A THEORY OF HOUSING DEMAND SHOCKS

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Abstract. Housing demand shocks are used in the standard macroeconomic models as a primary source of the house price fluctuation and, through the collateral channel, a driver of macroeconomic fluctuations. We provide a microeconomic foundation for these reduced-form shocks within a tractable heterogeneous-agent framework. Unlike a housing demand shock in the standard models, a credit supply shock in our micro-founded model generates a positive correlation between the trading volume and the house price, as well as an arbitrarily large fluctuation of the price-to-rent ratio. These theoretical implications are robust to alternative forms of heterogeneity and in line with empirical evidence.

I. Introduction

In the standard business cycle models, housing demand shocks are a primary driving force behind the fluctuation of the house price and, through the collateral channel, drive a large fraction of the business cycle fluctuation (Iacoviello and Neri, 2010; Liu et al., 2013, for example). These important shocks, however, are proxied by shifts in the representative agent's tastes for housing, which are of the reduced form without an explicit microeconomic foundation. Since a taste shock to housing impinges on both

Date: May 17, 2021.

Key words and phrases. Housing demand, house prices, price-rent ratio, heterogeneous agents, credit constraints.

JEL classification: E21, E27, E32.

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house prices and rents, it is challenging to account for the observed large fluctuations in the house price-to-rent ratio.

In this paper, we develop a tractable heterogeneous-agent framework that provides a microeconomic foundation for housing demand shocks. The tractability leads to a closed-form solution that allows us to uncover the underlying forces that drive fluctuations in aggregate housing demand. It also enables us to demonstrate that our theory can overcome the difficulty of the standard representative-agent model in explaining the observed large volatility of house prices relative that of rents.

The baseline model features a household family with a large number of members who hold heterogeneous beliefs about the future value of housing services. The members trade houses in a decentralized market, each endowed with net worth from the household family. Traders with optimistic beliefs about the future housing value choose to purchase houses with both internal net worth and external debt, subject to a credit constraint. Traders with pessimistic beliefs sell houses subject to a no-short-sale restriction. For a given loan-to-value (LTV) ratio, there exists a unique cutoff point in the support of the idiosyncratic belief distribution, and the marginal agent who holds the cutoff-level of belief is indifferent between purchasing and selling a house. The cutoff point is endogenous. It varies with changes in macroeconomic conditions, in particular with changes in credit supply conditions measured by the LTV value.

We show that the marginal agent's perceived future value of houses increases with the LTV ratio, such that a credit supply shock that raises the LTV ratio also raises the marginal agent's belief, boosting aggregate housing demand. The belief heterogeneity drives a wedge in the aggregate housing Euler equation, resembling a shift in the aggregate value of housing servies, which can be mapped to the reduced-form housing demand shock in the representative-agent model. Since aggregate housing demand is driven by changes in the marginal agent's belief about future housing value, a positive shock to credit supply boosts aggregate housing demand and the house price, but has no effect on the equilibrium rent. As a result, credit supply shocks in our heterogeneous-agent model are capable of generating an arbitrarily large volatility of the house price relative to that of the rent. Moreover, the house trading volume in our model increases with the LTV ratio, such that a positive credit supply shock raises both the house price and the trading volume, consistent with the prediction of the model of Stein (1995) and with the empirical evidence documented by Ortalo-Magné and Rady (2006).

Our model's theoretical implications are robust to alternative forms of heterogeneity. We show that the same qualitative results can be obtained in an environment with heterogeneous beliefs about future income growth (instead of about future value of housing services). Belief heterogeneity about income growth gives rise to an intertemporal wedge in the housing Euler equation for the marginal agent. Our closed-form solution reveals that this intertemporal wedge enters the effective discount rate and it resembles the growth rate in the dividend-discount model of Gordon (1959). The key difference is that the effective discount factor in our model is endogenous and, in particular, it increases with the LTV ratio. Through its impact on the discount factor, a positive credit supply shock raises the marginal agent's perceived income growth rate, boosting aggregate housing demand and the house price. Since the rent is independent of beliefs, this model can also generate arbitrarily large volatility of the house price relative to that of the rent. Consistent with Cochrane (2011), the fluctuations in the house price-rent ratio in our model are mainly driven by variations in the effective discount factor.

We further show that the main theoretical implications of our model survive in an environment with heterogeneities in fundamentals, such as heterogeneous tastes of housing services (see Appendix C).

The key theoretical predictions from our heterogeneous-agent model are supported by empirical evidence. We provide some cross-country cross-region evidence. For the cross-country data, we use an unbalanced panel of 25 advanced economies covering the period from 1965 to 2013. Following the approach of Mian et al. (2017), we construct a credit supply shock based on an accelerations in household credit growth in periods with low mortgage spreads, where the mortgage spread is the difference between the mortgage interest rate and the 10-year sovereign bond yield. We find that an increase in credit supply is followed by significant and persistent increases in both the house price and the price-to-rent ratio, while the impact on the rent is statistically insignificant. For the U.S. regional data, we use an unbalanced panel of 21 Metropolitan Statistical Areas (MSAs), and find that a credit supply shock generates dynamic responses of the house price, the rent, and the price-rent ratio very similar to those obtained for the cross-country data.[1]

[1]For more empirical studies that point to the importance of credit supply shocks for the boom-bust cycle in the housing market, see the survey of Mian and Sufi (2018) and the references therein.

The rest of the paper is organized as follows. Section II discusses our paper's contributions relative to the literature. Section III presents a simple representative-agent model to illustrate the role of aggregate housing demand shocks as well as the difficulty of such a model in generating the large volatility of house prices relative to that of rents. Section IV presents the baseline heterogenous-agent model, in which agents hold heterogeneous beliefs about the future value of housing services. We derive a mapping between the reduced-form housing demand shock in the representative-agent model and the effective housing demand in the heterogeneous-agent model. We show that an increase in the LTV value raises aggregate housing demands and the house price with no effect on the rent. A shock to the LTV value, which we call a credit supply shock, generates a positive correlation between the house price and the trading volume as in the data. In Section V, we present a heterogeneous-agent model with belief heterogeneity in future income growth, and show that the main results obtained in the baseline model are robust to this alternative setup. Section VI offers some empirical evidence in support of our model's prediction that a credit supply shock can have a large impact on the house price, but not on the rent. Section VII provides some concluding remarks. In the Appendix, we provide some proofs of the main theoretical results and some details of the data that we use. We also present a heterogeneous-agent model with heterogeneity in the tastes for housing services and show that the main theoretical results obtained in the baseline model with heterogeneous beliefs are robust.

II. Related Literature

Empirical studies suggest that belief heterogeneity is important for understanding financial markets. For example, based on surveys of a large panel of retail investors, Giglio et al. (2020) show that belief heterogeneity helps account for the direction and the magnitude of financial trades. Cheng et al. (2014) use personal home transaction data of midlevel managers in securitized finance to show that those managers were not aware of the looming crisis in the housing market during the boom years in 2004-2006. In light of this evidence, they argue that beliefs might be important for understanding the causes of the housing crisis. Bailey et al. (2019) study the relation between beliefs about future house price changes and mortgage leverage choices. Our paper focuses on the implications of belief heterogeneity for house price-rent dynamics.

The belief channel in our model shares the spirit of Scheinkman and Xiong (2003), who study a model with heterogeneous beliefs (or investor disagreements) stemming from overconfidence (e.g., each agent believes that "my information is better than the others"). They show that belief heterogeneity helps explain deviations of asset prices from their fundamental values, and the resulting asset bubbles lead to large asset price volatility. Furthermore, belief heterogeneity also helps explain the observed positive correlations between equity prices and trading volume. Our focus on the housing market complements the study of Scheinkman and Xiong (2003). We show that, with heterogeneous beliefs, credit constraints and aggregate credit supply shocks can generate large volatilities of the house price relative to that of rent and also positive correlations between the house price and trading volume.

In a related contribution, He et al. (2015) argue that house price booms can be partly driven by a liquidity premium stemming from collateralized home equity lending. In their model, liquidity depends on beliefs, giving rise to self-fulfilling equilibria and potentially large fluctuations in house prices. Although they do not model rental markets, renters in reality do not have access to home equity loans. Thus, in principle, their model can potentially generate large fluctuations in the house price-rent ratio. Our model has a different focus. The belief heterogeneity in our model leads to heterogeneity in the borrowing constraint. Traders with optimistic beliefs borrow to finance home purchases, and they face a binding credit constraint. Traders with pessimistic beliefs are unconstrained (they save) and they sell houses. Our model explicitly incorporates a frictionless rental market, and we show that the equilibrium rent does not vary with changes in beliefs driven by fundamental shocks such as credit supply shocks. Thus, our model can potentially generate arbitrarily large volatilities in the price-to-rent ratio.

Our work is related to Favilukis et al. (2016), who emphasize the importance of aggregate business cycle risks and wealth distribution driven by bequest heterogeneity in preferences for driving house price booms. They also find that credit supply expansion is important to account for the observed fluctuations in house prices, although they do not model rental markets. In their model, highly skewed wealth distribution stemming from bequest heterogeneity gives rise to a large set of credit-constrained agents, leading to high sensitivity of housing demand and house prices to credit supply conditions.[2]

We reach a similar conclusion through a different mechanism. In our model, heterogeneous beliefs about future economic conditions (either the future value of housing

[2]For other recent studies that emphasize the importance of credit supply shocks for house prices, see, for example, Justiniano et al. (2019) and Greenwald and Guren (2020).

or future income growth) give rise to natural buyers and sellers of houses, with buyers facing binding credit constraints. Credit supply shocks shifts the marginal trader's belief, driving changes in aggregate housing demand and house prices. Since the marginal trader's belief does not affect current marginal utility of housing, a credit supply shock does not affect rents. We further show that belief heterogeneity leads to positive correlations between house prices and trading volume.

Our study is also related to Kaplan et al. (2021), who argue that a shift in beliefs about future housing demand was a main driver of house price movements and the price-to-rent ratio around the Great Recession. In their model, a more optimistic forecast of future housing demand (i.e., a news shock) raises current housing demand and thus the house price because the price is forward looking; but the news shock has no effect on the current rent since no shocks have been materialized. Thus, conditional on a news shock about future housing demand, their model can generate arbitrarily large fluctuations in the house price relative to the rent. However, a credit supply shock (e.g., an increase in LTV) in the model of Kaplan et al. (2021) would change both the house price and the rent, because it is a materialized shock instead of a news shock. Even with news shocks, we show that such a representative-agent framework with no belief heterogeneity fails to generate the observed large *unconditional* volatility of the house price relative to that in the rent. This is true in general for a representative-agent framework, regardless of the information structure and the stochastic process of the housing demand shock. In contrast, belief heterogeneity in our model allows a fundamental shock such as a credit supply shock to generate arbitrarily large volatility of the house price relative to that of the rent.

Our model's mechanism works through heterogeneity in credit constraints stemming from heterogeneous beliefs or fundamentals. This mechanism is consistent with some existing empirical studies. For example, Landvoigt et al. (2015) use the micro-level data in the San Diego housing market to show that an increased credit availability for poor households with low-end homes was a major driver of the house price boom in the early 2000s. Households with lower-end homes have higher marginal utility of housing and thus face binding credit constraints. Rekkas et al. (2020) present a model of housing markets with price posting, directed search, and buyer heterogeneity. They show that the model's qualitative predictions are consistent with evidence from Vancouver's housing market. They further show that heterogeneity in buyer preferences helps explain the observed house price dispersion, while search frictions help explain price stickiness. In our model, agents with more optimistic beliefs about the future housing value have higher marginal utility of housing. They face binding credit constraints. An increase in credit supply turns some of these constrained agents into unconstrained ones while raising aggregate housing demands and the house price.

III. A representative-agent benchmark model

This section presents a stylized representative-agent model to illustrate the role of housing demand shocks in driving the house price. The model is intentionally kept simple to sharpen the exposition. In particular, we focus on an endowment economy such that house prices do not interact with consumption and production.[3]

The economy has one unit of housing supply (think about land) and an exogenous endowment of y_t units of consumption goods. The representative household has the expected utility function

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{\log c_t+\varphi_t\frac{h_t^{1-\theta}}{1-\theta}\right\}, \tag{1}$$

where c_t denotes consumption, h_t denotes housing, φ_t denotes a housing demand shock. The parameter $\beta \in (0,1)$ is the subjective discount factor and $\theta > 0$ is a parameter that measures the curvature of the utility function with respect to housing. The term $\mathbb{E}$ is an expectation operator.

The household chooses consumption, new housing purchases, and holdings of a risk-free bond denoted by b_t to maximize the utility function (1) subject to the flow of funds constraint

$$c_t+Q_t(h_t-h_{t-1})\leq y_t+\frac{b_t}{R_t}-b_{t-1}, \tag{2}$$

where Q_t denotes the house price and R_t denotes the risk free interest rate, both are taken as given by the household. The initial bond holdings b_{-1} and initial housing h_{-1} are also taken as given.

The optimizing decisions lead to the Euler equation for housing

$$\frac{Q_t}{c_t}=\beta\mathbb{E}_t\frac{Q_{t+1}}{c_{t+1}}+\varphi_t h_t^{-\theta}, \tag{3}$$

[3]The main insight about the importance of housing demand shocks for housing price fluctuations carries over to a more general environment with collateral constraints, as shown by Liu et al. (2013), provided that both constrained and unconstrained agents participate in the housing market. In the more general setup considered by Liu et al. (2013), the house price needs to satisfy the housing Euler equations of both types of agents, and the Euler equation for the unconstrained agent (the saver) in that model is qualitatively identical to that of the representative agent in the model studied here.

and for bond holdings

$$1 = \beta R_t \mathbb{E}_t \frac{c_t}{c_{t+1}}. \tag{4}$$

A competitive equilibrium consists of sequences of allocations $\{c_t, b_t, h_t\}$ and prices $\{Q_t, R_t\}$ that satisfy the Euler equations (3) and (4) and clear the markets for goods, bond, and housing. In particular, these market clearing conditions are given by

$$c_t = y_t, \tag{5}$$

$$b_t = 0, \tag{6}$$

$$h_t = 1. \tag{7}$$

The equilibrium house price is pinned down by iterating the housing Euler equation (3) forward. With the goods and housing market clearing conditions imposed, the housing Euler equation (3) implies that

$$\frac{Q_t}{y_t} = \beta \mathbb{E}_t Q_{t+1} \frac{1}{y_{t+1}} + \varphi_t, \tag{8}$$

Iterating forward, we obtain the equilibrium house price

$$Q_t = y_t \left[\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \varphi_{t+j} \right]. \tag{9}$$

The implicit (or shadow) rent is given by the household's marginal rate of substitution between housing and non-housing consumption, and it is given by

$$r_{ht} = \varphi_t y_t. \tag{10}$$

Thus, the price-to-rent ratio is given by

$$\frac{Q_t}{r_{ht}} = \frac{1}{\varphi_t} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \varphi_{t+j}. \tag{11}$$

Since we observe much larger fluctuations in house prices than in consumption or aggregate output, the house price solution (9) reveals that the large volatility of house prices stems primarily from shocks to housing demand (φ_t). In this model, housing demand shocks drive not just the house price fluctuations, but also rent fluctuations as is clear from Eq. (10). Thus, this representative agent model has difficulties in generating large volatilities in the price-to-rent ratio.

To see this more clearly, consider the stationary process for the housing demand shock

$$\hat{\varphi}_t = \rho \hat{\varphi}_{t-1} + e_t, \tag{12}$$

where $\hat{\varphi}_t \equiv \ln \frac{\varphi_t}{\varphi}$ denotes the log-deviations of the housing demand shock from steady state, and e_t is a white noise innovation to the shock.

Log-linearizing the solution to the house price in Eq. (9) around the deterministic steady state and imposing the shock process in Eq. (12), we obtain

$$\hat{Q}_t = \hat{y}_t + (1-\beta)\mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j}\right] = \hat{y}_t + \frac{1-\beta}{1-\beta\rho}\hat{\varphi}_t, \tag{13}$$

where $\hat{y}_t$ is an exogenous endowment process.

The log-linearized solution to the rent is given by

$$\hat{r}_{ht} = \hat{y}_t + \hat{\varphi}_t. \tag{14}$$

The log-linearized price-to-rent ratio is thus given by

$$\hat{Q}_t - \hat{r}_{ht} = -\frac{\beta(1-\rho)}{1-\beta\rho}\hat{\varphi}_t. \tag{15}$$

There are two counter-factual implications of this representative agent model. First, the model implies that the price-to-rent ratio falls when house price rises, as shown by Eq. (15). In the data, the price-rent ratio are highly positively correlated with house prices (see Figures 1-3).

Second, the model cannot generate larger volatility of house prices relative to rents. To see this, assume that the endowment is constant so that $\hat{y}_t = 0$. The model implies that

$$\frac{\mathbb{STD}(\hat{Q}_t)}{\mathbb{STD}(\hat{r}_{ht})} = \frac{1-\beta}{1-\beta\rho} < 1. \tag{16}$$

Thus, the model predicts that the house price is less volatile than the rent, while the opposite is true in the data.

The representative agent model generates counterfactual dynamics of the house price and the rent not only conditional on contemporaneous shocks to housing demand, but also conditional on news shocks. Consider the shock process

$$\hat{\varphi}_t = \rho\hat{\varphi}_{t-1} + e_t + z_{t-1}, \tag{17}$$

which contains both the contemporaneous shock e_t and the news shock z_t.

In this case, we have

$$\hat{Q}_t = \hat{y}_t + (1-\beta)\mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j}\right] = \hat{y}_t + \frac{1-\beta}{1-\beta\rho}\hat{\varphi}_t + \frac{(1-\beta)\beta}{1-\beta\rho}z_t, \tag{18}$$

and

$$\hat{r}_{ht} = \hat{y}_t + \hat{\varphi}_t. \tag{19}$$

Clearly, a positive news shock z_t would raise the house price, with no effect on the rent since it does not change the contemporaneous housing taste $\hat{\varphi}_t$. In this sense, the model can potential explain the relative volatility of house prices and rents conditional on news shocks (Kaplan et al., 2021).

However, even with news shocks, the model fails to generate the observed *unconditional* volatilities of the house price vs. the rent. To see this, consider the case without income shocks such that $\hat{y}_t = 0$. Under the shock process of $\hat{\varphi}_t$ specified in Eq. (17), the ratio of the unconditional volatility of the house price to that of the rent is given by

$$\frac{\mathbb{STD}(\hat{Q}_t)}{\mathbb{STD}(\hat{r}_{ht})} = \left(\frac{1-\beta}{1-\beta\rho}\right)\sqrt{1+\beta^2(1-\rho^2)\frac{\sigma_z^2}{\sigma_e^2+\sigma_z^2}} < 1, \tag{20}$$

where the last inequality obtains because $\rho < 1$ and $\sigma_e \geq 0$.

In the data, however, the relative volatility is much larger than one. To get a sence of the magnitude of the relative volatility implied by the representative agent model, consider the parameter values $\beta = 0.99$ and $\rho = 0.9$. These parameter values imply that $\frac{1-\beta}{1-\beta\rho} \approx 0.0917$. Thus, absent news shock (i.e, $\sigma_z = 0$), the upper bound of the relative volatility is about 0.0917, much smaller than that observed in the data. Introducing news shock amplifies the relative volatility, but the quantitative magnitude remains much smaller than that in the data. In particular, Eq (20) implies that, under the assumed parameter values, the upper bound of the relative volatility in the case with news shocks is $\frac{1-\beta}{1-\beta\rho}\sqrt{1+\beta^2(1-\rho^s)} \approx 0.0999$.

The failure of the representative agent framework in generating a large volatility of the house price relative to that of the rent emerges under very general assumptions about the agent's information set and the housing demand shock process. This result is formally stated in Proposition III.1 below.

Proposition III.1. *Assume that there is no income shock such that $\hat{y}_t = 0, \forall t$. For any arbitrary covariance-stationary process of the housing demand shock $\hat{\varphi}_t$ and any arbitrary information structure, the representative agent model implies that $\frac{\mathbb{STD}(\hat{Q}_t)}{\mathbb{STD}(\hat{r}_{ht})} < 1$.*

Proof. Regardless of the agent's information set and the shock processes, the equilibrium house price and the rent are given by

$$\hat{Q}_t = \hat{y}_t + (1-\beta)\mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j}\right], \tag{21}$$

$$\hat{r}_{ht} = \hat{y}_t + \hat{\varphi}_t. \tag{22}$$

For simplicity, assume that $\hat{y}_t = 0$. Then we have

$$\begin{aligned}\hat{Q}_t &= (1-\beta)\left[\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j}\right] \\ &\quad -(1-\beta)\left[\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j} - \mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j}\right]\right] \\ &\equiv Q_t^* - err_t,\end{aligned}$$

where $Q_t^* \equiv (1-\beta)\left[\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j}\right]$ and err_t denotes the present value of the expectation errors. Under rational expectations, $\mathbb{E}_t\hat{\varphi}_{t+j}$ and $\hat{\varphi}_{t+j} - \mathbb{E}_t\hat{\varphi}_{t+j}$ are independent, implying that $\hat{Q}_t$ and err_t are independent. Thus, we have $var(Q_t^*) = var(Q_t) + var(err_t) < var(Q_t^*)$, implying that

$$var(Q_t) < var(Q_t^*) \tag{23}$$

The unconditional variance of Q_t^* is given by

$$\begin{aligned}var(Q_t^*) &= \mathbb{E}\left[(1-\beta)\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j} - 0\right]^2 \\ &= (1-\beta)^2\mathbb{E}\left(\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j}\right)^2 \\ &= (1-\beta^2)\mathbb{E}(\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j}\sum_{j=0}^{\infty}\beta^j\hat{\varphi}_{t+j}) \\ &= (1-\beta)^2\frac{1}{1-\beta^2}[cov(\hat{\varphi}_t,\hat{\varphi}_t) + 2\beta cov(\hat{\varphi}_t,\hat{\varphi}_{t-1}) + 2\beta^2 cov(\hat{\varphi}_t,\hat{\varphi}_{t-2}) + ...],\end{aligned}$$

where we have used the stationarity property of $\hat{\varphi}_t$ such that $cov(\hat{\varphi}_{t+j},\hat{\varphi}_{t+\ell}) = cov(\hat{\varphi}_t,\hat{\varphi}_{t+j-\ell})$.

The Cauchy-Schwartz inequality implies that $cov(\hat{\varphi}_t,\hat{\varphi}_{t-j}) < cov(\hat{\varphi}_t,\hat{\varphi}_t)$. Thus, we have

$$var(Q_t^*) < (1-\beta)^2\frac{1}{1-\beta^2}\left[1+\frac{2\beta}{1-\beta}\right]cov(\hat{\varphi}_t,\hat{\varphi}_t) = var(\hat{\varphi}_t). \tag{24}$$

Since $var(\hat{r}_{ht}) = var(\hat{\varphi}_t)$, it follows that $var(\hat{Q}_t) < var(\hat{r}_{ht})$.

□

IV. A heterogenous-agent model of housing demand

The representative-agent model fails to generate the observe large volatilities of the house price relative to that of housing rent. This failure calls for a better understanding of the forces behind the reduced-form housing demand shock. We now present a tractable heterogeneous-agent framework that allows us to establish a microeconomic foundation for the reduced-form housing demand shock. We show that belief heterogeneity allows the model to generate arbitrarily large volatilities of the house price relative to that of the rent conditional on fundamental shocks such as credit supply shocks. Belief heterogeneity also gives rise to equilibrium trading in the housing markets. The model implies a positive correlation between the trading volume and the house price, in line with empirical evidence.

IV.1. **Model environment.** Consider a large household family with a continuum of members. The family has the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \tilde{\varphi}_t \frac{s_{ht}^{1-\theta}}{1-\theta} \right], \tag{25}$$

where c_t and s_{ht} denote consumption of goods and housing services, respectively, $\tilde{\varphi}_t$ denotes an i.i.d. shock to the utility value of housing services, the parameter $\beta \in (0,1)$ is a subjective discount factor, and $\mathbb{E}$ is an expectation operator.

In the beginning of period t, family members are dispersed to decentralized housing markets where they trade houses. Individual traders hold heterogeneous beliefs about the future value of housing services. In particular, an individual j's perceived marginal utility of housing services is given by $\tilde{\varphi}_{t+1} = \varepsilon_t^j$, where the belief ε_t^j is i.i.d. and is drawn from the distribution $F(\cdot)$. Since the only source of heterogeneity is belief shocks, an individual trader can be fully identified by her belief such that we can index the traders' house purchases and bond holdings in the decentralized markets by ε_t, without the j index.

A trader with the belief ε_t finances his spending on houses $Q_t h_t(\varepsilon_t)$ using both internal funds a_t that is received from the family before the decentralized markets are open, and external debt $b_t(\varepsilon_t)$ that she can borrow at the market interest rate R_t.

The trader ε_t in the decentralized housing market faces the flow-of-funds constraint

$$Q_t h_t(\varepsilon_t) \leq a_t + \frac{b_t(\varepsilon_t)}{R_t}. \tag{26}$$

As in Kiyotaki and Moore (1997), imperfect contract enforcement implies that the external debt cannot exceed a fraction of the collateral value. Thus, the trader ε_t faces the collateral constraint

$$\frac{b_t(\varepsilon_t)}{R_t} \leq \kappa_t Q_t h_t(\varepsilon_t), \tag{27}$$

where the loan-to-value ratio $\kappa_t \in [0,1]$ is exogenous and potentially time varying, representing an aggregate shock to credit conditions.

The trader ε_t also faces a short-selling constraint such that

$$h_t(\varepsilon_t) \geq 0. \tag{28}$$

At the end of period t, the traders return to the household family and pool their funds. All members of the family enjoy the same consumption of goods c_t and of housing services s_{ht}. The family optimizing decisions are subject to the budget constraint

$$c_t + r_{ht}s_{ht} + a_t = y_t + (Q_t + r_{ht}) \int h_{t-1}(\varepsilon_{t-1})dF(\varepsilon_{t-1}) - \int b_{t-1}(\varepsilon_{t-1})dF(\varepsilon_{t-1}), \tag{29}$$

where r_{ht} denotes the rental rate of housing services and Q_t denotes the house price.

Denote by $\eta_t(\varepsilon_t)$, $\pi_t(\varepsilon_t)$, $\mu_t(\varepsilon_t)$, and λ_t the Lagrangian multiplers associated with the constraints (26), (27), (28), and (29), respectively.

The first order conditions with respect to c_t and s_{ht} are given by

$$\frac{1}{c_t} = \lambda_t, \tag{30}$$

$$\lambda_t r_{ht} = \tilde{\varphi}_t s_{ht}^{-\theta} \tag{31}$$

$$\tag{32}$$

The first order condition with respect to a_t implies that

$$\lambda_t = \int \eta_t(\varepsilon_t)dF(\varepsilon_t). \tag{33}$$

A marginal unit of goods transferred to individual members for housing purchases reduces family consumption by one unit and hence the utility cost is λ_t. The utility gain from this transfer is the shadow value of newly purchased housing (i.e., $\eta_t(\varepsilon_t)$) averaged across all members.

The first order condition with respect to $h_t(\varepsilon_t)$ is given by

$$\eta_t(\varepsilon_t)Q_t = \beta\mathbb{E}_t\left[\lambda_{t+1}(Q_{t+1} + r_{h,t+1})|\tilde{\varphi}_{t+1} = \varepsilon_t\right] + \kappa_t Q_t \pi_t(\varepsilon_t) + \mu_t(\varepsilon_t). \tag{34}$$

If a household member with belief shock ε_t purchases an additional unit of housing, the utility cost is $Q_t\eta(\varepsilon_t)$. The extra unit of housing yields rental value $r_{h,t+1}$ and resale value Q_{t+1} in the next period. In addition, having the extra unit of housing helps relax the collateral constraint and the short-selling constraint, with the shadow utility gains of $\kappa_t Q_t \pi_t(\varepsilon_t) + \mu_t(\varepsilon_t)$.

The first order condition with respect to $b_t(\varepsilon_t)$ is given by

$$\eta_t(\varepsilon_t) = \beta R_t \mathbb{E}_t\left[\lambda_{t+1}|\tilde{\varphi}_{t+1} = \varepsilon_t\right] + \pi_t(\varepsilon_t). \tag{35}$$

Borrowing an extra unit of goods has the utility value of $\eta_t(\varepsilon_t)$ for the member with belief shock ε_t. The family needs to repay the debt next period at the interest rate R_t, with the utility cost of $\beta R_t \mathbb{E}_t \lambda_{t+1}$. The increase in borrowing also tightens the collateral constraint, with the utility cost of $\pi_t(\varepsilon_t)$. The optimal choice of $b_t(\varepsilon_t)$ equates the utility gains to the total costs.

A competitive equilibrium is a collection of allocations $\{c_t, s_{ht}, a_t, h_t(\varepsilon_t), b_t(\varepsilon_t)\}$ and prices $\{Q_t, R_t\}$ such that

(1) Taking the prices as given, the allocations solve the household's utility maximizing problem.
(2) Markets for goods, housing, and credit all clear, such that

$$c_t = y_t, \tag{36}$$

$$s_{ht} = 1, \tag{37}$$

$$\int h_t(\varepsilon_t) dF(\varepsilon_t) = 1, \tag{38}$$

$$\int b_t(\varepsilon_t) dF(\varepsilon_t) = 0, \tag{39}$$

where we assume that the aggregate supply of housing is fixed at one and the aggregate net supply of debt is 0.

IV.2. **Equilibrium characterization.** We now characterize the equilibrium. Intuitively, a trader with a higher belief ε_t assigns a higher value to future housing services and thus she would like to purchase more housing. Since such purchases are partly financed by external debt, traders with a sufficiently high ε_t would face binding borrowing constraints. Thus, we conjecture that there exists a cutoff level of the belief shock ε_t^*, such that $\pi_t(\varepsilon_t) \geq 0$ if $\varepsilon_t \geq \varepsilon_t^*$) and $\pi_t(\varepsilon_t) = 0$ otherwise. Traders with beliefs above the cutoff point ε_t^* hold relatively optimistic views of future housing values and are house buyers; those with beliefs below ε_t^* are pessimists and are sellers. The key step to find an equilibrium is to determine the identity of the marginal trader ε_t^*, which is established in Lemma IV.1 below.

Lemma IV.1. *There exists a unique cutoff point ε_t^* in the support of the distribution $F(\varepsilon)$ and it is given by*

$$F(\varepsilon_t^*) = \kappa_t. \tag{40}$$

Proof. For traders with $\varepsilon_t \geq \varepsilon_t^*$, the flow-of-funds constraint (26) and the collateral constraint (27) are both binding, implying that

$$Q_t h_t(\varepsilon_t) \leq \frac{a_t}{1-\kappa_t}. \tag{41}$$

Imposing the market clearing conditions (36) (39), we obtain $a_t = Q_t$. Thus, for all $\varepsilon_t \geq \varepsilon_t^*$, the equilibrium quantity of housing is given by

$$h(\varepsilon_t) = \frac{1}{1-\kappa_t}. \tag{42}$$

Thus, traders with $\varepsilon_t \geq \varepsilon_t^*$ are house buyers and they all buy the same quantity. Traders with $\varepsilon_t < \varepsilon_t^*$ are sellers with $h(\varepsilon_t) = 0$.

It follows from the house market clearing condition that

$$\frac{1}{1-\kappa_t}\int_{\varepsilon_t^*} dF(\varepsilon) = 1, \tag{43}$$

which gives the solution to ε_t^* in Eq. (40). □

From Eq. (40), it is clear that ε_t^* increases with κ_t. Thus, a credit supply expansion that raises κ_t also raises the marginal trader's perceived future utility value of housing (ε_t^*), boosting housing demand and the house price. The effective housing demand can be expressed as an implicit function of the LTV κ_t, as we show in the proposition below.

Proposition IV.2. *The equilibrium house price satisfies the aggregate Euler equation*

$$\lambda_t Q_t = \beta \mathbb{E}_t \lambda_{t+1} Q_{t+1} + \xi(\kappa_t), \tag{44}$$

where

$$\xi(\kappa_t) \equiv \frac{\beta}{1-F(\varepsilon_t^*)}\int_{\varepsilon_t^*} \varepsilon dF(\varepsilon), \tag{45}$$

which is a function of κ_t since ε_t^* is related to κ_t through $F(\varepsilon_t^*) = \kappa_t$.

Proof. For a house buyer with $\varepsilon_t \geq \varepsilon_t^*$, his housing Euler equation (34) can be written as

$$\begin{aligned}\eta_t(\varepsilon_t) Q_t &= \beta \mathbb{E}_t \lambda_{t+1} Q_{t+1} + \beta \mathbb{E}_t \left[\tilde{\varphi}_{t+1} s_{h,t+1}^{-\theta} \mid \tilde{\varphi}_{t+1} = \varepsilon_t\right] + \kappa_t Q_t \pi_t(\varepsilon_t) \\ &= \beta \mathbb{E}_t \lambda_{t+1} Q_{t+1} + \beta \varepsilon_t + \kappa_t Q_t \pi_t(\varepsilon_t), \quad \forall \varepsilon_t \geq \varepsilon_t^* \end{aligned} \tag{46}$$

where we have imposed the market clearing condition for rental housing $s_{ht} = 1$. His bond Euler equation (35) can be written as

$$\eta_t(\varepsilon_t) = \beta R_t \mathbb{E}_t \lambda_{t+1} + \pi_t(\varepsilon_t), \quad \forall \varepsilon_t \geq \varepsilon_t^*. \tag{47}$$

These conditions imply that

$$\pi_t(\varepsilon_t) = \frac{\beta}{1-\kappa_t}\left[\frac{\varepsilon_t + \mathbb{E}_t\lambda_{t+1}Q_{t+1}}{Q_t} - R_t\mathbb{E}_t\lambda_{t+1}\right], \quad \forall \varepsilon_t \geq \varepsilon_t^*. \tag{48}$$

Evaluating $\pi(\varepsilon_t)$ at ε_t^* and subtracting it from Eq (48), we obtain

$$\pi_t(\varepsilon_t) - \pi_t(\varepsilon_t^*) = \frac{\beta}{1-\kappa_t}\frac{\varepsilon - \varepsilon_t^*}{Q_t}.$$

Since $\pi(\varepsilon_t^*) = 0$, we have the solution for $\pi(\varepsilon_t)$

$$\pi_t(\varepsilon) = \frac{\beta}{1-\kappa_t}\max\left\{0, \frac{\varepsilon - \varepsilon_t^*}{Q_t}\right\}. \tag{49}$$

We can then solve for $\eta(\varepsilon_t)$ using Eq. (47), which yields

$$\eta_t(\varepsilon_t) = \beta R_t\mathbb{E}_t\lambda_{t+1} + \frac{\beta}{1-\kappa_t}\max\left\{0, \frac{\varepsilon - \varepsilon_t^*}{Q_t}\right\} \tag{50}$$

Given the solution for $\eta(\varepsilon_t)$, the first-order condition (33) then implies that

$$\lambda_t = \int \eta_t(\varepsilon_t)dF(\varepsilon_t) = \beta R_t\mathbb{E}_t\lambda_{t+1} + \frac{\beta}{1-\kappa_t}\frac{1}{Q_t}\int_{\varepsilon_t^*}\left[\varepsilon - \varepsilon_t^*\right]dF(\varepsilon) \tag{51}$$

Since $\pi(\varepsilon_t^*) = 0$, Equations (46) and (47) imply that

$$\begin{aligned} \eta_t(\varepsilon_t^*)Q_t &= \beta\mathbb{E}_t\lambda_{t+1}Q_{t+1} + \beta\varepsilon_t^* \\ \eta_t(\varepsilon_t^*) &= \beta R_t\mathbb{E}_t\lambda_{t+1} \end{aligned}$$

Substituting these relations into Eq. (51) yields

$$\begin{aligned} \lambda_t Q_t &= \beta\mathbb{E}_t\lambda_{t+1}Q_{t+1} + \beta\left[\varepsilon_t^* + \frac{1}{1-\kappa_t}\int_{\varepsilon_t^*}\left[\varepsilon - \varepsilon_t^*\right]dF(\varepsilon)\right] \\ &= \beta\mathbb{E}_t\lambda_{t+1}Q_{t+1} + \beta\frac{1}{1-F(\varepsilon_t^*)}\int_{\varepsilon_t^*}\varepsilon dF(\varepsilon), \\ &= \beta\mathbb{E}_t\lambda_{t+1}Q_{t+1} + \xi(\kappa_t), \end{aligned}$$

where $\xi(\kappa_t) \equiv \beta\frac{1}{1-F(\varepsilon_t^*)}\int_{\varepsilon_t^*}\varepsilon dF(\varepsilon)$ is a function of κ_t since Lemma IV.1 implies that $F(\varepsilon_t^*) = \kappa_t$. □

The next proposition draws an explicit mapping between the housing demand shock φ_t in the representative agent model and the housing demand shifter $\xi(\kappa_t)$ in the model with heterogeneous beliefs.

Proposition IV.3. *If $\varphi_t = \xi(\kappa_t)$, then the equilibrium house price in the heterogenous agent model coincides with that in the representative agent model.*

Proof. From Proposition IV.2, the housing Euler equation in the heterogeneous-agent economy is given by

$$\frac{Q_t}{c_t} = \beta \mathbb{E}_t \frac{Q_{t+1}}{c_{t+1}} + \xi(\kappa_t). \tag{52}$$

The housing Euler equation in the representative-agent model is given by Eq. (3) and is rewritten here for convenience of referencing:

$$\frac{Q_t}{c_t} = \beta \mathbb{E}_t \frac{Q_{t+1}}{c_{t+1}} + \varphi_t h_t^{-\theta} \tag{53}$$

In equilibrium, the housing market clears so that $h_t = 1$. Thus, if $\xi(\kappa_t) = \varphi_t$, then the housing Euler equations in the two different economies are formally identical. Furthermore, goods market clearing implies that $c_t = y_t$ in both models. Thus, the equilibrium house price is also identical. □

Proposition IV.3 provides a microeconomic foundation for the reduced-form housing demand shock. In this specific model, aggregate housing demand is a function of the loan-to-value shocks κ_t (as reflected in the $\xi(\kappa_t)$ function). In general, any shock that shifts the cutoff point ε_t^* also shifts aggregate housing demand, and thereby driving fluctuations in the house price.

We now show that an increase in the LTV ratio κ_t leads to an increase in housing demand $\xi(\kappa_t)$ and therefore an increase in the house price Q_t, but it has no effect on the housing rent r_{ht}. This is established in Proposition IV.4 below.

Proposition IV.4. *An increase in the LTV κ_t raises the house price Q_t but has no effect on the rent r_{ht}. That is,*

$$\frac{\partial Q_t}{\partial \kappa_t} > 0, \quad \frac{\partial r_{ht}}{\partial \kappa_t} = 0. \tag{54}$$

Proof. We first show that the housing demand shifter $\xi(\kappa_t)$ increases with κ_t. Since ε_t^* is strictly increasing in κ_t (see Lemma IV.1), it is sufficient to show that ξ_t strictly increases with ε_t^*. Differentiating ξ_t in Eq (45) with respect to ε_t^* to obtain

$$\begin{aligned} \frac{\partial \xi_t}{\partial \varepsilon_t^*} &= \frac{f(\varepsilon_t^*)}{[1-F(\varepsilon_t^*)]^2} \int_{\varepsilon_t^*} \varepsilon dF(\varepsilon) - \frac{\varepsilon_t^* f(\varepsilon_t^*)}{1-F(\varepsilon_t^*)} \\ &= \frac{f(\varepsilon_t^*)}{1-F(\varepsilon_t^*)} \left[\frac{\int_{\varepsilon_t^*} \varepsilon dF(\varepsilon)}{1-F(\varepsilon_t^*)} - \varepsilon_t^* \right] \\ &> \frac{f(\varepsilon_t^*)}{1-F(\varepsilon_t^*)} \left[\frac{\int_{\varepsilon_t^*} \varepsilon_t^* dF(\varepsilon)}{1-F(\varepsilon_t^*)} - \varepsilon_t^* \right] = 0, \end{aligned} \tag{55}$$

where $f(\varepsilon) \equiv F'(\varepsilon) > 0$ is the probability density function.

Given that $\frac{\partial \xi}{\partial \kappa_t} > 0$ and that $\lambda_t = 1/y_t$ is invariant to κ_t, the housing Euler equation (44) implies that $\frac{\partial Q_t}{\partial \kappa_t} > 0$.

The housing rent is given by

$$r_{ht} = \tilde{\varphi}_t \frac{s_{ht}^{-\theta}}{\lambda_t} = \tilde{\varphi}_t y_t, \tag{56}$$

which is independent of κ_t. □

Since changes in credit conditions (LTV) drive changes in the house price without affecting the rent, the heterogeneous agent model here is able to generate arbitrarily large volatility of the house price relative to that of the rent, as summarized in Proposition III.1).

The model with heterogeneous beliefs also generates positive correlations between house trading volumes and the house price through changes in credit conditions.

Define the house trading volume as

$$TV_t \equiv \frac{1}{2} \int \int |h_t(\varepsilon_t) - h_{t-1}(\varepsilon_{t-1})| dF(\varepsilon_t) dF(\varepsilon_{t-1}), \tag{57}$$

which measures the average number of houses that are either bought or sold from period $t-1$ to period t. Proposition IV.5 below shows that the trading volume is positively correlated with the contemporaneous value of the LTV κ_t.

Proposition IV.5. *The equilibrium house trading volume is given by*

$$TV_t = \max\{\kappa_t, \kappa_{t-1}\}. \tag{58}$$

Proof.

$$\begin{aligned}
TV_t &= \frac{1}{2} \int_{\varepsilon_t^*} \int_{\varepsilon_{t-1}^*} |\frac{1}{1-\kappa_t} - \frac{1}{1-\kappa_{t-1}}| dF(\varepsilon_t) dF(\varepsilon_{t-1}) \\
&\quad + \frac{1}{2} \int_{\varepsilon_t^*} \int_0^{\varepsilon_{t-1}^*} |\frac{1}{1-\kappa_t} - 0| dF(\varepsilon_t) dF(\varepsilon_{t-1}) \\
&\quad + \frac{1}{2} \int_0^{\varepsilon_t^*} \int_0^{\varepsilon_{t-1}^*} |0 - 0| dF(\varepsilon_t) dF(\varepsilon_{t-1}) \\
&\quad + \frac{1}{2} \int_0^{\varepsilon_t^*} \int_{\varepsilon_{t-1}^*} |0 - \frac{1}{1-\kappa_{t-1}}| dF(\varepsilon_t) dF(\varepsilon_{t-1}) \\
&= \frac{1}{2} \left\{ |\kappa_t - \kappa_{t-1}| + F(\varepsilon_{t-1}^*)[1 - F(\varepsilon_t^*)] \frac{1}{1-\kappa_t} + \frac{1}{1-\kappa_{t-1}} [1 - F(\varepsilon_{t-1}^*)] F(\varepsilon_{t-1}^*) \right\} \\
&= \frac{1}{2} [|\kappa_t - \kappa_{t-1}| + \kappa_{t-1} + \kappa_t] \\
&= \max\{\kappa_t, \kappa_{t-1}\}
\end{aligned}$$

□

Since both the trading volume and the house price increase with κ_t, the heterogeneous agent model predicts positive correlations between the trading volume and the house price, in line with the theoretical predictions of Stein (1995) and consistent with the empirical evidence documented by Ortalo-Magné and Rady (2006) (see also Clayton et al. (2009)).

V. Belief heterogeneity about future income growth

We have shown that belief heterogeneity provides a microeconomic foundation for the reduced-form housing demand shock. The benchmark heterogenous-agent model, unlike the representative-agent model, can generate large fluctuations of the house price relative to the rent. The theoretical insights from the benchmark model do not hinge upon the particular form of heterogeneity. In this section, we illustrate this point by studying a model that features heterogeneous beliefs in future income growth.[4]

V.1. **Model environment.** Suppose that aggregate output (i.e., income) grows at the rate $\frac{y_{t+1}}{y_t} = g_{t+1}$, where g_{t+1} is a random variable with the i.i.d. distribution $\tilde{F}$. The household consists of a large number of members, who are exte identical. Before entering the decentralized housing markets in the beginning of period t, the members each draws an i.i.d. belief shock about future income growth. In particular, member j believes that income growth will be $g_{t+1} = e_t^j$, where e_t^j is i.i.d. random variable drawn from the distribution $F(\cdot)$. Note that $\tilde{F}$ and F need not to be the same.[5]

Under perfect risk sharing, all family members enjoy the same consumption of goods and housing services. The household has the expected utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \varphi \frac{s_{ht}^{1-\theta}}{1-\theta} \right], \tag{59}$$

where, as in the previous model, c_t and s_{ht} denote the consumption of goods and housing services, respectively, $\beta \in (0,1)$ is a subjective discount factor, $\mathbb{E}$ is an expectation operator. We assume that there is no housing preference shock such that φ is a constant.

[4]Similar results can be obtained in a model with heterogeneous preferences for housing services, as we show in Appendix C.

[5]For simplicity, we focus on the simple model with heterogeneous beliefs about the point realizations of income growth. The model can be generalized to allow belief heterogeneity about the distribution ($\tilde{F}$) of income growth. Details are available from the authors upon request.

In the beginning of period t, all members receive a lump-sum transfer of a_t units of consumption goods from the family. The members are then dispersed to decentralized markets to trade houses based on their beliefs e_t about income growth g_{t+1}.[6]

In the decentralized housing markets, the member with belief e_t finances house purchases with both family transfer a_t and external debt $b_t(e_t)$, subject to the flow-of-funds constraint

$$Q_t h_t(e_t) \leq a_t + \frac{b_t(e_t)}{R_t}, \tag{60}$$

and the borrowing constraint

$$\frac{b_t(e_t)}{R_t} \leq \kappa_t Q_t h_t(e_t), \tag{61}$$

where, as in the benchmark heterogeneous-agent model, the risk-free interest rate R_t and the loan-to-value ratio κ_t are common for all borrowers. In addition, the housing purchase must be non-negative, such that

$$h_t(e_t) \geq 0. \tag{62}$$

At the end of the period, all members return to the household family. They receive the amount of consumption goods and housing services. The household faces the family budget constraint

$$c_t + r_{ht}s_{ht} + a_t = y_t + (Q_t + r_{ht}) \int h_{t-1}(e_{t-1}) dF(e_{t-1}) - \int b_{t-1}(e) dF(e_{t-1}), \tag{63}$$

where $h_{t-1}(e_{t-1})$ and $b_{t-1}(e_{t-1})$ denotes previous-period house purchase and bond holding of the family member who had idiosyncratic belief shock e_{t-1}. In addition to the endowment income y_t, the household receives the rental income and resale value of the houses that it carried over from period $t-1$ (i.e., $\int h_{t-1}(e_{t-1})dF(e_{t-1})$) at the rental rate r_{ht} and the house price Q_t. Using these sources of income net of repayments of the outstanding debt $\int b_{t-1}(e)dF(e_{t-1})$, the household finances consumption expenditures on goods and housing rental servides, as well as lump-sum transfers a_t to the family members.

Denote by $\eta_t(e_t)$, $\pi_t(e_t)$, $\mu_t(e_t)$, and λ_t the Lagrangian multiplers associated with the constraints (60), (61), (62), and (63), respectively. The first order condition with respect to c_t is given by

$$\frac{1}{c_t} = \lambda_t. \tag{64}$$

[6]As in the benchmark model, the members' house purchase and bond holding decisions can be fully identified by their beliefs e_t without carrying the j index.

The first order condition with respect to s_{ht} implies

$$\lambda_t r_{ht} = \varphi s_{ht}^{-\theta}. \tag{65}$$

The first order condition with respect to a_t implies

$$\lambda_t = \int \eta_t(e_t) dF(e_t). \tag{66}$$

The first order condition with respect to $h_t(e_t)$ is given by

$$\eta_t(e_t) Q_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \left(Q_{t+1} + r_{ht+1} \right) \mid \frac{y_{t+1}}{y_t} = e_t \right\} + \kappa_t Q_t \pi_t(e_t) + \mu_t(e_t). \tag{67}$$

The first order condition with respect to $b_t(e_t)$ is

$$\eta_t(e_t) = \beta R_t \mathbb{E}_t \left[\lambda_{t+1} | \frac{y_{t+1}}{y_t} = e_t \right] + \pi_t(e_t) \tag{68}$$

In these first-order conditions, the term $\mathbb{E}_t[\cdot \mid \frac{y_{t+1}}{y_t} = e_t]$ is the expectation operator for member with belief that $\frac{y_{t+1}}{y_t} = e_t$.

A competitive equilibrium is a collection of prices $\{Q_t, R_t, r_{ht}\}$ and allocations $\{c_t, a_t, s_{ht}, h_t(e_t), b_t(e_t)\}$, such that

(1) Taking the prices as given, the allocations solve the household's utility maximizing problem.
(2) Markets for goods, rental, housing, and credit all clear, so that

$$c_t = y_t, \tag{69}$$

$$s_{ht} = \int h_{t-1}(e_{t-1}) dF(e_{t-1}), \tag{70}$$

$$\int h_t(e_t) dF(\varepsilon_t) = 1, \tag{71}$$

$$\int b_t(e_t) dF(e_t) = 0. \tag{72}$$

Notice since $\int h_{t-1}(e_{t-1}) dF(e_{t-1}) = 1$, we then have $s_{ht} = 1$.

V.2. **Equilibrium characterization.** We now characterize the equilibrium.

After imposing the market clearing conditions that $c_t = y_t$ and $s_{ht} = 1$, Eq. (65) implies that

$$r_{ht} = \varphi y_t. \tag{73}$$

Thus, the equilibrium rent is a function of income alone, and does not depend on the credit supply conditions κ_t.

We conjecture that the equilibrium house price satisfies $Q_t = q(\kappa_t) y_t \equiv q_t y_t$. The price-rent ratio is then given by $\frac{Q_t}{r_{ht}} = \frac{q_t}{\varphi}$, which is proportional to q_t.

We also conjecture that there is a cutoff point e_t^* in the support of the distribution of beliefs $(F(e))$ such that those members (traders) with optimistic beliefs (i.e., $e_t \geq e_t^*$) buy houses and those with pessimistic beliefs (i.e., $e_t < e_t^*$) sell.

The marginal trader with belief e_t^* is a buyer, although the collateral constraint (61) is not binding. Thus, we have $\pi(e_t^*) = \mu(e_t^*) = 0$. The first-order condition (67) implies that the return on housing for the marginal agent is given by

$$\frac{\eta(e_t^*)}{\lambda_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{Q_{t+1} + r_{h,t+1}}{Q_t} \mid \frac{y_{t+1}}{y_t} = e_t^* \right\} = \beta \mathbb{E}_t \frac{q_{t+1} + \varphi}{q_t}, \tag{74}$$

where, to obtain the second equality, we have used the conditions that $\lambda_t = \frac{1}{y_t}$ and that $Q_t = q_t y_t$ and $r_{ht} = \varphi y_t$.

The marginal trader with belief e_t^* also trade bonds. The first-order condition (68) implies that the return on bond holding is given by

$$\frac{\eta(e_t^*)}{\lambda_t} = \beta R_t \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \mid \frac{y_{t+1}}{y_t} = e_t^* \right\} = \beta \frac{R_t}{e_t^*}. \tag{75}$$

In an equilibrium, the marginal trader is indifferent between the two types of assets: houses and bonds. Thus, the expected return on housing and on bond holdings should be equalized. In particular, from Equations (74) and (75), we obtain

$$q_t = \frac{e_t^*}{R_t} \mathbb{E}_t \left[q_{t+1} + \varphi \right]. \tag{76}$$

The term $\frac{e_t^*}{R_t}$ is analogous to the dividend discount model of Gordon (1959). Here, the marginal trader's belief about future income growth e_t^* can be interpreted as the dividend growth rate in the Gordon model. The difference is that e_t^* is endogenous, and it responds to changes in credit conditions summarized by κ_t.

Optimistic traders with $e_t \geq e_t^*$ are house buyers who face binding collateral constraints, whereas pessimistic traders with $e_t < e_t^*$ are sellers. Analogous to the benchmark heterogeneous-agent model in Section IV, the equilibrium cutoff point e_t^* in this model also increases with the LTV κ_t. This result is formally stated in the proposition below.

Proposition V.1. *The equilibrium cutoff point e_t^* is given by*

$$F(e_t^*) = \kappa_t, \tag{77}$$

where $F(\cdot)$ is the cumulative density function of the belief shocks. Clearly, the equilibrium cutoff point e_t^* is strictly increasing in the LTV κ_t. That is,

$$\frac{\partial e_t^*}{\partial \kappa_t} > 0. \tag{78}$$

Proof. The proof is similar to that in the benchmark heterogeneous-agent model. Pessimistic traders with $e_t < e_t^*$ are house sellers with $h(e_t) = 0$. Optimistic traders with $e_t \geq e_t^*$ face binding collateral constraints. After imposing the market clearing conditions in the family budget constraint (63), the binding collateral constraint (61) and the flow-of-funds constraint (60) for the optimistic traders imply that $h(e_t) = \frac{1}{1-\kappa_t}$. House market clearing implies that

$$1 = \int h(e)dF(e) = \int_{e_t^*} \frac{1}{1-\kappa_t} dF(e), \tag{79}$$

which leads to the solution to e_t^* in Eq. (77). Eq. (78) immediately follows from differenting Eq. (77). □

As in the benchmark model, a credit supply expansion that raises κ_t also raises the house price but has no effect on the rent. This result is formally stated in the next proposition.

Proposition V.2. *An increase in κ_t raises the house price Q_t but has no effect on the rent r_{ht}. That is,*

$$\frac{\partial Q_t}{\partial \kappa_t} > 0, \quad \frac{\partial r_{ht}}{\partial \kappa_t} = 0. \tag{80}$$

Proof. We provide a proof in the appendix. □

Following the same logics as in the benchmark model, it is straightforward to show that the house trading volume TV_t in this model is given by

$$TV_t = \max\{\kappa_t, \kappa_{t-1}\} \tag{81}$$

Thus, the trading volume is an increasing function of the LTV ratio κ_t and positively correlated with the house price.

VI. Empirical evidence

Our benchmark heterogeneous-agent model predicts that a credit supply shock can have a large impact on the house price, but not on rent. We now present some empirical evidence that supports this theoretical prediction.

We follow the approach in Mian et al. (2017) and identify a credit supply shock as an acceleration in credit growth during periods with low mortgage spreads. We use both international data and U.S. regional data. For the international data, we use an unbalanced panel of 25 advanced economies, with annual data covering the periods from 1965 to 2013. We measure credit growth by the year-over-year changes in the household debt-to-GDP ratio in each country, as in Mian et al. (2017). The mortgage spread for each country is the spread between the mortgage interest rates and the 10-year sovereign bond yields. For the U.S. regional data, we use an unbalanced panel of 21 Metropolitan Statistical Areas (MSAs) in the United States, with annual data covering the years from 1978 to 2017. Credit growth is measured by the year-over-year changes in the housing loan-to-price ratio in each MSA, and the mortgage spreads are the effective mortgage interest rates minus the 10-year U.S. Treasury yields.[7]

Using each set of panel data (international or regional), we estimate the dynamic responses of the housing market variables (price, rent, the price-rent ratio) to a credit supply shock using the local projections approach of Jorda (2005). In particular, we estimate the instrumental-variable local projections (IV-LP) model

$$\log Y_{i,t+h} - \log Y_{it} = \alpha_0^h + \sum_{j=0}^{8} \beta_j^h \Delta D_{i,t-j}^{HH} + \gamma_i^h + u_{i,t+h}^h, \tag{82}$$

where $Y_{i,t+h}$ denotes the variable of interest (the house price, rent, or the price-to-rent ratio) in country (or region) i and year $t+h$, ΔD_{it}^{HH} denotes the credit growth rate in country (or region) i in year t from $t-1$, γ_i^h captures the country (region) fixed effects, and the term $u_{i,t+h}^h$ denotes a regression residual. The parameters α_0^h and β_j^h are common for all countries (regions). Following Mian et al. (2017), we instrument credit growth by a dummy variable that equals one if the mortgage spread is below the median and zero otherwise. The F-statistics from the first-stage regressions suggest that we do not have a problem with weak instruments, because the instrumental variable (mortgage spread dummy) here is highly and positively correlated with the endogenous variable (credit growth) in the IV-LP regression, both for the international sample and for the regional sample (see Tables A2 and A4 in the appendix).

Figure 4 shows that the estimated dynamic responses of the house price, the rent price, and the price-to-rent ratio following a credit supply expansion, using both the international data (the left column) and the U.S. regional data (right column). In each case, a positive credit supply shock leads to large, persistent, and statistically significant increases in the house price. In both the international data and the MSA data, a one percentage point increase in credit supply growth leads to a roughly 7.5 percent increase in the house price at the peak, although the house price responses estimated from the international data are more persistent than those from the U.S. regional data. This finding is consistent with the literature (Mian et al., 2017; Jordà et al., 2016). In contrast, the responses of the rent to a credit supply shock is small

[7]Details of the data and summary statistics are presented in the Appendix B (Tables A1 and A3).

and statistically insignificant, as shown in the middle panels of the figure. The estimated rent responses from the MSA data become statistically significant after 3 years following the impact of the shock, but the magnitude of the rent responses is dwarfed by the house price responses.

Unlike a housing demand shock in the standard representative-agent model, a credit supply shock in our heterogeneous model can generate responses consistent with these empirical findings. That is, a credit supply shock leads to a large and persistent increase in the price-to-rent ratio (bottom panel of Figure 4).

VII. Conclusion

We provide a microeconomic foundation for aggregate housing demand shocks using a tractable heterogeneous-agent framework. The model predicts that a credit supply shock that raises the LTV ratio boosts aggregate housing demand and the house price, without generating counterfactually large fluctuations in the rent. The heterogeneous-agent framework also allows us to study the fluctuations of the house trading volume: a credit supply shock generates a positive correlation between the trading volume and the house price.

Housing demand shocks are popular reduced-form shocks used by the standard macroeconomic models to study the linkage between the house price and the macroeconomic activity. Understanding the microeconomic forces that underpin these reduced form shocks, as well as how the house price and the rent respond to these micro-founded factors, is an important first step for designing appropriate policy interventions in the housing market. We contribute to this important research area by providing a tractable framework whose key theoretical predictions are consistent with the data.

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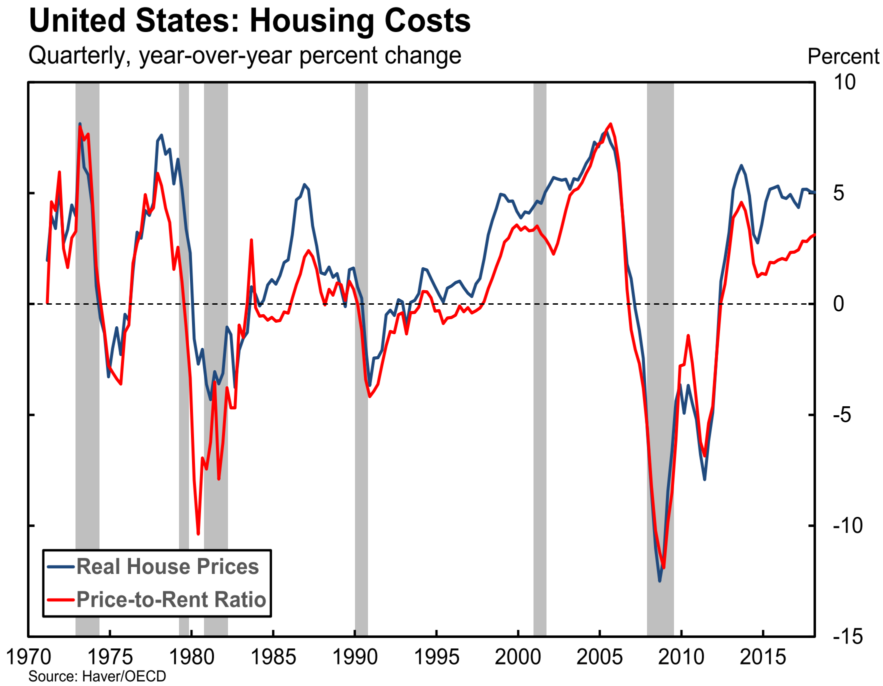

FIGURE 1. The real house prices and the price-to-rent ratio in the United States.

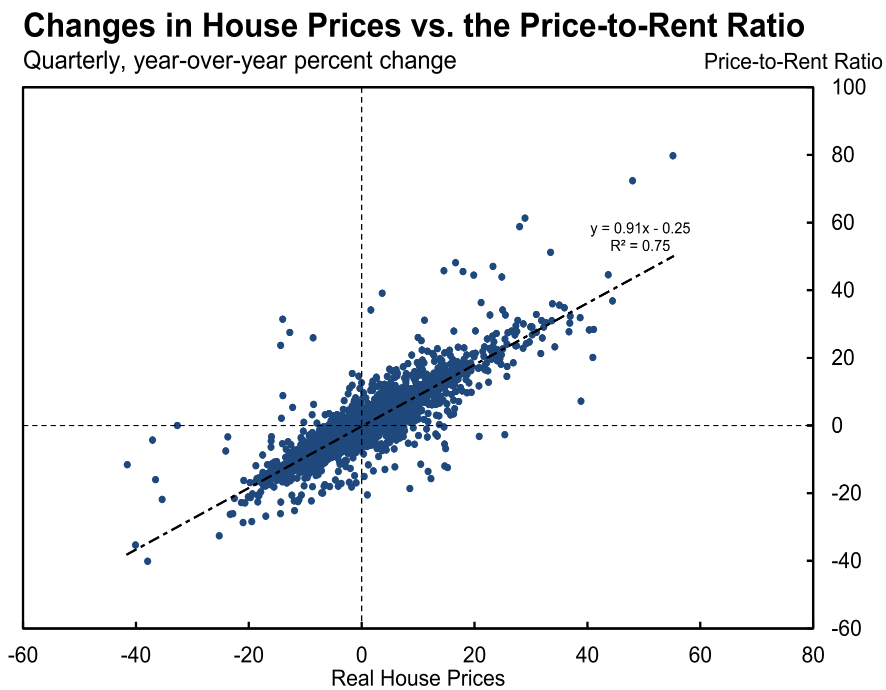

Figure 2. The real house prices and the price-to-rent ratio in OECD countries

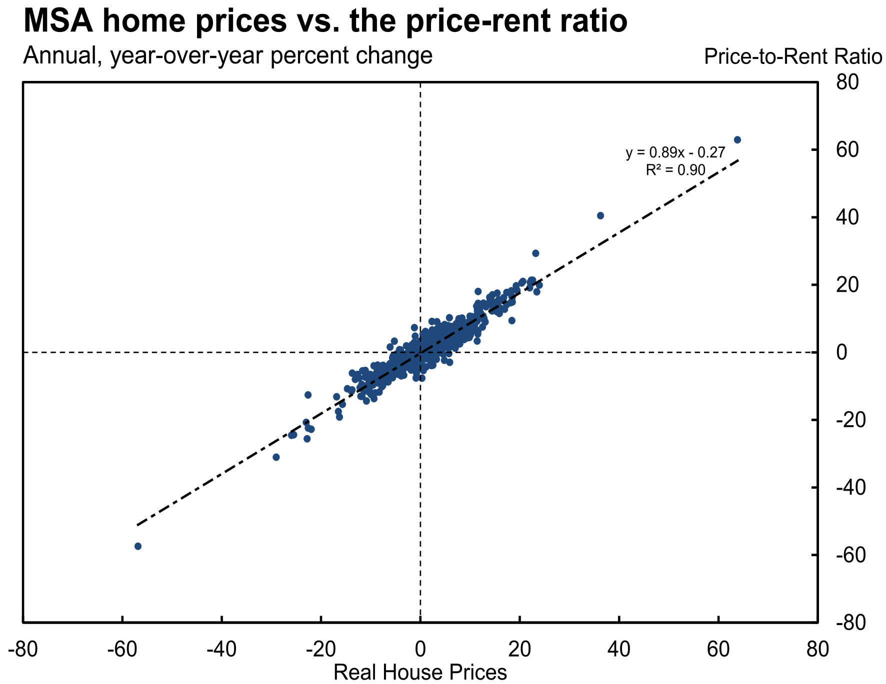

Figure 3. The real house prices and the price-to-rent ratio in U.S. MSA regions

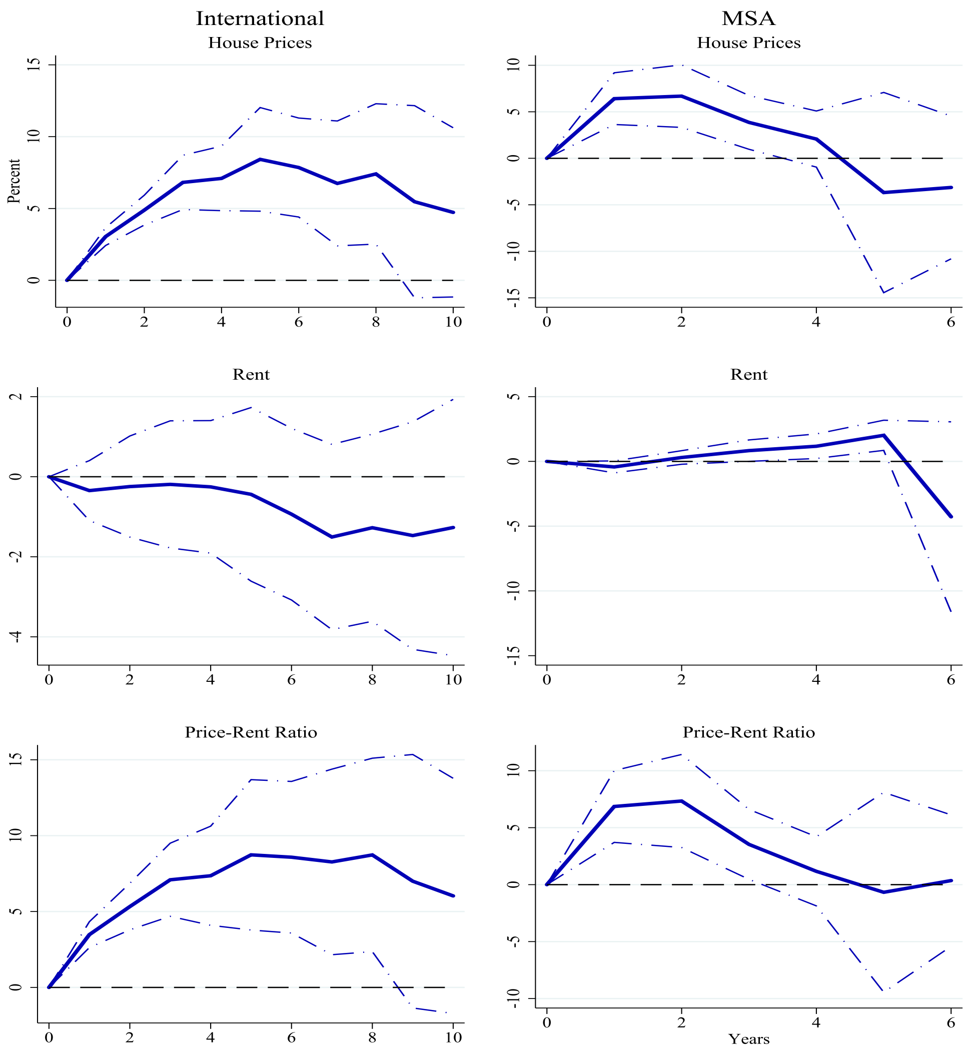

FIGURE 4. The dynamic responses of the house price, rent, and the price-to-rent ratio to a positive credit supply shock. The left column ("International") shows the responses estimated using data from 25 OECD economies. The right column ("MSA") shows the responses estimated using data from 21 U.S. MSAs. The solid line in each panel shows point estimate of the dynamic responses of each variable following an increase in credit supply using the local projection approach of Jorda (2005), and the dashed lines show the one standard-deviation confidence bands of the estimated responses.

Appendix

Appendix A. Proof of Proposition V.2

Proof. Eq (73) shows that $r_{ht} = \varphi y_t$ is independent of κ_t. It remains to show that $\frac{\partial Q_t}{\partial \kappa_t} > 0$.

Rewrite Eq. (76) here for convenience of referencing:

$$q_t = \frac{e_t^*}{R_t} \mathbb{E}_t \left[q_{t+1} + \varphi \right]. \tag{A.1}$$

We now consider two cases.

(1) First we consider $e_t \geq e_t^*$, we have $\pi_t(e_t) > 0$, implying that the borrowing constraint (61) is binding. Imposing market clearing conditions, we obtain $h_t(e_t) = \frac{1}{1-\kappa_t} > 0$. Thus, $\mu_t(e_t) = 0$. Equations (67) and (68) together imply that

$$\pi_t(e_t) = \frac{\beta}{(1-\kappa_t)\, y_t q_t} \left[\mathbb{E}_t(q_{t+1} + \varphi) - R_t \frac{q_t}{e_t} \right] \ , \tag{A.2}$$

and

$$\begin{aligned} \eta_t(e_t) &= \beta R_t \frac{1}{y_t e_t} + \frac{\beta}{(1-\kappa_t)\, y_t q_t} \left[\mathbb{E}_t \left[q_{t+1} + \varphi \right] - R_t \frac{q_t}{e_t} \right]. \\ &= \beta R_t \frac{1}{y_t e_t} + \frac{\beta R_t q_t}{(1-\kappa_t)\, y_t q_t} [\frac{1}{e_t^*} - \frac{1}{e_t}] \end{aligned} \tag{A.3}$$

where the second line has used the fact $\mathbb{E}_t(q_{t+1} + \varphi) = R_t \frac{q_t}{e_t^*}$.

(2) In this case with $e < e_t^*$, we have $\pi_t(e_t) = 0$, such that

$$\eta_t(e_t) = \beta R_t \frac{1}{y_t e_t}, \tag{A.4}$$

and

$$\begin{aligned} \mu_t(e_t) &= \beta R_t \frac{1}{e_t} q_t - \beta \mathbb{E}_t[q_{t+1} + \varphi] \\ &= \beta R_t q_t (\frac{1}{e_t} - \frac{1}{e_t^*}) > 0 \end{aligned} \tag{A.5}$$

Since $\mu_t(e_t) h_t(e_t) = 0$, we have $h_t(e_t) = 0$.

With the expression of $\eta_t(e_t)$, we can rewrite equation (66) as

$$\frac{1}{c_t} = \beta R_t \frac{1}{y_t} \int \frac{1}{e} dF(e) + \frac{\beta R_t}{(1-\kappa_t)\, y_t} \int_{e_t^*} [\frac{1}{e_t^*} - \frac{1}{e}] dF(e),$$

or

$$1 = \beta R_t \int_{e_{\min}}^{e_{\max}} \frac{1}{e} dF(e) + \frac{\beta R_t}{(1-\kappa_t)} \int_{e_t^*} [\frac{1}{e_t^*} - \frac{1}{e}] dF(\varepsilon), \tag{A.6}$$

Finally housing market clearing condition yields

$$\frac{1}{1-\kappa_t}\int_{e_t^*}^{\varepsilon_{\max}} dF(\varepsilon) = 1. \tag{A.7}$$

We then have

$$\begin{aligned}
\frac{e_t^*}{R_t} &= \beta e_t^* \int \frac{1}{e} dF(e) + \frac{\beta}{(1-\kappa_t)} \int_{e_t^*} [1 - \frac{e_t^*}{e}] dF(e) \\
&= \beta e_t^* \int \frac{1}{e} dF(e) + \frac{\beta}{1-F(e_t^*)} \int_{e_t^*} [1 - \frac{e_t^*}{e}] dF(e) \\
&= \beta + \beta e_t^* [\int \frac{1}{e} dF(\varepsilon) - \frac{1}{1-F(e_t^*)} \int_{e_t^*}^{e_{\max}} \frac{1}{e} dF(e)]. \qquad (A.8)
\end{aligned}$$

Denote $\phi(e_t^*) = \frac{1}{1-F(e_t^*)} \int_{e_t^*}^{e_{\max}} \frac{1}{e} dF(e)$, we have

$$\begin{aligned}
\frac{\phi'(e_t^*)}{\phi(e_t^*)} &= \frac{f(e_t^*)}{1-F(e_t^*)} - \frac{f(e_t^*)}{\int_{e_t^*}^{e_{\max}} \frac{e_t^*}{e} dF(e)} \\
&< \frac{f(e_t^*)}{1-F(e_t^*)} - \frac{f(e_t^*)}{\int_{e_t^*}^{e_{\max}} 1 dF(e)} \\
&= \frac{f(e_t^*)}{1-F(e_t^*)} - \frac{f(e_t^*)}{1-F(e_t^*)} = 0 \qquad (A.9)
\end{aligned}$$

Hence we have $\phi'(e_t^*)$. It is then obvious that

$$\frac{\partial\left(e_t^*/R_t\right)}{\partial e_t^*} > 0 \tag{A.10}$$

Finally, since $F(e_t^*) = \kappa_t$, it is clear that $\frac{\partial(e_t^*)}{\partial \kappa_t} > 0$.

It then follows from Eq. (A.1) that $\frac{\partial q_t}{\partial \kappa_t} > 0$, implying that $\frac{\partial Q_t}{\partial \kappa_t} > 0$ since $Q_t = q_t y_t$. $\square$

Appendix B. Data and regressions

In the empirical analysis in Section VI, we use both cross-country data and U.S. regional data.

B.1. **International data.** The cross-country data are an unbalanced panel of 25 advanced economies, covering the years from 1965 to 2013. The time series in each country includes the household debt-to-GDP ratio, the mortgage spread, the house price, and the rent. The household debt-to-GDP ratio and the mortgage spread are the same as those used by Mian et al. (2017). The house price and the rent series are taken from the OECD Main Economic Indicators through Haver Analytics. We deflate the nominal rent series using the consumer price index in each country (or the Harmonized Index of Consumer Prices for the European countries in our sample).

Table A1 presents the list of the countries and some summary statistics of the data.

TABLE A1. Summary of countries in the sample and key statistics

Country	Start Year	Mean ΔD^{HH}	SD ΔD^{HH}	Mean I^{MS}	SD I^{MS}	Mean $\Delta ln(P)$	SD $\Delta ln(P)$	Mean $\Delta ln(R)$	SD $\Delta ln(R)$
Australia	1979	2.22	2.58	1.14	1.65	3.19	6.60	0.56	2.03
Austria	1997	0.69	1.30	0.98	0.54	1.62	2.82	1.09	2.05
Belgium	1982	0.86	1.13	0.94	0.71	2.40	4.15	0.46	1.38
Canada	1971	1.44	2.40	2.10	0.72	2.51	6.28	-1.23	2.17
Czech Republic	2003	2.06	1.21	1.40	0.61	-2.88	2.71	2.98	4.85
Denmark	1996	3.65	4.06	0.31	0.52	2.87	7.90	0.58	0.77
Finland	1981	1.22	2.47	-0.33	1.28	2.19	9.40	0.09	3.57
France	1979	1.10	1.22	0.36	0.85	2.04	5.44	0.82	1.37
Germany	1972	0.50	1.82	0.98	0.69	-0.30	2.39	0.39	1.67
Greece	2000	4.00	2.18	1.23	0.36	0.06	8.67	-0.40	2.79
Hungary	2000	2.14	3.21	3.76	2.56	-7.07	5.03	1.46	3.32
Ireland	2004	5.01	8.46	0.54	0.73	-4.10	11.50	3.08	18.87
Italy	1996	1.59	1.16	1.52	1.06	0.57	5.20	0.62	1.67
Japan	1981	0.57	1.94	0.64	0.80	-0.19	4.02	0.46	1.20
Korea, Rep.	2001	2.86	3.28	-0.04	0.19	2.06	4.87	-0.38	1.69
Mexico	2005	0.51	0.65	7.26	0.78	0.01	2.07	-1.44	0.83
Netherlands	1992	3.76	2.73	1.24	0.72	3.16	6.17	1.17	1.65
Norway	1988	0.56	3.44	1.14	1.16	2.88	7.93	0.87	1.38
Poland	2003	2.18	2.46	0.96	0.87	-5.20	1.86	0.77	1.52
Portugal	1991	3.50	2.49	1.56	2.09	-1.22	4.03	1.05	3.08
Spain	1982	1.80	2.69	1.01	1.37	3.01	9.87	0.63	2.13
Sweden	1987	1.19	2.83	0.32	0.58	3.35	7.20	1.93	3.90
Switzerland	2001	1.25	3.22	0.98	0.58	2.91	2.88	0.87	0.88
U.K.	1974	1.55	2.52	0.70	0.67	2.24	9.31	1.61	3.13
U.S.	1965	0.69	2.21	1.74	0.54	1.22	4.19	0.66	1.48

Notes: This table lists the 25 countries and the years covered used in the international data sample. The variable ΔD^{HH} denotes the year-over-year changes in the household debt-to-GDP ratio, I^{MS} denotes the mortgage spread dummy, which equals one if the mortgage spread is below the median and zero otherwise (the mortgage spread is the difference between the mortgage interest rate and the 10-year sovereign bond yields), $\Delta ln(P)$ denotes the year-over-year log-changes in the real house price, and $\Delta ln(R)$ denotes the year-over-year log-changes in the real rent.

Table A2 presents the first-stage regression results in our instrumental variable local projection regression using the international data.

B.2. **U.S. regional data.** The U.S. regional data are an unbalanced panel, consisting of 21 MSAs, covering the years from 1978 to 2017. The time series in each MSA includes the housing loan-to-price ratio and the effective mortgage interest rate, both taken from the Federal Housing Finance Board (FHFB). The effective mortgage rate is defined as the contract mortgage rate plus fees and charges amortized over a 10-year period, the estimated average life of conventional mortgages. The mortgage spread used in our regression is the spread between the effective mortgage rates and the 10-year Treasury yields. The data include the house price index in each MSA from the Federal Housing Finance Agency (FHFA), the rent index, which is measured by the "rent of primary residence" in the expenditure categories of the consumer price index (CPI-All Urban Consumers) for each MSA. We convert the nominal house price and the nominal rent into real units by using the MSA-level CPI.

Table A3 presents the list of the MSAs and some summary statistics of the data.

Table A2. First-stage regression: Dependent variable is ΔD_{it}^{HH}

	Real Rent	Real Housing Prices
	(1)	(2)
I_t^{MS}	0.68***	0.71***
	(0.21)	(0.21)
Δd_{t-1}^{HH}	0.40***	0.39***
	(0.05)	(0.05)
Δd_{t-2}^{HH}	0.05	0.06
	(0.05)	(0.05)
Δd_{t-3}^{HH}	0.08**	0.08**
	(0.04)	(0.04)
Δd_{t-4}^{HH}	0.06	0.08
	(0.07)	(0.07)
Δd_{t-5}^{HH}	-0.09	-0.09
	(0.10)	(0.10)
Δd_{t-6}^{HH}	0.03	0.02
	(0.07)	(0.07)
Δd_{t-7}^{HH}	-0.15***	-0.14**
	(0.05)	(0.05)
Δd_{t-8}^{HH}	-0.16***	-0.15***
	(0.05)	(0.05)
Observations	513	494
F-Stat	30.02	30.99

Notes: This table shows the first-stage regression results for the local projections model specified in (82). The two columns correspond to the two different local projection specifications, one for the rent and the other for the house price. The first-stage regression in each case regress the log-growth rate of the household debt-to-GDP ratio ΔD_{it}^{HH} in country i and year t on its own lags and also on the instrumental variable, which is the mortgage spread dummy I_{it}^{MS} that equals one if the mortgage spread is below its median and zero otherwise.

Table A3. Summary of countries in the sample and key statistics

MSA	Start Year	Mean ΔD^{HH}	SD ΔD^{HH}	Mean I^{MS}	SD I^{MS}	Mean $\Delta ln(P)$	SD $\Delta ln(P)$	Mean $\Delta ln(R)$	SD $\Delta ln(R)$
ATL	1979	0.13	2.65	1.69	0.99	0.46	4.27	0.35	2.43
BON	1979	0.20	4.39	1.81	1.03	2.40	7.43	0.75	2.44
BTM	1979	0.10	2.39	1.69	0.95	1.04	5.78	1.36	1.86
CHG	1979	0.49	3.68	1.71	0.98	0.59	5.49	0.81	1.73
DAA	1979	0.25	3.12	1.73	0.95	-0.01	7.81	0.09	2.10
DFW	1979	-0.04	3.48	1.72	0.99	-0.01	4.12	0.21	2.68
DNV	1979	0.02	2.86	1.60	1.04	1.33	4.54	0.51	2.37
HON	1979	0.01	3.99	1.35	1.04	2.44	20.53	0.52	1.76
HTN	1979	-0.09	2.55	1.74	1.03	0.22	4.43	0.36	3.17
LNA	1979	0.00	2.87	1.53	1.06	1.84	9.50	0.97	2.19
MIM	1979	0.12	2.95	1.76	1.04	1.21	9.52	0.46	2.18
MSP	1979	0.36	3.06	1.60	1.00	0.53	5.08	0.25	2.16
NYT	1979	0.38	2.54	1.72	1.11	1.88	7.27	0.88	1.44
PHI	1979	0.26	2.36	1.79	1.02	1.59	5.45	0.53	2.04
PHO	2002	-0.04	2.40	2.01	0.68	1.69	14.77	0.60	2.86
SDI	1979	-0.26	3.54	1.40	1.13	1.19	8.80	0.65	3.01
SFC	1979	0.01	3.76	1.44	1.07	2.81	8.18	1.23	2.83
STL	1979	0.18	3.69	1.72	0.99	0.23	3.84	0.09	1.76
STW	1979	0.00	2.81	1.60	1.01	1.95	6.48	0.60	2.34
TMA	1997	-0.06	2.49	2.06	0.64	2.01	10.50	0.72	1.87
WSH	1979	0.10	2.39	1.69	0.95	1.43	6.86	1.30	1.39

Notes: This table lists the 21 MSAs and the years covered in the U.S. regional data sample. The variable ΔD^{HH} denotes the year-over-year changes in the housing loan-to-price ratio, I^{MS} denotes the mortgage spread dummy, which is one if the mortgage spread is below median (the mortgage spread is the difference between the effective mortgage interest rate and the 10-year Treasury yields), $\Delta ln(P)$ denotes the year-over-year log-changes in the real house price, and $\Delta ln(R)$ denotes the year-over-year log-changes in the real rent.

TABLE A4. First-Stage regression with dependent variable ΔD_{it}^{HH}: cross-MSA sample

	Rent	House price
	(1)	(2)
I_{it}^{MS}	0.47**	0.46**
	(0.20)	(0.19)
ΔD_{it-1}^{HH}	-0.11**	-0.12***
	(0.04)	(0.04)
ΔD_{it-2}^{HH}	-0.03	-0.04
	(0.05)	(0.05)
ΔD_{it-3}^{HH}	-0.15***	-0.15***
	(0.05)	(0.05)
ΔD_{it-4}^{HH}	-0.23***	-0.23***
	(0.03)	(0.03)
ΔD_{it-5}^{HH}	-0.06	-0.05
	(0.05)	(0.05)
ΔD_{it-6}^{HH}	-0.09**	-0.09**
	(0.03)	(0.03)
ΔD_{it-7}^{HH}	0.00	-0.01
	(0.03)	(0.03)
ΔD_{it-8}^{HH}	-0.13***	-0.14***
	(0.02)	(0.02)
Observations	574	596
F-Stat	20.39	28.28

Note: The table displays first-stage regression results in the local projection regression specified in (82) using the MSA panel data. Column (1) corresponds to the specification for rent and Column (2) for the house price. The first-stage regression in each case regresses the growth rate of the ratio of household debts to GDP ΔD_{it}^{HH} in country i and year t on its own lags as well as the instrumental variable, which is the mortgage spread dummy I_{it}^{MS} that equals one if the mortgage spread is below its median and zero otherwise.

Appendix C. Heterogeneous preferences for housing

The theoretical insights in our benchmark heterogeneous model are robust to alternative forms of heterogeneity. Here, we consider a simple model with heterogeneous preferences for housing.

C.1. **The model environment.** The model features a large household family with a continuum members, who are subject to an idiosyncratic shocks to utility value of housing services. Such idiosyncratic shocks are meant to capture potential heterogeneity in agents' desires to purchase homes for reasons such as job relocations, schooling choices, and health care needs. To obtain the key insight, we simplify the model by postulating the existence of some implicit financial arrangement to insure non-housing consumption against realizations of idiosyncratic taste shocks.

The idiosyncratic shock ε_t follows an i.i.d. process (across members and across time) drawn from the distribution $F(\cdot)$. Because of the complete insurance against non-housing consumption risks, all members enjoy the same consumption c_t. Housing services, however, must be indexed by ε. The expected utility function of the family is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \varphi \int_0^{\infty} h_t(\varepsilon_t) \varepsilon_t \, dF(\varepsilon_t) \right], \tag{C.1}$$

where $h_t(\varepsilon_t)$ denotes the housing stock owned by the member with shock ε_t and the parameter φ measures the utility weight for housing. Note that φ is constant so that there is no reduced-form aggregate housing demand shock in this heterogeneous-agent model.[8]

The household family faces the budget constraint

$$c_t + a_t = y_t + q_t \int h_{t-1}(\varepsilon_{t-1}) dF(\varepsilon_{t-1}) - \int b_{t-1}(\varepsilon_{t-1}) dF(\varepsilon_{t-1}), \tag{C.2}$$

where the variables have the same interpretations as in the benchmark model.

In the decentralized housing market, a family member with idiosyncratic shock ε_t finances her new house purchase $q_t h_t(\varepsilon_t)$ through both internal funds a_t and external borrowings $b_t(\varepsilon_t)$ at the interest rate R_t, facing the flow-of-funds constraint

$$q_t h_t(\varepsilon_t) \leq a_t + \frac{b_t(\varepsilon_t)}{R_t}. \tag{C.3}$$

[8] For simplicity, we assume that the utility function is linear in housing services. The main theoretical results do not hinge upon this simplifying assumption.

The member ε_t also faces the borrowing constraint

$$\frac{b_t(\varepsilon_t)}{R_t} \leq \kappa_t q_t h_t(\varepsilon_t), \tag{C.4}$$

and the short-sale constraint

$$h_t(\varepsilon_t) \geq 0. \tag{C.5}$$

The family chooses c_t and a_t and each member with ε_t chooses $h_t(\varepsilon_t)$ and $b_t(\varepsilon_t)$ to maximize the utility function (25), subject to the family budget constraint (C.2) as well as the individual member's flow-of-funds constraint (C.3), the borrowing constraint (C.4), and the short-sale constraint (C.5). The initial values of b_{-1} and h_{-1} and the prices q_t and R_t are taken as given.

Denote by $\lambda_t, \eta_t(\varepsilon_t)$, $\pi_t(\varepsilon_t)$ and $\mu_t(\varepsilon_t)$ the Lagrangian multiplers associated with constraints (C.2), (C.3), (C.4), and (C.5) respectively.

The first-order condition with respect to a_t is

$$\lambda_t = \int \eta_t(\varepsilon_t) dF(\varepsilon_t), \tag{C.6}$$

An extra unit of funds allocated to family members for purchasing new houses reduces family consumption by one unit with the utility cost λ_t. The utility gain from this allocation is the average shadow value of newly purchased houses across all members.

The first order condition with respect to $h_t(\varepsilon_t)$ is

$$\eta_t(\varepsilon_t) q_t = \varphi \varepsilon_t + \beta \mathbb{E}_t \lambda_{t+1} q_{t+1} + \kappa_t q_t \pi_t(\varepsilon_t) + \mu(\varepsilon_t). \tag{C.7}$$

For member ε, the utility cost of purchasing a unit of housing is $q_t \eta(\varepsilon_t)$. The extra unit of housing yields the utility gain $\varphi \varepsilon_t$ from housing services. The unit of housing can be sold at the price q_{t+1} next period, yielding the present value $\beta \mathbb{E}_t \lambda_{t+1} q_{t+1}$. Having an extra unit of housing also increases the collateral value and helps relax the collateral constraint, with the shadow utility value of $\kappa_t q_t \pi_t(\varepsilon_t)$. In addition, having an extra unit of housing also relaxes the short-sale constraint, with the shadow value of $\mu(\varepsilon_t)$. The optimal choice of $h_t(\varepsilon_t)$ implies that the marginal cost must equal the sum of these marginal benefits.

The first-order condition with respect to $b_t(\varepsilon_t)$ is

$$\eta_t(\varepsilon_t) = \beta R_t \mathbb{E}_t \lambda_{t+1} + \pi_t(\varepsilon_t). \tag{C.8}$$

Borrowing an extra unit of funds yields the utility value $\eta_t(\varepsilon_t)$. The family repays the debt next period at the interest rate R_t, with the present value of the utility cost of $\beta R_t \mathbb{E}_t \lambda_{t+1}$. The increase in borrowings tightens the collateral constraint with the utility cost $\pi_t(\varepsilon_t)$. For an optimal choice of $b_t(\varepsilon_t)$, the utility gain must equal to the sum of these two utility costs.

C.2. **Equilibrium.** A competitive equilibrium is a collection of prices $\{q_t, R_t\}$ and allocations $\{c_t, a_t, h_t(\varepsilon_t), b_t(\varepsilon_t)\}$ such that

(1) taking the prices as given, the allocations solve the household's utility maximizing problem;
(2) the markets for goods, housing, and credit all clear so that

$$c_t = y_t \tag{C.9}$$

$$\int h_t(\varepsilon)dF(\varepsilon) = 1 \tag{C.10}$$

$$\int b_t(\varepsilon)dF(\varepsilon) = 0. \tag{C.11}$$

C.3. **Characterizing the equilibrium.** Family members with sufficiently high marginal utility (i.e., high ε_t) are home buyers, and they finance their house purchases using both internal funds and external debt, facing binding borrowing constraints. Members with sufficiently low marginal utility of housing are sellers, facing binding short-sale constraints (given the linearity of preferences in housing services). There exists a cutoff point ε_t^* in the support of the distribution $F(\varepsilon)$ such that the marginal trader with idiosyncratic preference shock ε_t^* is indifferent between buying or selling houses.

As in the benchmark model, it is easy to show that traders that have sufficiently high valuations of housing services (i.e., those with $\varepsilon_t \geq \varepsilon_t^*$) face binding credit constraints and choose the same quantity of housing, whereas those with low valuation of housing services choose to sell, subject to the short-sale restriction. The equilibrium allocation of houses is given by

$$h_t(\varepsilon_t) = \begin{cases} \frac{1}{1-\kappa_t} & \text{if } \varepsilon \geq \varepsilon_t^* \\ 0 & \text{otherwise} \end{cases} \tag{C.12}$$

The housing allocations in Eq. (C.12) together with the house market clearing condition imply that the cutoff point is given by

$$F(\varepsilon_t^*) = \kappa_t. \tag{C.13}$$

Thus, an increase in the LTV ratio κ_t increases the cutoff point ε_t^*, such that the marginal trader assigns a higher value to housing and is willing to pay more. This would in turn boost aggregate housing demand and raise the house price.

The marginal agent with ε_t^* has the housing Euler equation

$$q_t \frac{\eta_t^*}{\lambda_t} = \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} + MRS_t^*, \tag{C.14}$$

where $\eta_t^* = R_t \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t}$ is an increasing function of the risk-free interest rate R_t and the term MRS_t^* denotes the marginal agent's MRS between housing and non-housing consumption given by

$$MRS_t^* = \frac{\varphi}{\lambda_t} \varepsilon_t^*. \tag{C.15}$$

Thus, for any given interest rate, the house price increases with MRS_t^*.

Since the cutoff point ε_t^* increases with κ_t [see Eq. (C.13)], a credit supply shock that relaxes the credit constraint raises the MRS of the new marginal agent from the original equilibrium. This increases the valuation of housing services for the marginal agent and all house buyers with $\varepsilon_t \geq \varepsilon_t^*$, boosting aggregate housing demand and the equilibrium house price.

We can obtain an explicit expression for aggregate housing demand. Integrating the individual housing Euler equation (C.7) across all members and using Eq. (C.6), we obtain the aggregate housing Euler equation

$$q_t \lambda_t = \mathbb{E}_t \beta \lambda_{t+1} q_{t+1} + \varphi \int_0^{\bar{\varepsilon}} \varepsilon dF(\varepsilon) + \kappa_t q_t \int_{\varepsilon_t^*}^{\bar{\varepsilon}} \pi_t(\varepsilon) dF(\varepsilon) + \int_0^{\bar{\varepsilon}} \mu_t(\varepsilon) dF(\varepsilon), \tag{C.16}$$

where the Lagrangian multiplier $\mu_t(\varepsilon)$ for the no short-sale constraint is given by $\mu_t(\varepsilon) = \varphi \max\{\varepsilon_t^* - \varepsilon, 0\}$.

Equation (C.16) can be written in the compact form

$$q_t \lambda_t = \mathbb{E}_t \beta \lambda_{t+1} q_{t+1} + \xi(\kappa_t), \tag{C.17}$$

where

$$\xi(\kappa_t) \equiv \underbrace{\varphi \mathbb{E}(\varepsilon)}_{\text{Average marginal utility}} + \underbrace{\kappa_t \frac{1}{1-\kappa_t} \int_{\varepsilon_t^*}^{\bar{\varepsilon}} \varphi (\varepsilon_t - \varepsilon_t^*) dF(\varepsilon_t)}_{\text{Liquidity premium } \Pi_t} + \underbrace{\varphi \int_0^{\varepsilon_t^*} (\varepsilon_t^* - \varepsilon_t) dF(\varepsilon_t)}_{\text{Option value } \mathcal{O}_t}. \tag{C.18}$$

The first term is the average marginal utility of housing across all agents. The implicit rent is given by

$$r_{ht} = \frac{\varphi}{\lambda_t} \int_0^{\bar{\varepsilon}} \varepsilon dF(\varepsilon) = \varphi c_t. \tag{C.19}$$

Equation (C.18) shows that aggregate housing demand and the house price depend on three forces: (i) the average MRS between housing services and non-housing consumption across all agents (the implicit rent), (ii) the liquidity premium deriving from binding collateral constraints for a subset of agents with high MRSs, and (iii) the option value for avoiding a binding short-sale constraint. It is the liquidity premium and the option value that drive a wedge between the house price and the rent, creating room for potentially large fluctuations in the price-to-rent ratio with changes in the leverage condition.

Clearly, if $\xi(\kappa_t) = \varphi_t$, then the equilibrium house price in the heterogeneous-agent model here coincides with that in the representative agent model, providing a microeconomic foundation for aggregate housing demand shocks.

As in the benchmark model with belief heterogeneity, the model here with heterogenous tastes for housing is also capable of generating large fluctuations in the house price relative to those in the rent. Consider an example with the uniform distribution $F(\varepsilon) = \frac{\varepsilon}{\bar{\varepsilon}}$ for the idiosyncratic taste shocks. We can obtain a closed-form expression for the aggregate housing demand shifter $\xi(\kappa_t)$. In particular, the liquidity premium is given by

$$\Pi_t = \varphi(1 - \kappa_t).$$

Thus, the liquidity premium decreases with κ_t. A relaxation of the credit constraint reduces the shadow value of internal funds, leading to a lower liquidity premium. On the other hand, it directly raises housing demand by increasing the borrowing capacity. The full impact is given by $\kappa\Pi(\kappa_t) = \varphi\kappa_t(1-\kappa_t)$. The increase in borrowing capacity offsets the decrease in liquidity premium and the net effect is concave in κ_t. The option value $\mathcal{O}_t$ is given by

$$\mathcal{O}_t = \varphi\kappa_t^2.$$

Thus, $\xi(\kappa_t)$ is given by

$$\xi(\kappa_t) = \varphi + \varphi\kappa_t(1 - \kappa_t) + \varphi\kappa_t^2 = \varphi(1 + \kappa_t), \tag{C.20}$$

Thus, housing demand $\xi(\kappa_t)$ increases with κ_t, implying that the house price increases with κ_t as well. Since the average marginal utility of housing (φ) is independent of κ_t, changes in κ_t do not affect the rent. Thus, the model here is capable of generating an arbitrarily large volatility of the house price relative to that of the rent, as in our benchmark model with heterogeneous beliefs.

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