Eaton Comments on

“Technological Diversification”

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- Complex model

- Connects Producer Level and Aggregate Shocks in a very clever way
• Connection to Klette Kortum

  – Continuum of products

  – Continuum of firms

  – Heterogeneity across firms in number of products

  – Over time firms take products from other firms or lose products to them
• Here

  – Continuum of firms

  – Firms combine an integer number of inputs

  – In the static version of the model we can think of each firm using its own set of inputs

  – Firms add or lose inputs but we don’t need to think of them taking inputs away from other firms or losing inputs to other firms.
• Each firm $j$ combines inputs to produce its output with elasticity of substitution $\varepsilon$

$$y(j) = \left[ \sum_{i=1}^{n(j)} l(i, j) (\varepsilon - 1) / \varepsilon \right]^{\varepsilon / (\varepsilon - 1)}$$

• Consumers combine outputs of a measure 1 of firms to produce utility with elasticity of substitution $\varepsilon$:

$$Y = \left[ \int_{0}^{1} y(j) (\varepsilon - 1) / \varepsilon \, dj \right]^{\varepsilon / (\varepsilon - 1)}$$

so that:

$$Y = \left[ \int \sum_{i=1}^{n(j)} l(i, j) (\varepsilon - 1) / \varepsilon \, dj \right]^{\varepsilon / (\varepsilon - 1)}$$
• It’s easy to show that each firm will employ the same measure of workers \( \bar{\ell} \) on each activity so that \( \bar{\ell} = L/N \) where:

\[
N = \int n(j) dj
\]

so that:

\[
Y = LN^{1/(\varepsilon-1)}e
\]

• Progress occurs as firms add inputs
• Up to this point we haven’t had to say anything about whether one firm’s input is the “same” as another’s or not.

• All that matters is the number of inputs that each firm is using.
• But now rank inputs so that all firms using 1 input use the input 1, all firms using 2 inputs use the input 1 and 2, etc.

• Allow for an input to “die” with hazard $\gamma$.

• If input 1 dies every firm loses it: $N$ falls by the measure of firms 1.

• If input $i$ dies then all the firms using $i' \geq i$ are affected. If the measure is $m(i)$ then $N$ falls by $m(i)$. Note that $m(i)$ is decreasing in $i$.

• Hence the death of an input creates an aggregate shock.
• The range of different inputs in use can increase when the frontier firm adds a new input.

• But passing the frontier isn’t any harder than acquiring an input that is already used by other firms.
• Much of the workings of the model can be described without the need for any dynamic optimization
  
  – A firm adds an input with hazard $\lambda$ which is independent of the number of firms using or that have ever used that input
  
  – All firms using input $i$ lose it with hazard $\gamma$

• But, as in KK, the authors introduce an activity that increases the probability of finding a new input for the firm

• The added difficulty here is that, since the economy is subject to aggregate shocks, the dynamic optimization is much more challenging
Here is where I got lost

• The authors specify demand for firm $j$ as:

$$ Y p(j)^{-\varepsilon} $$

where “aggregate output is taken as the numeraire.”

• But I would have specified it as:

$$ \frac{p(j)^{-\varepsilon}}{P^{-\varepsilon}} Y $$

where $P$ is the CES price index

$$ P = \left[ \int_{0}^{1} p(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)} $$

(See, e.g., Atkeson Burstein)
In Koren Tenreyro

\[ p(j) = \overline{m}wn(j)^{1/(1-\varepsilon)} \]

where:

\[ \overline{m} = \frac{\varepsilon}{\varepsilon - 1} \]

so that:

\[ P = \overline{m}wN^{1/(1-\varepsilon)} \]
With this difference I get that the profit of firm $j$ is:

$$\frac{wL}{(\varepsilon - 1)N} n(j)$$

as opposed to their:

$$\frac{1}{\varepsilon} N^{(2-\varepsilon)/(\varepsilon-1)} Ln(j)$$

So I can't find a value of $\varepsilon$ that eliminates feedback from $N$ to profits
• Whatever the resolution any flaw is not fatal

• I don’t think that the optimizing dynamics are playing that much of a role.

• The dynamics can be summarized in terms of the probability of gaining a new input

• I checked most of the rest of the paper and we agreed

• The key contribution, a connection between firm-level and aggregate shocks, remains
Suggestions

• Do much more with the simulations

• Compare the moments generated by the model with firm and aggregate correlations

• Get through regressions at the front faster: get to the model and simulations