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Mortgages as Recursive Contracts*

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Abstract: A recursive contract model of a fixed-rate mortgage contract is presented that captures the history-dependence of the refinancing decision on interest rates and house prices. Simulations of the model illustrate the following properties present in mortgage contracts: (i) mortgages provide rent-risk insurance to households; (ii) mortgage payments and the effective mortgage rate tend to ratchet down over the life of the contract; and (iii) households may choose to refinance into a lesser-valued contract (with a lower promised value) in order to extract equity from their home.

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I. Introduction.

A major change in the U. S. housing market over the past twenty years has been the surge in the incidence of mortgage refinancing during periods of declining interest rates and/or rising house prices. There have been three periods of "refinancing booms" since 1990. They occurred during 1991-1992, 1998, and 2000-2002. These boom periods are illustrated in Figure 1. The upper panel displays weekly data on the incidence of refinancing (the dashed line) and the 30-year fixed rate mortgage (the solid line). As shown, during each of the boom periods, mortgage rates were declining. However, the decline in rates was more dramatic in 1991-1992 than in the recent 2000-2002 period, yet the refinancing activity was far greater during the latter period. The bottom panel of Figure 1 illustrates why this occurred. The solid line is a smoothed quarterly average of the refinancing index shown in the upper panel. The boom periods are clearly identifiable. The dashed line is a real house price series. Note that the housing market during the 1991-1992 refinancing boom was flat to down, whereas during the 2000-2002 refinancing boom, house prices were rising at an extraordinary pace.[1] This strong housing market in the early 2000s contributed significantly to aggregate consumption expenditures, with "cash-out" refinancings adding over $100 billion to the U.S. economy annually.[2]

The purpose of this paper is to examine the role that the refinancing option plays in the evolution of a fixed-rate mortgage contract.[3] The approach that is taken here is to cast the mortgage as a one-sided recursive contract with an outside option for the borrower of terminating the mortgage, selling the home, realizing any capital gains on the sale of the

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[1]Bennett, Peach, and Peristiani (1998) indicate that points and fees on the average conventional loan fell by 150 basis points from 1983 to 1995, and provide evidence that this reduction in transaction costs also significantly affected refinancing activity.

[2]Canner, Dynan, and Passmore (2003) report the results of a survey of refinancing activity during the period from 2000 to mid-2002, which indicate that cash-outs added $141.6 billion to personal income, while the cumulative reduction in mortgage payments due to refinancings was $31.2 billion during this period.

[3]See Green and Shoven (1986) for empirical work on how changes in interest rates affect the probability that borrowers exercise the prepayment option.

house, and entering into a rental agreement, where future rental payments are stochastic. The refinancing option could be thought of as the "inside option." That is, the borrower may exercise this option, stay in the renegotiated contract and hence remain a homeowner, rather than terminating the contract and returning to renting.[4] Technically, this inside option is the mechanism that prevents the participation constraint from being violated. That is, whenever the outside option becomes sufficiently attractive for the household to terminate the contract and become a renter, the household has an incentive to exercise the inside option of refinancing, and thereby remain a homeowner.

The recursive contract framework developed by Green (1987) and Kocherlakota (1996) is used to capture the refinancing decision given the history-dependence of that decision. High house prices and low mortgage rates can precipitate a refinancing. However, the current house price and mortgage rate must be compared to their values at the time that the house was originally purchased or last refinanced. This history-dependence is clearly illustrated in the upper panel of Figure 1, where spikes in the incidence of refinancing are seen to coincide with recent new lows in the mortgage rate, such as the one in September, 1998.[5] Therefore, it is necessary to keep track of the "basis" that the household has in its house to determine whether there is sufficient movement in house prices and mortgage rates for them to choose optimally to refinance.[6]

This model captures several features of mortgage contracts. (i) Mortgages provide rent-risk insurance to homeowners. This feature is consistent with the empirical results of Sinai and Souleles (2003), which suggest that areas with highly volatile rents have higher house prices. (ii) Mortgage payments and the effective mortgage rate tend to ratchet

[4]The fact that refinancing could involve a new intermediary and a new contract is not essential to our model.

[5]See Bennett, Keane, and Mosser (1999) for a description of the phenomenon during the 1998 refinancing boom.

[6]This approach to modelling the refinancing provisions in the mortgage contract differs significantly from Hurst and Stafford (2002), who examine a model in which interest rates and house prices are fixed, but a stochastic income provides a "consumption-smoothing" rationale for refinancing, when households are liquidity constrained.

down over time. This result is due to both the one-sided nature of the contract, and the front-loading of the mortgage payments required to offset the aggregate interest-rate risk incurred by the lender – who must expect to realize nonnegative present value of profits from any new mortgage contract to which he agrees. (iii) The promised-value awards associated with the mortgage contract do not necessarily increase monotonically over the life of the mortgage. This result owes to the fact that there are opportunities for large equity extractions that could entice the borrower to refinance and realize the immediate consumption benefit from these "cash-outs," and as such he or she would agree to a lesser-valued contract going forward. Simulations of the evolution of the contract in a stationary environment indicate that declines in promised-values are most prominent early in the contract's life when the opportunities for significant capital gains are most prevalent.

The paper is organized as follows. The theoretical model is developed in Section II. The calibration of the model, the algorithm used in the simulations, and a report of the simulation results are presented in Section III. In Section IV, we discuss useful extensions of this model to capture other features of the housing market and the effects of the housing market on the aggregate economy.

II. A Theoretical Model of a Mortgage Contract.

This section presents a recursive contract model of a fixed-rate mortgage in which the household chooses between taking out a mortgage with a "seller," who finances the house purchase, versus renting. The rental rate is stochastic. There are two exogenous sources of changes in house prices. The first source is random supply and demand conditions in the housing market, which are modelled simply by introducing stochastic movements directly into the market's valuation of housing services. For simplicity, these shocks are assumed to be perfectly correlated with shocks to the rental price to reflect substitution between renting and owner-occupied housing.[7] The second source is the rate at which the market discounts housing service flows, such that a decline in the discount rate raises the present value of housing services, and hence the price of the house. Both of these factors may induce the household to exercise its option to refinance, thus entering into a new, renegotiated contract. It is noteworthy that in this setting, rising house prices due to supply and demand factors in the housing market may induce the household to choose a "cash-out" refinance, even when interest rates do not fall. Conversely, interest rates can decline, and if house prices fall (because service flow values fall), the household may choose not to refinance due to the capital loss that it would realize on the house.

II.1 The buyer.

The infinitely lived household purchases consumption goods, c_t, and housing services, z_t, from which it derives utility. It maximizes the expected present value of utility by choosing either to take out an infinitely lived "mortgage contract" with the "seller" of the house in order to purchase the house or to rent.

$$\max_{\{mortgage/rent\}} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_\tau, z_\tau), \quad \beta \in (0,1) \tag{1}$$

The household's initial endowments consist of a nonstochastic stream of income given by $\bar{y}$ each period and a stock of wealth, $g > 0$. If the household chooses to rent, it makes an indivisible investment in the amount g in a storage technology that yields a sure rate of return, $\bar{r}$, that coincides with the household's discount rate, or $\beta = (1+\bar{r})^{-1}$.[8] The endowment income and the interest income are used to purchase consumption, c^a, and to acquire housing services, z^a, by making a

[7]The basic properties of the model can be preserved when relaxing the assumption of perfect correlation between the shocks. It is only necessary that the correlation be positive.

[8]This assumption, along with the indivisible nature of the investment, precludes the household from self-insuring against stochastic rental payments, and renders the household indifferent between saving or consuming its wealth endowment.

rental payment, p. The rental payment is stochastic and given by a series of *iid* draws from an L-dimensional discrete distribution with $p_l \in [p_1, p_2, ..., p_L]$, where $p_1 > ... > p_L$, occurring with probability Π_l.

If the household chooses to rent, it is said to be living in *autarky* (reflecting its participation in the spot rental market). The *ex ante* expected lifetime utility of living in autarky is given by:

$$v_{aut} = E_{t-1}\Big[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_\tau^a, z^a)\Big], \tag{2}$$

where consumption satisfies the budget constraint

$$c_t^a + p_t = \bar{y} + \bar{r}g. \tag{3}$$

Note that, under the *iid* assumption on p_t, v_{aut} is a constant.

Rather than rent and live in autarky, the household may choose to purchase a house by entering into a mortgage contract with a seller. The contract requires the household to give up claims over its endowments to the seller in exchange for a commitment to deliver a stream of consumptions, $\tilde{c}_\tau$, that are net of the mortgage payments, and a constant stream of housing services, $z_\tau = \bar{z}, \tau = t, t+1, ..., \infty$, where the latter has a market price of p_τ^h. It is assumed that the services of living in a home owned by the household are at least as highly valued as those received from renting, or $\bar{z} \geq z^a$.

The household also acquires an equity claim on the house that is initially valued at $g = \delta H_0$, where H_0 is the initial house price and $(1 - \delta)$ is the maximum fraction of the sale price that the seller is willing to finance. Given the seller's down payment requirement, δ, the size of the household's wealth endowment, g, limits the value of the house that it can purchase, H_0. This down payment requirement also applies to refinancing in that, upon refinancing, the equity share that the household retains in the house is δ times the current house price.

Over time, the house price, H_t, varies stochastically, reflecting both the fluctuations in supply and demand factors in the housing market, that induce stochastic movements in the price of housing services, and exogenous changes in market interest rates that are reflected in the rate at which the housing service flows are being discounted. Express the house price as the expected present value of the stream of housing service flows, using the current value of the market's discount rate, r_t:

$$H_t = E_t\Big[\sum_{\tau=t}^{\infty} \frac{p_\tau^h}{(1+r_t)^\tau}\Big], \tag{4}$$

where the market price of housing services follows a sequence of *iid* draws that have the same support as rental prices with each period's draw given by $p^h \in [p_1^h, ..., p_L^h]$, where $p_1^h > ... > p_L^h$, occurring with probability Π_l. The house price can therefore be written as:

$$H_t = p_t^h + \frac{1-r_t}{r_t}\bar{p}^h, \tag{5}$$

where $\bar{p}^h = \sum_l \Pi_l p_l^h$. It is assumed that the rate at which the market discounts these housing service flows varies over time according to the sequence of *iid* draws, $r_q \in [r_1, ..., r_Q]$ with probability Π_q.

The household's *ex ante* expected lifetime utility under the contract at any date t is given by:

$$v_t = E_{t-1}\Big[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(\tilde{c}_\tau, \bar{z})\Big] \tag{6}$$

For the contract to be feasible, $v_t \geq v_{aut}$.

II.2 The seller.

The seller designs the contract with optimal refinancing provisions in order to maximize the expected present value of profits, which must be nonnegative for the contract to be feasible. The seller's expected present-value of profits, P_t, is computed as the discounted net cash payments that the seller receives from the household less the market value of the house at the time of the sale:

$$P_t = E_{t-1}\Big[\sum_{\tau=t}^{\infty}\beta^{\tau-t}(\bar{y}-\tilde{c}_\tau)\Big] - H_t \tag{7}$$

The consumption awards that the seller makes to the household vary over time, and are history-dependent. However, this history-dependence can be completely summarized by the house price at the time the house was initially purchased or was most recently refinanced. Denote this house price by $H^\dagger$. The state vector for determining the consumption awards is then given by $s_t = [p_t, r_t, p_t^h, H_t^\dagger]$.

For the contract to be *sustainable*, it must satisfy the following *ex post participation constraint* to ensure that the household will not terminate the contract and return to renting.

$$U[\tilde{c}_t, \bar{z}] + \beta E_t\Big[\sum_{j=1}^{\infty}\beta^j U[\tilde{c}_{t+j}, \bar{z}]\Big] \geq$$

$$U[\bar{y} + \bar{r}g + H_t - (1-\delta)H_t^\dagger - g - p_t, z^a] + \beta v_{aut}, \tag{8}$$

where the first argument in the $U[.,.]$ function on the right-hand side of equation (8) is the consumption that the household would realize in period t if it returned to autarky. It is assumed that the household would return its initial investment in the house, g, to the storage technology. Its consumption would then be given by its endowment income, $\bar{y}$, plus its interest income, $\bar{r}g$, plus its net capital gain after reinvesting in the storage technology, $H_t - (1-\delta)H_t^\dagger - g$, less its rental payment, p_t. Equation (8) thus states that the *ex post* value of staying in the contract is always at least as great as returning to autarky for any state and consumption history.

To write the contract recursively, we need the functions: $\tilde{c}_t = C(v_t, s_t)$ and $v_{t+1} = V(v_t, s_t)$, where the latter can be iterated from date $\tau = 0$ to t, to yield the current *ex ante* value of lifetime utility represented by the contract that is being offered to the household, v_t:

$$v_t = \tilde{V}(v_0, s_0, ..., s_{t-1}). \tag{9}$$

The contract then consists of the sequence of awards $\{\tilde{c}_t, w_t, \bar{z}\}$ which are made to the household each period, where w_t is next period's promised value of lifetime utility if the household remains in the contract, v_{t+1}.

The seller's problem can now be expressed as choosing decision rules for the consumption and promised-value awards that maximize its present value of profits from the contract, or:

$$P(v) = \max_{c_{q,l}, w_{q,l}} \sum_{q=1}^{Q} \sum_{l=1}^{L} \Pi_q \Pi_l \big[\bar{y} - c_{q,l} - H^{\dagger} + \beta P(w_{q,l})\big], \tag{10}$$

subject to:

$$\text{(promise-keeping):}\ \textstyle\sum_{q=1}^{Q} \sum_{l=1}^{L} \Pi_q \Pi_l \big\{U[c_{q,l}, \bar{z}] + \beta w_{q,l}\big\} = v \tag{11}$$

$$\text{(participation):}\quad U[c_{q,l}, \bar{z}] + \beta w_{q,l} \geq$$

$$U[\bar{y} + \bar{r}g + H_{q,l} - (1-\delta)H^{\dagger} - g - p_l, z^a] + \beta v_{aut}, \quad \forall q \text{ and } \forall l \tag{12}$$

$$\text{(feasibility condition):} \qquad P(v) \geq 0 \tag{13}$$

where the consumption and promised-value awards are bounded by $c \in [c_{min}, c_{max}]$ and $w \in [v_{aut}, \bar{v}]$. The *promise-keeping* constraint, (11), ensures that the household will always value the contract awards as highly as the current *ex ante* value of the contract. The *participation* constraint, (12), ensures that the household will never strictly prefer autarky *ex post*, regardless of the state. This latter constraint will not always bind. The *feasibility condition*, (13), restricts the set of feasible contracts offered by the seller to be those with nonnegative expected net present values.

The first-order and envelope conditions yield:

$$\beta P_w = P_v - \lambda/(\beta \Pi_q \Pi_l) \tag{14}$$

and

$$U_c = -1/P_w \tag{15}$$

where: $U_c > 0, P_w < 0$, and $P_v < 0$ are partial derivatives and λ is the multiplier on the participation constraint. When the participation constraint does not bind, $\lambda = 0$, and the consumption and promised-value awards are unaffected by the current draws on p, r, and p^h:

$$c = C^n(v), C^n_v \geq 0, \tag{16}$$

and

$$w = v. \tag{17}$$

When the participation constraint does bind, $\lambda > 0$, equation (14) indicates $P_v > P_w$, which implies that $w > v$. Then, from the participation constraint, the consumption enjoyed by the household under the contract is less than the consumption it would receive under autarky, $\tilde{c} < c^a$. Solving the binding participation constraint, (12), and the Euler equation, (15), implicitly for the consumption and promised-value awards gives:

$$c_{q,l} = C^b(v, s), \tag{18}$$

and

$$w_{q,l} = W^b(v, s). \tag{19}$$

The optimal contract is then given by:

$$(c^*, w^*) = \begin{cases} (C^n(v), v) & \text{if } \lambda = 0 \\ (C^b(v, s), W^b(v, s)) & \text{if } \lambda > 0 \end{cases} \tag{20}$$

The effective mortgage rate for the contract, denoted r^m, can be found from the household's budget constraint by noting that the income endowment less the consumption award is the household's mortgage payment to the seller.

$$r^m = \frac{\bar{y} - c^*}{(1-\delta)H^\dagger} \tag{21}$$

This mortgage rate does not change unless the participation constraint binds, in which case, the household refinances its house.

II.3 The refinancing decision.

When the household refinances its house, the consumption and the promised-value awards change in accordance with equations (18) and (19), as the "basis" on which the household's contract is written is recomputed. The new basis reflects the current market price of the house, as given by equation (5). This latter house price enters into the participation constraint, (12). Thus, the decision to refinance the house can be triggered by either a rise in the price of housing services, p^h, and/or a lower market interest rate, r.

To illustrate this process, suppose the household enters into the period with the prior state being $s_1 = [p_1, r_1, p_1^h, H_1^\dagger = H_{j-1}]$ under a contract $j-1$ periods old when the state was $[p_{j-1}, r_{j-1}, p_{j-1}^h, H_{j-1}]$. The value of the contract is given by:

$$d_1 = U[\bar{y} + (\bar{r} - 1)g + H_1 - (1-\delta)H_1^\dagger - p_1), z^a] + v_{aut}, \tag{22}$$

where H_1 is given by equation (5) evaluated at $s = s_1$. In the next period, suppose the draw yields $[p_2, r_2, p_2^h]$. If the household were to remain in the contract, now j periods old, without refinancing, the seller would have to compare the value of the contract under the new state, $s_2^j = [p_2, r_2, p_2^h, H_1^\dagger]$ with its value in the previous state to determine whether to raise the consumption and promised-value awards. (The superscript "j" on the state vector denotes the age of the existing contract, as reflected in the house price $H_1^\dagger$.) Denote the consumption and promised-value awards for this contract by $\{\tilde{c}_2^j, w_2^j\}$. If the inequality in (23) below holds, then the consumption and promised-value awards would remain unchanged:

$$U(\tilde{c}_2^j, \bar{z}) + \beta w_2^j = d_2 < d_1 = U(\tilde{c}_1, \bar{z}) + \beta w_1. \tag{23}$$

If the inequality in (23) were reversed, then the seller would have to increase the awards to keep the household in the contract as he is committed to do.

However, the conditions that would cause the inequality of equation (23) to reverse will in general lead to the household exercising the option to refinance the house. In this case, the basis would be reset and a new set of contract awards would apply. The household would choose this refinancing option if and only if the value of the newly modified contract, now $j = 0$ periods old, under the state $s_2^0 = [p_2, r_2, p_2^h, H_2]$ exceeded the value of the existing contract, after taking into account the additional consumption implied by the cash-out refinancing due to the capital gain, $(1-\delta)H_2 - H_1^\dagger$, or:

$$U\{\tilde{c}_2^0 + (1-\delta)[H_2 - H_1^\dagger], \bar{z}\} + \beta w_2^0 > d_1 \tag{24}$$

Note that if the capital gain is large, then the household is willing to enter into a new contract with a lower future promised value, ie., $w_2^0 < w_1$.

Therefore, when the household refinances its house, it may experience a reduction in its mortgage payment as well as a capital gain from cashing out a portion of the equity in the house. The capital gain is realized as a windfall boost to consumption that the household is not permitted in this model to carry forward, thereby adding lumpiness to the consumption profile. The ability to refinance the contract nonetheless insures the household against adverse house price movements, given that the decision to refinance is at the discretion of the household. Refinancing thus induces a ratcheting up of the *ex ante* value of the mortgage contract to the household over time, which coincides with a front-loading of the mortgage payments in the contract. The degree to which the mortgage contract is front-loaded can be affected by the desire of the household to avoid the rent risk associated with volatile rental payments. This risk avoidance sets an upper bound on the degree of front-loading. A lower bound is set by a binding feasibility condition, (13), which coincides with a zero expected present-value profit condition for the seller, who must absorb the aggregate risk of rising house prices. In particular, the higher is the variance of market interest rates, the greater will be the front-loading required by the seller to absorb this aggregate risk.

III. A Numerical Example of the Evolution of the Mortgage Contract.

To highlight the features of this contract, the model can be calibrated to determine the optimal contract awards for each state. Initial contract awards consistent with a zero expected present value profit condition, (13), and an initial state are found. The model is then simulated to illustrate how the contract evolves over time, when the household optimally exercises the refinancing option in the contract.

III.1 Calibration.

For the calibration of the model, period utility is given by: $U(c_t, z_t) = \log c_t + a \log z_t$, $a > 0$. The value of a is obtained by setting the intratemporal marginal rate of substitution between consumption and housing services, U_z/U_c, equal to the user cost of housing. The average value of the ratio of housing service consumption to consumption (U_z/U_c under log utility) is 15 percent, which is the average monthly expenditure share of housing in total consumption over the period 1959:1 to 2002:9. The assigned value for the user cost is taken to be 0.05, which is close to that obtained by Hendershott (1980). For simplicity, housing services are assumed to be equally valued whether the household is renting or owning the home, ie., $z^a = \bar{z}$.

The average value of consumption under autarky is the scale parameter in the model, and is arbitrarily set to 10. The average rental payment is then determined by setting the ratio of the rental payment to consumption equal to the shelter component's share of the CPI, or 0.315. From the household's budget constraint under autarky, equation (3), the sum of the average rental payment and the average consumption value equals the mean income endowment plus the mean interest income from the household's stock endowment, $\bar{r}g$. Here, g coincides with the original downpayment and is set equal to 20 percent of the initial house price, and $\bar{r}$ is determined by the choice of β as described below. The mean value of the market interest rate, r, is set equal to 10 percent, which is approximately equal to the average interest rate on 30-year fixed-rate mortgages over the past 30 years. The distribution of r is uniform with 30 increments of size 12.5 basis points, giving a range of rates from 8 to 12 percent.

To complete the calibration, the mean price of housing services, from equation (5) coincides with the initial mortgage payment that equals the average market interest rate times the initial house price. This initial mortgage payment is set equal to 0.3 times the mean autarkic income level, which is essentially the minimum requirement for conforming mortgage loans sponsored by the Federal Housing Administration (FHA).[9] The distribution of the price of housing services is assumed uniform with 20 increments along its support of size 0.1. Increments across the support for the rental payments are selected also to be of size 0.1. Finally, the discount factor is set to $\beta = 0.935$, which is the minimum value consistent with the initial expected present value of the seller's profits of zero.[10] A higher discount factor yields positive present value of profits to the seller; a lower β violates the feasibility

[9] This information is available at http://www.fhaloan.com under"Debt to Income Ratios."

[10] Krueger and Uhlig (2003) examine the relationship between the discount factor and the ability of a competitive intermediary to provide risk sharing in a random endowment economy.

condition, (13). This calibration produced the following set of values on which the terms of the initial mortgage contract were initially based: $c = 10; a = 0.0075; z = 1.5; \bar{y} = 12.60; g = 7.89; \bar{r} = 0.0695; r = 0.10; \beta = 0.935; H = 39.45; p = 3.15; p^h = 3.945$.

III.2 Solution algorithm.

To characterize the evolution of the consumption and refinancing activity, it is necessary to solve for the contract awards $\{\tilde{c}_{q,l}, w_{q,l}\}$ for all house prices, $H^{\dagger}$. The procedure begins by selecting a single initial house price and computing the *ex post* values of the contract, d_s, for all possible draws of $[p, r, p^h]$, assuming that the participation constraint just binds.

$$d_s = U\left\{\bar{y} + (\bar{r} - 1)g + (1 - \delta)[H_{q,l} - H^{\dagger}] - p_l, z^a\right\} + \beta v_{aut} \qquad (25)$$

These values are ordered, again for each initial house price. Denote the ordered (high-to-low) contract values by $\hat{d}_k$, where $k = 1, ..., QL$. The following recursive formula is then used to generate an ordered vector of promised-value awards for each state, denoted $\hat{w}_k$.[11]

$$\hat{w}_k = \sum_{k=2}^{QL} \frac{QL - k + 1}{QL} \hat{d}_k + \frac{1}{QL} \sum_{k=2}^{QL} \hat{w}_k, \qquad (26)$$

where: $\hat{w}_1 = \hat{d}_1$. The corresponding consumption awards are then computed from the participation constraint:

$$\hat{c}_k = \hat{d}_k - \beta \hat{w}_k \qquad (27)$$

To obtain the original contract at date $t = 0$, choose the initial state to be that state, denoted $\bar{s}$, that coincides with the mean house price. Then, select the initial contract awards that induce the household to enter the contract, satisfy the feasibility condition, and result in zero profits for the seller. That is, given $\bar{s}$, identify the state k_0 that solves

[11] This derivation follows Ljunqvist and Sargent (2000, p. 412.)

$$k_0 = \arg\min_k \{\hat{d}_k\} \quad s.t. \quad P(v) = 0. \tag{28}$$

Once the initial contract and contract awards are determined, simulations can be run by relying on criteria (24), given the ordering deduced from (25).

III.3 Simulation results.

The upper panel of Figure 2 depicts the profile of consumption awards over the course of 100 periods, when averaged over 10,000 simulations. For virtually the entire life of the contract, consumption awards paid out to the representative household are monotonically increasing. This feature reflects the provision of the contract for insurance against fluctuating rental payments. Over the life of a single contract, this consumption profile would appear as a step function. An increase in an individual's consumption award comes when the state of the economy triggers a refinancing of the contract. Therefore, the profile displayed in Figure 2 is an average of a large number (10,000) of these step functions.

In the middle panel, the time path of the promised value is displayed. Somewhat contrary to intuition, this profile is generally decreasing. The pattern is puzzling at first, because a downward-sloping path for the promised value would seem to indicate that the value of the contract is diminishing over time – a violation of the promise-keeping constraint. The puzzle is resolved by noting that the refinancing event represents a thorough renegotiation of the contract. When a household refinances, it cashes out some of the equity in the house and increases the amount of debt it is carrying. This effectively places the household in a new contract with a new downpayment and a new basis upon which to calculate capital gains. For any given basis, the path for the promised value is increasing. However, as the basis ratchets up, the promised value can decline. The household accepts this decline in the promised value in exchange for the lump-sum cash-out it gets when refinancing.

The increase in the basis is also visible in the bottom panel of Figure 2. As the simulations run their course, realizations of the housing service flow and interest rate variables make it profitable to refinance. Over time, the basis that the household attains in its mortgage eventually reaches a maximum (implied by the parameterization of the interest rate and service flow distributions). At this point, refinancing ceases and there is no further possibility for capital gains.

In this model, mortgage payments equate to the household's (nonstochastic) income less its consumption awards. The current effective mortgage rate that the household actually pays is computed from this current mortgage payment and the existing basis on the house, as given by equation (20). This effective mortgage rate is graphed in the bottom panel of Figure 2. It represents the model's purest measure of the economic effects of mortgage refinancing, and serves to illustrate the asymmetric way in which the refinancing option is exercised. That is, while mortgage interest rates fluctuate over time, those rates may bear little relation to actual borrowing costs, as households with existing contracts can simply ignore high market interest rates. The effective mortgage rate here represents actual borrowing costs. It tracks the changing mortgage payments due to falling market rates (through $\bar{y} - \tilde{c}_t$), while accounting for the fact that the household is becoming increasingly more leveraged through its cash-outs and the rising basis ($H^\dagger$). As leverage increases, effective borrowing costs decrease in this model.

In the beginning of the typical contract, the effective mortgage rate is higher than the assumed expected average interest rate of 10 percent. This feature is due to the low starting value for the consumption award. The household accepts high borrowing costs initially in exchange for the insurance against future fluctuations in rental rates. As time passes, however, effective borrowing costs decline steadily as the household refinances at lower current interest rates and increases its debt level. Like the contract awards, changes in the effective mortgage rate attenuate late in the life of the contract as the opportunities for profitable refinancing are eventually exhausted.

IV. Conclusion.

Recursive contract theory offers a natural way to model the history-dependence of the optimal refinancing decision on interest rates and house prices in a fixed-rate mortgage contract. It successfully captures several important features of mortgage contracts, including the provision of rent-risk insurance to homeowners, the tendency of the effective mortgage rate on any given mortgage contract to decline over time, and the evidence that homeowners often choose to extract equity from their house whenever they refinance.

We abstracted from several real-world features, such as finite-horizon contracts with declining principals, less-than-perfect correlation between house prices and rental rates, and persistence in the shocks, in order to obtain results. Relaxing these assumptions can present difficult technical issues, such as having to deal with the "curse of dimensionality" in the case of persistence in the shocks. However, it is unclear that these issues are very important to our results.

Rather than focusing on those modelling issues, we feel that useful extensions of the model could be made to capture other important "real-world" features of the housing market. For example, in a general equilibrium model, aspects of refinancing on the macroeconomy could be explored. We have ample evidence that the effect on economic activity, in terms of levels and composition of output, could be quite large and is asymmetric over the course of an interest rate cycle. We also know that households on average live in a given home only seven years, and that the timing of many of these moves is unanticipated. What effect does this uncertain duration of home ownership have on the provisions of an optimal mortgage contract? Since home ownership provides rent-risk insurance to households, how much does the effect get attenuated by the ownership of other assets that can also be used to help provide consumption-smoothing when rent and/or income are volatile? More generally, how does the housing market affect the price dynamics of other assets, given that housing is an asset with a unique property of providing housing service flows that cannot be driven to zero.

In any case, meaningful work on the role of the housing market in today's economy will necessarily have to deal with refinancing, and refinancing is a history-dependent decision that requires modelling. This paper suggests one promising avenue through which progress may be made on this line of research.

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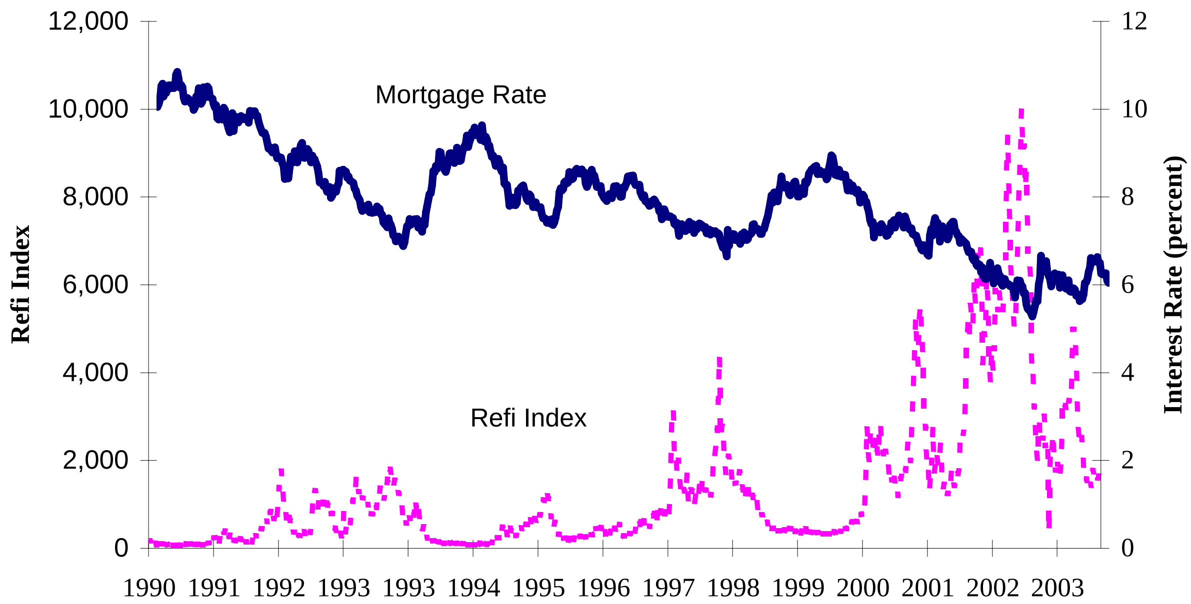
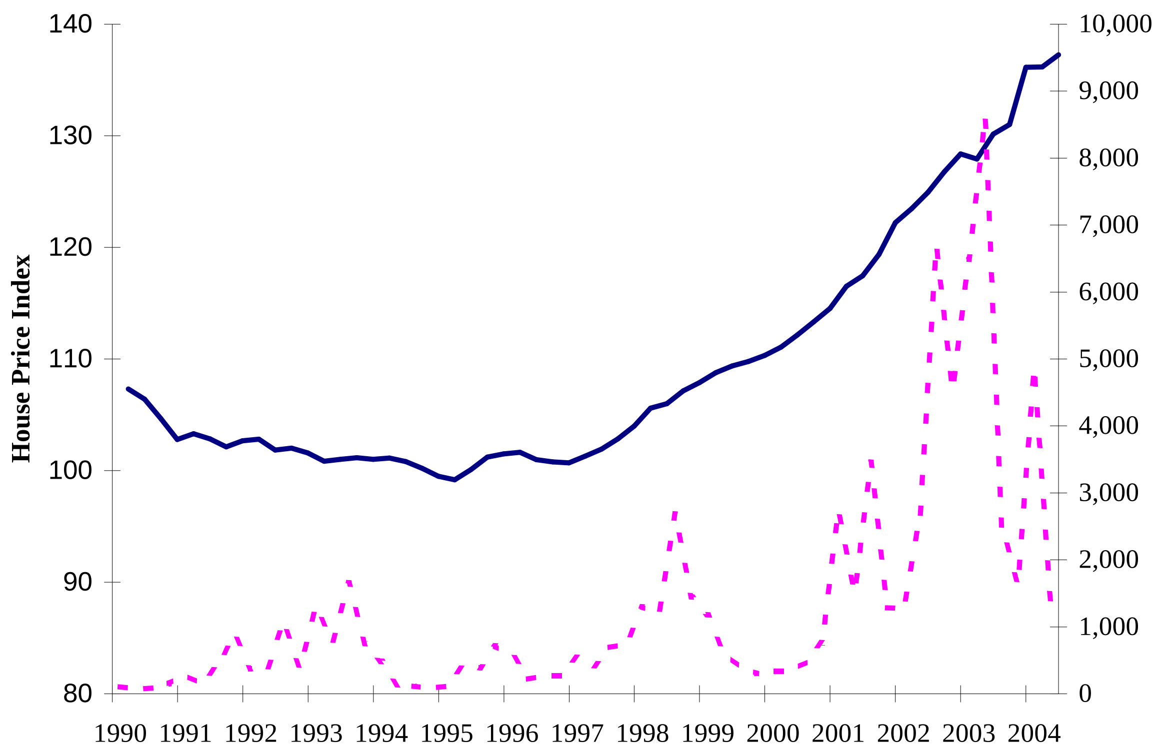
Figure 1



Source: Mortgage Banker's Association of America



Sources: Office of Federal Housing Enterprise Oversight and Mortgage Banker's Association of America

Figure 2: Evolution of Terms of Contract with Home Equity