Discussion of Comin and Mulani (2006)

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Recent Trends in Economic Volatility: Sources and Implications
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- Empirical relationships.
- Calibrated model predictions.
  - Social planner solution.
COMPUSTAT: Increase in firm size and productivity volatility.

LBD: Overall decrease in firm size and productivity volatility (Davis et al. (2006)).
  - Publicly held: increase in volatility.
  - Privately held: decrease in volatility.
  - Overall firm population trend dominated by privately held firms.

Comin and Mulani (2006) model is not a model of publicly held firms, only.

Aggregate growth and volatility measures include production from privately held firms.
  - Could consider producing aggregate measures on data from publicly held firms, only.
Authors find positive relationship between two-digit sectoral R&D intensity and within sector firm volatility. Adopt causal interpretation.

What causes cross-sector R&D intensity variation?

- Endogeneity bias?
- Even a casual “indicative evidence” usage of this regression is probably too strong.
Authors’ model calibration is,

<table>
<thead>
<tr>
<th></th>
<th>1950</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_h)</td>
<td>1.011</td>
<td>1.011</td>
</tr>
<tr>
<td>(\delta_q)</td>
<td>1.125</td>
<td>1.125</td>
</tr>
<tr>
<td>(\lambda^h)</td>
<td>2.070</td>
<td>1.036</td>
</tr>
<tr>
<td>(\lambda^q)</td>
<td>0.020</td>
<td>0.050</td>
</tr>
<tr>
<td>(\gamma_y)</td>
<td>0.025</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Growth implication for 2000 probably a bit low.

Mapping into model parameters? Existence?

- Production function parameters \(\alpha, \beta\).
- Mass of followers relative to leaders, \(m\).
- R&D cost and arrival process parameter, \(\lambda^q = \bar{\lambda} n^q/(1 - s)\).
- GI cost and arrival process parameters, \(\lambda^h = \bar{\lambda}^h (n^h)^{\rho^h}\).
Directly form Comin and Mulani (2006):

Optimal GI innovation condition,

\[
\frac{1}{\rho_h} (\bar{\lambda}^h) \frac{1}{\rho_h} (\lambda^h) \frac{1-\rho_h}{\rho_h} = \frac{(1 - s_t)(\delta_h - 1)}{\bar{\lambda}\delta_q} \tag{1}
\]

No arbitrage condition for R&D innovation

\[
(1 - s_t) = \bar{\lambda}\delta_q \frac{(1 - \alpha)\chi^l - c(\lambda^h)}{r + \lambda^q_t - \lambda^h_t(\delta_h - 1)}, \tag{2}
\]

where

\[
\chi^l = \left( \frac{(\beta\alpha^\alpha) \frac{1}{1-\alpha}}{(\beta\alpha^\alpha) \frac{1}{1-\alpha} + (1 - \beta) \frac{1}{1-\alpha}} \right).
\]

From footnote 30, sales of leaders are 70% higher than sales of followers,

\[
m = 1.7 \left( \frac{1 - \beta}{\beta\alpha^\alpha} \right) \frac{1}{1-\alpha} \Rightarrow \chi^l = \frac{1.7}{1.7 + m}. \tag{3}
\]
R&D subsidies, \( s_t \), driving process. Given the GI innovation cost specification, there exists a solution only if,

\[
\frac{1 - s_t}{1 - s_{t+1}} = \frac{r + \lambda^q_{t+1} - (1 - \rho_h)\lambda^h_{t+1}(\delta_h - 1)}{r + \lambda^q_t - (1 - \rho_h)\lambda^h_t(\delta_h - 1)}
\] (4)

Assume \( s_{1950} = 0 \). This implies \( s_{2000} = 0.3612 \).

The GI innovation cost curvature is given by,

\[
\rho_h = \left[1 + \frac{\ln (1 - s_t) - \ln (1 - s_{t+1})}{\ln \lambda_{h,t} - \ln \lambda_{h,t+1}}\right]^{-1} = 0.6070.
\] (5)

By \( \alpha \in (0, 1) \) it follows that \( (1 - \alpha)\chi^t \in (0, 1) \). This establishes a lower bound on \( \bar{\lambda}^h > 5.1 \).
Make the identifying assumption that \( \alpha = .5 \). In this case, I obtain

<table>
<thead>
<tr>
<th>( m )</th>
<th>( \lambda^h )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12.500</td>
<td>0.239</td>
</tr>
<tr>
<td>10</td>
<td>25.000</td>
<td>0.749</td>
</tr>
<tr>
<td>100</td>
<td>92.700</td>
<td>6.485</td>
</tr>
<tr>
<td>10,000</td>
<td>1502.100</td>
<td>637.883</td>
</tr>
</tbody>
</table>

Will use in social planner analysis.
Multiple products in a two-digit sector?

- is $m$ large?
- Taken literally, if a U.S. two-digit sector has only one leader, $m$ on the order of 40,000.
- Seems like a non-starter when concerned with explaining the great diversity in size, productivity, and dynamics at the firm level in a two-digit sector.
- Rather, consider multiple products, $J = 40,000/(m + 1)$. Each product has its own R&D process independent of the other products.
- In this case, variance of productivity growth within sector is,

$$ V(\gamma_{ys}) = \frac{\lambda_s^q}{J} \ln(\delta_q)^2 + \lambda^h \ln(\delta_h)^2. $$

- If $J$ is large, all of sector volatility due to GI innovation process $\Rightarrow$ perfect co-movement. Cannot explain lower co-movement through increases in $\lambda_s^q$. 

Lentz - Discussion of Comin and Mulani (2006).
Hamiltonian,

\[
H = \ln (1 - n^q - n^h) + \ln q_t + \ln h_t + \frac{1}{\alpha} \ln \left[ \beta (x_l)^\alpha + (1 - \beta) (mx_f)^\alpha \right] \\
+ \omega_1 [L - x_l - mx_f] \\
+ \omega_2 \bar{\lambda} n^q \ln (\delta_q) \\
+ \omega_3 (1 + m) \bar{\lambda}^h \left( \frac{n^h}{1 + m} \right)^\rho \ln (\delta_h)
\]

Given calibration, corner solution where \( n^q = 0 \). Optimal \( n^h \) given by,

\[
\rho \bar{\lambda}^h (n^h)^{\rho - 1} \ln (\delta_h) = \frac{r}{1 - (m + 1) n^h}.
\]

Optimal growth rates,

<table>
<thead>
<tr>
<th>( m )</th>
<th>0</th>
<th>2</th>
<th>10</th>
<th>100</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_y )</td>
<td>0.024</td>
<td>0.165</td>
<td>0.653</td>
<td>6.171</td>
<td>613.453</td>
</tr>
</tbody>
</table>
- Extreme planner results partly a feature of $c'(0) = 0$.