# Macro Risks and the Term Structure

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# Question?

- The paper constructs a novel term structure model
- Factors of this model are related to macro fundamentals
- Some of these fundamentals are obtained as a by-product of estimating aggregate supply and demand shocks

# The model

- I will ignore unconditional means throughout

- The real (log) pricing kernel is

  $$m_{t+1} - E_t m_{t+1} = \lambda_{pd}\omega^d_{p,t+1} + \lambda_{nd}\omega^d_{n,t+1} + \lambda_{ps}\omega^s_{p,t+1} + \lambda_{ns}\omega^s_{n,t+1},$$

  where the shocks $\omega$ are de-meaned Gamma variables with time-varying shape parameter, e.g.,

  $$\omega^d_{n,t} \sim \Gamma(n^d_{t-1}, 1) - n^d_{t-1}, \quad n^d_t \text{ is AR(1)}$$

- The real short interest rate is

  $$\begin{aligned} y_{1,t} &= a_g E_t g_{t+1} + a_\pi E_t \pi_{t+1} + a_{pd}\hat{p}^d_t + a_{nd}\hat{n}^d_t + a_{ns}\hat{n}^s_t + z_t, \\ E_t g_{t+1} &= \hat{b}_g g_t + \hat{b}_\pi \pi_t, \\ E_t \pi_{t+1} &= \hat{c}_e \pi^e_t + \hat{c}_\pi \pi_t, \end{aligned}$$

  and $z_t$ is AR(1).

# Estimation and fit

- The model is estimated with 1, 5, and 10-year nominal, and 2, 5, and 10-year real zero-coupon Treasuries
- Time-series plots of the actual and fitted 5-year nominal yield
- Unconditional correlations of the data and model for others
- The paper also offers a prequel to the model where bond yields and excess returns are regressed on the shape parameters

# AD/AS shocks

- Quick and dirty "textbook" model of AD/AS
- Assume that current potential output is equal to the output from the previous period and inflation target is constant

$$\begin{aligned} \pi_t &= \pi_{t-1} + \beta g_t + \sigma_s u_t^s \\ g_t &= \alpha \pi_t + \sigma_d u_t^d \end{aligned}$$

- The corresponding VAR is:

$$\begin{aligned} \pi_t &\propto \pi_{t-1} + \sigma_s u_t^s + \beta \sigma_d u_t^d \\ g_t &\propto \alpha \sigma_s u_t^s + \sigma_d u_t^d \end{aligned}$$

# Shock extraction

- What are these $\omega$'s and how are they related to AS/AD shocks, $u_t = (u_t^s, u_t^d)^\top$?

- Consider a standard VAR in $f_t = (\pi_t, g_t)^\top$ :

$$f_t = \sum_{i=1}^{k} \Phi_i f_{t-i} + \varepsilon_t, \quad E\varepsilon_t \varepsilon_t^\top = \Sigma$$

- Then the disturbances and shocks have the usual relationship:

$$A_0 \varepsilon_t = u_t, \quad A_0^{-1} = \begin{pmatrix} -\sigma_{\pi s} & \sigma_{\pi d} \\ \sigma_{gs} & \sigma_{gd} \end{pmatrix}$$

- Identification?
  - In general, cannot estimate in the traditional setup with normal $\varepsilon_t$
  - The sign restriction restriction tells us which shock is AS

# The approach

- Switch from normal to non-normal shocks
- Variance is not enough to characterize them, so need more equations – easier to identify
- The specification, $\varepsilon_t = A_0^{-1} u_t$ and

$$\begin{aligned} u_t^s &= \sigma_p^s \omega_{p,t}^s - \sigma_n^s \omega_{n,t}^s \\ u_t^d &= \sigma_p^d \omega_{p,t}^d - \sigma_n^d \omega_{n,t}^d \end{aligned}$$

- Can express moments of $\varepsilon_t$ via $A_0$ and higher-order moments of $u_t$
  - Infinite amount of moments depend on 4 + infinite number of parameters
  - The authors pick 7 moments that depend on 4+2 parameters

# The Wold representation and non-iid shocks

- In order to estimate $\varepsilon_t$ the authors construct conditional means of $\pi$ and $g$ via a linear projection on a bunch of stuff

- The preferred specification is used in their term structure model:

$$\begin{aligned} E_t g_{t+1} &= \hat{b}_g g_t + \hat{b}_\pi \pi_t, \\ E_t \pi_{t+1} &= \hat{c}_e \pi_t^e + \hat{c}_\pi \pi_t \end{aligned}$$

- Appears as an implicit appeal to the Wold representation

- The residuals need not coincide with shocks

- Put differently, shock structure determines conditional expectations of variables

# Conclusion

- Potentially useful approach towards identification of shocks
- Seems like the approach has a potential for a lot of applications
- Not sure (yet) if term structure is the best application