Appendix for "Productivity and Unemployment over the Business Cycle"

Regis Barnichon

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A1. Data

Raw statistics

The data on labor productivity, unemployment, employment and hours per worker are taken from the U.S. Bureau of Labor Statistics (BLS). Labor productivity is measured as real average output per hour in the non-farm business sector and unemployment is the quarterly average of the monthly unemployment rate series constructed by the BLS from the Current Population Survey. The employment series is the number of employed workers (in millions) in the non-farm business sector. Hours per worker is derived from subtracting (log) employment from (log) total hours in the non-farm business sector (the results in the paper are robust to using total private average weekly hours from the Current Employment Survey (BLS)). The vacancy series is the composite Conference Board help advertising index (see Barnichon, 2009). All series are expressed as deviations from an HP-filter with smoothing parameter 1600, and the conclusions from the paper are independent of the smoothing parameter.

The construction of the TFP series is relatively standard and extends the series constructed by Beaudry and Portier (2006). Using nonfarm private business sector data from the BLS, I retrieve two annual series: labor share and capital services, and I interpolate the capital services series a quarterly series, assuming constant growth within the quarters of the same year. Output and total hours are quarterly and seasonally adjusted nonfarm business measures, from 1948Q1 to 2008Q4 (also from the BLS). I then construct a measure of (log) TFP as $TFP_t = \log(Y_t / H_t^{\bar{s}_h} K_t^{1-\bar{s}_h})$, where $\bar{s}_h$ is the average level of the labor share over the period.

VAR evidence

I use quarterly data taken from the U.S. Bureau of Labor Statistics (BLS) covering the period 1948:Q1 to 2008:Q4. Labor productivity x_t is measured as real average output per hour in the non-farm business sector, and unemployment u_t is the quarterly average of the monthly unemployment rate series constructed by the BLS from the Current Population Survey. Following Fernald (2007), I allow for two breaks in Δx_t, 1973:Q1 and 1997:Q1. The next section describes the details of the estimation method.

A2. Estimation of technology and non-technology shocks

I am interested in estimating the system

$$\begin{pmatrix} \Delta x_t \\ u_t \end{pmatrix} = C(L) \begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^d \end{pmatrix} = C(L)\varepsilon_t \tag{1}$$

where x_t is labor productivity defined as output per hours, u_t unemployment, $C(L)$ an invertible matrix polynomial and ε_t the vector of structural orthogonal innovations comprised of ε_t^a technology shocks and ε_t^d non-technology shocks. I use the estimation method of Shapiro and Watson (1988) and Francis and Ramey (2003) to allow for time-varying variance of the structural innovations.

Without loss of generality, (1) can be written

$$\begin{aligned} \Delta x_t &= \sum_{j=1}^{p} \beta_{xx,j} \Delta x_{t-j} + \sum_{j=0}^{p} \tilde{\beta}_{xu,j} u_{t-j} + \varepsilon_t^a \\ u_t &= \sum_{j=1}^{p} \beta_{uu,j} u_{t-j} + \sum_{j=1}^{p} \beta_{ux,j} \Delta x_{t-j} + \alpha \varepsilon_t^a + \varepsilon_t^m \end{aligned}$$

As discussed in Shapiro and Watson (1988), imposing the long run restriction that only technology shocks have a permanent effect on x_t is equivalent to restricting the variable u_t to enter the first equation in differences. Consequently, the system reduces to

$$\Delta x_t = \sum_{j=1}^{p} \beta_{xx,j} \Delta x_{t-j} + \sum_{j=0}^{p-1} \beta_{xu,j} \Delta u_{t-j} + \varepsilon_t^a \quad (2)$$

$$u_t = \sum_{j=1}^{p} \beta_{uu,j} u_{t-j} + \sum_{j=1}^{p} \beta_{ux,j} \Delta x_{t-j} + \alpha \varepsilon_t^a + \varepsilon_t^m \quad (3)$$

Since Δu_{t-j} is correlated with ε_t^a, equation (2) must be estimated with instrumental variables. I use lags 1 to $p = 4$ of Δx_t and u_t as instruments. The residual from this IV regression is the estimated technology shock $\hat{\varepsilon}_t^a$. The second equation can be identified by OLS but using $\hat{\varepsilon}_t^a$ in place of ε_t^a. Finally to allow for time-varying variance of the structural innovations (or more generally heteroskedasticity), I follow Francis and Ramey (2003) and estimate both equations jointly using GMM. That way, I can estimate the variance-covariance matrix of the estimates and generate the standard error bands for the impulse response functions. The error bands are derived by generating random vectors from a multivariate normal distribution with mean equal to the coefficient estimates and variance-covariance matrix equal to the estimated one, and then calculating the impulse response functions.

A3. Robustness of the empirical findings

VAR evidence

A number of researchers question the robustness of Gali's (1999) findings, and my approach may suffer from similar critiques. In this section, I discuss the robustness of my results in light of their findings. Christiano, Eichenbaum and Vigfusson (2003) argue that the negative response of labor input to a technology shock may be the result of a mistreatment of labor input in the empirical model. Depending on the filter applied to hours, the response of hours can change sign. In the paper, I follow Fernald (2007), remove the low-frequency movements in productivity by allowing for two breaks in Δx_t, 1973:Q1 and 1997:Q1, and my results do not depend on the method used to detrend unemployment.[1] Chang and Hong (2006) question Gali's (1999) use of output per hour as a measure of productivity. They argue that, because output per hour, unlike TFP, is influenced by permanent shifts in input mix (e.g. shocks affecting permanently the capital-labor ratio), Gali (1999) mislabels changes in input mix as technology shocks and does not properly identify the response of total hours worked to technology shocks. In Figure 1, I reproduce my VAR exercise using TFP instead of output per hour. Encouragingly, the impulse responses look very similar to the ones using output per hour, and technology shocks increase unemployment temporarily. Finally, Chang and Hong (2006) and Holly and Petrella (2008) show that while technology shocks may decrease hours worked at the aggregate level, this finding does not necessarily hold at the industry level. Unfortunately, the focus on unemployment does not allow me to test the validity of my results in this dimension.

I also estimate a higher dimensional (4 variable) VAR with the (logged) job finding probability and the (logged) employment exit probability as additional variables. Figure 2 plots the impulse responses to a positive technology shock. The responses of unemployment and output per hour are similar to the ones obtained from a bivariate VAR. The job finding probability declines significantly on impact and after two quarters displays a similar behavior to that of unemployment. The employment exit probability increases on impact before reverting quickly to its long run value. However, the initial response is only marginally significant.

The volatility drop in demand shocks

There is reassuring evidence (see Galí and Rabanal, 2004) that technology shocks are correctly identified by long run restrictions but, since I emphasize the role played by aggregate demand shocks, I also look at the Romer and Romer (2004) monetary shocks and verify that they

[1] The results are available upon request. Fernald (2007) showed that the presence of a low-frequency correlation between labor productivity growth and unemployment, while unrelated to cyclical phenomena, could significantly distort the estimates of short run responses obtained with long run restrictions. An alternative proposed by Fernald (2007) would be to separately analyze subsamples with no breaks in technology growth. In a robustness check, I restrict the sample period to 1973-1997 where there is no clear trend break. Results remain very similar.

experienced a volatility drop similar to the one used in the simulation. Those shocks are identified with a different method, but we can see in Figure 3 that, notwithstanding the large volatility increase in the late 70s, their volatility in 1975 is twice as high as that in 1990, a volatility drop similar to the one used in the simulation.

A4. Derivation of the model

Hours/effort decision and procyclical productivity

When a firm and a worker meet, I assume that both parties negotiate the hours/effort decision by choosing the optimal allocation. More precisely, they solve

$$\min_{h_{it},e_{it}} \frac{\lambda_h}{1+\sigma_h} h_t^{1+\sigma_h} + h_t \frac{\lambda_e}{1+\sigma_e} e_t^{1+\sigma_e} \tag{4}$$

subject to satisfying demand $A_t n_{it} h_{it}^{\alpha} e_{it}^{\alpha} = y_{it}^{d}$ at date t. The first-order conditions imply that effort per hour is a function of hours per worker $e_{it} = e_0 h_{it}^{\frac{\sigma_h}{1+\sigma_e}}$ where $e_0 = \left(\frac{1+\sigma_e}{\sigma_e}\frac{\lambda_h}{\lambda_e}\right)^{\frac{1}{1+\sigma_e}}$ is a positive constant. Thus, changes in hours can proxy for changes in effort, and I can rewrite the firm production function as

$$y_{it} = y_0 A_t n_{it} h_{it}^{\varphi} \tag{5}$$

with $y_0 = e_0^{\alpha}$ and $\varphi = \alpha\left(1 + \frac{\sigma_h}{1+\sigma_e}\right)$.

Wage bargaining

As is usual in the search literature, firms and workers bargain individually about the real wage and split the surplus in shares determined by an exogenous bargaining weight γ (see e.g. Krause and Lubik, 2007 and Trigari, 2009).

On the firm's side, the surplus $J_i(w_{it})$ obtained from a marginal worker equals his marginal contribution to profits so

$$
\begin{aligned}
J_i(w_{it}) &= \frac{\partial\left(\frac{P_{it}}{P_t}y_{it} - w_{it}n_{it}\right)}{\partial n_{it}} + E_t\beta_{t+1}(1-\lambda)J_i(w_{it+1}) \\
&= \frac{h_{it}}{\varphi}\frac{\partial w_{it}}{\partial h_{it}} - w_{it} + E_t\beta_{t+1}(1-\lambda)J_i(w_{it+1}) \qquad (6)
\end{aligned}
$$

with w_{it} the wage, λ_t the marginal utility of consumption and $\beta_{t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$ the stochastic discount factor. In a context of monopolistic competition and infrequent price adjustment, once the firm has set a price, its revenue is independent of n_{it}. Therefore, the contribution of the marginal worker to flow profits is given, not by the marginal revenue product of the worker $\left(\left(\frac{\partial\left(\frac{P_{it}}{P_t}y_{it}\right)}{\partial n_{it}} = \frac{\partial\left(\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}Y_t\right)}{\partial n_{it}} = 0\right)\right)$, but by the marginal reduction in the wage bill $\left(-\frac{\partial(w_{it}n_{it})}{\partial n_{it}} = -n_{it}\frac{\partial(w_{it})}{\partial h_{it}}\frac{\partial h_{it}}{\partial n_{it}} - w_{it} = \frac{h_{it}}{\varphi}\frac{\partial w_{it}}{\partial h_{it}} - w_{it}\right)$. If the worker walked away from the job, and given the impossibility of hiring a replacement immediately, the firm would need to increase the number of hours of (and therefore the wage payments to) all other workers in order to meet its demand.

A vacancy is filled with probability $q(\theta_t)$ and remains open otherwise. With c_t the cost of keeping a vacancy open at date t, the value $V_i(w_{it})$ of posting a vacancy in terms of current consumption is given by

$$
V_i(w_{it}) = -c_t + E_t\beta_{t+1}[q(\theta_t)J_i(w_{it+1}) + (1-q(\theta_t))V_i(w_{it+1})] \qquad (7)
$$

Note that the firm will post vacancies as long as the value of a vacancy is greater than zero. In equilibrium, $V_i(w_{it}) = 0$ so that

$$
\frac{c_t}{q(\theta_t)} = E_t\beta_{t+1}[J_i(w_{it+1})]. \qquad (8)
$$

Turning to the worker's problem, denote $W_i(w_{it})$ and U_t the value of being respectively employed and unemployed in units of consumption goods. The worker's asset value of being matched to firm i is

$$W_i(w_{it}) = w_{it} - \frac{1}{\lambda_t}\left(\frac{\lambda_h}{1+\sigma_h}h_{it}^{1+\sigma_h} + h_t\frac{\lambda_e}{1+\sigma_e}e_{it}^{1+\sigma_e}\right) + E_t\beta_{t+1}[(1-\lambda)W_i(w_{it+1}) + \lambda U_t] \quad (9)$$

and the value of being unemployed U_t is

$$U_t = b_t + E_t\beta_{t+1}\left[\int_0^1 \theta_t q(\theta_t)\frac{v_{jt}}{v_t}W_j(w_{it+1})dj + (1-\theta_t q(\theta_t))U_{t+1}\right] \quad (10)$$

with b_t the value of home production or unemployment benefits. A worker receives wage payments minus the disutility of labor, and has a probability λ of becoming unemployed next period. When unemployed, a worker receives b_t, has a probability $\theta_t q(\theta_t)\frac{v_{jt}}{v_t}$ to find a job next period with firm j and a probability $1-\theta_t q(\theta_t)$ to remain unemployed.

The equilibrium wage w_{it} satisfies $w_{it} = \underset{w_{it}}{\operatorname{argmax}} \ (W_i(w_{it}) - U_t)^{\gamma}(J_i(w_{it}))^{1-\gamma}$ so that the surplus-sharing rule implies

$$W_i(w_{it}) - U_t = \frac{\gamma}{1-\gamma}J_i(w_{it}). \quad (11)$$

Denoting the worker's surplus $S_{it} = W_i(w_{it}) - U_t$, I can write

$$\begin{aligned} E_t S_{it+1} &= E_t\beta_{t+1}[\int_0^1 \theta_t q(\theta_t)\frac{v_{jt}}{v_t}S_{jt+1}dj + (1-\lambda)E_t\beta_{t+1}S_{it+1} \\ &= E_t\beta_{t+1}\frac{\gamma}{1-\gamma}[\int_0^1 \theta_t q(\theta_t)\frac{v_{jt}}{v_t}J_j(w_{it+1})dj + (1-\lambda)J_i(w_{it+1})] \\ &= \frac{\gamma}{1-\gamma}\frac{c_t}{q(\theta_t)}(1-\lambda-\theta_t q(\theta_t)) \text{ with } (8) \end{aligned} \quad (12)$$

Rewriting (11) using (6), (9), (10) and (12), the equilibrium wage satisfies

$$\begin{aligned} & w_{it} - b_t - \tfrac{1}{\lambda_t}\left(\tfrac{\lambda_h}{1+\sigma_h}h_t^{1+\sigma_h} + h_t\tfrac{\lambda_e}{1+\sigma_e}e_t^{1+\sigma_e}\right) \\ & \qquad + \tfrac{\gamma}{1-\gamma}\tfrac{c_t}{q(\theta_t)}(1-\lambda-\theta_t q(\theta_t)) = \tfrac{\gamma}{1-\gamma}\left(-w_{it} + \tfrac{h_{it}}{\varphi}\tfrac{\partial w_{it}}{\partial h_{it}} + (1-\lambda)\tfrac{c_t}{q(\theta_t)}\right) \end{aligned}$$

or after rearranging,

$$w_{it} = \gamma \left(\frac{h_{it}}{\varphi} \frac{\partial w_{it}}{\partial h_{it}} + c_t \theta_t \right) + (1-\gamma) \left(b_t + \frac{g(h_t, e_t)}{\lambda_t} \right). \tag{13}$$

While the wage equation (13) is a weighted average of both parties surpluses and is similar to other bargained wages derived in e.g. Krause and Lubik (2007) or Trigari (2009), the firm's surplus is *not* given by the marginal product of labor. Indeed, once the firm has chosen its price, it is demand constrained and a marginal worker will not increase the firm's revenue. Instead, the first term of (13) is given by $-\frac{\partial w_{it}}{\partial n_{it}} = \frac{h_{it}}{\varphi} \frac{\partial w_{it}}{\partial h_{it}}$, the change in the wage bill caused by substituting the intensive margin (hours and effort) with the extensive one (employment).

A solution to (13) is given by

$$w_{it} = \gamma c_t \theta_t + (1-\gamma) b_t + (1-\gamma) \varkappa \frac{h_{it}^{1+\sigma_h}}{\lambda_t} \tag{14}$$

with $\varkappa = \frac{\lambda_h \frac{1+\sigma_h+\sigma_e}{(1+\sigma_h)\sigma_e}}{1-\frac{\gamma}{\varphi}(1+\sigma_h)}$.

0.1 Deriving the first-order conditions

Given the aggregate price level, firm i will choose a sequence of price $\{P_{it}\}$ and vacancies $\{v_{it}\}$ to maximize the expected present discounted value of future profits subject to the demand constraint, the Calvo price setting rule, the hours/effort choice and the law of motion for employment. Formally, the firm maximizes its value

$$E_t \sum_j \beta^j \frac{u'(C_{t+j})}{u'(C_t)} \left[\frac{P_{i,t+j}}{P_{t+j}} y_{i,t+j}^d - n_{i,t+j} w_{i,t+j} - c_{t+j} v_{i,t+j} \right]$$

subject to

$$\begin{cases} y_{it}^d = (\frac{P_{i,t}}{P_t})^{-\varepsilon} Y_t \\ y_{it} = y_0 A_t n_{it} h_{it}^{\varphi} \\ n_{it+1} = (1-\lambda) n_{it} + q(\theta_t) v_{it} \\ w_{it} = \gamma c_t \theta_t + (1-\gamma) b_t + (1-\gamma) \varkappa \frac{h_{it}^{1+\sigma_h}}{\lambda_t} \end{cases}$$

The optimal vacancy posting condition takes the form

$$\frac{c_t}{q(\theta_t)} = E_t \beta_{t+1} \left[\chi_{it+1} + \frac{c_{t+1}}{q(\theta_{t+1})}(1-\lambda) \right] \tag{15}$$

with χ_{it}, the shadow value of a marginal worker, given by

$$\begin{aligned} \chi_{it} &= -\frac{\partial n_{it} w_{it}}{\partial n_{it}} = -w_{it} + \frac{1}{\varphi} h_{it} \frac{\partial w_{it}}{\partial h_{it}} \\ &= -w_{it} + (1-\gamma) \varkappa \frac{1+\sigma_h}{\varphi} \frac{h_{it}^{1+\sigma_h}}{\lambda_t}. \end{aligned} \tag{16}$$

Using the wage equation, I can rewrite the marginal worker's value as

$$\chi_{it} = -\gamma \frac{c}{\lambda_t} \theta_t - (1-\gamma) \frac{b}{\lambda_t} + (1-\gamma) \left(\frac{1+\sigma_h}{\varphi} - 1 \right) \varkappa h_{it}^{1+\sigma_h} Y_t. \tag{17}$$

The level of hours per worker drives the firm's incentives to post vacancies. With $\varphi < 1 + \sigma_h$, the longer hours are, the larger is the wage bill reduction obtained with an extra worker. As hours increase because of a higher demand for the firm's products, the worker's marginal value increases, and the firm post more vacancies to increase employment.[2]

With Calvo-type price setting, a firm i resetting its price at date t will satisfy the standard Calvo price setting condition:

$$E_t \sum_{j=0}^{\infty} \nu^j \beta_j \left[\frac{P_{it}^*}{P_{t+j}} - \mu s_{it+j} \right] Y_{t+j} P_{t+j}^{\varepsilon} = 0 \tag{18}$$

[2] Note that this mechanism is different from the one at play in models with a retail sector and a wholesale sector as in Trigari (2004) and Walsh (2004). In those models, hiring firms are not demand constrained and the contribution of an additional worker is given by the marginal product of labor minus the wage bill.

where the optimal mark-up is $\mu = \frac{\varepsilon}{\varepsilon - 1}$ and the firm's real marginal cost

$$s_{it} = \frac{1+\sigma_h}{\varphi}(1-\gamma)\varkappa\frac{Y_t}{A_t}h_{it}^{1+\sigma_h-\varphi}. \tag{19}$$

The firm will choose a price P_{it}^* that is, in expected terms, a constant mark-up μ over its real marginal cost for the expected lifetime of the price.

Finally, the household first-order conditions for consumption and money holding take the usual form $\frac{1}{C_t} = \beta E_t(1+i_t)\frac{P_t}{P_{t+1}}\frac{1}{C_{t+1}}$ and $\frac{M_t}{P_t} = \frac{1}{C_t}\frac{i_t}{1+i_t}$.

(Non-stationary) Equilibrium

In this non-stationary model economy, I rescale the non-stationary variables with the technology index A_t. Denoting rescaled variables with lower-case letters, the frictionless economy is described by the following system with 5 equations and 5 unknowns θ^*, y^*, h^*, e^* and n^*:

$$\begin{aligned}
y^* &= \left(\frac{Y_t}{A_t}\right)^* = y_0 n^* h^{*\varphi} \\
e^* &= e_0\left(h^*\right)^{\frac{\sigma_h}{1+\sigma_e}} \\
\beta\chi^* &= \frac{c}{q(\theta^*)}\left(1-\beta(1-\lambda)\right) \\
\chi^* &= -\gamma c\theta^* - (1-\gamma)b + (1-\gamma)\left(\frac{1+\sigma_h}{\varphi} - 1\right)\varkappa h^{*1+\sigma_h} y^* \\
1 &= \mu\frac{1+\sigma_h}{\varphi}(1-\gamma)\varkappa y^* h^{*1+\sigma_h-\varphi} \\
n^* &= \frac{\theta^* q(\theta^*)}{\lambda + \theta^* q(\theta^*)}
\end{aligned}$$

where y_0, e_0 and $\varkappa$ are positive constants defined previously.

Log-linearization and the New-Keynesian Phillips curve

Log-linearizing the vacancy posting condition equation around the (zero-inflation) steady state, I get

$$\frac{c\eta}{q(\theta^*)}\hat{\theta}_t = E_t\beta\left[\chi^*\hat{\chi}_{it+1} + \frac{c(1-\lambda)\eta}{q(\theta^*)}\hat{\theta}_{t+1}\right] \tag{20}$$

where the value of a marginal worker $\hat{\chi}_{it}$ is given by

$$\chi^*\hat{\chi}_{it} = -\gamma c\theta\hat{\theta}_t + \frac{\varphi}{n\mu}(\frac{1+\sigma_h}{\varphi} - 1)\hat{h}_{it} \tag{21}$$

$$= -\gamma c\theta\hat{\theta}_t + \frac{1}{n\mu}(\frac{1+\sigma_h}{\varphi} - 1)\left(\hat{y}_{it} - \hat{n}_{it}\right) \tag{22}$$

where $\hat{\chi}_{it} = \ln(\frac{\chi_{it}/A_t}{\chi^*})$, $\hat{\theta}_t = \ln\left(\frac{\theta_t}{\theta^*}\right)$, $\hat{h}_{it} = \ln\left(\frac{h_{it}}{h^*}\right)$, $\hat{n}_{it} = \ln\left(\frac{n_{it}}{n^*}\right)$, and $\hat{y}_{it} = \ln\left(\frac{Y_t/A_t}{y^*}\right)$.

To derive the New-Keynesian Phillips curve, I log-linearize the price-setting condition around the zero inflation equilibrium. However, because employment is a state variable and because of firms' ex-post heterogeneity, the derivation is not straightforward. I follow Woodford's (2004) similar treatment of endogenous capital in a New-Keynesian model with Calvo price rigidity. In my case, employment is the state variable and plays the role of capital in Woodford's model. Log-linearizing the price-setting condition gives me

$$\sum_{k=0}^{\infty}(\nu\beta)^k\hat{E}_t^i\left[\tilde{p}_{it+k} - \hat{s}_{it+k}\right] = 0 \tag{23}$$

with

$$\hat{s}_{it+k} = \hat{n}_{it+k} + \frac{1+\sigma_h}{\varphi}\left(\hat{y}_{it+k} - \hat{n}_{it+k}\right) - \hat{y}_{it+k} + \hat{y}_{t+k} \tag{24}$$

The notation $\hat{E}_t^i$ denotes an expectation conditional on the state of the world at date t but integrating only over future states in which firm i has not reset its price since period t. $\tilde{p}_{it} \equiv \log\left(\frac{P_{it}}{P_t}\right)$ is the firm's relative price.

Denoting log prices by lower-case letters and p_{it}^* the optimal (log) price for firm i at t, the demand curve for firm i at date $t+1$ can be written $\hat{y}_{it+1} = \hat{y}_{t+1} - \varepsilon(p_{it} - p_{t+1})$ if it cannot reset its price at $t+1$ and $\hat{y}_{it+1} = \hat{y}_{t+1} - \varepsilon(p^*_{it+1} - p_{t+1})$ if it can reset its price.

Averaging across all firms, I get

$$\begin{aligned} \int_0^1 \hat{y}_{it+1} di &= \hat{y}_{t+1} - \varepsilon \left[\nu(\int_0^1 p_{it} di - p_{t+1}) + (1-\nu)(\int_0^1 p^*_{it+1} di - p_{t+1}) \right] \\ &= \hat{y}_{t+1} - \varepsilon \left[\nu(p_t - p_{t+1}) + (1-\nu)(p^*_{t+1} - p_{t+1}) \right] \end{aligned} \tag{25}$$

where $p^*_{t+1} = \int_0^1 p^*_{it+1} di$ is the average price chosen by all price setters at date $t+1$.

With Calvo price-setting, I can write

$$p_{t+1} = \left((1-\nu) p^{*1-\varepsilon}_{t+1} + \nu p_t^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

or

$$1 = (1-\nu) \left(\frac{p^*_{t+1}}{p_{t+1}} \right)^{1-\varepsilon} + \nu \left(\frac{p_t}{p_{t+1}} \right)^{1-\varepsilon}.$$

Log-linearizing around the zero-inflation equilibrium gives $-\nu(p_{t+1} - p_t) = (1-\nu)(p^*_{t+1} - p_{t+1})$ and combining with (25) gives $\int_0^1 \hat{y}_{it+1} di = \hat{y}_{t+1}$. Further, $\int_0^1 \hat{n}_{it} di = \hat{n}_t$.

Averaging (24) across all firms, I can rewrite the real marginal cost as

$$\hat{s}_{it+k} = \hat{s}_{t+k} + \left(\frac{1+\sigma_h}{\varphi} - 1 \right) (-\varepsilon \tilde{p}_{it+k} - \tilde{n}_{it+k}) \tag{26}$$

where $\tilde{n}_{it+k} = n_{it+k} - n_{t+k}$ is the relative employment of firm i.

Using that $\hat{E}^i_t \tilde{p}_{it+k} = p_{it} - E_t p_{t+k}$ and (26) in (24) yields

$$\begin{aligned} &\left(1 + \varepsilon \left(\tfrac{1+\sigma_h}{\varphi} - 1 \right) \right) p^*_{it} \\ &\quad = (1-\nu\beta) \sum_{k=0}^{\infty} (\nu\beta)^k \hat{E}^i_t \left[\hat{s}_{t+k} + \left(1 + \varepsilon \left(\tfrac{1+\sigma_h}{\varphi} - 1 \right) \right) p_{t+k} - \left(\tfrac{1+\sigma_h}{\varphi} - 1 \right) \tilde{n}_{it+k} \right] \end{aligned} \tag{27}$$

Moreover, subtracting (20) from its average, I get

$$\begin{aligned} \tilde{n}_{it+1} &= E_t(\hat{y}_{it+1} - \hat{y}_{t+1}) \\ &= -\varepsilon E_t\left[\nu(p_{it} - p_{t+1}) + (1-\nu)(p^*_{it+1} - p_{t+1}\right] \\ &= -\varepsilon\nu\tilde{p}_{it} - \varepsilon(1-\nu)(p^*_{it+1} - p^*_{t+1}) \end{aligned} \tag{28}$$

since $p_{t+1} = \nu p_t + (1-\nu)p^*_{t+1}$.

The firm's pricing decision depends on its employment level and the economy's aggregate state. But to a first order, the log-linearized equations are linear so that the difference between p^*_{it} and p^*_t, the average price chosen by all price setters, is independent from the economy's aggregate state and depends only on the relative level of employment $n_{it} - n_t = \tilde{n}_{it}$. So as in Woodford (2004), I guess that the firm's pricing decision takes the form

$$p^*_{it} - p^*_t = -\epsilon\tilde{n}_{it} \tag{29}$$

with ϵ a constant to be determined. Hence, (28) becomes

$$\tilde{n}_{it+1} = \frac{-\varepsilon\nu}{1-\varepsilon(1-\nu)\epsilon}\tilde{p}_{it} = -f(\epsilon)\tilde{p}_{it}$$

Since this was shown for any $t > 0$, I also get $\tilde{n}_{it+k} = -f(\epsilon)\tilde{p}_{it+k-1}$, $\forall k > 0$ so that I can rewrite (27) as

$$\phi p^*_{it} = (1-\nu\beta)\sum_{k=0}^{\infty}(\nu\beta)^k\hat{E}^i_t\left[\hat{s}_{t+k} + \left(1+\varepsilon\left(\frac{1+\sigma_h}{\varphi} - 1\right)\right)p_{t+k}\right] - (1-\nu\beta)\left(\frac{1+\sigma_h}{\varphi} - 1\right)\tilde{n}_{it} \tag{30}$$

with $\phi = \left(1+\varepsilon\left(\frac{1+\sigma_h}{\varphi} - 1\right) - \nu\beta\left(\frac{1+\sigma_h}{\varphi} - 1\right)f(\epsilon)\right)$.

Subtracting (30) from its average, I obtain

$$\phi(p^*_{it} - p^*_t) = -(1-\nu\beta)\left(\frac{1+\sigma_h}{\varphi} - 1\right)\tilde{n}_{it}. \tag{31}$$

This equation is of the conjectured form (29) if and only if ϵ satisfies

$$\epsilon = \frac{(1-\nu\beta)\frac{1+\sigma_h}{\varphi} - 1}{1 + \varepsilon\left(\frac{1+\sigma_h}{\varphi} - 1\right) - \nu\beta\left(\frac{1+\sigma_h}{\varphi} - 1\right) f(\epsilon)}. \tag{32}$$

Averaging (30) and using $\pi_t = \frac{1-\nu}{\nu}(p_t^* - p_t)$, I obtain the New-Keynesian Phillips curve

$$\pi_t = \delta.\hat{s}_t + \beta E_t \pi_{t+1} \tag{33}$$

with $\delta = \frac{(1-\nu)(1-\nu\beta)}{\nu\phi}$.[3]

Finally, log-linearizing the household's first-order conditions and denoting $\hat{m}_t = \ln\left(\frac{M_t/P_t A_t}{(M/P)^*}\right)$ the log-deviation of real rescaled money from its constant value in the zero-inflation equilibrium, I get $\hat{y}_t = E_t \hat{y}_{t+1} - (\hat{\imath}_t - E_t \pi_{t+1})$ and $\hat{m}_t = \hat{y}_t - \eta_i \hat{\imath}_t$ with $\hat{\imath}_t = \ln\left(\frac{1+i_t}{1+i*}\right)$. The log-linearized law of motion for employment can be written $\hat{n}_{t+1} = (1 - \lambda - \theta q(\theta))\hat{n}_t + \frac{1-n}{n}(1-\eta)\theta q(\theta)\hat{\theta}_t$.

A5. Embodied Technology and creative destruction

In this section, I discuss an alternative interpretation of the empirical relationship between labor productivity and unemployment studied in Section 2 of the paper. In this alternative, one could ignore aggregate demand altogether and emphasize instead the Schumpeterian aspect of technological progress. Indeed, Michelacci and Lopez-Salido (2007) and Canova, Michelacci and Lopez-Salido (2007) argue that a technology shock with a permanent impact on productivity may increase unemployment through creative destruction. The introduction of new technologies brings about a simultaneous increase in the destruction of technologically obsolete jobs which prompts a contractionary period during which employment temporarily falls. In that framework, the sign flip of ρ in the mid 80s could be due to an acceleration of creative destruction spawned by the Information Technology (IT) revolution. Put differently, technology

[3] In independent work, Thomas (2009) shows that the New-Keynesian Phillips curve (30) is flatter than the Phillips curve of a standard New-Keynesian model (for which $\delta = \frac{(1-\nu)(1-\nu\beta)}{\nu}$). He shows quantitatively that such real rigidities strongly increase the persistence of inflation compared to standard New-Keynesian models.

was disembodied before 1984 but became more embodied with the IT revolution, and there is no need to appeal to aggregate demand to explain the sign switch. However, I see a number of arguments suggesting that creative destruction is not the most plausible explanation.

First, if technical progress had become more embodied, this should have appeared in the empirical impulse responses. But as we saw, technology shocks had a quantitatively *smaller* negative impact on unemployment after 1984; exactly the opposite of what more creative destruction would imply but consistent with an improvement in the conduct of monetary policy.

Second, in a world with creative destruction, when productivity increases, unemployment goes up temporarily because firms destroy old and less productive jobs. If technology had become more embodied after 1984, movements in the separation rate should contribute to a larger fraction of unemployment fluctuations. However, Shimer (2007) finds that the proportion of unemployment fluctuations accounted for by variations in the separation rate actually decreased from 21% to only 5% after 1985.[4]

Moreover, with embodied technology, firms need to post vacancies to create new matches with the latest level of technology. Hence, an acceleration of creative destruction in the mid 80s could have caused the correlation between productivity and unemployment to become positive, but it could not have caused the correlation between productivity and vacancies to become negative. In contrast, an explanation emphasizing the role of aggregate demand shocks is consistent with large movements in both correlations: when productivity increases because of a technology shock, aggregate demand does not adjust immediately, firms post fewer vacancies, *and* unemployment goes up. Looking again at Figure 2 of the paper, the 10-year rolling correlation of labor productivity and vacancies displays a sign switch similar to ρ and favors the latter explanation.

Finally, it is important to distinguish between embodiment in new jobs and embodiment in new capital. For example, technology may be embodied in capital but disembodied in jobs.[5]

[4] The job-finding rate accounting for the residual.

[5] An example given by Pissarides and Vallanti (2005) is the one of a secretary using Microsoft Windows. A

In order to explain the sign switch of ρ with creative destruction, technological progress needs to be embodied in new jobs. Studying the impact of productivity growth on unemployment, Pissarides and Vallanti (2005) find that technology embodied in jobs and creative destruction play no role in the dynamics of unemployment.

new version of Windows may require a more powerful computer but the secretary keeps the job and only needs to learn how to use the new version: technology is embodied in new capital but not in new jobs.

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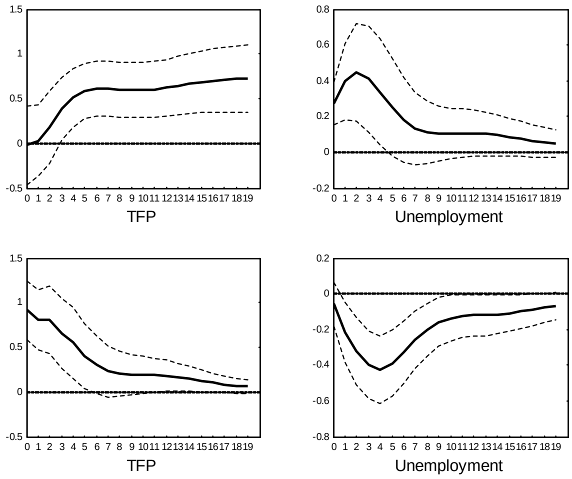

Figure 1: Impulse response functions to technology and non-technology shocks. Productivity is measured with TFP unadjusted for capacity utilization. Dashed lines represent the 95% confidence interval.

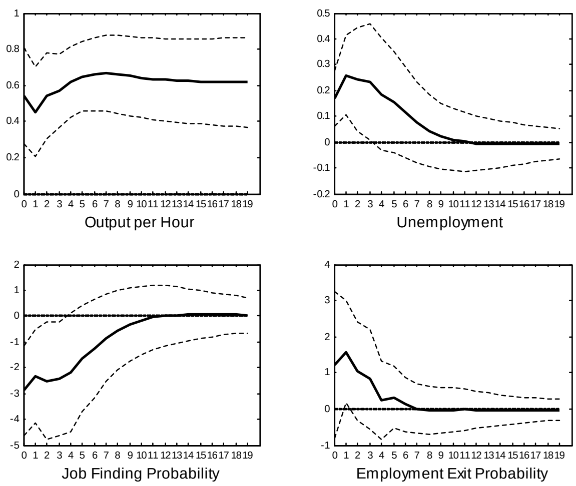

Figure 2: Impulse response functions to a technology shock in a 4 variables VAR over 1948-2006. Dashed lines represent the 95% confidence interval.

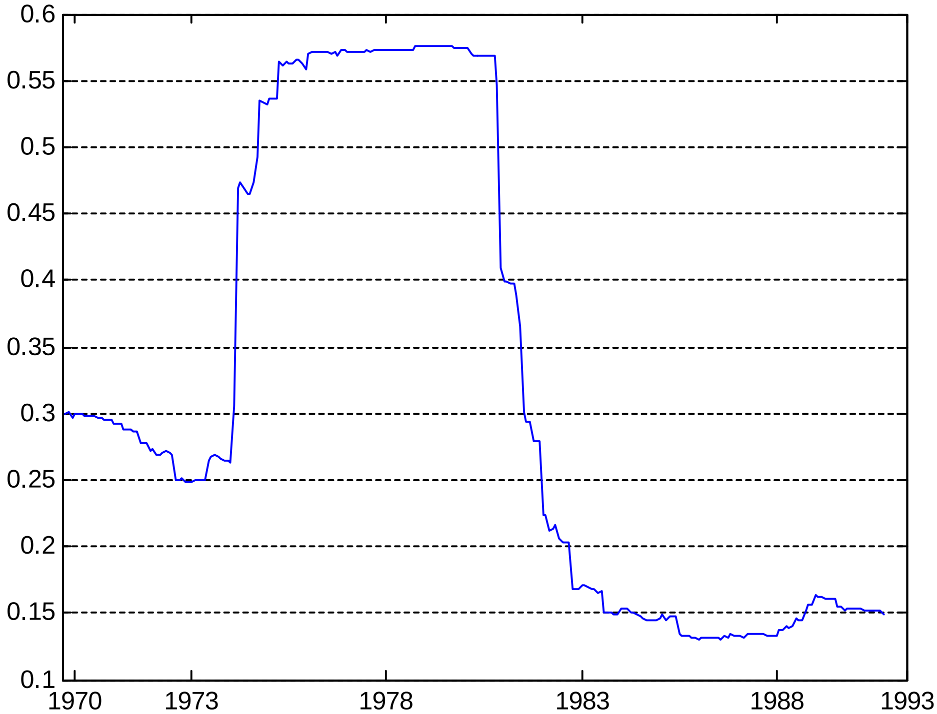

Figure 3: 5-year rolling standard-deviation of Romer and Romer monetary shocks. 1969:Q1-1996:Q4.