Asset Pricing with Concentrated Ownership of Capital and Distribution Shocks*

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Abstract

This paper develops a production-based asset pricing model with two types of agents and concentrated ownership of physical capital. A temporary but persistent “distribution shock” causes capital’s share of income to fluctuate in a procyclical manner, consistent with U.S. data. The concentrated ownership model significantly magnifies the equity risk premium relative to a representative-agent model because the capital owners’ consumption is more-strongly linked to volatile dividends from equity. With a steady-state risk aversion coefficient around 6, the model delivers an equity premium of nearly 5 percent relative to short-term bonds and a premium of about 2 percent relative to long-term bonds.

Keywords: Asset Pricing, Equity Premium, Term Premium, Distribution Shocks, Income Inequality.

JEL Classification: E25, E32, E44, G12, O40.

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1 Introduction

1.1 Overview

The distribution of wealth in the U.S. economy is highly skewed. The top decile of U.S. households owns approximately 80 percent of financial wealth and about 70 percent of total wealth including real estate.¹ Shares of corporate stock are an important component of financial wealth, representing claims to the physical capital of firms. As recently as 1995, the lowest 75% of U.S. households sorted by wealth owned less than 10% of stocks.²

While the degree of wealth inequality in the U.S. economy has remained relatively steady over time (Kopczuk and Saez, 2004), measures of pre-tax income inequality display a large amount of volatility. Over the sample period from 1918 to 2012, the share of total pre-tax income, including capital gains, going to the top decile of U.S. households exhibits a mean of 40% and a standard deviation of 5.6% (top left panel of Figure 1).³ Capital’s share of income from 1929 to 2012 exhibits a mean of 37% and a standard deviation of 2% (bottom left panel of Figure 1).⁴

The right-hand panels of Figure 1 show that the U.S. historical equity premium is positively correlated with annual changes in both of the income share variables.⁵ Given the concentration of capital ownership in the top decile, fluctuations in the income share variables would be expected to impact stockholder consumption. A presumed link between stockholder consumption and equity prices is the foundation of consumption-based asset pricing models. The correlation coefficient between the equity premium and the change in capital’s share of income (lower right panel) is around 0.3 and statistically significant.⁶ While Figure 1 is suggestive, a recent empirical study by Greenwald, Lettau, and Ludvigson (2014) finds that temporary but persistent “factor share shocks” that redistribute income between stockholders and non-stockholders are an important driver of U.S. stock prices. Motivated by these observations, this paper develops a production-based model of asset pricing with the following features: (1) a stable but highly-skewed distribution of physical capital wealth, and (2) temporary but persistent fluctuations in income shares.

The framework for the analysis is a real business cycle model with two types of agents,

¹See Wolff (2006), Table 4.2, p. 113.
²See Heaton and Lucas (2000), Figure 3, p. 224.
⁴Capital’s share is measured as 1 minus the ratio of employee compensation to gross value added of the corporate business sector. Both series are from the Bureau of Economic Analysis, National Income and Product Accounts, Table 1.14.
⁵The equity premium in U.S. data is measured by the difference between the real return on equity and the real return on short-term bills, from Dimson, Marsh, and Staunton (2002), updated through 2011.
⁶This result is even more striking considering that employee compensation derived from the exercise of company-awarded stock options is labeled as labor income, not capital income. See Lequiller (2002).
called capital owners and workers. Capital owners represent the top decile of earners in the economy. These agents own 100% of the productive capital stock—a setup that roughly approximates the highly concentrated ownership distribution of U.S. stock market wealth. I associate capital owners in the model with U.S. stockholders. The consumption of the capital owners is funded from dividends and wage income. I associate workers in the model with U.S. non-stockholders. The consumption of the workers is funded only from wage income. Since workers do not save, all assets (equity and bonds) are priced by the capital owners. The labor supply of the capital owners is inelastic, consistent with the idea that asset prices are determined in securities markets by agents who remain fully-employed at all times. For simplicity, I also assume that the workers’ labor supply is inelastic.7

I consider two types of shocks: (1) a standard labor-augmenting productivity shock that evolves as a random walk with drift, and (2) a temporary but persistent “distribution shock” that causes capital’s share of total income to fluctuate in a manner consistent with the data plotted in Figure 1. Along similar lines, Young (2004) and Ríos-Rull and Santeulàlia-Llopis (2010) allow for stochastic variation in the capital income share parameter to account for various business cycle facts, but they do not examine the asset pricing implications of this shock.8

I calibrate the volatility of the productivity shock innovation in the model to match the 2.6% standard deviation of real per capita aggregate consumption growth from 1930 to 2012.9 I calibrate the volatility of the distribution shock innovation so that the model matches the 2% standard deviation of capital’s share of income from 1929 to 2012. The AR(1) coefficient for the distribution shock is 0.8, corresponding to the persistence of the U.S. capital income share. Under this baseline calibration, the model exhibits the property that dividend growth is about two times more volatile than aggregate consumption growth. The baseline calibration can be viewed as conservative given that real dividend growth for the S&P 500 stock index is over four times more volatile than real aggregate consumption growth.10 I employ a conservative calibration regarding the volatility of dividend growth in order to match another empirical observation, namely, the relative volatilities of consumption growth for stockholders versus non-stockholders. A study by Malloy, Moskowitz, and Vissing-Jørgensen (2009) finds that the volatility of consumption growth for U.S. stockholders is roughly twice that of non-stockholders.

7 Allowing for elastic labor supply on the part of workers would not change the model’s asset pricing results because workers do not participate in financial markets. Allowing for elastic labor supply on the part of capital owners would introduce an additional mechanism for these agents to smooth their consumption, making it more difficult for the model to achieve a sizeable equity premium.

8 Lansing and Markiewicz (2013) examine the welfare consequences of permanent shifts in the income share parameters of a CES production function.

9 Data on real consumption expenditures on nondurables and services are from the Federal Reserve Bank of St. Louis’ FRED data base.

for the period 1982 to 2004. This calibration target can also be viewed as conservative; a study by Aït-Sahalia, Parker, and Yogo (2004) suggests a much higher relative volatility of stockholder consumption growth based on retail sales data for luxury goods over the period 1961 to 2001.

With a steady state risk aversion coefficient of around 6, the concentrated ownership model delivers an annual equity risk premium of nearly 5% relative to short-term bonds and a premium of about 2% relative to long-term bonds. The corresponding risk premia in U.S. data for the period 1900 to 2011 are higher at 7% and 5% respectively, as documented by Dimson, Marsh, and Staunton (2002, updated). Nevertheless, an otherwise similar representative-agent version of the model delivers equity risk premia of only 1.5% and 0.75%, respectively. Capital owners in the concentrated ownership model demand a high equity premium because their consumption is strongly linked to volatile dividends from equity. The capital owners' consumption growth is more volatile than aggregate consumption growth, while the reverse is true for workers. The higher volatility of the capital owners' consumption growth serves to magnify the equity risk premium for any given level of risk aversion. In a representative-agent economy with iid aggregate consumption growth, the equity risk premium relative to one-period bonds is given by the product of the coefficient of relative risk aversion and the variance of aggregate consumption growth.11 The concentrated ownership model links the equity risk premium to stockholders' consumption growth rather than aggregate consumption growth.

Along the lines of Rudebusch and Swanson (2008), a long-term bond is modeled as a decaying-coupon consol with a Macaulay duration of 10 years. The model's underprediction of the equity risk premium relative to long-term bonds reflects the fact that long-term bonds in the model behave too much like equity—a result also typical of endowment economies.12 Nevertheless, the concentrated ownership model is able to match the 20% standard deviation of real equity returns in long-run U.S. data and delivers about one-third of the observed volatility in the price-dividend ratio for the S&P 500 index. The corresponding standard deviations in the representative agent model are substantially lower. Both models overpredict the volatility of long-term bond returns, again because these bonds behave too much like equity.

As part of the analysis, I investigate how some key model parameters influence the size of the mean equity premium. These include: (1) the curvature parameter in the law of motion for capital that governs the strength of the capital adjustment costs, (2) the standard deviation of the distribution shock innovation, (3) a utility curvature parameter that influences the degree of risk aversion, and (4) a utility habit parameter that allows for time-varying risk aversion.

11 Specifically, we have \( \log \left[ E (R_{t+1}^e) / E (R_{t+1}^d) \right] = \alpha \Var[\log (c_{t+1}^e / c_{t+1}^d)] \), where \( R_{t+1}^e \) is the gross return on equity, \( R_{t+1}^d \) is the gross return on a one-period discount bond (the risk free rate), \( \alpha \) is the coefficient of relative risk aversion, and \( c_t^e \) is real aggregate consumption per capita. For the derivation, see Abel (1994, p. 353).

12 See, for example, Abel (2008), Table 2.
Stronger capital adjustment costs impair the capital owner's ability to smooth consumption, thereby raising the mean equity premium. I show that increasing the curvature of the capital law of motion can produce a sizeable equity premium (of about 1.5%) even in the representative agent version of the model. Since the distribution shock is an important source of consumption risk for capital owners, increasing its volatility raises the mean equity premium. The effect of this shock on the equity premium is much stronger in the concentrated ownership model than in the representative model. Larger values for either the utility curvature parameter or the habit formation parameter serve to raise the mean equity premium. However, if the values become too large, the model will overpredict the volatility of the equity return in the data.

On the macro side, I show that the concentrated ownership model performs reasonably well in matching the business cycle moments of aggregate variables, including the pro-cyclical behavior of capital’s share of income in U.S. data. A positive innovation to the capital income share induced by the distribution shock causes an increase in real output. In simulations, the model delivers a contemporaneous correlation of 0.3 between capital's share of income and the growth rate of real output—consistent with U.S. data over the period 1930 to 2012.

Finally, I show that the equity premium generated by the concentrated ownership model is predictable using the preceeding period’s dividend yield (i.e., the inverse of the price dividend ratio). The estimated coefficient in the predictability regression is similar in magnitude to that obtained using long-run U.S. financial market data.

1.2 Related Literature

The model developed here is most closely related to Danthine and Donaldson (2002) who also employ a setup with capital owners and workers. In their model, workers are not paid their marginal product but instead enter into long-term wage contracts with capital owners. The wage contract is designed to smooth workers’ consumption streams by providing insurance against aggregate shocks, a mechanism they describe as “operational leverage.” A persistent shock to the relative bargaining power of the two groups creates an additional source of risk that must be borne by the capital owners and contributes to a higher equity premium. When the bargaining power shocks are positively correlated with (temporary) productivity shocks, the model can produce an equity premium relative to one-period bonds close to 6%, but the result is accompanied by too much volatility in the one-period bond return, i.e., a standard deviation in excess of 10%. Other counterfactual implications of their model are: (1) the consumption growth of stockholders is 10 times more volatile than aggregate consumption growth and, (2) the standard deviation of model-implied dividend growth is nearly 20%—

13 Further elaboration on the Danthine-Donaldson model can be found in Danthine et al. (2008).
14 See Table 4, Panel B, p. 59 in Danthine and Donaldson (2002).
about twice the volatility of S&P 500 dividend growth.\textsuperscript{15} The model developed here avoids these counterfactual predictions while still delivering a sizeable equity premium.

Guvenen (2009) also develops a model with concentrated ownership of capital. Stockholders price equity while non-stockholders price one-period bonds. As buyers of the one-period bonds, non-stockholders have a very low elasticity of intertemporal substitution which makes them heavily dependent on the bond market to smooth their consumption, thereby producing a low equilibrium bond return, i.e., a low risk free rate. As sellers of the bonds, stockholders have a high elasticity of intertemporal substitution coupled with a risk aversion coefficient equal to 6. Stockholders must bear the risk of countercyclical interest payments to non-stockholders which amplifies the volatility of the stockholders’ consumption streams, thereby raising their required rate of return on equity.\textsuperscript{16} For the baseline model with inelastic labor supply, Guvenen’s model delivers an equity premium relative to one-period bonds of about 5.5\%, but he does not investigate the model’s implications for long-term bonds. It is not clear how long-term bonds would be priced in Guvenen’s model, since it appears that both types of agents would be willing to buy these bonds.

De Graeve et al. (2010) develop a model that combines elements from both Danthine and Donaldson (2002) and Guvenen (2009). They allow for three types of agents, all with elastic labor supply: stockholders who price equity and long-term bonds, bondholders who price one-period bonds, and workers who do not save. They find that the stockholder-bondholder interaction from the Guvenen model is much less effective in generating a large equity premium when the model also includes the stockholder-worker wage bargaining shocks from the Danthine-Donaldson model. De Graeve et al. assume that while one-period bonds are priced by the bondholders, long-term bonds are priced by the stockholders—a setup that seems hard to justify. An important limitation of all the foregoing models is that they abstract from long-run growth—a feature that affects the change in consumption from one period to the next. In contrast, the model developed here is calibrated to match both the mean and volatility of per capita consumption growth in long-run U.S. data.

Papers by Polkovnichenko (2004) and Walentin (2010) show that a permanent increase in the share of dividend income in stockholders’ total income serves to increase the equity premium in endowment economies. A similar mechanism is at work here, except that the distribution shock delivers temporary but persistent fluctuations in the share of dividend income in stockholders’ total income (which consists of dividends and wage income).

As a caveat, it should be noted that model comparisons with the U.S. equity return data pertain only to publically-traded firms. A study by Davis, et al. (2006), p. 119 finds that

\textsuperscript{15}See Table 6, Panel A, p. 62 in Danthine and Donaldson (2002). They do not report the volatility of consumption growth for workers. To address the excessive volatility of dividend growth, they introduce an ad hoc mechanism for smoothing paid out dividends.

\textsuperscript{16}Guo (2004) develops a similar mechanism in the context of an endowment economy.
privately-held firms account for more than two-thirds of total private business employment. The inclusion of privately-held firms in the equity return data would provide a broader measure of the equity risk premium. Moskowitz and Vissing-Jørgensen (2002), p. 765 find that while average equity returns for public and private firms are similar, private equity returns exhibit a lower standard deviation relative to public-firms’ market equity returns. According to these measures, the inclusion of private equity return data would increase the magnitude of the Sharpe ratio in the data that any model would seek to explain.

2 Model

The model consists of workers, capital owners, and competitive firms. There are \( n \) times more workers than capital owners, with the total number of capital owners normalized to one. Naturally, the firms are owned by the capital owners. Workers and capital owners both supply labor to the firms inelastically, but in different amounts.\(^{17}\)

2.1 Workers

Workers are assumed to incur a transaction cost for saving or borrowing small amounts which prohibits their participation in financial markets. As a result, workers simply consume their labor income each period such that

\[
c_t^w = w_t^w \ell_t,\]

where \( c_t^w \) is the individual worker’s consumption, \( w_t^w \) is the worker’s competitive market wage, and \( \ell_t^w = \ell^w \) is the constant supply of labor hours per worker.

2.2 Capital Owners

The capital owner’s decision problem is to maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t}{\mu_t} - \mu \frac{C_t}{\mu_{t-1}} \right)^{1-\alpha} - 1, \tag{1}
\]

subject to the budget constraint

\[
c_t + p_t^a q_{t+1}^a + p_t^b q_{t+1}^b + p_t^c q_{t+1}^c = (p_t^a + d_t) q_t^a + q_t^b + (\delta p_t^c + 1) q_t^c + w_t^a \ell_t, \tag{2}
\]

where \( E_t \) represents the mathematical expectation operator, \( \beta \) is the subjective time discount factor, \( c_t \) is the individual capital owner’s consumption, and \( \alpha \geq 0 \) is a curvature parameter that influences the coefficient of relative risk aversion. Along the lines of Abel (1999), an

\(^{17}\)The model setup is similar to a standard framework that is often used to study optimal redistributive capital taxation. See, for example, Judd (1985), Lansing (1999), and Krusell (2002). In these examples, however, capital owners do not supply labor.
individual capital owner derives utility from consumption relative to an exogenously-growing living standard index \( H_t = \exp(\mu t) \), where \( \mu \) is the economy’s trend growth rate. This setup implies that capital owners today are not substantially “happier” (as measured in utility terms) than they were a hundred years ago because individual consumption is measured relative to an ever-improving living standard. The net effect of \( H_t \) is to change the effective time discount factor which turns out to be useful in the calibration procedure.\(^{18}\) To allow for time-varying risk aversion, I assume that an individual capital owner’s felicity is also measured relative to the lagged per capita consumption basket \( C_{t-1}/H_{t-1} \), which the agent views as outside of his control.\(^{19}\) The parameter \( \kappa \geq 0 \) governs the importance of the external habit stock. When \( \alpha = 1 \), the within-period utility function can be written as \( \log \left( c_t/H_t - \kappa C_{t-1}/H_{t-1} \right) \).

Capital owners derive labor income in the amount \( w_t^c \ell_t^c \), where \( \ell_t^c = \ell^c \) is the constant supply of labor hours per person. Capital owners may purchase the firm’s equity shares in the amount \( q_{t+1}^c \) at the ex-dividend price \( p_t^c \). Shares purchased in the previous period yield a dividend \( d_t \). One-period discount bonds purchased in the previous period yield a single payoff of one consumption unit per bond. Capital owners may also purchase long-term bonds (consols) in the amount \( q_{t+1}^b \) at the ex-coupon price \( p_t^b \). A long-term bond purchased in period \( t \) yields the following stream of decaying coupon payments (measured in consumption units) starting in period \( t+1 \): 1, \( \delta \), \( \delta^2 \), ..., where \( \delta \) is the decay parameter that governs the Macaulay duration of the bond, i.e., the present-value weighted average maturity of the bond’s cash flows.\(^{20}\) When \( \delta = 0 \), the long-term bond collapses to a one-period bond. Equity shares are assumed to exist in unit net supply while both types of bonds exist in zero net-supply. Market clearing therefore implies \( q_t^a = 1 \) and \( q_t^b = q_t^c = 0 \) for all \( t \).

The capital owner’s first-order conditions with respect to \( q_{t+1}^a \), \( q_{t+1}^b \), and \( q_{t+1}^c \) are as follows:

\[
p_t^a = E_t \beta \exp(-\phi \mu) \left( \frac{c_{t+1} + \kappa \exp(\mu) c_t}{c_t - \kappa \exp(\mu) c_{t-1}} \right)^{-\alpha} \left( p_{t+1}^a + d_t \right), \tag{3}
\]

\[
p_t^b = E_t \beta \exp(-\phi \mu) \left( \frac{c_{t+1} + \kappa \exp(\mu) c_t}{c_t - \kappa \exp(\mu) c_{t-1}} \right)^{-\alpha}, \tag{4}
\]

\[
p_t^c = E_t \beta \exp(-\phi \mu) \left( \frac{c_{t+1} + \kappa \exp(\mu) c_t}{c_t - \kappa \exp(\mu) c_{t-1}} \right)^{-\alpha} \left( 1 + \delta p_{t+1}^c \right), \tag{5}
\]

where \( \phi = 1 - \alpha \) and I have made the substitutions \( (H_{t+1}/H_t)^{(1-\alpha)} = \exp(-\phi \mu) \) and \( c_t = C_t \) for all \( t \). In equilibrium, the capital owner’s budget constraint becomes \( c_t = d_t + w_t^c \ell_t^c \), which

\(^{18}\)The value of \( \beta \) is chosen to match the mean price-dividend ratio in long-run U.S. data. The presence of \( H_t \) in (1) allows the calibration target to be achieved with \( \beta < 1 \), even if steady-state risk aversion is high.

\(^{19}\)Maurer and Meier (2008) find strong empirical evidence for “peer-group effects” on individual consumption decisions using panel data on U.S. household expenditures.

\(^{20}\)Rudebusch and Swanson (2008) employ a similar setup except that a long-term bond purchased in period \( t \) yields a declining coupon stream of 1, \( \delta \), \( \delta^2 \) starting in period \( t \) rather than in period \( t+1 \).
shows that the capital owner’s consumption is funded from dividends and wage income.

2.3 Firms

The firm’s output is produced according to the technology

$$y_t = k_t^{\theta_t} \left[ \exp \left( z_t \right) \left( \ell_t^\mu \right)^a \left( n \ell_t^\nu \right)^{1-a} \right]^{1-\theta_t}, \quad a \in (0, 1)$$

(6)

$$z_t = z_{t-1} + \mu + \varepsilon_t, \quad \varepsilon_t \sim N \left( 0, \sigma_\varepsilon^2 \right),$$

(7)

$$\theta_t = \theta \exp (u_t), \quad \theta \in (0, 1),$$

(8)

$$v_t = \rho v_{t-1} + u_t, \quad |\rho| < 1, \quad u_t \sim N \left( 0, \sigma_u^2 \right),$$

(9)

with $z_0$ and $v_0$ given. The symbol $k_t$ is the firm’s stock of physical capital and $z_t$ is a labor-augmenting “productivity shock” that evolves as a random walk with drift. The drift parameter $\mu$ determines the trend growth rate of output. The parameter $a$ governs the relative productivity of the two types of labor inputs. Along the lines of Young (2004) and Ríos-Rull and Santaeulàlia-Llopis (2010), capital’s share of total income $\psi_t$ can fluctuate over time in response to a “distribution shock” $v_t$ which evolves as a stationary AR(1) process. As described later, the parameters $\theta$, $\rho$ and $\sigma_u$ are chosen so that capital’s share of income in the model exhibits the same mean, persistence, and volatility as the U.S. capital share data plotted in Figure 1.

Resources devoted to investment augment the firm’s stock of physical capital according to the law of motion

$$k_{t+1} = B \left\{ (1 - \lambda) \left[ k_t (1 - \delta) \right]^{\psi_k} + \lambda t_t^{\psi_k} \right\}^{1/\psi_k}, \quad B > 0, \quad \lambda \in (0, 1), \quad \delta \in (0, 1)$$

(10)

$$\psi_k \equiv (\sigma_k - 1)/\sigma_k$$

$$\sigma_k \in (0, \infty)$$

with $k_0$ given. The parameter $\delta$ is the capital depreciation rate. The parameter $\psi_k$ depends on the elasticity of substitution $\sigma_k$ between undepreciated capital and new investment in the production of new capital. As $\sigma_k \to 0$ (or $\psi_k \to -\infty$), new investment and existing capital becomes more complimentary (i.e., more tightly coupled) which raises the implicit cost of adjusting the capital stock from one period to the next. Kim (2003) shows that the intertemporal adjustment cost specification (10) can also be interpreted as a multisectoral adjustment cost that imposes a nonlinear transformation between consumption and investment in the national income identity. A convenient feature of the above specification is that it nests the standard linear law of motion with no adjustment costs as a special case. The standard
linear law of motion can be recovered by imposing the following parameter settings: \( \sigma_k = \infty \) (or \( \psi_k = 1 \)), \( B = 2 \), and \( \lambda = \frac{1}{2} \).

Under the assumption that the labor market is perfectly competitive, firms take \( \ell_t^c \) and \( \ell_t^w \) as given and choose sequences of \( \ell_{t+j}^c \), \( \ell_{t+j}^w \), and \( k_{t+1+j} \), to maximize the following discounted stream of expected dividends:

\[
E_0 \sum_{j=0}^{\infty} M_{t+j} \left[ y_{t+j} - w_{t+j}^c \ell_{t+j}^c - n w_{t+j}^w \ell_{t+j}^w - i_{t+j} \right],
\]

subject to the production function (6) and the capital law of motion (10). Firms act in the best interests of their owners such that dividends in period \( t+j \) are discounted using the capital owner’s stochastic discount factor \( \beta^j \exp (-\phi \mu j) \left[ c_{t+j} - \kappa \exp (\mu) c_{t+j-1} \right]^{-\alpha} \).

The firm’s first-order conditions are:

\[
w_t^c = \frac{(1 - \theta_t) a y_t}{\ell_t^c},
\]

\[
w_t^w = \frac{(1 - \theta_t) (1 - a) y_t}{n \ell_t^w},
\]

\[
i_t g (k_{t+1}/k_t) = E_t M_{t+1} \left[ \theta_{t+1} y_{t+1} - i_{t+1} + i_{t+1} g (k_{t+2}/k_{t+1}) \right],
\]

where \( g (k_{t+1}/k_t) \equiv 1 + \frac{1 - \lambda}{\left[ k_{t+1}/k_t \right]^{\psi_k} - (1 - \lambda)} \),

which reflect the constant labor supplies \( \ell^c \) and \( \ell^w \). Equations (13) and (14) show that each type of labor is paid its marginal product. The share of total income going to capital owners is \( \theta_t + (1 - \theta_t) a \), while the share of total income going to workers is \( (1 - \theta_t) (1 - a) \). Comparing the first-order condition (15) to the equity pricing equation (3), we see that the ex-dividend price of an equity share is given by \( p_t^s = i_t g (k_{t+1}/k_t) \).

The equity share is a claim to a perpetual stream of dividends \( d_{t+1} = \theta_{t+1} y_{t+1} - i_{t+1} \) starting in period \( t+1 \). In the version of the model with no capital adjustment costs (\( \psi_k = 1 \), \( B = 2 \), \( \lambda = \frac{1}{2} \)), we have \( p_t^s = k_{t+1} \). When \( \sigma_k = 1 \) such that \( \psi_k = 0 \), the capital law of motion (10) takes a Cobb-Douglas form and we have \( p_t^s = i_t/\lambda \). These examples demonstrate that the degree of curvature in the capital law of motion can influence the volatility of the equity price and hence the volatility of the equity return.

\[\text{21 After taking the derivative of the profit function (11) with respect to } k_{t+1}, \text{ I have multiplied both sides of the resulting first-order condition by } k_{t+1}, \text{ which is known at time } t.\]
3 Model Calibration

A time period in the model is taken to be one year. The baseline parameters are chosen simultaneously to match various empirical targets, as summarized in Table 1. In addition to the concentrated ownership model, I similarly calibrate a representative-agent version of the same model. Analytical moment formulas derived from the log-linear approximate solution of both models are used in the calibration procedure.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Concentrated Ownership</th>
<th>Representative Agent</th>
<th>Description/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>9</td>
<td>–</td>
<td>Capital owners = top income decile.</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.37</td>
<td>0.37</td>
<td>Mean value of capital’s share of income.</td>
</tr>
<tr>
<td>( a )</td>
<td>0.048</td>
<td>–</td>
<td>Mean top decile income share = 0.4.</td>
</tr>
<tr>
<td>( \ell^c/\ell^w )</td>
<td>0.225</td>
<td>–</td>
<td>Mean relative wage ( w^c/w^w = 2 ).</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.07</td>
<td>0.07</td>
<td>Annual capital depreciation rate.</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.02</td>
<td>0.02</td>
<td>Mean per capita consumption growth = 2.0%.</td>
</tr>
<tr>
<td>( B )</td>
<td>1.141</td>
<td>1.141</td>
<td>Mean ( k_t/y_t \simeq 2.9 ).</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.00042</td>
<td>0.00042</td>
<td>Mean ( i_t/k_t = \exp(\mu) + 1 - \delta \simeq 0.09 ).</td>
</tr>
<tr>
<td>( \sigma_k )</td>
<td>0.3</td>
<td>0.3</td>
<td>Curvature parameter for capital law of motion.</td>
</tr>
<tr>
<td>( \sigma_u )</td>
<td>0.0324</td>
<td>0.0324</td>
<td>Std. dev. capital’s share = 2.0%.</td>
</tr>
<tr>
<td>( \sigma_\varepsilon )</td>
<td>0.0367</td>
<td>0.0280</td>
<td>Std. dev. consumption growth = 2.6%.</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.8</td>
<td>0.8</td>
<td>AR(1) for capital’s share of income.</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.953</td>
<td>0.960</td>
<td>Mean ( p_t^s/d_t \simeq 27.3 ).</td>
</tr>
<tr>
<td>( \delta_c )</td>
<td>0.964</td>
<td>0.956</td>
<td>Consol duration = 10 years.</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>5</td>
<td>5</td>
<td>Curvature parameter for utility function.</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.2</td>
<td>0.2</td>
<td>Utility habit parameter.</td>
</tr>
</tbody>
</table>

The number of workers per capital owner is set to \( n = 9 \) so that capital owners represent the top income decile of households in the concentrated ownership model. The steady state capital income share \( \theta \) is set to match the sample mean of 0.37, as plotted in the bottom left panel of Figure 1. The production elasticity of the capital owner’s labor supply is set to \( a = 0.048 \). This value implies a top decile income share in steady state of \( \theta + (1 - \theta) a = 0.4 \), corresponding to the sample mean, as plotted in the top left panel of Figure 1. Given these values, the labor supply ratio \( \ell^c/\ell^w \) is set so that the steady state wage ratio is \( w^c/w^w = 2 \). For comparison, Heathcote, Perri, and Violante (2010, p. 686) report a male college wage premium of about 1.4 in 1980, whereas Gottschalk and Danziger (2005, p. 238) report a male wage ratio of about 4 when comparing the top decile to the bottom decile. The wage ratio \( w^c/w^w \) in this model compares the top decile to the remainder of households, so one would expect it to fall somewhere in between the values reported by the two studies, but likely closer to the value reported by Heathcote, Perri, and Violante (2010). The quantitative results are
not sensitive to the value of this wage ratio.

The capital depreciation rate is set to $\delta = 0.07$, a typical value. The drift parameter $\mu$ of the random walk productivity process (7) is set to achieve a steady-state per capita growth rate of 2%. The capital law of motion parameters $B$ and $\lambda$ are chosen to deliver realistic target values for the steady capital-output ratio and the steady-state investment-capital ratio. The steady-state target for $i_t/k_t$ corresponds to the value implied by a model with no capital adjustment costs. The values for $B$ and $\lambda$ depend on the chosen value for the curvature parameter $\sigma_k$. Each time $\sigma_k$ is changed, the values of $B$ and $\lambda$ are adjusted to maintain the same steady-state ratios for $k_t/y_t$ and $i_t/k_t$ as before. In this way, changes in $\sigma_k$ identify a family of CES production functions that are distinguished only by the elasticity parameter, and not by the steady-state ratios for $k_t/y_t$ and $i_t/k_t$.\footnote{This methodology follows the standard normalization procedure that is used when comparing CES production models with different parameterizations. See Klump and Saam (2008).}

As will be demonstrated in the sensitivity analysis, smaller values for $\sigma_k$ (implying stronger capital adjustment costs) result in a higher mean equity premium and a higher standard deviation of the equity return. The baseline value of $\sigma_k = 0.3$ delivers a sizeable equity premium in the concentrated ownership model together with a realistic amount of volatility in the equity return.

The standard deviations $\sigma_u$ and $\sigma_e$ for the two shock innovations are chosen to match the 2% volatility of capital’s share of income and the 2.6% volatility of aggregate consumption growth. The value for $\sigma_u$ is the same for both models. The concentrated ownership model requires a higher value of $\sigma_e$ because the investment decision of the top-decile agents has a smaller proportional impact on aggregate consumption volatility versus the investment decision of a representative agent.

The discount factor $\beta$ is chosen to achieve a mean price-dividend ratio of about 27, consistent with long-run average for the S&P 500 stock index. The consol coupon decay parameter $\delta_c$ is set so that the Macauly duration of the long-term bond is $D = 10$ years. The Macauly duration is the present-value-weighted average maturity of the bond’s cash flows, computed as follows:

$$D = \frac{\sum_{t=0}^{\infty} (\bar{M} \delta)^t (t+1)}{\sum_{t=0}^{\infty} (\bar{M} \delta)^t} = \frac{1}{1 - \bar{M} \delta},$$

(16)

where $\bar{M} \equiv \exp [E \log (M_{t+1})] = \beta \exp (-\mu)$ from equation (12).

The utility parameters $\alpha$ and $\kappa$ both influence the degree of risk aversion. The capital owner’s time-varying coefficient of relative risk aversion (CRRA$_t$) is given by

$$\text{CRRA}_t = - \frac{c_t U_{cc}}{U_c} = \frac{\alpha}{1 - \kappa (c_t/c_{t-1}) \exp (\mu)},$$

(17)
which collapses to $\alpha/(1 - \kappa)$ in steady state. The baseline values of $\alpha = 5$ and $\kappa = 0.2$ imply a steady-state risk aversion coefficient of 6.25. These values deliver a sizeable equity risk premium in the concentrated ownership model without overpredicting the volatility of the equity return in the data.

In the sensitivity analysis, I investigate the effects of changing the values of the parameters $\sigma_k$, $\alpha$, and $\kappa$. I also investigate the effects of changing the relative volatility of the two shock innovations, as measured by the ratio $\sigma_u/\sigma_\varepsilon$.


<table>
<thead>
<tr>
<th>Statistic</th>
<th>U.S. Data</th>
<th>Concentrated Ownership Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top decile share</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>40%$^1$</td>
<td>40%</td>
</tr>
<tr>
<td>Financial wealth</td>
<td>80%$^2$</td>
<td>100%</td>
</tr>
<tr>
<td>Gini coefficient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>0.32 - 0.42$^3$</td>
<td>0.30</td>
</tr>
<tr>
<td>Financial wealth</td>
<td>0.89 - 0.93$^2$</td>
<td>0.90</td>
</tr>
</tbody>
</table>


4 **Quantitative Results**

4.1 **Impulse Response Functions**

Details regarding the model solution are contained in the appendix. Both the concentrated ownership model and the representative agent model are solved in the same way. The capital growth rate $x_t \equiv k_{t+1}/k_t$ is the only decision variable. There are four state variables: (1) the normalized capital stock $k_{n,t}$ which subsumes the productivity shock $z_t$, (2) the distribution shock $v_t$, (3) the lagged consumption-capital ratio $c_{t-1}/k_{t-1}$, and (4) the lagged decision variable $x_{t-1}$. The last two state variables summarize the influence of the external habit stock. An approximate log-linear solution is used as a starting value for an alternative solution method that preserves the model’s nonlinear equilibrium conditions. The alternative solution employs a version of the parameterized expectation algorithm (PEA) described by Den Haan and Marcet (1990). The results obtained using the PEA solution are not much different from those generated by the log-linear solution.
Figures 2 and 3 plot the concentrated ownership model’s response to a one standard deviation innovation of the distribution shock (solid blue line) and the productivity shock (dashed red line). Both figures plot the percentage deviation of the variables from the no-shock trend. The effects of the distribution shock are highly persistent but temporary, whereas the effects of the productivity shock are permanent due to the unit root in the law of motion (7). On impact, both shocks move the macro variables (Figure 2) and the asset pricing variables (Figure 3) in the same direction.

A positive distribution shock innovation raises capital’s share of total income $\theta_t$ and induces a 6% increase in investment and an 18% increase in the equity price relative to the no-shock trend. The higher value of $\theta_t$ signals an increase in the productivity of physical capital. The capital owner reacts to this signal by devoting more resources to capital investment. But even with more resources devoted to investment, the higher value of $\theta_t$ combined with the resulting increase in aggregate output $y_t$ still allows for a 4% increase in dividends relative to trend, where dividends are given by $d_t = \theta_t y_t - i_t$. The increase in dividends combined with movements in the capital owners’s stochastic discount deliver a higher equity price, in accordance with equation (3). The response of the equity price is linked to the response of investment via the equilibrium relationship $p^e_t = i_t g(k_{t+1}/k_t)$, where both $i_t$ and the nonlinear function $g(k_{t+1}/k_t)$ are influenced by the curvature parameter $\sigma_k$ which governs capital adjustment costs. A positive distribution shock also causes bond prices to increase so as to satisfy the no-arbitrage condition across the different asset classes.

A positive productivity shock yields qualitatively similar results, except that now the increases in the variables are all permanent. The equity and bond prices are less sensitive to a permanent shock.

The fact that a temporary distribution shock can induce a large move in the equity price helps the model to match the volatility of equity returns in U.S. data. However, as we shall see in the simulations, the volatility of the model price-dividend ratio is still below that observed in the data.

4.2 Sensitivity of Equity Premium to Key Parameters

Figure 4 plots the mean equity premium relative to short-term bonds $E(R^a_{t+1} - R^b_{t+1})$ as some key parameters are varied in the concentrated ownership model (solid blue line) and the representative agent model (dashed red line).

I examine the effects of: (1) the capital-investment substitution elasticity $\sigma_k$ which governs the strength of capital adjustment costs, (2) the relative volatility of the distribution shock innovation, as measured by the ratio $\sigma_u/\sigma_v$, (3) the utility curvature parameter $\alpha$, and (4) the utility habit parameter $\kappa$. The vertical dashed line in each panel marks the baseline calibra-
tion in the concentrated ownership model. The return moments are computed analytically using the approximate log-linear solution of the model. When either $\sigma_k$, $\alpha$, or $\kappa$ is changed, the remaining non-curvature parameters are adjusted to maintain the same empirical targets shown in Table 1. To vary the ratio $\sigma_u/\sigma_\varepsilon$, I choose $\sigma_u$ to be a fixed multiple of $\sigma_\varepsilon$ while the latter continues to be chosen in each model to match the volatility of U.S. aggregate consumption growth. Hence, for the plot that varies $\sigma_u/\sigma_\varepsilon$, the models do not match the volatility of capital’s share of income in U.S. data, except at their respective baseline values for $\sigma_u/\sigma_\varepsilon$.

The top left panel of Figure 4 shows the effect of changing $\sigma_k$. Smaller values for $\sigma_k$ imply more curvature in the capital law of motion (10). More curvature implies that current-period investment is more complimentary (i.e., more tightly coupled) to existing capital, thereby increasing the cost of adjusting next period’s capital stock via changes in current-period investment. Stronger capital adjustment costs impair the capital owner’s ability to smooth consumption, thereby raising the mean equity premium.

Figure 5 shows how smaller values of $\sigma_k$ imply more curvature in the relationship between the investment-capital ratio $i_t/k_t$ and gross capital growth $k_{t+1}/k_t$. During model simulations, the realized capital adjustments costs are generally small but can occasionally be large if realizations of $i_t/k_t$ fall outside a two standard deviation range surrounding the mean.

When $\sigma_k = 0.3$, the mean equity premium in the concentrated ownership is 4.84% versus 1.53% in the representative agent model. When $\sigma_k = 1$, capital law of motion takes the form of a Cobb-Douglas function. In this case, the mean equity premium in the concentrated ownership model is only about 1% versus 0.7% in the representative agent model. When $\sigma_k = 2$, the equity premium is less than 0.5% in both models. As $\sigma_k \to \infty$, the capital adjustment costs become vanishingly small and the equity premium in both models is tiny—in line with vast literature on the equity premium puzzle. As $\sigma_k$ becomes smaller, the size of the equity premium increases more rapidly in the concentrated ownership model. Given that the capital owner’s consumption is more strongly linked to volatile dividends from equity, imposing restrictions on the capital owner’s ability to smooth consumption results in a higher utility cost, thus justifying a higher equity premium relative to the representative agent model.

The top right panel of Figure 4 shows the effect of changing the relative volatility of the distribution shock innovation. Higher values of the ratio $\sigma_u/\sigma_\varepsilon$ raise the equity premium in both models. Again, the equity premium in the concentrated ownership model is more sensitive to the parameter shift because the ratio $\sigma_u/\sigma_\varepsilon$ strongly impacts the volatility of dividends and hence the volatility of the capital owner’s consumption growth. At the far right when $\sigma_u/\sigma_\varepsilon = 1.6$, the equity premium in the concentrated ownership model exceeds 11%

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23 The baseline values for $\sigma_\varepsilon$, $\alpha$, $\kappa$, and $\sigma_u$ are the same for both models. However, the representative agent model requires a lower baseline value for $\sigma_\varepsilon$ in order to match the standard deviation of U.S. aggregate consumption growth. See Table 1.
while it remains below 2% in the representative agent model.

Table 3 below shows how changes in the ratio \( \sigma_u / \sigma_\varepsilon \) affect the standard deviations of selected variables in the concentrated ownership model. Given the other parameter settings, values of \( \sigma_u / \sigma_\varepsilon \) that exceed the baseline ratio 0.88 cause the model to start significantly overpredicting the volatility of the equity return in long-run U.S. data.

<table>
<thead>
<tr>
<th>( \sigma_u / \sigma_\varepsilon )</th>
<th>( \theta_t )</th>
<th>( \Delta \log (c_t^d) )</th>
<th>( \Delta \log (d_t) )</th>
<th>( \Delta \log (c_t) )</th>
<th>( \Delta \log (c_t^w) )</th>
<th>( R_{t+1}^s )</th>
<th>( R_{t+1}^s - R_{t+1}^\beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0%</td>
<td>2.6%</td>
<td>3.7%</td>
<td>3.4%</td>
<td>2.4%</td>
<td>6.2%</td>
<td>1.3%</td>
</tr>
<tr>
<td>0.5</td>
<td>1.2%</td>
<td>2.6%</td>
<td>4.4%</td>
<td>3.9%</td>
<td>2.4%</td>
<td>13.5%</td>
<td>2.5%</td>
</tr>
<tr>
<td>0.88</td>
<td>2.0%</td>
<td>2.6%</td>
<td>5.5%</td>
<td>4.7%</td>
<td>2.3%</td>
<td>21.6%</td>
<td>4.8%</td>
</tr>
<tr>
<td>1.0</td>
<td>2.2%</td>
<td>2.6%</td>
<td>5.9%</td>
<td>5.0%</td>
<td>2.3%</td>
<td>24.1%</td>
<td>5.8%</td>
</tr>
<tr>
<td>1.2</td>
<td>2.6%</td>
<td>2.6%</td>
<td>6.6%</td>
<td>5.4%</td>
<td>2.3%</td>
<td>28.5%</td>
<td>7.5%</td>
</tr>
</tbody>
</table>

U.S. Data | 2.0% | 2.6% | 11.8% | — | — | 20.2% | 7.1% |

Note: Moments of concentrated ownership model computed analytically from log-linear solution.
The baseline calibration from Table 1 implies \( \sigma_u / \sigma_\varepsilon = 0.88 \).

The bottom two panels in Figure 4 show that higher values for either \( \alpha \) or \( \kappa \) lead to a higher mean equity premium in both models. This is not surprising given that an increase in either parameter contributes to a higher coefficient of relative risk aversion, as shown by equation (17). For any given level of risk aversion, the higher volatility of the capital owner’s consumption growth serves to magnify the equity risk premium in the concentrated ownership model relative to the representative agent model. The baseline equity premium of 4.84% in the concentrated ownership model is still below the long-run average equity premium in U.S. data which is roughly 7%. The concentrated ownership model can deliver a mean equity premium in excess of 6% if \( \alpha \) is increased to around 8 or if \( \kappa \) is increased to around 0.6. However, as with increasing the ratio \( \sigma_u / \sigma_\varepsilon \), increasing either \( \alpha \) or \( \kappa \) will cause the model to start significantly overpredicting the volatility of the equity return in U.S. data. At the baseline values of \( \alpha = 5 \), \( \kappa = 0.2 \), \( \sigma_k = 0.3 \) and \( \sigma_u / \sigma_\varepsilon = 0.88 \), the concentrated ownership comes very close to matching the 20% standard deviation of the real equity return in the data.

### 4.3 Model Simulations

Table 4 provides some direct evidence in support of the model’s main mechanism, namely an empirical link between distribution risk, as measured by movements in capital’s share of income, and the contemporaneous equity risk premium. I regress the equity premium in the data on capital’s share of income and the change in capital’s share of income. The U.S. data regressions employ the same equity premium and capital income share data plotted earlier in Figure 1. The first U.S. data regression shows a positive estimated coefficient \( b = 1.80 \), which
is close to significant with a \( t \)-statistic of \( 1.80/1.19 = 1.5 \). The second U.S. data regression uses the change in capital’s share of income to eliminate the influence of the persistent uptrend in the U.S. capital income share since around 1980. Now the estimated coefficient \( b = 4.71 \) is positive and statistically significant with a \( t \)-statistic of \( 4.71/1.80 = 2.6 \). These results complement the empirical findings of Greenwald, Lettau, and Ludvigson (2014) who employ a vector autoregression analysis on quarterly U.S. data over the period 1952.Q2 to 2012.Q4. Their study identifies a statistically significant impact of “factor share shocks” on detrended stock market wealth and detrended stock prices.

The regressions in Table 4 on model-generated data show that an increase in either \( \theta_t \) or \( \Delta \theta_t \) serves to raise the contemporaneous equity premium \( R^e_t - R^b_t \). The estimated link between the distribution shock and the equity premium is stronger in the concentrated ownership model than in the representative agent model, consistent with the sensitivity results plotted earlier in the top right panel of Figure 4.

Table 4: Equity Premium Regressions

<table>
<thead>
<tr>
<th>Regression</th>
<th>U.S. Data 1930-2011</th>
<th>Concentrated Ownership</th>
<th>Representative Agent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^e_t - R^b_t = a + b \theta_t + \eta_{t+1} )</td>
<td>1.80</td>
<td>6.37</td>
<td>3.45</td>
</tr>
<tr>
<td>( a + b \theta_t + \eta_{t+1} )</td>
<td>(1.19)</td>
<td>(0.08)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>( R^e_t - R^b_t = a + b \Delta \theta_t + \omega_{t+1} )</td>
<td>4.71</td>
<td>18.6</td>
<td>10.3</td>
</tr>
<tr>
<td>( a + b \Delta \theta_t + \omega_{t+1} )</td>
<td>(1.80)</td>
<td>(0.05)</td>
<td>(0.04)</td>
</tr>
</tbody>
</table>

Notes: \( \theta_t \) = capital’s share of total income. \( \Delta \theta_t = \theta_t - \theta_{t-1} \).

Standard errors in parentheses. Model regressions based on data from a 20,000 period simulation.

Table 5 presents unconditional moments of asset pricing variables computed from model simulations using the baseline parameter values shown in Table 1. The table also shows the corresponding statistics from U.S. data. Figures 6 through 8 provide a visual comparison between the U.S. data and the model-generated data for selected variables.

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24 Data on the price-dividend ratio are from Robert Shiller’s website <http://www.econ.yale.edu/~shiller/>. The price-dividend ratio in year \( t \) is defined as the value of the S&P 500 stock index at the beginning of year \( t + 1 \), divided by the accumulated dividend over year \( t \). The U.S. real return statistics shown in Table 4 are for equity, long-term bonds, and short term bills, from Dimson, et al. (2002), updated through 2011.
Table 5: Unconditional Asset Pricing Moments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dates</th>
<th>Statistic</th>
<th>U.S. Data</th>
<th>Concentrated Ownership</th>
<th>Representative Agent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t^d/d_t$</td>
<td>1871-2012</td>
<td>Mean</td>
<td>27.3</td>
<td>27.5</td>
<td>27.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Std. Dev.</td>
<td>14.2</td>
<td>5.95</td>
<td>2.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Corr. Lag 1</td>
<td>0.93</td>
<td>0.73</td>
<td>0.93</td>
</tr>
<tr>
<td>$R_{t+1}^d$</td>
<td>1900-2011</td>
<td>Mean</td>
<td>8.2%</td>
<td>8.0%</td>
<td>6.4%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Std. Dev.</td>
<td>20.2%</td>
<td>21.4%</td>
<td>11.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Corr. Lag 1</td>
<td>-0.01</td>
<td>-0.14</td>
<td>-0.21</td>
</tr>
<tr>
<td>$R_{t+1}^c$</td>
<td>1900-2011</td>
<td>Mean</td>
<td>0.99%</td>
<td>3.3%</td>
<td>5.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Std. Dev.</td>
<td>4.7%</td>
<td>7.4%</td>
<td>4.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Corr. Lag 1</td>
<td>0.62</td>
<td>0.71</td>
<td>0.50</td>
</tr>
<tr>
<td>$R_{t+1}$</td>
<td>1900-2011</td>
<td>Mean</td>
<td>2.7%</td>
<td>6.3%</td>
<td>5.9%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Std. Dev.</td>
<td>10.3%</td>
<td>16.9%</td>
<td>8.3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Corr. Lag 1</td>
<td>0.06</td>
<td>-0.13</td>
<td>-0.23</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>1900-2011</td>
<td>$E(R_{t+1}^d-R_{t+1}^c)/SD(R_{t+1}^d-R_{t+1}^c)$</td>
<td>0.359</td>
<td>0.185</td>
<td>0.095</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>1900-2011</td>
<td>$E(R_{t+1}^d-R_{t+1}^c)/SD(R_{t+1}^d-R_{t+1}^c)$</td>
<td>0.275</td>
<td>0.064</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Note: Model results are computed from a 20,000 period simulation.

Table 4 and the top panels of Figure 7 show that the concentrated ownership model underpredicts the volatility of the price-dividend ratio in the data but delivers about twice as much volatility as the representative agent model. The standard deviation of the price-dividend ratio is about 14 in the data versus about 6 in the concentrated ownership model and about 3 in the representative agent model. The volatility of the U.S. price-dividend ratio is influenced by a dramatic bubble-like run-up starting in the mid-1990s that is partially retraced by the end of the data sample in 2012. A large literature finds evidence that real-world stock prices exhibit “excess volatility” when compared to the discounted stream of ex post realized dividends.25 If findings of excess volatility in the data are genuine, then one would not expect a fully rational model like this one to be able to match the volatility of the price dividend ratio in the data. An extension of the present model that allows for boundedly-rational expectations on the part of capital owners could potentially magnify the volatility of the price-dividend ratio, providing a better match with the data.26

Despite underpredicting the volatility of the price-dividend ratio, the concentrated ownership model provides a good match with mean and volatility of the U.S. equity return, which are around 8% and 20%, respectively. Recall that the later statistic is strongly influenced by the volatility of the distribution shock, as shown earlier in Table 3. The distribution shock

25 Lansing and LeRoy (2012) provide a recent update on this literature.
26 For examples along these lines, see Bansal and Shaliastovich (2010), Fuster, Hebert, and Laibson (2012), Lansing (2010, 2012), and Hirshleifer and Yu (2013), among others.
volatility is pinned down by the standard deviation of capital’s share of income in the data, which is about 2%.

The concentrated ownership model overpredicts the mean and volatility of the U.S. short-term bond return, although it should be noted that the return data constructed by Dimson, Marsh, and Staunton (2002, updated) pertain to a 3-month “bill” whereas the short-term bond in the model has a one-year maturity. The prediction of too much volatility in the short term bond return is a typical shortcoming of models with habit formation (Jermann 1998 and Abel 2008). It is possible, however, to reverse-engineer more complicated laws of motion for the stochastic discount factor (12) so that the expected stochastic discount factor $E_t M_{t+1}$ exhibits little or no volatility, thereby reducing or even eliminating the volatility in the short term bond return (Campbell and Cochrane 1999). The reverse-engineering approach has the unfortunate side effect of magnifying the degree of steady-state risk aversion that is needed to generate a sizeable equity premium.

As noted in the introduction, the concentrated ownership model’s long-term bond behaves too much like equity such that the mean and volatility of the consol are too high relative to the mean and volatility of the U.S. long-term bond return. This deficiency in the model is well-summarized by the Sharpe ratio comparison at the bottom of Table 5. The concentrated ownership model does capture the fact that returns on equity and long-terms bonds exhibit near-zero autocorrelation in the data while returns on short-term bonds exhibit strong positive autocorrelation. Overall, the concentrated ownership model outperforms the representative agent model in matching the majority of the data statistics in Table 5.

The bottom panel of Figure 7 shows that the equity premium $R_{t+1}^e - R_{t+1}^b$ in the concentrated ownership model exhibits a correlation coefficient with aggregate consumption growth of 0.50 versus a value of 0.20 in the data. The correlation coefficient in the representative agent model is much higher at 0.86. Movements in the equity premium are determined in part by movements in the stochastic discount factor. In the concentrated ownership model, the stochastic discount factor (12) depends on the capital owner’s consumption growth which responds differently to shocks than does aggregate consumption growth (see Figure 2). In contrast, the stochastic discount factor in the representative agent model depends on aggregate consumption growth which helps to explain the representative agent model’s counterfactual prediction of a strong positive correlation between aggregate consumption growth and the equity premium.

Figure 8 shows that asset returns in U.S. data and the models exhibit time-varying means and volatilities. The time-varying behavior in the data suggests the presence of nonlinearities. The time-varying behavior in models is wholly endogenous, owing to the nonlinear nature of the various functional forms and equilibrium conditions. In contrast, Bansal and Yaron (2004) introduce exogenous time-varying volatility via the stochastic process for consumption growth.
within an endowment economy.

Tables 6 and 7 show that the concentrated ownership model performs reasonably well in matching the business cycle moments of aggregate macro variables. Both models capture the procyclical movement of capital’s share of income $\theta_t$. As shown earlier in Figure 2, a positive innovation to $\theta_t$ induces a temporary but persistent increase in real output. In simulations, both models deliver a correlation coefficient around 0.3 between $\theta_t$ and the growth rate of real output $\Delta \log (y_t)$. The corresponding correlation in the data is 0.32 over the period 1930 to 2012.

Table 6 shows that the model macro variables all exhibit strong correlations with output growth—a typical feature of productivity-shock driven real business cycle models. The model financial variables $\Delta \log \left( \frac{p_t^e}{\sigma_{t+1}} \right)$ and $R_{t+1}^e - R_{t+1}^b$ also exhibit strong correlations with output growth, but the corresponding correlations in the data are weak. This observation suggests the presence of additional fundamental or non-fundamental factors that induce movements in real-world asset prices but are missing from the models. Along these lines, Greenwald, Lettau, Ludvigson (2014) argue that “risk aversion shocks” which are unrelated to either aggregate consumption or aggregate labor income account for the bulk of short-term fluctuations in U.S. stock prices. Such a shock could be introduced into the present model by allowing for exogenous stochastic variation in the capital owner’s utility curvature parameter $\alpha$ which appears in expression for the risk aversion coefficient (17). Another possibility is to allow for so-called “valuation risk” that is driven by exogenous stochastic variation in the

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dates</th>
<th>U.S. Data</th>
<th>Concentrated Ownership</th>
<th>Representative Agent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \log (y_t)$</td>
<td>1930-2012</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>1930-2012</td>
<td>0.32</td>
<td>0.28</td>
<td>0.29</td>
</tr>
<tr>
<td>$\Delta \log (c_t^e)$</td>
<td>1930-2012</td>
<td>0.81</td>
<td>0.89</td>
<td>0.99</td>
</tr>
<tr>
<td>$\Delta \log (c_t^{w})$</td>
<td>–</td>
<td>–</td>
<td>0.98</td>
<td>–</td>
</tr>
<tr>
<td>$\Delta \log (d_t)$</td>
<td>1930-2012</td>
<td>0.57</td>
<td>0.97</td>
<td>0.89</td>
</tr>
<tr>
<td>$\Delta \log (i_t)$</td>
<td>1930-2012</td>
<td>0.85</td>
<td>0.84</td>
<td>0.93</td>
</tr>
<tr>
<td>$\Delta \log (p_t^e)$</td>
<td>1930-2012</td>
<td>0.11</td>
<td>0.83</td>
<td>0.92</td>
</tr>
<tr>
<td>$R_{t+1}^e - R_{t+1}^b$</td>
<td>1930-2011</td>
<td>0.15</td>
<td>0.83</td>
<td>0.93</td>
</tr>
<tr>
<td>$R_{t+1}^e - R_{t+1}^c$</td>
<td>1930-2011</td>
<td>0.15</td>
<td>0.97</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Note: Model results computed from a 20,000 period simulation.

27 For consistency with the model, data on real per capita output $y_t$ from 1930-2012 are constructed as the sum of real per capita aggregate consumption $c_t^e$ (nondurables and services) and real per capita investment $i_t$ (business fixed investment plus durables consumption), where all series are from the Federal Reserve Bank of St. Louis’ FRED data base.

stockholder’s subjective discount factor $\beta$, along the lines of the endowment model developed by Albuquerque, Eichenbaum, and Rebelo (2012).

Table 7: Standard Deviation of Macro Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dates</th>
<th>U.S. Data</th>
<th>Concentrated Ownership</th>
<th>Representative Agent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \log (y_t)$</td>
<td>1930-2012</td>
<td>3.66%</td>
<td>3.07%</td>
<td>2.73%</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>1930-2012</td>
<td>1.95%</td>
<td>1.98%</td>
<td>1.98%</td>
</tr>
<tr>
<td>$\Delta \log (c_t^p)$</td>
<td>1930-2012</td>
<td>2.60%</td>
<td>2.59%</td>
<td>2.59%</td>
</tr>
<tr>
<td>$\Delta \log (c_t)$</td>
<td>–</td>
<td>–</td>
<td>4.65%</td>
<td>–</td>
</tr>
<tr>
<td>$\Delta \log (e_t^p)$</td>
<td>–</td>
<td>–</td>
<td>2.33%</td>
<td>–</td>
</tr>
<tr>
<td>$\Delta \log (d_t)$</td>
<td>1872-2012</td>
<td>11.8%</td>
<td>5.49%</td>
<td>11.8%</td>
</tr>
<tr>
<td>$\Delta \log (i_t)$</td>
<td>1930-2012</td>
<td>16.2%</td>
<td>6.23%</td>
<td>3.45%</td>
</tr>
<tr>
<td>$\Delta \log (p_t^e)$</td>
<td>1872-2012</td>
<td>17.9%</td>
<td>19.9%</td>
<td>11.0%</td>
</tr>
</tbody>
</table>

| Std. Dev. $[\Delta \log(c_t)]$ | 1982-2004 | 1.63 |
| Std. Dev. $[\Delta \log(c_t^p)]$ | –        | 1.99 |

Note: Model results computed from a 20,000 period simulation.

In Table 7, the concentrated ownership model underpredicts the volatility of investment growth $\Delta \log (i_t)$ in the data. Due to capital adjustment costs, investment growth in the concentrated ownership model is only about 1.8 times more volatile than output growth, whereas investment growth in the data is around 4 times more volatile than output growth over the period 1930 to 2012. Barlevy (2004, p. 983) notes the difficulty of generating sufficient investment volatility in real business cycle models with capital adjustment costs. Nevertheless, the concentrated ownership model does a good job of matching the volatility of equity price growth $\Delta \log (p_t)$ which is about 20% in the model versus about 18% in the data.

The representative agent model also underpredicts the volatility of investment growth, but it exhibits much higher volatility for dividend growth $\Delta \log (d_t)$, to the point of actually matching the standard deviation of 11.8% in the data. Model dividends $d_t = \theta_t y_t - i_t$ are influenced by both the temporary distribution shock $v_t$ and the permanent productivity shock $z_t$. The temporary distribution shock has less impact on investment growth volatility in the representative agent model because the agent’s consumption-investment decision pertains to aggregate consumption which is a much larger base than the capital owner’s consumption in the concentrated ownership model. Moreover, recall from Table 1 that the representative agent model employs a baseline calibration of $\sigma_u/\sigma_\varepsilon = 1.16$ to simultaneously match the standard deviations of the capital income share and aggregate consumption growth in the data. The concentrated ownership model employs a baseline calibration of $\sigma_u/\sigma_\varepsilon = 0.88$ to match the same empirical targets. Since $\sigma_u/\sigma_\varepsilon$ measures the relative volatility of temporary versus permanent shock innovations, a higher value for $\sigma_u/\sigma_\varepsilon$ translates into higher short-term volatility in dividends, which in turn implies a higher standard deviation for the growth rate of dividends in the representative agent model.
The bottom row of Table 7 shows that the capital owner’s consumption growth $\Delta \log (c_t)$ is about two times more volatile than the worker’s consumption growth $\Delta \log (c^w_t)$. The source of the extra volatility for capital owners is their heavy reliance on volatile dividends to fund their consumption. The procyclical behavior of capital’s share of income implies that labor’s share $(1 - \theta_t)$ is countercyclical. The countercyclical nature of labor’s share helps to smooth the consumption of the workers relative to that of capital owners.

Malloy, Moskowitz, and Vissing-Jörgensen (2009) study consumption growth data for stockholders versus non-stockholders over the period 1982 to 2004. Using their data, the consumption growth volatility ratio for the two groups is 1.63, as shown in bottom row of Table 6. The corresponding volatility ratio in the model is a bit higher at 1.99. Expanding the Malloy, Moskowitz, and Vissing-Jörgensen (2009) sample period to include the Great Depression and other volatile stock market episodes would likely magnify the volatility of stockholders’ consumption growth relative to that of non-stockholders. Aït-Sahalia, Parker, and Yogo (2004) argue that luxury goods sales provide a much better proxy for the consumption of U.S. stockholders than does aggregate consumption. They find (p. 2974) that luxury retail sales growth is about 4 times more volatile than aggregate consumption growth and that sales of luxury goods covary positively with excess stock market returns over the period 1961 to 2001. By comparison, Table 7 shows that the capital owner’s consumption growth is only 1.8 times more volatile than aggregate consumption growth.

In the model of Guvenen (2009), the source of extra volatility for stockholders is the bond market; stockholders make interest payments to bondholders which smooths the bondholders’ consumption but magnifies the volatility of stockholders’ consumption. Guvenen’s model delivers a consumption growth volatility ratio for stockholders relative to non-stockholders of 2.4. In the model of Danthine and Donaldson (2002), the source of extra volatility for capital owners is the wage contract which smooths workers’ consumption at the expense of larger fluctuations in capital owners’ consumption. In the version of their model that delivers an equity premium approaching 6%, the capital owners’ consumption growth is 10 times more volatile than aggregate consumption growth.30

Finally, Table 8 shows the results of model forecasting regressions that seek to predict either the excess return on equity relative to short-term bonds $R^*_t - R^b_{t+1}$ or the gross dividend growth rate $d_{t+1}/d_t$ using the prior year’s value of the dividend yield $d_t/p_t$ (i.e., the inverse of the price-dividend ratio). For the model regressions, the table reports results for the baseline calibration with $\alpha = 5$ and $\kappa = 0.2$ and an alternative calibration with $\alpha = 3$ and $\kappa = 0.52$. The alternative calibration has the same steady-state coefficient of relative risk aversion as

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29 The data are available from <www.kellogg.northwestern.edu/faculty/vissing/htm/research1.htm>
30 See Table 6, Panel A, p. 62 in Danthine and Donaldson (2002). They do not report the volatility of the worker’s consumption growth.
the baseline, but the higher value of the habit parameter $\kappa$ implies more time-variation in the risk aversion coefficient in response to shocks, as governed by equation (17). The signs and magnitudes of the model-generated regression coefficients are influenced by the values of $\alpha$ and $\kappa$.

Both U.S. data regressions imply that a higher dividend yield predicts higher excess returns on equity. Put another way: when equity prices are temporarily low relative to dividends (i.e., a high value for $d_t/p_t$) future equity prices will tend to rise faster than dividends, pushing the dividend yield back down towards its long-run mean and in so doing, delivering a higher return on equity relative to bonds. The U.S. data results for the excess return regression in Table 4 are similar to those reported by Cochrane (2008), p. 1534. He estimates a statistically significant value of $b_r = 3.83$ (standard error = 1.47) using U.S. stock market data for the period 1929 to 2004.

Like the data, the concentrated ownership model delivers a positive estimated coefficient in the excess return regression, with $b_r = 2.91$ for the baseline calibration and $b_r = 5.24$ for the alternative calibration. The higher value of the habit parameter $\kappa$ in the alternative calibration delivers more time variation in the capital owner’s risk aversion coefficient and hence more time variation in the excess return which compensates the capital owner for undertaking the risk of holding equity. The representative agent model delivers a negative estimated coefficient $b_r = -0.82$ for the baseline calibration but a positive estimated coefficient $b_r = 2.56$ for the alternative calibration. Since the representative agent’s stochastic discount factor is driven by aggregate consumption, it is less volatile than the capital owner’s stochastic discount factor for any given value of $\alpha$ and $\kappa$. The mean excess return in the representative agent model is also much smaller than in the concentrated ownership model. The alternative calibration regression coefficient of $b_r = 2.56$ for the representative agent model, is much closer to the results in the data.

For the dividend growth regression, Table 8 shows that the data yield a negative estimated coefficient $b_d = -2.54$ for sample period from 1900 to 2011. This result implies that when equity prices are temporarily low relative to dividends (i.e., a high value for $d_t/p_t$) future dividend growth rates will tend to be lower on average, thus helping to justify the current state of low equity prices relative to dividends. However, in the more recent sample period from 1948 to 2011, the estimated coefficient in the dividend growth regression is positive but statistically insignificant, i.e., $b_d = 0.28$ (standard error = 0.59). Cochrane (2008) estimates $b_d = 0.07$ (standard error = 1.16) using data for the period 1929 to 2004. Hence, the more recent data imply that a low dividend yield is not predictive of higher future dividend growth.

Table 8 shows that the concentrated ownership model can deliver either a positive or negative value of $b_d$, depending on the calibration. The baseline calibration yields $b_d = 1.38$ while the alternative calibration yields $b_d = -0.52$. This pattern can once again be traced to
the relative impact of temporary versus permanent shocks on the dividend growth rate versus the dividend yield. The relative volatility of the two shock innovations is influenced by the values of \( \alpha \) and \( \kappa \) because the productivity shock innovation \( \sigma_e \) must be recalibrated each time in order to match the 2.6% standard deviation of per capita real aggregate consumption growth in U.S. data. Overall, Table 8 shows that the concentrated ownership model can produce regression results that are similar to those in the data.

### Table 8: Forecasting Regressions

<table>
<thead>
<tr>
<th>Regression</th>
<th>Concentrated Ownership</th>
<th>Representative Agent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{t+1} - R^*_t )</td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td>( \alpha = 5 )</td>
<td>( \alpha = 3 )</td>
<td>( \kappa = 0.2 )</td>
</tr>
<tr>
<td>U.S. Data 1900-2011</td>
<td>1.82</td>
<td>3.57</td>
</tr>
<tr>
<td>U.S. Data 1948-2011</td>
<td>(1.11)</td>
<td>(1.51)</td>
</tr>
<tr>
<td>( d_{t+1}/d_t )</td>
<td>-2.54</td>
<td>0.28</td>
</tr>
<tr>
<td>( a_d + b_d ) (( g_t/p_t )) + ( \eta_{t+1} )</td>
<td>(0.61)</td>
<td>(0.59)</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses. Model regressions based on data from a 20,000 period simulation.

### 5 Conclusion

A long history of research since Mehra and Prescott (1985) has sought to develop models that can account for the high mean and high volatility of observed equity returns relative to bond returns. One branch of this research has focused on investigating modifications to agents’ preferences that govern attitudes towards risk or intertemporal substitution. Another branch has focused on investigating changes to the structure of the cash flows that are priced by agents in the model. This paper falls mainly into the second category. The basic intuition for the results is that capital owners demand a high equity premium to compensate for the risk of linking their consumption to a volatile dividend stream. Since ownership of stock market wealth in the U.S. economy is highly concentrated at the top end of the income distribution, the owners of this wealth must bear a disproportionate share of the risk from factors that cause dividend-type income to fluctuate.

In the model, volatility derives from two sources: (1) a random walk productivity shock and (2) a temporary but persistent distribution shock that shifts capital’s share of total income in a pro-cyclical manner, consistent with U.S. data. As the volatility of the distribution shock increases, the mean equity premium rises and the return on equity becomes more volatile (Table 3). With reasonable levels of risk aversion (coefficients of relative risk aversion around 6), the concentrated ownership model delivers an equity premium relative to short-term bonds.
of nearly 5%. The model can approximately match other quantitative features of U.S. data under the assumption of fully-rational expectations, but it notably underpredicts the volatility of the price-dividend ratio and the volatility of investment growth. Both deficiencies could potentially be addressed by a richer model that allows more than two fundamental shocks or perhaps allows a role for non-fundamental factors.
A Appendix: First-Order Condition in Stationary Variables

To facilitate a solution for the equilibrium allocations, the first-order condition (15) must be rewritten in terms of stationary variables. Given that labor supply is inelastic, the combined entity of the firm and capital owner must only decide the fraction of available resources to be devoted to investment, with the remaining fraction devoted to consumption. The investment-capital ratio \( i_t / k_t \) is uniquely pinned down by the growth rate of capital. Hence, I employ \( x_t \equiv k_{t+1} / k_t \) as the capital owner’s single decision variable. There are four stationary state variables: (1) the normalized capital stock defined as \( k_{n,t} \equiv k_t / \left[ \exp \left( z_t \right) (\ell^c)_t \left( n \ell^w \right)^{1-a} \right] \), (2) the distribution shock \( v_t \), (3) the lagged consumption-capital ratio \( c_{t-1} / k_{t-1} \), and (4) the lagged decision variable \( x_{t-1} \). The last two state variables summarize the influence of the external habit stock. From the definition of \( k_{n,t} \) and the production function (6), it follows that \( y_t / k_t = (k_{n,t})^{\beta t-1} \).

Dividing both sides of the firm’s first-order condition (15) by \( k_{t+1} \) and then employing the definitions of \( x_t \) and \( k_{n,t} \) yields:

\[
(i_t / k_t) \cdot g(x_t) \cdot x_t^{-1} = \frac{E_t M_{t+1} \left\{ \theta_{t+1} (k_{n,t+1})^{\beta_{t+1}-1} - (i_{t+1} / k_{t+1}) \left[ 1 - g(x_{t+1}) \right] \right\}}{\theta_{t+1} (k_{n,t+1})^{\beta_{t+1}-1} - (i_{t+1} / k_{t+1}) \left[ 1 - g(x_{t+1}) \right]},
\]

(A.1)

where \( i_t / k_t = (1 - \delta) \left\{ \frac{1}{\lambda} \left[ \frac{x_t}{B (1 - \delta)} \right] \frac{1}{\psi_k} - \frac{1 - \lambda}{\lambda} \right\}^{1/\psi_k} \) for all \( t \),

\[
g(x_t) \equiv 1 + \frac{1 - \lambda}{B(1-\delta)} \frac{\psi_k}{\left\{ B(1-\delta) \right\} - (1 - \lambda)} \text{ for all } t,
\]

\[
M_{t+1} \equiv \beta \exp(-\phi \mu) \left[ \frac{c_{t+1} / k_{t+1} - \kappa \exp(\mu) (c_t / k_t) x_t^{-1}}{c_t / k_t - \kappa \exp(\mu) (c_{t-1} / k_{t-1}) x_{t-1}^{-1}} \right] - \alpha x_t^{-\alpha},
\]

\[
c_t / k_t = [\theta_t + a (1 - \theta_t)] (k_{n,t})^{\beta_t-1} - i_t / k_t \text{ for all } t.
\]

Using the definitions of \( k_{n,t} \) and \( x_t \), the law of motion for the normalized capital stock is

\[
k_{n,t+1} \equiv \frac{k_{t+1}}{\exp \left( z_{t+1} \right) (\ell^c)_t \left( n \ell^w \right)^{1-a}} = \frac{k_t x_t \exp (z_t - z_{t+1})}{\exp (z_t) (\ell^c)_t \left( n \ell^w \right)^{1-a}} = k_{n,t} x_t \exp(-\mu - \varepsilon_{t+1}) , \quad (A.2)
\]

which is conveniently log-linear before undertaking any approximation.

An expression for the capital owner’s consumption growth in terms of stationary variables is given by

\[
c_{t+1} / c_t = x_t \left[ \frac{\theta_{t+1} + a (1 - \theta_{t+1}) (k_{n,t+1})^{\beta_{t+1}-1} - i_{t+1} / k_{t+1}}{\theta_t + a (1 - \theta_t) (k_{n,t})^{\beta_t-1} - i_t / k_t} \right] , \quad (A.3)
\]

where \( i_{t+1} / k_{t+1} \) and \( i_t / k_t \) depend on the decision variables \( x_{t+1} \) and \( x_t \), respectively, as shown in (A.1). It is straightforward to derive analogous expressions for dividend growth \( d_{t+1} / d_t \), output growth \( y_{t+1} / y_t \), aggregate consumption growth \( c_{t+1}^a / c_t^a \), and the worker’s consumption growth \( c_{t+1}^w / c_t^w \).
A.1 Asset Pricing Variables

Given the equilibrium relationships \( p_t = i_t g(x_t) \) and \( d_t = \theta_t y_t - i_t \), it is straightforward to derive the following expressions for the equity price-dividend ratio and the gross equity return in terms of stationary variables:

\[
p_t^s / d_t = \frac{\left(i_t / k_t\right) g(x_t)}{\theta_t (k_{n,t})^{\theta_t - 1} - i_t / k_t},
\]

\[
R_{t+1}^s = \frac{p_{t+1}^s + d_{t+1}}{p_t^s} = \frac{\left(p_t^s / d_t + 1\right)}{\theta_t (k_{n,t})^{\theta_t - 1} - i_t / k_t},
\]

where \( i_{t+1} / k_{t+1} \) and \( i_t / k_t \) depend on the decision variables \( x_{t+1} \) and \( x_t \), respectively, as shown in (A.1). The above expressions show that the distribution shock (which drives movement in \( \theta_t \)) has a direct impact on the volatility of the equity return.

The remaining asset pricing variables are the one-period bond return \( R_{t+1}^b \) (the risk free rate) and the long-term bond return \( R_{t+1}^c \) which are defined as follows:

\[
R_{t+1}^b = \frac{1}{p_t^b} = \frac{1}{E_t M_{t+1}},
\]

\[
R_{t+1}^c = \frac{1 + \delta p_{t+1}^c}{p_t^c} = \frac{1 + \delta p_{t+1}^c}{E_t M_{t+1} (1 + \delta p_{t+1}^c)},
\]

where \( M_{t+1} \) is shown in (A.1). Approximate solutions for the stationary bond prices \( p_t^b \) and \( p_t^c \) take the form of log-linear decision rules as a function of the four state variables \( k_{n,t}, v_t, c_{t-1} / k_{t-1}, \) and \( x_{t-1} \). The approximate solutions are used as a starting values for the PEA solution described in Appendix C.

B Appendix: Approximate Log-linear Solution

An approximate solution to the transformed first-order condition (A.1) takes the form of the following log-linear decision rule for \( x_t \) as a function of the four state variables:

\[
x_t = \bar{x} \left[ k_{n,t} \right]^{s_1} \exp \left[ s_2 v_t \right] \left[ \frac{c_t / k_t}{c / k} \right]^{s_3} \left[ \frac{x_{t-1}}{\bar{x}} \right]^{s_4},
\]

where \( s_1 \) through \( s_4 \) are solution coefficients. The Taylor-series approximation is taken around the ergodic mean such that \( \bar{x} \equiv \exp \{ E[\log(x_t)] \} = \exp(\mu) \), \( \bar{k}_n \equiv \exp \{ E[\log (k_{n,t})] \} \), and \( \bar{c} / k \equiv \exp \{ E[\log (c_t / k_t)] \} \).

After substituting in the various laws of motion, including (A.2), into the transformed first-order condition (A.1), I take logarithms and apply a first-order Taylor series approximation.
to each side to obtain the following expression

\[
a_0 \left[ \frac{x_t}{\bar{x}} \right]^{a_1} \exp \left[ a_2 v_t \right] \left[ \frac{k_{n,t}}{k_n} \right]^{a_3} \left[ \frac{c_{t-1}/k_{t-1}}{c/k} \right]^{a_4} \left[ \frac{x_{t-1}}{\bar{x}} \right]^{a_5} = E_t B_0 \left[ \frac{x_t}{\bar{x}} \right]^{b_1} \exp \left[ b_2 v_t \right] \left[ \frac{k_{n,t}}{k_n} \right]^{b_3} \exp \left( b_4 v_{t+1} + b_5 \varepsilon_{t+1} \right) \left[ \frac{x_{t+1}}{\bar{x}} \right]^{b_6}, \quad (B.2)
\]

where \( a_i \) and \( b_i \) for \( i = 0, 1, 2, \ldots \) are Taylor series coefficients. The Taylor-series coefficients are functions of the ergodic-mean approximation points \( \bar{x}, \bar{k}_n, \) and \( c/k. \) Similarly, the laws of motion governing the evolution of the endogenous state variables \( k_{n,t} \) and \( c_{t-1}/k_{t-1} \) are approximated as

\[
\frac{k_{n,t+1}}{k_n} = \left[ \frac{x_t}{\bar{x}} \right] \left[ \frac{k_{n,t}}{k_n} \right] \exp (-\mu - \varepsilon_{t+1}), \quad (B.3)
\]

\[
\frac{c_t/k_t}{c/k} = \left[ \frac{x_t}{\bar{x}} \right] f_t \exp \left[ f_2 v_t \right] \left[ \frac{k_{n,t}}{k_n} \right]^{f_1}, \quad (B.4)
\]

where (B.3) follows directly from (A.2).

The conjectured form of the solution (B.1) is iterated ahead one period and then substituted into the right-side of equation (B.2) together with the approximate laws of motion (B.3) and (B.4) and the law of motion for the distribution shock (9). After evaluating the conditional expectation and then collecting terms, we have

\[
x_t = \bar{x} \left[ \frac{b_0}{a_0} \right] \left[ \frac{b_3 - a_3 + b_6(s_1 + f_3 s_3)}{a_1 - b_1 - b_6(s_1 + f_1 s_3 + s_4)} \right] \exp \left[ \frac{1}{2} (b_4 + b_6 s_2)^2 \sigma_s^2 + \frac{1}{2} (b_5 - b_6 s_1)^2 \sigma_s^2 \right]^{a_1} \left[ \frac{x_t}{\bar{x}} \right]^{a_2} \left[ \frac{k_{n,t}}{k_n} \right]^{a_3} \exp \left[ \frac{b_2 - a_2 + b_6 f_2 s_3 + \rho(b_4 + b_6 s_2)}{a_1 - b_1 - b_6(s_1 + f_1 s_3 + s_4)} \right]^{a_4} \left[ \frac{x_{t-1}}{\bar{x}} \right]^{a_5} \left[ \frac{x_t}{\bar{x}} \right]^{a_6} \left[ \frac{k_{n,t}}{k_n} \right]^{a_7} \left[ \frac{c_{t-1}/k_{t-1}}{c/k} \right]^{a_8} \left[ \frac{x_{t-1}}{\bar{x}} \right]^{a_9}, \quad (B.5)
\]

which yields four equations in the four solution coefficients \( s_1 \) through \( s_4. \)

From the transformed first-order condition (A.1), the Taylor-series coefficients \( a_0 \) and \( b_0 \) are given by

\[
a_0 = \tilde{i}/k \bar{g}(\bar{x}) / \bar{x}, \quad (B.6)
\]

\[
b_0 = \tilde{M} \left\{ \theta \left( \bar{k}_n \right)^{\theta-1} - \tilde{i}/k \left[ 1 - \bar{g}(\bar{x}) \right] \right\}, \quad (B.7)
\]
where $\tilde{x} = \exp(\mu), \tilde{i}/k = \text{func}(\tilde{x}), \tilde{c}/k = \text{func}(\tilde{k}_n, \tilde{x}),$ and $\tilde{M} = \text{func}(\tilde{c}/k, \tilde{x})$. Given these relationships, the constant term in (B.5) yields a fifth equation that pins down the approximation point $\tilde{k}_n$.

C Appendix: Nonlinear Model Solution

The impulse response functions in Figures 2 and 3 and model simulation results are generated using the solution method outlined below that preserves the model’s nonlinear equilibrium conditions. The method employs a version of the parameterized expectation algorithm (PEA) described by Den Haan and Marcet (1990).

After substituting in the various laws of motion, the transformed first-order condition (A.1) can be represented as:

$$f(x_t, k_{n,t}, v_t, c_{t-1}/k_{t-1}, x_{t-1}) = E_t h(x_t, k_{n,t}, v_t, x_{t+1}, u_{t+1}, \varepsilon_{t+1})$$

where $h(\cdot)$ is the nonlinear object to be forecasted. For purposes of constructing the conditional expectation, the function $h(\cdot)$ is approximated as

$$h(\cdot) \simeq d_0 [k_{n,t}]^{d_1} \exp [d_2 v_t] [c_{t-1}/k_{t-1}]^{d_3} [x_{t-1}]^{d_4} \exp [d_5 u_{t+1} + d_6 \varepsilon_{t+1}],$$

where $d_0$ through $d_6$ are regression coefficients that are obtained by projecting the true nonlinear function $h(\cdot)$ onto the form (C.2) during repeated simulations of the model, as described below. The initial guesses for $d_0$ through $d_6$ are computed using the approximate log-linear solution from Appendix B.

Given a set of initial guesses for $d_0$ through $d_6$, a simulation is run where the conditional expectation on the right side of (B.1) is constructed each period as

$$E_t h(\cdot) = d_0 [k_{n,t}]^{d_1} \exp [d_2 v_t] [c_{t-1}/k_{t-1}]^{d_3} [x_{t-1}]^{d_4} \exp \left[ \frac{1}{2} (d_5 \sigma_u)^2 + \frac{1}{2} (d_6 \sigma_\varepsilon)^2 \right].$$

Given the forecast $E_t h(\cdot)$, the nonlinear function (C.1) is solved each period for the decision variable $x_t$ using a nonlinear equation solver. The endogenous state variables $k_{n,t}$ and $c_{t-1}/k_{t-1}$ evolve according to their exact nonlinear laws of motion. The endogenous state variable $x_{t-1}$ is simply the lagged decision variable. During the simulation, realized values of the nonlinear function $h(\cdot)$ are constructed. At the end of the simulation, the realized values of $h(\cdot)$ are projected onto the form (C.2) to obtain new guesses for $d_0$ through $d_6$. The simulation is then repeated using the new guesses for $d_0$ through $d_6$ with the same sequence of draws for the shock innovations $u_{t+1}$ and $\varepsilon_{t+1}$. The procedure is stopped when the guesses for $d_0$ through $d_6$ do not change from one simulation to the next. In practice, convergence to five decimal places occurs after about 250 simulations.

An analogous procedure is used to construct the conditional expectations in the bond pricing equations (4) and (5) to solve for $p^b_t$ and $p^c_t$ each period. Specifically, the nonlinear objects to be forecasted are approximated by power functions of the state variables and shock innovations as follows:
\[ p_t^b = E_t M_{t+1}, \]

where \( M_{t+1} \approx m_0 [k_{n,t}]^{m_1} \exp [m_2 v_t] [c_{t-1}/k_{t-1}]^{m_3} [x_{t-1}]^{m_4} \exp [m_5 u_{t+1} + m_6 \varepsilon_{t+1}] \),

\[ (C.4) \]

\[ p_t^c = E_t M_{t+1} + E_t \delta M_{t+1} \hat{p}_{t+1}^c \]

where \( \delta M_{t+1} \hat{p}_{t+1}^c \approx n_0 [k_{n,t}]^{n_1} \exp [n_2 v_t] [c_{t-1}/k_{t-1}]^{n_3} [x_{t-1}]^{n_4} \exp [n_5 u_{t+1} + n_6 \varepsilon_{t+1}] \).

\[ (C.5) \]

The initial guesses for the regression coefficients \( m_0 \) through \( m_6 \) and \( n_0 \) through \( n_6 \) are computed using the approximate log-linear solution of the model. After each simulation, new guesses for the regression coefficients are obtained by projecting the realized values of the nonlinear functions \( M_{t+1} \) and \( \delta M_{t+1} \hat{p}_{t+1}^c \) onto the forms shown in (C.4) and (C.5) until convergence is achieved.
References


Walentin, K. 2010 Earnings inequality and the equity premium, *B.E. Journal of Macroeconomics* 10 (Contributions), article 36.


Figure 1: The U.S. top decile income share and capital’s share of total income both exhibit significant volatility, motivating a model with distribution shocks. Horizontal dashed lines show the sample means. The equity premium (return on stocks minus the return on short-term bonds) is positively correlated with the change in both income share variables.
Figure 2: Concentrated ownership model: Impulse responses to one standard deviation innovation of the temporary distribution shock (solid blue line) or the permanent productivity shock (dashed red line).
Figure 3: Concentrated ownership model: Impulse responses to one standard deviation innovation of the temporary distribution shock (solid blue line) or the permanent productivity shock (dashed red line).
Figure 4: Sensitivity of the equity premium $E \left( R_{t+1} - R_{t+1}^b \right)$ to parameter values in the concentrated ownership model (solid blue line) and the representative agent model (dashed red line). Vertical lines mark the baseline calibration in the concentrated ownership model.
Figure 5: Effect of curvature parameter $\sigma_k$ on capital adjustment costs in the concentrated ownership model. Smaller values of $\sigma_k$ imply more complementarity between new investment and existing capital, which raises adjustment costs.
Figure 6: Simulated variables in the concentrated ownership model (solid blue line) are considerably more volatile than those in the representative agent model (dashed red line).
Figure 7: Asset pricing variables: Data versus models.
Figure 8: Asset returns in the concentrated ownership model exhibit time-varying means and volatilities, similar to that observed in the data.