# Macro Risks and the Term Structure

Geert Bekaert[1] Eric Engstrom[2] Andrey Ermolov[3]

2015

The views expressed herein do not necessarily reflect those of the Federal Reserve System, its Board of Governors, or staff.

[1]Columbia University and NBER

[2]Federal Reserve Board of Governors

[2]Gabelli School of Business, Fordham University

# Introduction (1/2)
# Existing Literature

- Affine models
  - Latent variables
  - Macro variables (Ang and Piazzesi; 2003)
  - Less focus on economics; “Fits” yield curve data

- DSGE models
  - Optimizing agents
  - Complex equations
  - Lots of economics; tightly parameterized

- Many models are still (conditionally) Gaussian

# Introduction (2/2)
# Contributions of this paper

- Macroeconomics
  - New device to model real activity and inflation
  - Evidence for non-Gaussian shocks with time-varying distributions

- Asset Pricing
  - Macro variables drive 70 percent of variation in yields
  - Non-Gaussian macro risk factors drive substantial variation in risk premiums
  - Novel TS model in the affine class (work in progress)

# Roadmap for Presentation

- 2 Key modeling assumptions
- 3 Methodological steps
- Some reduced-form asset pricing results
- Plan for the formal TS model

# Key Modeling Assumption (1/2) "Device" for Macroeconomic Shocks

- Consider shocks to real growth and inflation

$$g_t = E_{t-1}[g_t] + u_t^g,$$

$$\pi_t = E_{t-1}[\pi_t] + u_t^\pi,$$

- Model shocks as functions of supply/demand shocks

$$u_t^\pi = -\sigma_{\pi s} u_t^s + \sigma_{\pi d} u_t^d,$$

$$u_t^g = \sigma_{gs} u_t^s + \sigma_{gd} u_t^d,$$

$$Cov(u_t^d, u_t^s) = 0, Var(u_t^d) = Var(u_t^s) = 1.$$

# Key Modeling Assumption (1/2)
# "Device" for Macroeconomic Shocks

- If supply/demand shocks are heteroskedastic

$$Cov_{t-1}[u_t^g, u_t^\pi] = -\sigma_{\pi s}\sigma_{gs}Var_{t-1}u_t^s + \sigma_{\pi d}\sigma_{gd}Var_{t-1}u_t^d.$$

– Demand shock environment

$Cov_{t-1}[u_t^g, u_t^\pi] > 0$

nominal assets hedge real risk

– Supply shock environment

$Cov_{t-1}[u_t^g, u_t^\pi] < 0$

nominal bonds exacerbate real risk

# Key Modeling Assumption (2/2)
# Non-Gaussian Distributions for Shocks

- Demand (and supply) shocks are "BEGE"-distributed

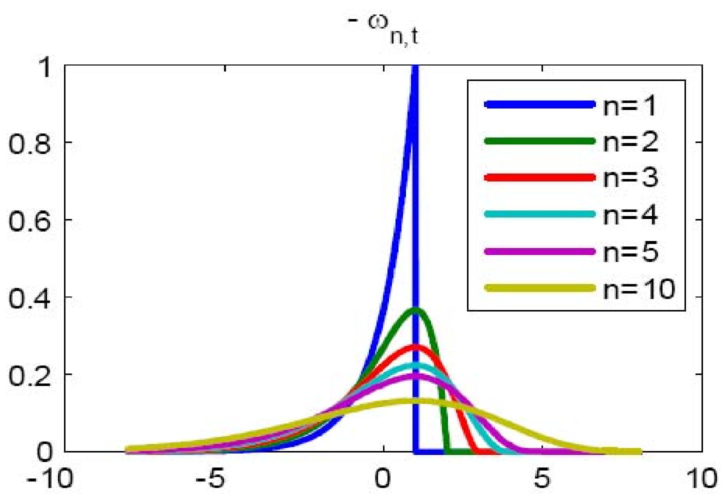
$$u_t^d = \sigma_p^d \omega_{p,t}^d - \sigma_n^d \omega_{n,t}^d$$

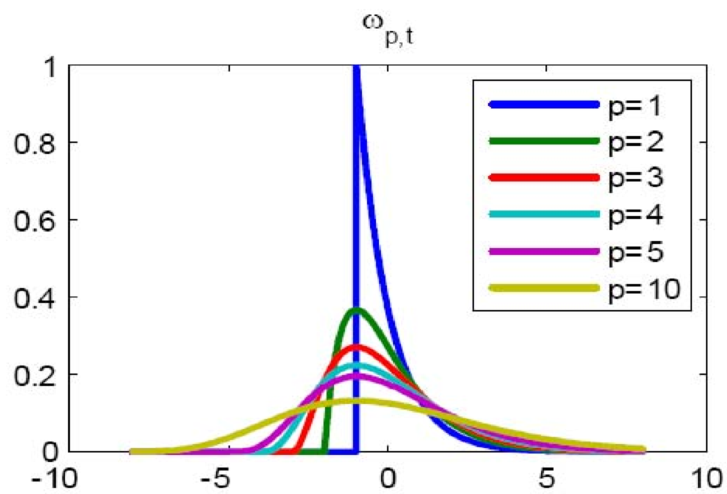
- $\omega_{p,t}^d$ and $\omega_{n,t}^d$ follow gamma distributions

$$\omega_{p,t}^d \sim \Gamma(p_{t-1}^d, 1)$$
$$\omega_{n,t}^d \sim \Gamma(n_{t-1}^d, 1)$$

– $\Gamma(p_{t-1}^d, 1)$ denotes a demeaned gamma distribution with time-varying shape parameter $p_{t-1}^d$ and unit size parameter

# Digression on the Gamma Distribution

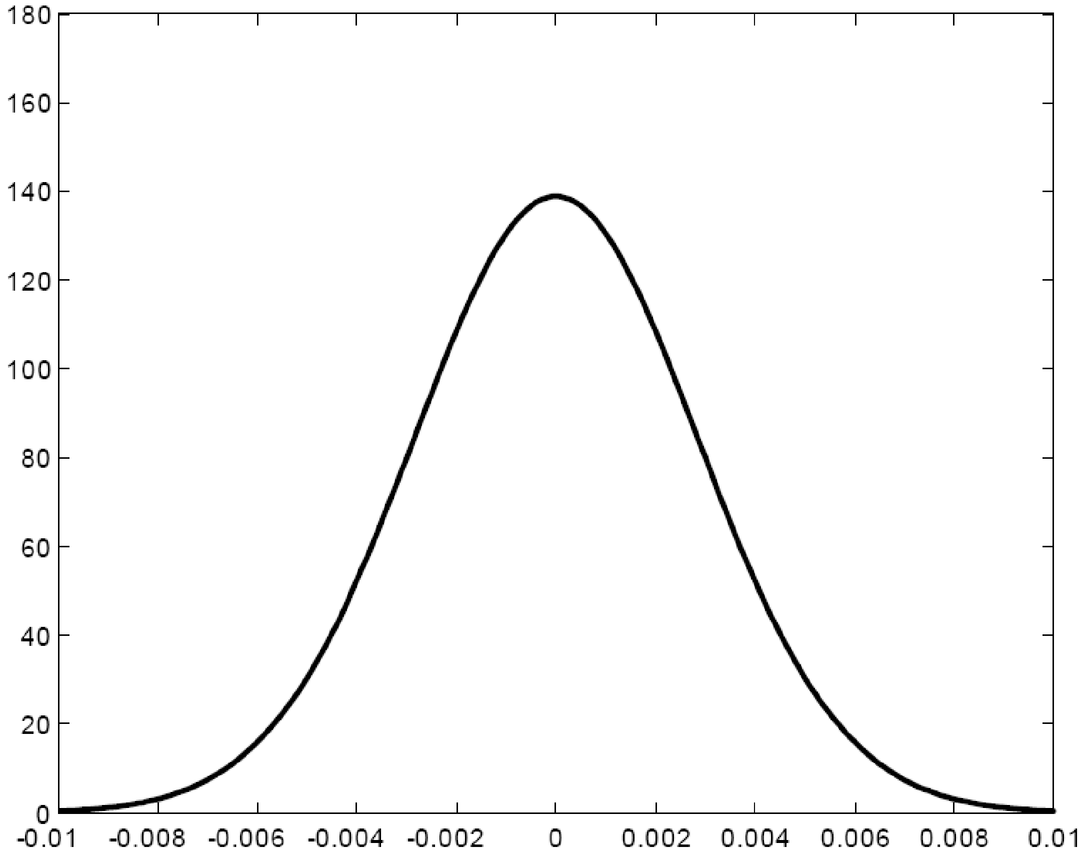



| $-\omega_{n,t}$ | | $\omega_{p,t}$ |
|---|---|---|
| $n_t$ | Variance$_t$ | $p_t$ |
| $-2/\sqrt{n_t}$ | Skewness$_t$ | $2/\sqrt{p_t}$ |
| $6/n_t$ | Excess Kurtosis$_t$ | $6/p_t$ |

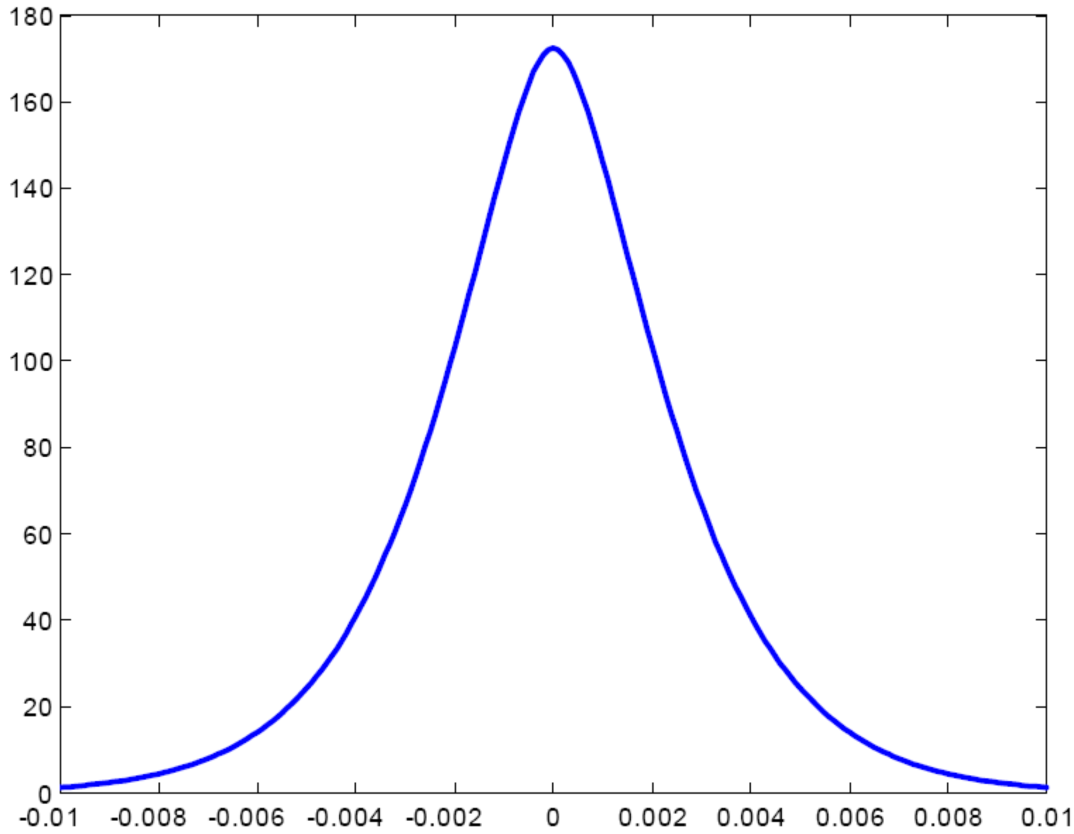
# BEGE Distributions

1) “Large” and equal $p_t$ and $n_t$: Gaussian limit



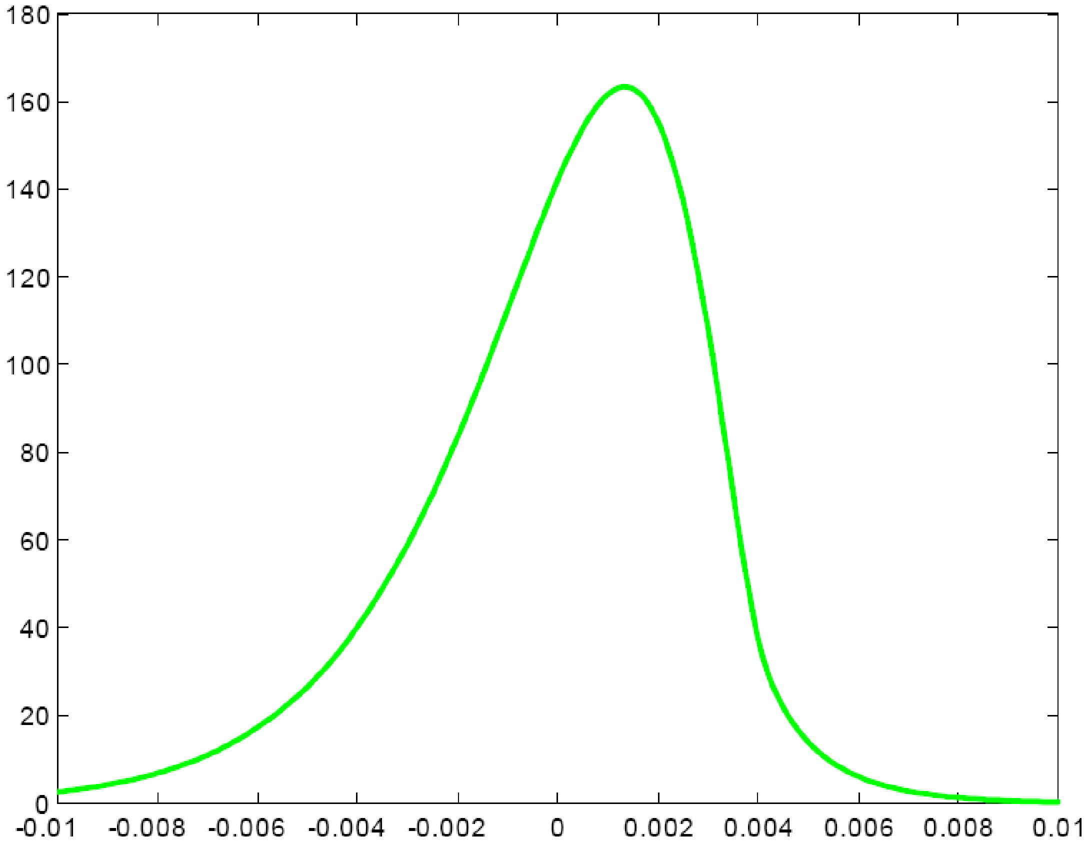
# BEGE Distributions

2) "Small" but still equal $p_t$ and $n_t$: excess kurtosis



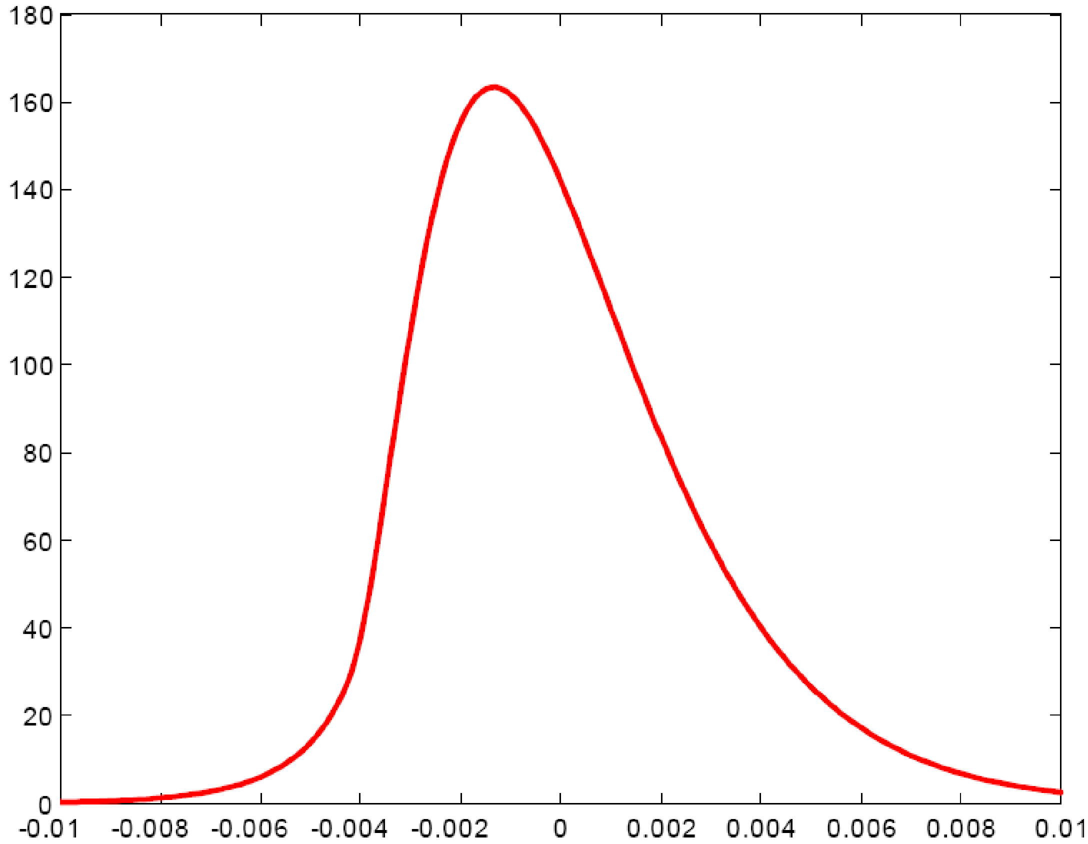
# BEGE Distributions

3) Relatively large $n_t$: negative skewness: "Bad Environment"



# BEGE Distributions

4) Relatively large $p_t$: positive skewness "Good Environment"



# BEGE Distributions

- The BEGE distribution has some advantages…
  - Flexible
  - Realistic
    - Fits some financial and macro economic data well
  - Tractable
    - Moments are affine in $p_t$ and $n_t$
    - Fits in the affine class of asset pricing models

# BEGE Distributions

… and some disadvantages

# 3 Methodological Steps to Assemble a set of Macro Factors

- We assemble six factors for use in term structure modeling that we identify using (*only*) macro data

| | |
|---|---|
| $E_t[g_{t+1}] =$ | *Expected growth* |
| $E_t[\pi_{t+1}] =$ | *Expected inflation* |
| $p_t^d =$ | *"Good" (positive skew) demand variance* |
| $p_t^s =$ | *"Good" (positive skew) supply variance* |
| $n_t^d =$ | *"Bad" (negative skew) demand variance* |
| $n_t^s =$ | *"Bad" (negative skew) supply variance* |

# 3 Methodological Steps

1. Identify conditional means versus shocks in growth and inflation data → $E_t[g_{t+1}], E_t[\pi_{t+1}]$
2. Recover supply and demand shocks
3. Estimate BEGE processes → $[p_t^d, n_t^d, p_t^s, n_t^d]$

# Methodological Steps (1/3)
# Measuring Expected Growth and Inflation

- Use simple predictive regressions
  - LHS: quarterly U.S. GDP growth and CPI inflation from 1970
  - RHS: lagged LHS, survey-based (SPF) forecasts
    - Try many possible combinations of RHS variables and lag structures
    - Use BIC to choose

# Methodological Steps (1/3)
# Measuring Expected Growth and Inflation

➢ Results

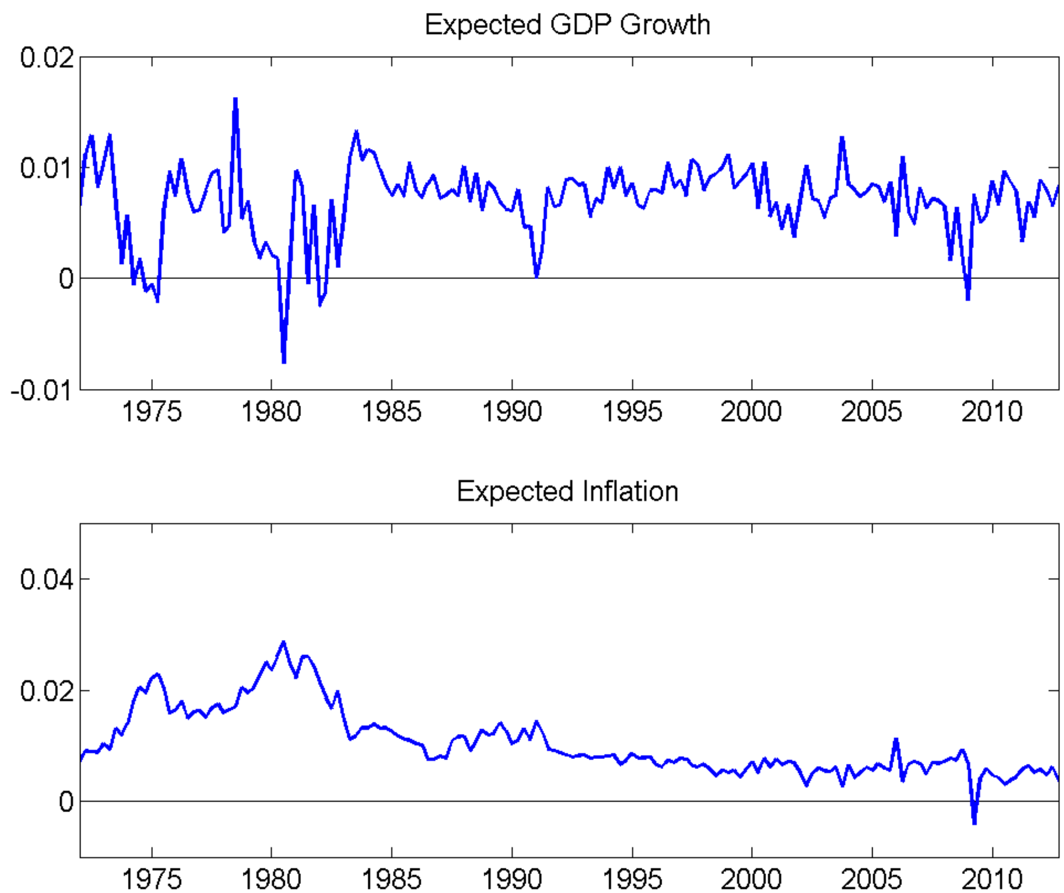
$$g_{t+1} = \underset{(0.0014)}{0.0064^{***}} + \underset{(0.0951)}{0.3401^{***}}\, g_t + \underset{(0.0783)}{-0.1721^{**}}\, \pi_t$$

$$\pi_{t+1} = \underset{(0.0010)}{-0.0002} + \underset{(0.1772)}{0.9055^{***}}\, \pi^e_{t,t+1} + \underset{(0.1202)}{0.2355^{**}}\, \pi_t$$

– GDP growth expectations consistent with VAR(1)

– Inflation expectations load on survey measures

# Methodological Steps (1/3)
# Measuring Expected Growth and Inflation



# Methodological Steps (2/3)
# Recover Supply/Demand Shocks

- Fundamental identification problem with Gaussian DGP

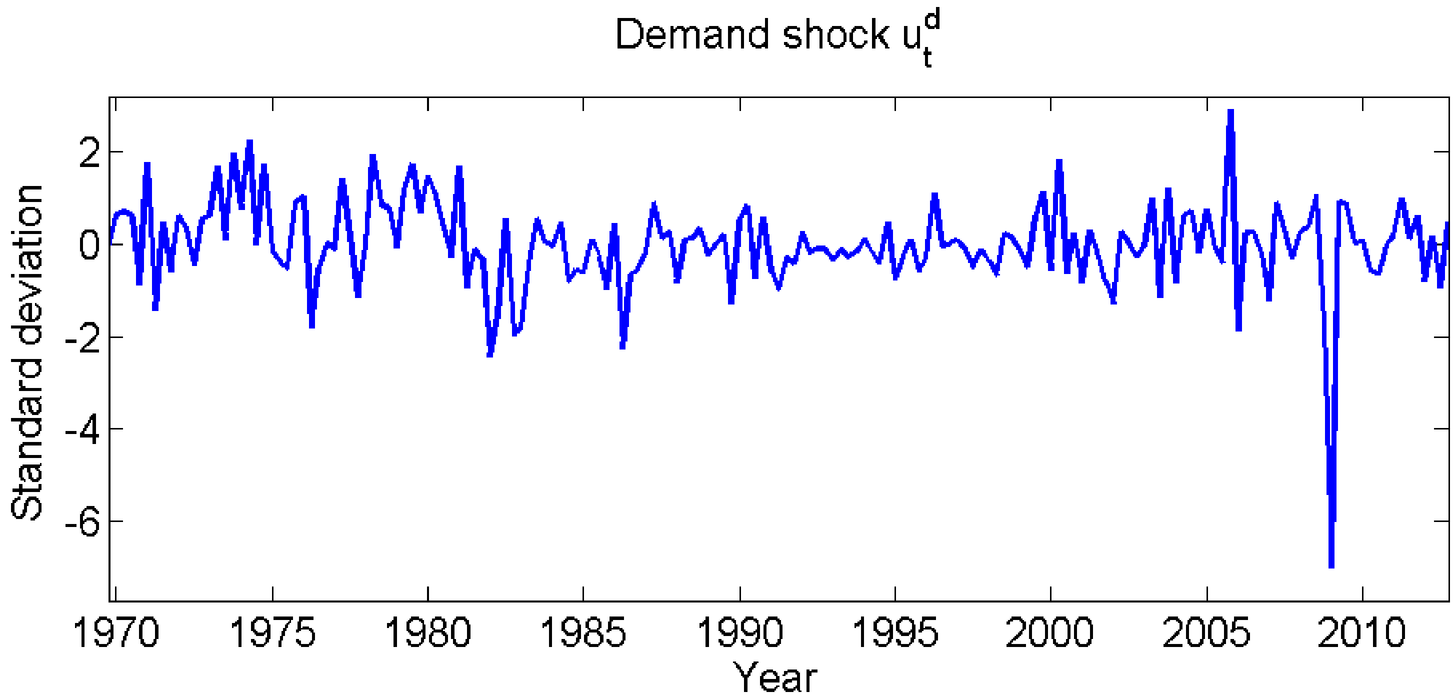
$$u_t^\pi = -\sigma_{\pi s} u_t^s + \sigma_{\pi d} u_t^d,$$

$$u_t^g = \sigma_{gs} u_t^s + \sigma_{gd} u_t^d,$$

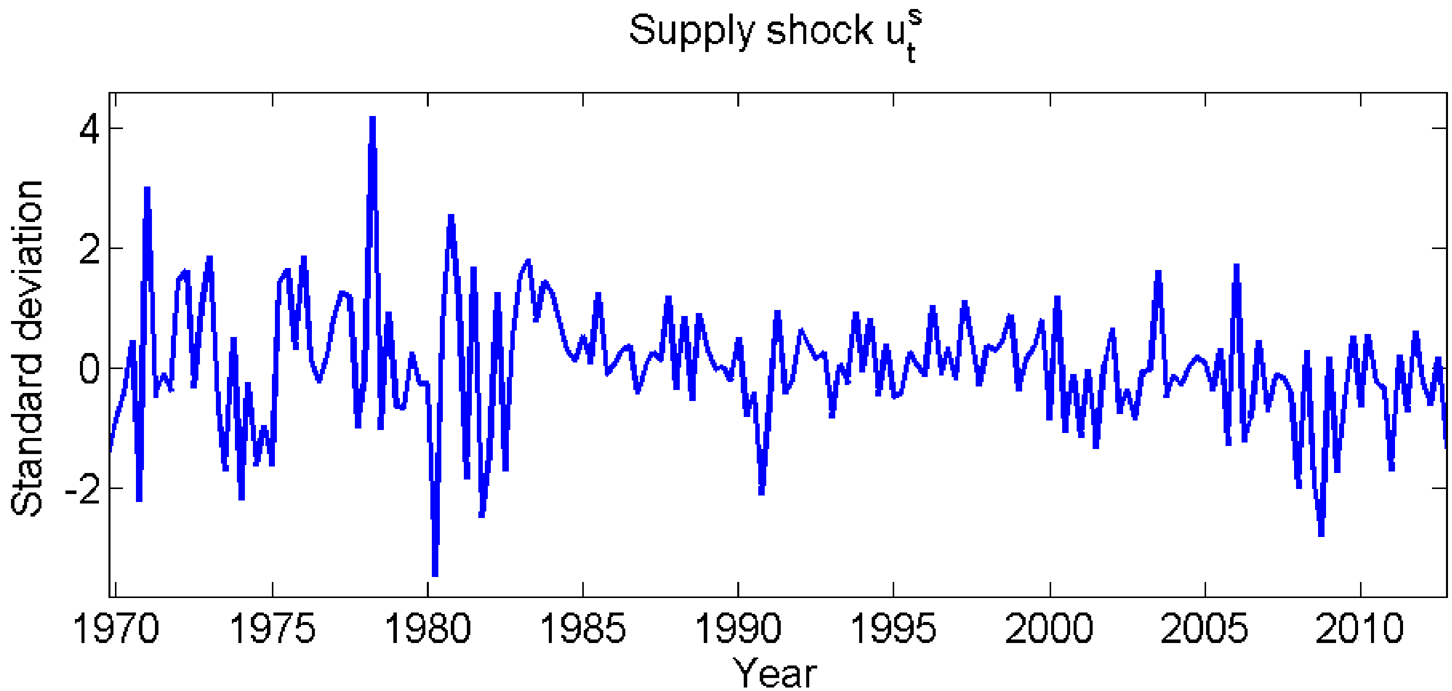
$$Cov(u_t^d, u_t^s) = 0, Var(u_t^d) = Var(u_t^s) = 1.$$

- The BEGE structure is consistent with identification using non-Gaussian features of data
  - Use 2nd 3rd 4th order moments to identify "σ"s
  - Then "invert" supply and demand shocks

# Methodological Steps (2/3)
# Recover Supply/Demand Shocks



# Methodological Steps (2/3)
# Recover Supply/Demand Shocks



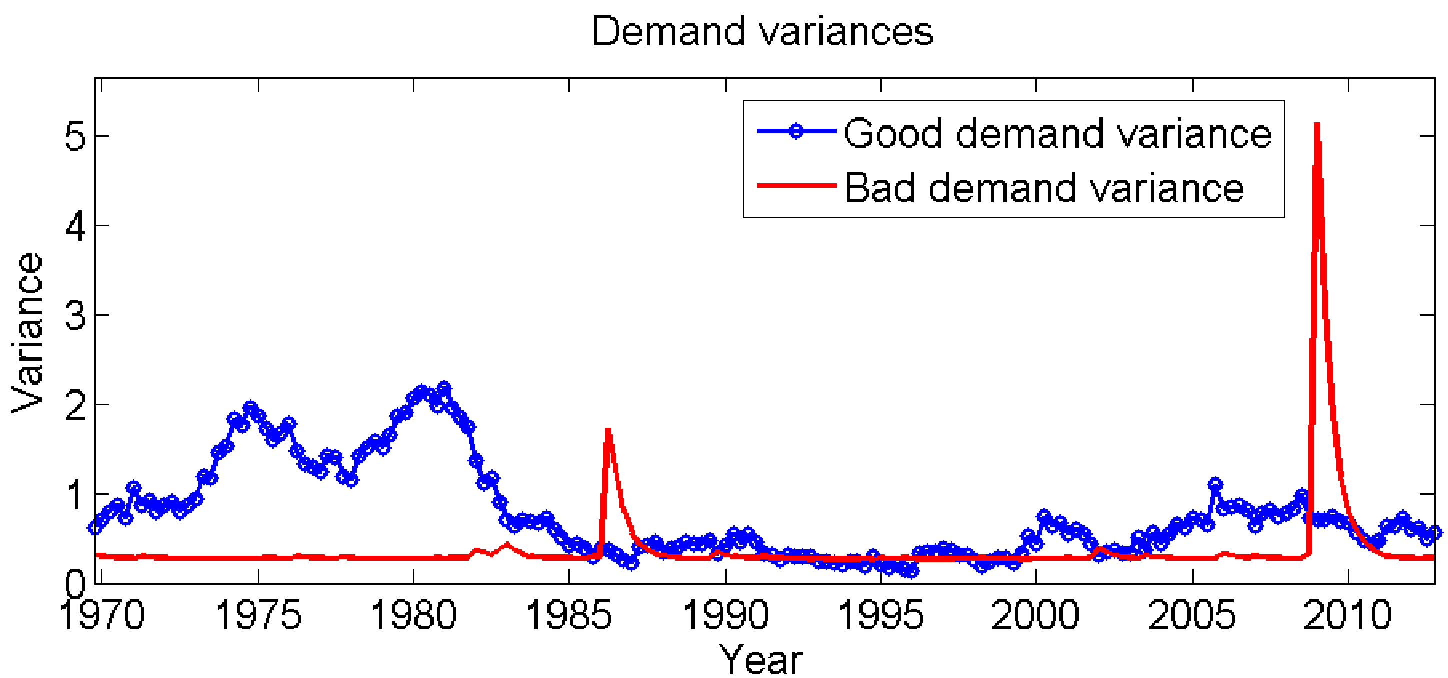
# Methodological Steps (3/3)
# Filter BEGE Factors

- Assume autoregressive, square root-like processes for the four BEGE factors

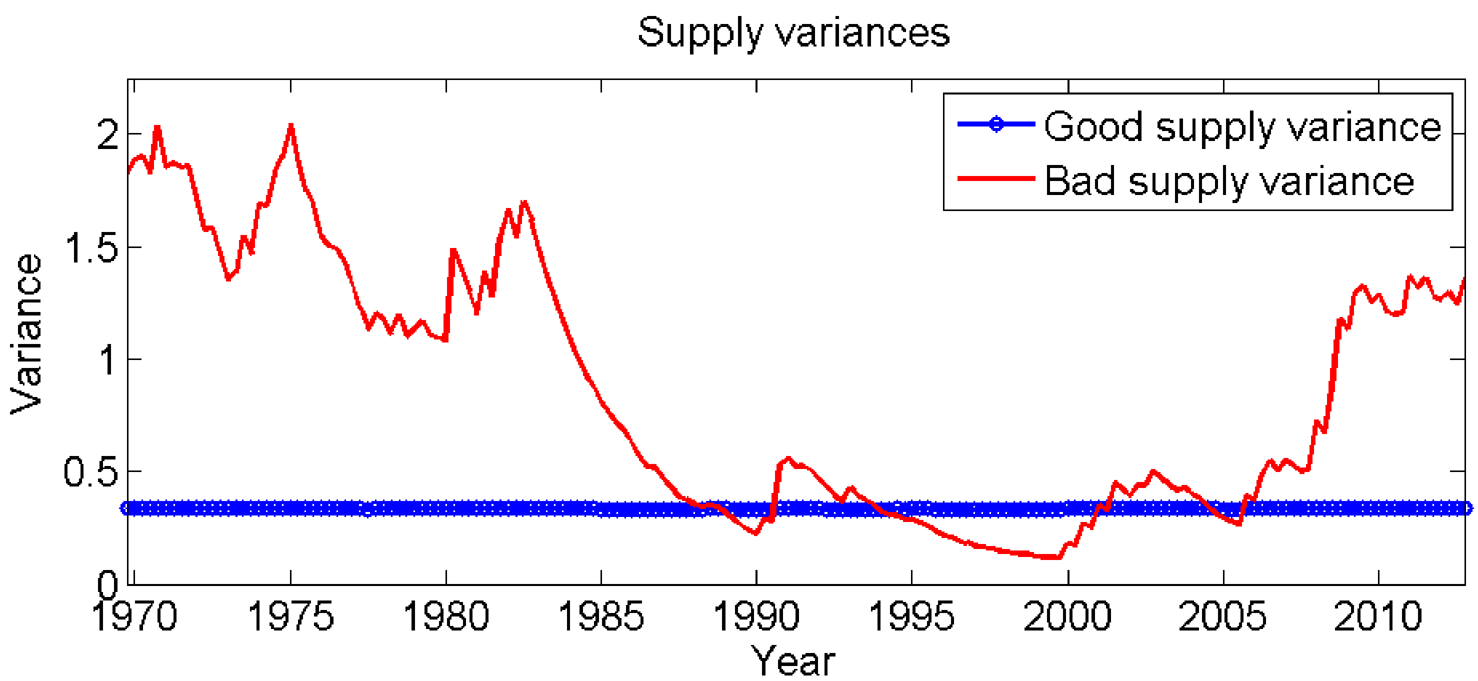
$$
\begin{aligned}
p_t^d &= \bar{p}^d(1-\phi_p^d) + \phi_p^d p_{t-1}^d + \sigma_p^d \omega_{p,t}^d, \\
p_t^s &= \bar{p}^d(1-\phi_p^s) + \phi_p^s p_{t-1}^s + \sigma_p^s \omega_{p,t}^s, \\
n_t^d &= \bar{n}^d(1-\phi_n^d) + \phi_n^d n_{t-1}^d + \sigma_n^d \omega_{n,t}^d, \\
n_t^s &= \bar{s}^d(1-\phi_n^s) + \phi_n^s p_{t-1}^s + \sigma_n^s \omega_{n,t}^s.
\end{aligned}
$$

- Use Bates filter to estimate parameters and filter
  - accommodates non-Gaussian processes

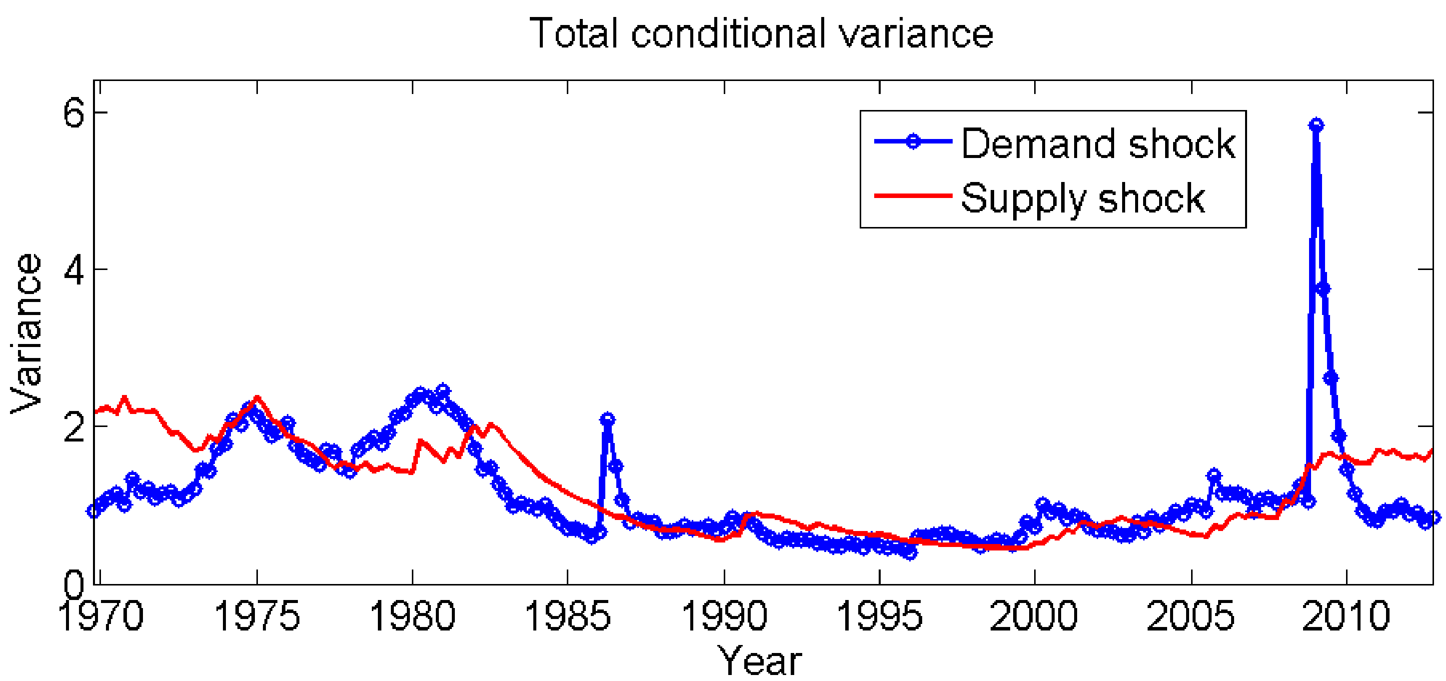
# Methodological Steps (3/3)
## Filter BEGE Factors



# Methodological Steps (3/3)
## Filter BEGE Factors

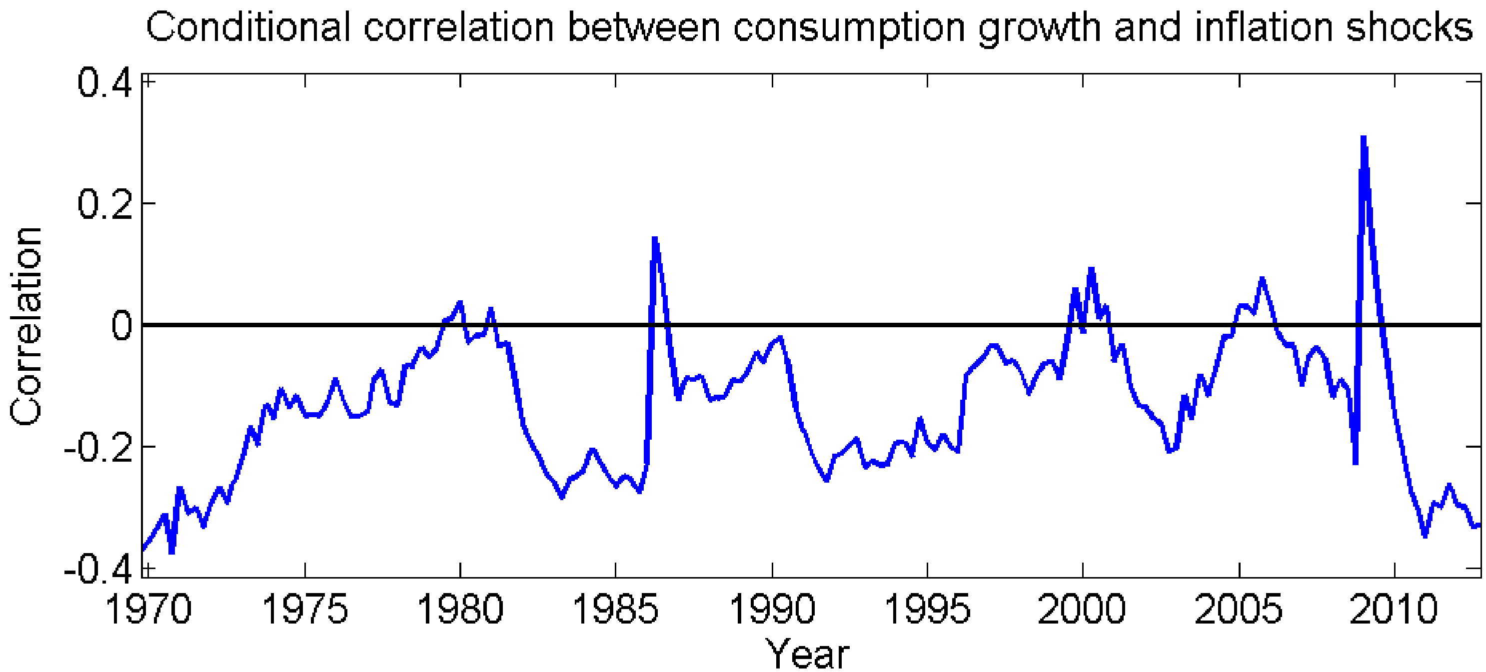


# Methodological Steps (3/3)
## Filter BEGE Factors



# Methodological Steps (3/3)
## Filter BEGE Factors

- We can recover the implied correlation between real growth and inflation



# Macro Risks and the Term Structure: Reduced-from evidence

- So far, we have (purposefully) not looked at asset price data
- Do the macro factors show signs of life for helping to explain yields and risk premiums?

# Macro Risks and the Term Structure: Reduced-from evidence 1

1 Quarter nominal interest rate

| Constant | $E_t g_{t+1}$ | $E_t \pi_{t+1}$ | $p_t^d$ | $n_t^d$ | $n_t^s$ | $R^2$ |
|---|---|---|---|---|---|---|
| -0.0022 | 0.4944*** | 1.5208*** | -0.0001 | 0.0193 | -0.0008* | 0.7074 |
| (0.0027) | (0.1849) | (0.2205) | (0.0001) | (0.0149) | (0.0004) | |

1 Year nominal interest rate

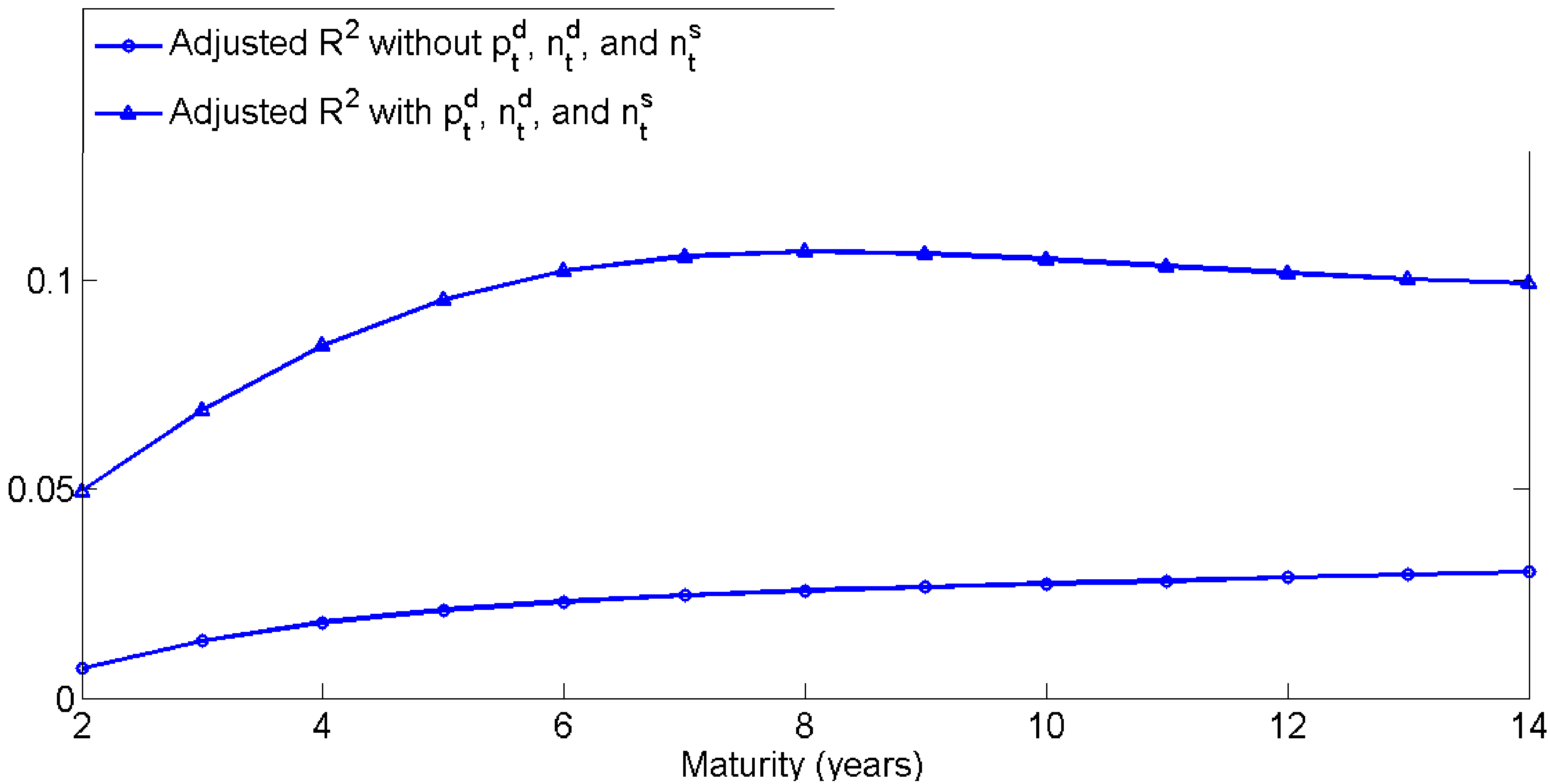
| Constant | $E_t g_{t+1}$ | $E_t \pi_{t+1}$ | $p_t^d$ | $n_t^d$ | $n_t^s$ | $R^2$ |
|---|---|---|---|---|---|---|
| -0.0022 | 0.5645*** | 1.6767*** | -0.0001 | 0.0206 | -0.0006 | 0.7174 |
| (0.0028) | (0.1936) | (0.2393) | (0.0001) | (0.0178) | (0.0006) | |

10 Year nominal interest rate

| Constant | $E_t g_{t+1}$ | $E_t \pi_{t+1}$ | $p_t^d$ | $n_t^d$ | $n_t^s$ | $R^2$ |
|---|---|---|---|---|---|---|
| 0.0036* | 0.5011*** | 1.5623*** | -0.0003** | 0.0261* | 0.0001 | 0.7284 |
| (0.0022) | (0.1583) | (0.2100) | (0.0001) | (0.0143) | (0.0004) | |

# Macro Risks and the Term Structure: Reduced-form evidence 2

1-year holding period excess returns, predictability



# Macro Risks and the Term Structure: Reduced-form evidence 2

1-qtr holding period excess returns, predictability

| | Excess return on 2 year bond | Excess return on 10 year bond |
|---|---|---|
| Constant | -0.0003 | 0.0228*** |
| | (0.0012) | (0.0081) |
| $E_t g_{t+1}$ | 0.1810** | 0.1797 |
| | (0.0846) | (0.8231) |
| $E_t \pi_{t+1}$ | 0.0967 | -0.8669 |
| | (0.1252) | (0.5974) |
| $p_t^d$ | -0.0002*** | -0.0012*** |
| | (0.0001) | (0.0003) |
| $n_t^d$ | -0.0150 | -0.2448*** |
| | (0.0103) | (0.0719) |
| $n_t^s$ | 0.0006* | 0.0055*** |
| | (0.0003) | (0.0014) |

# Formal Term Structure Model Aspirations

- Specify real short rate as function of macro factors

$$y_{1,t} = a_0 + a_g E_t[g_{t+1}] + a_\pi E_t[\pi_{t+1}] + a_{pd} p_t^d + a_{nd} n_t^d + a_{ns} n_t^s + z_t$$

  – $z_t$ is a latent factor (Gaussian)

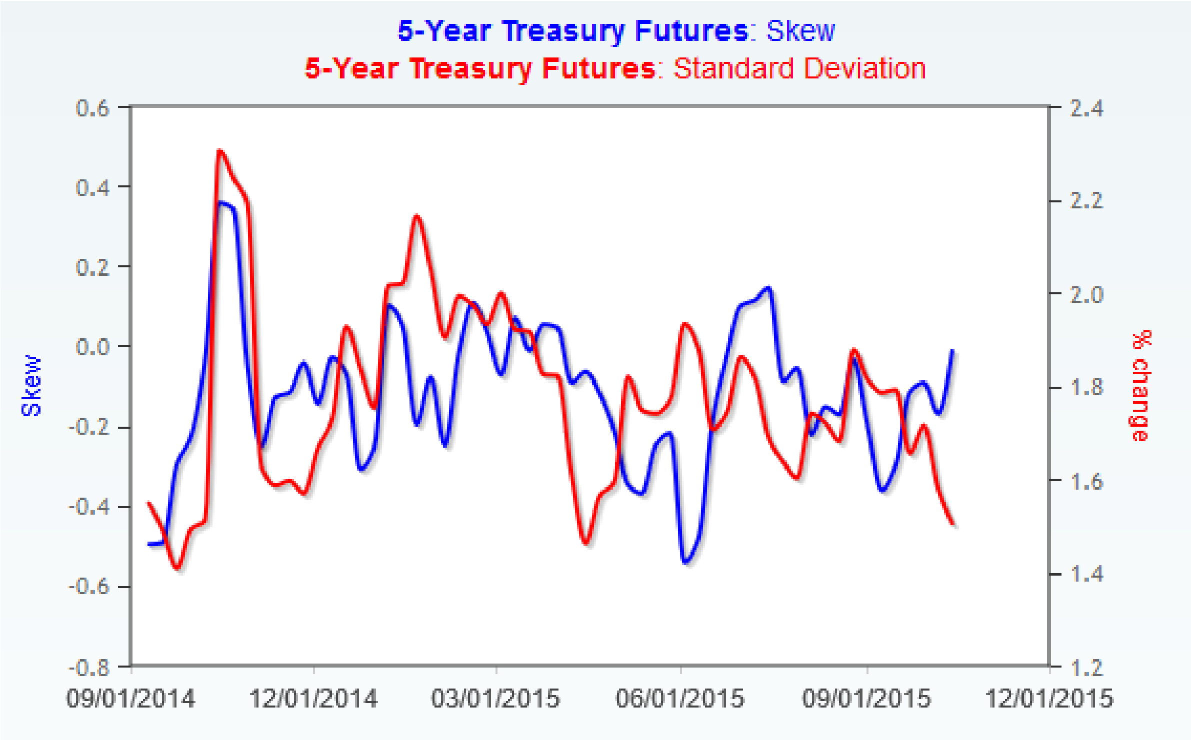
- Specify an "empirical" pricing kernel

$$(m_{t+1} - E_t[m_{t+1}]) = \lambda_{pd} \omega_{p,t+1}^d + \lambda_{nd} \omega_{n,t+1}^d + \lambda_{ps} \omega_{p,t+1}^s + \lambda_z \varepsilon_{t+1}^z$$

  – Constant prices of risk → model is in the affine class

# Formal Term Structure Model Aspirations

- Can the model explain using macro factors
  - yield dynamics?
  - apparent non-Gaussianities in option prices?



# Conclusions

- Supply and demand shocks
  - Relative variances change considerably over time
  - Evidence of non-Gaussianity

- Asset prices
  - Macro factors drive significant portion of variation in yields
  - Non-Gaussian macro risks are important drivers of risk premiums for nominal bonds