Forward-Looking Behavior and Optimal Discretionary Monetary Policy*

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Abstract

This paper derives a closed-form solution for the optimal discretionary monetary policy in a small macroeconomic model that allows for varying degrees of forward-looking behavior. We show that a more forward-looking aggregate demand equation serves to attenuate the response to inflation and the output gap in the optimal interest rate rule. In contrast, a more forward-looking real interest rate equation serves to magnify the reponse to both variables. A more forward-looking Phillips curve serves to attenuate the response to inflation but magnify the response to the output gap.

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1 Introduction

This paper develops a simple tractable model to investigate how forward-looking behavior affects the central bank's response to inflation and the output gap under optimal discretionary monetary policy. The framework for our analysis is a generalized version of the purely backward-looking model of Svensson (1997). Our version adds three separate parameters that govern the role of forward-looking expectations in the IS (or aggregate demand) equation, the real-interest rate equation, and the short-run Phillips curve. Particular settings for these parameters allow our model to loosely approximate some commonly-used specifications in the literature.

2 The Model

We generalize the model of Svensson (1997) to allow for varying degrees of forward-looking behavior.[1] The equations that describe the model are as follows:

$$y_{t+1} = \beta_y \left[(1-\mu_y) y_t + \mu_y E_t y_{t+1} \right] - \beta_r (\rho_t - \overline{\rho}) + v_{t+1}, \quad v_{t+1} \sim N\left(0, \sigma_v^2\right), \tag{1}$$

$$\rho_t = (1-\mu_r)(i_t - \pi_t) + \mu_r E_t (i_{t+1} - \pi_{t+1}), \tag{2}$$

$$\pi_{t+1} = (1-\mu_\pi)\pi_t + \mu_\pi E_t \pi_{t+1} + \alpha_y y_t + z_{t+1}, \quad z_{t+1} \sim N\left(0, \sigma_z^2\right). \tag{3}$$

Equation (1) is the IS equation, where y_t is the deviation of real output from trend, i.e., the output gap, ρ_t is the real interest rate that matters for aggregate demand, v_{t+1} is a demand shock, and E_t is the expectation operator conditional on information available at time t. The parameter $\beta_y > 0$ governs the sensitivity of next period's output gap to a weighted combination of the current gap and the expected gap, where $\mu_y \in [0,1]$ is the weight assigned to the expected gap. The parameter $\beta_r > 0$ governs the sensitivity of next period's gap to the real interest rate. In steady-state, the output gap is zero which implies that $\overline{\rho}$ is the steady-state real interest rate.

Equation (2) defines the real interest rate that matters for aggregate demand. The variable i_t represents the one-period *nominal* interest rate which is under the control of the central bank and π_t is the inflation rate. The parameter $\mu_r \in [0,1]$ governs the degree to which agents' expectations of the future nominal interest rate influence the relevant real rate. In steady-state, equation (2) implies the Fisher relationship: $\overline{i} = \overline{\rho} + \overline{\pi}$.

Equation (3) is the short-run Phillips curve, where z_{t+1} is a cost-push shock and the parameter $\alpha_y \geq 0$ governs the slope of the curve. The parameter $\mu_\pi \in [0,1]$ governs the degree to which next period's inflation rate is influenced by current-period expectations of inflation. Equation (3) combines elements of a backward-looking Phillips curve with an expectational timing feature that is motivated by the "sticky information" Phillips curve described by Mankiw and Reis (2002).

Standard techniques yield the following reduced-form versions of the IS equation and the Phillips curve:

[1] Svensson (1999, section 7) considers forward-looking behavior as an extension to his original model. However, his analysis is non-quantitative and he focuses mainly on a special case where the central bank does not care about output fluctuations.

$$y_{t+1} = \left[\frac{\beta_y(1-\mu_y)+\beta_r\mu_r\left(\frac{\alpha_y}{1-\mu_\pi}\right)}{1-\beta_y\mu_y}\right] y_t - \left(\frac{\beta_r}{1-\beta_y\mu_y}\right)\left[(1-\mu_r)(i_t-\overline{\rho}-\overline{\pi})-(\pi_t-\overline{\pi})\right]$$

$$- \left(\frac{\beta_r\mu_r}{1-\beta_y\mu_y}\right)(E_t i_{t+1}-\overline{\rho}-\overline{\pi}) + v_{t+1}, \tag{4}$$

$$\pi_{t+1} = \pi_t + \left(\frac{\alpha_y}{1-\mu_\pi}\right) y_t + z_{t+1}. \tag{5}$$

When $\mu_y = \mu_\pi = \mu_r = 0$, equations (4) and (5) are identical to those in the purely backward-looking model of Svensson (1997).

3 Optimal Monetary Policy Under Discretion

Following Svensson (1997), we assume that the central bank's decision problem can be written as

$$\min_{\{i_t\}_{t=0}^{\infty}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \delta^t \left\{ (\pi_t - \overline{\pi})^2 + \lambda y_t^2 \right\}, \tag{6}$$

subject to the structural constraints (4) and (5). The symbol $\overline{\pi}$ is the exogenously-specified inflation target, $\delta \in (0,1)$ is the central bank's subjective discount factor, and $\lambda \geq 0$ is the subjective weight assigned to stabilizing output fluctuations relative to stabilizing inflation fluctuations.

Central bank policy operates through changes in i_t or $E_t i_{t+1}$. The constraint (5) shows that π_{t+1} and $E_t\pi_{t+1}$ cannot be influenced by policy at time t because π_t and y_t are predetermined state variables. The other constraint (4) shows that changes in i_t or $E_t i_{t+1}$ *can* influence y_{t+1} which, in turn, influences π_{t+2} and $E_{t+1}\pi_{t+2}$. Thus, the constraints imply that the central bank exerts control over the output gap with a one-period lag and the inflation rate with a two-period lag.

We make the realistic assumption that current policymakers cannot bind future policymakers to abide by the optimal policy rule computed today. In other words, we assume that future policymakers are free to reoptimize given the state of the economy that exists at each future decision point. Rational private-sector agents will of course anticipate these actions and adjust their behavior to take into account the policymakers' incentive to reoptimize. This anticipation effect influences the nature of the policy rule that ultimately prevails in a rational expectations equilibrium.

Proposition. *The central bank's optimal interest rate rule under discretion is given by*

$$i_t = \overline{\rho} + \overline{\pi} + g_\pi^* (\pi_t - \overline{\pi}) + g_y^* y_t,$$

where

$$g_\pi^* = 1 + \frac{\left[1 - \frac{\beta_y(\mu_y-\mu_r)}{1-\mu_r}\right] \delta \left(\frac{\alpha_y}{1-\mu_\pi}\right) k}{\beta_r \left[\lambda + (1-\mu_r)\,\delta \left(\frac{\alpha_y}{1-\mu_\pi}\right)^2 k\right]},$$

$$g_y^* = \frac{\left(1-\beta_y\mu_y\right)\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2 k + \frac{\beta_y\left(1-\mu_y\right)}{1-\mu_r}\left[\lambda+\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2 k\right]}{\beta_r\left[\lambda+\left(1-\mu_r\right)\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2 k\right]},$$

$$\textit{and } k = \frac{1}{2}\left\{1-\frac{\lambda(1-\delta)}{\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2}+\sqrt{\left(1+\frac{\lambda(1-\delta)}{\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2}\right)^2+\frac{4\lambda}{\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2}}\right\} \geq 1.$$

The proof of the above proposition follows the basic methodology outlined in Svensson (1997).[2] Notice that all three forward-looking parameters, μ_y, μ_π, and μ_r, can influence the magnitude of the optimal response coefficients. When $\mu_y = \mu_\pi = \mu_r = 0$, the expressions for g_π^* and g_y^* are identical to those derived by Svensson (1997) for a purely backward-looking model.

An increase in the parameter μ_y leads to a weaker policy response, i.e., $\frac{\partial g_\pi^*}{\partial \mu_y} < 0$ and $\frac{\partial g_y^*}{\partial \mu_y} < 0$. The intuition is straightforward. An increase in the forward-looking component of the IS equation serves to reduce the persistence of the equilibrium output gap in response to shocks. From the reduced-form IS equation (4), we can see that an increase in μ_y leads to a smaller reduced-form coefficient on the current gap y_t. This means that the output gap can exhibit a more-pronounced jump in response to a change in current nominal rate i_t (the central bank's policy variable). The enhanced impact of the policy variable allows the output gap to be brought back to zero (the central bank's target level) with a smaller interest rate response. Moreover, given that the central bank exerts control over inflation via interest-rate induced changes in the output gap, improved control over the gap allows inflation to be brought back to target with a smaller response as well.

While our results regarding μ_y pertain to the optimal policy rule under discretion, Rudebusch (2002) obtains qualitatively similar results for an optimal policy rule under commitment. In particular, he finds that an increase in μ_y reduces the magnitude of the response coefficients g_π^* and g_y^* in an optimized version of the Taylor rule.[3] Our results are also consistent with those of Söderlind, Söderström, and Vredin (2002) who compute the optimal policy rule under discretion. Both of these studies adopt a different timing convention for agents' expectations.[4] Nevertheless, their specification and ours share the property that increasing the weight assigned to the expectational term in the IS equation reduces the persistence of the equilibrium output gap.

An increase in the parameter μ_r leads to a stronger policy response, i.e., $\frac{\partial g_y^*}{\partial \mu_r} > 0$ and $\frac{\partial g_y^*}{\partial \mu_r} > 0$. This result is consistent with the findings of Eijffinger Schaling, and Verhagen (2000) who introduce a forward-looking term structure into the model of Svensson (1997). When $\mu_r = 1/2$, equation (2) can be viewed as an approximation to the expectations theory of the term structure where ρ_t corresponds to a two-period real rate.[5] Eijffinger, Schaling, and Verhagen (2000) consider a more-general term structure equation where $\rho_t = \left(1-\mu_r\right)\left(i_t - E_t\pi_{t+1}\right) + \mu_r E_t\rho_{t+1}$ and $\mu_r = D/\left(1+D\right)$. The parameter D can be interpreted as the duration of a real consol that is used to approximate

[2] The details of the proof are contained in the appendix.

[3] Compare Table 1 $\left(\mu_y = \mathbf{0}\right)$ with Table A1 $\left(\mu_y = \mathbf{0.3}\right)$ in Rudebusch (2002).

[4] Both studies employ an IS equation of the form:

$$y_t = \beta_y\left[\left(1-\mu_y\right)y_{t-1} + \mu_y E_{t-1}y_{t+1}\right] - \beta_r\left(i_{t-1} - E_{t-1}\pi_{t+1}\right) + v_t.$$

[5] The exact version of the expectations theory would imply $\rho_t = \frac{1}{2}\left(i_t - E_t\pi_{t+1}\right) + \frac{1}{2}E_t\left(i_{t+1} - \pi_{t+2}\right)$.

a finite maturity long-term bond. In both of these setups, the central bank does not have direct control over the "longer-term" interest rate ρ_t because this rate is partly determined by private-sector expectations of *future* short-term rates. As μ_r increases, the effective bond maturity lengthens and the central bank is forced to move the current short rate more aggressively to produce the same desired impact on ρ_t.

It not obvious how an increase in μ_π affects the strength of the policy response because this parameter enters the expressions for g_π^* and g_y^* in a rather complicated way. We can gain some insight by considering a special case of the model when $\lambda = 0$, that is, when the central bank cares only about minimizing deviations of inflation from target.[6] When $\lambda = 0$, the above proposition implies $k = 1$ and we obtain $\frac{\partial g^*_{\pi|\lambda=0}}{\partial \mu_\pi} < 0$ and $\frac{\partial g^*_{y|\lambda=0}}{\partial \mu_\pi} = 0$. Hence, an increase in μ_π leads, on balance, to a weaker policy response. The increase in μ_π serves to reduce the persistence of inflation in response to shocks. From the reduced-form Phillips curve (5), we see that an increase in μ_π causes next period's inflation rate π_{t+1} to be determined less by current inflation π_t and more by the current output gap y_t. Given that the central bank exerts control over the output gap with only a one-period lag, an increase in μ_π makes future inflation more responsive to the interest rate. This, in turn, allows inflation to brought back to $\overline{\pi}$ with smaller interest rate changes. When $\lambda = 0$, the direct response to the output gap does not change with μ_π because this special case implies that the central bank does not care about output fluctuations. The more realistic case of $\lambda > 0$ is investigated numerically in the next section.

4 Quantitative Results

We now turn to a quantitative assessment of the optimal response coefficients in a calibrated version of the model. The time period is taken to be one quarter. Notice that the optimal response coefficients do not depend on the values of $\overline{\pi}$ and $\overline{\rho}$. For the remaining parameters, we adopt a set of baseline values that are drawn from studies that estimate models which resemble ours. In particular, we choose $\mu_y = \mu_\pi = \mu_r = 0.5$, $\beta_y = 1$, $\beta_r = 0.2$, $\alpha_y = 0.04$, $\delta = 0.99$, and $\lambda = 1$. The basic nature of our results does not hinge on any particular calibration of the model.

For the baseline parameter settings, the optimal response coefficients take on the values $g_\pi^* = 5.7$ and $g_y^* = 5.4$. These coefficients are considerably larger in magnitude than the Taylor (1993) rule coefficients of $g_\pi = 1.5$ and $g_y = 0.5$. Hence our baseline calibration confirms a common result in the literature that the optimal policy rule calls for a stronger response to inflation and the output gap than is recommended by "Taylor-type" rules estimated from macroeconomic data.[7]

It is worth noting that our values for g_π^* and g_y^* are also somewhat larger than those typically reported in the literature for "optimized" Taylor rules. For example, Rudebusch and Svensson (1999, Table 5.3) and Rudebusch (2001, Table 1) obtain $g_\pi^* \approx 3$ and $g_y^* \approx 2$ using purely backward-looking models.[8] Unlike our study, these authors adopt a central bank loss function that specifically penalizes movements in the nominal interest rate. In particular, the response coefficients cited above correspond to a within-period loss function that takes the form $L_t = (\pi_t - \overline{\pi})^2 + \lambda y_t^2 +$

[6] This case is termed "strict inflation targeting" by Svensson (1997).
[7] To our knowledge, this point was first made by Ball (1999).
[8] The results from these studies are not strictly comparable to ours because the authors compute "optimal simple rules" that involve a restricted number of state variables.

$\nu(i_t - i_{t-1})^2$ with $\lambda = 1$ and $\nu = 0.5$. The presence of the penalty term $\nu(i_t - i_{t-1})^2$ serves to reduce the magnitude of the optimal response coefficients in comparison to our setup which imposes $\nu = 0$. While the above authors do not consider the $\nu = 0$ case, they do compute the optimal response coefficients when the penalty-term weight is reduced to $\nu = 0.1$. In this case, Rudebusch and Svensson (1999, Table 5.6) and Rudebusch (2001, Table 1) obtain somewhat larger response coefficients: $g_\pi^* \approx 3.5$ and $g_y^* \approx 2.5$. Our results for the $\nu = 0$ case suggest that the impact of the penalty term on the magnitude of the optimal response coefficients is highly nonlinear.[9]

When $\mu_y = \mu_\pi = \mu_r = 0$, our model collapses to the purely backward-looking framework of Svensson (1997) and we obtain $g_\pi^* = 5.3$ and $g_y^* = 5.2$ (with the other parameters held constant at the baseline values). Another interesting benchmark is $\mu_y = 0$ and $\mu_\pi = \mu_r = 0.5$ which can be viewed as a simplified version of the Fuhrer and Moore (1995) model. In this case, we obtain $g_\pi^* = 10.7$ and $g_y^* = 10.4$.

Figures 1 through 3 plot g_π^* and g_y^* as each forward-looking parameter is varied while holding the remaining parameters constant at the baseline values given earlier. A vertical dashed line marks the baseline value of each parameter.

Figure 1 shows that an increase in μ_y causes both g_π^* and g_y^* to decline in a linear fashion. Figure 2 shows an increase in μ_r causes both g_π^* and g_y^* to rise in a nonlinear fashion. Both figures confirm the theoretical results presented earlier.

Figure 3 shows that g_π^* and g_y^* do not react very much until μ_π reaches a value of about 0.9. Beyond this value, the optimal response coefficients diverge, with g_π^* dropping sharply towards 1.0 and g_y^* shooting up to above 10. This phenomenon can be understood from the reduced-form Phillips curve (5). As μ_π increases (holding α_y constant), future inflation is determined less by current inflation and more by the current output gap. The optimal response, then, is for the central bank to react less to inflation and more to the output gap. Recall that when $\lambda = 0$, our theoretical results showed that the optimal response is for the central bank to react less to inflation and react *the same* to the output gap.

5 Concluding Remarks

This paper investigated how forward-looking behavior affects the nature of the optimal discretionary monetary policy. We showed that a more forward-looking IS equation serves to attenuate the optimal response to inflation and the output gap. A more forward-looking real interest rate equation serves to magnify the optimal response to both variables. A more forward-looking Phillips curve serves to attenuate the optimal response to inflation but magnify the optimal response to the output gap. Our results have implications for studies that attempt to reconcile estimated versions of the central bank's policy rule with optimal discretionary policy. In particular, a successful reconciliation is likely to require a different degree of forward-looking behavior in each part of the model economy. While our analysis employed a simple stylized model to permit a closed-form solution for the optimal policy rule, the basic intuition for our results should extend to more complicated frameworks.

[9] This point is supported by the analytical results of Svensson (1999, section 5).

A Appendix

A.1 Proof of Proposition 1

The state variables for the central bank's dynamic programming problem are π_t and y_t. However, following Svensson (1997), we can treat $E_t\pi_{t+1}$ as the single state variable because $E_t\pi_{t+1} = \pi_t + \left(\frac{\alpha_y}{1-\mu_\pi}\right) y_t$ for all t. We can also treat $E_t y_{t+1}$ as the single control variable. Given the central bank's choice for $E_t y_{t+1}$, the structural constraint (4) can be used to recover the interest rate rule that implements the optimal allocations. With these formulations, the central bank's dynamic programming problem can be written as

$$V\left(E_t\pi_{t+1}\right) = \min_{E_t y_{t+1}} \left\{ \frac{1}{2}\left[\left(E_t\pi_{t+1} - \overline{\pi}\right)^2 + \lambda\left(E_t y_{t+1}\right)^2\right] + \delta E_t V\left(E_{t+1}\pi_{t+2}\right) \right\}, \tag{A.1}$$

subject to:

$$E_{t+1}\pi_{t+2} = \pi_{t+1} + \left(\frac{\alpha_y}{1-\mu_\pi}\right) y_{t+1}, \tag{A.2}$$

where $V(\cdot)$ is the value function and equation (A.2) is obtained from the structural constraint (5) by updating all variables by one time step and then taking expectations at time $t+1$.

A.1.1 The Central Bank's Value Function

Since the central bank's loss function is quadratic and the constraints are linear, the value function will take the form

$$V\left(E_t\pi_{t+1}\right) = k_0 + \frac{k}{2}\left(E_t\pi_{t+1} - \overline{\pi}\right)^2, \tag{A.3}$$

where k_0 and k are coefficients to be determined. The linear-quadratic nature of the optimal control problem gives rise to the property of certainty equivalence, i.e., the solution does not depend on the variances σ_v^2 and σ_z^2 that govern the stochastic shocks.

The first order condition of (A.1) with respect to the control variable $E_t y_{t+1}$ is

$$\lambda E_t y_{t+1} + \delta E_t \frac{\partial V\left(E_{t+1}\pi_{t+2}\right)}{\partial E_{t+1}\pi_{t+2}} \frac{\partial E_{t+1}\pi_{t+2}}{\partial E_t y_{t+1}} = \lambda E_t y_{t+1} + \delta k\left(E_t\pi_{t+2} - \overline{\pi}\right)\left(\frac{\alpha_y}{1-\mu_\pi}\right) = 0, \tag{A.4}$$

where we have made use of the law of iterated mathematical expectations. The first order condition can be rewritten as

$$E_t\pi_{t+2} - \overline{\pi} = \frac{-\lambda}{\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)k} E_t y_{t+1}. \tag{A.5}$$

The constraint (A.2) implies

$$E_t y_{t+1} = \left(\frac{\alpha_y}{1-\mu_\pi}\right)^{-1}\left(E_t\pi_{t+2} - E_t\pi_{t+1}\right). \tag{A.6}$$

where we have once again made use of the law of iterated mathematical expectations.

Substituting equation (A.6) into equation (A.5) to eliminate $E_t y_{t+1}$ and then rearranging yields

$$\left(E_t\pi_{t+2} - \overline{\pi}\right) = \frac{\lambda}{\lambda + \delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2 k}\left(E_t\pi_{t+1} - \overline{\pi}\right) \tag{A.7}$$

which shows that the central bank should adjust its policy variable to gradually close the gap between the two-period ahead inflation forecast and the long-run inflation target. When $\lambda = 0$, equation (A.7) simplifies to $E_t \pi_{t+2} = \overline{\pi}$. In this case, the central back should adjust its policy variable so that the two-period ahead inflation forecast is equal to the long-run inflation target.

To determine the coefficient k, we proceed as follows. From equation (A.3), we have

$$\frac{\partial V\left(E_t \pi_{t+1}\right)}{\partial E_t \pi_{t+1}} = k\left(E_t \pi_{t+1} - \overline{\pi}\right). \tag{A.8}$$

From equation (A.1) we have

$$\frac{\partial V\left(E_t \pi_{t+1}\right)}{\partial E_t \pi_{t+1}} = \left(E_t \pi_{t+1} - \overline{\pi}\right) + \delta E_t \underbrace{\frac{\partial V\left(E_{t+1} \pi_{t+2}\right)}{\partial E_{t+1} \pi_{t+2}}}_{= \; k(E_{t+1}\pi_{t+2} - \overline{\pi})} \underbrace{\frac{\partial E_{t+1} \pi_{t+2}}{\partial E_{t+1} \pi_{t+1}}}_{= \; 1}, \tag{A.9}$$

where the two partial derivatives on the right-hand side are computed using equations (A.3) and (A.2), respectively.

Making use of the envelope theorem, we can equate (A.8) and (A.9) to obtain

$$k\left(E_t \pi_{t+1} - \overline{\pi}\right) = \left(E_t \pi_{t+1} - \overline{\pi}\right) + \frac{\delta \lambda k}{\lambda + \delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2 k}\left(E_t \pi_{t+1} - \overline{\pi}\right), \tag{A.10}$$

where we have used equation (A.7) to substitute out $(E_t \pi_{t+2} - \overline{\pi})$ from equation (A.9). The above expression implies

$$k = 1 + \frac{\delta \lambda k}{\lambda + \delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2 k}, \tag{A.11}$$

which can be rearranged to yield the following quadratic equation

$$k^2 - \left[1 - \frac{\lambda(1-\delta)}{\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2}\right] k - \frac{\lambda}{\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2} = 0. \tag{A.12}$$

Following Svensson (1997, Appendix B), equation (A.12) can be solved for the unique positive value of k shown in the proposition.

A.1.2 The Optimal Interest Rate Rule

To solve for the optimal interest rate rule, we rearrange the first-order condition (A.5) to obtain

$$\begin{aligned}
E_t y_{t+1} &= \frac{-\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right) k}{\lambda}\left(E_t \pi_{t+2} - \overline{\pi}\right), \\
&= \frac{-\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right) k}{\lambda}\left[E_t \pi_{t+1} - \overline{\pi} + \left(\frac{\alpha_y}{1-\mu_\pi}\right) E_t y_{t+1}\right], \\
&= \frac{-\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right) k}{\lambda}\left[\pi_t - \overline{\pi} + \left(\frac{\alpha_y}{1-\mu_\pi}\right) y_t + \left(\frac{\alpha_y}{1-\mu_\pi}\right) E_t y_{t+1}\right],
\end{aligned} \tag{A.13}$$

where we have used the structural constraint (5) to compute the expectations $E_t\pi_{t+2}$ and $E_t\pi_{t+1}$. Solving equation (A.13) for $E_t y_{t+1}$ yields

$$E_t y_{t+1} = \frac{-\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)k}{\lambda+\delta\left(\frac{\alpha_y}{1-\mu_\pi}\right)^2 k}\left[\pi_t-\overline{\pi}+\left(\frac{\alpha_y}{1-\mu_\pi}\right)y_t\right]. \quad \text{(A.14)}$$

The structural constraint (4), which is derived from the IS equation, implies

$$\begin{aligned} E_t y_{t+1} &= \left[\frac{\beta_y(1-\mu_y)+\beta_r\mu_r\left(\frac{\alpha_y}{1-\mu_\pi}\right)}{1-\beta_y\mu_y}\right]y_t - \left(\frac{\beta_r}{1-\beta_y\mu_y}\right)\left[(1-\mu_r)\left(i_t-\overline{\rho}-\overline{\pi}\right)-\left(\pi_t-\overline{\pi}\right)\right] \\ &\quad - \left(\frac{\beta_r\mu_r}{1-\beta_y\mu_y}\right)\left[E_t i_{t+1}-\overline{\rho}-\overline{\pi}\right]. \quad \text{(A.15)} \end{aligned}$$

Following the common practice in the literature, we restrict our attention to so-called "Markov perfect equilibria" where the policy rule is a stationary function of current state variables. We make the conjecture that the optimal interest rate rule takes the form

$$i_t-\overline{\rho}-\overline{\pi} = g_\pi^*\left(\pi_t-\overline{\pi}\right)+g_y^* y_t, \quad \text{(A.16)}$$

for all t. Conditional on this rule, private-sector agents form rational expectations such that

$$\begin{aligned} E_t i_{t+1}-\overline{\rho}-\overline{\pi} &= g_\pi^*\left(E_t\pi_{t+1}-\overline{\pi}\right)+g_y^* E_t y_{t+1}, \\ &= g_\pi^*\left[\pi_t-\overline{\pi}+\left(\frac{\alpha_y}{1-\mu_\pi}\right)y_t\right]+g_y^* E_t y_{t+1}. \quad \text{(A.17)} \end{aligned}$$

Substituting equations (A.16) and (A.17) into equation (A.15) and then collecting terms yields

$$E_t y_{t+1} = \left[\frac{\beta_y(1-\mu_y)-\beta_r\mu_r\left(\frac{\alpha_y}{1-\mu_\pi}\right)(g_\pi^*-1)-\beta_r(1-\mu_r)g_y^*}{1-\beta_y\mu_y+\beta_r\mu_r g_y^*}\right]y_t - \left[\frac{\beta_r(g_\pi^*-1)}{1-\beta_y\mu_y+\beta_r\mu_r g_y^*}\right]\left(\pi_t-\overline{\pi}\right), \quad \text{(A.18)}$$

By equating the coefficients on $(\pi_t-\overline{\pi})$ and y_t in the two expressions for $E_t y_{t+1}$ given by (A.14) and (A.18), we have two equations that can be solved for the two optimal response coefficients g_π^* and g_y^*. The two equations are linear so the solution is unique. After some tedious but straightforward algebra, we obtain the solution shown in the proposition.

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Fig 1: OPTIMAL POLICY RULE COEFFICIENTS
Effect of Forward Looking Parameter in IS Equation

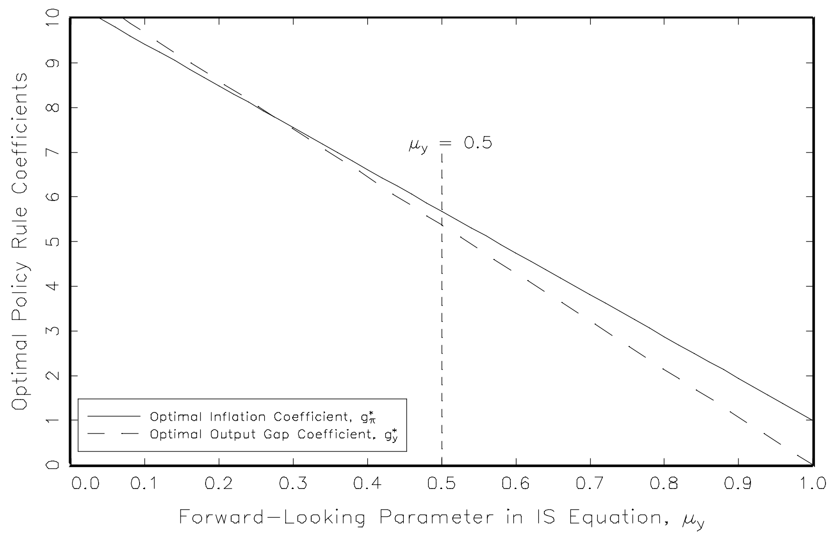

Fig 2: OPTIMAL POLICY RULE COEFFICIENTS
Effect of Forward Looking Parameter in Real Interest Rate Equation

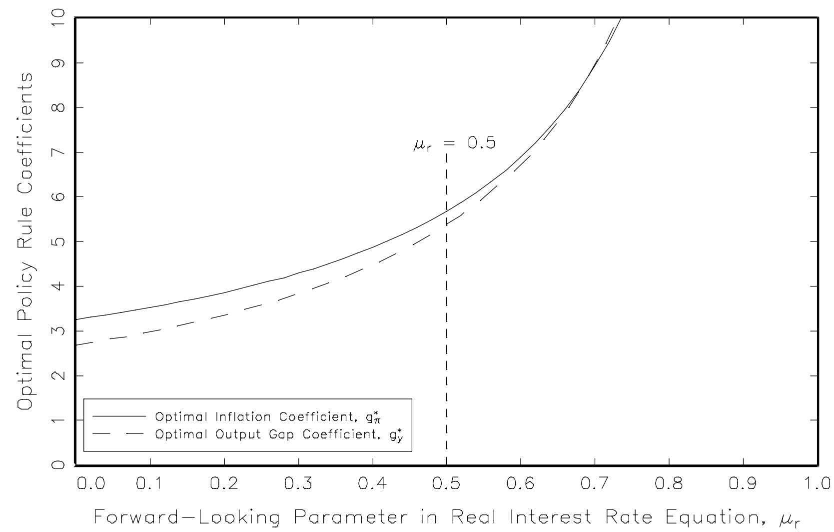

Fig 3: OPTIMAL POLICY RULE COEFFICIENTS
Effect of Forward Looking Parameter in Phillips Curve