Is the Potential for International Diversification Disappearing?  
A Dynamic Copula Approach

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Motivation

Understanding the evolution of co-movements in international markets is crucial for asset pricing and portfolio selection.

Research Questions

1. How and has cross-country dependence changed through time?
   - Cross-country linear correlations have not increased (Bekaert, Hodrick, and Zhang (2009))

2. Is correlation a satisfactory dependence measure in international markets?
   - Correlations are higher in down markets (Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002))

3. How does the diversification benefit of emerging markets compare to developed countries?
   - Differences in the evolution of correlations?
   - Differences in tail dependence?
Average Linear Rolling Correlation on Weekly Returns
Average Threshold Correlations on Weekly Returns

16 Developed Markets
1973–2009

13 Emerging Markets
1989–2008

Empirical Gaussian Distribution

Christoffersen, Errunza, Jacobs and Langlois (2012)
Our key contributions are

1. We develop a model which
   - can be estimated on a large set of countries
   - can accommodate for
     - dynamic dependence
     - a trend in correlation
     - positive tail dependence
     - univariate and multivariate asymmetries

2. We develop a diversification benefit measure that takes into account higher order moments
Our key results are

1. Cross-country dependence has significantly increased over time
   - Dependence for emerging markets is still a lot lower than for developed countries

2. We find overwhelming evidence of non-normalities in dependence
   - Tail dependence is both positive and asymmetric for developed and emerging markets

3. We confirm with different panel regressions that
   - dependence is positively linked to volatility
   - although dependence is related to market integration, financial, and macro variables, the time trend is still significant and remains unexplained
Multivariate Model

We decompose the conditional multivariate log-likelihood function as

\[ L = \sum_{t=1}^{T} \sum_{i=1}^{N} \log \left( f_{i,t}(R_{i,t}) \right) + \sum_{t=1}^{T} \log \left( c_{t}(F_{1,t}(R_{1,t}), F_{2,t}(R_{2,t}), \ldots, F_{N,t}(R_{N,t})) \right) \]

where

- \( T \) is the number of weeks in our sample
  - developed markets 1973-2009
  - emerging markets 1989-2008
  - investable emerging market 1995-2009

- \( N \) is the number of countries used in the estimation
  - 16 developed markets
  - 13 emerging markets
  - 17 investable emerging markets
Volatility Model for Each Country

We decompose the conditional multivariate log-likelihood function as

\[
L = \sum_{t=1}^{T} \sum_{i=1}^{N} \log \left( f_{i,t}(R_{i,t}) \right) + \sum_{t=1}^{T} \log \left( c_t \left( F_{1,t}(R_{1,t}), F_{2,t}(R_{2,t}), \ldots, F_{N,t}(R_{N,t}) \right) \right)
\]

where

\[
f_{i,t}(R_{i,t}) \text{ is given by a AR-NGARCH model}
\]

\[
R_{i,t} = \mu_{i,t} + \sigma_{i,t} z_{i,t}
\]

\[
\sigma_{i,t}^2 = \omega_i + \alpha_i \left( \varepsilon_{i,t-1} - \gamma_i \sigma_{i,t-1} \right)^2 + \beta_i \sigma_{i,t-1}^2
\]

2 sources of univariate asymmetry

1. leverage effect \( \Rightarrow \gamma_i \)

2. residual asymmetry \( \Rightarrow z_{i,t} \) comes from an asymmetric t distribution

Dependence Model

We decompose the conditional multivariate log-likelihood function as

\[ L = \sum_{t=1}^{T} \sum_{i=1}^{N} \log \left( f_{i,t} \left( R_{i,t} \right) \right) \quad + \sum_{t=1}^{T} \log \left( c_{t} \left( F_{1,t} \left( R_{1,t} \right), F_{2,t} \left( R_{2,t} \right), \ldots, F_{N,t} \left( R_{N,t} \right) \right) \right) \]

where

\[ c_{t} \left( F_{1,t} \left( R_{1,t} \right), \ldots \right) \] comes from a skewed t copula with

- \( \Psi_{t} \) a time-varying correlation matrix
- \( \nu \) a degree-of-freedom parameter
- \( \lambda \) an asymmetry parameter
The Dynamic Asymmetric Copula Model

The copula correlation matrix is **time-varying**

At time $t$, it is given by a weighted average of 3 components

$$
\Gamma_t = (1 - \beta_\Gamma - \alpha_\Gamma) [(1 - \varphi_\Gamma) \Omega + \varphi_\Gamma \Upsilon_t] + \beta_\Gamma \Gamma_{t-1} + \alpha_\Gamma z_{t-1}^* z_{t-1}^{*\top}
$$

where

$$
\Upsilon_t \text{ captures a deterministic trend } \Upsilon_t = \frac{\delta^2 t^2}{1 + \delta^2 t^2}
$$

$\Gamma_{t-1}$ is the lagged correlation matrix

$z_{t-1}^* z_{t-1}^{*\top}$ is the cross-product of copula shocks
Estimation for Many Countries

Estimation on many countries is made possible by two improvements

1. We use a moment estimator for $\Omega$

$$\hat{\Omega} = \frac{1}{T} \sum_{t=1}^{T} \tilde{z}_t^* \tilde{z}_t^{*\top} - \varphi \frac{1}{T} \sum_{t=1}^{T} \Upsilon_t \frac{1}{1 - \varphi}$$

where $\frac{1}{T} \sum_{t=1}^{T} \tilde{z}_t^* \tilde{z}_t^{*\top}$ is the sample copula correlation

2. From Engle, Shephard and Sheppard (2008), we maximize the composite log-likelihood

$$CL(\theta) = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j>i}^{N} \ln c_t(\eta_{i,t}, \eta_{j,t}; \theta)$$

Bivariate log-likelihood for countries $i$ and $j$
Evolution of Average Copula Correlation

1989–2008
16 Developed Markets

13 Emerging Markets, IFCG

All 29 Markets

16 DMs vs 13 EMs Cross–Correlation

1995–2009
16 Developed Markets

17 Emerging Markets, IFCI

All 33 Markets

16 DMs vs 17 EMs Cross–Correlation

Dynamic
Constant
Long–run
90% C.I.
Evolution of Copula Correlation for Developed Markets

- 1973–2009
- 1989–2008
- 1995–2009

Dynamic
Long-run
Constant
90% C.I.

Christoffersen, Errunza, Jacobs and Langlois (2012) Is the Potential for International Diversification Disappearing?
Evolution of Regional Copula Correlation

European Union (EU)
Developed Non–EU
Latin America
Emerging Eurasia
Evolution of Average Tail Dependence

16 Developed Markets

13 Emerging Markets

All 29 Markets

Christoffersen, Errunza, Jacobs and Langlois (2012)
Model Implied Threshold Correlation

16 Developed Markets
1973–2009

13 Emerging Markets
1989–2008

Empirical
Gaussian Distribution
NS–DCD t Copula
NS–DCD Skewed t Copula
NS–DCD Skewed t Copula with calibrated $\lambda$
To take into account higher order moments in the portfolio return distribution, we construct a diversification benefit measure based on expected shortfall

$$ES_t^q(R_i,t) = -E \left[ R_{i,t} | R_{i,t} \leq F_{i,t}^{-1}(q) \right]$$

Note that

$$ES_t^q = \underbrace{VaR_t^q(w_t^T R_t)}_{\text{Perfect diversification}} \leq \underbrace{ES_t^q(w_t^T R_t)}_{\text{Portfolio expected shortfall}} \leq \underbrace{\sum_{i=1}^N w_{i,t} ES_t^q(R_{i,t})}_{\text{No diversification}}$$

We define

$$CDB_t(w_t, q) = \frac{ES_t^q - ES_t^q(w_t^T R_t)}{ES_t^q - ES_t^q}$$
A Conditional Diversification Benefit Measure

The conditional diversification benefit measure

1. lies between 0 and 1
2. does not depend of expected returns

The Special Case of Normality

If returns are multivariate normal and \( q = 50\% \), then \( CDB_t \) reduces to

\[
CDB_t(w_t, q) = 1 - \frac{\sqrt{w_t^\top \Sigma_t w_t}}{w_t^\top \sigma_t}
\]

portfolio’s volatility

upper bound for portfolio’s volatility

Christoffersen, Errunza, Jacobs and Langlois (2012)
Evolution of Diversification Benefit

16 Developed Markets, 1973–2009


All 29 Markets, 1989–2008

Christoffersen, Errunza, Jacobs and Langlois (2012)
1. We propose a new model capturing dynamic trending copula correlation, tail dependence, and multivariate asymmetries

2. We propose a conditional diversification benefit measure which takes into account higher order moments

We find that

1. Cross-country dependence has significantly increased over time

2. But dependence for emerging markets is still lower than for developed countries