin{cases} \Psi_1^M \left[\log\left(E_t R_{t+1}^m - R_{t+1}\right) - \log\left(spread_{SS}^m\right)\right] & \text{if } \left(E_t R_{t+1}^m - R_{t+1}\right) > spread_{ss}^m \\ 0 & \text{otherwise} \end{cases} \tag{61}$$

where

$$spread_{ss}^m = R_{ss}^m - R_{ss}$$

Therefore, the central bank will start intermediating assets only when it makes a positive excess expected return. The parameter ψ_1^M will determine the intensity of the intervention.

3 Numerical Experiments

3.1 Calibration

The model is solved by log-linearization, and it is calibrated to have a steady state where the bank incentive constraint is always binding.

In Table 1 I summarize the parameter values used for the numerical simulations. The time horizon is quarterly and the calibration is aimed at matching some aggregate quantities as a share of GDP. In particular, I consider GDP as also comprising the value of rents, so that $GDP_t = Y_t + r_t \bar{H}$.

The preference parameters for patient households are standard. As regards impatient households β and ρ are calibrated in order to obtain an household leverage η equal to .65 and a value of rents equal to 16% of gdp. Jeske et al. (2014) report a median leverage of .61 from the Survey of Consumer Finance in 2004, but considering the very low down-payments of the subprime boom my number seems conservative.

I set γ equal to .8, in line with the average foreclosure losses reported in Jeske et al. I use a log-normal distribution for the depriaciation shock, $\ln(\xi_t) \sim N\left(-\frac{\lambda^2}{2}, \lambda^2\right)$ and I calibrate the steady state value of λ in order to have a default rate of 1%, a number in line with the foreclosure rate before the crisis.

The parameters of the financial sector $\theta_{ss}^k, \theta_{ss}^h$, σ and $\bar{\omega}$ are calibrated to hit the moments of some specific financial variables. In particular they imply a bank leverage ratio of 10, an annual spread on loans of 50 basis points, a spread of MBS of 20 basis points and an average life for the bankers of 5 years. The leverage ratio should represent an average of the leverages of commercial banks, investment banks and firms . The spread on R^k should refer to the spread on a Baa bond, whereas the one on R^m should capture the lower spread on MBS securities before the crisis. As a result $\theta^m_{ss} < \theta^k_{ss}$, implying that it is more difficult for the banker to divert mortgage-re lated assets than loans. What I try to capture is the idea that, before the crisis, because of several factors, not directly modeled here, including government support and financial innovation, mortgage-backed-securities were perceived as a safer and more liquid type of asset. In addition such calibration implies a spread on mortgages, $(1/Q - R)$, of 120 basis points annually.

As regards the technological environment, I assume $\alpha = .33$ and $\mu = .5$, so that labor income is equally distributed across the two groups of households. I use $\delta_k = .025$ and obtain that investments account respectively for 15% of GDP. The fixed supply of housing is calibrated to imply a value of houses equal to 1.05 of annual GDP, whereas capital will be equal to 1.5 annual GDP.

Finally, as regards the monetary policy parameters they mostly follow from Gertler and Karadi (2011). The calibration of the credit policy parameters will be discussed below.

3.2 Model Behavior

I begin by considering a first set of conventional shocks to illustrate the model behavior in comparison to a corrisponding model without financial frictions in the banking sector. In such model patient households can frictionlessly invest in mortgages or capital, so that bankers and their net worth play no role for aggregate fluctuations. For this reason we label this model as the "no-bank" model. In particular this will also imply the following relationship for the returns on both types of bank assets

$$E_t \hat{\beta} \Lambda_{t,t+1} \left(R^j_{t+1} - R_{t+1} \right) = 0 \quad \text{for } j = m, k \tag{62}$$

The objective of this exercise is to show how constrained bankers can influence the way in which shocks affect the real economy and the role played by housing variables. In particular, it is useful to define the following annualized spreads

$$spread^k_t = 4 \left(E_t R^k_{t+1} - R_{t+1} \right)$$

$$spread^m_t = 4 \left(E_t R^m_{t+1} - R_{t+1} \right)$$

$$spread^h_t = 4 \left(1/Q_t - R_{t+1} \right)$$

The first fwo variables represent the the difference between the expected return that the bank is making on its assets and its cost of funding. Therefore we can refer to $spread^k_t$ as loan spread and $spread^m_t$ as MBS spread. In addition $spread^h_t$ represents the spread between the mortgage rate and the risk free rate; in particular this variable will be affected both by a default premium and by the MBS spread.

Figure (1) shows the response of the baseline model, in absence of credit policy, to three shocks to productivity, housing preferences and nominal interest rates, all calibrated to generate a downturn. The solid line is the baseline model, whereas the dotted line is the "no-bank model".

The TFP shock is a 1% drop with a persistence of .95. One driver behind the amplification in the output drop is the fact that real investments decline twice as much in the baseline model. This is the well known "financial accelerator" mechanism that operates through a drop in bankers net worth and a consequent fall in capital demand that increases the related spread. However, in this first experiment we can already see an additional channel that increases the correlation between house prices and investments in capital, and causes a drop both in house prices and mortgage prices. Such connection operates first of all through the balance sheet of financial intermediaries, that react to the decline in their net worth by reducing the demand for both loans and mortgages. However, as I will explain in detail in the next section, two new channels are at play in this model. The first one is related to the endogenous increase in defaults following a drop in house prices, further affecting banks net worth. The second one, operates through the wealth of impatient agents, that will decrease together with q_t^h reducing borrowers demand for consumption good.

The housing preference shock is again a 1% drop with a persistence of .95. In this case the crisis is initiated by a drop in the demand for houses.

Finally, the monetary policy shock is a 10 basis points increase in the short term nominal interest rate. The amplification in this case comes from the fact that such shock directly tightens banks incentive constraint by increasing the real interest rate. This will cause downward pressure on asset prices, and, through the consequent drop in net worth, a reduction in both types of bank assets.

In all 3 cases, the model with banks produces comovements between the three spreads, a larger drop in business investments and a larger drop in output.

3.3 A Housing Risk Shock

The first type of shock, peculiar to this model, that I study, is a housing risk shock. In Figure 2, I consider a 10% increase in the variance of the idiosyncratic depreciation distribution, λ_t with a peristence of .7. Such shock is aimed at capturing the impact of the increase in subprime delinquencies on house prices during the financial crisis. The first effect of such disturbance is to increase mortgage defaults, since it increases the mass of agents below the threshold $\bar{\xi}_t$. In particular I calibrate the shock to obtain a 1% increase in mortgage defaults.

The first channel through which the effects of such shock are amplified is the bank balance sheet. In fact, financial intermediaries suffer losses on their mortgages and, because of the leverage constraint, once their net worth drops they begin divesting from both mortgages and loans.

These fire sales imply two additional negative feedback mechanisms, compared with a model with unconstrained intermediaries. First, there is a spillover effect that depresses investments and consequently output, as we can see from the fact that q_t^k drops in the baseline model whereas it barely moves in the "no-bank" model.

In addition, the tightening of the leverage constraint implies a decrease in the stochastic discount factor of the bank $\tilde{\Omega}_{t,t+1}$, resulting in a lower price paid for mortgages Q_t as we can see from (36).

This entails a lower demand for houses by impatient agents, as we can see from the policy function in (26) so that q_t^h drops as well. But a lower q_t^h also means an increase in the default threshold $\bar{\xi}_t = \eta_{t-1}/q_t^h$, so that the initial increase in defaults is reinforced. Compared to the model without financial intermediaries the prices of houses and mortgages drop approximately 20% and 50% more.

With respect to the financial accelerator in Gertler and Karadi (2011), this model presents also an additional mechanism that allows asset prices to affect real variables, operating through the net worth of impatient agents. In fact, as I showed in Proposition 1, borrowers aggregate consumption is linear in their aggregate wealth, whose value is increasing in house prices. As a consequence, a housing bust, by reducing NW_t^{imp} also depresses borrowers consumption, causing a reduction in the demand for output. In presence of nominal rigidities, this also entails a drop in wages and labor, because of the increase in mark-ups, so that borrowers wealth is affected even further. It has to be noted that this demand channel is present also in the model without banks, but in that case, because of the absence of banks deleveraging, it implies a drop in consumption and output that is approximately 20% and 30% smaller.

3.4 An MBS Crisis

In the previous section I have analyzed a crisis generated by an increase in mortgage riskiness. In figure (3) I study the effect of a tightening in banks funding conditions specific to the financing of mortgage securities.

The idea behind this exercise is that of capturing the turmoil in the market for asset-backed securities that followed the melt-down of the securitization market. As a result of these events, the collateral value of MBS deteriorated consistently and almost permanently. In particular, I consider a 15% increase in θ_t^m, with persistence .95 where the shock is calibrated to reproduce a 1% increase in the MBS spread ($spread_t^m$) on impact.

The initial effect of an increase in θ_t^m is that of negatively affecting banks demand for mortgages as it is clear from equation (36). Again Q_t drops and $spread_t^h$ increases, reducing house prices and increasing defaults. The consequent drop in NW_t^b initiates a sell-off of mortgages and capital, producing the amplification channels described in the previous experiment.

However, it has to be noted that in this case banks spread on mortgages, $spread_t^m$ increases much more and is the main driver of the increase in $spread_t^h$. In addition, since banks are the only agent able to intermediate capital, for them to keep providing loans to goods producers, $spread_t^k$ has to increase as well, as indicated by (33). As a result, in this experiment investments and q_t^k drop considerably more, so that the total drop in the aggregate net worth of financial intermediaries is twice as large as the one occurring with the housing risk shock.

3.5 Crisis Experiments with Government Asset Purchases

In Figure (4) and (5) I study the effects of a credit intervention similar to the purchase of mortgage-backed-securities that the Federal Reserve implemented at the height of the financial crisis. The two-sector framework of the model allows for a more realistic representation of the Fed's asset purchase program, that was mainly targeted at housing-related assets.

In particular I consider again the two exogenous shocks reported in figure (2) and (3), and compare the response of aggregate variables in presence of unconventional monetary policy. I assume that the efficiency cost of MBS purchases τ, is equal to 10 basis points and adjust the policy intensity Ψ_1 in each experiment, to obtain a central bank containing the increase in the MBS spread approximately by 50% (Ψ_{low}) or 95%(Ψ_{high}) on impact[17].

Figure (4) considers the effects of a housing risk shock when the central bank intervenes to purchase mortgages from the financial sector. Central intermediation reduces the downturn by dampening the rise in the spreads of the assets held by financial intermediaries; in particular the aggressive intervention is able to keep $spread_t^m$ almost unchanged. As a result, Q_t and consequently q_t^h drop less, causing a smaller drop in banks net worth. Importantly this process implies that banks demand for capital will also decrease by a smaller amount, causing a higher q_t^k that will have a positive effect on banks balance sheet as well. Therefore, central bank intervention in the housing market during a crisis has also positive spillover effects on business investments. However it has to be noticed that in this case MBS purchases have a limited impact on defaults and on $spread_t^h$, the reason being that the rise in defaults is mainly driven by the variance shock in this experiment. As a result the wealth of borrowers is not sheltered by the crisis as much as the one of bankers.

On the other hand, if we consider the consequences of this policy in the case of an MBS shock in figure (5), we see how it is more effective at reducing the increase in mortgage spreads and the drop in Q_t. The reason is that in this experiment, unlike the previous case, the rise in $spread_t^h$ is mainly due to the increase in $spread_t^m$. As a result, in this case the MBS purchases are more effective in preventing a drop in q_t^h causing a smaller drop in NW_t^{imp} and consequently in consumption and output.

4 Concluding Remarks

This paper presents a new framwork to study the interaction between mortgage defaults, house prices, and banks balance sheets in a macroeconomic model. All these elements have been important ingredients for the Great Recession. In particular the presence of constrained intermediaries and endogenous defaults can create negative feed-back mechanisms that can amplify the response of business investments, output and house prices during a financial crisis.

When these episodes occur, unconventional monetary policy in the form of central bank asset purchases can be particularly beneficial, especially when the downturn is generated by tighter constraints on bank funding for mortgage securities.

Several elements can be added to this model to improve its realism and quantitative performance. For example, the introduction of long-term mortgages might considerably strenghten the

[17] This implies a Ψ_1^{low} =.1 and Ψ_1^{high} =1 in the housing risk experiment, whereas a Ψ_1^{low} =.01 and Ψ_1^{high} =.025 in the MBS crisis experiment.

amplification mechanism, through the movements in the value of outstanding mortgages present on banks balance sheets.

In addition, as regards policy experiments, this model could be used to evaluate the impact of other types of interventions, aimed either at financial intermediaries, like for example a direct transfer to banks during a crisis (bank bailout), or to homeowners, like a default guarantee on mortgages. All these are interesting topics for future research.

5 Appendix

In this appendix I provide the details for the solution of the optimization problems of impatient households and bankers

5.1 Solution of the Impatient Household Problem

The original problem to be solved is

$$V_t(\omega_t) = \max_{\tilde{c}_t, h_t, \eta_t} \{U(c_t, x_t) + \beta E_t V_{t+1}(\omega_{t+1})\}$$

$$c_t + r_t x_t + h_t \left[q_t^h - Q_t(\eta_t)\eta_t\right] \leq \omega_t + w_t$$

$$\omega_{t+1} = \begin{cases} h_t \left[\left(q_{t+1}^h \xi_{t+1} + r_{t+1}\right) - \eta_t\right] & \text{if} \quad \xi_{t+1} \geq \bar{\xi}_{t+1}(\eta_t) = \eta_t / q_{t+1}^h \\ h_t r_{t+1} \quad \text{if } \xi_{t+1} < \bar{\xi}_{t+1}(\eta_t) = \eta_t / q_{t+1}^h \end{cases}$$

As in the main text, I begin by solving the static expenditures problem, that is

$$u(\tilde{c}_t, r_t) = \max \{\rho_t \log(c_t) + (1-\rho_t)\log(x_t)\} \text{ s.t.}$$

$$c_t + r_t x_t = \tilde{c}_t$$

The first order conditions imply
$$\frac{c_t}{r_t x_t} = \frac{\rho_t}{(1-\rho_t)}$$
and using this together with the constraint implies

$$c_t = \rho_t \tilde{c}_t \tag{63}$$

$$r_t x_t = (1-\rho_t)\tilde{c}_t \tag{64}$$

Then substituting these two equations in the objective function we obtain

$$\begin{aligned} u(\tilde{c}_t, r_t) &= \log(c_t) + \{\rho_t \log(\rho_t) + (1-\rho_t)[\log(1-\rho_t) - \log(r_t)]\} \\ &= \log(\tilde{c}) + \Theta(\rho_t, r_t) \end{aligned}$$

At this point we can rewrite the problem as

$$V_t(\omega_t) = \max_{\tilde{c}_t, h_t, \eta_t} \{u(\tilde{c}_t, r_t) + \beta E_t V_{t+1}(\omega_{t+1})\}$$

$$\tilde{c}_t + h_t \left[q_t^h - Q_t(\eta_t)\eta_t\right] \leq \omega_t + w_t$$

$$\omega_{t+1} = \begin{cases} h_t \left[\left(q^h_{t+1} \xi_{t+1} + r_{t+1} \right) - \eta_t \right] & \text{if} \quad \xi_{t+1} \geq \bar{\xi}_{t+1} \left(\eta_t \right) \\ h_t r_{t+1} \quad \text{if } \xi_{t+1} < \bar{\xi}_{t+1} \left(\eta_t \right) \end{cases}$$

Next, we introduce labor claims l_t and rewrite the value function in terms of effective wealth $a_t = \omega_{t+1} + l_t \left(w_t + p_t \right)$ where p_t represents the present discounted value of wages. In addition, if define savings as $s_t = h_t \left[q^h_t - Q_t \left(\eta_t \right) \eta_t \right] + l_t p_t$ we can write the problem as

$$V_t \left(a_t \right) = \max_{c_t, \varphi_t, \eta_t, s_t} \left\{ u \left(\tilde{c}_t, r_t \right) + \beta E_t V_{t+1} \left(a_{t+1} \right) \right\} \qquad \text{s.t.}$$

$$\tilde{c}_t + s_t = a_t$$

$$a_{t+1} = s_t R^s_{t+1} \left(\eta_t, \varphi_t \right)$$

$$R^s_t \left(\eta_{t-1}, \varphi_{t-1}, \xi_t \right) = \left[\varphi_{t-1} R^l_t + \left(1 - \varphi_{t-1} \right) R^h_t \left(\eta_{t-1}, \xi_t \right) \right]$$

$$R^l_t = \frac{w_t + p_t}{p_{t-1}}$$

$$R^h_t \left(\eta_{t-1}, \xi_t \right) = \max \left(R^{h,d}_t \left(\eta_{t-1} \right), R^{h,nd}_t \left(\eta_{t-1}, \xi_t \right) \right) = \frac{\max \left(r_t, \left(q^h_{t+1} \xi_{t+1} + r_{t+1} \right) - \eta_t \right)}{\left[q^h_t - Q_t \left(\eta_t \right) \eta_t \right]}$$

The FOC for φ_t, s_t and η_t are

$$\beta E_t \left\{ V'_{t+1} \left(R^l_{t+1} - R^h_t \left(\eta_t, \xi_{t+1} \right) \right) \right\} = 0$$

$$\beta E_t \left\{ \frac{V'_{t+1}}{u_{c,t}} R^s_{t+1} \left(\eta_t, \varphi_t, \xi_{t+1} \right) \right\} = 1$$

$$E_t \left\{ V'_{t+1} \left(a_{t+1} \right) R^h_{t+1} \left(\eta_t, \xi_{t+1} \right) \right\} \frac{d \left[Q_t \left(\eta_t \right) \eta_t \right]}{d\eta} = E_t \left\{ V'_{t+1} 1 (\xi_{t+1} > \frac{\eta_t}{q_{t+1}}) \right\}$$

Then we guess the policy function $\tilde{c}_t = \left(1 - \chi \right) a_t$ so that from the BC and evolution of wealth we obtain

$$s_t = \chi a_t$$

$$a_{t+1} = \chi a_t R^s_{t+1} \left(\eta_t, \varphi_t, \xi_{t+1} \right)$$

Also guess a value function form as $V_t \left(a_t \right) = A_t + B \log \left(a_t \right)$. From the envelope theorem this implies

$$V'_t \left(a_t \right) = u_{c,t}$$

$$\implies B = \frac{1}{\left(1 - \chi \right)}$$

Therefore, substituting into the FOC for s_t gives

$$\beta E_t \left\{ \frac{V'_{t+1}}{u_{c,t}} R^s_{t+1} \left(\eta_t, \varphi_t, \xi_{t+1} \right) \right\} = 1$$

$$\Longrightarrow \beta E_t \left\{ \frac{B\left(1-\chi\right)a_t}{\chi a_t R^s_{t+1}\left(\eta_t, \varphi_t, \xi_{t+1}\right)} R^s_{t+1}\left(\eta_t, \varphi_t, \xi_{t+1}\right) \right\} = 1$$

$$\Longrightarrow \chi = \beta$$

In addition, if we subtract the FOC for φ_t from the FOC for s_t we obtain

$$\beta E_t \left\{ \frac{V'_{t+1}}{u_{c,t}} \left[R^s_{t+1}\left(\eta_t, \varphi_t, \xi_{t+1}\right) - \varphi_t \left(R^l_{t+1} - R^h_t\left(\eta_t, \xi_{t+1}\right)\right)\right] \right\} = 1$$

$$\Longrightarrow \beta E_t \left\{ \frac{V'_{t+1}}{u_{c,t}} \left[R^h_t\left(\eta_t, \xi_{t+1}\right)\right] \right\} = 1$$

so that the FOC for η_t becomes

$$\frac{d\left[Q_t\left(\eta_t\right)\eta_t\right]}{d\eta} = E_t \left\{ \frac{1}{R^s_{t+1}\left(\eta_t, \varphi_t, \xi_{t+1}\right)} 1(\xi_{t+1} > \frac{\eta_t}{q_{t+1}}) \right\}$$

In addition, the FOC for φ reduces to

$$E_t \left\{ \frac{\left(R^l_{t+1} - R^h_t\left(\eta_t, \xi_{t+1}\right)\right)}{R^s_{t+1}\left(\eta_t, \varphi_t, \xi_{t+1}\right)} \right\} = 0$$

As a result, the system of equations solving the impatient agent problem is

$$\frac{d\left[Q_t\left(\eta_t\right)\eta_t\right]}{d\eta} = E_t \left\{ \frac{1}{R^s_{t+1}\left(\eta_t, \varphi_t\right)} 1(\xi_{t+1} > \frac{\eta_t}{q_{t+1}}) \right\}$$

$$E_t \left\{ \frac{\left(R^l_{t+1} - R^h_t\left(\eta_{t-1}, \xi_t\right)\right)}{R^s_{t+1}\left(\eta_t, \varphi_t\right)} \right\} = 0$$

$$\tilde{c}_t = \left(1-\beta\right)a_t$$

$$\tilde{c}_t + s_t = a_t$$

Finally, if we use the definition of s_t and φ_t, together with the policies for c_t and x_t from the static problem we obtain the equations from Proposition 1.

$$\frac{d\left[Q_t\left(\eta_t\right)\eta_t\right]}{d\eta} = E_t \left\{ \frac{1}{R^s_{t+1}\left(\eta_t, \varphi_t, \xi_{t+1}\right)} 1(\xi_{t+1} > \frac{\eta_t}{q_{t+1}}) \right\}$$

$$E_t \left\{ \frac{\left(R^l_{t+1} - R^h_t\left(\eta_{t-1}, \xi_t\right)\right)}{R^s_{t+1}\left(\eta_t, \varphi_t, \xi_{t+1}\right)} \right\} = 0$$

$$c_t = \rho_t\left(1-\beta\right)a_t$$

$$r_t x_t = \left(1-\rho_t\right)\left(1-\beta\right)a_t$$

$$l_t p_t = \varphi_t \beta a_t$$
$$h_t \left[q_t^h - Q_t(\eta_t, s_t)\eta_t \right] = (1-\varphi_t)\beta a_t$$
$$a_{t+1} = \chi_t a_t R_{t+1}^s(\eta_t, \varphi_t, \xi_{t+1})$$

5.2 Solution to the Banker's Problem

Therefore the banker's problem can be written as

$$V_t(n_t) = \max_{k_t, \{m_t(\eta_t)\}_{\eta_t}} E_t \hat{\beta}\Lambda_{t,t+1}\{(1-\sigma)n_{t+1} + \sigma V_{t+1}(n_{t+1})\} \text{ s.t.}$$

$$q_t^k z_t + \int Q_t(\eta_t) m_t(\eta_t) d\eta_t = n_t + d_t$$
$$n_{t+1} = q_t^k z_t R_{t+1}^k + \int \{Q_t(\eta_t) m_t(\eta_t) R_{t+1}^m(\eta_t)\} d\eta_t - R_{t+1} d_t \tag{65}$$
$$V_t(n_t) \geq \theta_t^m \left[\int Q_t(\eta_t) m_t(\eta_t) d\eta_t \right] + \theta_t^k q_t^k z_t$$

If we define μ_t as the multiplier on the incentive constraint, and guess a value function of the form $V_t(n_t) = \varphi_t n_t$. Then the FOCs for k_t, $m_t(\eta_t)$ and μ_t are

$$E_t \hat{\beta}\Lambda_{t,t+1}\left\{ [(1-\sigma) + \sigma\varphi_{t+1}] \left(R_{t+1}^k - R_{t+1} \right) \right\} = \mu_t \theta^k$$

$$E_t \hat{\beta}\Lambda_{t,t+1}\left\{ [(1-\sigma) + \sigma\varphi_{t+1}] \left(R_{t+1}^m(\eta_t) - R_{t+1} \right) \right\} = \mu_t \theta^m \quad \forall \eta_t$$
$$\mu_t \left\{ \varphi_t n_t - \left[\theta^m \left(\int Q(\eta_t, s_t) m_t(\eta_t) d\eta_t \right) + \theta^k q_t^k k_t \right] \right\} = 0$$

where the first two equations imply that

$$\frac{E_t \hat{\beta}\Lambda_{t,t+1}\left\{ [(1-\sigma) + \sigma\varphi_{t+1}] \left(R_{t+1}^k - R_{t+1} \right) \right\}}{\theta_t^k} = \frac{E_t \hat{\beta}\Lambda_{t,t+1}\left\{ [(1-\sigma) + \sigma\varphi_{t+1}] \left(R_{t+1}^m(\eta_t) - R_{t+1} \right) \right\}}{\theta_t^m} \quad \forall \eta_t$$

Plugging the guess into the value function we obtain

$$\begin{aligned} V_t(n_t) &= \varphi_t n_t \\ &= E_t \hat{\beta}\Lambda_{t,t+1} \left\{ \begin{array}{c} [1-\sigma+\sigma\varphi_{t+1}] \left[q_t^k z_t \left(R_{t+1}^k - R_{t+1} \right) + \int Q_t(\eta_t) m_t(\eta_t) \left(R_{t+1}^m(\eta_t) - R_{t+1} \right) d\eta_t \right] \\ +R_{t+1} n_t \end{array} \right\} \end{aligned}$$

and using the relationship between the spreads, this becomes

$$\varphi_t n_t = E_t \hat{\beta}\Lambda_{t,t+1} \left\{ [1-\sigma+\sigma\varphi_{t+1}] \left[\left(R_{t+1}^k - R_{t+1} \right) \left(q_t^k k_t + \frac{\theta^m}{\theta^k} \int Q(\eta_t, s_t) m_t(\eta_t) d\eta_t \right) \right] + R_{t+1} n_t \right\}$$

As a result, the marginal value of net-worth will have to satisfy

$$\varphi_t = E_t\hat{\beta}\Lambda_{t,t+1}\left\{\left[1-\sigma+\sigma\varphi_{t+1}\right]\left[\left(R^k_{t+1}-R_{t+1}\right)\phi_t + R_{t+1}\right]\right\}$$

where

$$\phi_t = \left[q^k_t k_t + \frac{\theta^m_t}{\theta^k_t}\int Q\left(\eta_t, s_t\right) m_t\left(\eta_t\right) d\eta_t\right]/n_t$$

In addition, if the constraint binds

$$\varphi_t n_t = \left\{\theta^m_t\left[\int Q\left(\eta_t, s_t\right) m_t\left(\eta_t\right) d\eta_t\right] + \theta^k_t q^k_t z_t\right\}$$

$$\Longrightarrow \varphi_t = \phi_t\theta^k$$

that implies

$$\phi_t\theta^k = E_t\hat{\beta}\Lambda_{t,t+1}\left\{\left[1-\sigma+\sigma\phi_{t+1}\theta^k\right]\left[\left(R^k_{t+1}-R_{t+1}\right)\phi_t + R_{t+1}\right]\right\}$$

and consequently a value for leverage

$$\phi_t = \frac{E_t\hat{\beta}\Lambda_{t,t+1}\left[1-\sigma+\sigma\phi_{t+1}\theta^k\right]R_{t+1}}{\theta^k_t - E_t\hat{\beta}\Lambda_{t,t+1}\left[1-\sigma+\sigma\phi_{t+1}\theta^k\right]\left(R^k_{t+1}-R_{t+1}\right)}$$

In addition, by rewriting the FOC for m_t we obtain the mortgage pricing equation

$$Q_t\left(\eta_t, s_t\right) = \frac{E_t\hat{\beta}\tilde{\Lambda}_{t,t+1}}{E_t\hat{\beta}\tilde{\Lambda}_{t,t+1}R_{t+1} + \theta^m\mu_t}\Psi^m_{t+1}\left(\eta_t, \xi_{t+1}\right)$$

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Table 1

Parameter	Value	Description
Patient Household		
$\hat{\beta}$	0.99	Discount rate patient HH
γ_n	.276	Inverse Frisch Elasticity
Impatient Household		
β	.9521	Discount rate impatient HH
$1-\rho$	.5	Housing Preference
Intermediate Good Firms		
α	.33	Capital Share in Production
μ	.5	Impatient Labor Share
δ_k	.025	Capital Depreciation Rate
Capital Producing Firms		
γ_i	.25	Elasticity of Price to Investments
Retail Firms		
ϵ	4,167	Elasticity of Substitution
ξ	.833	Probability of Fixed Price
Bankers		
θ^m_{ss}	0.053	Divertable MBS
θ^k_{ss}	0.1324	Divertable Capital
$\bar{\omega}$	.007	Transfer to Entering Bankers
σ	.96	Bankers survival probability
Mortgages		
$1-\gamma$	.2	Default Cost
λ	.17	Housing Risk Variance
Monetary Policy		
ρ_i	.8	Smoothing parameter
κ_π	1.5	Inflation Coefficient
κ_y	.50/4	Output Coefficient

Figure 1: Model Behavior

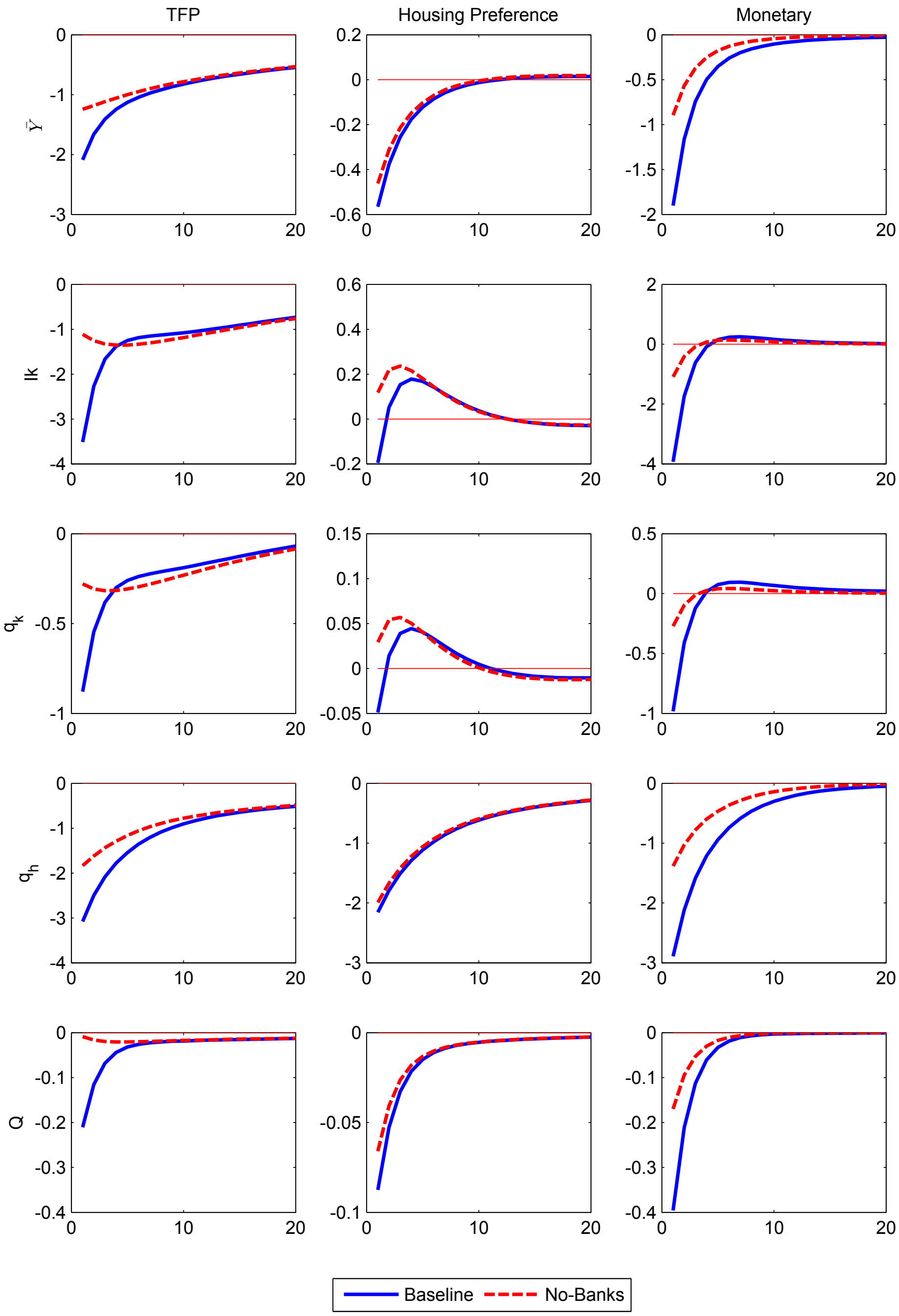

Figure 2: Housing Risk Shock

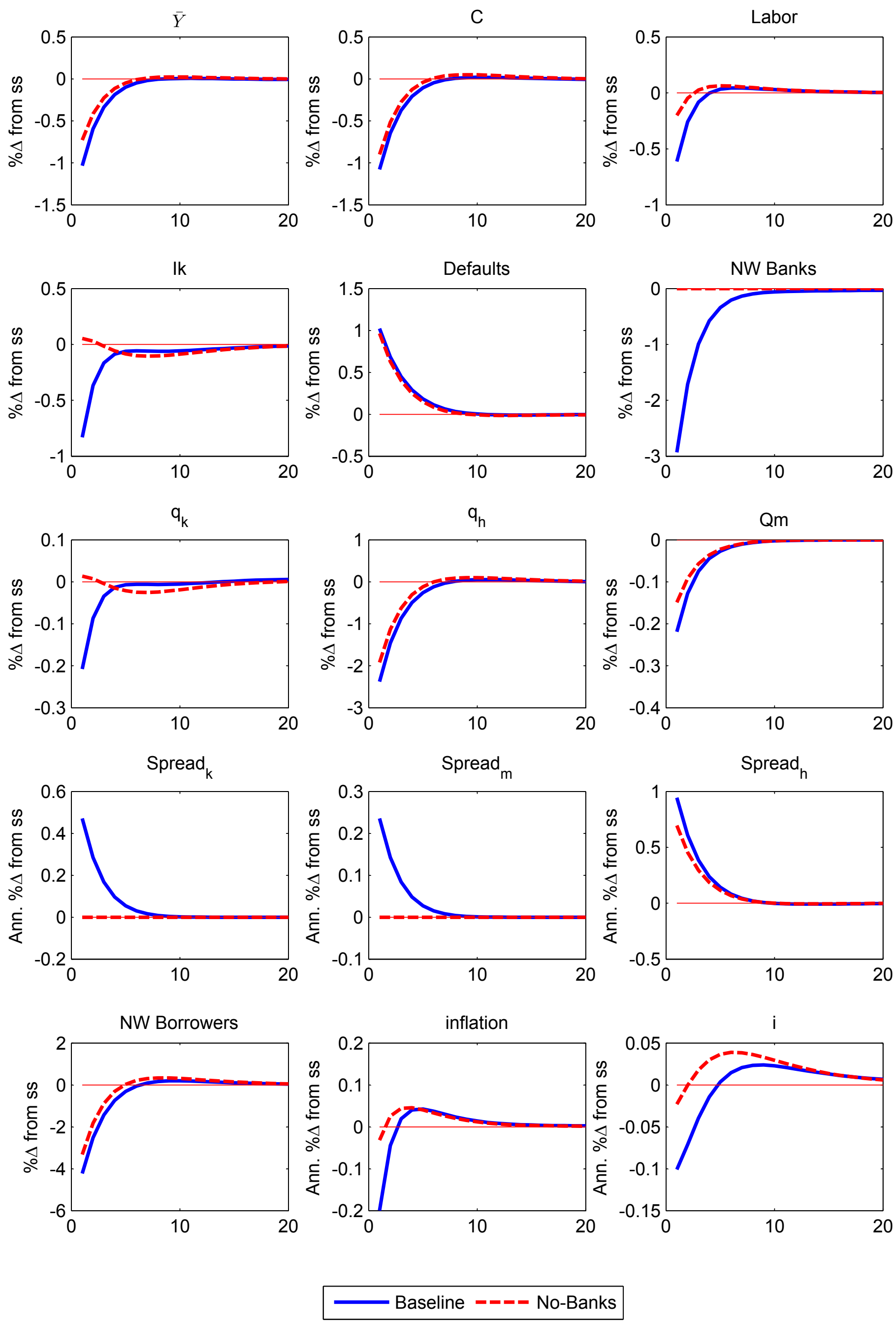

Figure 3: MBS Shock

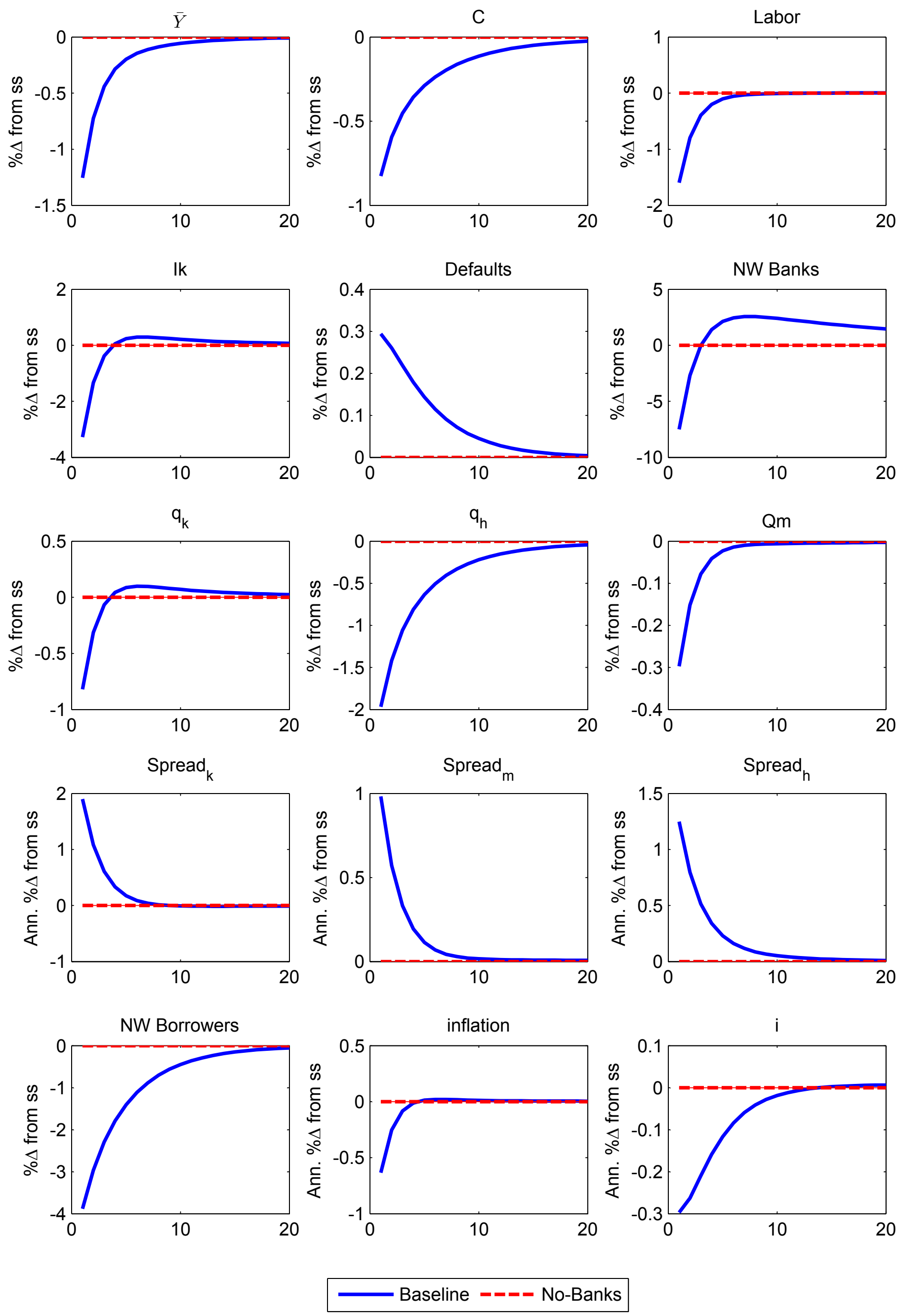

Figure 4: Housing Risk Shock with Government Policy

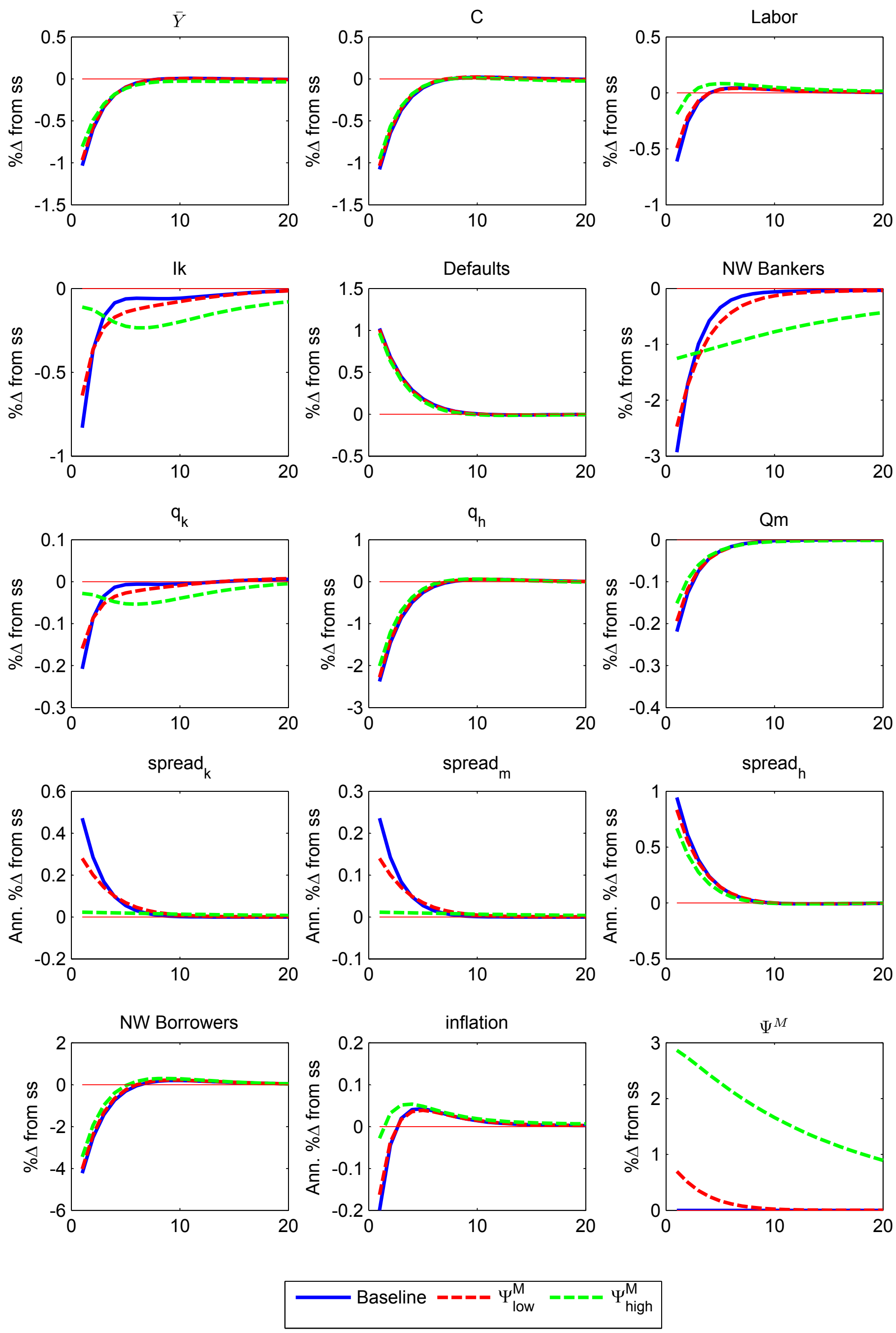

Figure 5: MBS Shock with Government Policy