Macro Risks and the Term Structure
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The paper constructs a novel term structure model.

Factors of this model are related to macro fundamentals.

Some of these fundamentals are obtained as a by-product of estimating aggregate supply and demand shocks.
The model

- I will ignore unconditional means throughout.

- The real (log) pricing kernel is

  \[ m_{t+1} - E_t m_{t+1} = \lambda_{pd}\omega^d_{p,t+1} + \lambda_{nd}\omega^d_{n,t+1} + \lambda_{ps}\omega^s_{p,t+1} + \lambda_{ns}\omega^s_{n,t+1}, \]

  where the shocks \( \omega \) are de-meaned Gamma variables with time-varying shape parameter, e.g.,

  \[ \omega^d_{n,t} \sim \Gamma(n^d_{t-1}, 1) - n^d_{t-1}, \quad n^d_t \text{ is AR}(1) \]

- The real short interest rate is

  \[ y_{1,t} = a_g E_t g_{t+1} + a_\pi E_t \pi_{t+1} + a_{pd}\hat{p}_t + a_{nd}\hat{n}_t + a_{ns}\hat{n}_s + z_t, \]

  \[ E_t g_{t+1} = \hat{b}_g g_t + \hat{b}_\pi \pi_t, \]

  \[ E_t \pi_{t+1} = \hat{c}_e \pi^e_t + \hat{c}_\pi \pi_t, \]

  and \( z_t \) is AR(1).
Estimation and fit

- The model is estimated with 1, 5, and 10-year nominal, and 2, 5, and 10-year real zero-coupon Treasuries

- Time-series plots of the actual and fitted 5-year nominal yield

- Unconditional correlations of the data and model for others

- The paper also offers a prequel to the model where bond yields and excess returns are regressed on the shape parameters
AD/AS shocks

- Quick and dirty “textbook” model of AD/AS

- Assume that current potential output is equal to the output from the previous period and inflation target is constant

\[
\pi_t = \pi_{t-1} + \beta g_t + \sigma_s u_t^s \\
g_t = \alpha \pi_t + \sigma_d u_t^d
\]

- The corresponding VAR is:

\[
\pi_t \propto \pi_{t-1} + \sigma_s u_t^s + \beta \sigma_d u_t^d \\
g_t \propto \alpha \sigma_s u_t^s + \sigma_d u_t^d
\]
Shock extraction

- What are these $\omega$’s and how are they related to AS/AD shocks, $u_t = (u_t^s, u_t^d)\top$?

- Consider a standard VAR in $f_t = (\pi_t, g_t)\top$:

  $$f_t = \sum_{i=1}^k \Phi_i f_{t-i} + \varepsilon_t, \quad E\varepsilon_t\varepsilon_t\top = \Sigma$$

- Then the disturbances and shocks have the usual relationship:

  $$A_0\varepsilon_t = u_t, \quad A_0^{-1} = \begin{pmatrix} -\sigma_{\pi s} & \sigma_{\pi d} \\ \sigma_{gs} & \sigma_{gd} \end{pmatrix}$$

- Identification?
  - In general, cannot estimate in the traditional setup with normal $\varepsilon_t$
  - The sign restriction restriction tells us which shock is AS
The approach

- Switch from normal to non-normal shocks

- Variance is not enough to characterize them, so need more equations – easier to identify

- The specification, $\varepsilon_t = A_0^{-1} u_t$ and

  $$
  u^s_t = \sigma^s_p \omega^s_{p,t} - \sigma^s_n \omega^s_{n,t} \\
  u^d_t = \sigma^d_p \omega^d_{p,t} - \sigma^d_n \omega^d_{n,t}
  $$

- Can express moments of $\varepsilon_t$ via $A_0$ and higher-order moments of $u_t$
  - Infinite amount of moments depend on $4 +$ infinite number of parameters
  - The authors pick 7 moments that depend on $4+2$ parameters
The Wold representation and non-iid shocks

- In order to estimate $\varepsilon_t$ the authors construct conditional means of $\pi$ and $g$ via a linear projection on a bunch of stuff.

- The preferred specification is used in their term structure model:

$$E_t g_{t+1} = \hat{b}_g g_t + \hat{b}_\pi \pi_t,$$
$$E_t \pi_{t+1} = \hat{c}_e \pi_t^e + \hat{c}_\pi \pi_t$$

- Appears as an implicit appeal to the Wold representation.

- The residuals need not coincide with shocks.

- Put differently, shock structure determines conditional expectations of variables.
Conclusion

- Potentially useful approach towards identification of shocks
- Seems like the approach has a potential for a lot of applications
- Not sure (yet) if term structure is the best application