Conditional probabilities for Euro area sovereign default risk

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Disclaimer: Not necessarily the views of ECB or ESCB.
Contributions

We propose a novel modeling framework to infer conditional and joint probabilities for sovereign default risk from observed CDS.

**Novel framework?** Based on a dynamic GH skewed–t multivariate density/copula with time-varying volatility and correlations.

Multivariate model is sufficiently flexible to be calibrated daily to credit market expectations. Not an "official opinion".

Analysis is based on Euro area CDS data from 2008M1 to 2011M6. **Event study:** SMP/EFSF announcement & initial impact on risk.
1. **Sovereign credit risk**: e.g. Pan and Singleton (2008), Longstaff, Pan, Pedersen, and Singleton (2011), Ang and Longstaff (2011).

2. **Contagion**, see e.g. Forbes and Rigobon (2002), Caporin, Pelizzon, Ravazzolo, Rigobon (2012).


4. **Non-Gaussian dependence/copula/credit modeling**, see e.g. Demarta and McNeil (2005), Patton and Oh (2011).
Empirical questions

(Q1) Financial stability information: Based on credit market expectations, what is ...

- $\Pr(\text{two or more credit events in Euro area})$?
- $\Pr(i|j) - \Pr(i)$, for any $i,j$?
- Spillovers, e.g., $\Pr(PT|GR) - \Pr(PT|\text{not GR})$?
- $\text{Corr}_t(i,j)$ at time $t$?

(Q2) Model risk: For answering (a), how important are parametric assumptions? *Normal vs Student-t vs GH skewed-t*. 

(Q3) Event study: did the May 09, 2010 Euro area rescue package change risk dependence? How?
The GHST copula framework

Sovereign defaults iff benefits \( (v_{it}) \) exceed a cost \( (c_{it}) \), where

\[
v_{it} = (\zeta_t - \mu_\zeta) \tilde{L}_{it} \gamma + \sqrt{\zeta_t} \tilde{L}_{it} \epsilon_t, \quad i = 1, \ldots, n,
\]

\( \epsilon_t \sim N(0, I_n) \) is a vector of risk factors,
\( \tilde{L}_{it} \) contains risk factor loadings,
\( \gamma \in \mathbb{R}^n \) determines skewness,
\( \zeta_t \sim IG \) is an additional scalar risk factor for, say, interconnectedness.

A default occurs with probability \( p_{it} \), where

\[
p_{it} = \Pr[v_{it} > c_{it}] = 1 - F_i(c_{it}) \iff c_{it} = F_i^{-1}(1 - p_{it}),
\]

where \( F_i \) is the CDF of \( v_{it} \).

Focus on conditional probability \( \Pr[v_{it} > c_{it} | v_{jt} > c_{jt}], \ i \neq j \).
Data: skewed, fat tailed, tv vol’s and correlation

- Average of rolling window correlations

- Squared differences, AT

- Squared differences, GR
The GH skewed-t multivariate distribution

\[ y_t = \mu + L_t e_t, \quad t = 1, \ldots, T, \quad e_t \sim \text{GHST}, \quad \mathbb{E}[e_t e_t'] = I_n, \]

\[
p(y_t; \cdot) = \frac{\nu^\frac{v}{2} 2^{1-\frac{v+n}{2}}}{\Gamma \left( \frac{v}{2} \right) \pi^{\frac{n}{2}}} \frac{K_{v+n} \left( \sqrt{d(y_t) \cdot (\gamma' \gamma)} \right)}{|\tilde{\Sigma}_t|^{\frac{1}{2}}} \cdot \frac{\left( d(y_t) \cdot (\gamma' \gamma) \right)^{-\frac{v+n}{4}}}{d(y_t)^{\frac{v+n}{2}}} e^{\gamma' \tilde{L}_t^{-1}(y_t - \tilde{\mu}_t)},
\]

where

\[
d(y_t) = \nu + (y_t - \tilde{\mu}_t)' \tilde{\Sigma}_t^{-1} (y_t - \tilde{\mu}_t),
\]

\[
\tilde{\mu}_t = -\nu / (v - 2) \tilde{L}_t \gamma,
\]

\[
\tilde{\Sigma}_t = \tilde{L}_t \tilde{L}_t' \quad \text{is scale matrix}
\]

If \( \gamma = 0 \), then GH skewed-\( t \) simplifies to Student’s \( t \) density.

If in addition \( v^{-1} \rightarrow 0 \), then multivariate Gaussian density.

\( \tilde{\Sigma}_t(f_t) = \tilde{L}_t(f_t) \tilde{L}_t(f_t)' \) is driven by 1st and 2nd derivative of the pdf.
The model with time varying parameters

Assume that \( \Sigma_t = D_t R_t D_t = L_t(f_t)L_t(f_t)' \) and that

\[
f_{t+1} = \omega + \sum_{i=0}^{p-1} A_i s_{t-i} + \sum_{j=0}^{q-1} B_j f_{t-j},
\]

where \( s_t = S_t \nabla_t \) is the scaled score

\[
\nabla_t = \partial \ln p(y_t; \tilde{\Sigma}(f_t), \gamma, \nu)/\partial f_t \\
S_t = E_{t-1}[\nabla_t \nabla_t' | y_{t-1}, y_{t-2}, \ldots]^{-1},
\]

Scaling matrix \( S_t \) is inverse conditional Fisher information matrix.
Time varying parameters: score

Important: first two derivatives are available in closed form.

$$\nabla_t = \partial \ln p(y_t; \tilde{\Sigma}(f_t), \gamma, v) / \partial f_t$$

$$= \frac{\partial \text{vech}(\Sigma_t)'}{\partial f_t} \frac{\partial \text{vech}(L_t)'}{\partial \text{vech}(\Sigma_t)} \frac{\partial \text{vec}(\tilde{L}_t)'}{\partial \text{vec}(L_t)} \frac{\partial \ln p_{GH}(y_t | f_t)}{\partial \text{vec}(\tilde{L}_t)}$$

$$= \ldots$$

$$= \Psi'_t H'_t \text{vec} \left\{ w_t y_t y'_t - \tilde{\Sigma}_t - \left( 1 - \frac{v}{v - 2 w_t} \right) \tilde{L}_t \gamma y'_t \right\}$$

where \( \Psi_t = \frac{\partial \text{vech}(\Sigma_t)}{\partial f_t'} \)

\( H_t = \) messy

\( w_t = \frac{v + n}{2 \cdot d(y_t)} - \frac{k'_{y+n}}{2} \left( \sqrt{d(y_t) \cdot (\gamma' \gamma)} \right) \); \( k'_a(b) = \frac{\partial \ln K_a(b)}{\partial b} \).
Extracting marginal pd’s from CDS

We equate the premium and default leg of CDS given a default intensity.

\[ p_{it} \approx \frac{s_{it}(1 + r_t)}{1 - rec_i}, \quad (*) \]

where

\[ s_{it} = CDS \text{ annual fee, country } i, \text{ time } t \]
\[ r_t = \text{LIBOR 1 year rate, flat} \]
\[ rec_i = 25\% \text{ expected recovery, stressed.} \]

Eqn (\*) is exact if the term structures for pd’s and interest rates are flat, \( s_{it} \) is paid to the seller continuously, and there is no counterparty credit risk, see Brigo and Mercurio (2007).
Marginal pd’s from CDS

- Austria
- Germany
- France
- Ireland
- Netherlands
- Belgium
- Spain
- Greece
- Italy
- Portugal

2008 2009 2010 2011
Volatility estimates

2008 2009 2010 2011
0.05
0.15 Austria CDS changes squared Austria Gaussian Est. vol Austria GHST Est. vol
0.025
0.075 Belgium CDS changes squared Belgium Gaussian Est. vol Belgium GHST Est. vol
0.005
0.075 Germany CDS changes squared Germany Gaussian Est. vol Germany GHST Est. vol
0.025
0.050 Spain CDS changes squared Spain Gaussian Est. vol Spain GHST Est. vol
0.025
0.050 Ireland CDS changes squared Ireland Gaussian Est. vol Ireland GHST Est. vol
0.025
0.050 Portugal CDS changes squared Portugal Gaussian Est. vol Portugal GHST Est. vol
Dynamic correlations

- Gaussian correlation
- Rolling window correlation
- GHST correlation
- $t$ correlation
- Rolling window correlation

2008 2009 2010 2011
0.25 0.50 0.75

Gaussian correlation
$G_{HST}$ correlation
$G_{HST}$ rolling window correlation
$t$ correlation
$t$ rolling window correlation
The probability of two or more failures
The probability of $k=0,1,2,...$ failures
Conditional pds: $\Pr(\text{all } i | \text{ all } j)$
Conditional pds: $\text{Pr}(i|\text{GR})$

- Gaussian
- Symmetric $t$
- GH Skewed
- GH Skewed $t$ with zero correlation
GHST spillovers: Pr(i|GR) - Pr(i|not GR)
### The May 09, 2010 package

#### Joint risk, \( \Pr(i \cap j) \)

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The May 09, 2010 package

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Bottom line: joint risks ↓↓↓, but dependence ↑. "Firewall"-analogy?
Conclusion

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