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Risk Aversion, the Labor Margin, and Asset Pricing in DSGE Models

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Abstract

In dynamic stochastic general equilibrium (DSGE) models, the household's labor margin as well as consumption margin affects Arrow-Pratt risk aversion. This paper derives simple, closed-form expressions for risk aversion that take into account the household's labor margin. Ignoring the labor margin can lead to wildly inaccurate measures of the household's true attitudes toward risk. We show that risk premia on assets computed using the stochastic discount factor are proportional to Arrow-Pratt risk aversion, so that measuring risk aversion correctly is crucial for understanding asset prices. Closed-form expressions for risk aversion in DSGE models with generalized recursive preferences and internal and external habits are also derived.

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1. Introduction

In a static, one-period model with household utility $u(\cdot)$ defined over the quantity c of a single consumption good, Arrow (1964) and Pratt (1965) defined the coefficients of absolute and relative risk aversion, $-u''(c)/u'(c)$ and $-c\,u''(c)/u'(c)$. The former is the natural measure of household aversion to gambles over consumption units, and the latter the natural measure for gambles over a fraction of total household consumption.

Difficulties immediately arise, however, when one attempts to generalize these concepts from the case of a one-period, one-good model to the case of many periods or many goods (e.g., Kihlstrom and Mirman, 1974). To measure risk aversion when multiple goods are present, Stiglitz (1969) proposed using the indirect utility function rather than the utility function itself. This reformulates the problem from measuring risk aversion with respect to multiple goods to measuring risk aversion with respect to a single good, wealth. Then $-v''(a)/v'(a)$ and $-av''(a)/v'(a)$ are the natural measures of the household's absolute and relative risk aversion with respect to gambles over wealth, where $v(\cdot)$ denotes the household's indirect utility as a function of wealth a.

In the dynamic setting, Constantinides (1990) proposed measuring risk aversion using the household's value function, which again collapses the problem of measuring risk aversion with respect to an infinity of goods across time and states of nature into the much simpler problem of measuring risk aversion with respect to a single good, beginning-of-period household wealth. Boldrin, Christiano, and Fisher (1997) apply Constantinides' definition to very simple endowment economy models for which they can derive closed-form expressions for the value function, and thereby compute risk aversion.

The present paper builds on Boldrin, Christiano, and Fisher (1997) by deriving Arrow-Pratt risk aversion for dynamic stochastic general equilibrium (DSGE) models in general. We show that risk aversion depends on the partial derivatives of the household's value function with respect to assets. Even though the value function typically cannot be computed in closed form, we nevertheless are able to derive closed-form expressions for risk aversion because derivatives of the household's value function are much easier to compute than the value function itself, by the envelope theorem. For example, in many DSGE models the derivative of the value function with respect to wealth equals the current-period marginal utility of consumption (Benveniste and Scheinkman, 1979). Building on this insight allows us to compute simple, closed-form expressions for risk aversion.

The importance of measuring risk aversion in DSGE models has increased as researchers work to bring these models into closer agreement with asset prices (e.g., Boldrin, Christiano, and Fisher (2001), Tallarini (2000), Rudebusch and Swanson (2008, 2009), Van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2008)). When matching asset prices, household risk aversion is a crucial parameter of the model—indeed, risk premia computed using the household's stochastic discount factor are proportional to Arrow-Pratt risk aversion, as we show in section 2. It is therefore surprising that so little attention has been paid to computing this coefficient accurately in DSGE models. The present paper aims to fill that void.

A central result of the paper is that risk aversion depends on both the household's consumption and labor margins. When faced with a stochastic shock to income or wealth, the household may absorb that shock either through changes in consumption, changes in hours worked, or some combination of the two. Measuring risk aversion without taking into account the household's labor margin, as is common in the DSGE literature, can lead to wildly inaccurate estimates of the household's true attitudes toward risk. For example, if the household's period utility kernel is given by $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \chi l_t$, the quantity $-c\, u_{11}/u_1 = \gamma$ is often referred to as the household's coefficient of relative risk aversion, but the household is in fact *risk neutral* with respect to gambles over income or wealth—the proper measure of risk aversion—as we will show in section 2, below. More generally, when $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \chi_0 l_t^{1+\chi}/(1+\chi)$, risk aversion equals $(\gamma^{-1} + \chi^{-1})^{-1}$, a combination of the parameters on the household's consumption and labor margins, reflecting that the household absorbs shocks using both margins.

A corollary of this result is that risk aversion and the intertemporal elasticity of substitution are not inverse to each other when the household's labor margin is nontrivial, even for the case of expected utility preferences.[1] There is a wedge between the two concepts that depends on the household's labor margin and how it interacts with consumption in household utility.

[1] Generalized recursive preferences, of course, completely separate these two concepts.

The remainder of the paper proceeds as follows. Section 2 works through the main ideas of the paper, deriving Arrow-Pratt risk aversion in DSGE models for the simplest case, time-separable expected utility preferences, and demonstrating the importance of risk aversion for asset pricing. Section 3 extends the analysis to the case of generalized recursive preferences (Epstein and Zin, 1989), which have been the focus of much recent research at the boundary between macroeconomics and finance. Section 4 extends the analysis to the case of internal and external habits, two of the most common intertemporal nonseperabilities in preferences in both the macroeconomics and finance literatures. Section 5 discusses some general implications and concludes. An Appendix provides details of derivations that are outlined in the main text.

2. Time-Separable Expected Utility Preferences

To highlight the intuition in the paper, consider first the case where the household has additively time-separable expected utility preferences.

2.1 The Household's Optimization Problem and Value Function

The household seeks to maximize the expected present discounted value of utility flows:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau, l_\tau), \tag{1}$$

subject to the sequence of asset accumulation equations:

$$a_{\tau+1} = (1+r_\tau)a_\tau + w_\tau l_\tau + d_\tau - c_\tau, \quad \tau = t, t+1, \ldots \tag{2}$$

and transversality condition:

$$\lim_{T\to\infty} \prod_{\tau=t}^{T} (1+r_\tau)^{-1} a_{T+1} \geq 0, \tag{3}$$

where E_t denotes the mathematical expectation conditional on the household's information set at the beginning of period t, $\beta \in (0,1)$ denotes the household's discount factor, $c_t \geq 0$ and $l_t \geq 0$ the household's choice of consumption and labor in period t, a_t the household's beginning-of-period assets, and w_t, r_t, and d_t denote the exogenous (to the household) real wage, interest rate, and transfer payments at time t. The function u is assumed to be increasing in its first argument, decreasing in its second, twice-differentiable, and concave. Note that since u is increasing in consumption (i.e., there is no satiation), condition (3) will hold with equality at the optimum.

Let $V(a_t;\theta_t)$ denote the value function for the household's problem, where θ_t denotes the vector of exogenous (to the household) state variables governing the processes for w_t, r_t, and d_t. That is, V satisfies the Bellman equation:

$$V(a_t;\theta_t) = \max_{c_t, l_t} \; u(c_t, l_t) + \beta E_t V(a_{t+1};\theta_{t+1}), \tag{4}$$

where a_{t+1} is given by (2). Letting $c_t^* \equiv c^*(a_t;\theta_t)$ and $l_t^* \equiv l^*(a_t;\theta_t)$ denote the household's optimal choices of c_t and l_t as functions of the state a_t and θ_t, V can be written as:

$$V(a_t;\theta_t) = u(c_t^*, l_t^*) + \beta E_t V(a_{t+1}^*;\theta_{t+1}), \tag{5}$$

where $a_{t+1}^* \equiv (1+r_t)a_t + w_t l_t^* + d_t - c_t^*$.

2.2 Representative Household and Steady State Assumptions

So far we have considered the case of a single household, leaving the other households of the model and the production side of the economy unspecified. Implicitly, the other households and production sector jointly determine the processes for θ_t, w_t, r_t, and d_t in the DSGE model, and much of the analysis below does not to be any more specific about these processes than this. However, to move from general expressions for household risk aversion to concrete, closed-form expressions, we incorporate two standard assumptions from the DSGE literature.[2]

First, we assume that the household described above is *representative.* This allows the variables w_t and r_t to be expressed in terms of derivatives of the household's own utility function $u(\cdot,\cdot)$ in equilibrium.

[2] This is not to say that these assumptions are *necessary*—alternative assumptions about the nature of the other households in the model or the production sector may also allow for closed-form expressions for risk aversion. However, the assumptions used here are standard in the literature and thus the most natural to pursue.

Second, we assume that the model has a *nonstochastic steady state,* or a balanced growth path that can be renormalized to a nonstochastic steady state after a suitable change of variables. At the steady state, $x_t = x_{t+1} = x_{t+k}$ for $k = 1, 2, \ldots$, and $x \in \{c, l, a, w, r, d, \theta\}$, and we drop the subscript t to denote the steady-state value.

It is important to note that the nonstochastic steady state does not rule out the possibility that an individual household faces a hypothetical gamble of the types discussed below—the steady state of the model serves only as a reference point around which the *aggregate* variables w, r, d, and θ and the *other households'* choices of c, l, and a can be predicted with certainty. This reference point is important because it makes it much easier to compute closed-form expressions for many features of the model.

2.3 The Coefficient of Absolute Risk Aversion

The household's risk aversion at time t generally depends on the household's state vector at time t, $(a_t; \theta_t)$. Given this state, we consider the household's aversion to a hypothetical one-shot gamble in period t of the form:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \varepsilon_{t+1}, \tag{6}$$

where ε_{t+1} is a random variable with mean 0 and variance $d\sigma^2$ that represents the gamble.[3] Following Arrow (1964) and Pratt (1965), we can ask what one-time fee $d\mu$ the household would be willing to pay in period t in order to avoid the gamble. The quantity $2d\mu/d\sigma^2$ is the household's coefficient of absolute risk aversion, which we show in the Appendix is given by:[4]

$$\frac{-E_t V_{11}(a^*_{t+1}; \theta_{t+1})}{E_t V_1(a^*_{t+1}; \theta_{t+1})}, \tag{7}$$

where V_1 and V_{11} denote the first and second partial derivatives of V with respect to its first argument. Equation (7) is essentially the Constantinides (1990) definition of risk aversion, and has obvious similarities to Arrow (1964) and Pratt (1965). Here, of course,

[3] Dating the gamble $t + 1$ helps to clarify that its outcome is not in the household's information set at time t.

[4] We defer discussion of relative risk aversion until the next subsection because defining total household wealth is complicated by the presence of human capital—that is, the household's labor income.

it is the curvature of the value function V with respect to assets that matters, rather than the curvature of the utility kernel u with respect to consumption.[5]

Deriving the coefficient of absolute risk aversion (7) is simple enough, but the problem with (7) is that closed-form expressions for V do not exist in general, even for the simplest DSGE models. This difficulty may help to explain the widespread popularity of "shortcut" appraoches to measuring risk aversion, notably $-u_{11}(c_t^*, l_t^*)/u_1(c_t^*, l_t^*)$, which has no clear relationship to (7) except in the one-good one-period case. Boldrin, Christiano, and Fisher (1997) derive closed-form solutions for V—and hence risk aversion—for some very simple endowment economy models, but these models all exclude labor. Their approach is thus a nonstarter for even the simplest DSGE models.

We solve this problem by observing that V_1 and V_{11} often can be computed even when closed-form solutions for V cannot be. The case of V_1 is straightforward, following from the Benveniste-Scheinkman equation:

$$V_1(a_t; \theta_t) = (1 + r_t)\, u_1(c_t^*, l_t^*), \tag{8}$$

which states that the marginal value of a dollar of beginning-of-period assets equals the marginal utility of consumption times $1 + r_t$ (the interest rate appears because beginning-of-period assets generate income in period t). In (8), u_1 is a known function. Although closed-form solutions for the functions c^* and l^* are not known in general, the points c_t^* and l_t^* often are known—for example, when they are evaluated at the nonstochastic steady state, c and l.

We can compute V_{11} by noting that equation (8) holds for general a_t; hence we can differentiate (8) to yield:

$$V_{11}(a_t; \theta_t) \;=\; (1 + r_t)\left[u_{11}(c_t^*, l_t^*)\, \frac{\partial c_t^*}{\partial a_t} \;+\; u_{12}(c_t^*, l_t^*)\, \frac{\partial l_t^*}{\partial a_t} \right]. \tag{9}$$

All that remains is to find the derivatives $\partial c_t^* / \partial a_t$ and $\partial l_t^* / \partial a_t$.

We solve for $\partial l_t^* / \partial a_t$ by differentiating the household's intratemporal optimality condition:

$$-u_2(c_t^*, l_t^*) \;=\; w_t\, u_1(c_t^*, l_t^*), \tag{10}$$

[5] In their discussions, Arrow (1964) and Pratt (1965) refer to utility as being defined over "money", so one could argue that they always intended for risk aversion to be measured using indirect utility or the value function.

with respect to a_t, and rearranging terms to yield:

$$\frac{\partial l_t^*}{\partial a_t} = -\lambda_t \frac{\partial c_t^*}{\partial a_t}\,, \tag{11}$$

where

$$\lambda_t \;\equiv\; \frac{w_t u_{11}(c_t^*, l_t^*) + u_{12}(c_t^*, l_t^*)}{u_{22}(c_t^*, l_t^*) + w_t u_{12}(c_t^*, l_t^*)} \;=\; \frac{u_1(c_t^*, l_t^*) u_{12}(c_t^*, l_t^*) - u_2(c_t^*, l_t^*) u_{11}(c_t^*, l_t^*)}{u_1(c_t^*, l_t^*) u_{22}(c_t^*, l_t^*) - u_2(c_t^*, l_t^*) u_{12}(c_t^*, l_t^*)}\,. \tag{12}$$

Note that, if consumption and leisure in period t are normal goods, then λ_t must be positive. To compute risk aversion in the model, it only remains to solve for the derivative $\partial c_t^*/\partial a_t$.

Intuitively, $\partial c_t^*/\partial a_t$ should not be too difficult to compute: it is just the household's marginal propensity to consume today out of a change in assets, which we can deduce from the household's Euler equation and budget constraint. Differentiating the household's Euler equation:

$$u_1(c_t^*, l_t^*) \;=\; \beta E_t (1 + r_{t+1})\, u_1(c_{t+1}^*, l_{t+1}^*), \tag{13}$$

with respect to a_t yields:[6]

$$u_{11}(c_t^*, l_t^*) \frac{\partial c_t^*}{\partial a_t} + u_{12}(c_t^*, l_t^*) \frac{\partial l_t^*}{\partial a_t} = \beta E_t (1+r_{t+1}) \left[u_{11}(c_{t+1}^*, l_{t+1}^*) \frac{\partial c_{t+1}^*}{\partial a_t} + u_{12}(c_{t+1}^*, l_{t+1}^*) \frac{\partial l_{t+1}^*}{\partial a_t} \right] \tag{14}$$

Substituting in for $\partial l_t^*/\partial a_t$ gives:

$$(u_{11}(c_t^*, l_t^*) - \lambda_t u_{12}(c_t^*, l_t^*)) \frac{\partial c_t^*}{\partial a_t} = \beta E_t (1+r_{t+1}) \left(u_{11}(c_{t+1}^*, l_{t+1}^*) - \lambda_{t+1} u_{12}(c_{t+1}^*, l_{t+1}^*) \right) \frac{\partial c_{t+1}^*}{\partial a_t}\,. \tag{15}$$

Evaluating (15) at steady state, $\beta = (1+r)^{-1}$, $\lambda_t = \lambda_{t+1} = \lambda$, and the u_{ij} cancel, giving:

$$\frac{\partial c_t^*}{\partial a_t} \;=\; E_t \frac{\partial c_{t+1}^*}{\partial a_t} \;=\; E_t \frac{\partial c_{t+k}^*}{\partial a_t}\,, \quad k = 1, 2, \ldots \tag{16}$$

$$\frac{\partial l_t^*}{\partial a_t} \;=\; E_t \frac{\partial l_{t+1}^*}{\partial a_t} \;=\; E_t \frac{\partial l_{t+k}^*}{\partial a_t}\,, \quad k = 1, 2, \ldots \tag{17}$$

[6] By $\partial c_{t+1}^*/\partial a_t$ we mean:

$$\frac{\partial c_{t+1}^*}{\partial a_t} \;=\; \frac{\partial c_{t+1}^*}{\partial a_{t+1}} \frac{da_{t+1}^*}{da_t} \;=\; \frac{\partial c_{t+1}^*}{\partial a_{t+1}} \left[1 + r_{t+1} + w_t \frac{\partial l_t^*}{\partial a_t} - \frac{\partial c_t^*}{\partial a_t} \right],$$

and analogously for $\partial l_{t+1}^*/\partial a_t$, $\partial c_{t+2}^*/\partial a_t$, $\partial l_{t+2}^*/\partial a_t$, etc.

In other words, whatever the change in the household's consumption today, it must be the same as the expected change in consumption tomorrow, and the expected change in consumption at each future date $t + k$.[7]

The household's budget constraint is implied by asset accumulation equation (2) and transversality condition (3). Differentiating (2) with respect to a_t, evaluating at steady state, and applying (3), (16), and (17) gives:

$$\frac{1+r}{r}\,\frac{\partial c_t^*}{\partial a_t} \;=\; 1+\frac{1+r}{r}\,w\,\frac{\partial l_t^*}{\partial a_t}\,. \tag{18}$$

That is, the budget constraint implies that the expected present value of changes in consumption must equal the change in assets plus the expected present value of changes in labor income.

Combining (18) with (11), we can now solve for $\partial c_t^*/\partial a_t$:

$$\frac{\partial c_t^*}{\partial a_t} \;=\; \frac{r}{1+r}\,\frac{1}{1+w\lambda}\,. \tag{19}$$

In response to an increase in assets, the household raises consumption in every period by the extra asset income, $r/(1+r)$, adjusted downward by an amount $1+w\lambda$ that takes into account the household's decrease in hours worked.

We are now in a position to compute the household's coefficient of absolute risk aversion. As shown in the Appendix, evaluating (7) at steady state yields:

$$\frac{-\,E_t V_{11}(a_{t+1}^*;\theta_{t+1})}{E_t V_1(a_{t+1}^*;\theta_{t+1})} \;=\; \frac{-V_{11}(a;\theta)}{V_1(a;\theta)}\,. \tag{20}$$

Substituting (8), (9), (11), and (19) into (20), we have:

$$\frac{-V_{11}(a;\theta)}{V_1(a;\theta)} \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{1}{1+w\lambda}\,\frac{r}{1+r}\,. \tag{21}$$

When there is no labor in the model—that is, $u_{12} = u_2 = 0$ and $w = 0$—the household's coefficient of absolute risk aversion is just the traditional measure, $-u_{11}/u_1$, times the ratio $r/(1+r)$, which translates assets into current-period consumption. This observation

[7] Note that this equality does not follow from the steady state assumption. For example, in a model with internal habits, which we will consider in Section 4, the individual household's optimal consumption response to a change in assets increases with time, even starting from steady state.

is actually quite remarkable: for *any* utility kernel u, the traditional, static measure of risk aversion is also the correct measure in the dynamic context (without labor). This is true regardless of whether u or the rest of the model is homothetic, and no matter what the functional form of V.

More generally, when household preferences include labor, risk aversion is less than the traditional measure by the factor $1 + w\lambda$, which takes into account the household's ability to partially absorb shocks to income through changes in hours worked. When $u_{12} \neq 0$, risk aversion is further attenuated or amplified depending on the sign and size of the interaction u_{12}. As we show in the examples in Section 2.5, below, the household's ability to absorb shocks to income through changes in hours worked can have dramatic effects on the household's attitudes toward risk. We turn to these examples after first defining relative risk aversion.

2.4 The Coefficient of Relative Risk Aversion

The difference between absolute and relative risk aversion is the size of the hypothetical gamble faced by the household. If the household faces a one-shot gamble of size A_t in period t, that is:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + A_t \varepsilon_{t+1}, \tag{22}$$

or the household can pay a one-time fee $A_t d\mu$ in period t to avoid this gamble, then the household's coefficient of risk aversion, $2d\mu/d\sigma^2$, for this gamble is given by:

$$\frac{-A_t E_t V_{11}(a^*_{t+1}; \theta_{t+1})}{E_t V_1(a^*_{t+1}; \theta_{t+1})}. \tag{23}$$

The natural definition of A_t, considered by Arrow (1964) and Pratt (1965), is the household's wealth at time t. In this case, the gamble in (22) is over a fraction of the household's wealth and (23) is referred to as the household's coefficient of relative risk aversion.

In the multiple-good, multi-period context of the present paper, household wealth is more difficult to define than in Arrow (1964) and Pratt (1965). In particular, household wealth consists not just of financial assets a_t and the present value of transfers d_t, but also human wealth—the present value of the household's ability to generate labor income, $w_t l_t$.

Defining human wealth in the DSGE framework is not always straightforward, so we consequently define two measures of household wealth A_t and hence two coefficients of relative risk aversion (23), which differ only in their definition of A_t. When the household's time endowment is not well-defined in the model—as, for example, when the household's utility kernel is given by $c_t^{1-\gamma}/(1-\gamma) - l_t^{1+\chi}$ and no upper bound on l_t is specified—then we define the household's total wealth A_t to be the present discounted value of consumption, c_t^*; that is, $A_t = c_t^* + \beta E_t\big(u_1(c_{t+1}^*, l_{t+1}^*)/u_1(c_t^*, l_t^*)\big)A_{t+1}$. Equivalently, A_t equals financial assets a_t plus the present discounted value of labor income $w_t l_t^*$ and transfers d_t, which follows from the household's budget constraint (2) and (3). In steady state, $A = c/(1-\beta) = c(1+r)/r$. Under this definition of A_t, the gamble in (22) is over a fraction of the household's lifetime consumption, and we refer to (23) as the household's consumption-based coefficient of relative risk aversion.

When the household's time endowment $\bar{l}$ is well defined, then we can also consider an alternative definition of household wealth, $\tilde{A}_t$, that incorporates leisure as well as goods consumption. In this case, we define the household's leisure-and-consumption-based coefficient of relative risk aversion by setting $\tilde{A}_t$ equal to the present discounted value of household leisure $w_t(\bar{l} - l_t^*)$ plus consumption c_t^*. From (2) and (3), this equals financial assets a_t plus the present discounted value of the household's time endowment $w_t\bar{l}$ and transfers d_t. Thus, the only difference between the consumption-based and leisure-and-consumption-based measures of wealth is whether human capital is measured using $\bar{l}$ or l_t^*. In steady state, $\tilde{A} = (c + w(\bar{l} - l))/(1-\beta)$.

From (21) and (23), the household's consumption-based coefficient of relative risk aversion, evaluated at steady state, is:

$$\frac{-A\, V_{11}(a;\theta)}{V_1(a;\theta)} = \frac{-u_{11} + \lambda u_{12}}{u_1} \, \frac{c}{1 + w\lambda}\,, \tag{24}$$

while the leisure-and-consumption-based coefficient of relative risk aversion is given by:

$$\frac{-\tilde{A}\, V_{11}(a;\theta)}{V_1(a;\theta)} = \frac{-u_{11} + \lambda u_{12}}{u_1} \, \frac{c + w(\bar{l} - l)}{1 + w\lambda}\,. \tag{25}$$

Of course, (24) and (25) are related by the ratio of the two gambles, $(c + w(\bar{l} - l))/c$. Other definitions of relative risk aversion, corresponding to alternative definitions of wealth and the size of the gamble A_t, are also possible, but the above two definitions are the most natural for several reasons. First, both definitions reduce to the usual present discounted value of income or consumption when there is no human capital in the model. Second, both measures of relative risk aversion reduce to the traditional $-c\,u_{11}/u_1$ when there is no labor in the model—that is, when $u_2 = 0$ and $w = 0$. Third, in steady state the household consumes exactly the flow of income from its wealth, $Ar/(1+r)$, consistent with standard permanent income theory (where one must include the value of leisure $w(\bar{l} - l)$ as part of consumption when the value of leisure is included in wealth).

2.5 Examples

Some simple examples illustrate how ignoring the household's labor margin can lead to wildly inaccurate measures of the household's true attitudes toward risk in a DSGE model.

Example 2.1. Consider the additively separable utility kernel:

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi}\,, \tag{26}$$

where $\gamma,\ \chi,\ \chi_0 > 0$. The traditional measure of risk aversion for this utility kernel is $-c\,u_{11}/u_1 = \gamma$, but the household's actual consumption-based coefficient of relative risk aversion is given by (24):

$$\frac{-AV_{11}}{V_1} = \frac{-cu_{11}}{u_1} \frac{1}{1 + w\,\frac{wu_{11}}{u_{22}}} = \frac{\gamma}{1 + \frac{\gamma}{\chi}\frac{wl}{c}}\,. \tag{27}$$

The household's leisure-and-consumption-based coefficient of relative risk aversion (25) is not well defined in this example (the household's risk aversion can be made arbitrarily large or small just by varying the household's time endowment $\bar{l}$), so we focus only on the consumption-based measure (27).

In steady state, $c \approx wl$,[8] so (27) can be written as:

$$\frac{-AV_{11}}{V_1} \approx \frac{1}{\frac{1}{\gamma} + \frac{1}{\chi}}\,. \tag{28}$$

[8] In steady state, $c = ra + wl + d$, so $c = wl$ holds exactly if there is neither capital nor transfers in the model. In any case, $ra + d$ is typically small for standard calibrations in the literature.

Note that (28) is less than the traditional measure of risk aversion, γ, by a factor of $1 + \gamma/\chi$. Thus, if $\gamma = 2$ and $\chi = 1$—parameter values that are well within the range of estimates in the literature—then the household's true risk aversion is less than the traditional measure by a factor of about three. If χ is very large, then the bias from using the traditional measure is small because household labor supply is essentially fixed. However, as χ approaches 0, a common benchmark in the literature, the bias explodes and true risk aversion approaches zero—that is, the household becomes risk neutral. Intuitively, households with linear disutility of work are risk neutral with respect to gambles over wealth because they can completely offset those gambles at the margin by working more or fewer hours, and households with linear disutility of work are clearly risk neutral with respect to gambles over hours.

Expression (28) also helps to clarify several points. First, risk aversion in the model is a combination of both parameters γ and χ, reflecting that the household absorbs income gambles along both of its two margins, consumption and labor. Second, for any given γ, actual risk aversion in the model can lie anywhere between 0 and γ, depending on χ. That is, having an additional margin with which to absorb income gambles reduces the household's aversion to risk. Third, (28) is symmetric in γ and χ, reflecting that labor and consumption enter essentially symmetrically into u in this example and play an essentially equal role in absorbing income shocks (equal, that is, before taking into account the importances γ and χ). Put differently, ignoring the labor margin in this example would be just as erroneous as ignoring the consumption margin.

Example 2.2. Consider the King-Plosser-Rebelo-type (1988) utility kernel:

$$u(c_t, l_t) \;=\; \frac{c_t^{1-\gamma}\,(1-l_t)^{\chi(1-\gamma)}}{1-\gamma}, \tag{29}$$

where $\gamma > 0$, $\gamma \neq 1$, $\chi > 0$, $l_t < 1$, and $\chi(1-\gamma) < \gamma$ for concavity. The traditional measure of risk aversion for (29) is γ, but the household's actual leisure-and-consumption-based coefficient of relative risk aversion is given by:

$$\frac{-\tilde{A}\,V_{11}}{V_1} \;=\; \frac{-u_{11} + \lambda u_{12}}{u_1}\;\frac{c + w(1-l)}{1 + w\lambda} \;=\; \gamma - \chi(1-\gamma). \tag{30}$$

Note that concavity of (29) implies that (30) is positive. As in the previous example, (30) depends on both γ and χ, and can lie anywhere between 0 and the traditional measure γ, depending on χ. In this example, risk aversion is less than the traditional measure by the amount $\chi(1-\gamma)$. As χ approaches $\gamma/(1-\gamma)$—that is, as utility approaches Cobb-Douglas—the household becomes risk neutral; in this case, household utility along the line $c_t = w_t(1-l_t)$ is linear, so the household finds it optimal to absorb shocks to wealth along that line.

The household's consumption-based coefficient of relative risk aversion is a bit more complicated than (30):

$$\frac{-A\,V_{11}}{V_1} = \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{c}{1+w\lambda} = \frac{\gamma-\chi(1-\gamma)}{1+\chi}\,. \tag{31}$$

Again, (31) is a combination of the parameters γ and χ, and can lie anywhere between 0 and γ, depending on χ. Neither (30) nor (31) equals the traditional measure γ, except for the special case $\chi = 0$.

2.6 Discussion

In the preceding sections, we showed that the labor margin has important implications for Arrow-Pratt risk aversion with respect to gambles over income or wealth. We now show that Arrow-Pratt risk aversion with respect to these gambles is the right concept for asset pricing.

2.6.1 Measuring Risk Aversion with V As Opposed to u

Some comparison of the expressions $-V_{11}/V_1$ and $-u_{11}/u_1$ helps to clarify why the former measure is the relevant one for pricing assets, such as stocks or bonds, in the model.

As described in Sections 2.3 and 2.4, and derived in detail in the Appendix, the expression $-V_{11}/V_1$ is the Arrow-Pratt coefficient of (absolute) risk aversion for gambles over income or wealth in period t. In contrast, the expression $-u_{11}/u_1$ in a DSGE model is the Arrow-Pratt coefficient of risk aversion for a hypothetical gamble in which the household is *forced to consume* in period t the outcome of the gamble.

Clearly, it is the former concept that corresponds to the stochastic payoffs of a standard asset, such as a stock or bond. When the household purchases such a security in period t, that security is resaleable in period $t+1$ and the proceeds can be reinvested, saved at the risk-free rate, or consumed, as the household sees fit—that is, the proceeds contribute directly to the household's income or wealth in period $t+1$. The household should thus evaluate those securities in the same way that it does the hypothetical gambles over income or wealth in Sections 2.3 and 2.4.

In order for $-u_{11}/u_1$ to be the relevant measure for pricing a security, it is not enough that the security pay off in units of consumption in period $t+1$. The household would additionally have to be prevented from adjusting its consumption and labor choices in period $t+1$ in response to the security's payoffs, so that the household is forced to absorb those payoffs into period $t+1$ consumption. Examples of such a security are difficult to imagine—all standard securities (such as stocks, bonds, options, etc.) correspond to gambles over income or wealth, for which the $-V_{11}/V_1$ measure of risk aversion is the appropriate one.

2.6.2 Arrow-Pratt Risk Aversion and the Stochastic Discount Factor

Assets in a DSGE model are priced using the household's stochastic discount factor, which is tied to the household's marginal utility of consumption. One might then wonder, how does the labor margin enter into this equation? Here, we show the tight link between Arrow-Pratt risk aversion, the labor margin, and the stochastic discount factor.

Let m_t denote the household's stochastic discount factor and p_t the cum-dividend price at time t of a risky asset, with $E_t p_{t+1}$ normalized to unity. The difference between the risk-neutral price of the asset and its actual price:

$$E_t m_{t+1} E_t p_{t+1} - E_t m_{t+1} p_{t+1} \;=\; -\text{Cov}_t(m_{t+1}, p_{t+1}) \;=\; -\text{Cov}_t(dm_{t+1}, dp_{t+1}), \qquad (32)$$

measures the risk premium on the asset, where Cov_t denotes the covariance conditional on information at time t, and $dx \equiv x_{t+1} - E_t x_{t+1}$, $x \in \{m, p\}$. Since $m_{t+1} = \beta u_1(c^*_{t+1}, l^*_{t+1})/u_1(c^*_t, l^*_t)$, for small changes dc^*_{t+1} and dl^*_{t+1}, we have:

$$dm_{t+1} \approx \frac{\beta}{u_1(c^*_t, l^*_t)} \left[u_{11}(c^*_{t+1}, l^*_{t+1}) dc^*_{t+1} + u_{12}(c^*_{t+1}, l^*_{t+1}) dl^*_{t+1} \right], \qquad (33)$$

conditional on information at time t. Equation (33) shows how the household's labor margin as well as consumption margin matter for the stochastic discount factor.

We can relate (33) to Arrow-Pratt risk aversion by solving for dc^*_{t+1} and dl^*_{t+1} as in the previous sections. From $w_{t+1} = -u_2(c^*_{t+1}, l^*_{t+1})/u_1(c^*_{t+1}, l^*_{t+1})$, we have, to first order:

$$dl^*_{t+1} = -\lambda_{t+1} dc^*_{t+1} - \frac{u_1}{u_{22} + w_{t+1}u_{12}}\, dw_{t+1}. \tag{34}$$

In the Appendix, we show that the household's Euler equation and budget constraint, together with (34), imply:

$$\begin{aligned} dc^*_{t+1} \;=\; & \frac{r}{1+r}\,\frac{1}{1+\lambda w}\left[da_{t+1} + \sum_{k=0}^{\infty}\frac{1}{(1+r)^k}\, l\, dw_{t+1+k} + \sum_{k=0}^{\infty}\frac{1}{(1+r)^{2k}}(c - wl)dR_{t+1,t+1+k}\right] \\ & +\; \Psi_1 dw_{t+1} \;+\; \frac{r}{1+r}\,\frac{1}{1+\lambda w}\sum_{k=0}^{\infty}\frac{1}{(1+r)^k}\big(\Psi_2 dR_{t+1,t+1+k} + \Psi_3 dw_{t+1+k}\big), \end{aligned} \tag{35}$$

where Ψ_1, Ψ_2, and Ψ_3 are constants reported in the Appendix, and where $R_{t+1,t+k} \equiv \prod_{i=1}^{k}(1+r_{t+i})$. The first term in brackets describes the change in present value of household income, and thus the first line of (35) describes the income effect. The second line of (35) describes the substitution effect: changes in consumption due to changes in current and future interest rates and wages.

Substituting (34)–(35) into (33) yields:

$$\begin{aligned} dm_{t+1} \;\approx\; & \beta\,\frac{u_{11} - \lambda u_{12}}{u_1}\,\frac{1}{1+w\lambda}\,\frac{r}{1+r}\left[da_{t+1} + \sum_{k=0}^{\infty}\frac{1}{(1+r)^k}\, l\, dw_{t+1+k}\right. \\ & \left. +\; \sum_{k=0}^{\infty}\frac{c - wl}{(1+r)^{2k}}\, dR_{t+1,t+1+k} \;+\; \sum_{k=0}^{\infty}\frac{1}{(1+r)^k}\big(\Psi_2 dR_{t+1,t+1+k} + \Psi_3 dw_{t+1+k}\big)\right]. \end{aligned} \tag{36}$$

Note that the term before the brackets in (36) is exactly the Arrow-Pratt coefficient of absolute risk aversion (times β).[9]

Comparing (36) to (32) shows the extremely tight link between Arrow-Pratt risk aversion and the risk premium on the asset: the latter equals the Arrow-Pratt coefficient of absolute risk aversion times the sum of covariances of the asset price with household assets, aggregate wages, and aggregate interest rates. This link should not be too surprising: Arrow-Pratt risk aversion describes the risk premium for very simple gambles over household income or wealth. Here we have shown that this risk aversion coefficient can also be derived through the standard stochastic discounting equation applied to gambles that may be correlated with aggregate variables such as interest rates and wages.

[9] The factor β disappears if we consider $d\log m_{t+1}$ rather than dm_{t+1}.

3. Generalized Recursive Preferences

We now turn to the case of generalized recursive preferences, as in Epstein and Zin (1989) and Weil (1989). The household's asset accumulation equation (2) and transversality condition (3) are the same as in Section 2, but now instead of maximizing (1), the household chooses c_t and l_t to maximize the recursive expression:[10]

$$V(a_t;\theta_t) = \max_{c_t,l_t} u(c_t,l_t) + \beta \left(E_t V(a_{t+1};\theta_{t+1})^{1-\alpha}\right)^{1/(1-\alpha)}, \tag{37}$$

where α can be any real number. Note that (37) is the same as (4), but with the value function "twisted" and "untwisted" by the coefficient $1-\alpha$. When $\alpha = 0$, the preferences given by (37) reduce to the special case of expected utility.

If $u \geq 0$ everywhere, then the proof of Theorem 3.1 in Epstein and Zin (1989) shows that there exists a solution V to (37) with $V \geq 0$. If $u \leq 0$ everywhere, then it is natural to let $V \leq 0$ and reformulate the recursion as:

$$V(a_t;\theta_t) = \max_{c_t,l_t} u(c_t,l_t) - \beta \left(E_t(-V(a_{t+1};\theta_{t+1}))^{1-\alpha}\right)^{1/(1-\alpha)}. \tag{38}$$

The proof in Epstein and Zin (1989) also demonstrates the existence of a solution V to (38) with $V \leq 0$ in this case.[11] When $u \geq 0$, higher values of α correspond to greater degrees of risk aversion, and when $u \leq 0$, the opposite is true: higher values of α correspond to lesser degrees of risk aversion.

The main advantage of generalized recursive preferences (37) is that they allow for greater flexibility in modeling risk aversion and the intertemporal elasticity of substitution. In (37), the intertemporal elasticity of substitution over deterministic consumption paths is exactly the same as in (4), but the household's risk aversion to gambles can be amplified (or attenuated) by the additional parameter α.

[10] Note that, traditionally, Epstein-Zin preferences over consumption streams have been written as:

$$\widetilde{V}(a_t;\theta_t) = \max_{c_t} \left[c_t^\rho + \beta \left(E_t \widetilde{V}(a_{t+1};\theta_{t+1})^{\widetilde{\alpha}}\right)^{\rho/\widetilde{\alpha}}\right]^{1/\rho},$$

but by setting $V = \widetilde{V}^\rho$ and $\alpha = 1 - \widetilde{\alpha}/\rho$, this can be seen to correspond to (37).

[11] In this paper, we exclude the case where u is sometimes positive and sometimes negative, in order to avoid complications related to complex numbers.

3.1 Coefficients of Absolute and Relative Risk Aversion

We confront the household with the same hypothetical gamble as in (6). As shown in the Appendix, the Arrow-Pratt coefficient of absolute risk aversion for these preferences is given by:

$$\frac{-E_t V(a_{t+1}^*;\theta_{t+1})^{-\alpha}\left[V_{11}(a_{t+1}^*;\theta_{t+1}) \;-\; \alpha\,\dfrac{V_1(a_{t+1}^*;\theta_{t+1})^2}{V(a_{t+1}^*;\theta_{t+1})}\right]}{E_t V(a_{t+1}^*;\theta_{t+1})^{-\alpha} V_1(a_{t+1}^*;\theta_{t+1})}\,. \tag{39}$$

As also shown in the Appendix, this simplifies to:

$$\frac{-V_{11}(a;\theta)}{V_1(a;\theta)} \;+\; \alpha\,\frac{V_1(a;\theta)}{V(a;\theta)}\,, \tag{40}$$

when evaluated at steady state. The first term in (40) is the same as the expected utility case (7). The second term in (40) reflects the amplification or attenuation of risk aversion from the additional curvature parameter α. Note that when $\alpha = 0$, (40) reduces to (7). When $u \geq 0$ and hence $V \geq 0$, higher values of α correspond to greater degrees of risk aversion, and when u and $V \leq 0$, higher values of α correspond to lesser degrees of risk aversion.

Similarly, the household's coefficient of relative risk aversion is given by A_t times (39), which, evaluated at steady state, simplifies to:

$$\frac{-A_t V_{11}(a;\theta)}{V_1(a;\theta)} \;+\; \alpha\,\frac{A_t V_1(a;\theta)}{V(a;\theta)}\,. \tag{41}$$

We define the household's total wealth A_t, based on lifetime consumption or lifetime leisure and consumption, as in the previous section, and we refer to (41) as the consumption-based coefficient of relative risk aversion or the leisure-and-consumption-based coeffcient of relative risk aversion, depending on the definition of A_t.[12]

Expressions (40) and (41) highlight an important feature of risk aversion with generalized recursive preferences: it is not invariant with respect to level shifts of the utility

[12] Note that, with generalized recursive preferences, the household's discount factor is given by:

$$\frac{\beta u_1(c_{t+1}^*, l_{t+1}^*)}{u_1(c_t^*, l_t^*)}\left(\frac{V(a_{t+1}^*;\theta_{t+1})}{(E_t V(a_{t+1}^*;\theta_{t+1})^{1-\alpha})^{1/(1-\alpha)}}\right)^{-\alpha},$$

which must be used to compute household wealth. At steady state, however, this simplifies to the usual β.

kernel, except for the special case of expected utility ($\alpha = 0$). That is, the utility kernel $u(\cdot,\cdot)$ and $u(\cdot,\cdot) + k$, where k is a constant, lead to different household attitudes toward risk. The household's preferences are invariant, however, with respect to multiplicative shifts of the utility kernel.

When it comes to computing the risk aversion coefficients (40) and (41), expressions (8) through (19) for V_1, V_{11}, $\partial l_t^*/\partial a_t$, and $\partial c_t^*/\partial a_t$ continue to apply in the current context. Moreover, $V = u(c,l)/(1-\beta)$ at the steady state. Thus, (40) can be written as:

$$\frac{-V_{11}}{V_1} + \alpha \frac{V_1}{V} \;=\; \frac{-u_{11} + \lambda u_{12}}{u_1} \, \frac{1}{1 + w\lambda} \, \frac{r}{1+r} \;+\; \alpha \, \frac{u_1}{u} \, \frac{r}{1+r}, \tag{42}$$

and (41) as:

$$\frac{-AV_{11}}{V_1} + \alpha \frac{AV_1}{V} \;=\; \frac{-u_{11} + \lambda u_{12}}{u_1} \, \frac{c}{1 + w\lambda} \;+\; \alpha \, \frac{c\, u_1}{u}\,, \tag{43}$$

taking $A = \frac{1+r}{r}\, c$. Ignoring the household's labor margin biases both the first and second terms in (42) and (43). The bias in the first term is the same as for expected utility. Bias in the second term can arise from ignoring the presence of labor in steady-state utility, $u(c,l)$. This is not to mention the bias from excluding the value of leisure in wealth if it is the household's leisure-and-consumption-based coefficient of relative risk aversion that is of interest.

3.2 Examples

Example 3.1. Consider the additively separable utility kernel:

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi}\,, \tag{44}$$

with generalized recursive preferences and $\chi > 0$, $\chi_0 > 0$, and $\gamma > 1$, which was used by Rudebusch and Swanson (2009) to analyze the bond premium puzzle in a DSGE model.[13] In this case, where $u(\cdot,\cdot) < 0$, risk aversion is decreasing in α, and $\alpha < 0$ corresponds to preferences that are more risk averse than expected utility.

[13] For the technical reasons discussed above, we require $u(\cdot,\cdot) < 0$; hence for simplicity we restrict attention here to the case $\gamma > 1$. The case $\gamma \leq 1$ can be considered if we place restrictions on the domain of c_t and l_t such that $u(\cdot,\cdot) < 0$ on that domain. One can always choose units for c_t and l_t in such a way that this doesn't represent much of a constraint in practice. Of course, one can also consider alternative utility kernels with $\gamma \leq 1$ for which $u(\cdot,\cdot) > 0$ holds.

In models without labor, preferences $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma)$ imply a coefficient of relative risk aversion of $\gamma + \alpha(1-\gamma)$, which we will refer to as the traditional measure.[14] However, when we take into account both the consumption and labor margins of the more general preferences (44), the household's consumption-based coefficient of relative risk aversion (43) is given instead by:

$$\begin{aligned} \frac{-AV_{11}}{V_1} + \alpha\frac{AV_1}{V} &= \frac{\gamma}{1 + \frac{\gamma}{\chi}\frac{wl}{c}} + \frac{\alpha(1-\gamma)}{1 + \frac{\gamma-1}{1+\chi}\frac{wl}{c}}\,, \\ &\approx \frac{\gamma}{1 + \frac{\gamma}{\chi}} + \frac{\alpha(1-\gamma)}{1 + \frac{\gamma-1}{1+\chi}}\,, \end{aligned} \tag{45}$$

using $c \approx wl$. As in Example 2.1, the household's leisure-and-consumption-based coefficient of relative risk aversion is not well defined in this example, so we restrict attention to the consumption-based measure (45).

As χ becomes large, household labor becomes less flexible and the bias from ignoring the labor margin shrinks to zero (that is, (45) approaches $\gamma + \alpha(1-\gamma)$). As χ approaches zero, (45) decreases to $\alpha(1-\gamma)/\gamma$, which is close to zero if we think of γ as being small (not much greater than unity). Thus, for given values of γ and α, actual household risk aversion can lie anywhere between about zero and $\gamma + \alpha(1-\gamma)$, depending on the value of χ.

Example 3.2. Van Binsbergen et al. (2008) and Backus, Routledge, and Zin (2008) consider generalized recursive preferences with $u(c_t, l_t)$ given by a Cobb-Douglas aggregate over consumption and leisure:

$$u(c_t, l_t) = \frac{\left(c_t^{\nu}(1-l_t)^{1-\nu}\right)^{1-\gamma}}{1-\gamma}\,, \tag{46}$$

where $\gamma > 0$ and $\nu \in (0,1)$. Van Binsbergen et al. use $\gamma+\alpha(1-\gamma)$ to measure risk aversion, while Backus et al. use $\gamma\nu + \alpha(1-\gamma)\nu + (1-\nu)$, after mapping each study's notation over to the present paper's. The former measure effectively treats consumption and leisure as a single composite commodity, while the latter measure allows the parameter ν—the

[14] Set $\chi_0 = 0$ and $w = 0$ and substitute (44) into (43). This is the case, for example, in Epstein and Zin (1989) and Boldrin, Christiano, and Fisher (1997), which do not have labor. In models with variable labor, Rudebusch and Swanson (2009) refer to $\gamma + \alpha(1-\gamma)$ as the *quasi* coefficient of relative risk aversion.

importance of the household's labor margin—to affect the household's attitudes toward risk.

Substituting (46) into (41), the household's consumption-based coefficient of relative risk aversion in this example is given by:

$$\frac{-AV_{11}}{V_1} + \alpha\frac{AV_1}{V} \;=\; \gamma\nu + \alpha(1-\gamma)\nu, \tag{47}$$

while the leisure-and-consumption-based coefficient of relative risk aversion is given by:

$$\frac{-\tilde{A}V_{11}}{V_1} + \alpha\frac{\tilde{A}V_1}{V} \;=\; \gamma + \alpha(1-\gamma). \tag{48}$$

The latter agrees with the Van Binsbergen et al. (2008) measure of risk aversion, while the former is similar to (though not quite the same as) the Backus et al. (2008) measure. In this paper, we have provided the formal justification for both measures, (47) and (48).[15]

4. Internal and External Habits

Many studies in macroeconomics and finance assume that households derive utility not from consumption itself, but from consumption relative to some reference level, or habit stock. Habits, in turn, can have substantial effects on the household's attitudes toward risk, as discussed by Campbell and Cochrane (1999) and Boldrin, Christiano, and Fisher (1997). In this section, we investigate how habits affect risk aversion in the DSGE framework.

We thus generalize the household's utility kernel in this section to the specification $u(c_t - h_t, l_t)$, where h_t denotes the household's reference level of consumption, or habits. We focus on an additive rather than multiplicative role for habits because the implications for risk aversion are typically more interesting in the additive case.

If the habit stock h_t is external to the household ("keeping up with the Joneses" utility), then the parameters that govern the process for h_t can be incorporated into the exogenous state vector θ_t, and the analysis proceeds much as in the previous sections. However, if the habit stock h_t is a function of the household's own past levels of consumption, then the state variables of the household's optimization problem must be augmented to include the state variables that govern h_t. We consider each of these cases in turn.

[15] Note that as ν decreases to zero, the ratio of wages to consumption becomes infinite and consumption becomes trivial to insure with variations in labor supply, which is why the consumption-based coefficient of relative risk aversion (47) approaches zero.

4.1 External Habits

When the household's reference consumption level h_t in the utility kernel $u(c_t - h_t, l_t)$ is external to the household, then the parameters that govern h_t can be incorporated into the exogenous state vector θ_t and the analysis of the previous sections can be carried over essentially as before. In particular, the coefficient of absolute risk aversion continues to be given by (7) in the case of expected utility and (39) in the case of generalized recursive preferences; the household's intratermporal optimality condition still implies:

$$\frac{\partial l_t^*}{\partial a_t} \;=\; -\lambda_t \frac{\partial c_t^*}{\partial a_t}\,, \tag{49}$$

where λ_t is still given by (12), and the household's Euler equation still implies that, at the steady state:

$$\frac{\partial c_t^*}{\partial a_t} \;=\; E_t \frac{\partial c_{t+1}^*}{\partial a_t} \;=\; E_t \frac{\partial c_{t+k}^*}{\partial a_t}\,, \quad k = 1, 2, \ldots \tag{50}$$

$$\frac{\partial l_t^*}{\partial a_t} \;=\; E_t \frac{\partial l_{t+1}^*}{\partial a_t} \;=\; E_t \frac{\partial l_{t+k}^*}{\partial a_t}\,, \quad k = 1, 2, \ldots \tag{51}$$

Together with the household's budget constraint, it again follows that:

$$\frac{\partial c_t^*}{\partial a_t} \;=\; \frac{r}{1+r}\,\frac{1}{1+w\lambda}\,, \tag{52}$$

at the steady state, as before.

The only real differences that arise relative to the case without habits is, first, that the steady-state point at which the derivatives of $u(\cdot,\cdot)$ are evaluated is $(c-h, l)$ rather than (c, l), and second, that relative risk aversion confronts the household with a hypothetical gamble over c rather than $c - h$, which has a tendency to make the household more risk averse for a given functional form $u(\cdot,\cdot)$, because the stakes are larger.

Example 4.1. Consider the case of expected utility with the additively separable utility kernel:

$$u(c_t - h_t, l_t) \;=\; \frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi}\,, \tag{53}$$

where γ, χ, $\chi_0 > 0$. The traditional measure of risk aversion for this example is $-cu_{11}/u_1 = \gamma c/(c-h)$, which exceeds γ by a factor that depends on the importance of habits relative to consumption. The household's consumption-based coefficient of relative risk aversion is given by:

$$\frac{-AV_{11}}{V_1} = \frac{-cu_{11}}{u_1} \frac{1}{1 + w\,\frac{wu_{11}}{u_{22}}}\,,$$
$$= \frac{\gamma c}{(c-h)} \frac{1}{1 + \frac{\gamma c}{\chi(c-h)} \frac{wl}{c}}\,. \tag{54}$$

When labor is absent from the model ($u_2 = 0$ and $w = 0$), the consumption-based measure agrees with the traditional measure. When labor is present in the model, the household's consumption-based coefficient of relative risk aversion (54) is less than the traditional measure by the factor $1 + \frac{\gamma c}{\chi(c-h)}$, using $wl \approx c$. Ignoring the labor margin in (54) thus leads to an even greater bias in the model with habits ($h > 0$) than in the model without habits ($h = 0$). If $\gamma = 2$, $\chi = 1$, and $h = .8c$, then the household's true risk aversion is smaller than the traditional measure by a factor of more than ten.

When the household has generalized recursive preferences rather than expected utility preferences, the consumption-based coefficient of relative risk aversion for the utility kernel (53) is given by:

$$\frac{\gamma c}{(c-h)} \frac{1}{1 + \frac{\gamma c}{\chi(c-h)} \frac{wl}{c}} + \frac{\alpha(1-\gamma)c}{(c-h)} \frac{1}{1 + \frac{c}{(c-h)} \frac{\gamma - 1}{1+\chi} \frac{wl}{c}}\,. \tag{55}$$

Again, the bias from ignoring the labor margin in (55) is even greater in the model with habits ($h > 0$) than without habits ($h = 0$).

4.2 Internal Habits

When habits are internal to the household, we must specify how the household's actions affect its future habits. In order to minimize notation and emphasize intuition, in the present section we focus on the case where habits are proportional to last period's consumption:

$$h_t = bc_{t-1}, \tag{56}$$

$b \in (0, 1)$, and we assume the household has expected utility preferences. In the Appendix, we derive closed-form expressions for the more complicated case where the household has generalized recursive preferences and the habit stock evolves according to:

$$h_t = \rho h_{t-1} + bc_{t-1}, \tag{57}$$

$\rho \in (-1, 1)$, which allows for longer-memory habits.

With internal habits, the value of h_{t+1} depends on the household's choices in period t, so we write out the dependence of the household's value function on h_t explicitly:

$$V(a_t, h_t; \theta_t) = u(c_t^* - h_t, l_t^*) + \beta E_t V(a_{t+1}^*, h_{t+1}^*; \theta_{t+1}), \tag{58}$$

where $c_t^* \equiv c^*(a_t, h_t; \theta_t)$ and $l_t^* \equiv l^*(a_t, h_t; \theta_t)$ denote the household's optimal choices for consumption and labor in period t as functions of the household's state vector, and a_{t+1}^* and h_{t+1}^* denote the optimal stocks of assets and habits in period $t+1$ that are implied by c_t^* and l_t^*; that is, $a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t^* + d_t - c_t^*$ and $h_{t+1}^* \equiv bc_t^*$.

The household's coefficient of absolute risk aversion can be derived in the same way as before, and results in the same basic expression:

$$\frac{-E_t V_{11}(a_{t+1}^*, h_{t+1}^*; \theta_{t+1})}{E_t V_1(a_{t+1}^*, h_{t+1}^*; \theta_{t+1})}. \tag{59}$$

However, computing the derivatives V_1 and V_{11} is more complicated in the case of internal habits, because of the dynamic relationship between the household's current consumption and its future habits. We now turn to computing these derivatives.

The household's first-order conditions for (58) with respect to consumption and labor are given by:

$$u_1(c_t^* - h_t, l_t^*) \;=\; \beta E_t V_1(a_{t+1}^*, h_{t+1}^*; \theta_{t+1}) \;-\; \beta b E_t V_2(a_{t+1}^*, h_{t+1}^*; \theta_{t+1}), \tag{60}$$

$$u_2(c_t^* - h_t, l_t^*) \;=\; -\beta w_t E_t V_1(a_{t+1}^*, h_{t+1}^*; \theta_{t+1}) \tag{61}$$

Equation (61) is essentially the same as in the case without habits. The first-order condition (60), however, includes the future effect of consumption on habits in the second term on the right-hand side.

Differentiating (58) with respect to its first two arguments and applying the envelope theorem yields:

$$V_1(a_t, h_t; \theta_t) \;=\; \beta(1 + r_t)\, E_t V_1(a_{t+1}^*, h_{t+1}^*; \theta_{t+1}), \tag{62}$$

$$V_2(a_t, h_t; \theta_t) \;=\; -u_1(c_t^* - h_t, l_t^*). \tag{63}$$

Equations (61) and (62) can be used to solve for V_1 in terms of current-period utility:

$$V_1(a_t, h_t; \theta_t) \;=\; -\frac{(1 + r_t)}{w_t}\, u_2(c_t^* - h_t, l_t^*), \tag{64}$$

which states that the marginal value of wealth equals the marginal utility of working fewer hours.[16] This solves for V_1.

To solve for V_{11}, differentiate (64) with respect to a_t to yield:

$$V_{11}(a_t, h_t; \theta_t) \;=\; -\frac{(1+r_t)}{w_t}\Big(u_{12}\,\frac{\partial c_t^*}{\partial a_t} + u_{22}\,\frac{\partial l_t^*}{\partial a_t}\Big), \tag{65}$$

where we drop the arguments of the u_{ij} to reduce notation. It now remains to solve for $\partial c_t^*/\partial a_t$ and $\partial l_t^*/\partial a_t$, which we do in the same manner as before, except that the dynamics of internal habits will require us to solve for $\partial c_\tau^*/\partial a_t$ and $\partial l_\tau^*/\partial a_t$ for all dates $\tau \geq t$ at the same time. To better keep track of these dynamics, we henceforth let a time subscript $\tau \geq t$ denote a generic future date and reserve the subscript t to denote the date of the current period—the period in which the household faces the hypothetical one-shot gamble.

We solve for $\partial l_\tau^*/\partial a_t$ in terms of $\partial c_\tau^*/\partial a_t$ in much the same way as the case without habits. The household's intratemporal optimality condition ((60) combined with (61)) implies:

$$-u_2(c_\tau^* - h_\tau^*, l_\tau^*) \;=\; w_\tau\big[u_1(c_\tau^* - h_\tau^*, l_\tau^*) \;+\; b\beta E_\tau V_2(a_{\tau+1}^*, h_{\tau+1}^*; \theta_{\tau+1})\big]. \tag{66}$$

$$\;=\; w_\tau(1-\beta bF)\,u_1(c_\tau^* - h_\tau^*, l_\tau^*), \tag{67}$$

where F denotes the forward operator, that is $Fx_\tau \equiv E_\tau x_{\tau+1}$ for any expression x dated τ. Differentiating (67) with respect to a_t yields:

$$-u_{12}\Big(\frac{\partial c_\tau^*}{\partial a_t} - \frac{\partial h_\tau^*}{\partial a_t}\Big) - u_{22}\frac{\partial l_\tau^*}{\partial a_t} \;=\; w_\tau(1-\beta bF)\Big[u_{11}\Big(\frac{\partial c_\tau^*}{\partial a_t} - \frac{\partial h_\tau^*}{\partial a_t}\Big) + u_{12}\frac{\partial l_\tau^*}{\partial a_t}\Big], \tag{68}$$

where $Fu_{11}\,\partial c_\tau^*/\partial a_t$ denotes $E_\tau u_{11}(c_{\tau+1}^* - h_{\tau+1}^*, l_{\tau+1}^*)\,\partial c_{\tau+1}^*/\partial a_t$, and $\partial h_\tau^*/\partial a_t = 0$ for $\tau = t$ since h_t is given. Evaluating (68) at steady state and solving for $\partial l_\tau^*/\partial a_t$ yields:

$$\frac{\partial l_\tau^*}{\partial a_t} \;=\; -\frac{u_{12} + wu_{11} - \beta bwu_{11}F}{u_{22} + wu_{12}}\Big[1 - \frac{\beta bwu_{12}}{u_{22} + wu_{12}}F\Big]^{-1}(1-bL)\,\frac{\partial c_\tau^*}{\partial a_t}\,. \tag{69}$$

where the u_{ij} are evaluated at steady state, L denotes the lag operator—that is, $Lx_\tau \equiv x_{\tau-1}$ for any expression x dated τ—and we assume $|\beta bwu_{12}/(u_{22} + wu_{12})| < 1$ in order to

[16] Using the marginal utility of labor is simpler than using the marginal utility of consumption in (64) because it avoids having to keep track of future habits and the value function next period. However, in steady state it is also true that $V_1 = u_1(1-\beta b)/\beta$, which we will use to express risk aversion in terms of u_1 and u_{11} below.

ensure convergence. Note that when $b = 0$, (69) reduces to $-\frac{wu_{11}+u_{12}}{u_{22}+wu_{12}} \frac{\partial c_\tau^*}{\partial a_t}$, as in Section 2. This solves for $\partial l_\tau^*/\partial a_t$ in terms of (current and future) $\partial c_\tau^*/\partial a_t$.

As before, we solve for $\partial c_\tau^*/\partial a_t$ using the household's Euler equation and budget constraint. Differentiating the household's Euler equation:

$$\frac{1}{w_\tau} u_2(c_\tau^* - h_\tau^*, l_\tau^*) \;=\; \beta E_\tau \frac{1 + r_{\tau+1}}{w_{\tau+1}} u_2(c_{\tau+1}^* - h_{\tau+1}^*, l_{\tau+1}^*), \tag{70}$$

with respect to a_t and evaluating at steady state yields:

$$u_{12}\big[(1+b) - F - bL\big] \frac{\partial c_\tau^*}{\partial a_t} \;=\; -u_{22}(1-F)\frac{\partial l_\tau^*}{\partial a_t}\,. \tag{71}$$

Substituting (69) into (71) yields the following difference equation for c_τ:

$$\Big[u_{12}\big(u_{22} + wu_{12} - \beta bwu_{12}F\big)\big[(1+b) - F - bL\big] - \\ u_{22}(1-F)\big(u_{12} + wu_{11} - \beta bwu_{11}F\big)(1-bL)\Big]\frac{\partial c_\tau^*}{\partial a_t} \;=\; 0. \tag{72}$$

Since $FL = 1$,[17] equation (72) simplifies to:

$$(1-\beta bF)(1-F)(1-bL)\,\frac{\partial c_\tau^*}{\partial a_t} \;=\; 0, \tag{73}$$

which, from (71), also implies:

$$(1-\beta bF)(1-F)\,\frac{\partial l_\tau^*}{\partial a_t} \;=\; 0. \tag{74}$$

Equations (73) and (74) hold for all $\tau \geq t$, hence we can invert the $(1-\beta bF)$ operator forward to get:

$$(1-F)(1-bL)\,\frac{\partial c_\tau^*}{\partial a_t} \;=\; 0, \tag{75}$$

$$(1-F)\,\frac{\partial l_\tau^*}{\partial a_t} \;=\; 0. \tag{76}$$

In other words, whatever the initial responses $\partial c_t^*/\partial a_t$ and $\partial l_t^*/\partial a_t$ are, we must have:

$$E_t\frac{\partial c_{t+1}^*}{\partial a_t} \;=\; (1+b)\,\frac{\partial c_t^*}{\partial a_t}\,,$$

$$E_t\frac{\partial c_{t+2}^*}{\partial a_t} \;=\; (1+b+b^2)\,\frac{\partial c_t^*}{\partial a_t}\,,$$

$$E_t\frac{\partial c_{t+k}^*}{\partial a_t} \;=\; (1+b+\cdots+b^k)\,\frac{\partial c_t^*}{\partial a_t}\,, \tag{77}$$

$$\text{and}\quad E_t\frac{\partial l_{t+k}^*}{\partial a_t} \;=\; \frac{\partial l_t^*}{\partial a_t}, \quad k = 1, 2, \ldots \tag{78}$$

[17] To be precise, $FLx_\tau = E_{\tau-1}x_\tau$, but since the household evaluates these expressions from the perspective of the initial period t, $E_tFLx_\tau = E_tx_\tau$. Formally, take the expectation of (72) at time t and then apply $E_tFL = E_t$ to get (73).

Because of habits, consumption responds only gradually to a surprise change in wealth, asymptoting over time to its new steady-state level. Labor, in contrast, moves immediately to its new steady-state level.

From (77), we can now solve (70) to get:

$$\frac{\partial l_t^*}{\partial a_t} = -\lambda \frac{\partial c_t^*}{\partial a_t}, \tag{79}$$

where

$$\lambda \equiv \frac{w(1-\beta b)u_{11} + u_{12}}{u_{22} + w(1-\beta b)u_{12}} = \frac{u_1 u_{12} - u_2 u_{11}}{u_1 u_{22} - u_2 u_{12}}, \tag{80}$$

and where the latter equality follows because $w = -(1-\beta b)^{-1} u_2/u_1$ in steady state. Note that equations (79)–(80) are essentially identical to (11)–(12) for the model without habits.[18] Again, λ must be positive if leisure and consumption are normal goods.

From the household's budget constraint and condition (78), we have:

$$E_t \sum_{\tau=t}^{\infty} (1+r)^{-(\tau-t)} \frac{\partial c_\tau^*}{\partial a_t} = 1 + w\frac{1+r}{r} \frac{\partial l_t^*}{\partial a_t}, \tag{81}$$

which, from (78), (79), and (80) implies:

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1+r} \frac{1-\beta b}{1+(1-\beta b)w\lambda}. \tag{82}$$

Without habits or labor, an increase in assets would cause consumption to rise by the amount of the income flow from the change in assets—the first term on the right-hand side of (82). The presence of habits attenuates this change by the amount βb in the numerator of the second term, and the consumption response is further attenuated by the household's change in hours worked, which is accounted for by the denominator of the second term in (82).

Substituting (64), (65), (79), (80), and (82) into (54) gives us the household's coefficient of absolute risk aversion:[19]

$$\frac{-V_{11}}{V_1} = \frac{-u_{11} + \lambda u_{12}}{u_1} \frac{1-\beta b}{1+(1-\beta b)w\lambda} \frac{r}{1+r}, \tag{83}$$

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[18] Unlike in the model without habits, equations (79)–(80) only hold here in steady state, but the expression is otherwise the same.

[19] In order to express (83) in terms of u_1 and u_{11} instead of u_2 and u_{22}, we use $V_1 = (1-\beta b)u_1/\beta$ and differentiate the first-order condition:

$$u_1(c_t^* - h_t, l_t^*) = \frac{1}{1+r_t} V_1(a_t, h_t; \theta_t) + \beta b E_t u_1(c_{t+1}^* - h_{t+1}^*, l_{t+1}^*)$$

with respect to a_t to solve for V_{11} using (77), (79), and (82).

and consumption-based coefficient of relative risk aversion:

$$\frac{-AV_{11}}{V_1} = \frac{-u_{11} + \lambda u_{12}}{u_1} \frac{(1-\beta b)c}{1+(1-\beta b)w\lambda}. \tag{84}$$

Equations (83) and (84) have essentially the same form as the corresponding expressions in the model without habits.

Example 4.2. Consider the utility kernel of example 4.1:

$$u(c_t - h_t, l_t) = \frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi}, \tag{85}$$

where $\gamma,\ \chi,\ \chi_0 > 0$, but now with the habit stock $h_t = bc_{t-1}$ internal to the household rather than external. In thise case, the household's consumption-based coefficient of relative risk aversion is given by:

$$\begin{aligned} \frac{-AV_{11}}{V_1} &= \frac{-cu_{11}}{u_1} \frac{1-\beta b}{1+(1-\beta b)w\lambda}, \\ &= \gamma \frac{1-\beta b}{1-b} \frac{1}{1+\frac{\gamma}{\chi}\frac{1-\beta b}{1-b}\frac{wl}{c}}, \\ &\approx \frac{\gamma}{1+\frac{\gamma}{\chi}}, \end{aligned} \tag{86}$$

where the last line uses $\beta \approx 1$ and $wl \approx c$.

The most striking feature of equation (86) is that it is independent of b, the importance of habits. This is in sharp contrast to the case where habits are external, where risk aversion is strongly increasing in b (cf. equation (49)).

5. Conclusions

This paper has shown that many studies in the macroeconomics, finance, and international finance literatures substantially overstate risk aversion in their models. The traditional measure of risk aversion, $-cu_{11}/u_1$, ignores the household's ability to partially offset shocks to income with changes in hours worked. For reasonable parameterizations in the literature, the traditional measure can easily overstate risk aversion by a factor of three or more.

Measuring risk aversion matters for understanding asset prices. Indeed, the risk premium on assets derived using the consumption-based stochastic discount factor is proportional to Arrow-Pratt risk aversion, as we showed above. If risk aversion is not measured correctly, then the risk premia on assets in the model are more likely to be surprising or puzzling. For example, some specifications of household preferences imply that the household is risk neutral—implying zero risk premia—even though the traditional measure of risk aversion is far from zero.

Another implication of the labor margin is that risk aversion and the intertemporal elasticity of substitution are not inverses of each other, even for the case of expected utility preferences.

More generally, we have derived simple, closed-form expressions for Arrow-Pratt risk aversion in DSGE models. The class of models for which these solutions are valid is quite general and includes models with generalized recursive preferences and internal or external habits as well as models with time-separable expected utility preferences. These expressions, and the methods of the paper more generally, should be useful to researchers interested in pricing any asset—e.g., stocks, bonds, or futures, in foreign or domestic currency—within the framework of dynamic equilibrium models. Since these models represent one of the main workhorses of research in academia, at central banks, and international financial institutions, the applicability of this paper's results should be widespread.

Appendix: Mathematical Derivations

Expected Utility Preferences

As discussed in the text, we offer an individual household in Section 2 a hypothetical one-shot gamble in period t of the form:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1}, \tag{A1}$$

where ε_{t+1} is independent of the exogenous state variables θ_τ for all times τ, independent of the household's variables a_τ, c_τ, and l_τ for $\tau \leq t$, and ε_{t+1} has zero mean and unit variance. Alternatively, we consider charging the household a one-time fee of μ in period t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu. \tag{A2}$$

To determine the household's coefficient of absolute risk aversion, we must find what size fee $d\mu$ makes the household indifferent between the fee and a gamble of size $d\sigma$, where $d\mu$ and $d\sigma$ are infinitesimals.

For an infinitesimal fee $d\mu$, the change in household welfare (5) is given by:

$$-\beta E_t V_1(a^*_{t+1}; \theta_{t+1})\, d\mu\,, \tag{A3}$$

which follows from the envelope theorem (terms involving $dc^*_t/d\mu$ and $dl^*_t/d\mu$ cancel).

Turning now to the gamble, note first that the household's optimal choices for consumption and labor in period t, c^*_t and l^*_t, will generally depend on the size of the gamble σ—for example, the household may undertake precautionary saving when faced with this gamble. Thus, in this section we write $c^*_t \equiv c^*(a_t; \theta_t; \sigma)$ and $l^*_t \equiv l^*(a_t; \theta_t; \sigma)$ to emphasize this dependence on σ.

Because c^*_t and l^*_t depend on σ, the household's value-to-go at time t also depends on σ. We write this dependence out explicitly as well, so that:

$$\widetilde{V}(a_t; \theta_t; \sigma) = u(c^*_t, l^*_t) + \beta E_t V(a^*_{t+1}; \theta_{t+1}), \tag{A4}$$

where $a^*_{t+1} \equiv (1 + r_t)a_t + w_t l^*_t + d_t - c^*_t$. Because (A1) describes a one-shot gamble in period t, it affects assets a^*_{t+1} in period $t + 1$ but otherwise does not affect the household's optimization problem from period $t + 1$ onward; as a result, the household's value-to-go at time $t + 1$ is just $V(a^*_{t+1}; \theta_{t+1})$, which does not depend on σ except through a^*_{t+1}. The tilde over the V on the left-hand side of (A4) emphasizes that the form of the value function itself is different in period t due to the presence of the one-shot gamble in that period.

Differentiating (A4) with respect to σ, the first-order effect of the gamble on household welfare is:

$$\left[u_1 \frac{\partial c^*}{\partial\sigma} + u_2 \frac{\partial l^*}{\partial\sigma} + \beta E_t V_1 \cdot \left(w_t \frac{\partial l^*}{\partial\sigma} - \frac{\partial c^*}{\partial\sigma} + \varepsilon_{t+1}\right)\right] d\sigma, \tag{A5}$$

where the arguments of u_1, u_2, and V_1 are suppressed to simplify notation. Optimality of c^*_t and l^*_t implies that the terms involving $\partial c^*/\partial\sigma$ and $\partial l^*/\partial\sigma$ in (A5) cancel, as in the usual envelope theorem (these derivatives vanish at $\sigma = 0$ anyway, for the reasons discussed below). Moreover, $E_t V_1(a^*_{t+1}; \theta_{t+1})\varepsilon_{t+1} = 0$ because ε_{t+1} is independent of θ_{t+1} and a^*_{t+1}, evaluating the latter at $\sigma = 0$. Thus, the first-order cost of the gamble is zero, as in Arrow (1964) and Pratt (1965).

To second order, the effect of the gamble on household welfare is:

$$\begin{aligned}\Bigg[u_{11}\left(\frac{\partial c^*}{\partial\sigma}\right)^2 + 2u_{12}\frac{\partial c^*}{\partial\sigma}\frac{\partial l^*}{\partial\sigma} + u_{22}\left(\frac{\partial l^*}{\partial\sigma}\right)^2 + u_1\frac{\partial^2 c^*}{\partial\sigma^2} + u_2\frac{\partial^2 l^*}{\partial\sigma^2} \\ + \beta E_t V_{11}\cdot\left(w_t\frac{\partial l^*}{\partial\sigma} - \frac{\partial c^*}{\partial\sigma} + \varepsilon_{t+1}\right)^2 + \beta E_t V_1\cdot\left(w_t\frac{\partial^2 l^*}{\partial\sigma^2} - \frac{\partial^2 c^*}{\partial\sigma^2}\right)\Bigg]\frac{d\sigma^2}{2}\,.\end{aligned} \tag{A6}$$

The terms involving $\partial^2 c^*/\partial\sigma^2$ and $\partial^2 l^*/\partial\sigma^2$ cancel due to the optimality of c_t^* and l_t^*. The derivatives $\partial c^*/\partial\sigma$ and $\partial l^*/\partial\sigma$ vanish at $\sigma = 0$ (there are two ways to see this: first, the linearized version of the model is certainty equivalent; alternatively, the gamble in (A1) is isomorphic for positive and negative σ, hence c^* and l^* must be symmetric about $\sigma = 0$, implying the derivatives vanish). Thus, for infinitesimal gambles, (A6) simplifies to:

$$\beta E_t V_{11}(a_{t+1}^*; \theta_{t+1})\, \varepsilon_{t+1}^2 \frac{d\sigma^2}{2}\,. \tag{A7}$$

Finally, ε_{t+1} is independent of θ_{t+1} and a_{t+1}^*, evaluating the latter at $\sigma = 0$. Since ε_{t+1} has unit variance, (A7) reduces to:

$$\beta E_t V_{11}(a_{t+1}^*; \theta_{t+1}) \frac{d\sigma^2}{2}\,. \tag{A8}$$

Equating (A3) to (A8), the Arrow-Pratt coefficient of absolute risk aversion, $2d\mu/d\sigma^2$, is:

$$\frac{-\, E_t V_{11}(a_{t+1}^*; \theta_{t+1})}{E_t V_1(a_{t+1}^*; \theta_{t+1})}\,. \tag{A9}$$

The coefficient of relative risk aversion is computed similarly, except that instead of (A1), the hypothetical one-shot gamble is given by:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + A_t \sigma \varepsilon_{t+1}, \tag{A10}$$

and, instead of (A2), the one-time fee is given by:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - A_t \mu, \tag{A11}$$

where A_t is the size of the gamble (taken in the main text to be the household's total wealth by either of the two measures described there). Since A_t is known at time t, it is trivial to modify the equations above to get the coefficient of relative risk aversion:

$$\frac{-\, A_t E_t V_{11}(a_{t+1}^*; \theta_{t+1})}{E_t V_1(a_{t+1}^*; \theta_{t+1})}\,. \tag{A12}$$

Recall that (A9) and (A12) are already evaluated at $\sigma = 0$, so to evaluate them at steady state, we simply set $a_{t+1} = a$ and $\theta_{t+1} = \theta$ to get:

$$\frac{-\, V_{11}(a; \theta)}{V_1(a; \theta}\,, \tag{A13}$$

and

$$\frac{-\, A V_{11}(a; \theta)}{V_1(a; \theta}\,. \tag{A14}$$

Arrow-Pratt Risk Aversion and the Stochastic Discount Factor

As in the text, let m_t denote the household's stochastic discount factor and p_t the cum-dividend price at time t of a risky asset, with $E_t p_{t+1}$ normalized to unity. The difference between the risk-neutral price of the asset and its actual price:

$$E_t m_{t+1} E_t p_{t+1} - E_t m_{t+1} p_{t+1} \;=\; -\mathrm{Cov}_t(m_{t+1}, p_{t+1}) \;=\; -\mathrm{Cov}_t(dm_{t+1}, dp_{t+1}), \tag{A15}$$

measures the risk premium on the asset, where Cov_t denotes the covariance conditional on information at time t, and $dx \equiv x_{t+1} - E_t x_{t+1}$, $x \in \{m, p\}$. Since $m_{t+1} = \beta u_1(c^*_{t+1}, l^*_{t+1})/u_1(c^*_t, l^*_t)$, for small changes dc^*_{t+1} and dl^*_{t+1}, we have, to first order:

$$dm_{t+1} = \frac{\beta}{u_1(c^*_t, l^*_t)} \left[u_{11}(c^*_{t+1}, l^*_{t+1}) dc^*_{t+1} + u_{12}(c^*_{t+1}, l^*_{t+1}) dl^*_{t+1} \right], \tag{A16}$$

conditional on information at time t.

Differentiating $w_t = -u_2(c^*_t, l^*_t)/u_1(c^*_t, l^*_t)$ gives, to first order:

$$dl^*_t = -\lambda_t dc^*_t - \frac{u_1}{u_{22} + w_t u_{12}} \, dw_t, \tag{A17}$$

at each time t. Differentiating the household's Euler equation and evaluating at steady state yields:

$$u_{11}(dc^*_t - E_t dc^*_{t+1}) + u_{12}(dl^*_t - E_t dl^*_{t+1}) = \beta E_t u_1 dr_{t+1}, \tag{A18}$$

which, applying (A17), becomes:

$$(u_{11} - \lambda u_{12})(dc^*_t - E_t dc^*_{t+1}) - \frac{u_1 u_{12}}{u_{22} + w u_{12}} (dw_t - E_t dw_{t+1}) = \beta E_t u_1 dr_{t+1}. \tag{A19}$$

Note that (A19) implies:

$$E_t dc^*_{t+k} \; = \; dc^*_t - \frac{u_1 u_{12}}{(u_{11} - \lambda u_{12})(u_{22} + w u_{12})} (dw_t - E_t dw_{t+k}) - \frac{\beta u_1}{u_{11} - \lambda u_{12}} E_t dR_{t,t+k}, \tag{A20}$$

for $k = 1, 2, \ldots$, where $R_{t,t+k} \equiv \prod_{i=1}^{k} (1 + r_{t+i})$.

Differentiating the household's flow budget constraint, evaluating at steady state, and imposing the transversality condition yields, to first order:

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k} (dc^*_{t+k} + w dl^*_{t+k} + l dw^*_{t+k}) - \sum_{k=0}^{\infty} \frac{1}{(1+r)^{2k}} (c - wl) dR_{t,t+k} \; = \; da_t. \tag{A21}$$

Substituting (A17) and (A20) into (A21) and solving for dc^*_t yields:

$$\begin{aligned} dc^*_t \; = \; & \frac{r}{1+r} \, \frac{1}{1+\lambda w} \left[da_t + \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} \, l \, dw_{t+k} + \sum_{k=0}^{\infty} \frac{1}{(1+r)^{2k}} (c - wl) dR_{t,t+k} \right] \\ & + \frac{u_1 u_{12}}{(u_{11} - \lambda u_{12})(u_{22} + w u_{12})} \, dw_t \\ & + \frac{r}{1+r} \, \frac{1}{1+\lambda w} \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} \left[\frac{\beta u_1}{u_{11} - \lambda u_{12}} \, dR_{t,t+k} + \frac{\lambda w u_{22}/u_2}{(u_{11} - \lambda u_{12})(u_{22} + w u_{12})} \, dw_{t+k} \right]. \end{aligned} \tag{A22}$$

Finally, combining (A16), (A17), and (A22) gives:

$$\begin{aligned} dm_{t+1} \; = \; & \beta \, \frac{u_{11} - \lambda u_{12}}{u_1} \, \frac{1}{1 + w\lambda} \, \frac{r}{1+r} \Bigg[da_{t+1} + \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} \, l \, dw_{t+1+k} + \sum_{k=0}^{\infty} \frac{c - wl}{(1+r)^{2k}} \, dR_{t+1,t+1+k} \\ & + \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} \left[\frac{\beta u_1}{u_{11} - \lambda u_{12}} \, dR_{t+1,t+1+k} + \frac{\lambda w u_{22}/u_2}{(u_{11} - \lambda u_{12})(u_{22} + w u_{12})} \, dw_{t+1+k}) \right] \Bigg]. \end{aligned} \tag{A23}$$

Generalized Recursive Preferences

For generalized recursive preferences, the hypothetical one-shot gamble and one-time fee faced by the household are the same as for the case of expected utility described above. However, the household's optimality conditions for c_t^* and l_t^* (and, implicitly, a_{t+1}^*) are slightly more complicated:

$$u_1(c_t^*, l_t^*) = \beta\big(E_t V(a_{t+1}^*; \theta_{t+1})^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V(a_{t+1}^*; \theta_{t+1})^{-\alpha} V_1(a_{t+1}^*; \theta_{t+1}), \tag{A24}$$

$$u_2(c_t^*, l_t^*) = -\beta w_t\big(E_t V(a_{t+1}^*; \theta_{t+1})^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V(a_{t+1}^*; \theta_{t+1})^{-\alpha} V_1(a_{t+1}^*; \theta_{t+1}). \tag{A25}$$

Note that (A24) and (A25) are related by the usual $u_2(c_t^*, l_t^*) = -w_t u_1(c_t^*; l_t^*)$, and when $\alpha = 0$, (A24) and (A25) reduce to the standard optimality conditions for expected utility.

For an infinitesimal fee $d\mu$, the change in welfare for the household with generalized recursive preferences is:

$$-V_1(a_t; \theta_t)\,\frac{d\mu}{1 + r_t}\,, \tag{A26}$$

which, applying the envelope theorem, can be rewritten as:

$$-\beta\big(E_t V(a_{t+1}^*; \theta_{t+1})^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V(a_{t+1}^*; \theta_{t+1})^{-\alpha} V_1(a_{t+1}^*; \theta_{t+1})\, d\mu\,. \tag{A27}$$

Turning now to the gamble, the first-order effect of the gamble on household welfare is:

$$\left[u_1 \frac{\partial c^*}{\partial \sigma} + u_2 \frac{\partial l^*}{\partial \sigma} + \beta\big(E_t V^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V^{-\alpha} V_1 \cdot \big(w_t \frac{\partial l^*}{\partial \sigma} - \frac{\partial c^*}{\partial \sigma} + \varepsilon_{t+1}\big)\right] d\sigma, \tag{A28}$$

where we have dropped the arguments of u_1, u_2, V, and V_1 to simplify notation. As before, optimality of c_t^* and l_t^* implies that the terms involving $\partial c^*/\partial\sigma$ and $\partial l^*/\partial\sigma$ cancel, and $E_t V^{-\alpha} V_1 \varepsilon_{t+1} = 0$ because ε_{t+1} is independent of θ_{t+1} and a_{t+1}^*, evaluating the latter at $\sigma = 0$. Thus, the first-order cost of the gamble is zero.

To second order, the effect of the gamble on household welfare is:

$$\begin{aligned}
&\Bigg\{ u_{11}\left(\frac{\partial c^*}{\partial \sigma}\right)^2 + 2u_{12}\frac{\partial c^*}{\partial \sigma}\frac{\partial l^*}{\partial \sigma} + u_{22}\left(\frac{\partial l^*}{\partial \sigma}\right)^2 + u_1 \frac{\partial^2 c^*}{\partial \sigma^2} + u_2 \frac{\partial^2 l^*}{\partial \sigma^2} \\
&\qquad + \alpha\beta\big(E_t V^{1-\alpha}\big)^{(2\alpha-1)/(1-\alpha)} \left[E_t V^{-\alpha} V_1 \cdot \left(w_t \frac{\partial l^*}{\partial \sigma} - \frac{\partial c^*}{\partial \sigma} + \varepsilon_{t+1}\right)\right]^2 \\
&\qquad - \alpha\beta\big(E_t V^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V^{-\alpha-1} \left[V_1 \cdot \left(w_t \frac{\partial l^*}{\partial \sigma} - \frac{\partial c^*}{\partial \sigma} + \varepsilon_{t+1}\right)\right]^2 \\
&\qquad + \beta\big(E_t V^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V^{-\alpha} V_{11} \cdot \left(w_t \frac{\partial l^*}{\partial \sigma} - \frac{\partial c^*}{\partial \sigma} + \varepsilon_{t+1}\right)^2 \\
&\qquad + \beta\big(E_t V^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V^{-\alpha} V_1 \cdot \left(w_t \frac{\partial^2 l^*}{\partial \sigma^2} - \frac{\partial^2 c^*}{\partial \sigma^2}\right) \Bigg\} \frac{d\sigma^2}{2}\,.
\end{aligned} \tag{A29}$$

The derivatives $\partial c^*/\partial\sigma$ and $\partial l^*/\partial\sigma$ vanish at $\sigma = 0$, the terms involving $\partial^2 c^*/\partial\sigma^2$ and $\partial^2 l^*/\partial\sigma^2$ cancel due to the optimality of c_t^* and l_t^*, and ε_{t+1} is independent of θ_{t+1} and a_{t+1}^* (evaluating the latter at $\sigma = 0$). Thus, (A29) simplifies to:

$$\beta\big(E_t V^{1-\alpha}\big)^{\alpha/(1-\alpha)} \big(E_t V^{-\alpha} V_{11} - \alpha E_t V^{-\alpha-1} V_1^2\big) \frac{d\sigma^2}{2}\,. \tag{A30}$$

Equating (A27) to (A30), the Arrow-Pratt coefficient of absolute risk aversion is:

$$\frac{-E_t V^{-\alpha} V_{11} + \alpha E_t V^{-\alpha-1} V_1^2}{E_t V^{-\alpha} V_1}. \tag{A31}$$

The coefficient of relative risk aversion is:

$$\frac{-A_t E_t V^{-\alpha} V_{11} + \alpha A_t E_t V^{-\alpha-1} V_1^2}{E_t V^{-\alpha} V_1}. \tag{A32}$$

Since (A31) and (A32) are already evaluated at $\sigma = 0$, to evaluate them at steady state, we simply set $a_{t+1} = a$, $\theta_{t+1} = \theta$, yielding:

$$\frac{-V_{11}(a;\theta)}{V_1(a;\theta)} + \alpha \frac{V_1(a;\theta)}{V(a;\theta)}, \tag{A33}$$

and

$$\frac{-AV_{11}(a;\theta)}{V_1(a;\theta)} + \alpha \frac{AV_1(a;\theta)}{V(a;\theta)}. \tag{A34}$$

Internal Habits

We consider here the case of generalized recursive preferences:

$$V(a_t, h_t; \theta_t) = u(c_t^* - h_t, l_t^*) + \beta \left(E_t\, V(a_{t+1}^*, h_{t+1}^*; \theta_{t+1})^{1-\alpha} \right)^{1/(1-\alpha)}, \tag{A35}$$

and a longer-memory specification for habits:

$$h_t = \rho h_{t-1} + b c_{t-1}, \tag{A36}$$

with $|\rho| < 1$, and we assume $\rho + b < 1$ in order to ensure $h < c$. As in the main text, we write out the dependence of the household's value function on h_t explicitly, $c_t^* \equiv c^*(a_t, h_t; \theta_t)$ and $l_t^* \equiv l^*(a_t, h_t; \theta_t)$ denote the household's optimal choices for consumption and labor in period t as functions of the household's state vector, and a_{t+1}^* and h_{t+1}^* denote the optimal stocks of assets and habits in period $t+1$ that are implied by c_t^* and l_t^*; that is, $a_{t+1}^* \equiv (1+r_t)a_t + w_t l_t^* + d_t - c_t^*$ and $h_{t+1}^* \equiv \rho h_t + b c_t^*$.

By the same analysis as in the model without habits, the household's coefficient of absolute risk aversion is given by:

$$\frac{-E_t V(a_{t+1}^*, h_{t+1}^*; \theta_{t+1})^{-\alpha} \left[V_{11}(a_{t+1}^*, h_{t+1}^*; \theta_{t+1}) \;-\; \alpha \, \dfrac{V_1(a_{t+1}^*, h_{t+1}^*; \theta_{t+1})^2}{V(a_{t+1}^*, h_{t+1}^*; \theta_{t+1})} \right]}{E_t V(a_{t+1}^*, h_{t+1}^*; \theta_{t+1})^{-\alpha} V_1(a_{t+1}^*, h_{t+1}^*; \theta_{t+1})}, \tag{A37}$$

which, evaluated at steady state, simplifies to:

$$\frac{-V_{11}(a,h;\theta)}{V_1(a,h;\theta)} \;+\; \alpha \, \frac{V_1(a,h;\theta)}{V(a,h;\theta)}, \tag{A38}$$

where $h = bc/(1-\rho)$ follows from (A36).

It remains to compute V_1 and V_{11}. As in the main text, these derivatives are more complicated to compute in the case of internal habits, due to the dynamic relationship between current consumption and future habits.

The household's first-order conditions for (A35) with respect to consumption and labor are given by:

$$u_1 \;=\; \beta\big(E_t V^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V^{-\alpha}[V_1 - bV_2], \tag{A39}$$
$$u_2 \;=\; -\beta w_t\big(E_t V^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V^{-\alpha} V_1, \tag{A40}$$

where we have dropped the arguments of u and V to reduce notation. Equations (A39) and (A40) are the same as in the main text except that the discounting of future periods involves the value function V when $\alpha \neq 0$.

Differentiating (A35) with respect to its first two arguments and applying the envelope theorem yields:

$$V_1 \;=\; \beta(1+r_t)\,\big(E_t V^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V^{-\alpha} V_1, \tag{A41}$$
$$V_2 \;=\; -u_1 \;+\; \rho\beta\big(E_t V^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_t V^{-\alpha} V_2. \tag{A42}$$

As in the main text, (A40) and (A41) can be used to solve for V_1 in terms of current-period utility:

$$V_1(a_t, h_t; \theta_t) \;=\; -\frac{(1+r_t)}{w_t}\, u_2(c_t^* - h_t, l_t^*). \tag{A43}$$

To solve for V_{11}, differentiate (A43) with respect to a_t to yield:

$$V_{11}(a_t, h_t; \theta_t) \;=\; -\frac{(1+r_t)}{w_t}\left(u_{12}\,\frac{\partial c_t^*}{\partial a_t} + u_{22}\,\frac{\partial l_t^*}{\partial a_t}\right), \tag{A44}$$

It remains to solve for $\partial c_t^*/\partial a_t$ and $\partial l_t^*/\partial a_t$. As in the main text, we solve for $\partial c_\tau^*/\partial a_t$ and $\partial l_\tau^*/\partial a_t$ for all dates $\tau \geq t$ at the same time. We henceforth let a time subscript $\tau \geq t$ denote a generic future date and reserve the subscript t to denote the date of the current period—the period in which the household faces the hypothetical one-shot gamble.

We solve for $\partial l_\tau^*/\partial a_t$ in terms of $\partial c_\tau^*/\partial a_t$ in the same manner as in the main text, except that the expressions are more complicated due to the persistence of habits and the household's more complicated discounting of future periods. Note first that (A42) can be used to solve for V_2 in terms of current and future marginal utility:

$$V_2 = -(1-\rho\beta F)^{-1}\, u_1, \tag{A45}$$

where now F denotes the "generalized recursive" forward operator; that is,

$$F x_\tau \equiv \big(E_\tau V^{1-\alpha}\big)^{\alpha/(1-\alpha)} E_\tau V^{-\alpha} x_{\tau+1}. \tag{A46}$$

The household's intratemporal optimality condition ((A39) combined with (A40)) implies:

$$-u_2(c_\tau^* - h_\tau^*, l_\tau^*) \;=\; w_\tau\big[u_1(c_\tau^* - h_\tau^*, l_\tau^*) \;+\; b\beta E_\tau V_2(a_{\tau+1}^*, h_{\tau+1}^*; \theta_{\tau+1})\big]. \tag{A47}$$
$$\;=\; w_\tau\big(1 - \beta b F(1-\beta\rho F)^{-1}\big)\, u_1(c_\tau^* - h_\tau^*, l_\tau^*), \tag{A48}$$

Differentiating (A48) with respect to a_t and evaluating at steady state yields:

$$-u_{12}\left(\frac{\partial c_\tau^*}{\partial a_t} - \frac{\partial h_\tau^*}{\partial a_t}\right) - u_{22}\frac{\partial l_\tau^*}{\partial a_t} \;=\; w\,\big(1-\beta b F(1-\beta\rho F)^{-1}\big)\left[u_{11}\left(\frac{\partial c_\tau^*}{\partial a_t} - \frac{\partial h_\tau^*}{\partial a_t}\right) + u_{12}\frac{\partial l_\tau^*}{\partial a_t}\right], \tag{A49}$$

where we have used the fact that:

$$\frac{\partial}{\partial a_t}\, F x_\tau \;=\; F\, \frac{\partial x_\tau}{\partial a_t}\,, \tag{A50}$$

when the derivative is evaluated at steady state. Solving (A49) for $\partial l_\tau^* / \partial a_t$ yields:

$$\begin{aligned}\frac{\partial l_\tau^*}{\partial a_t} \;=\; & -\frac{u_{12} + w u_{11} - \beta(\rho u_{12} + (\rho+b) w u_{11})F}{u_{22} + w u_{12}} \times \\ & \left[1 - \frac{\beta(\rho u_{22} + (\rho + b) w u_{12})}{u_{22} + w u_{12}} F\right]^{-1} \left(1 - bL(1-\rho L)^{-1}\right) \frac{\partial c_\tau^*}{\partial a_t}\,.\end{aligned} \tag{A51}$$

where we've used $h_\tau = bL(1-\rho L)^{-1} c_\tau$ and we assume $\left|\beta\big(\rho u_{22} + (\rho+b) w u_{12}\big)/(u_{22} + w u_{12})\right| < 1$ to ensure convergence. This solves for $\partial l_t^* / \partial a_t$ in terms of (current and future) $\partial c_\tau^* / \partial a_t$.

We now turn to solving for $\partial c_\tau^* / \partial a_t$. The household's intertemporal optimality (Euler) condition is given by:

$$\frac{1}{w_\tau}\, u_2(c_\tau^* - h_\tau^*, l_\tau^*) \;=\; \beta F\, \frac{1 + r_\tau}{w_\tau}\, u_2(c_\tau^* - h_\tau^*, l_\tau^*). \tag{A52}$$

Differentiating (A52) with respect to a_t and evaluating at steady state yields:

$$u_{12}(1-F)\left[1 - bL(1-\rho L)^{-1}\right] \frac{\partial c_\tau^*}{\partial a_t} \;=\; -u_{22}(1-F) \frac{\partial l_\tau^*}{\partial a_t}\,. \tag{A53}$$

Using (A51) and noting $FL = 1$ at steady state, (A53) simplifies to:

$$\left[1 - \beta(\rho+b)F\right](1-F)\left[1 - bL(1-\rho L)^{-1}\right] \frac{\partial c_\tau^*}{\partial a_t} \;=\; 0, \tag{A54}$$

which, from (A53), also implies:

$$\left[1 - \beta(\rho+b)F\right](1-F)\, \frac{\partial l_\tau^*}{\partial a_t} \;=\; 0. \tag{A55}$$

Equations (A54) and (A55) hold for all $\tau \geq t$, hence we can invert the $\left[1 - \beta(\rho+b)F\right]$ operator forward to get:

$$(1-F)\left[1 - bL(1-\rho L)^{-1}\right] \frac{\partial c_\tau^*}{\partial a_t} \;=\; 0, \tag{A56}$$

$$(1-F)\, \frac{\partial l_\tau^*}{\partial a_t} \;=\; 0. \tag{A57}$$

Finally, we can apply $(1-\rho L)$ to both sides of (A56) to get:

$$(1-F)\left[1 - (\rho+b)L\right] \frac{\partial c_\tau^*}{\partial a_t} \;=\; 0, \tag{A58}$$

which then holds for all $\tau \geq t+1$. Thus, whatever the initial responses $\partial c_t^* / \partial a_t$ and $\partial l_t^* / \partial a_t$, we must have:

$$\begin{aligned} E_t \frac{\partial c_{t+1}^*}{\partial a_t} \;&=\; (1+b)\, \frac{\partial c_t^*}{\partial a_t}\,, \\ E_t \frac{\partial c_{t+2}^*}{\partial a_t} \;&=\; \big(1 + b(\rho+b)\big) \frac{\partial c_t^*}{\partial a_t}\,, \\ E_t \frac{\partial c_{t+k}^*}{\partial a_t} \;&=\; \left(1 + b(\rho+b)^{k-1}\right) \frac{\partial c_t^*}{\partial a_t}\,, \end{aligned} \tag{A59}$$

$$\text{and} \quad E_t \frac{\partial l_{t+k}^*}{\partial a_t} \;=\; \frac{\partial l_t^*}{\partial a_t}, \quad k = 1, 2, \ldots \tag{A60}$$

Consumption responds gradually to a surprise change in wealth, while labor moves immediately to its new steady-state level.

From (A59), we can now solve (A51) to get:

$$\frac{\partial l_t^*}{\partial a_t} \;=\; -\lambda \, \frac{\partial c_t^*}{\partial a_t}\,. \tag{A61}$$

where

$$\lambda \;\equiv\; \frac{w(1-\beta(\rho+b))u_{11} \;+\; (1-\beta\rho)u_{12}}{(1-\beta\rho)u_{22} \;+\; w(1-\beta(\rho+b))u_{12}} \;=\; \frac{u_1 u_{12} - u_2 u_{11}}{u_1 u_{22} - u_2 u_{12}}\,, \tag{A62}$$

where the latter equaltiy follows because $w = -\frac{u_2}{u_1}\frac{1-\beta\rho}{1-\beta(\rho+b)}$ in steady state. Again, λ must be positive if leisure and consumption are normal goods.

The household's intertemporal budget constraint implies:

$$E_t \sum_{\tau=t}^{\infty} (1+r)^{-(\tau-t)} \, \frac{\partial c_\tau^*}{\partial a_t} \;=\; 1 + w\frac{1+r}{r}\,\frac{\partial l_t^*}{\partial a_t}\,, \tag{A63}$$

which, from (A59), (A61), and (A62), implies:

$$\frac{\partial c_t^*}{\partial a_t} \;=\; \frac{r}{1+r}\,\frac{1-\frac{\beta b}{1-\beta\rho}}{1+(1-\frac{\beta b}{1-\beta\rho})w\lambda}\,. \tag{A64}$$

Without habits or labor, an increase in assets would cause consumption to rise by the amount of the income flow from the change in assets—the first term on the right-hand side of (A64). The presence of habits attenuates this change by the amount $\beta b/(1-\beta\rho)$ in the numerator of the second term, and the consumption response is further attenuated by the household's change in hours worked, which is accounted for by the denominator of the second term in (A64).

Together, (A62) and (A64) allow us to compute the household's coefficient of absolute risk aversion (A38):[20]

$$\frac{-V_{11}}{V_1} + \alpha\frac{V_1}{V} \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{1-\frac{\beta b}{1-\beta\rho}}{1+(1-\frac{\beta b}{1-\beta\rho})w\lambda}\,\frac{r}{1+r} \;+\; \alpha\frac{u_1}{u}\left(1-\frac{\beta b}{1-\beta\rho}\right)\frac{r}{1+r}\,, \tag{A65}$$

and consumption-based coefficient of relative risk aversion:

$$\frac{-AV_{11}}{V_1} + \alpha\frac{AV_1}{V} \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{c(1-\frac{\beta b}{1-\beta\rho})}{1+(1-\frac{\beta b}{1-\beta\rho})w\lambda} \;+\; \alpha\frac{c\,u_1}{u}\left(1-\frac{\beta b}{1-\beta\rho}\right). \tag{A66}$$

Equations (A65) and (A66) have obvious similarities to the corresponding expressions without habits and with expected utility preferences.

[20] In order to express (A65) in terms of u_1 and u_{11} instead of u_2 and u_{22}, we use $V_1 = (1-\beta(\rho+b))u_1/(\beta(1-\beta\rho))$ and differentiate the first-order condition:

$$V_1(a_t, h_t; \theta_t) \;=\; (1+r_t)\,(1-\beta b F(1-\beta\rho F)^{-1})\,u_1(c_\tau^* - h_\tau, l_\tau^*),$$

with respect to a_t to solve for V_{11}.

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