A Macroeconomic Framework for Quantifying Systemic Risk

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Financial Crisis in the Model

Note: Capital constraint binds for $e < 0.435$
Non-linearity: State-dependent Impulse Response: -1% Shock

Investment

Sharpe ratio

Land price

He and Krishnamurthy (Chicago, Stanford)
Global Solution: Steady State Distribution

steady state distribution

scaled intermediary reputation $e$

He and Krishnamurthy (Chicago, Stanford)
Model-based stress test

- Pick initial condition to match 2007Q2 asset prices
- Probability of crisis over horizon:
  - 1 year: 0.32%
  - 2 year: 3.57%
  - 5 year: 17.30%
- Initial condition + rational forward looking agents = can’t see around corners!
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- Probability of crisis over horizon:
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- Initial condition + rational forward looking agents = can’t see around corners!
- Stress test:
  - Add $2 trillion of shadow banking liabilities, with close to 0% capital.
  - This information was not in 2007Q2 asset prices
- Probability of crisis over horizon:
  - 1 year: 6.73%
  - 2 year: 23.45%
  - 5 year: 57.95%
Nonlinear macro model of a financial crisis

- Recent work on financial intermediaries: He-Krishnamurthy, Brunnermeier-Sannikov, Rampini-Viswanathan, Adrian-Boyarchenko, Gertler-Kiyotaki
- Our approach: occasionally binding constraint; global solution method (similar to Brunnermeier-Sannikov, Adrian-Boyarchenko)
Outline of Presentation

1. Nonlinear macro model of a financial crisis
   - Recent work on financial intermediaries: He-Krishnamurthy, Brunnermeier-Sannikov, Rampini-Viswanathan, Adrian-Boyarchenko, Gertler-Kiyotaki
   - Our approach: occasionally binding constraint; global solution method (similar to Brunnermeier-Sannikov, Adrian-Boyarchenko)

2. Calibration and results

3. Quantify systemic risk and stress test
Model

- Two classes of agents: households and bankers
  - Households:
    \[ E \left[ \int_0^\infty e^{-\rho t} \frac{1}{1-\gamma} C_t^{1-\gamma} dt \right] , \quad C_t = (c_t^y)^{1-\phi} (c_t^h)^{\phi} \]

- Two types of capital: productive capital \( K_t \) and housing capital \( H \).
  - Fixed supply of housing \( H \equiv 1 \)
  - Price of capital \( q_t \) and price of housing \( P_t \) determined in equilibrium
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- Production \(Y = AK_t\), with \(A\) being constant

- Fundamental shocks: stochastic capital quality shock \(dZ_t\). TFP shocks
  \[\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t\]
Model

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  \[ \frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t \]

- Investment/Capital \( i_t \), quadratic adjustment cost
  \[ \Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2} (i_t - \delta)^2 K_t \]
  \[ \max_{i_t} q_t i_t K_t - \Phi(i_t, K_t) \Rightarrow i_t = \delta + \frac{q_t - 1}{\kappa} \]
Aggregate Balance Sheet

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$

Equity $E_t$

Housing $P_t H$

Debt $W_t - E_t$

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$

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Financial Wealth

$W_t = q_tK_t + P_tH$

$(1 - \lambda)W_t$

$\lambda W_t = "Liquid balances"$

benchmark capital structure
Equity Dynamics in GE

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$
-10%

Housing $P_t H$

Equity $E_t$
-10% $\times$ Lev

Debt $W_t - E_t$

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$

$(1 - \lambda) W_t$

$\lambda W_t = \text{"Liquid balances"}$
Equity Constraint

Aggregate intermediary equity constraint $E_t$

\[ \frac{dE_t}{E_t} = m \times \text{ROE, ROE is endogenous} \]

Financial Wealth

\[ W_t = q_t K_t + P_t H \]

Household Sector

\[ \lambda W_t = "\text{Liquid balances}" \]

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Equity constraint: $\epsilon_t$

- Bank can raise equity up to $\epsilon_t$ at zero cost.
- Cost of raising equity more than $\epsilon_t$ is infinite.
- $\epsilon_t$ linked to intermediary performance (constant $m$)

$$\frac{d\epsilon_t}{\epsilon_t} = md\tilde{R}_t.$$
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$$\frac{d\epsilon_t}{\epsilon_t} = md\tilde{R}_t.$$

- $\epsilon_t$ as “reputation” of the banker
- $\epsilon_t$ as banker’s “net worth” fluctuating with past returns
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\]

- $\epsilon_t$ as "reputation" of the banker
- $\epsilon_t$ as banker's "net worth" fluctuating with past returns

- Aggregate dynamics of $\mathcal{E}_t = \int \epsilon_t$
## Calibration: Baseline Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Choice</th>
<th>Targets (Unconditional)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Intermediation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>Performance sensitivity</td>
<td>2</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Debt ratio</td>
<td>0.67</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Banker exit rate</td>
<td>13%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Entry trigger</td>
<td>6.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Entry cost</td>
<td>2.43</td>
</tr>
<tr>
<td><strong>Panel B: Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Capital quality shock</td>
<td>3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>10%</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Adjustment cost</td>
<td>3</td>
</tr>
<tr>
<td>$A$</td>
<td>Productivity</td>
<td>0.133</td>
</tr>
<tr>
<td><strong>Panel C: Others</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Time discount rate</td>
<td>2%</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1/EIS</td>
<td>0.15</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Housing share</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Results(1): State variable is $e_t = \mathcal{E}_t / K_t$

- Capital constraint binds for $e < 0.435$
Capital constraint binds for $e < 0.435$

Without the possibility of the capital constraint, all of these lines would be flat. Model dynamics would be i.i.d., with vol=3%
State-dependent Impulse Response: -1% Shock ($= \sigma dZ_t$)

- **Investment**
  - Plot shows the response of investment to a -1% shock over 8 quarters.
  - Two lines indicate different states: crisis and normal.

- **Sharpe ratio**
  - Plot shows the response of Sharpe ratio to a -1% shock over 8 quarters.
  - Two lines indicate different states: crisis and normal.

- **Land price**
  - Plot shows the response of land price to a -1% shock over 8 quarters.
  - Two lines indicate different states: crisis and normal.

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Matching the 2007-2009 Crisis

Pick initial condition for intermediary state variable \((e)\) to match asset prices in 2007Q2

- Asset price = Gilchrist-Zakrajsek credit spread
- Note: this spread (as with most spreads) was low in 2007Q2
- Data from 1975 to 2010; compute histogram of spread variable
- Match percentile of spread in the data to the same percentile in model implied distribution for risk premium

Answer: In 2007Q2, \(e = 1.27\).
Set initial condition of \( e = 1.27 \) in 2007Q2.

Then choose \((Z_{t+1} - Z_t)\) shocks to match realized intermediary equity series.

<table>
<thead>
<tr>
<th></th>
<th>07QIII</th>
<th>07QIV</th>
<th>08QI</th>
<th>08QII</th>
<th>08QIII</th>
<th>08QIV</th>
<th>09QI</th>
<th>09QII</th>
<th>09QIII</th>
<th>09QIV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.5%</td>
<td>-4.2</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-0.7</td>
<td>-1.6</td>
<td>-1.8</td>
<td>-1.8</td>
<td>-0.9</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

- Total -15.5%. Capital constraint binds after 07Q4—systemic risk state
- In the model (data), land price falls by 50% (55%)
- In the model (data), investment falls by 23% (25%)
What is the likelihood of the constraint binding ("systemic crisis") assuming $e = 1.27$ currently:

- 0.32% in next 1 years
- 3.57% in next 2 years
- 17.30% in next 5 years
What is the likelihood of the constraint binding ("systemic crisis") assuming \( e = 1.27 \) currently:

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Lessons:

- Initial condition calibrated to asset prices + rational forward looking agents = can’t see around corners!
- Even with a highly non-linear model
- Could abandon RE. Credit growth unusually high, crash likely, even though asset markets don’t see it
Stress testing: Leverage test

- Financial sector aggregate leverage fixed at 3 in model
  - We measure across commercial banks, broker/dealers, hedge funds in 2007:
    - Assets = $15,703 billion; Liabilities = $10,545 billion

- Suppose a stress test uncovered leverage:
  - ABCP (SIVs): $1,189 billion; Liabilities $1,189 billion
  - Repo (MMFs and Sec Lenders): $1,020 billion; Liabilities $1,000 billion (assumed 2% haircut)

- Leverage is “hidden” in sense that agents take equilibrium functions as given based on leverage=3
  - 1 year: 6.73%
  - 2 year: 23.45%
  - 5 year: 57.95%
Stress testing plus a model

- In current practice, work goes into estimating exposure (i.e. true leverage in example)

With a model:

1. Stress may trigger macro and asset price feedbacks, second round,... third round...
   - Model computes the fixed point

2. Model translates stress event into a probability of a systemic crisis

3. Model can help calibrate corrective actions (i.e. capital raising) based on target:
   - How much capital is needed to ensure probability of crisis < X%?
   - “Macro-VAR"
Stress testing

Key step: Need to map from stress scenario into underlying shock, \( dZ_t \).

- Say stress scenario \( \Rightarrow -30\% \) Return on equity
- Naive partial eqbm: leverage of 3, \( \sigma(Z_{t+0.25} - Z_t) = -30/3 = -10\% \).
- Feed in \(-10\%\) shock into the model over one quarter.
- Result: Beginning at \( e = 1.27 \) in 2007Q2, economy is immediately moved into crisis region, \( e < 0.435 \)

our model helps in figuring out the right shock \( dZ_t \)

In US stress tests, scenario was over 6 quarters. Feed in shocks quarter-by-quarter, over 6 quarters:

<table>
<thead>
<tr>
<th>Return on Equity</th>
<th>6 QTR Shocks</th>
<th>Prob(Crisis within next 2 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2%</td>
<td>-1.16%</td>
<td>5.25 %</td>
</tr>
<tr>
<td>-5</td>
<td>-2.53%</td>
<td>8.90</td>
</tr>
<tr>
<td>-10</td>
<td>-4.69%</td>
<td>22.88</td>
</tr>
<tr>
<td>-15</td>
<td>-6.71%</td>
<td>48.90</td>
</tr>
<tr>
<td>-30</td>
<td>-8.72%</td>
<td>100.00</td>
</tr>
</tbody>
</table>
Map “stress test” into a shock to $e$. 
Conclusion

- We develop a fully stochastic model of a systemic crisis, with an equity capital constraint on the intermediary sector
- Is able to replicate 2007/2008 period with only intermediary capital shocks
- The model quantitatively matches the differential comovements in distress and non-distress periods
- Offers a way of mapping macro-stress tests into probability of systemic states.
Other crises

Panel A: Savings and Loan Crisis

Panel B: 1998 Hedge Fund Crisis

Panel C: 2002 Corporate Bond Market Crisis
Panel A: Distress Periods

Panel B: Non Distress Periods