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Monetary Policy Response to Oil Price Shocks

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Abstract

How should monetary authorities react to an oil price shock? The New Keynesian literature has concluded that ensuring complete price stability is the optimal thing to do. In contrast, this paper argues that a *meaningful* trade-off between stabilizing inflation and the welfare relevant output gap arises in a distorted economy once one recognizes (*i*) that oil (energy) cannot be easily substituted by other factors in the short-run, (*ii*) that there is no fiscal transfer available to policymakers to neutralize the steady-state distortion due to monopolistic competition, and (*iii*) that increases in oil prices also directly affect consumption by raising the price of fuel, heating oil, and other energy sources. While the first two conditions are necessary to introduce a microfounded monetary policy trade-off, the third one makes it *quantitatively* significant.

The optimal precommitment monetary policy relies on unobservables and is therefore hard to implement. To address this concern, I derive a simple interest rate feedback rule that mimics the optimal plan in all relevant dimensions but that depends only on observables, namely core inflation, oil price inflation, and the growth rate of output.

Keywords: optimal monetary policy, oil shocks, divine coincidence, simple rules
JEL Class: E32, E52, E58

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1 Introduction

In the last ten years a new macroeconomic paradigm has emerged centered around the New Keynesian (NK henceforth) model, which is at the core of the more involved and detailed dynamic stochastic general equilibrium (DSGE) models used for policy analysis at many central banks. Despite its apparent simplicity, the NK model has solid theoretical foundations and has therefore been used to draw normative conclusions on the appropriate response of monetary policy to economic shocks.

One result that stands out is that optimal monetary policy should aim at replicating the real allocation under flexible prices and wages, or *natural* output, which features constant markups and no inflation. In the case of an oil price shock, the canonical NK prescription to policymakers is then fairly simple. Central banks must *perfectly* stabilize inflation,[1] even if that leads to large drops in output and employment. Because the latter are considered efficient, monetary policy should focus on minimizing inflation volatility. There is a "divine coincidence,"[2] i.e., an absence of trade-off between stabilizing inflation and stabilizing the "welfare relevant" output gap.

The contrast between theory and practice is striking, however. When confronted with rising commodity prices, policymakers in inflation-targeting central banks do indeed perceive a trade-off. They typically favor a long run approach to price stability by avoiding second-round effects — where wage inflation affects inflation expectations and ultimately leads to upward spiralling inflation — but by letting first-round effects play out.

So why the difference? Do policymakers systematically conduct irrational, suboptimal policies? Or should we reconsider some of the assumptions embedded in the NK model? In a recent paper, Blanchard and Galí (2007) (henceforth BG07) argued that dropping the assumption of perfectly flexible *real* wages drives a time-varying wedge between *natural* and *efficient* output (i.e., the undistorted level of output that would prevail in the absence of nominal frictions in a perfectly competitive economy). Therefore, stabilizing prices — which amounts to targeting the *natural* level of output — introduces inefficient output variations and the divine coincidence does not hold

[1] As noted by Galí (2008 chapter 6), different assumptions on nominal rigidities give rise to different definitions of target inflation. Goodfriend and King (2001) and Aoki (2001) argue that monetary policy should stabilize the stickiest price. By introducing sticky wages alongside sticky prices, Erceg et al. (2000) and Bodenstein, Erceg and Guerrieri (2008) find that optimal monetary policy should *perfectly* stabilize a weighted average of core prices and (negative) wage inflation.

[2] The expression is from Blanchard and Galí (2007).

anymore.

In this paper, I focus on an alternative explanation that does not hinge on real rigidities but on the specification of technology and its interaction with the assumption of monopolistic competition, standard in NK models. The canonical NK model relies on the assumption of Cobb-Douglas production functions. Cobb-Douglas production functions greatly simplify the analysis and permit nice closed-form solutions, but because they assume a unitary elasticity of substitution between factors they feature constant cost shares over the cycle *regardless of the degree of market distortion*. A 1-percent increase in the relative price of a factor immediately leads to a 1-percent decrease in its relative use. Yet, the case for a unitary elasticity between oil and other factors is not particularly compelling, especially at business cycle frequency.[3] When oil is considered a gross complement to other factors, at least in the short run, the response of output to an oil price shock will depend on the degree of market distortion in the economy, which typically varies with the extent of firms' monopolistic power in NK models. The larger the distortion, the more important is the impact of a given oil price shock on the oil cost share — and therefore on output — in the flexible price and wages equilibrium. Like in BG07, this creates a time-varying wedge between the *natural* (distorted) and the *efficient* levels of output, which implies that strictly stabilizing inflation in the face of an oil price shock is no longer the optimal policy to follow.[4]

The first contribution of this paper is to show that increases in oil prices lead to a *meaningful* monetary policy trade-off once it is acknowledged (*i*) that oil cannot easily be substituted by other factors[5] in the short run, (*ii*) that there is no fiscal transfer available to policymakers to neutralize the steady-state distortion due to monopolistic competition, and (*iii*) that oil is an input both to production and to consumption (via the impact of the price of crude oil on the prices of gasoline, heating oil, and electricity).

While the first two conditions are *necessary* to introduce a microfounded monetary policy trade-off, they are not *sufficient* to explain the policymakers' concern for the real activity consequences of oil price shocks. Hence, this paper stresses that perfectly

[3] A voluminous empirical literature (Hughes et al., 2008) has documented the fact that the share of oil (or gasoline) in production and expenditures is highly correlated with its price. In other words, it is difficult to substitute oil in the short run.

[4] Monetary authorities can aim at a higher level of welfare by trading some of the costs of inefficient output fluctuations against the distortion resulting from more inflation, which is in line with the general theory of the second best (see Lipsey and Lancaster, 1956).

[5] This issue has also been considered in the recent analysis of Montoro (2007) and Castillo et al. (2007) in an "oil-in-production-only" framework where oil is a gross complement to labor.

stabilizing inflation becomes particularly costly when the impact of higher oil prices on households' overall consumption is also taken into account. In a nutshell, changes in oil prices directly affect the cost of consumption, and then act as a distortionary tax on labor income. The lower the elasticity of substitution between energy and other consumption goods, the larger is the tax effect, and the more detrimental are the consequences on employment and output of a given increase in oil prices.

One problem with utility-based optimal policies is that they rely on unobservables, such as the efficient level of output or various shadow prices. This makes them difficult to communicate and to implement. The second contribution of this paper is to propose a simple interest rate rule that mimics the optimal plan in all relevant dimensions but relies only on observables: core inflation, oil price inflation, and the growth rate of output.[6] Interestingly, I find that the optimal monetary policy response to a persistent increase in oil price resembles the typical response of inflation targeting central banks. While long-term price stability is ensured by a credible commitment to stabilize inflation and inflation expectations, short-term real rates drop right after the shock to help dampen real output fluctuations. By managing expectations efficiently, central banks can improve on both the flexible price equilibrium solution and the recommendation of simple Taylor rules.

The rest of the paper is structured as follows. In Section 2, I start by building a two-sector NK model where oil enters as an input to both production and consumption in a low elasticity CES framework, and which, therefore, features both core and headline inflation. Section 3 solves a log-linearized version of the model in the flexible price and wage equilibrium and shows that the cost-push shock leading to the monetary policy trade-off is increasing in the degree of steady-state distortion and is inversely related to the elasticities of substitution, both at the production and the consumption level. In Section 4, I derive an analytical linear-quadratic solution to the optimal policy problem in a timeless perspective. I show that the optimal weight on inflation in the policymaker's loss function decreases with the production elasticity of substitution and increases with the degree of real wage and nominal price stickiness. Section 5 derives a simple implementable interest rate feedback rule that replicates the optimal plan.

Oil shocks are rare but costly events. To give a sense of the costs incurred by following suboptimal monetary policies, Section 6 revisits the 1979 oil shock and computes

[6] See Orphanides and Willliams (2003) for a thorough discussion of implementable monetary policy rules.

the welfare losses associated with alternative policy rules. I reckon that following a Taylor rule instead of the optimal precommitment policy could have cost the United States 2.1 percent of one year's consumption (or about 200 billions dollars in terms of 2008 private consumption).

Finally, as the long run price elasticity of oil demand is deemed much higher than its short-run counterpart[7], Section 7 shows that this paper's findings are robust to a production framework that assumes gross complementarity in the short run but gross substitutability in the long run, in the spirit of "putty-clay" models of energy use.[8] Section 8 summarizes the main findings and sketches directions for future research.

2 The model

Following Bodenstein, Erceg and Guerrieri (2008) (thereafter BEG08), I assume a two-layer closed-economy setting[9] composed of a core consumption good, which takes labor and oil as inputs, and a consumption basket consisting of the core consumption good and oil. In order to keep the notations as simple as possible, there is only one source of nominal rigidity in this economy: core goods prices[10] are sticky and firms set prices according to a Calvo scheme.

In contrast to BEG08, however, I relax the assumption of a unitary elasticity of substitution between oil and other goods and factors. I also explicitly consider a distorted economy: there is no fiscal transfer to neutralize the monopolistic competition distortion.

2.1 Households

There exists a unit mass continuum of infinitely lived households indexed by $j \in [0,1]$, which maximize the discounted sum of present and expected future utilities defined as follows

$$\mathbb{E}_t \sum_{t=s}^{\infty} \beta^{t-s} \left\{ \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \nu \frac{H_t(j)^{1+\phi}}{1+\phi} \right\}, \tag{1}$$

[7] See Pindyck and Rotemberg (1983) for an empirical investigation.

[8] See Atkeson and Kehoe (1999).

[9] This assumption allows one to ignore income distribution and international risk-sharing related issues.

[10] Introducing nominal wage stickiness would not change the thrust of the argument. As shown by Woodford (2003) and Gali (2008), one can always define a composite index of wage and price inflation such that there is no trade-off between stabilizing the composite index and the welfare-relevant output gap in the canonical NK model.

where $C_t(j)$ is the consumption goods bundle, $H_t(j)$ is the (normalized) quantity of hours supplied by household of type j, the constant discount factor β satisfies $0 < \beta < 1$ and ν is a parameter calibrated to ensure that the typical household works eight hours a day in steady state.

In each period, the representative household j faces a standard flow budget constraint

$$P_t B_t(j) + P_t C_t(j) = R_{t-1} B_{t-1}(j) + W_t H_t(j) + \widetilde{\Pi}_t(j) + T_t(j), \tag{2}$$

where $B_t(j)$ is a non-state-contingent one period bond, R_t is the nominal gross interest rate, P_t is the CPI, $\widetilde{\Pi}_t(j)$ is the household j share of the firms' dividends and $T_t(j)$ is a lump sum fiscal transfer to the household of the profits from sovereign oil extraction activities.

Because the labor market is perfectly competitive, I drop the index j such that $H_t \equiv H_t(j) = \int_0^1 H_t(j)\, dj$, and I write the consumption goods bundle[11] C_t as a CES aggregator of the core consumption goods basket $C_{Y,t}$ and the household's demand for oil $O_{C,t}$

$$C_t = \left((1-\omega_{oc}) C_{Y,t}^{\frac{\chi-1}{\chi}} + \omega_{oc} O_{C,t}^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}, \tag{3}$$

where ω_{oc} is the oil quasi-share parameter and χ is the elasticity of substitution between oil and non-oil consumption goods.

Households determine their consumption, savings, and labor supply decisions by maximizing (1) subject to (2). This gives rise to the traditional Euler equation

$$1 = \beta \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{R_t}{\Pi_{t+1}} \right\}, \tag{4}$$

which characterizes the optimal intertemporal allocation of consumption and where Π_t represents headline inflation.

Allowing for real wage rigidity (which may reflect some unmodeled imperfection in the labor market as in BG07), the labor supply condition relates the marginal rate of substitution between consumption and leisure to the geometric mean of real wages in periods t and $t-1$.

$$\left(C_t^{\sigma} \nu H_t^{\phi} \right)^{(1-\eta)} = \frac{W_t}{P_t} \left(\frac{W_{t-1}}{P_{t-1}} \right)^{-\eta}. \tag{5}$$

[11] The consumption basket can be regarded as produced by perfectly competitive consumption distributors whose production function mirrors the preferences of households over consumption of oil and non-oil goods.

In the benchmark calibration, i.e., unless stated otherwise, $\eta = 0$; real wages are perfectly flexible and equal to the marginal rate of substitution between labor and consumption in all periods.

Finally, households optimally divide their consumption expenditures between core and oil consumption according to the following demand equations:

$$C_{Y,t} = P_{y,t}^{-\chi} (1 - \omega_{oc})^{\chi} C_t, \tag{6}$$

$$O_{C,t} = P_{o,t}^{-\chi} \omega_{oc}^{\chi} C_t, \tag{7}$$

where $P_{y,t} \equiv \frac{P_{Y,t}}{P_t}$ is the relative price of the core consumption good and $P_{o,t} \equiv \frac{P_{O,t}}{P_t}$ is the relative price of oil in terms of the consumption good bundle and where

$$P_t = \left((1 - \omega_{oc})^{\chi} P_{Y,t}^{1-\chi} + \omega_{oc}^{\chi} P_{O,t}^{1-\chi} \right)^{\frac{1}{1-\chi}} \tag{8}$$

represents the overall consumer price index (CPI).

2.2 Firms

2.2.1 Core goods producers

I assume that the core consumption good is produced by a continuum of perfectly competitive producers indexed by $c \in [0, 1]$ that use a set of imperfectly substitutable intermediate goods indexed by $i \in [0, 1]$. In other words, core goods are produced via a Dixit-Stiglitz aggregator

$$Y_t (c) = \left(\int_0^1 Y_t (i)^{\frac{\varepsilon - 1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon - 1}}, \tag{9}$$

where ε is the elasticity of substitution between intermediate goods. Given the individual intermediate goods prices, $P_{Y,t} (i)$, cost minimization by core goods producers gives rise to the following demand equations for individual intermediate inputs:

$$Y_t (i) = \left(\frac{P_{Y,t} (i)}{P_{Y,t}} \right)^{-\varepsilon} Y_t (c), \tag{10}$$

where $P_{Y,t} = \left(\int_0^1 P_{Y,t} (i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is the core price index.

Aggregating (10) over all core goods firms, the total demand for intermediate goods $Y_t (i)$ is derived as a function of the demand for core consumption goods Y_t

$$Y_t (i) = \left(\frac{P_{Y,t} (i)}{P_{Y,t}} \right)^{-\varepsilon} Y_t, \tag{11}$$

using the fact that perfect competition in the market for core goods implies $Y_t(c) \equiv Y_t = \int_0^1 Y_t(c)\, dc$.

2.2.2 Intermediate goods firms

Each intermediate goods firm produces a good $Y_t(i)$ according to a constant returns-to-scale technology represented by the CES production function

$$Y_t(i) = \left((1-\omega_{oy}) \left(\mathcal{H}_t H_t(i) \right)^{\frac{\delta-1}{\delta}} + \omega_{oy} \left(O_{Y,t}(i) \right)^{\frac{\delta-1}{\delta}} \right)^{\frac{\delta}{\delta-1}}, \tag{12}$$

where $\mathcal{H}_t$ is the exogenous Harrod-neutral technological progress whose value is normalized to one, for I am here only interested in the dynamic response of the economy to an oil price shock. $O_{Y,t}(i)$ and $H_t(i)$ are the quantities of oil and labor required to produce $Y_t(i)$ given the quasi-share parameters, ω_{oy}, and the elasticity of substitution between labor and oil, δ.

Each firm i operates under perfect competition in the factor markets and determines its production plan so as to minimize its total cost

$$TC_t(i) = \frac{W_t}{P_{Y,t}} H_t(i) + \frac{P_{O,t}}{P_{Y,t}} O_{Y,t}(i), \tag{13}$$

subject to the production function (12) for given W_t, $P_{Y,t}$, and $P_{O,t}$. Their demands for inputs are given by

$$H_t(i) = \left(\frac{W_t}{MC_t(i) P_{Y,t}} \right)^{-\delta} (1-\omega_{oy})^{\delta} Y_t(i) \tag{14}$$

$$O_{Y,t}(i) = \left(\frac{P_{O,t}}{MC_t(i) P_{Y,t}} \right)^{-\delta} \omega_{oy}^{\delta} Y_t(i), \tag{15}$$

where the real marginal cost in terms of core consumption goods units is given by

$$MC_t(i) \equiv MC_t = \left((1-\omega_{oy})^{\delta} \left(\frac{W_t}{P_{Y,t}} \right)^{1-\delta} + \omega_{oy}^{\delta} \left(\frac{P_{O,t}}{P_{Y,t}} \right)^{1-\delta} \right)^{\frac{1}{1-\delta}}. \tag{16}$$

2.2.3 Price setting

Final goods producers operate under perfect competition and therefore take the price level $P_{Y,t}$ as given. In contrast, intermediate goods producers operate under monopolistic competition and face a downward-sloping demand curve for their products, whose price elasticity is positively related to the degree of competition in the market. They set prices so as to maximize profits following a sticky price setting scheme à la Calvo. Each firm contemplates a fixed probability θ of not being able to change its price next period and therefore sets its profit-maximizing price $\overline{P_{Y,t}}\,(i)$ to solve

$$\arg\max_{\overline{P_{Y,t}}(i)} \left\{ \mathbb{E}_t \sum_{n=0}^{\infty} \theta^n \mathcal{D}_{t,t+n} \widetilde{\Pi}_{t,t+n}\,(i) \right\},$$

where $\mathcal{D}_{t,t+n}$ is the stochastic discount factor defined by $\mathcal{D}_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+n}}$ and profits are

$$\widetilde{\Pi}_{t,t+n}\,(i) = \overline{P_{Y,t}}\,(i)\, Y_{t+n}\,(i) - MC_{t+n} P_{t+n}^{Y} Y_{t+n}\,(i)\,.$$

The solution to this intertemporal maximization problem yields

$$\frac{\overline{P_{Y,t}}\,(i)}{P_{Y,t}} = \frac{K_t}{F_t}, \tag{17}$$

where

$$K_t \equiv \left(\frac{\varepsilon}{\varepsilon-1}\right) \mathbb{E}_t \sum_{n=0}^{\infty} (\beta\theta)^n \left(X_{t+n}^{Y}\right)^{-\varepsilon} \left(\frac{Y_{t+n}}{C_{t+n}^{\sigma}}\right) \left(\frac{P_{t+n}^{Y}}{P_{t+n}}\right) MC_{t+n},$$

and

$$F_t \equiv \mathbb{E}_t \sum_{n=0}^{\infty} (\beta\theta)^n \left(X_{t+n}^{Y}\right)^{1-\varepsilon} \left(\frac{Y_{t+n}}{C_{t+n}^{\sigma}}\right) \left(\frac{P_{t+n}^{Y}}{P_{t+n}}\right).$$

Since only a fraction $(1-\theta)$ of the intermediate goods firms are allowed to reset their prices every period while the remaining firms update them according to the steady-state inflation rate (which is optimally zero in the present context), it can be shown that the overall core price index dynamics is given by the following equation

$$\left(P_{Y,t}\right)^{1-\varepsilon} = \theta \left(P_{Y,t-1}\right)^{1-\varepsilon} + (1-\theta) \left(\overline{P_{Y,t}}\,(i)\right)^{1-\varepsilon} \tag{18}$$

Following Benigno and Woodford (2005), I rewrite equation (18) in terms of the core inflation rate $\Pi_{Y,t}$

$$\theta \left(\Pi_{Y,t}\right)^{\varepsilon-1} = 1 - (1-\theta) \left(\frac{K_t}{F_t}\right)^{1-\varepsilon}, \tag{19}$$

for

$$K_t = \frac{\varepsilon}{\varepsilon-1} \left(\frac{Y_t}{C_t^{\sigma}} \frac{P_{Y,t}}{P_t}\right) MC_t + \beta\theta \mathbb{E}_t \left\{ \left(\Pi_{Y,t+1}\right)^{\varepsilon} K_{t+1} \right\},$$

and

$$F_t = \frac{Y_t}{C_t^{\sigma}} \frac{P_{Y,t}}{P_t} + \beta\theta \mathbb{E}_t \left\{ \left(\Pi_{Y,t+1}\right)^{\varepsilon-1} F_{t+1} \right\}.$$

2.3 Government

To close the model, I assume that oil is extracted with no cost by the government, which sells it to the households and the firms and transfers the proceeds in a lump sum fashion to the households. I abstract from any other role for the government and assume that it runs a balanced budget in each and every period so that its budget constraint is simply given by

$$T_t = P_{O,t} O_t,$$

for O_t the total amount of oil supplied.

2.4 Market clearing and aggregation

In equilibrium, goods, oil, and labor markets clear. In particular, given the assumption of a representative household and competitive labor markets, the labor market clearing condition is

$$H_t^D \equiv \int_0^1 H_t(i)\, di = \int_0^1 H_t(j)\, dj \equiv H_t.$$

Because I assume that the real price of oil $P_{o,t}$ is exogenous in the model, the government supplies all demanded quantities at the posted price. The oil market clearing condition is then given by

$$\int_0^1 O_{C,t}(j)\, dj + \int_0^1 O_{Y,t}(i)\, di = O_t,$$

for O_t the total amount of oil supplied.

As there is no net aggregate debt in equilibrium,

$$\int_0^1 B_t(j)\, dj = B_t = 0,$$

we can consolidate the government's and the household's budget constraints to get the overall resource constraint

$$C_{Y,t} = Y_t.$$

Finally, Calvo price setting implies that in a sticky price equilibrium there is no simple relationship between aggregate inputs and aggregate output, i.e., there is no aggregate production function. Namely, defining the efficiency distortion related to price stickiness $P_t^* \equiv \frac{P_t^{disp}}{P_t^Y}$ for $P_t^{disp} \equiv \left(\int_0^1 (P_{Y,t}(i))^{-\varepsilon} di\right)^{-\frac{1}{\varepsilon}}$, I follow Yun (1996) and write the aggregate production relationship

$$Y_t = \left((1-\omega_{oy}) H_t^{\frac{\delta-1}{\delta}} + \omega_{oy} O_{Y,t}^{\frac{\delta-1}{\delta}}\right)^{\frac{\delta}{\delta-1}} P_t^*, \tag{20}$$

where price dispersion leads to an inefficient allocation of resources given that

$$P_t^* : \begin{cases} \leq 1 & \\ = 1 & P_{Y,t}(r) = P_{Y,t}(s), \quad \text{all } r = s. \end{cases}$$

The inefficiency distortion P_t^* is related to the rate of core inflation $\Pi_{Y,t}$ by making use of the definition

$$P_t^* = \left(\theta \left(P_{t-1}^{disp}\right)^{-\varepsilon} + (1-\theta)\left(\overline{P_{Y,t}}(i)\right)^{-\varepsilon}\right)^{-\frac{1}{\varepsilon}},$$

and equations (19) and (17) to get

$$P_t^* = \left((1-\theta)\left(\frac{K_t}{F_t}\right)^{-\varepsilon} + \frac{\theta\left(\Pi_{Y,t}\right)^{\varepsilon}}{P_{t-1}^*}\right)^{-1}.$$

2.5 Calibration

For the sake of comparability, the model calibration closely follows BEG08. The quarterly discount factor β is set at 0.993, which is consistent with an annualized real interest rate of 3 percent. The consumption utility function is chosen to be logarithmic ($\sigma = 1$) and the Frish elasticity of labor supply is set to unity ($\phi = 1$).

In the baseline calibration, I set the consumption, χ, and production, δ, oil elasticities of substitution at 0.3, a low number that corresponds to the average of estimates found in the empirical literature. Following BEG08, ω_{oc} is set such that the energy component of consumption (gasoline and fuel plus gas and electricity) equals 6 percent, which is in line with US NIPA data, and ω_{oy} is chosen such that the share of energy in production is 2 percent.

Prices are assumed to have a duration of four quarters, so that $\theta = 0.75$. The core goods elasticity of substitution parameters ε is set to 6, which implies a 20 percent markup of (core) prices over marginal costs.

Finally, the logarithm of the real price of oil in terms of the consumption goods bundle $po_t = \log(P_{o,t})$ is supposed to follow a persistent AR(1) process ($\rho_o = 0.95$).

3 Divine coincidence?

Because of monopolistic competition in intermediate goods markets, the economy's steady state is distorted. Production and employment are suboptimally low. Fully acknowledging this feature of the economy instead of subsidizing it away for convenience, as is often done, entails important consequences for optimal policy.

In particular, the divine coincidence breaks down when Cobb-Douglas production is replaced by CES. Cobb-Douglas production functions greatly simplify the analysis and permit nice closed-form solutions, but because they assume a unitary elasticity of substitution between factors they feature constant cost shares over the cycle *regardless of the degree of market distortion.* In a nutshell, when oil is considered a gross complement to labor in production, the oil price elasticity of real marginal costs is increasing in the oil cost share, which depends on the economy overall distortion. The less competitive the economy, the larger is the steady-state share of oil, and the more sensitive are real marginal costs to increases in oil prices. Because perfect price stability is the result of constant real marginal costs, the more distorted the economy, the bigger is the real wage (and then labor demand and output) drop required to compensate for higher oil prices. As in BG07, the drop in natural output is not efficient, which introduces a cost push shock in the NK Phillips curve (as shown in Section 4) and a trade-off for monetary policy.

Moreover, this section shows that perfectly stabilizing inflation becomes particularly costly when the impact of higher oil prices on households' overall consumption is *also* taken into account. As stated in the introduction, increases in oil prices act as a tax on labor income; the lower the elasticity of substitution, the larger the tax effect which compounds with the production effect on marginal costs and amplifies the trade-off faced by monetary authorities.

3.1 Flexible price and wage equilibrium (FPWE)

Before analyzing, in the next sub-section, how the wedge between efficient and natural output reacts to an oil price shock, I first describe the properties of the system at the FPWE in the log-linearized economy (see Appendix IV for details). Note that lowercase letters denote the percent deviation of each variable with respect to its steady state (e.g., $c_t \equiv \log\left(\frac{C_t}{C}\right)$).

Solving the system for $mc_t = 0$ (because $MC_t = MC$ in the FPWE) and assuming

$\sigma = 1$ and $\eta = 0$ for simplicity, I get

$$h_t = -\left[\frac{\widetilde{\omega_{oy}}\,(1-\delta)}{\Lambda} + \Theta\right] po_t, \tag{21}$$

$$y_t = -\left[\frac{\widetilde{\omega_{oy}}\,(1+\delta\phi)}{\Lambda} + \Theta\right] po_t, \tag{22}$$

and

$$w_t = -\frac{\widetilde{\omega_{oy}}\,(1-\widetilde{\omega_{oc}}) + \widetilde{\omega_{oc}}}{(1-\widetilde{\omega_{oc}})\,(1-\widetilde{\omega_{oy}})} po_t \tag{23}$$

for $\Theta = \frac{\widetilde{\omega_{oc}}\left[1-\chi sy^{-1}(1-\widetilde{\omega_{oc}})\right]}{(1-\widetilde{\omega_{oc}})(\phi+1)}$, $\Lambda = (1-\widetilde{\omega_{oy}})\,(1-\widetilde{\omega_{oc}})\,(\phi+1)$, $0 < MC \equiv \frac{\varepsilon-1}{\varepsilon} < 1$, and where $\widetilde{\omega_{oy}} \equiv \omega_{oy}^{\delta}\left(\frac{P_o}{MC\cdot P_y}\right)^{1-\delta}$ is the share of oil in the real marginal cost, $\widetilde{\omega_{oc}} \equiv \omega_{oc}^{\chi} P_o^{1-\chi}$ is the share of oil in the CPI, and $sy \equiv (1-\omega_{oc})\left(\frac{Y}{C}\right)^{\frac{\chi-1}{\chi}}$ is the share of the core good in the consumption goods basket.

Looking at equations (21), (22) and (23), we first see that the response of employment, output and the real wage will depend on $\widetilde{\omega_{oy}}$, the oil price elasticity of real marginal costs (see equation (A43) in Appendix IV), which is itself a function of the elasticity of substitution, δ, and the steady-state markup, $\frac{1}{MC}$. The lower δ and MC, the larger are $\widetilde{\omega_{oy}}$ and the effect of changes in oil prices on real wages, employment and output.

Second, equation (21) shows that when $\delta = \chi = 1$, which occurs when the production functions for intermediate and final goods are Cobb-Douglas, substitution and income effects compensate one another on the labor market and employment is constant after an oil price shock ($\Theta = h_t = 0$).

Third, assuming imperfect substitution between oil and other consumption goods *amplifies* the responses of both employment and output to the shock as Θ is decreasing, and Λ is increasing in the elasticity of substitution between oil and other consumption goods, χ. As stated in the introduction, increases in oil prices act as a tax on labor income when $\chi < 1$; the lower the elasticity of substitution, the larger the tax effect which compounds with the effect on marginal costs. Note that the tax effect tends to zero when the elasticity of substitution $\chi \to \infty$ as in this case, $\widetilde{\omega_{oc}} \to 0$, and the solution of the model collapses to the one when oil is an input to production only.

3.2 Analyzing the cyclical wedge between efficient and natural output[12]

To analyze the cyclical wedge between the natural and efficient levels of output after an oil price shock, it suffices to compare the log-linearized, flex-price output responses in the distorted (natural), y_t^N, and undistorted (efficient) y_t^* economies .

Starting from equation (22), the cyclical wedge can be written as

$$y_t^N - y_t^* = -(1+\delta\phi)\left(\widetilde{\omega_{oy}}^N/\Lambda^N - \widetilde{\omega_{oy}}^*/\Lambda^*\right) po_t, \tag{24}$$

where I assume $MC = 1$ in $\widetilde{\omega_{oy}}^*$ and Λ^* .

The first thing to notice is that the wedge is constant ($y_t^N - y_t^* = 0$) and the divine coincidence holds when a fiscal transfer is available to offset the steady state distortion. In this case, $\widetilde{\omega_{oy}}^N = \widetilde{\omega_{oy}}^*$ and $\Lambda^N = \Lambda^*$. There is no cost-push shock and then no policy trade-off.

Second, when production functions are Cobb-Douglas ($\delta = \chi = 1$), $\widetilde{\omega_{oy}}^* = \widetilde{\omega_{oy}}^N = \omega_{oy}$ and $\widetilde{\omega_{oc}} = \omega_{oc}$ so that $\widetilde{\omega_{oy}}^N/\Lambda^N - \widetilde{\omega_{oy}}^*/\Lambda^* = 0$ and there is again no-trade-off. Cobb-Douglas production implies constant cost shares regardless of the degree of steady-state distortion. Therefore, the flex-price reaction of output, which implies constant real marginal costs, will be independent of the level of overall distortion and $y_t^N = y_t^*$.

Third, allowing for low substitutability in a world without fiscal transfer, y_t^N will drop more than y_t^* after an oil shock as $\widetilde{\omega_{oy}}^N/\Lambda^N > \widetilde{\omega_{oy}}^*/\Lambda^*$ because $MC^N < MC^* = 1$. Clearly, the lower δ and the larger the steady-state distortion (the lower MC^N), the larger is the cyclical wedge between y_t^N and y_t^*. Perfectly stabilizing prices (by aiming at y_t^N in each and every period) requires relatively large drops in real wages that arise out of large drops in labor demand and output in equilibrium.

Looking more closely at equation (24), one notices that the elasticity of substitution between energy and other consumption goods, χ, plays an important role in amplifying the effect of oil prices on the gap between y_t^N and y_t^*: the lower χ, the lower is P_y and the larger are $\widetilde{\omega_{oc}}$ and the amplification effect.

This result can be quite easily understood. Accounting for the direct effect of an increase in oil prices on headline inflation creates a discrepancy between real oil

[12] In the NK model, the social planner's *efficient* allocation is the same as the one in the decentralized economy when prices and wages are flexible and there is no steady-state distortion ($MC = 1$). The *natural* allocation, on the other hand, corresponds to the flex-price and wage equilibrium in a distorted economy ($MC < 1$).

prices faced by consumers, po_t, and (higher) real oil prices faced by firms $(po_t - py_t)$.[13] Moreover, this distortion is compounded by the fact that the real wage pocketed by households (the consumption real wage w_t) is lower than the real wage faced by firms (the production real wage $w_t - py_t$). The lower the elasticity of substitution between energy and other consumption goods, χ, the larger is the effect of a given increase in oil prices on py_t and the larger is the required drop in real wages w_t (and output) to stabilize real marginal costs.

Figure 1 shows the instantaneous response of the gap between natural (YN) and efficient (Y*) output (as defined in equation (24)) to a (one period) 1-percent increase in the real price of oil as a function of δ, the production elasticity of substitution, and for different values of the consumption elasticity of substitution, χ.[14] The gap is exponentially decreasing in both the elasticities δ and χ. Looking at the northeastern extreme of the figure, where both elasticities are equal to one (the Cobb-Douglas case), we see that the reaction of natural and efficient outputs are the same, the gap is zero. Stabilizing inflation or output at its natural level is welfare maximizing. Lowering the production elasticity only (along the curve CHI=1) gives rise to a monetary policy trade-off[15]. Yet, the wedge becomes really large when both the consumption and production elasticity are compounded (like on the curve labeled CHI=0.3).

< Figure 1 >

Figure 2 performs a similar exercise, but varies the degree of net steady-state markups $(\frac{1}{MC} - 1)$ for different values of the elasticities δ and χ. Again, the wedge between efficient and natural output swells for large distortions and low elasticities.

< Figure 2 >

4 Optimal monetary policy

What weight should the central bank attribute to inflation over output gap stabilization? Rotemberg and Woodford (1997) and Benigno and Woodford (2005) have shown

[13] Because immediatly after an increase in oil prices, the ratio core to headline prices deteriorates $(py_t < 0)$.

[14] Note that the amplitude of the gap also depends on the Frish-elasticity of labor supply as measured by $\frac{1}{\phi}$. The smaller ϕ (the larger the elasticity), the larger are the labor demand and output drops needed to stabilize the real marginal cost, and the larger is the cyclical gap between efficient and natural output.

[15] Recal that the cyclical gap between effcient and natural output drives the cost-push shock in the New Keynesian Phillips Curve (NKPC, see section 4), and as such governs the trade-off faced by monetary policy.

that the central bank's loss function could be derived from the households utility function, thereby setting a natural criterion to answer this question. Indeed, it is possible to reformulate the central bank's optimal strategy of maximizing household utility into the equivalent problem (to a second order of approximation) of minimizing a quadratic loss function defined as a weighted sum of inflation and the welfare relevant output gap. Therefore, the importance of inflation stabilization over output gap stabilization depends explicitly on preferences and technology parameters (see Appendix I for details).

Following Woodford (2003), I define optimal policy as the optimal precommitment policy in a timeless perspective. In a nutshell, monetary authorities try to minimize the objective $\Upsilon \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \lambda x_t^2 + \pi_{y,t}^2 \right\}$ subject to the sequence of constraints $\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y x_t + \mu_t$ (and a constraint on the initial inflation rate), where $\pi_{y,t}$ is the core inflation rate, x_t is the welfare relevant output gap $x_t = y_t - y_t^*$ and $\mu_t = k_y \left(y_t^* - y_t^N \right)$ is the cost push shock that depends on the gap between efficient and natural output (see Appendix II for details).

In Section 4.1, I show that the parameters governing the nominal and real rigidities in the model interact with the elasticities of substitution (that we assume smaller than one) and have important consequences on the choice of policy. For reasonable parameters, however, the weight on inflation stabilization remains larger than the one on the output gap, a result also obtained by Woodford (2003) in a more constrained environment. Section 4.2 contrasts the dynamic transmission of oil price shocks under strict inflation targeting and under optimal policy[16], and shows the importance for optimal policy of assuming Cobb-Douglas technology.

4.1 Lambda

Figure 3 describes the variation of λ, the relative weight assigned to output gap stabilization as a function of the elasticity of substitution δ and the degree of price stickiness θ. Stickier prices (larger θ) result in larger price dispersion and therefore larger inflation costs. In this case, monetary authorities will be less inclined to stabilize output and, for given elasticities of substitution, λ decreases when θ becomes larger. But λ

[16] Note that when solving for the optimal precommitment policy, I implicitly assume that the central bank can choose the levels of output gap and inflation that would maximize households welfare. Of course, in practice, central banks do not set any of these variables directly but adjust their policy instrument (typically the short-term interest rate) until the required optimal relation between the welfare relevant variables is attained given the IS and NKPC constraints.

also depends crucially on the elasticities of substitution. The lower the elasticities, the larger is the cost-push shock, and the flatter is the New Keynesian Phillips curve (NKPC), which implies a relatively large sacrifice ratio.[17] With a large cost-push shock and a large sacrifice ratio, the central bank will be more concerned with the distortionary cost of inflation and λ will be smaller. Assuming perfectly flexible real wages ($\eta = 0$), our baseline calibration ($\theta = 0.75$, $\delta = 0.3$) leads to $\lambda = 0.028$, which implies a targeting rule that places a larger weight on inflation stabilization than on the output gap (in annual inflation terms, the ratio output gap to inflation stabilization is $\sqrt{0.022} \times 4 = 0.59$).[18] Note that the focus of policy is very sensitive to the degree of price stickiness. Setting $\theta = 0.5$ results in $\lambda = 0.138$ and a policy that sets a larger weight on output gap stabilization ($\sqrt{0.138} \times 4 = 1.48$).

< Figure 3 >

BG07 argue that the optimal policy choice depends crucially on the degree of real wage stickiness. Figure 4 verifies this claim by letting the degree of real wage stickiness vary between $\eta = 0$ and $\eta = 0.9$. The larger the real wage stickiness, the larger is the cost-push shock, and the larger is the sacrifice ratio as a relatively larger drop in labor demand and output is necessary to engineer the required drop in real wages that stabilizes the real marginal cost and inflation. The central bank will tend to be more concerned with inflation stabilization and λ will be smaller when η is high. Assuming $\eta = 0.9$, Figure 4 shows that, for our baseline calibration, $\lambda = 0.002$ ($\sqrt{0.002} \times 4 = 0.18$).

< Figure 4 >

4.2 Analyzing the trade-off

Figures 5 and 6 illustrate the transmission of a persistent oil price shock to the economy under different assumptions on elasticities by comparing the natural, FPWE allocation that implies strict inflation targeting, to the one implied by the optimal precommitment policy. Figure 5 assumes Cobb-Douglas technologies and shows that in this case the optimal policy replicates the FPWE allocation perfectly; the dashed and solid lines are on top of each other. The divine coincidence holds as policy can perfectly stabilize both the welfare relevant output gap and core inflation.

Figure 6 is based on the baseline calibration and contrasts strict inflation targeting

[17] When the NKPC is flat, a large change in output is required to affect inflation.

[18] The traditional Taylor rule that places equal weights on output stabilization and inflation stabilization would imply $\lambda = 1/16$ with quarterly inflation.

with optimal policy. Assuming low substitution ($\delta = \chi = 0.3$), a policy trade-off appears. The responses under the optimal policy differ quite substantially from those under strict inflation targeting. While the latter implies an increase in real interest rates (which corresponds to the expected growth of future consumption), the optimal policy recommends a temporary drop for one year following the shock. Consequently, the drop in output on impact is more than three times larger in the FPWE allocation, which is the price for stabilizing core inflation perfectly.

< Figure 5 >

< Figure 6 >

Finally, Figure 7 shows how acute the policy trade-off is by displaying the differences in both the welfare relevant output gaps and inflation reactions to a 1-percent increase in the price of oil under optimal policy and strict inflation targeting. The "oil-in-production-only" case (dotted line) is compared with the case where energy is an input to both consumption and production (solid line). In both cases, optimal policy lets inflation increase and the welfare relevant output gap decrease. But the difference with strict inflation targeting is three times as large when oil is both an input to production and consumption, as could be inferred from Section 3.

< Figure 7 >

5 Simple rules

Optimal monetary policy plans may not be very transparent, nor easy to communicate, as they rely on the real-time calculation of the welfare relevant output gap, an abstract, non-observable theoretical construct. Therefore, accountability-related issues could be raised, which may cast doubt on the overall credibility of the assumption of precommitment that underlies the analysis.

As an alternative, some authors (McCallum, 1999, Söderlind, 1999, and Dennis, 2004) have advocated the use of simple optimal interest rate rules. Those rules should approximate the allocation under the optimal plan but should not rely on an over-stretched information set. In what follows, I first derive such a rule analytically and show that it is based on core inflation and on current and lagged deviations of output and the real price of oil from the steady state.

As the mere notion of steady state can also be subject to uncertainty in real-time policy exercises, I then show that the optimal simple rule can be approximated by a 'speed limit'-type interest rate rule (see Walsh, 2003 and Orphanides and Williams, 2003) that relies only on the rate of change of the variables, i.e., on current core inflation, oil price inflation, and the growth rate of output, and that this rule remains close to optimal even when real wages are sticky.

5.1 The optimal precommitment simple rule

Using the minimal state variable (MSV) approach pioneered by McCallum (1999b), one can conjecture the (no bubble) solution to the dynamic system relating the optimal allocation under the timeless perspective optimal plan, equation (A23), and the private-sector relation between the welfare relevant output gap and (core) inflation, equation (A27) (see Appendix II) and get:

$$\pi_{y,t} = \alpha_{11} x_{t-1} + \alpha_{12} \mu_t, \tag{25}$$

$$x_t = \alpha_{21} x_{t-1} + \alpha_{22} \mu_t, \tag{26}$$

where α_{ij} for $i, j = 1, 2$ are functions of β, k_y, and λ.

Combining (25) and (26) with the Euler equation (4), I then solve for r_t, the nominal interest rate, and derive the optimal simple rule consistent with the optimal plan (see Appendix III):

$$r_t = \Phi \alpha_{11}^{-1} \pi_{y,t} + \Omega y_t - \Gamma y_{t-1} + (\Xi + \Psi\Omega)\, po_t - \Psi\Gamma po_{t-1}, \tag{27}$$

for $\Phi \equiv \rho_o - \sigma\alpha_{22}\alpha_{12}^{-1}(1-\rho_o)$, $\Omega \equiv \alpha_{11} + \sigma\alpha_{21}$, $\Gamma \equiv \Phi + \sigma\alpha_{21}$ and $\Xi \equiv (\rho_o - 1)\left(\frac{\widetilde{\omega_{oc}}}{1-\widetilde{\omega_{oc}}} - \Psi\sigma\right)$.

The optimal interest rate rule is a function of core inflation, current and lagged output, and current and lagged real oil price, all taken as log deviations from their respective steady states. Its parameters are functions of households preferences, technology, and nominal frictions.

For a permanent shock, $\rho_o = 1$, the rule simplifies to

$$r_t = \alpha_{11}^{-1} \pi_{y,t} + \Omega\left(y_t - \Gamma\Omega^{-1} y_{t-1}\right) + \Psi\Omega\left(po_t - \Gamma\Omega^{-1} po_{t-1}\right),$$

as $\Phi = 1$, and $\Xi = 0$. Looking at Γ and Ω shows that the closer α_{11} is to 1, the more precisely a speed limit policy (a rule based on the rate of growth of the variables) replicates the optimal policy.

In the next section I show that for $\rho_o = 0.95$, a degree of persistence which corresponds closely to the 1979 oil shock (see Section 6), the speed limit policy still approximates almost perfectly the optimal feedback rule despite a value of α_{11} clearly below 1.

5.2 Optimized simple rules

The analytical solution to the kind of problem described in Section 5.1 rapidly becomes intractable once one considers a vector of shocks or once the number of lagged state variables is increased (e.g., by allowing for the possibility of real wage rigidity). An alternative is to resort to a numerical approach that would search within a predetermined space of simple interest rate rules for the one that minimizes the central bank loss function, and to compare the loss under the optimal plan and the proposed simple rule (Söderlind, 1999 and Dennis, 2004).

Because different combinations of output gaps and inflation variability could, in principle, produce the same welfare loss, I follow a different strategy here. My goal is to find a simple rule that mimics the optimal plan along *all* relevant dimensions, and where the success criterion is the rule's ability to produce the same real allocation after an oil shock. I then rely on a distance minimization algorithm defined over the n impulse response functions of m variables of interest to the policymakers. More specifically, the algorithm searches the space of (monetary policy) parameters for the interest rate rule that minimizes the distance criterion

$$\arg\min_{\vartheta} \left(IRF_{SR}(\vartheta) - IRF_O\right)' \left(IRF_{SR}(\vartheta) - IRF_O\right),$$

where $IRF_{SR}(\vartheta)$ is an $mn \times 1$ vector of impulses under the postulated simple interest rate rule, and IRF_O is its counterpart under the optimal plan. The algorithm matches the responses of eight variables (output, consumption, hours, headline inflation, core inflation, real marginal costs, and nominal and real interest rates) over a 20-quarter period using constrained versions of the following general specification of the simple interest rate rule

$$r_t = g_\pi \pi_{y,t} + g_y y_t + g_{y1} y_{t-1} + g_{po} po_t + g_{po_1} po_{t-1} + g_{w1} w_{t-1}, \quad (28)$$

where $\vartheta = (g_\pi, g_y, g_{y1}, g_{po}, g_{po_1}, g_{w1})'$.

I start with a version of the model that assumes perfect real wage flexibility and run the minimum distance algorithm on an unconstrained version of equation (28) and on a speed limit version where $g_y + g_{y1} = 0$, $g_{po} + g_{po_1} = 0$, $g_{w1} = 0$ and $g_y, g_{po} \geq 0$. Figure 8 shows the response to a 1 percent shock to oil prices under the optimal precommitment policy (solid line), the optimized simple rule (OR, dotted line) and the speed limit rule (SLR, dashed line). The responses under the OR stand exactly on top of the ones under the optimal policy, which is not surprising given that an analytical solution to the problem can be derived (see previous sub-section). More remarkable, however, is how well the speed limit rule (dashed line) is able to match the optimal precommitment policy (solid line). For most variables they are almost indistinguishable.

The coefficients of the different rules are reported in Table 1. They are quite large compared to the coefficients typically found for Taylor-type interest rate rules, but they are not unusual when compared to the literature on optimal simple rules (McCallum and Nelson, 2005). Both the OR and the SLR would react strongly to demand shocks that push inflation and the output gap in the same direction, but they imply quite a balanced response to cost-push shocks.

< Figure 8 >

How robust are these findings to the assumption of real wage stickiness? I assume $\eta = 0.9$ and run the minimal distance algorithm again. Figure 9 shows that, again, the OR (dotted line) is almost on top of the optimal plan benchmark (solid line). The speed-limit policy that sets $g_y + g_{y1} = 0$, $g_{po} + g_{po_1} = 0$, $g_{w1} = 0$, and $g_y, g_{po} \geq 0$ seems to be a good approximation to the optimal precommitment simple rule in this case too.

< Figure 9 >

The estimated parameters (see Table 1, OR_W) show that the monetary authorities react strongly to both inflation and the output gap (defined as deviation from steady state), but also to changes in oil prices. This means that the rules tends toward perfect prices stability in the case of a demand shock, but acknowledge the trade-off in the case of an oil price shock.

< Table 1 >

Following a suboptimal rule can be costly in periods of large oil shocks. In the next section I revisit the 1979 oil shock to quantify these costs using a welfare criterion.

6 1979 oil price shock and US monetary policy

All US recessions since the end of World War II — and the latest vintage is no exception — have been preceded by a sharp increase in oil prices and an increase in interest rates.[19] But are US recessions really caused by oil shocks, or should the monetary policy responses to the shocks be blamed for this outcome? Empirical evidence seems to suggest a role for monetary policy (Bernanke et al. 2004), but its importance remains difficult to assess.

One major stumbling block is the role of expectations. To evaluate the effect of different monetary policies in the event of an oil price shock one has to take into account the effect of those policies on the agents' expectations, which is typically not feasible using reduced-form time series models whose estimated parameters are not invariant to policy (see Lucas 1976, and Bernanke et al. 2004 for a discussion in the context of an oil shock).

An alternative approach is to rely on a structural, microfounded model to analyze the implications of different monetary policies for output, inflation, and welfare in a precisely defined experiment. Since increases in oil prices are particularly challenging for central banks when they are both large and persistent, I have chosen to look at the 1979 oil shock resulting from the Iranian revolution.[20] In one year, oil prices increased by 126 percent in real terms and did not return to their preshock level before 1986. During this period, the three-month Treasury bill rate rose from 6.7 percent in June 1978 to 15.3 percent in March 1980, and economic activity slumped with a trough of −7.8 percent quarter-on-quarter real GDP growth in the second quarter of 1980.[21]

Could another monetary policy have improved this dismal outcome? One way to answer this question is simply to compare the courses of real activity and inflation under different monetary policy rules for an oil price shock similar to the 1979 episode.

6.1 Dynamic analysis under different policies

Figure 10 below shows that an AR(1) process $po_t = \rho_o po_{t-1} + \varepsilon_{o,t}$ for $\rho_o = 0.95$ and a shock $\varepsilon_{o,t}$ that leads to a 100 percent log-increase on impact in the real price of oil is very close to the 1979-1986 historical pattern.

< Figure 10 >

The model baseline calibration assumes that US monetary policy can be described by a traditional Taylor rule based on headline inflation (HTR henceforth) with coefficients $g_\pi = 1.5$ on headline inflation and $g_y = 0.5$ on the deviation of output with

[19] See Hamilton (2009) for a recent analysis.

[20] This oil shock was clearly exogenous to economic activity and as such corresponds perfectly to the model definition of an oil price shock.

[21] This is based on chained 2000 dollars (Bureau of Economic Analysis).

respect to steady-state output as in Taylor (1993).[22] Admittedly, the HTR is only a rough approximation of the actual Federal Reserve behavior, but it seems sufficiently accurate to describe how US monetary policy has been conducted on average over the last three decades, and in particular during the oil shock of 1979 (see Orphanides 2000).

Before turning to counterfactual experiments with alternative monetary policies, it is important to check the empirical properties of the model against the available empirical evidence. The following simulations are made under the baseline calibration where I also follow BG07 in setting the parameter governing the degree of real wage stickiness η, to 0.9. Figure 11 shows the response of the model economy to a 1979-like 100 percent log increase in oil prices, where the dashed line represents the response under the baseline HTR calibration. In this case, the shock leads to a -8 percent drop in output on impact, a 1000 basis point tightening of the three-month nominal interest rate and a 7.4 percent pickup in headline inflation implying a cumulated 9 percent increase in the price level after two years.

Although the responses might seem somewhat excessive, they are in the same ballpark as the existing VAR evidence on the transmission of oil shocks to the economy.[23] To be sure, the lag structure of the structural model is too simple to reproduce the VARs' hump-shaped responses. But its quantitative predictions under the baseline HTR calibration are close enough to their empirical counterparts to serve as meaningful benchmark when discussing alternative monetary policies.

Figure 11 also compares the responses under optimal policy (OR henceforth, solid line) and the traditional Taylor rule (HTR, dashed line). The top two panels show the responses of the welfare relevant output gap and core inflation, the two determinants of the central bank's loss function. Under optimal policy, the central bank credibly commits to a state-contingent path for future interest rates that involves holding real interest rates slightly above what would be justified by output gap and inflation consid-

[22] Our benchmark Taylor rule is based, as in Taylor (1993), on annual inflation measured as the rate of price changes between $t-3$ and t (for quarterly observations), and the ouput gap defined as the deviation of output from its long-run trend.

[23] Following a 100 percent increase in oil prices, the maximal effect on real GDP is supposed to lie somewhere between -4 and -11 percent, depending on the identification and the measurement of the oil price shock. For example, Bernanke et al. (2004) find a maximal drop of US real GDP of -7 percent four quarters after the shock. As for prices, the uncertainty is comparable, with estimates ranging from a cumulated increase of 1 percent (in Bernanke et al. 2004) to 8 percent (Blanchard and Galí 2007) two years after the shock. There are also discrepancies between the different estimations of the monetary policy reactions. Carlstrom and Fuerst (2005) find a maximal increase of 500 basis points three quarters after the shock, while Bernanke et al. (2004) find an increase of 1500 basis points.

erations in the next five years. In so doing, it is able to dampen inflation expectations without having to resort to large movements in real interest rates, and therefore it reaches a much better outcome in terms of both inflation and the output gap in the short to medium run. At the peak, output falls twice as much and core inflation is five times larger under HTR than under OR. Because inflation never really takes off under OR, nominal interest rates remain practically constant over the whole period. This first result suggests that, if monetary policy had been conducted according to OR during the oil shock of 1979, the recession would not have been averted but it would have been much milder with almost no increase of core inflation beyond steady-state inflation.

Many observers, including the US Federal Reserve, have emphasized core inflation as a guide to monetary operational decisions.[24] In Figure 11, I represent this alternative by a Taylor rule based on core inflation (CTR henceforth, dotted line). Looking at the consequences for output and the output gap only, it seems that such a rule is preferable to the HTR and is indeed also preferable to the OR, for it would imply less contraction in real activity over the whole five-year period. In the context of the 1979 oil shock, a CTR would have led to a 2 percent drop in output on impact (instead of 4 percent with the OR and 8 percent with HTR). Still, as the upper right panel clearly shows, the inflation cost of this policy would have been large. Core inflation would have risen almost as much as under the HTR for about a year after the shock and would have remained elevated throughout. Assuming some degree of inertia ($\rho = 0.8$) brings HTR closer to CTR, but the general pattern of responses, if smoother, remains largely unchanged. HTR continues to imply too much variation in both output and inflation, and CTR leads to more inflation compared to OR, which is the price for a more accommodating monetary policy.

$<$ Figure 11 $>$

Some authors (Bernanke et al. 1999) have argued that monetary policy should be framed with respect to a *forecast* of inflation rather than *realized* inflation on the grounds that the former approach is better able to deal with supply-side shocks implying a temporary trade-off between stabilizing inflation and the output gap. And, indeed, many inflation-targeting central banks communicate their policy by referring to an explicit goal for their forecast of inflation to revert to some target within a spec-

[24] Although they usually acknowledge that it may be sensible to express longer-run objectives in terms of headline inflation.

ified period. Like BEG08, I define a forecast-based rule as a Taylor-type rule where realized inflation has been replaced by a one-quarter-ahead forecast of core or headline inflation; the parameters remain the same with $g_\pi = 1.5$ and $g_y = 0.5$.

Figure 12 illustrates the implications of forecast-based rules and compares them with the optimal rule in the context of the 1979 oil price shock. In the long term, forecast-based rules fulfill their goal of stabilizing both headline and core inflation. In the shorter run, however, they appear to be much more accommodative than the OR. As the oil price shock is temporary, oil price inflation will be negative next period, which pushes down headline inflation due to the direct effect of energy costs on the CPI. A Taylor rule based on a forecast of headline inflation would have (almost) completely eliminated the output consequence of the 1979 oil shock, as can be seen in the upper left panel, but with dire effects on inflation in the short to medium run, as shown on the upper right panel where core inflation remains at about 3 percent above target for two years after the shock with a peak at 4 percent one year after the shock. Compared to the benchmark HTR policy in Figure 12, this is 50 percent more inflation!

< Figure 12 >

6.2 Welfare costs from suboptimal policies

Having characterized the responses of macroeconomic variables under popular alternative monetary policies, I will devote the rest of this section to quantifying the costs involved when following those suboptimal policies. Table 2 summarizes the main results.[25] Its first column shows the cumulative welfare loss from following alternative policies for 1979 Q1 to 1983 Q4 expressed as a percent of one year steady-state consumption. The second and third columns report the λ-weighted decomposition of the loss arising from volatility in the output gap or in core inflation. The numbers seem to be unusually large. They are about 100 times larger than the ones reported by Lucas (1987). However, it must be kept in mind that our calculation refers to the cumulative welfare loss associated with *one* particularly painful episode, and not the *average* cost from garden variety oil price shocks where the cost of severe oil price increases would be diluted in long periods of very low volatility. Indeed, Galí et al. (2007) reckon

[25] Note that our calibration implies a relatively low λ, which means that policymakers attribute a lot of importance to minimizing the distortions associated with high inflation. Thus, despite the policy trade-off that emerges following an oil price shock, a policy that stabilizes inflation will tend to be favored over a policy that stabilizes the welfare relevant output gap.

that the welfare costs of recessions can be quite large.[26] Their typical estimate for the cumulative cost of a 1980-type recession is in the range of 2 to 8 percent of one year steady-state consumption, depending on the elasticities of labor supply and of intertemporal substitution.[27]

< Table 2 >

The message arising from the welfare calculations is in line with the dynamic analysis as shown by the IRFs. Table 2 shows that because of their inflationary consequences, forecasting rules are particularly costly. For example, despite a very good performance in terms of the output gap, forecast-based HTR has the worst result among the rules considered because of higher core inflation. Taylor rules based on contemporaneous headline inflation are also quite costly if there is no inertia in interest rate decisions.

The results suggest that, having followed a policy closer to the benchmark Taylor rule (HTR) during the 1979 oil shock instead of the optimal policy may have cost the equivalent of 2.1 percent of one year steady-state consumption to the representative household. The overall cost would have been 40 percent smaller if monetary policy had been based on an inertial interest rate rule such as CTR or HTR with $\rho = 0.8$.

As mentioned above, our utility-based welfare metric tends to weigh heavily inflation deviations as a source of welfare costs. Assuming $\theta = 0.75$ and $\eta = 0.9$ amounts to setting λ to 0.02, which means that the central bank attributes about twice as much importance to inflation stabilization as to output gap stabilization when inflation is expressed in annual terms.[28] This notwithstanding, the results suggest that welfare losses under the perfect price stability policy are three times as large as under optimal policy and amount to 1.8 percent of one year steady-state consumption due to disproportionately large fluctuations in output.

[26] Once one recognizes the distorted nature of the steady state, first-order welfare costs due to business cycle fluctuations must be taken into account. When studying a particular recessionary episode, these first-order welfare costs are not averaged out and can be quite large.

[27] They also aknowledge that their estimates are probably a lower bound as they ignore the costs of fluctuations in price and wage inflation resulting from nominal rigidities.

[28] This is not an unusual result. New Keynesian models typically attribute a much larger cost to inefficiencies in the *composition* of output due to relative price distortions when prices are sticky, than to inefficiencies in the *level* of output. As allocative distortions get larger when inflation rises, monetary authorities tend to assign a large weight to stabilizing inflation. As a basis of comparison, Rotemberg and Woodford (1997) compute a value of λ equivalent to 0.003 when translated into quarterly units.

7 Time-varying elasticities of substitution

It is a well-know empirical fact that the demand for energy is almost unrelated to changes in its relative price in the short run. In the long run however, persistent changes in prices have a significant bearing on the demand for energy. Pindyck and Rotemberg (1983), for example, report a cross-section price elasticity of oil demand close to one.

How are the result of the precedent sections affected by the possibility of time-varying elasticities of substitution. Is the short run monetary trade-off after an oil price shock the mere reflection of some CES-related specificity, or is it a more general argument related to low short-term substitutability in a distorted economy ?

To allow for time-varying elasticities of substitution, I transform the production processes of Section 2 by introducing a convex adjustment cost of changing the input mix in production. More specifically, I follow Bodenstein et al. (2007) and redefine equations (12) and (3) as

$$Y_t = \left((1-\omega_{oy}) H_t^{\frac{\delta-1}{\delta}} + \omega_{oy} \left[\varphi_{OY,t} O_{Y,t} \right]^{\frac{\delta-1}{\delta}} \right)^{\frac{\delta}{\delta-1}}, \tag{29}$$

and

$$C_t = \left((1-\omega_{oc}) C_{Y,t}^{\frac{\chi-1}{\chi}} + \omega_{oc} \left[\varphi_{OC,t} O_{C,t} \right]^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}. \tag{30}$$

The variables $\varphi_{OY,t}$ and $\varphi_{OC,t}$ represent the costs of changing the oil intensity in the production of the core good and the consumption basket, and are supposed to take the following quadratic form

$$\varphi_{OY,t} = \left[1 - \frac{\varphi_{OY}}{2} \left(\frac{O_{Y,t}/H_t}{O_{Y,t-1}/H_{t-1}} - 1 \right)^2 \right] \tag{31}$$

$$\varphi_{OC,t} = \left[1 - \frac{\varphi_{OC}}{2} \left(\frac{O_{C,t}/C_{Y,t}}{O_{C,t-1}/C_{Y,t-1}} - 1 \right)^2 \right]. \tag{32}$$

This specification allows for oil demand to respond quickly to changes in output and consumption, while responding slowly to relative price changes. In the long-run, the elasticity of substitution is determined by the value of δ and χ. Although somewhat ad hoc, this form of adjustment costs introduces a time-varying elasticity of substitution for oil, an important characteristic of putty-clay models such as in Atkeson and Kehoe

(1999) or Gilchrist and Williams (2005).[29] The presence of adjustment costs transforms the static cost-minimization problem of the representative intermediate firms and final consumption goods distributors into forward-looking dynamic ones. They can be regarded as choosing contingency plans for $O_{Y,t}$, H_t, $O_{C,t}$, and $C_{Y,t}$ that minimize their discounted expected cost of producing Y_t and C_t subject to the constraints represented by equations (29) to (32).

I calibrate φ_{OY} and φ_{OC} such that the instantaneous price elasticities of demand for oil correspond to the baseline calibration chosen in the CES setting of the previous sections Namely, the short-term elasticities of substitution are set to 0.3. In the long run, I assume a unitary elasticity of substitution ($\delta = \chi = 1$) such that (29) and (30) are *de facto* Cobb-Douglas functions when $t \rightarrow \infty$.

Figure 13 shows impulse responses to a 1 percent shock to the price of oil in the flex-price equilibrium and according to the optimal precommitment policy when there is no steady-state distortion. Because of the adjustment costs — which add two state variables to the problem — the IRFs are not exactly similar to the ones obtained under CES production (see Figure 6) However, the main message remains the same: price stability is the optimal policy in an efficient economy.

Figure 14 performs the same exercise but allows for the same degree of monopolistic competition as in previous sections (leading to a 20 percent net markup of core prices over marginal costs). Again, it shows that allowing for time-varying elasticities of substitution (converging to Cobb-Douglas in this case) does not affect this paper's main finding: in a distorted equilibrium, an oil price shock introduces a significant monetary policy trade-off if the elasticity of substitution is lower than 1 in the short run.

$<$ Figure 13 $>$

$<$ Figure 14 $>$

8 Conclusion

Most inflation targeting central banks understand their mandate to be ensuring long-term price stability. Following an oil price shock, however, none of them would be ready to expose the economy to the type of output and employment drops recommended in

[29] In putty-clay models of energy use, a large variety of types of capital goods are combined with energy in different fixed proportions, making the short-term elasticity of substitution low. In the longer run, the elasticity goes up as firms invest in capital goods with different fixed energy intensities.

standard theory to stabilize prices in the short term.

This paper has shown that the contrast between theory and practice can be explained by the type of restrictive assumption to technology and preferences typically made in the New Keynesian literature. In particular, increases in oil prices imply a *meaningful* monetary policy trade-off between stabilizing output and stabilizing inflation once it is acknowledged (*i*) that oil cannot be easily substituted by other factors in the short run, (*ii*) that there is no fiscal transfer available to neutralize the steady-state distortion due to monopolistic competition, and (*iii*) that oil is an input both to production and to consumption (via the impact of the price of crude oil on the price of fuel, heating oil, and electricity). In this case, policies that *perfectly* stabilize inflation entail significant welfare costs, which may explain the reluctance of policymakers to enforce them.

Interestingly, I find that the optimal monetary policy response to a persistent increase in oil price resembles the typical response of inflation targeting central banks. While long-term price stability is ensured by a credible commitment to keep inflation and inflation expectations in check, short-term real rates drop right after the shock to help dampen real output fluctuations. By managing expectations efficiently, central banks can improve on both the flexible price equilibrium solution and the recommendation of simple Taylor rules.

This finding, however, is based on the assumptions that monetary policy is perfectly credible and transparent and that agents and the central banks have the right (and the same) model of the economy. Further work should explore how robust these policy conclusions are to the incorporation of imperfect information and learning in the analysis.

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9 Appendix I : Deriving a quadratic loss function

The policy problem originally defined as maximizing households utility can be rewritten in terms of a quadratic loss function defined over the welfare relevant output gap $y_t - y_t^*$ and core inflation $\pi_{y,t}$. This appendix describes the steps involved following Benigno and Woodford (2005) and Montoro (2007).

Second-order approximation of the model supply side

Starting with the labor market, I can rewrite the labor demand

$$H_t = \left(\frac{W_t}{MC_t p_{y,t}}\right)^{-\delta} (1-\omega_{oy})^{\delta} \frac{Y_t}{P_t^*},$$

and the labor supply

$$C_t^{\sigma(1-\eta)} \nu H_t^{\phi(1-\eta)} = \frac{W_t}{P_t}\left(\frac{W_{t-1}}{P_{t-1}}\right)^{-\eta}$$

in log-deviations from steady state as

$$h_t = y_t - \delta\left(w_t - mc_t - py_t\right) + \Delta_t$$

$$w_t = \eta w_{t-1} + (1-\eta)\left(\phi h_t + \sigma c_t\right)$$

where Δ_t is the log deviation of the price dispersion measure $\frac{1}{P_t^*}$ from its steady state and measures the distortion due to inflation. Note that these two log-linear equations are exact transformations of the nonlinear equations.

Combining the labor demand and supply with a second-order approximation of the real marginal cost

$$mc_t = \left(1-\widetilde{\omega_{oy}}\right)\left[w_t - py_t\right] + \widetilde{\omega_{oy}}\left[po_t - py_t\right] + \frac{1}{2}\widetilde{\omega_{oy}}\left(1-\widetilde{\omega_{oy}}\right)(1-\delta)\left[w_t - po_t\right]^2 + O\left(\|\xi\|^3\right)$$

and a first-order approximation of the demand for consumption (where the demand for energy consumption has been substituted out)

$$c_t = -\chi\frac{\widetilde{\omega_{oc}}}{sy} p_{o,t} + y_t + O\left(\|\xi\|^2\right), \tag{A0}$$

I obtain a second-order accurate equilibrium relation linking total hours to output and the real price of oil

$$h_t = \left(1 - \mathcal{D}\left(\sigma + \nu\right)\right) y_t + \frac{\mathcal{D}}{(1-\eta)} \mathcal{M} p_{o,t} + \frac{\mathcal{W}}{(1-\widetilde{\omega_{oy}})} \Delta_t$$
$$+ \frac{1}{2} \frac{\mathcal{D}}{(1-\eta)} \mathcal{W}^2 \frac{(1-\delta)}{(1-\widetilde{\omega_{oy}})} \left[\mathcal{J} y_t + \mathcal{L} po_t\right]^2 + O\left(\|\xi\|^3\right) + t.i.p \tag{A1}$$

where

$\mathcal{M} \equiv \frac{\left[\widetilde{\omega_{oc}}(1-\eta)\frac{\sigma\chi}{sy} - \frac{\widetilde{\omega_{oc}}}{1-\widetilde{\omega_{oc}}} + \mathcal{B}\right]}{1-\mathcal{W}}$

$\mathcal{B} \equiv \frac{1-\mathcal{W}+\mathcal{W}(1+(1-\eta)\delta\nu)[\widetilde{\omega_{oc}}(1+(1-\eta)\mathcal{A})-(1-\eta)\mathcal{A}]}{1-\widetilde{\omega_{oc}}}$,

A$\equiv \frac{\nu\delta}{(1+(1-\eta)\nu\delta)} \left(\frac{\widetilde{\omega_{oc}}}{1-\widetilde{\omega_{oc}}}\right) + \frac{\chi\sigma}{(1+(1-\eta)\nu\delta)} \left(\frac{\widetilde{\omega_{oc}}}{sy}\right)$,

$\mathcal{W} \equiv \frac{(1-\widetilde{\omega_{oy}})}{1+\widetilde{\omega_{oy}}(1-\eta)\nu\delta}$,

$1 - \mathcal{W} \equiv \frac{\widetilde{\omega_{oy}}(1+(1-\eta)\nu\delta)}{1+\widetilde{\omega_{oy}}(1-\eta)\nu\delta}$,

$\mathcal{J} \equiv (1-\eta)(\sigma+\nu)$,

$\mathcal{L} \equiv \frac{(1-\eta)\nu\delta}{(1+(1-\eta)\delta\nu\mathcal{W})} \mathcal{B} - \left(1 + \widetilde{\omega_{oy}}(1-\eta)\nu\delta\right)\left(1 + (1-\eta)\mathcal{A}\right)$,

$\mathcal{D} = (1-\eta)\,\delta\mathcal{W} \frac{\widetilde{\omega_{oy}}}{(1-\widetilde{\omega_{oy}})}$.

As the price dispersion measure can be written as

$$P_t^* = \left((1-\theta) \left(\frac{K_t}{F_t}\right)^{-\varepsilon} + \frac{\theta \left(\Pi_{Y,t}\right)^{\varepsilon}}{P_{t-1}^*} \right)^{-1},$$

Benigno and Woodford (2004) demonstrate that Δ_t — the log deviation of the price dispersion measure — has a second-order approximation that depends only on second-order inflation terms and lagged dispersion

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta_t = f\left(\Delta_{t_0-1}\right) + \frac{1}{2}\frac{\varepsilon}{k} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\pi_{y,t}^2}{2} + O\left(\|\xi\|^3\right). \tag{A2}$$

Second-order approximation to NKPC

In this section I derive a second-order approximation to the NKPC, which can be used to substitute out the term linear in y_t in the second-order approximation to utility when the steady state is distorted.

I start by writing a second-order approximation to the model inflation/marginal cost nexus. For convenience, I rewrite from the main text

$$K_t = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{Y_t}{C_t^{\sigma}}\right) P_{y,t} MC_t + \beta\theta E_t \left(\Pi_{Y,t+1}\right)^{\varepsilon} K_{t+1}$$

$$F_t = \left(\frac{Y_t}{C_t^{\sigma}}\right) P_{y,t} + \beta\theta E_t \left(\Pi_{Y,t+1}\right)^{\varepsilon-1} F_{t+1}$$

$$\frac{K_t}{F_t} = \left[\frac{1-\theta\left(\Pi_{Y,t}\right)^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}.$$

Taking a second-order approximation of the three preceding equations, I follow Benigno and Woodford (2004) and Castillo et al. (2007) and express the NKPC as

$$V_t = kmc_t + \frac{1}{2}kmc_t\left[2\left(y_t - \sigma c_t + p_{y,t}\right) + mc_t\right] + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t V_{t+1} + O\left(\|\xi\|^3\right) \tag{A3}$$

where I define the auxiliary variable V_t

$$V_t = \Pi_{Y,t} + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\left(\Pi_{Y,t}\right)^2 + \frac{1}{2}\left(1-\theta\beta\right)\Pi_{Y,t}z_t$$

and the linear expansion of z_t

$$z_t = 2\left(y_t - \sigma c_t + p_{y,t}\right) + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\Pi_{Y,t+1} + z_{t+1}\right).$$

Using the first-order[30] approximation of c_t and $p_{y,t}$ and a second-order approximation of mc_t, I write

$$\begin{aligned} mc_t = {} & \mathcal{W}\eta w_{t-1} + (1-\eta)(\sigma+\nu)\mathcal{W}y_t \\ & + (1-\eta)\nu\mathcal{W}\Delta_t + \mathcal{B}po_t \\ & + \frac{1}{2}\frac{(1-\delta)}{(1-\widetilde{\omega_{oy}})}\mathcal{W}^2(1-\mathcal{W})\left[(1-\eta)(\sigma+\nu)y_t + \mathcal{L}p_{o,t}\right]^2 \\ & + O\left(\|\xi\|^3\right) + t.i.p. \end{aligned}$$

which I substitute in (A3) to get

$$\begin{aligned} V_t = {} & k_y y_t + k_p p_{o,t} + k\mathcal{W}\nu\Delta_t \\ & + \frac{1}{2}k\left(c_{yy}y_t^2 + 2c_{yp}y_t p_{o,t} + c_{pp}{p_{o,t}}^2\right) \\ & + \frac{1}{2}\varepsilon\pi_{y,t}^2 + \beta E_t V_{t+1} + O\left(\|\xi\|^3\right) \end{aligned} \tag{A4}$$

[30] A second-order approximation is not necessary here as these two variables enter multiplicatively with mc_t.

for

$k_y \equiv k(1-\eta)(\sigma+\nu)\mathcal{W}$

$k_p \equiv k\mathcal{B}$

$c_{yy} \equiv \digamma(1-\eta)^2(\sigma+\nu)^2 + 2(1-\eta)(\sigma+\nu)(1-\sigma)\mathcal{W} + (1-\eta)^2(\sigma+\nu)^2\mathcal{W}^2$

$c_{yp} \equiv (1-\eta)(\sigma+\nu)\mathcal{W}(\Sigma+\mathcal{B}) - \digamma(\sigma+\nu)(1-\eta)\mathcal{L} + \mathcal{B}(1-\sigma)$

$c_{pp} \equiv \digamma\mathcal{L}^2 + 2\Sigma\mathcal{B} + \mathcal{B}^2$

$\digamma \equiv \frac{1-\delta}{1-\widetilde{\omega_{oy}}}\mathcal{W}^2(1-\mathcal{W})$

$\Sigma \equiv \sigma\chi\frac{\widetilde{\omega_{oc}}}{sy} - \frac{\widetilde{\omega_{oc}}}{1-\widetilde{\omega_{oc}}}$.

Note that the natural level of output can be found from the preceding equation by rewriting it as

$$V_t = k_y\left\{y_t + \overbrace{k_y^{-1}k_p p_{o,t}}^{-Y_t^N} + k_y^{-1}k\mathcal{W}\nu\Delta_t + \frac{1}{2}k_y^{-1}k\left(c_{yy}y_t^2 + 2c_{yp}y_t p_{o,t} + c_{pp}{p_{o,t}}^2\right) + \frac{1}{2}k_y^{-1}\varepsilon\pi_t^2\right\}$$
$$+\beta E_t V_{t+1} + O\left(\|\xi\|^3\right)$$

and ignoring all second-order terms.

Using the law of iterated expectation and (A2), equation (A4) can be rewritten as an infinite discounted sum

$$\begin{aligned}\sum_{t=t_0}^{\infty}\beta^{t-t_0}y_t &= \sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{k_y}\left(V_{t_0} - f\left(\Delta_{t_0-1}\right)\right)\\ &\quad -\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\begin{array}{c} k_y^{-1}k_p p_{o,t} + \frac{1}{2}k_y^{-1}k\left(c_{yy}y_t^2 + 2c_{yp}y_t p_{o,t} + c_{pp}{p_{o,t}}^2\right)\\ +\frac{1}{2}k_y^{-1}\varepsilon\left(1+\nu\mathcal{W}\right)\pi_{y,t}^2\end{array}\right\}\\ &\quad + O\left(\|\xi\|^3\right).\end{aligned} \tag{A5}$$

A second-order approximation to utility

I take a second-order approximation of the representative household utility function in t_o

$$U_{t_o} = E_{t_o}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left\{\mathfrak{u}\left(C_t\right) - \upsilon\left(H_t\right)\right\}. \tag{A6}$$

The second-order approximation of the first term is given by

$$\mathfrak{u}\left(C_t\right) = C\mathfrak{u}_c\left\{c_t + \frac{1}{2}(1-\sigma)c_t^2\right\} + O\left(\|\xi\|^3\right) + t.i.p. \tag{A7}$$

Substituting (A0) and its square into (A7), I get

$$\mathfrak{u}(C_t) = C\mathfrak{u}_c \left\{ \mathfrak{u}_y y_t + \frac{1}{2}\mathfrak{u}_{yy} y_t^2 + \mathfrak{u}_{yp} y_t p_{o,t} \right\} + O\left(\|\xi\|^3\right) + t.i.p. \tag{A8}$$

for $\mathfrak{u}_y \equiv 1$, $\mathfrak{u}_{yy} \equiv 1-\sigma$, $\mathfrak{u}_{yp} \equiv -\chi\frac{\widetilde{\omega_{oc}}}{sy}(1-\sigma)$ and $t.i.p$ stands for terms independent of policy.

The second term in household utility is approximated by

$$v(H_t) = Hv_h \left\{ h_t + \frac{1}{2}(1+\nu) h_t^2 \right\} + O\left(\|\xi\|^3\right) + t.i.p. \tag{A9}$$

Substituting (A1) and its square in (A9) and getting rid of variables independent of policy, I obtain

$$v(H_t) = Hv_h \left\{ v_y y_t + v_\Delta \Delta_t + \frac{1}{2} v_{yy} y_t^2 + v_{yp} p_{o,t} y_t \right\} + O\left(\|\xi\|^3\right) + t.i.p. \tag{A10}$$

for
$v_y \equiv 1 - \mathcal{D}(\nu+\sigma)$,
$v_\Delta \equiv \frac{\mathcal{W}}{1-\widetilde{\omega_{oy}}}$,
$v_{yy} \equiv \frac{\mathcal{D}}{(1-\eta)}\mathcal{W}^2\frac{(1-\delta)}{(1-\widetilde{\omega_{oy}})}\mathcal{J}^2 + (1+\nu)(1-\mathcal{D}(\nu+\sigma))^2$
$v_{yp} \equiv \frac{\mathcal{D}}{(1-\eta)}\left[(1+\nu)(1-\mathcal{D}(\nu+\sigma))\mathcal{M} + \mathcal{W}^2\frac{(1-\delta)}{(1-\widetilde{\omega_{oy}})}\mathcal{J}\mathcal{L}\right]$.

Now, since the technology is constant returns to scale, the share of labor in total cost is equivalent to its share in marginal cost and the following equilibrium relationship at the steady state

$$Y\mathfrak{u}_c MC(1-\widetilde{\omega_{oy}}) = Hv_h,$$

which can be used to rewrite total utility U_{t_0} by substituting (A10) and (A8) into (A6), to get

$$U_{t_0} = (Y\mathfrak{u}_c)\, E_{to} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ u_y y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yp} p_{o,t} y_t + \frac{1}{2} u_\pi \pi_t^2 \right\} + O\left(\|\xi\|^3\right) + t.i.p., \tag{A11}$$

where
$u_y = \frac{C}{Y} - MC\left(1 - \widetilde{\omega_{oy}}\right) v_y,$
$u_{yy} = \frac{C}{Y}\left(1-\sigma\right) - MC\left(1 - \widetilde{\omega_{oy}}\right) v_{yy},$
$u_{yp} = -\frac{C}{Y}\left(\frac{\widetilde{\omega_{oc}}}{sy}\right)\chi\left(1-\sigma\right) - MC\left(1 - \widetilde{\omega_{oy}}\right) v_{yp}$
$u_\Delta = -MC\left(1 - \widetilde{\omega_{oy}}\right) v_\Delta = -MC\mathcal{W}$
$u_\pi = \frac{\varepsilon}{k} u_\Delta = \frac{\varepsilon}{k} MC\mathcal{W}.$

For the last step, substituting the expression (A5) for $\sum_{t=t_0}^{\infty} \beta^{t-t_0} y_t$ in (A11) obtains

$$\begin{aligned} U_{t_0} = & -Y\mathfrak{u}_c E_{to} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2}\left(u_y k_y^{-1} k c_{yy} - u_{yy}\right) y_t^2 \\ & + \frac{1}{2}\left(2u_y k_y^{-1} k c_{yp} - 2u_{yp}\right) y_t p_{o,t} \\ & + \frac{1}{2}\left(u_y k_y^{-1} \varepsilon\left(1+\nu\mathcal{W}\right) - u_\pi\right)\pi_t^2 + O\left(\|\xi\|^3\right) + t.i.p., \end{aligned}$$

that can be rewritten equivalently as

$$\begin{aligned} U_{t_0} = & -Y\mathfrak{u}_c E_{to} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2}\left(u_y k_y^{-1} k c_{yy} - u_{yy}\right)\left[y_t - y_t^*\right]^2 \\ & + \frac{1}{2}\left(u_y k_y^{-1} \varepsilon\left(1+\nu\mathcal{W}\right) - u_\pi\right)\pi_t^2 + O\left(\|\xi\|^3\right) + t.i.p. \end{aligned}$$

which ends up as the central banks's loss function to minimize

$$U_{t_0} = \Upsilon \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0}\left\{\lambda x_t^2 + \pi_{y,t}^2\right\} + O\left(\|\xi\|^3\right) + t.i.p.$$

for$\Upsilon \equiv \frac{1}{2}\lambda_{\pi_y} Y \mathfrak{u}_c$, $\Psi \equiv \frac{k u_y k_y^{-1} c_{yp} - u_{yp}}{k u_y k_y^{-1} c_{yy} - u_{yy}}$, and where $\lambda \equiv \frac{\lambda_y}{\lambda_{\pi_y}}$ for $\lambda_y = u_y k_y^{-1} k c_{yy} - u_{yy}$ and $\lambda_{\pi_y} = u_y k_y^{-1} \varepsilon\left(1+\phi\mathcal{W}\right) - u_\pi$. The output gap $x_t = y_t - y_t^*$ is now the percent deviation of output with respect to the welfare relevant output y_t^* itself defined as

$$y_t^* \equiv -\frac{k u_y k_y^{-1} c_{yp} - u_{yp}}{k u_y k_y^{-1} c_{yy} - u_{yy}} p_{o,t} = -\Psi p o_t.$$

The values of λ_y and λ_{π_y} are functions of the model parameters and describe the weights assigned by the central bank to stabilize the welfare relevant output gap and core inflation. In what follows I summarize this information with $\lambda \equiv \frac{\lambda_y}{\lambda_{\pi_y}}$, which determines how concerned about the output gap a central bank should be after an oil price shock. Typically, λ decreases with the sacrifice ratio and the degree of price stickiness.

10 Appendix II: Characterizing optimal policy

Following Woodford (2003)[31], I circumvent the usual time consistency issues associated with fully optimal monetary policies by assuming that the central bank is able to commit with full credibility to an optimal policy plan which specifies a full set of state-contingent sequences $\{x_t, \pi_{y,t}\}_{t=0}^{\infty}$ that minimize

$$\Upsilon \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \lambda x_t^2 + \pi_{y,t}^2 \right\}$$

subject to the following sequence of constraints

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y x_t + \mu_t \tag{A21}$$

and a constraint on the initial inflation rate

$$\pi_{y,t_0} = \overline{\pi}_{y,t_0}, \tag{A22}$$

where $\overline{\pi}_{y,t_0}$ is defined as the inflation rate in time t_0 that is consistent with optimal policy in a "timeless perspective" or, in other words, the inflation rate that would have been chosen a long time ago and which is consistent with the optimal precommitment plan.

Solving this problem under the timeless perspective gives rise to the following set of first-order conditions

$$x_t = x_{t-1} - \frac{k_y}{\lambda} \pi_{y,t}, \tag{A23}$$

which are supposed to hold for all $t = 0, 1, 2, 3, \ldots$ and which characterize the central bank's optimal policy response.

As shown in Section 3, acknowledging the low level of short-term substitutability between oil and other factors gives rise to a cyclical distortion coming from the interaction between the steady-state efficiency distortion and the oil price shock. In terms of the model equations, this cyclical distortion is translated into a cost-push shock that enters the New Keynesian Phillips curve (NKPC henceforth).

Taking a log-linear approximation of equation (19) around the zero inflation steady-state yields the standard result that (core) inflation is a function of next period inflation and this period real marginal cost: the NKPC

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k mc_t, \tag{A24}$$

[31] See Woodford (2003), chapter 7 for a discussion.

where mc_t is the log-deviation of real marginal cost from its (distorted) steady state and $k = \left(\frac{1-\theta}{\theta}\right)(1-\theta\beta)$ is the elasticity of inflation to the real marginal cost.

Substituting the labor market clearing level of the real wage into the real marginal cost equation (16), we can rewrite (A24) as

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y gap_t, \tag{A25}$$

where the output gap $gap_t = y_t - y_t^N$ measures the deviation between current output and the natural level of output, and where

$$y_t^N = -\frac{k\mathcal{B}}{k_y} po_t, \tag{A26}$$

for $\mathcal{B}$ a decreasing function of δ and χ, the oil production and consumption elasticities of substitution defined in Appendix I.

But (A25) can be equivalently rewritten as

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y x_t + \mu_t, \tag{A27}$$

for $\mu_t = k_y \left(y_t^* - y_t^N\right)$ the cost-push shock that arises as a direct function of the cyclical wedge between the natural and the welfare maximizing level of output.

Obviously, the divine coincidence obtains when $y_t^* = y_t^N$, which is the case for $\chi = \delta = 1$, as shown in Section 3.

Appendix III : Deriving an optimal simple rule

In the timeless perspective equilibrium, inflation and the output gap behave according to the rational expectation solution of the model consisting of the NKPC (A27) and the policy rule (A23). Following McCallum (1999b) and McCallum and Nelson (2004), the (no bubble) MSV solution takes the following form:

$$\pi_{y,t} = \alpha_{11} x_{t-1} + \alpha_{12} \mu_t \tag{A31}$$

$$x_t = \alpha_{21} x_{t-1} + \alpha_{22} \mu_t, \tag{A32}$$

where α_{ij} for $i, j = 1, 2$ are functions of β, k_y and λ.

Because the supply of oil is supposed perfectly elastic at a given exogenous real price, one can write the following definitions:

$$c_t - c_t^* = y_t - y_t^* \tag{A33}$$

$$\pi_{c,t} = \pi_{y,t} + \frac{\widetilde{\omega_{oc}}}{1-\widetilde{\omega_{oc}}}\left(po_t - po_{t-1}\right) \tag{A34}$$

that are used to rewrite the consumption Euler equation in deviation from efficient consumption as follows:

$$x_t = -\frac{1}{\sigma}\left(r_t - \mathbb{E}_t\pi_{y,t+1} - \frac{\widetilde{\omega_{oc}}}{1-\widetilde{\omega_{oc}}}\mathbb{E}_t\left(po_{t+1} - po_t\right) - rr_t^*\right) + \mathbb{E}_t x_{t+1} \tag{A35}$$

for $rr_t^* = \sigma\mathbb{E}_t\left\{\Delta y_{t+1}^*\right\} = -\Psi\sigma\left(1-\rho_o\right)po_t$.

Combining (A31), (A32), and (A35) leads to

$$\alpha_{21}x_{t-1} + \alpha_{22}\mu_t = -\frac{1}{\sigma}\left(\begin{array}{c} r_t - \alpha_{11}x_t - \alpha_{12}\rho_o\mu_t \\ -\frac{\widetilde{\omega_{oc}}}{1-\widetilde{\omega_{oc}}}\left(\rho_o - 1\right)po_t - \Psi\sigma\left(1-\rho_o\right)po_t \end{array}\right) + \alpha_{21}x_t + \alpha_{22}\rho_o\mu_t,$$

which can be solved for r_t :

$$\begin{aligned} r_t &= \left(\alpha_{11} + \sigma\alpha_{21}\right)x_t - \sigma\alpha_{21}x_{t-1} - \left(\sigma\alpha_{22} - \alpha_{12}\rho_o - \sigma\alpha_{22}\rho_o\right)\mu_t \\ &\quad + \underbrace{\left(\rho_o - 1\right)\left(\frac{\widetilde{\omega_{oc}}}{1-\widetilde{\omega_{oc}}} - \Psi\sigma\right)}_{\Xi}po_t. \end{aligned}$$

As shown from (A31), $\mu_t = \frac{\pi_{y,t} - \alpha_{11}x_{t-1}}{\alpha_{12}}$, the above equation can be rewritten as:

$$r_t = \frac{\Phi}{\alpha_{11}}\pi_{y,t} + \left(\alpha_{11} + \sigma\alpha_{21}\right)x_t - \left(\Phi + \sigma\alpha_{21}\right)x_{t-1} + \Xi po_t,$$

for $\Phi = \left(\rho_o - \sigma\alpha_{22}\alpha_{12}^{-1}\left(1-\rho_o\right)\right)\alpha_{11}$.

Finally, substituting $y_t^* = -\Psi po_t$ in the above, I get:

$$\begin{aligned} r_t &= \frac{\Phi}{\alpha_{11}}\pi_{y,t} + \left(\alpha_{11} + \sigma\alpha_{21}\right)\left(y_t + \Psi po_t\right) - \left(\Phi + \sigma\alpha_{21}\right)\left(y_{t-1} + \Psi po_{t-1}\right) + \Xi po_t \\ &= \frac{\Phi}{\alpha_{11}}\pi_{y,t} + \left(\alpha_{11} + \sigma\alpha_{21}\right)y_t - \left(\Phi + \sigma\alpha_{21}\right)y_{t-1} \\ &\quad + \left(\left(\alpha_{11} + \sigma\alpha_{21}\right)\Psi + \Xi\right)po_t - \left(\Phi + \sigma\alpha_{21}\right)\Psi po_{t-1} \\ &= \Phi\alpha_{11}^{-1}\pi_{y,t} + \Omega y_t - \Gamma y_{t-1} + \left(\Xi + \Psi\Omega\right)po_t - \Psi\Gamma po_{t-1}. \end{aligned}$$

11 Appendix IV: log-linearized economy

The allocation in the decentralized economy can be summarized by the following five equations. Log-linearizing the labor supply equation (5) (and setting $\eta = 0$ for flexible real wages), the labor demand equation (14), and the real marginal cost (16), gives equations (A41), (A42), and (A43). Substituting out oil consumption (7) in (3) and making use of the overall resource constraint gives (A44). Finally, equation (A45) is the log-linear version of (8) and describes the evolution of the ratio of core to headline price indices as a function of the real price of oil in consumption units. Lowercase letters denote the percent deviation of each variable with respect to their steady states (e.g., $c_t \equiv \log\left(\frac{C_t}{C}\right)$):

$$w_t = \phi h_t + \sigma c_t \tag{A41}$$

$$h_t = y_t - \delta\left(w_t - mc_t - py_t\right) + \Delta_t \tag{A42}$$

$$mc_t = \left(1 - \widetilde{\omega_{oy}}\right)\left(w_t - py_t\right) + \widetilde{\omega_{oy}}\left(po_t - py_t\right) \tag{A43}$$

$$c_t = -\chi \frac{\widetilde{\omega_{oc}}}{sy} po_t + y_t \tag{A44}$$

$$py_t = -\frac{\widetilde{\omega_{oc}}}{1 - \widetilde{\omega_{oc}}} po_t \tag{A45}$$

where $w_t = \log(\frac{W_t P}{P_t W})$ is the consumption real wage, $po_t = \log(\frac{P_{o,t}}{P_o})$ is the real oil price in consumption units, $py_t = \log(\frac{P_{y,t}}{P_y})$ is the relative price of the core goods in terms of consumption goods, $\widetilde{\omega_{oy}} \equiv \omega_{oy}^{\delta}\left(\frac{P_o}{MC \cdot P_y}\right)^{1-\delta}$ is the share of oil in the real marginal cost, $\widetilde{\omega_{oc}} \equiv \omega_{oc}^{\chi} P_o^{1-\chi}$ is the share of oil in the CPI, and $sy \equiv (1 - \omega_{oc})\left(\frac{Y}{C}\right)^{\frac{\chi-1}{\chi}}$ is the share of the core good in the consumption goods basket.

Also, the real marginal cost is equal to the inverse of the desired gross markup in the steady state, itself determined by the degree of monopolistic competition as measured by the elasticity of substitution between goods ε. So $MC = \frac{\varepsilon-1}{\varepsilon}$in the steady state and $MC \to 1$ when $\varepsilon \to \infty$ in the perfect competition limit.

12 Tables

Table 1: Optimized simple rule (OR) and speed limit rules (SLR)

Simple rule	g_π	g_y	g_{y1}	g_{po}	g_{po_1}	g_{w1}
OR	5.123	4.742	-4.731	0.007	-0.014	-
SLR	5.101	4.742	-4.742	0.008	-0.008	-
OR_w ($\eta = 0.9$)	5.134	8.708	-7.884	0.276	-0.240	0.088
SLR_w ($\eta = 0.9$)	2.054	3.404	-3.404	0.096	-0.096	-

note: all coefficients are consistent with annualized interest rates and inflation

Table 2: Welfare costs under alternative policies (percent of annual consumption)

Policy		Total Loss	y Loss	π_y Loss
optimal		0.6	0.4	0.2
core	Strict inflation target	1.8	1.8	0
	CTR	1.9	0.2	1.7
	CTR inertia ($\rho = 0.8$)	1.7	0.2	1.5
	Forecasting CTR	4.4	0.2	4.2
headline	HTR	2.7	0.4	2.3
	HTR inertia ($\rho = 0.8$)	1.7	0.3	1.4
	Forecasting HTR	9.4	0.1	9.3

13 Figures

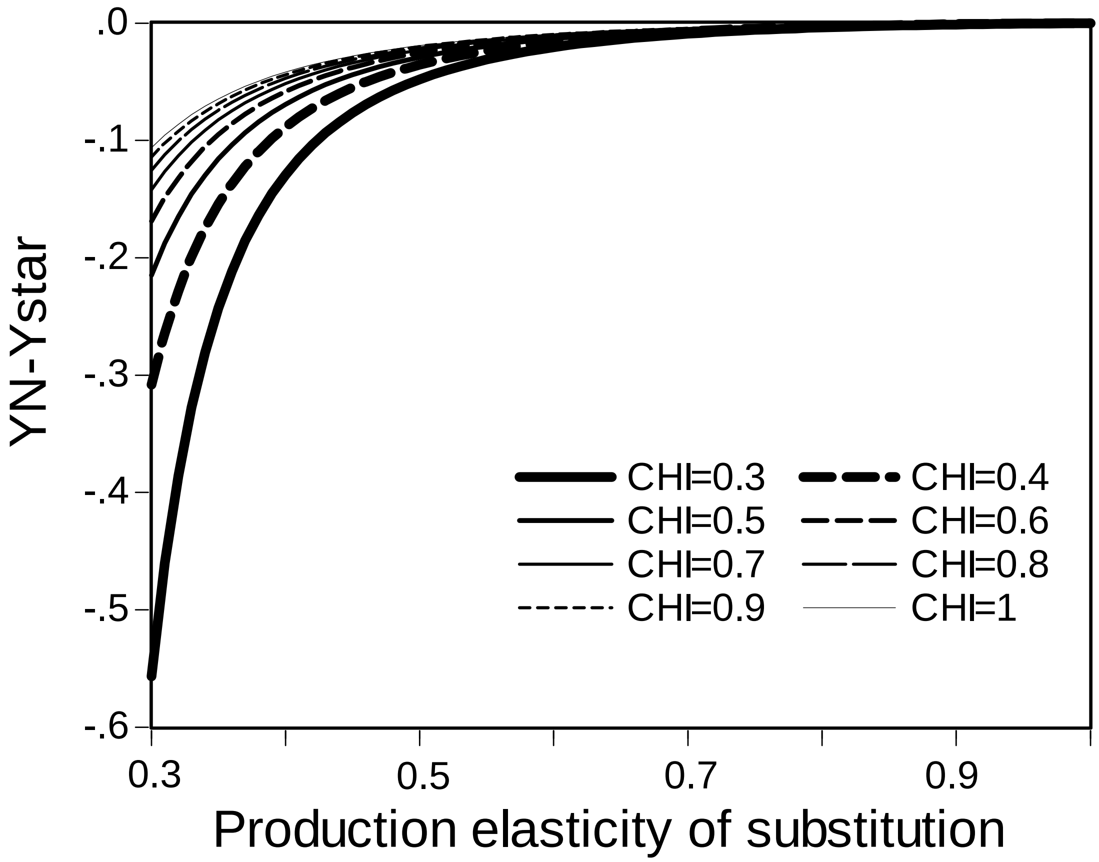

Figure 1: (YN) and efficient (Ystar) output to a 1-percent increase in oil price as a function of the production and the consumption elasticity of substitution (Chi); baseline calibration with 2-percent oil share of output and 6-percent energy component of consumption.

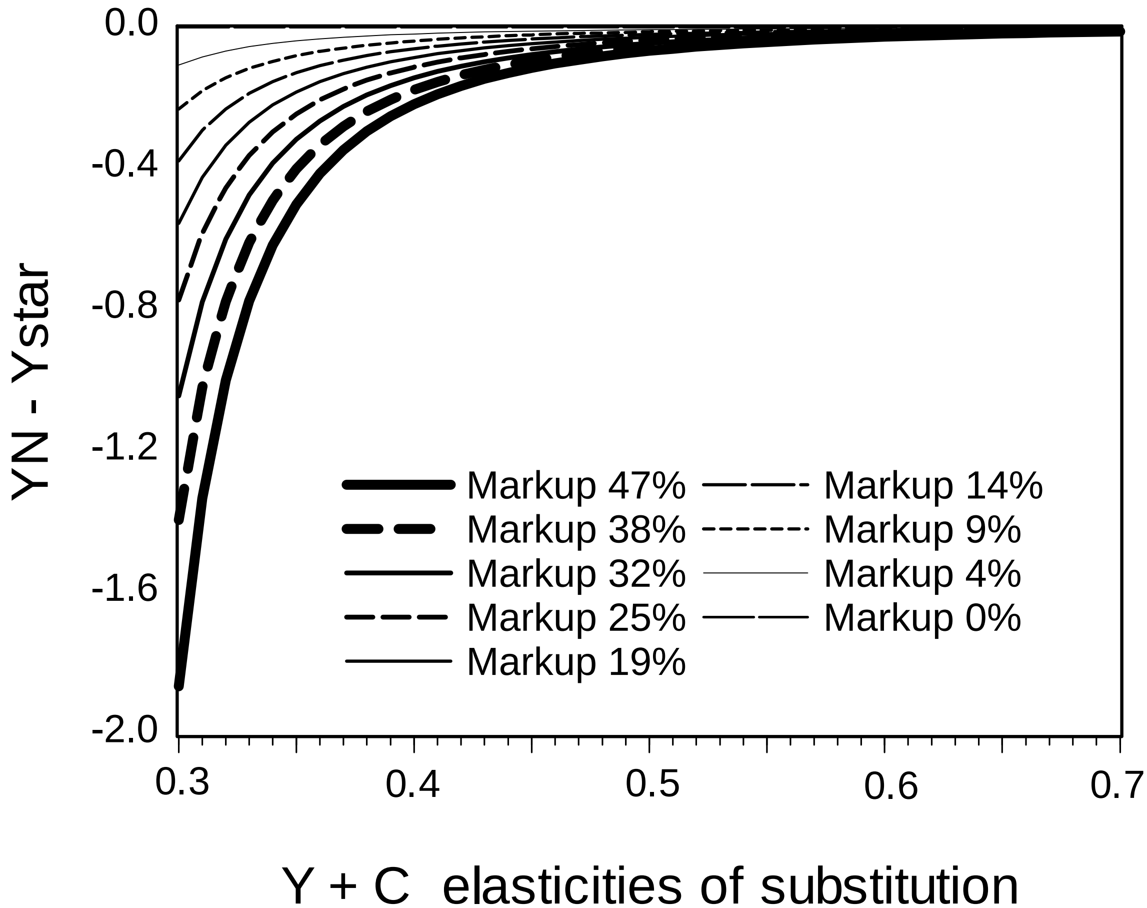

Figure 2: Response of the gap between natural (YN) and efficient (Ystar) output to a 1-percent increase in oil price as a function of the degree of monopolistic competition; baseline calibration with 2-percent oil share of output and 6-percent energy component of consumption.

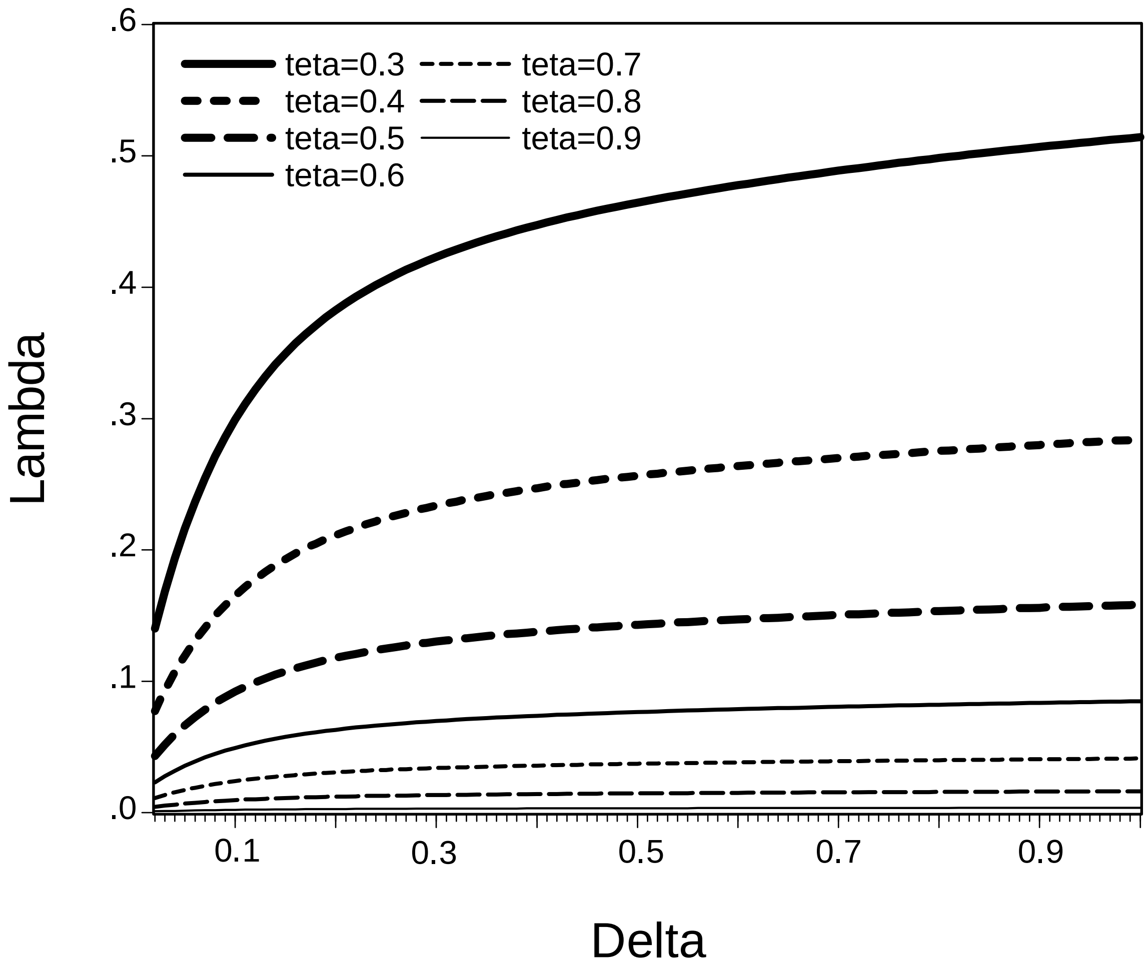

Figure 3: Change in the weight (Lambda) assigned to output gap stabilization as a function of the elasticities of substitution (Delta) and the degree of price stickiness (teta)

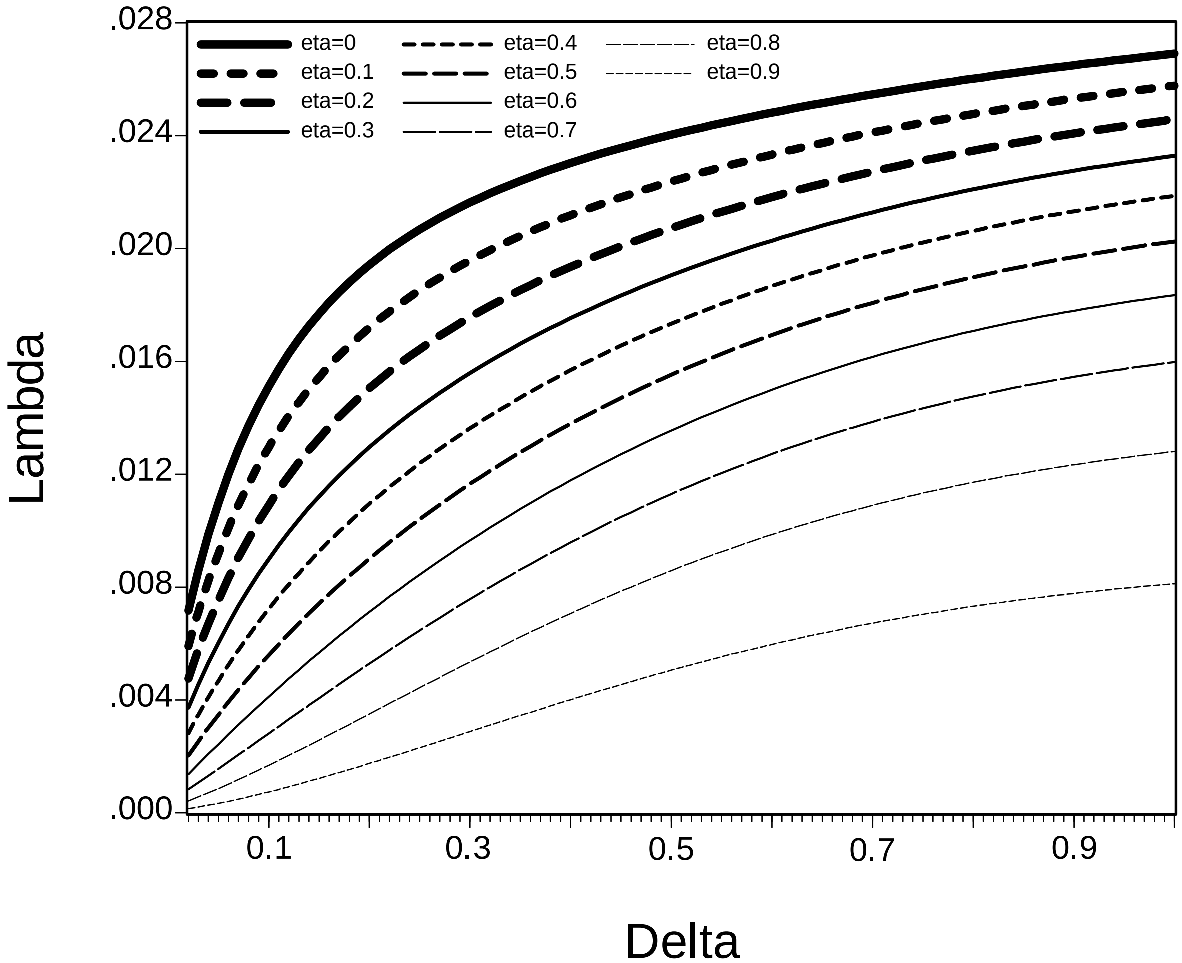

Figure 4: Change in the weight (Lambda) assigned to output gap stabilization as a function of the elasticities of substitution (Delta) and the degree of real wage rigidity (eta)

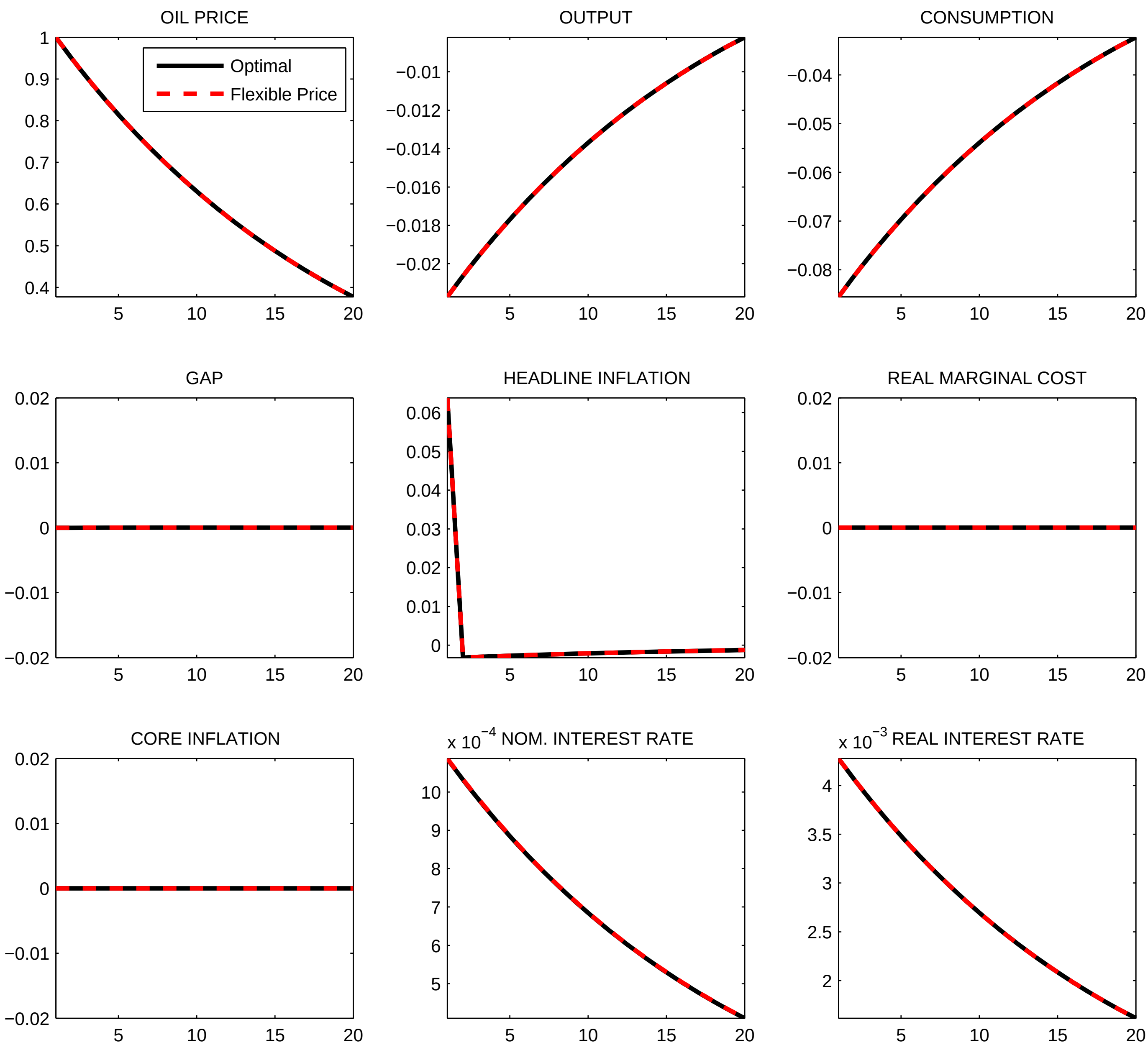

Figure 5: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with flexible price equilibrium; Cobb-Douglas technology; baseline calibration.

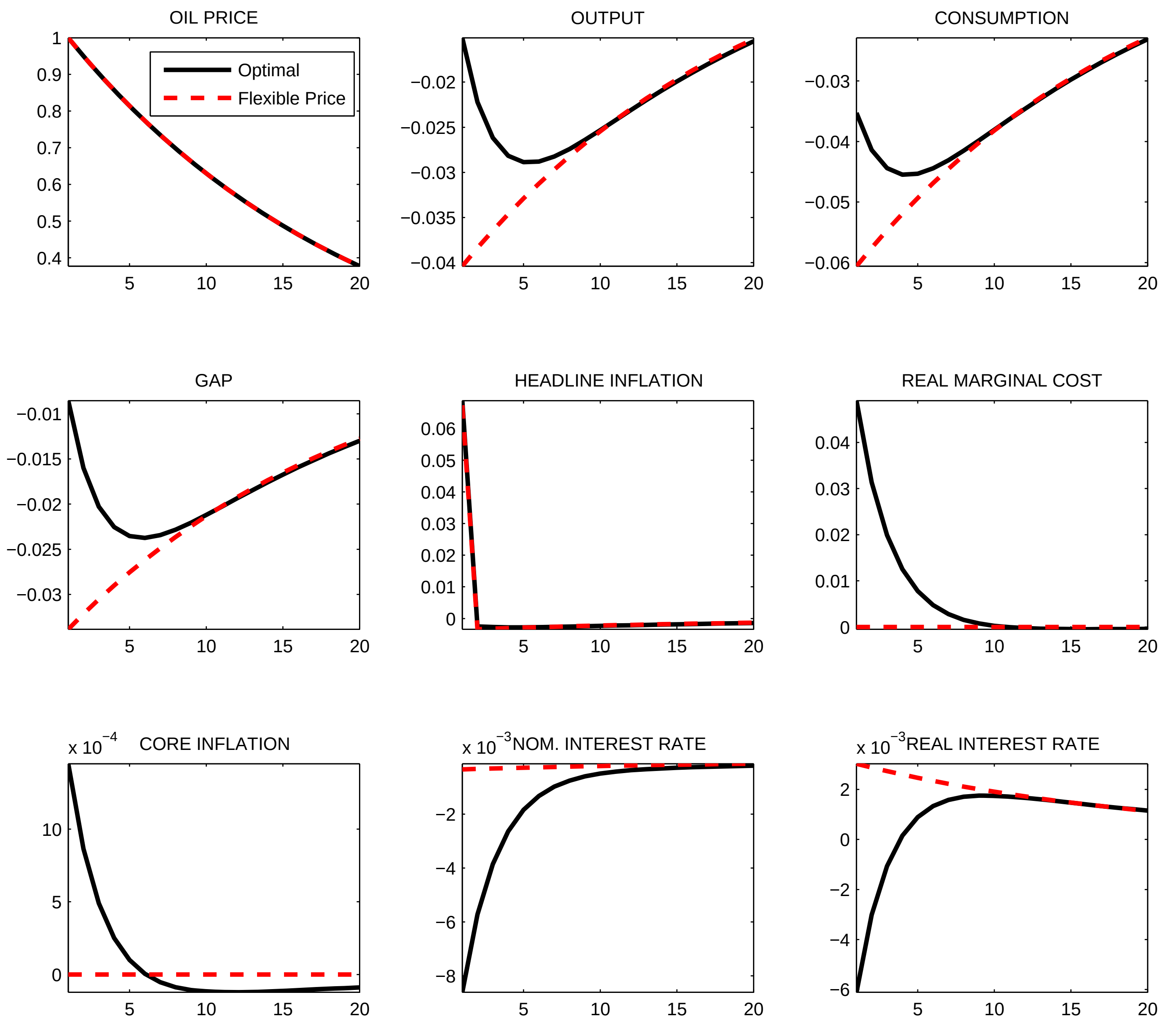

Figure 6: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with flexible price equilibrium; CES technology with low elasticity ($\psi = \chi = 0.3$); baseline calibration.

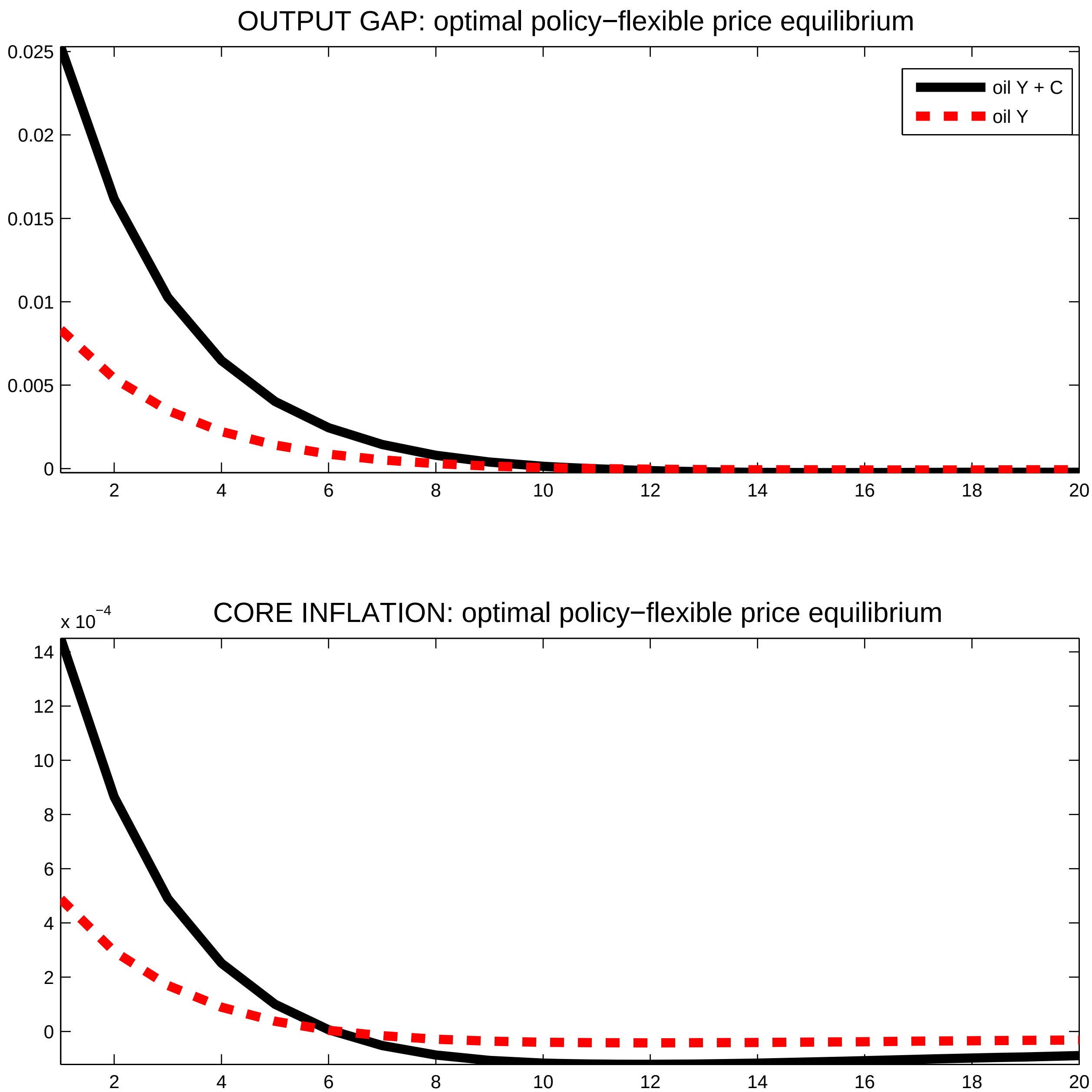

Figure 7: Tradeoff magnification effect; difference between optimal policy and FPWE when oil is an input to production only (dashed line) and when oil is an input to both production and consumption (solid line); baseline calibration.

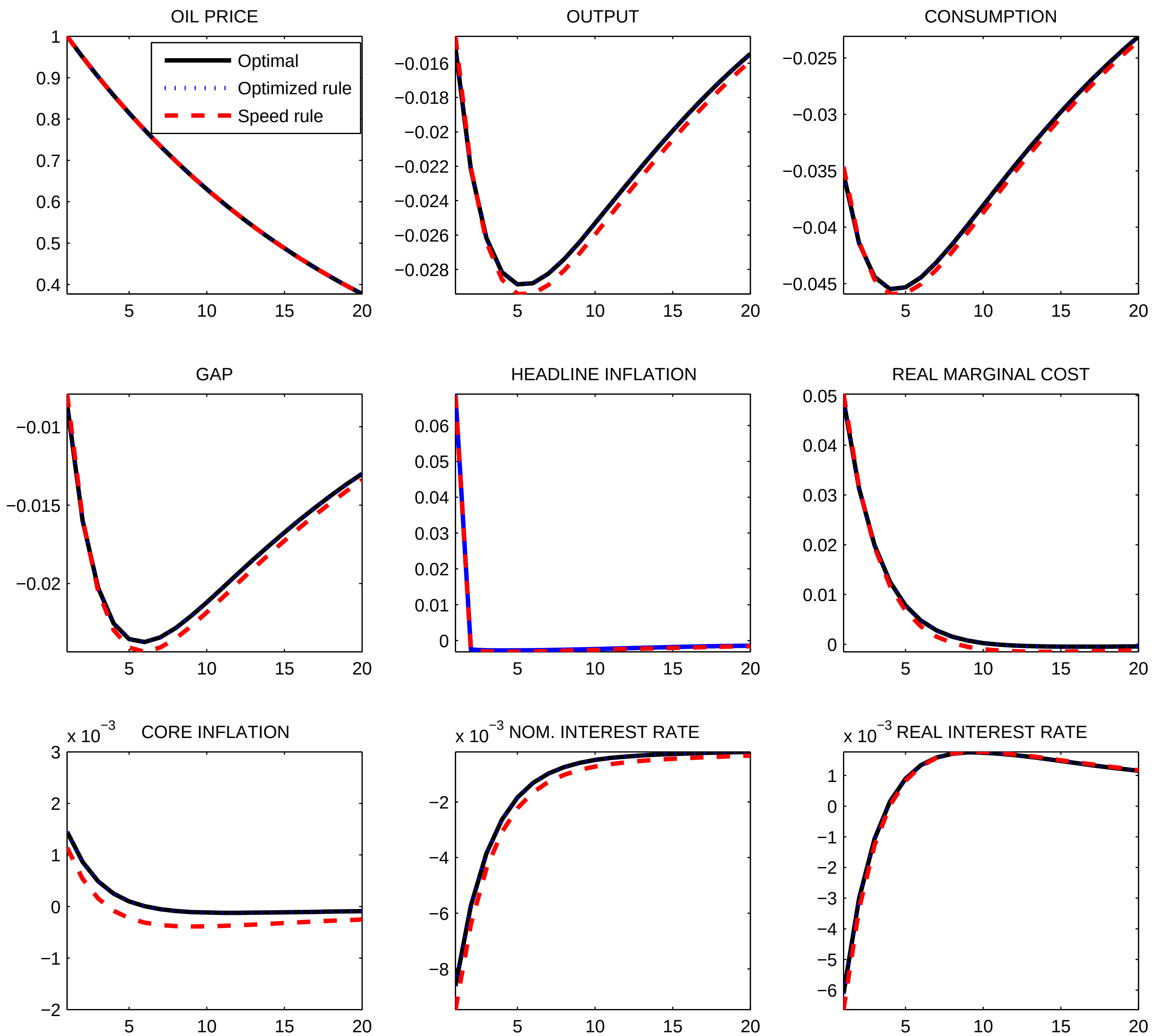

Figure 8: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with optimized simple rule and speed limit policy; CES technology with low elasticity ($\psi = \chi = 0.3$); baseline calibration.

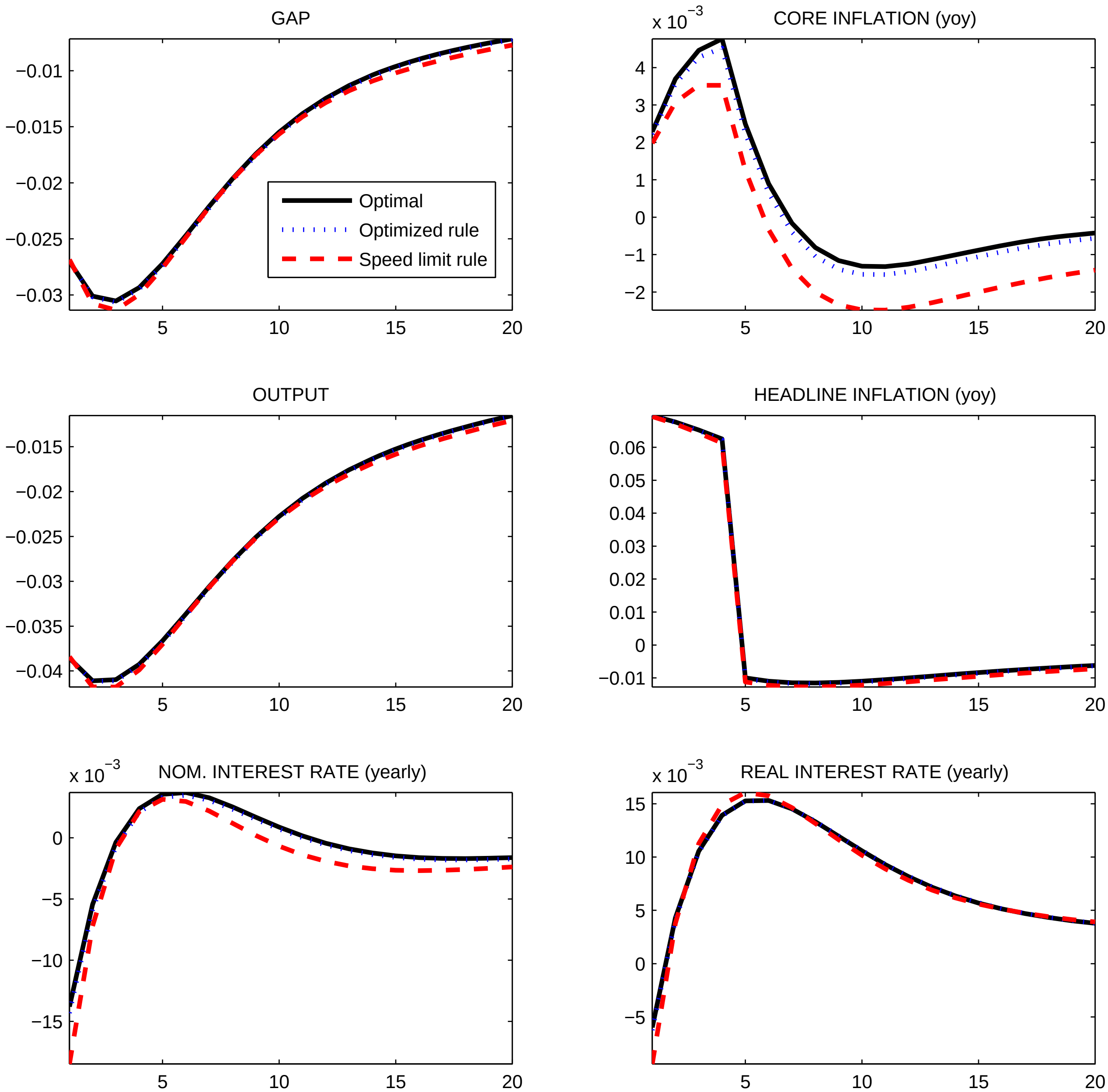

Figure 9: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with optimized simple rule and speed limit policy based on four quarters moving average of core inflation; CES technology with low elasticity ($\psi = \chi = 0.3$); real wage rigidity ($\eta = 0.9$).

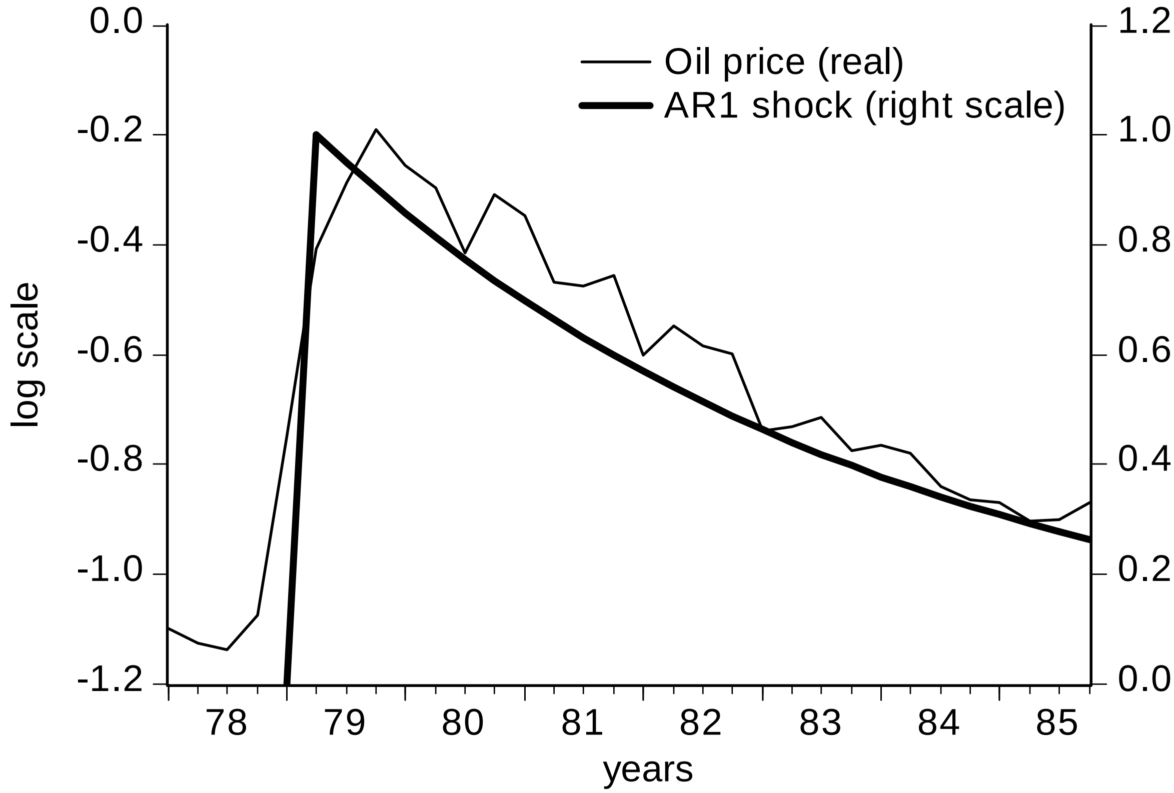

Figure 10: 1979 oil price shock and comparable AR(1) exogenous process for the real price of oil; log scales

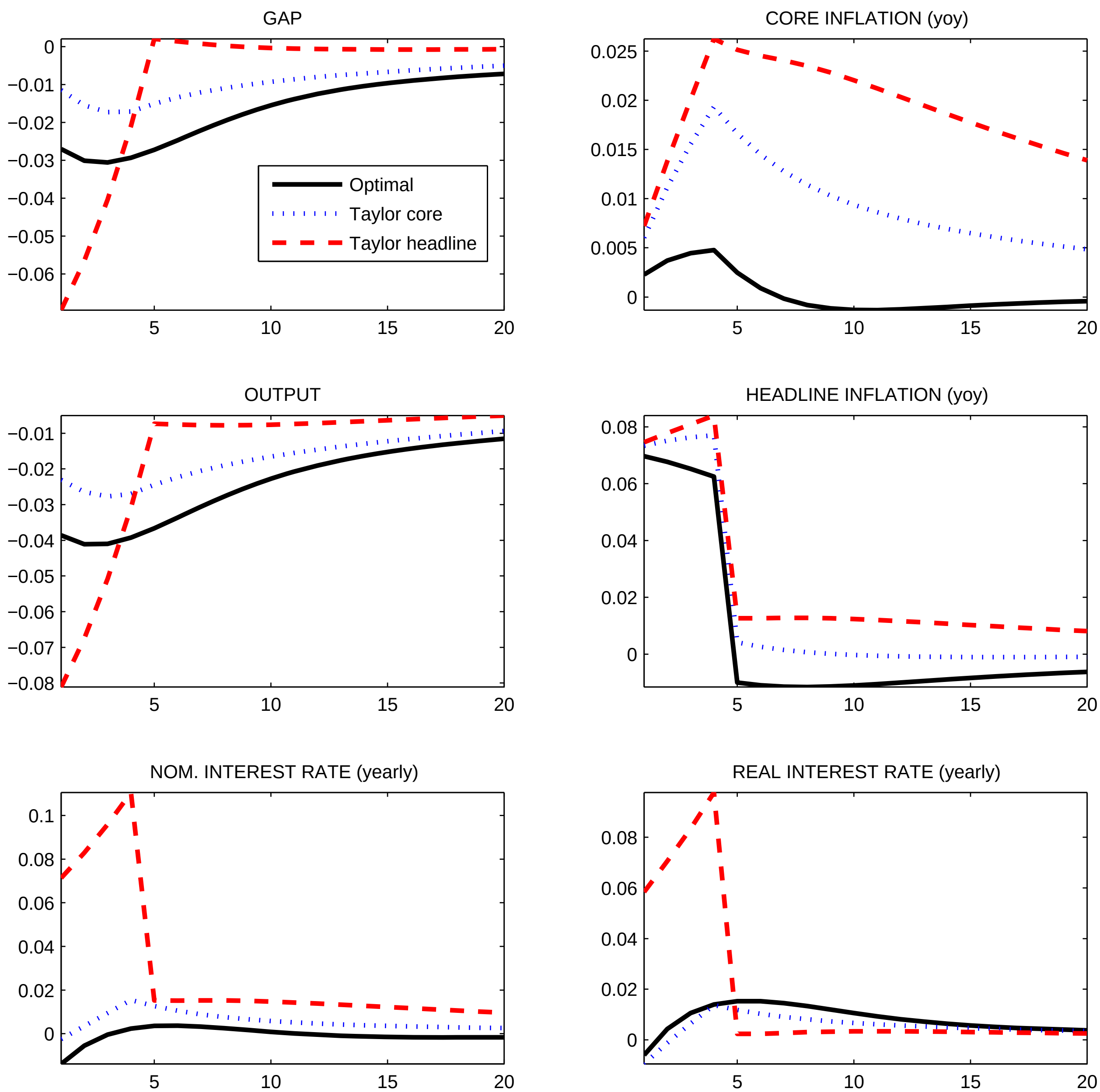

Figure 11: Impulse response functions to a 1979-like 100% log-increase in oil price; comparison of optimal precommitment monetary policy with simple Taylor rules based on four quarters moving average values of core or headline inflation; baseline calibration; real wage stickiness ($\eta = 0.9$).

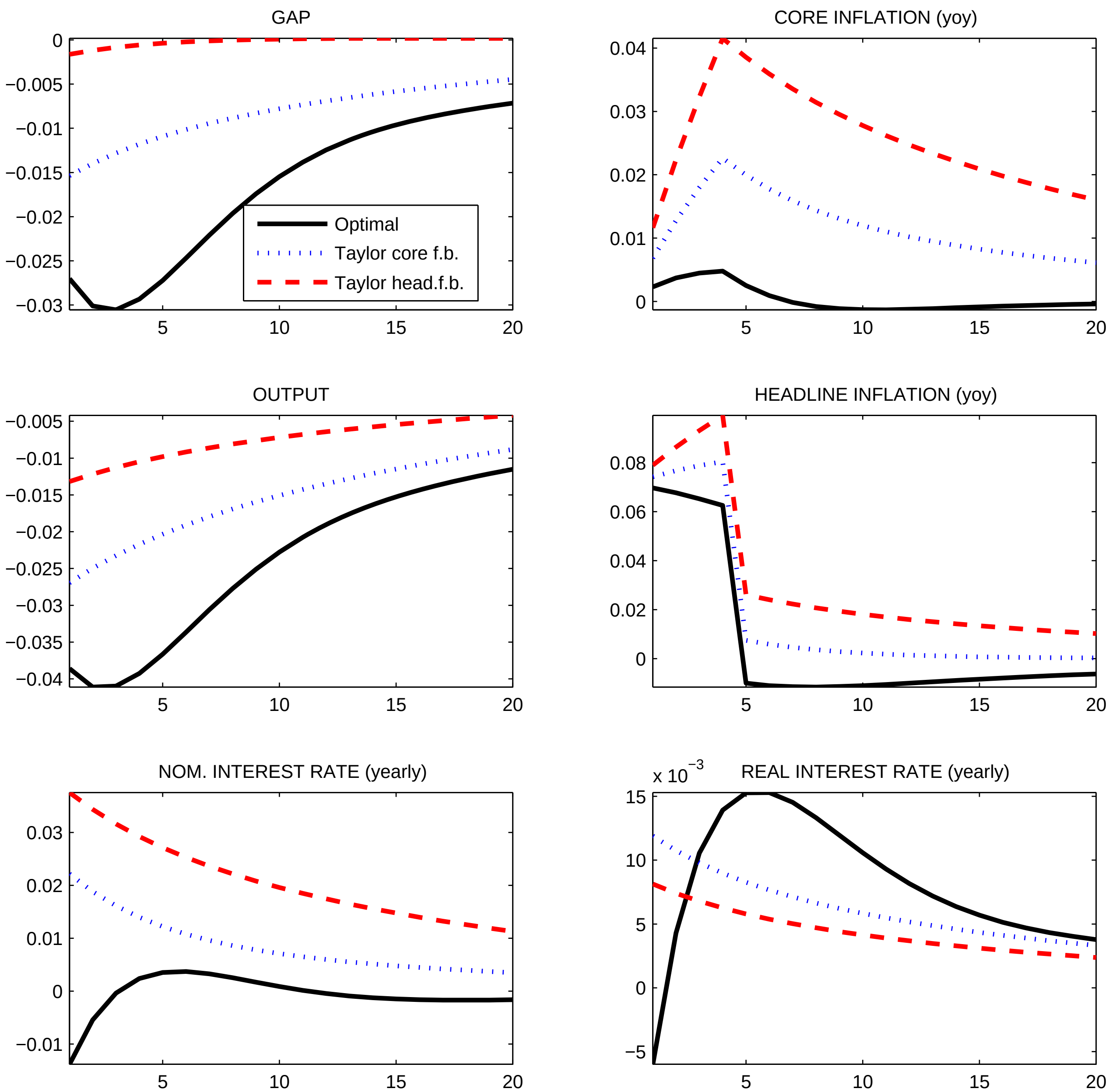

Figure 12: Impulse response functions to a 1979-like 100% log-increase in oil price; comparison of optimal precommitment monetary policy with simple forecast based Taylor rules; baseline calibration; real wage stickiness ($\eta = 0.9$).

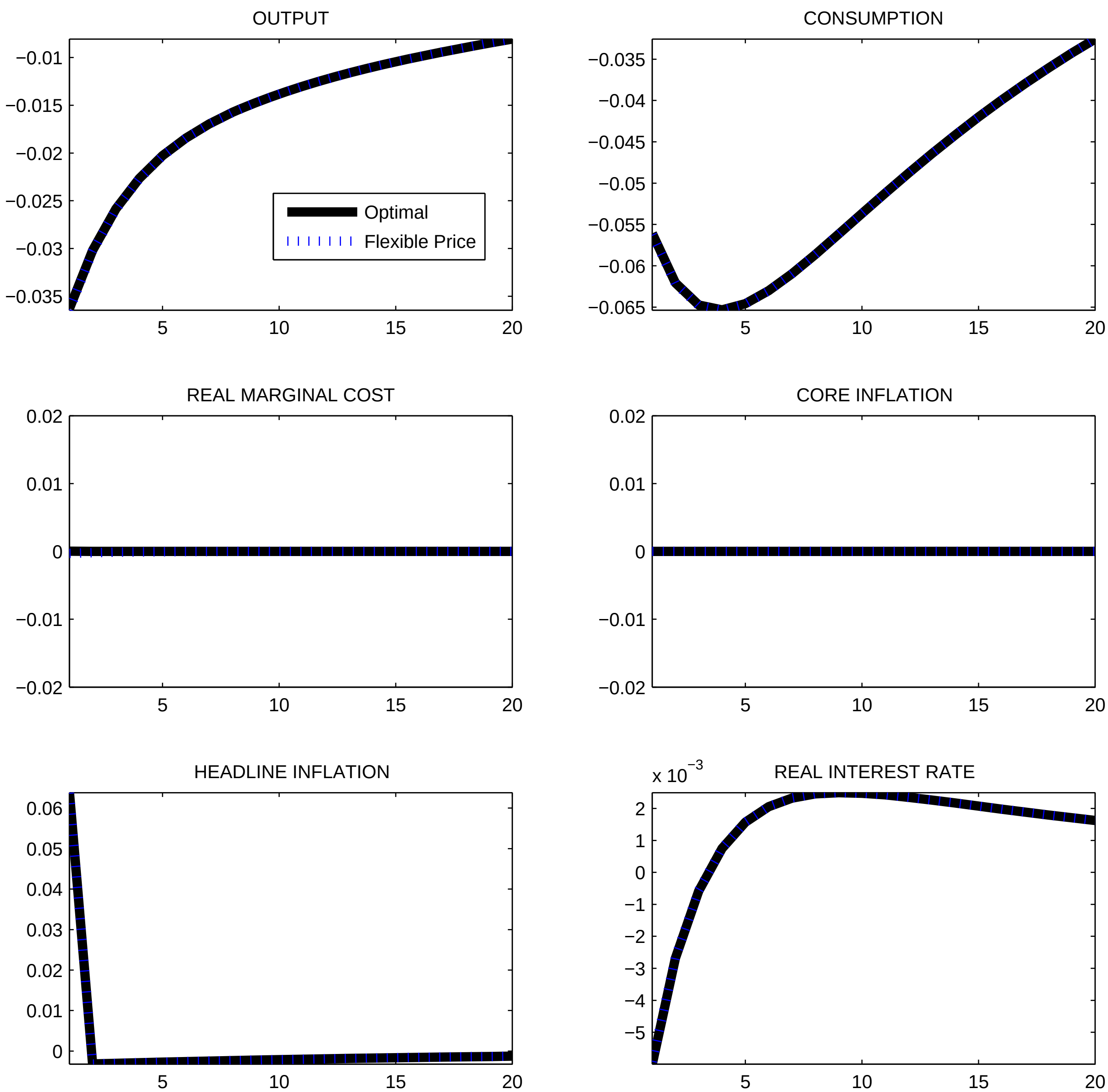

Figure 13: Impulse response functions to a 1-percent log-increase in oil price; time-varying elasticities; comparison of optimal precommitment monetary policy with flexible price equilibrium; undistorted equilibrium.

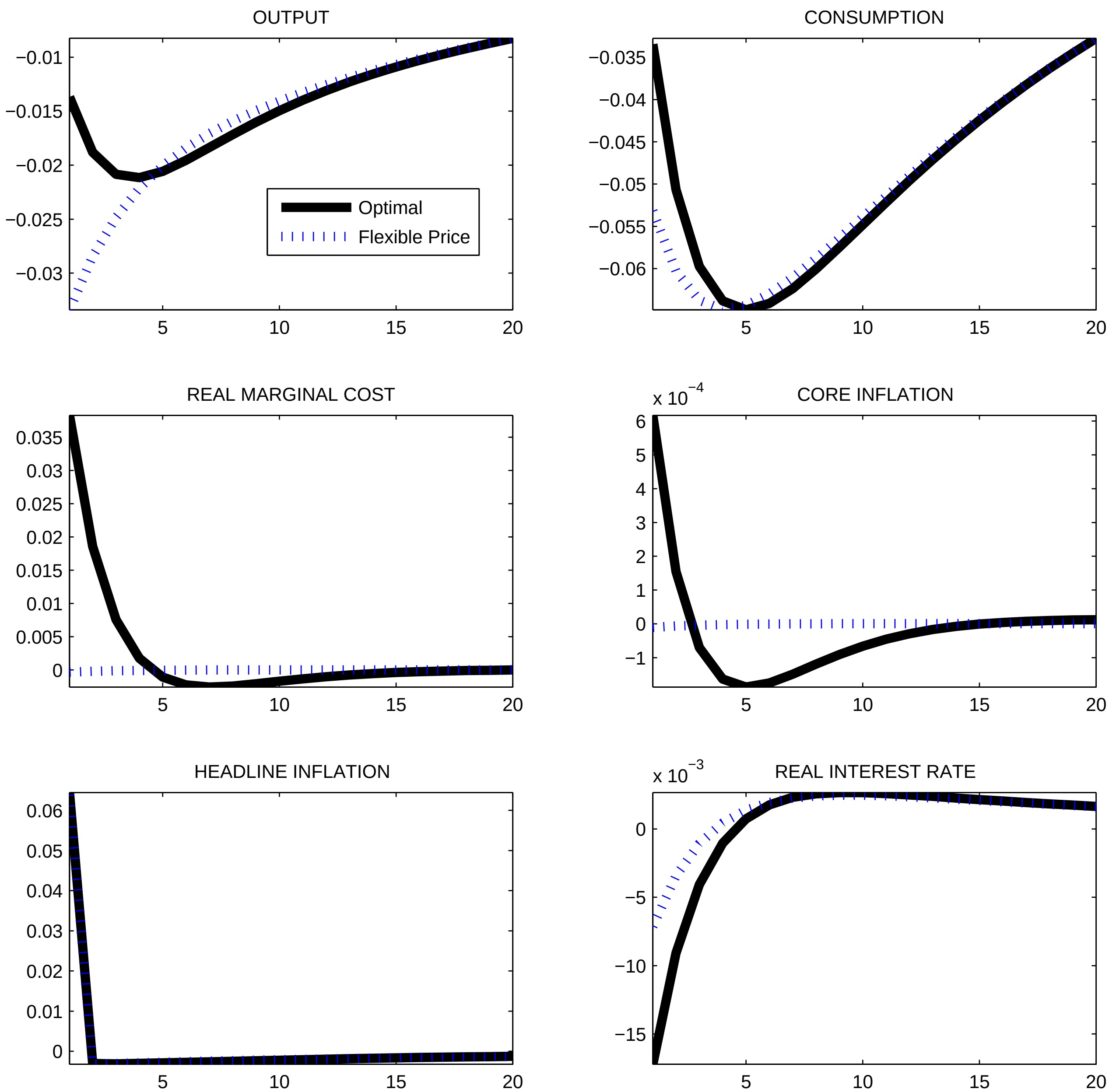

Figure 14: Impulse response functions to a 1-percent log-increase in oil price; Time varying elasticities; comparison of optimal precommitment monetary policy with flexible price equilibrium; distorted equilibrium (markup 20%).