Macro Risks and the Term Structure

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Existing Literature

- **Affine models**
  - Latent variables
  - Macro variables (Ang and Piazzesi; 2003)
  - Less focus on economics; “Fits” yield curve data

- **DSGE models**
  - Optimizing agents
  - Complex equations
  - Lots of economics; tightly parameterized

- Many models are still (conditionally) Gaussian
Introduction (2/2)
Contributions of this paper

Macroeconomics
- New device to model real activity and inflation
- Evidence for non-Gaussian shocks with time-varying distributions

Asset Pricing
- Macro variables drive 70 percent of variation in yields
- Non-Gaussian macro risk factors drive substantial variation in risk premiums
- Novel TS model in the affine class (work in progress)
Roadmap for Presentation

- 2 Key modeling assumptions
- 3 Methodological steps
- Some reduced-form asset pricing results
- Plan for the formal TS model
Key Modeling Assumption (1/2)

“Device” for Macroeconomic Shocks

- Consider shocks to real growth and inflation

\[
g_t = E_{t-1}[g_t] + u_t^g, \\
\pi_t = E_{t-1}[\pi_t] + u_t^\pi,
\]

- Model shocks as functions of supply/demand shocks

\[
u_t^\pi = -\sigma_{\pi s} u_t^s + \sigma_{\pi d} u_t^d, \\
u_t^g = \sigma_{gs} u_t^s + \sigma_{gd} u_t^d, \\
Cov(u_t^d, u_t^s) = 0, Var(u_t^d) = Var(u_t^s) = 1.
\]
Key Modeling Assumption (1/2)
“Device” for Macroeconomic Shocks

➢ If supply/demand shocks are heteroskedastic

\[ Cov_{t-1}[u_t^g, u_t^\pi] = -\sigma_{\pi_s} \sigma_{gs} Var_{t-1} u_t^s + \sigma_{\pi_d} \sigma_{gd} Var_{t-1} u_t^d. \]

– Demand shock environment

\[ Cov_{t-1}[u_t^g, u_t^\pi] > 0 \]

nominal assets hedge real risk

– Supply shock environment

\[ Cov_{t-1}[u_t^g, u_t^\pi] < 0 \]

nominal bonds exacerbate real risk
Key Modeling Assumption (2/2)
Non-Gaussian Distributions for Shocks

- Demand (and supply) shocks are “BEGE”-distributed
  \[ u_t^d = \sigma_p^d \omega_{p,t}^d - \sigma_n^d \omega_{n,t}^d \]

- \( \omega_{p,t}^d \) and \( \omega_{n,t}^d \) follow gamma distributions
  \[ \omega_{p,t}^d \sim \Gamma(p_{t-1}^d, 1) \]
  \[ \omega_{n,t}^d \sim \Gamma(n_{t-1}^d, 1) \]

- \( \Gamma(p_{t-1}^d, 1) \) denotes a demeaned gamma distribution with
  time-varying shape parameter \( p_{t-1}^d \) and unit size parameter
Digression on the Gamma Distribution

\begin{align*}
-\omega_{n,t} &= n_t \\
\text{Variance}_t &= \frac{-2}{\sqrt{n_t}} \\
\text{Skewness}_t &= \frac{6}{n_t} \\
\omega_{p,t} &= p_t \\
\text{Excess Kurtosis}_t &= \frac{2}{\sqrt{p_t}} \\
\end{align*}
1) “Large” and equal $p_t$ and $n_t$: Gaussian limit
BEGE Distributions

2) “Small” but still equal $p_t$ and $n_t$: excess kurtosis
3) Relatively large $n_t$: negative skewness: “Bad Environment”
4) Relatively large $p_t$: positive skewness “Good Environment”
The BEGE distribution has some advantages...

- Flexible
- Realistic
  - Fits some financial and macro economic data well
- Tractable
  - Moments are affine in $p_t$ and $n_t$
  - Fits in the affine class of asset pricing models
BEGE Distributions

... and some disadvantages
3 Methodological Steps to Assemble a set of Macro Factors

- We assemble six factors for use in term structure modeling that we identify using *(only)* macro data

\[
\begin{align*}
E_t [g_{t+1}] &= \text{Expected growth} \\
E_t [\pi_{t+1}] &= \text{Expected inflation} \\
p^d_t &= \text{“Good” (positive skew) demand variance} \\
p^s_t &= \text{“Good” (positive skew) supply variance} \\
n^d_t &= \text{“Bad” (negative skew) demand variance} \\
n^s_t &= \text{“Bad” (negative skew) supply variance}
\end{align*}
\]
3 Methodological Steps

1. Identify conditional means versus shocks in growth and inflation data \( \rightarrow E_t[g_{t+1}], E_t[\pi_{t+1}] \)

2. Recover supply and demand shocks

3. Estimate BEGE processes \( \rightarrow [p_t^d, n_t^d, p_t^s, n_t^d] \)
Methodological Steps (1/3)
Measuring Expected Growth and Inflation

➢ Use simple predictive regressions
  
  — LHS: quarterly U.S. GDP growth and CPI inflation from 1970

  — RHS: lagged LHS, survey-based (SPF) forecasts
    • Try many possible combinations of RHS variables and lag structures
    • Use BIC to choose
Results

\[
g_{t+1} = 0.0064^{**} + 0.3401^{***} g_t + -0.1721^{**} \pi_t \\
\pi_{t+1} = -0.0002 + 0.9055^{***} \pi_{t,t+1} + 0.2355^{**} \pi_t
\]

- GDP growth expectations consistent with VAR(1)
- Inflation expectations load on survey measures
Methodological Steps (1/3)
Measuring Expected Growth and Inflation

**Expected GDP Growth**

**Expected Inflation**
Fundamental identification problem with Gaussian DGP

\[ u_i^\pi = -\sigma_{\pi s} u_i^s + \sigma_{\pi d} u_i^d, \]
\[ u_i^q = \sigma_{gs} u_i^s + \sigma_{gd} u_i^d, \]
\[ Cov(u_i^d, u_i^s) = 0, Var(u_i^d) = Var(u_i^s) = 1. \]

The BEGE structure is consistent with identification using non-Gaussian features of data

- Use 2\textsuperscript{nd} 3\textsuperscript{rd} 4\textsuperscript{th} order moments to identify “\( \sigma \)”s
- Then “invert” supply and demand shocks
Methodological Steps (2/3)
Recover Supply/Demand Shocks
Methodological Steps (2/3)
Recover Supply/Demand Shocks

Supply shock $u_t^s$
Methodological Steps (3/3)
Filter BEGE Factors

➢ Assume autoregressive, square root-like processes for the four BEGE factors

\[
\begin{align*}
    p_t^d &= \bar{p}^d (1 - \phi_p^d) + \phi_p^d p_{t-1}^d + \sigma_p^d \omega_{p,t}^d, \\
    p_t^s &= \bar{p}^s (1 - \phi_p^s) + \phi_p^s p_{t-1}^s + \sigma_p^s \omega_{p,t}^s, \\
    n_t^d &= \bar{n}^d (1 - \phi_n^d) + \phi_n^d n_{t-1}^d + \sigma_n^d \omega_{n,t}^d, \\
    n_t^s &= \bar{s}^d (1 - \phi_n^s) + \phi_n^s p_{t-1}^s + \sigma_n^s \omega_{n,t}^s.
\end{align*}
\]

➢ Use Bates filter to estimate parameters and filter
   – accommodates non-Gaussian processes
Methodological Steps (3/3)

Filter BEGE Factors

Demand variances

- Good demand variance
- Bad demand variance

Year:
- 1970
- 1975
- 1980
- 1985
- 1990
- 1995
- 2000
- 2005
- 2010

Variance:
- 0
- 1
- 2
- 3
- 4
- 5
Methodological Steps (3/3)
Filter BEGE Factors

Supply variances

- Blue line: Good supply variance
- Red line: Bad supply variance

Year

Variance

0 0.5 1 1.5 2

Methodological Steps (3/3)
Filter BEGE Factors

Total conditional variance

- Blue dots: Demand shock
- Red line: Supply shock

Year

Variance

We can recover the implied correlation between real growth and inflation.
Macro Risks and the Term Structure: Reduced-from evidence

- So far, we have (purposefully) not looked at asset price data
- Do the macro factors show signs of life for helping to explain yields and risk premiums?
Macro Risks and the Term Structure: Reduced-from evidence 1

<table>
<thead>
<tr>
<th>1 Quarter nominal interest rate</th>
<th>Constant</th>
<th>$E_t g_{t+1}$</th>
<th>$E_t \pi_{t+1}$</th>
<th>$p_t^d$</th>
<th>$n_t^d$</th>
<th>$n_t^s$</th>
<th>$R^2$</th>
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<tbody>
<tr>
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<td>-0.0022</td>
<td>0.4944***</td>
<td>1.5208***</td>
<td>-0.0001</td>
<td>0.0193</td>
<td>-0.0008*</td>
<td>0.7074</td>
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<td>(0.0027)</td>
<td>(0.1849)</td>
<td>(0.2205)</td>
<td>(0.0001)</td>
<td>(0.0149)</td>
<td>(0.0004)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>1 Year nominal interest rate</th>
<th>Constant</th>
<th>$E_t g_{t+1}$</th>
<th>$E_t \pi_{t+1}$</th>
<th>$p_t^d$</th>
<th>$n_t^d$</th>
<th>$n_t^s$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.0022</td>
<td>0.5645***</td>
<td>1.6767***</td>
<td>-0.0001</td>
<td>0.0206</td>
<td>-0.0006</td>
<td>0.7174</td>
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<td>(0.0028)</td>
<td>(0.1936)</td>
<td>(0.2393)</td>
<td>(0.0001)</td>
<td>(0.0178)</td>
<td>(0.0006)</td>
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</table>

<table>
<thead>
<tr>
<th>10 Year nominal interest rate</th>
<th>Constant</th>
<th>$E_t g_{t+1}$</th>
<th>$E_t \pi_{t+1}$</th>
<th>$p_t^d$</th>
<th>$n_t^d$</th>
<th>$n_t^s$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0036*</td>
<td>0.5011***</td>
<td>1.5623***</td>
<td>-0.0003**</td>
<td>0.0261*</td>
<td>0.0001</td>
<td>0.7284</td>
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<td>(0.0022)</td>
<td>(0.1583)</td>
<td>(0.2100)</td>
<td>(0.0001)</td>
<td>(0.0143)</td>
<td>(0.0004)</td>
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</tr>
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</table>
Macro Risks and the Term Structure: Reduced-form evidence 2

1-year holding period excess returns, predictability

- Adjusted $R^2$ without $p_t^d$, $n_t^d$, and $n_t^s$
- Adjusted $R^2$ with $p_t^d$, $n_t^d$, and $n_t^s$
### Macro Risks and the Term Structure: Reduced-form evidence 2

#### 1-qtr holding period excess returns, predictability

<table>
<thead>
<tr>
<th></th>
<th>Excess return on 2 year bond</th>
<th>Excess return on 10 year bond</th>
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</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.0003</td>
<td>0.0228***</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0081)</td>
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<tr>
<td>$E_t g_{t+1}$</td>
<td>0.1810**</td>
<td>0.1797</td>
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<td></td>
<td>(0.0846)</td>
<td>(0.8231)</td>
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<tr>
<td>$E_t \pi_{t+1}$</td>
<td>0.0967</td>
<td>-0.8669</td>
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<tr>
<td></td>
<td>(0.1252)</td>
<td>(0.5974)</td>
</tr>
<tr>
<td>$p_t^d$</td>
<td>-0.0002***</td>
<td>-0.0012***</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$n_t^d$</td>
<td>-0.0150</td>
<td>-0.2448***</td>
</tr>
<tr>
<td></td>
<td>(0.0103)</td>
<td>(0.0719)</td>
</tr>
<tr>
<td>$n_t^s$</td>
<td>0.0006*</td>
<td>0.0055***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0014)</td>
</tr>
</tbody>
</table>
Formal Term Structure Model

Aspirations

Specify real short rate as function of macro factors

\[ y_{1,t} = a_0 + a_g E_t [g_{t+1}] + a_\pi E_t [\pi_{t+1}] + a_{pd} p^d_t + a_{nd} n^d_t + a_{ns} n^s_t + z_t \]

- \( z_t \) is a latent factor (Gaussian)

Specify an “empirical” pricing kernel

\[ (m_{t+1} - E_t [m_{t+1}]) = \lambda_{pd} \omega^d_{p,t+1} + \lambda_{nd} \omega^d_{n,t+1} + \lambda_{ps} \omega^s_{p,t+1} + \lambda_z \varepsilon^z_{t+1} \]

- Constant prices of risk \( \rightarrow \) model is in the affine class
Can the model explain using macro factors
– yield dynamics?
– apparent non-Gaussiananities in option prices?
Conclusions

Supply and demand shocks
- Relative variances change considerably over time
- Evidence of non-Gaussianity

Asset prices
- Macro factors drive significant portion of variation in yields
- Non-Gaussian macro risks are important drivers of risk premiums for nominal bonds