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SHOULD THE CENTRAL BANK BE CONCERNED ABOUT HOUSING PRICES?

KARSTEN JESKE AND ZHENG LIU

ABSTRACT. Housing is an important component of the consumption basket. Since both rental prices and goods prices are sticky, the literature suggests that optimal monetary policy should stabilize both types of prices, with the optimal weight on rental inflation proportional to the housing expenditure share. In a two-sector DSGE model with sticky rental prices and goods prices, however, we find that the optimal weight on rental inflation in the Taylor rule is small—much smaller than that implied by the housing expenditure share. We show that the asymmetry in policy responses to rent inflation versus goods inflation stems from the asymmetry in factor intensity between the two sectors.

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I. Introduction

Recent studies suggest several lessons for optimal monetary policy. First, achieving price stability is socially desirable. By stabilizing inflation, monetary policy helps alleviate distortions caused by nominal rigidities (Goodfriend and King, 1997; Woodford, 2003). Second, price stability can be achieved through simple feedback interest rate rules (Rotemberg and Woodford, 1999; Erceg, Henderson, and Levin, 2000; Huang and Liu, 2005). Third, optimal monetary policy should target the "core" price inflation that excludes sectors with flexible prices such as commodities, asset prices, and housing prices (Aoki, 2001). In practice, however, many central banks target the inflation rate measured by the consumer price index (CPI), specifically the core CPI of all items excluding food and energy. An important component of the core CPI is housing.[1] To what extent does the CPI-inflation targeting policy resemble optimal policy? In particular, should the central bank be concerned about housing price inflation? These issues are important, especially in light of the large cyclical fluctuations in housing prices.

This article examines the issue of optimal monetary policy in a two-sector dynamic stochastic general equilibrium (DSGE) model with a housing service sector and a non-housing goods sector and with sticky prices in each. We find that, although optimal policy assigns a positive weight to rental price inflation since the rental prices are sticky, the optimal weight for the housing component is small, much smaller than its expenditure share in the consumption basket. This result stems from an asymmetry in factor intensity of the production technologies in the two sectors, as we elaborate below.

Our model features a representative household who consumes goods, housing services, and leisure. The household purchases an investment good to accumulate physical capital and the housing stock. The household supplies labor and capital to a continuum of intermediate goods producers, each of which produces a differentiated good. The household supplies a fraction of the housing stock to a continuum of real estate firms, each of which produces a differentiated housing service. The non-housing goods consumption is a Dixit-Stiglitz composite of the intermediate goods. The housing service consumption consists of owner occupied housing and a Dixit-Stiglitz composite of the rental services. Differentiated intermediate goods are produced using a constant returns technology with labor and capital as inputs. Intermediate goods producers face monopolistic competition in the product markets and set prices in a staggered fashion

[1] In the United States, the CPI weight on housing services is about 30% for shelters and about 40% for shelters and utilities.

(Calvo, 1983). Differentiated housing services are produced using housing stocks as the only input. Housing service producers face monopolistic competition in the rental markets and set rental prices in a staggered fashion. The monetary authority commits to a feedback interest rate rule, under which the nominal interest rate responds to the lagged nominal interest rate, non-housing goods price inflation, rental price inflation, and detrended output.

Since both the goods prices and the rental prices are sticky, cyclical fluctuations in the relative price of housing services are inefficient. Thus, optimal monetary policy faces a trade-off between stabilizing the inflation rates in the two sectors and cannot replicate the flexible-price equilibrium allocations. In the standard two-sector model, the optimal monetary policy literature suggests that the optimal relative weight of a particular sector's inflation should depend on the relative nominal rigidity in and the expenditure share of that sector (Woodford, 2003; Benigno, 2004; Huang and Liu, 2005). This result is typically obtained in models with some kind of symmetry imposed across sectors. In particular, factor intensities in the production technologies are typically assumed to be identical across sectors (Erceg and Levin, 2006; Benigno, 2004).

In our model, we allow for potential asymmetry in factor intensity across sectors. We assume that the housing service sector is less labor intensive than the goods sector. We find that optimal policy assigns a small weight on rental price inflation—much smaller than the expenditure share of housing services in the consumption basket. Under calibrated parameters, the optimal weight on rental inflation is about 10%; in contrast, the actual weight of housing in the consumer price index (CPI) is between 30% and 40%.

This result stems from the asymmetry in factor intensity. In particular, low labor intensity for producing differentiated housing services gives rise to lower optimal weight on rent inflation than the housing expenditure share. Under staggered price setting, firms with lower relative prices need to hire more workers to meet higher demand whereas in a flexible-price economy, all firms make identical employment decisions. The price dispersion stemming from sticky prices thus leads to misallocation of labor and welfare losses relative to a flexible-price economy. This form of misallocation is less severe, the lower the share of labor in production. As the labor intensity in the housing sector is lower, the nominal rigidities in that sector contribute less to aggregate welfare losses from sticky prices. Accordingly, optimal policy assigns a smaller weight to rent inflation than the housing expenditure share, as we find in the paper.

In what follows, we discuss our contribution relative to the literature in Section II, present our model in Section III, describe our calibration and solution methods in Section IV, discuss optimal monetary policy rules in Section V, and conclude in Section VI. We gather some detailed derivations of the equilibrium dynamics in the Appendix A.

II. Related Literature

Our work belongs to a general class of DSGE models that study optimal monetary policy in the presence of nominal rigidities. In this class of models, many authors argue that stabilizing the price level helps close the output gap that measures the deviations of equilibrium output from its natural rate. To the extent that fluctuations in the natural rate of output resemble efficient responses of the economy's output to various sources of shocks, optimal policy calls for stabilizing the price level (King and Wolman, 1999; Woodford, 2003). The basic DSGE framework can be generalized to study optimal monetary policy in the presence of several sources of nominal rigidities that stem from, for example, sticky prices in multiple sectors (Mankiw and Reis, 2003; Huang and Liu, 2005) or multiple countries (Benigno, 2004; Liu and Pappa, 2008), or sticky prices and sticky nominal wages (Erceg, Henderson, and Levin, 2000). With multiple sources of nominal rigidities, exclusively stabilizing the price level does not necessarily close the output gap since monetary policy faces a tradeoff between stabilizing various measures of inflation. In particular, optimal monetary policy should stabilize an inflation index that is a weighted average of the sectoral inflation rates. The optimal weight assigned to a particular sector's inflation is an increasing function of the sector's price stickiness and expenditure share in the final consumption basket. This logic implies that optimal policy should not assign any weight to the sectors with flexible prices and it should instead target core inflation (Aoki, 2001).

Our model builds on this strand of multi-sector DSGE literature. As a key point of departure from the literature, we highlight the importance of rental price stickiness and we emphasize the implications of the asymmetry of factor intensity between housing and non-housing sectors for optimal monetary policy rules. In this sense, our work is closely related to Eusepi, Hobijn, and Tambalotti (2009), who study optimal monetary policy in a multi-sector model in which the labor share differs across sectors. In their model, labor is the only variable input factor and production technology exhibits decreasing returns to scale.[2] As a consequence, the labor share in their model is tied to

[2]Their model can be interpreted as one with firm-specific factors, such as those studied by Chari, Kehoe, and McGrattan (2000), Huang and Liu (2002), and Altig, Christiano, Eichenbaum, and Linde (2004).

the curvature of the production function. They find that a larger labor share (or a less concave production function) implies a flatter Phillips curve and thus a smaller welfare loss from staggered price setting. In contrast, our model features constant return technologies, with capital (or housing) as an additional variable input factor. Thus, unlike Eusepi, Hobijn, and Tambalotti (2009), the labor share in our model is decoupled from the curvature of the production function. Our results suggest that, given the production function curvature, a smaller labor share implies less severe misallocation and thus lower welfare loss stemming from nominal rigidities. Our work is also related to Lombardo (2006), who examines optimal monetary policy in a model with different degrees of market power in different member countries of a monetary union; and Liu and Pappa (2008), who examine the gains from monetary policy coordination in a two-country economy in which the share of non-traded goods in the consumption baskets differs across countries. Our work adds to this literature by pointing out that asymmetries in factor intensity across sectors can have important consequences for optimal monetary policy.

To the extent that housing is a type of durable goods, our work is related to Barsky, House, and Kimball (2007), who examine the transmission of monetary shocks in a two-sector DSGE model with durable and non-durable goods. They argue that the pricing behavior in the durable goods sector (i.e., whether or not the durable goods prices are sticky) is more important for understanding the transmission of monetary shocks than the pricing behavior in the non-durable goods sector. Unlike Barsky, House, and Kimball (2007), who focus on the transmission of monetary shocks, we focus on optimal monetary policy in a model with durable (housing) and non-durable goods.

Our work is more closely related to Erceg and Levin (2006), who study optimal monetary policy in a two-sector DSGE model with durable and non-durable goods and nominal rigidities in each sector. Although their durable goods sector can be broadly interpreted as corresponding to our housing service sector, they reach a different conclusion that optimal monetary policy assigns a large weight to the durable goods sector relative to its small weight in the economy. The difference between their results and ours stems from the difference in the assumptions about production technologies. In their model, production of durable goods uses the same technology as that of non-durable goods. Thus, production technologies in the two sectors have identical factor intensity. In our model, however, the housing service sector uses a more "capital intensive" technology than the non-housing sector (indeed, in our model, production of housing services uses housing stocks as the only input). Our finding suggests that, in general, in a multi-sector economy, the relative weight on a sector's inflation under optimal policy depends not only on the sector's relative price stickiness and expenditure share, but also on the relative factor intensity in the production technology.

III. The Model

In this section, we present a DSGE model with a housing sector and a non-housing sector. We use this model as a context to examine the issue of optimal monetary policy designs in the presence of nominal rigidities in both rental prices and goods prices. In the model, time is discrete. In each period t, the economy experiences a realization of shocks s_t. The history of events up to date t are given by $s^t = (s_0, \ldots, s_t)$ with probability $\pi(s^t)$. The initial realization s_0 is given.

III.1. **Aggregation.** There is an aggregate technology that transforms a continuum of differentiated goods $\{X(j, s^t)\}_{i\in[0,1]}$ into a composite final good $X(s^t)$ to be used by the households for consumption and investment. There is also an aggregation technology that transforms a continuum of differentiated housing services $\{H_r(i, s^t)\}_{j\in[0,1]}$ into a composite housing service $H_r(s^t)$ to be rented to the households. In particular, the aggregation technologies are given by

$$X(s^t) = \left[\int_0^1 X(j, s^t)^{\frac{\theta_f - 1}{\theta_f}} dj\right]^{\frac{\theta_f}{\theta_f - 1}}, \quad H_r(s^t) = \left[\int_0^1 H_r(i, s^t)^{\frac{\theta_h - 1}{\theta_h}} di\right]^{\frac{\theta_h}{\theta_h - 1}}, \tag{1}$$

where θ_f and θ_h are the elasticities of substitution between the differentiated goods and the differentiated housing services, respectively.

Cost-minimizing implies that the demand functions for good i and for housing service j are given by

$$X^d(j, s^t) = \left[\frac{P_f(j, s^t)}{\bar{P}_f(s^t)}\right]^{-\theta_f} X(s^t), \quad H_r^d(i, s^t)) = \left[\frac{R_h(i, s^t)}{\bar{R}_h(s^t)}\right]^{-\theta_h} H_r(s^t), \tag{2}$$

where the price index $\bar{P}_f(s^t)$ of the composite good is related to the prices $\{P_f(j, s^t)\}_{j\in[0,1]}$ of the differentiated goods by $\bar{P}_f(s^t) = \left[\int_0^1 P_f(j, s^t)^{1-\theta_f)} dj\right]^{\frac{1}{1-\theta_f}}$ and the rental price index $\bar{R}_h(s^t)$ is related to the rental prices $\{R_h(i, s^t)\}_{i\in[0,1]}$ of the differentiated goods by $\bar{R}_h(s^t) = \left[\int_0^1 R_h(i, s^t)^{1-\theta_h)} di\right]^{\frac{1}{1-\theta_h}}$.

III.2. **The representative household.** The economy is populated by a continuum of infinitely lived, identical households with the population size normalized to one. The representative household has an initial endowment of $K(s_{-1})$ units of capital stock and $S(s_{-1})$ units of housing stock. The household is also endowed with 1 unit of time in each period. The household has preferences over non-housing consumption $C(s^t)$, housing consumption $H(s^t)$, and leisure $1 - L(s^t)$. The preferences are represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) U(C(s^t), H(s^t), 1 - L(s^t)), \tag{3}$$

where $\beta \in (0,1)$ is a subjective discount factor. The household purchases investment goods to accumulate capital and the housing stocks. The law of motions for the capital and housing stocks are given by

$$K(s^t) = (1-\delta_k)K(s^{t-1}) + I_k(s^t)\left[1 - \Psi_k\left(\frac{I_k(s^t)}{I_k(s^{t-1})}\right)\right], \tag{4}$$

$$S(s^t) = (1-\delta_s)S(s^{t-1}) + I_s(s^t)\left[1 - \Psi_s\left(\frac{I_s(s^t)}{I_s(s^{t-1})}\right)\right], \tag{5}$$

where δ_k (δ_s) denotes the depreciation rate of the capital (housing) stock, $I_k(s^t)$ ($I_s(s^t)$) denotes the gross investment in capital (housing), and $\Psi_k(\cdot)$ ($\Psi_s(\cdot)$) denotes the adjustment cost for changing the flow of capital (housing) investment. We follow Christiano, Eichenbaum, and Evans (2005) and assume that $\Psi_j(1) = \Psi_j'(1) = 0$ and that $\Psi_j'' > 0$ for $j \in \{k, s\}$.[3]

The representative household consumes a combination of owner-occupied and rental housing services. Specifically, we assume that the household supplies a share $\gamma \in (0,1)$ of its housing stock to real estate firms (i.e., landlords) and consumes a share $(1-\gamma)$ of its own housing stock and also rents additional composite housing services from the rental market.[4] The effective housing services that the household consumes is thus given by

$$H(s^t) = (1-\gamma)S(s^{t-1}) + H_r(s^t). \tag{6}$$

The household faces the budget constraint

$$\begin{aligned} & C(s^t) + \frac{\bar{R}_h(s^t)}{\bar{P}_f(s^t)} H_r(s^t) + I_k(s^t) + I_s(s^t) + \sum_{s^{t+1}} Q\left(s^{t+1}|s^t\right) \frac{B\left(s^{t+1}\right)}{\bar{P}_f(s^t)} \\ \leq \; & w(s^t)L(s^t) + r_k(s^t)K(s^{t-1}) + \gamma r_s(s^t) S(s^{t-1}) + \frac{\Pi(s^t)}{\bar{P}_f(s^t)} + \frac{B(s^t)}{\bar{P}_f(s^t)}, \end{aligned} \tag{7}$$

[3]Since the accumulations of both capital and housing use the same investment good, the adjustment costs here are important to rule out "bang-bang" solutions with extreme (and counterfactual) flows of investment goods between sectors in response to sector specific shocks.

[4]One can interpret this feature as an economy in which each household consists of a dynasty of individuals with a fraction γ of renters and a fraction $(1-\gamma)$ of house owners.

where $\bar{R}_h(s^t)$ denotes the rental price; $B(s^{t+1})$ denotes the holdings of a state contingent nominal bond that pays off one dollar in period $t+1$ if s^{t+1} is realized and the bond costs $Q\left(s^{t+1}|s^t\right)$ dollars at s^t; $w(s^t)$ denotes the real wage; $r_k(s^t)$ and $r_s(s^t)$ denotes the real rental rates for capital and housing stocks; $\Pi(s^t)$ denotes the aggregate profit shares; and $T(s^t)$ denotes a lump-sum transfer from the government.[5]

The household maximizes the expected utility in Equation (3) subject to the constraints in Equations (4) through (7), taking as given the prices $\bar{R}_h(s^t)$, $\bar{P}_f(s^t)$, $Q\left(s^{t+1}|s^t\right)$, $w(s^t)$, $r_k(s^t)$, and $r_s(s^t)$ as well as the initial conditions on $K(s^{-1})$, $S(s^{-1})$, and $B(s^0)$. In the utility maximizing problem, the household also faces the borrowing constraint $B(s^t) \geq -\bar{B}$ for some large $\bar{B}$. The household's optimal decisions are summarized by the following first-order necessary conditions:

$$\frac{U_h(s^t)}{U_c(s^t)} = \frac{\bar{R}_h(s^t)}{\bar{P}_f(s^t)}, \tag{8}$$

$$\frac{-U_l(s^t)}{U_c(s^t)} = w(s^t), \tag{9}$$

$$\begin{aligned}\frac{1}{q_k(s^t)} &= 1-\Psi_k\left(\lambda_{I_k}(s^t)\right)-\lambda_{I_k}(s^t)\Psi_k'\left(\lambda_{I_k}(s^t)\right)+ \\ &\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\beta\frac{U_c(s^{t+1})}{U_c(s^t)}\frac{q_k(s^{t+1})}{q_k(s^t)}\Psi_k'\left(\lambda_{I_k}(s^{t+1})\right)\lambda_{I_k}(s^{t+1})^2,\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{1}{q_s(s^t)} &= 1-\Psi_s\left(\lambda_{I_s}(s^t)\right)-\lambda_{I_s}(s^t)\Psi_s'\left(\lambda_{I_s}(s^t)\right)+ \\ &\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\beta\frac{U_c(s^{t+1})}{U_c(s^t)}\frac{q_s(s^{t+1})}{q_s(s^t)}\Psi_s'\left(\lambda_{I_s}(s^{t+1})\right)\lambda_{I_s}(s^{t+1})^2,\end{aligned} \tag{11}$$

$$q_k(s^t) = \sum_{s^{t+1}}\pi(s^{t+1}|s^t)\beta\frac{U_c(s^{t+1})}{U_c(s^t)}\left[(1-\delta_k)q_k(s^{t+1})+r_k(s^{t+1})\right], \tag{12}$$

$$q_s(s^t) = \sum_{s^{t+1}}\pi(s^{t+1}|s^t)\beta\frac{U_c(s^{t+1})}{U_c(s^t)}\left[(1-\delta_s)q_s(s^{t+1})+\gamma r_s(s^{t+1})+(1-\gamma)r_h(s^{t+1})\right], \tag{13}$$

$$Q(s^{t+1}|s^t) = \pi(s^{t+1}|s^t)\beta\frac{U_c(s^{t+1})}{U_c(s^t)}\frac{\bar{P}_f(s^t)}{\bar{P}_f(s^{t+1})}, \tag{14}$$

[5]Although owner-occupied housing S and rental housing H_r are perfect substitutes in our model (see (6)), their prices are not necessarily equalized in equilibrium because the supply of each type of housing is restricted: a constant fraction γ of housing stocks is used for producing rental housing services and the remaining fraction $1-\gamma$ is used for direct consumption. In a more general (and more realistic) setup, one could model choices between renting and owning housing, where the two types of housing services are imperfect substitutes. In our view, examining optimal monetary policy in such a more general model economy should be important enough to deserve a separate investigation.

where $\lambda_{I_k}(s^t) \equiv \frac{I_k(s^t)}{I_k(s^{t-1})}$ and $\lambda_{I_s}(s^t) \equiv \frac{I_s(s^t)}{I_s(s^{t-1})}$ and the terms $q_k(s^t)$ and $q_s(s^t)$ are the shadow prices of capital and housing stocks (or Tobin's Q) given by

$$q_k(s^t) \equiv \frac{\mu_k(s^t)}{U_c(s^t)}, \quad q_s(s^t) \equiv \frac{\mu_s(s^t)}{U_c(s^t)}, \tag{15}$$

with $\mu_k(s^t)$ and $\mu_s(s^t)$ denoting the Lagrangian multipliers for (4) and (5), respectively.

The first two equations (8) and (9) are intratemporal optimizing conditions that equate the marginal rate of substitution (MRS) between housing and non-housing consumption to the real rental price and the MRS between leisure and consumption to the real wage rate. The remaining five equations are intertemporal optimizing conditions with respect to capital investment, housing investment, the capital stock, the housing stock, and the state-contingent nominal bond. Equation (10) describes the benefit and the cost for having a marginal unit of capital investment. The left-hand side of the equation is the cost of purchasing a marginal unit of investment good and the right-hand side is the benefit, which consists of the value of the increased level of new capital net of adjustment cost and the expected present value of reduced adjustment cost in the next period for having more capital in place. Absent adjustment costs, (10) implies that $q_k = 1$. Equation (11) can be similarly interpreted. Equation (12) is the intertemporal Euler equation for capital accumulation. The cost of adding an additional unit of capital is given by $q_k(s^t)$ (in units of consumption good) and the benefit of having this additional unit of capital in the current period includes the discounted resale value and rental value of the capital stock in the next period. Equation (13) is the intertemporal Euler equation for the housing stock. It can be interpreted similarly as the capital Euler equation, except that the rental value for housing consists of a γ fraction of the rental value of the owner-occupied housing stock and a $1-\gamma$ fraction of the rental value of the composite housing service. Finally, equation (14) describes the tradeoff for purchasing a marginal unit of the nominal bond.

Denote by $R(s^t)$ the nominal interest rate on a one-period risk-free bond. No arbitrage implies that $R(s^t) = [\sum_{s^{t+1}} Q(s^{t+1}|s^t)]^{-1}$. Equation (14) then implies that

$$\frac{1}{R(s^t)} = \sum_{s^{t+1}} \pi(s^{t+1}|s^t)\beta \frac{U_c(s^{t+1})}{U_c(s^t)} \frac{\bar{P}_f(s^t)}{\bar{P}_f(s^{t+1})}. \tag{16}$$

III.3. **Firms.** There is a measure one of intermediate firms indexed by $j \in [0,1]$. Firm j produces an intermediate good of type j using capital and labor as inputs with a

Cobb-Douglas production technology given by

$$X\left(j, s^t\right) = Z_{ft} L^f \left(j, s^t\right)^{1-\alpha} K^f \left(j, s^t\right)^{\alpha}, \tag{17}$$

where $L^f(j, s^t)$ and $K^f(j, s^t)$ are the primary inputs of labor and capital and Z_{ft} is a productivity shock common to all goods-producing firms and follows the stationary process

$$\ln Z_{ft} = (1-\rho_f) \ln Z_f + \rho_f \ln Z_{f,t-1} + \varepsilon_{ft}, \tag{18}$$

where $\rho_f \in (-1, 1)$ is the persistence parameter and ε_{ft} is a white noise process with a zero mean and a finite variance σ_f^2.

Firms are price takers in input markets and monopolistically competitive in their output market where they set their prices. The pricing decisions are staggered across firms (Calvo, 1983). Specifically, in each period, a fraction $1-\alpha_f$ of all firms can renew their price contracts. If a firm cannot set a new price, its price is mechanically updated by the steady-state rate of inflation $\bar{\pi}$. If a firm can set a new price, it chooses the price $P_f(j, s^t)$ to solve

$$\text{Max}_{P_f(j,s^t)} \quad \sum_{k=0}^{\infty} \alpha_f^k \sum_{s^{t+k}} Q(s^{t+k}|s^t)[P_f(j, s^t)\bar{\pi}^k - \bar{P}_f(s^{t+k})\phi(s^{t+k})] X^d(j, s^{t+k}), \tag{19}$$

subject to the demand function for goods in (2). In (19), $\phi(s^t)$ denotes the real marginal cost of production which, from the cost-minimizing problem, is given by

$$\phi(s^t) = \tilde{\alpha} w(s^t)^{1-\alpha} r_k(s^t)^{\alpha} / Z_{ft}, \tag{20}$$

with $\tilde{\alpha} = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$. The conditional factor demand functions from solving the cost-minimizing problem implies that

$$\frac{w(s^t)}{r_k(s^t)} = \frac{1-\alpha}{\alpha} \frac{K^f(j, s^t)}{L^f(j, s^t)}, \quad \forall j \in [0, 1]. \tag{21}$$

Solving the firm's profit-maximizing problem (19) yields the optimal price setting rule

$$P_f(j, s^t) = \frac{\theta_f}{\theta_f - 1} \frac{\sum_{k=0}^{\infty} \alpha_f^k \sum_{s^{t+k}} Q(s^{t+k}|s^t) X^d(j, s^{t+k}) \bar{P}_f(s^{t+k}) \phi(s^{t+k})}{\sum_{k=0}^{\infty} \alpha_f^k \sum_{s^{t+k}} Q(s^{t+k}|s^t) X^d(j, s^{t+k}) \bar{\pi}^k}. \tag{22}$$

Thus, the optimal price set in period t is a markup over a weighted average of the current and future marginal costs. If price adjustments are flexible (i.e., $\alpha_f = 0$), then the pricing rule (22) implies that the optimal price is a constant markup over the contemporaneous marginal cost.

III.4. **Real Estate Firms.** There is a measure one of real estate firms indexed by $i \in [0, 1]$. The real estate firm i produces a differentiated housing service $H_r(i, s^t)$ using the homogeneous housing stock $S_h(i, s^t)$ supplied by the households. The production function for housing services is given by

$$H_r(i, s^t) = Z_{ht} S_h(i, s^t), \tag{23}$$

where Z_{ht} is a housing sector-specific productivity shock, which follows a stationary process given by

$$\ln Z_{ht} = (1 - \rho_h) \ln Z_h + \rho_h \ln Z_{h,t-1} + \varepsilon_{ht}, \tag{24}$$

where $\rho_h \in (-1, 1)$ is the persistence parameter and ε_{ht} is a white noise process with a zero mean and a finite variance σ_h^2.

As in the regular goods sector, the real estate firms are price takers in the input market and monopolistic competitors in the output market where they set rental prices for the differentiated housing services.[6] In each period, a constant fraction $1-\alpha_h$ of the real estate firms can adjust their prices. If a real estate firm cannot set a new rental price, its price is mechanically updated by the steady-state inflation rate $\bar{\pi}$. If a real estate firm can set a new rental price, it chooses $R_h(i, s^t)$ to solve

$$\text{Max}_{R_h(i,s^t)} \quad \sum_{k=0}^{\infty} \alpha_h^k \sum_{s^{t+k}} Q(s^{t+k}|s^t) \left[R_h(i, s^t)\bar{\pi}^k - \frac{\bar{P}_f(s^{t+k}) r_s(s^{t+k})}{Z_{h,t+k}} \right] H^d(i, s^{t+k}), \tag{25}$$

subject to the demand function for housing services in (2). Note that all real estate firms face the same marginal cost (i.e., $r_s(s^t)$), regardless of whether or not they are setting a new rent for their housing services. Profit maximizing implies that the optimal rental price-setting rule is given by

$$R_h(i, s^t) = \frac{\theta_h}{\theta_h - 1} \frac{\sum_{k=0}^{\infty} \alpha_h^k \sum_{s^{t+k}} Q(s^{t+k}|s^t) H_r^d(i, s^{t+k}) \bar{P}_f(s^{t+k}) r_s(s^{t+k}) / Z_{h,t+k}}{\sum_{k=0}^{\infty} \alpha_h^k \sum_{s^{t+k}} Q(s^{t+k}|s^t) H_r^d(i, s^{t+k}) \bar{\pi}^k}. \tag{26}$$

This equation takes a similar form as the optimal pricing rule (22) for goods-producing firms and bears similar interpretations. A key difference is that the marginal cost facing rental service producers is the price of housing, while the marginal cost for the goods producers is a weighted average of the wage rate and the capital rental rate. If the fluctuations in the housing price differ from those of the primary factor prices, then the fluctuations in the rental price inflation will be different from those in the goods

[6] One can think that the pricing power of real estate firms (or landlords) arise for similar reasons as in the product market. Even in the same geographic area, there can be many landlords who provide differentiated housing services through, for instance, "brand-names," amenities, and maintenance services. Such product differentiation provides the landlords with some pricing power.

price inflation. Such difference has implications for optimal monetary policy, as we demonstrate in Section V.

III.5. **Monetary Policy.** Monetary policy follows a feedback interest-rate rule. Specifically, we generalize the interest rate rule considered in Clarida, Galí, and Gertler (2000) by allowing, but not requiring, the monetary policy to respond to changes in rental price inflation. The interest rate rule is given by

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1-\rho_r)\left[\phi_\pi\left(\gamma_f \hat{\pi}_{ft} + (1-\gamma_f)\hat{\pi}_{ht}\right) + \phi_y \hat{y}_t\right], \tag{27}$$

where $\hat{R}_t \equiv \ln\left(\frac{R(s^t)}{R}\right)$ denotes the log-deviation of the nominal interest rate from steady state, $\hat{\pi}_{ft} \equiv \ln\left(\frac{\bar{P}_f(s^t)}{\bar{P}_f(s^{t-1})\bar{\pi}}\right)$ and $\hat{\pi}_{ht} \equiv \ln\left(\frac{\bar{R}_h(s^t)}{\bar{R}_h(s^{t-1})\bar{\pi}}\right)$ denote the log-deviations of the goods price inflation and housing rental price inflation from the steady state, and $\hat{y}_t \equiv \ln\left(\frac{Y(s^t)}{Y}\right)$ denotes the log-deviation of real GDP from steady state.

III.6. **Market clearing and equilibrium.** In equilibrium, the markets for the final good, the housing stock, labor, capital, and bond all clear. The final-good market clearing implies that

$$X(s^t) = C(s^t) + I_k(s^t) + I_s(s^t). \tag{28}$$

Market clearing for the housing stock implies that

$$\gamma S(s^{t-1}) = \int_0^1 S_h(i, s^t)di. \tag{29}$$

Market clearing for the primary factors (labor and capital) implies that

$$L(s^t) = \int_0^1 L^f(j, s^t)dj, \quad K(s^{t-1}) = \int_0^1 K^f(j, s^t)dj. \tag{30}$$

Finally, market clearing for the nominal bond implies that $B(s^{t+1}) = 0$.

The real GDP (denoted by $Y(s^t)$) includes both the output of the final good and the imputed rental value of final housing services. In particular, the real GDP is given by

$$Y(s^t) = X(s^t) + r_h(s^t)H(s^t). \tag{31}$$

An equilibrium consists of allocations $C(s^t)$, $H(s^t)$, $L(s^t)$, $I_k(s^t)$, $I_s(s^t)$, $K(s^t)$, $S(s^t)$, and $B(s^{t+1})$; allocations $X(j, s^t)$, $K^f(j, s^t)$, and $L^f(j, s^t)$ and price $P_f(j, s^t)$ for firm $j \in [0, 1]$; allocations $H_r(i, s^t)$ and $S_h(i, s^t)$ and price $R_h(i, s^t)$ for real estate firm $i \in [0, 1]$, along with the prices $\bar{P}_f(s^t)$, $\bar{R}_h(s^t)$, $Q(s^{t+1}|s^t)$, $w(s^t)$, $r_k(s^t)$, and $r_s(s^t)$ such that

(1) Taking prices as given, the household's allocations solve the utility maximizing problem;

(2) Taking all prices but its own as given, the firm's allocations and price solve its profit maximizing problem;
(3) Taking all prices but its own as given, the real estate firm's allocations and price solve its profit maximizing problem;
(4) Markets for the final good, the housing stock, labor, capital, and the nominal bond all clear;
(5) Monetary policy is specified in (27).

IV. Calibration and Solution

We calibrate the model parameters based on quarterly U.S. data and microeconomic evidence. First, we calibrate the preference parameters. We set the subjective discount factor to $\beta = 0.99$ so that the steady-state annualized real interest rate is 4%. We assume that the representative household's period utility function takes the form

$$u(C, H, L) = \log C^{1-\rho} H^{\rho} - \eta \frac{L^{1+\xi}}{1+\xi}, \tag{32}$$

where $\rho \in (0, 1)$ corresponds to the expenditure share of housing services, $\xi > 0$ is the inverse Frisch elasticity of labor supply, and $\eta > 0$ is the utility weight for leisure. The utility function implies the housing expenditure share is constant, which is consistent with empirical evidence. In the data, the ratio of housing expenditures to total personal consumption expenditures (PCE) remains stable at about 15% during the sample period from 1987 to 2007 in the United States.[7] Thus, we set $\rho = 0.15$. Since housing service consists of both owner occupied housing and rental housing, we set $\gamma = 0.268$, which corresponds to the steady-state share of rental housing in total housing expenditure. Microeconomic studies suggest that the Frisch elasticity of labor supply is small. Thus, we set $\xi = 2$, corresponding to a Frisch elasticity of 0.5, which is in line with empirical evidence (Pencavel, 1986). We set $\eta = 26.99$ so that the steady state hours worked is 30% of the time endowment.

Second, we calibrate the technology parameters, including α, the elasticity of output with respect to capital and θ_f and θ_h, the elasticities of substitution between differentiated goods and between rental services. We set $\alpha = 0.3$ so that the cost share of labor input is 70%. We set $\theta_f = \theta_h = 11$ so that the steady-state markup is 10% in both sectors.

[7] Housing has a smaller share in PCE than in CPI because PCE covers the consumption expenditures in the entire U.S. economy while CPI covers only urban households.

Third, we calibrate the depreciation rates of capital stock and housing stock. We follow Davis and Heathcote (2005) and set the capital depreciation rate to $\delta_k = 0.0139$ and the housing depreciation rate to $\delta_s = 0.0035$, corresponding to annual depreciate rates of 5.57% and 1.41%, respectively.

Fourth, we calibrate the nominal rigidity parameters. We set $\alpha_h = 0.75$ so that rental contracts last on average for four quarters, consistent with microeconomic evidence (Genesove, 2003). We set $\alpha_f = 0.75$ so that goods price contracts also last for four quarters on average, as in the macroeconomic literature. For robustness, we have also examined the case with more frequent adjustments in goods prices as recent microeconomic studies suggest. In particular, we examine the case with $\alpha_f = 0.5$ so that goods price contracts last on average for six months, in line with the studies by Bils and Klenow (2004) and Nakamura and Steinsson (2008).

Fifth, we calibrate the monetary policy parameters. We follow Lubik and Schorfheide (2004) and set $\rho_r = 0.84$, $\phi_\pi = 2.19$, and $\phi_y = 0.30$ in the benchmark Taylor rule. Further, we set $\gamma_f = 0.7$, reflecting the expenditure on the shelter component of housing services of about 30% as in the CPI.[8]

Sixth, we calibrate the parameters in the shock processes. We set the AR(1) coefficients in the productivity shock processes to $\rho_f = 0.95$ and $\rho_h = 0.95$. We set the standard deviation of the housing service productivity shock to $\sigma_h = 0.02$. We then calibrate the standard deviation of the productivity shock in the goods-producing sector σ_f so that the model implies a standard deviation of real GDP of 0.016, the same as that calculated from the HP-filtered U.S. data for the sample period 1950:Q1-2009:Q2.

Finally, we calibrate the adjustment cost parameters through simulation of the model. We assume that the adjustment cost functions take the form

$$\Psi_j\left(\frac{I_{jt}}{I_{j,t-1}}\right) = \frac{\omega_j}{2}\left(\frac{I_{jt}}{I_{j,t-1}} - 1\right)^2, \quad j \in \{k, s\}. \tag{33}$$

We assign values for ω_k and ω_s so that the model under the benchmark Taylor rule generates the same standard deviations of non-residential and residential fixed investments relative to the volatility of real GDP as those observed in the postwar U.S. data in the sample period from 1950:Q1 to 2009:Q2 (these relative volatilities based on HP-filtered data are 3.11 and 6.05, respectively). Table 1 summarizes the calibrated parameters.

[8]We have examined the model's implications when we set $\gamma_f = 0.6$, corresponding to a housing expenditure share of about 40% (including shelter and utilities) in the CPI. The qualitative results do not change.

Using the calibrated parameter values, we solve the model based on second-order approximations of the equilibrium conditions around the deterministic steady state. To evaluate welfare under a particular set of Taylor rule parameters, we follow Sutherland (2002) by including the representative household's value function as an additional variable to be solved in the system of equations. We describe the second-order approximations to the equilibrium dynamics in the Appendix.

V. Optimal monetary policy

In this section, we examine optimal monetary policy in the class of feedback interest rate rules. We also discuss an alternative policy rule that exclusively focuses on stabilizing the price level.

V.1. **Optimal Taylor rule.** To study optimal monetary policy, we focus on the class of simple, implementable feedback interest rate rules as in (27). We evaluate the welfare loss under a particular set of Taylor rule parameters by computing the consumption equivalence in the economy with sticky prices relative to the economy with flexible prices. We start with evaluating the welfare losses under our benchmark calibration. We then update the Taylor rule parameters recursively to minimize the welfare loss.[9] The optimal monetary policy rule is defined as the rule with the set of parameters ρ_r, ϕ_π, ϕ_y, and γ_f that minimize the welfare loss.

Table 2 displays the optimal Taylor-rule coefficients and the welfare loss under the optimal rule. For comparison, the table also displays the welfare loss under the benchmark Taylor rule, where the coefficients are fixed at their calibrated values. The welfare losses are expressed in terms of consumption equivalence, which is calculated as the percentage of steady-state non-housing consumption required to compensate the representative household so that she is indifferent between living in the economy with nominal rigidities and the one without.

The Table shows that the welfare loss under optimal policy is substantially smaller than that under the benchmark Taylor rule (0.006 vs. 2.126 percent of consumption). The optimal rule calls for a greater response of the nominal interest rate to the composite inflation rate (i.e., the average of the goods and rental inflation rates) than does the benchmark Taylor rule, with a coefficient on average inflation of about 7.06, compared

[9] In this optimizing process, some parameters can enter the region of indeterminacy where the Taylor principle is violated and multiple equilibria can arise. To avoid the indeterminacy problem, we check the eigenvalues and, when indeterminacy is detected, we set the welfare loss to infinity to force a restart of the search process.

to 2.19 under the benchmark rule. The optimal rule also suggests that the monetary authority should mainly respond to inflation fluctuations and should put little weight on output gap or the lagged interest rate.

An important and perhaps surprising result is that the optimal rule assigns a much smaller weight on rental price inflation than the housing expenditure share in the consumption basket (0.10 vs. 0.30). This finding is new relative to the literature. Existing studies suggest that, in a multi-sector model with multiple sources of nominal rigidities, optimal monetary policy faces a trade-off in stabilizing the sectoral inflation rates and cannot replicate the flexible-price equilibrium allocations. Thus, optimal (second-best) policy should assign positive weights to each sector's inflation, with the relative weights determined by the relative price stickiness and the expenditure shares of the sectors' goods in the final consumption basket (Woodford, 2003; Benigno, 2004; Huang and Liu, 2005).

As in the literature, optimal policy in our model with two sources of nominal rigidities faces a tradeoff between stabilizing the inflation rates in the two sectors. This tradeoff is illustrated in Figure 1. The figure plots the welfare loss the relative weight assigned to non-housing price inflation (i.e., γ_f) varies in the range between 0 and 1, while the other Taylor rule parameters remain at their values under the optimal rule. Since optimal policy stabilizes a weighted average of the two sectors' inflation rates, the welfare loss is a hyperbolic function of γ_f, as shown in the figure. For small values of γ_f, the welfare loss decreases with γ_f; for large enough values of γ_f, the welfare loss increases with γ_f. The minimum welfare loss obtains when $\gamma_f = 0.90$, that is, when the rental price inflation receives a relative weight of 0.10 in the consumption basket.

Unlike the results reported in the literature, however, we find that the optimal relative weight on rental price inflation does *not* fully reflect the housing expenditure share even though the two sectors have equal durations of price contracts. This finding stems from a particular source of asymmetry in our model: the production of differentiated housing services is less labor intensive than the production of non-housing goods. The lower labor intensity in producing housing services is important for optimal policy and welfare losses. Under staggered price-setting, the price dispersion and the associated misallocation of labor across firms give rise to welfare losses relative to an economy with flexible prices, in which all firms make identical pricing and employment decisions. This form of misallocation is less severe, the lower the labor share in production. As the labor intensity in the housing service sector is lower, the housing sector receives a lower weight than its expenditure share under optimal inflation-targeting policy, as we find in this paper.

Our result can be generalized to an economy with more frequent adjustments in goods prices than rental prices. In particular, when we set $\alpha_f = 0.5$ while keeping $\alpha_h = 0.75$ so that rental price contracts last twice as long as goods price contracts, the optimal weight for rental price inflation becomes larger (0.21) than that under our baseline calibration (0.10). Nonetheless, it is still substantially smaller than the housing expenditure share in the consumption basket.

V.2. **Relative volatility of rental price inflation.** The asymmetry in factor intensity has implications for the relative volatility of rental price inflation versus goods price inflation. In the model, production of housing services uses housing stocks intensively. As the adjustment in housing stocks is costly, the price of housing stocks needs to adjust rapidly in response to shocks. Thus, marginal cost facing real estate firms adjusts rapidly. Although rental contracts are renewed infrequently, large cyclical movements in housing prices force those real estate firms that can re-optimize to change their rental prices rapidly. Thus, the rental price index, which is a weighted average of the reset rental prices and those remain fixed, should adjust rapidly. As the goods sector is more labor intensive than the housing sector, the adjustments in marginal cost and the optimal reset prices in the goods sector should be smaller and less rapid than those in the housing service sector. With identical durations of price contracts, the model implies that rental price inflation should be more volatile than goods price inflation.

This implication of the model is consistent with empirical evidence. Figure 2 shows that rental price inflation has displayed larger swings than core CPI inflation in the U.S. economy. During the sample period from the second quarter of 1959 to the second quarter of 2010, rent inflation has a standard deviation of 3.72%, while core inflation has a much smaller standard deviation of 2.72%. Rent inflation is thus about 1.37 times as volatile as core inflation in the data. In comparison, the model under the benchmark Taylor rule generates a relative volatility of rent inflation of about 1.29, which is close to the relative volatility observed in the data.

V.3. **Extreme inflation targeting.** We have examined optimal implementable monetary policy within the class of Taylor rules under which the monetary authority sets the nominal interest rate to respond to deviations of inflation and output from their targets. In a standard one-sector sticky-price model, however, a policy that exclusively focuses on stabilizing the price level is optimal since it brings equilibrium output to its potential. This result is known as the "divine coincidence" (Blanchard and Gali, 2007) and it provides theoretical support for the monetary policy practice of inflation targeting.

In our two-sector model with two sources of nominal rigidities, monetary policy faces a trade-off between stabilizing goods price inflation and rental price inflation so that the divine coincidence breaks down. Nonetheless, it is natural to examine whether or not an extreme inflation targeting policy (i.e., a policy that exclusively aims at stabilizing the price level) brings equilibrium allocations close to optimal.

To address this issue, we compute the welfare loss under an extreme inflation targeting policy. In particular, we replace the Taylor rule in Equation (27) with a constant-inflation rule

$$\gamma_f \hat{\pi}_{ft} + (1 - \gamma_f)\hat{\pi}_{ht} = 0, \tag{34}$$

where $\hat{\pi}_{ft}$ and $\hat{\pi}_{ht}$ denote goods price inflation and rental price inflation. We set $\gamma_f = 0.7$ as in our benchmark calibration, so that the left-hand side of Equation (34) corresponds to changes in the consumer price index.

The bottom row of Table 2 shows that the welfare loss under the constant-inflation rule is about 0.012 percent of consumption equivalence. This welfare loss here is greater than that under the optimal Taylor rule (0.006), but the difference is small. Thus, as in the standard one-sector sticky-price model, a policy that exclusively focuses on stabilizing the price level in our model with housing and multiple sources of nominal rigidities is close to optimal. In this sense, the trade-off between stabilizing goods price inflation and rental price inflation is small. Our earlier analysis suggests a reason for this result. Since the housing services sector is less labor intensive than the goods sector, nominal rigidities and the associated misallocation of labor in that sector do not introduce a quantitatively important trade-off for monetary policy.

V.4. **Implications of factor intensity for optimal policy.** We have argued that our finding about optimal monetary policy responses to rental price inflation stems from the asymmetry in factor intensity. We have assumed in the benchmark model that producing differentiated housing services is less labor intensive than producing non-housing goods. In our benchmark specification, the housing service sector does not require any labor input. In general, however, it is plausible that some labor input is needed to differentiate housing services. If our intuition is correct, then one should expect that, all else equal, optimal policy in an economy with a greater labor share in the housing sector should assign a larger weight on rental price inflation. This is indeed the case, as we show below.

To examine the implications of labor share in the housing sector for optimal inflation-targeting policy, we consider the more general production function for real estate firms:

$$H_r(i, s^t) = Z_{ht} S_h(i, s^t)^a L_h(i, s^t)^{1-a}, \tag{35}$$

where $L_h(i, s^t)$ denotes the labor input. The special case with $a = 1$ corresponds to our benchmark specification. To maintain minimal deviations from the benchmark, we assume that labor is firm specific.[10] We replace the housing-service production function (23) with this more general production function and search for optimal coefficients in the Taylor rule (27) under calibrated parameter values, taking the value of a as given. For each value of a, we recalibrate the adjustment-cost parameters ω_k and ω_s such that the model under the benchmark Taylor rule generates the same standard deviations of non-residential and residential fixed investment relative to the standard deviation of real GDP in the data.

The results we obtain from this exercise confirm our intuition: the optimal weight on rent inflation is a decreasing function of a. When a goes down from 1 (corresponding to the benchmark specification) to 0.8, the optimal weight on rent inflation goes up from 0.10 to 0.14. When a goes further down to 0.3, the optimal weight goes further up to 0.26, which is close to the housing expenditure share of 0.3. Thus, the asymmetry in factor intensity is a key driver of our findings about optimal monetary policy responses to rental price inflation.

VI. Conclusion

Optimal monetary policy analysis suggests that it is socially desirable to achieve price-level stability. In practice, many central banks choose to target, either explicitly or implicitly, some core measures of consumer price inflation. Since housing is an important component of the consumption basket, such inflation targeting policies implicitly assign a weight to rental price inflation that is identical to the housing expenditure share.

In this paper, we find that optimal monetary policy should place a much smaller weight on rental price inflation than that implied by the housing expenditure share. This result stems from the differences in factor intensity between the two sectors. In

[10]This assumption biases the result towards assigning a larger weight to rental price inflation under optimal policy since incorporating firm-specific factors in general helps dampen price adjustments for firms that re-optimize (Chari, Kehoe, and McGrattan, 2000; Altig, Christiano, Eichenbaum, and Linde, 2004).

particular, since housing-service production is less labor intensive than goods production, nominal rigidities and the associated price dispersion cause less severe misallocation of labor in the housing sector than in the goods sector. Thus, optimal inflation-targeting policy should place a smaller weight on rental price inflation than the housing expenditure share.

To help focus our discussion on the trade-off between rental price inflation and goods price inflation, we abstract from several other sources of frictions that might be important in the actual economy. For instance, we do not model nominal wage rigidities. Generalizing our model to allow for nominal wage rigidities is unlikely to change our qualitative results. Indeed, it may likely strengthen our results. Since the rental housing sector is more capital intensive than the goods sector, introducing nominal wage rigidities would make goods price adjustments even more sluggish and thus optimal policy should assign an even smaller weight on rental price inflation.

Another potentially important direction to generalize our work is to introduce credit constraints in the housing market along the lines of Kiyotaki and Moore (1997) and study optimal monetary policy in response to booms and busts in housing prices. As shown by Liu, Wang, and Zha (2009), this class of models contains a financial multiplier that plays a quantitatively important role in amplifying and propagating small economic shocks and transforming these shocks into large business cycle fluctuations. In our view, studying optimal monetary policy in a model with housing and financial frictions is an important and promising direction for future research.

Appendix A. Equilibrium Dynamics

In this Appendix, we summarize the equations that characterize the equilibrium dynamics in our model and we describe our solution method. To simplify notations, we drop the state notation s^t and denote the variable $X(s^t)$ by X_t. According, we denote by $E_t X_{t+k} \equiv \sum_{s^{t+k}} \pi(s^{t+k}|s^t) X(s^{t+k})$ the conditional expectation of the variable X_{t+k}.

A.1. **Equilibrium conditions.**

The firm's optimal pricing rule (22) implies that

$$P_{ft}^* = \frac{\theta_f}{\theta_f - 1} \frac{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha_f \beta)^k U_{c,t+k} X_{t+k} \phi_{t+k} \left(\prod_{i=1}^{k} \pi_{f,t+i}/\bar{\pi}_f \right)^{\theta_f}}{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha_f \beta)^k U_{c,t+k} X_{t+k} \left(\prod_{i=1}^{k} \pi_{f,t+i}/\bar{\pi}_f \right)^{\theta_f - 1}}, \tag{A1}$$

where $P_{ft}^* \equiv \frac{P_{ft}(j)}{\bar{P}_{ft}}$ denotes the relative price for the optimizing firm j and we have substituted for the demand schedule $X_{t+k}^d(j)$ using the relation

$$X_{t+k}^d(j) = P_{ft}^{*\ -\theta_f} \left[\frac{\bar{P}_{ft}}{\bar{P}_{f,t+k}}\right]^{-\theta_f} X_{t+k} = P_{ft}^{*\ -\theta_f} \left(\prod_{i=1}^{k} \pi_{f,t+k}/\bar{\pi}_f\right)^{\theta_f} X_{t+k}.$$

Similarly, the real estate firm's optimal pricing rule (26) implies that

$$R_{ht}^* = \frac{\theta_h}{\theta_h - 1} \frac{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha_h \beta)^k U_{c,t+k} H_{r,t+k} \frac{r_{s,t+k}}{Z_{h,t+k}} \left(\prod_{j=1}^{k} \pi_{h,t+j}/\bar{\pi}_f\right)^{\theta_h}}{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha_h \beta)^k U_{c,t+k} H_{r,t+k} r_{h,t+k} \left(\prod_{j=1}^{k} \pi_{h,t+j}/\bar{\pi}_f\right)^{\theta_h - 1}}, \tag{A2}$$

where $R_{ht}^* \equiv \frac{R_{ht}(i)}{\bar{R}_{ht}}$ denotes the relative rental price for the optimizing real estate firm i.

The price index relations imply that

$$1 = \alpha_f \left(\frac{\bar{\pi}}{\pi_{ft}}\right)^{1-\theta_f} + (1-\alpha_f)\left(P_{ft}^*\right)^{1-\theta_f}, \tag{A3}$$

$$1 = \alpha_h \left(\frac{\bar{\pi}}{\pi_{ht}}\right)^{1-\theta_h} + (1-\alpha_h)\left(R_{ht}^*\right)^{1-\theta_h}. \tag{A4}$$

The real marginal cost for firms in the goods-production sector is given by

$$\phi_t = \frac{\tilde{\alpha}}{Z_{ft}} w_t^{1-\alpha} r_{kt}^{\alpha}. \tag{A5}$$

The firm's cost-minimizing problem implies the factor demand equations

$$\frac{w_t}{r_{kt}} = \frac{1-\alpha}{\alpha}\frac{K_{t-1}}{L_t}, \tag{A6}$$

$$r_{kt} = \phi_t \alpha \frac{G_{ft} X_t}{K_{t-1}}, \tag{A7}$$

where we have used the factor market clearing conditions (30) and the term $G_{ft} \equiv \int_0^1 \left(\frac{P_{ft}(j)}{\bar{P}_{ft}}\right)^{-\theta_f} dj$ measures the price dispersion across goods-producing firms. In a symmetric equilibrium, the term G_{ft} is given by

$$G_{ft} = \alpha_f \left(\frac{\bar{\pi}}{\pi_{ft}}\right)^{-\theta_f} + (1-\alpha_f)\left(P_{ft}^*\right)^{-\theta_f}. \tag{A8}$$

The housing service production function (23) implies that

$$S_{ht} = \frac{G_{ht} H_{rt}}{Z_{ht}}, \tag{A9}$$

where the term $G_{ht} \equiv \int_0^1 \left(\frac{R_{ht}(i)}{\bar{R}_{ht}}\right)^{-\theta_h} di$ measures the rental price dispersion across real estate firms, which, in a symmetric equilibrium, can be written as

$$G_{ht} = \alpha_h \left(\frac{\bar{\pi}}{\pi_{ht}}\right)^{-\theta_h} + (1-\alpha_h)\left(R_{ht}^*\right)^{-\theta_h}. \tag{A10}$$

The household's optimal choice of housing and non-housing consumption services implies that

$$\frac{U_{ht}}{U_{ct}} = r_{ht}. \tag{A11}$$

The household's optimal labor supply decision is given by

$$\frac{-U_{lt}}{U_{ct}} = w_t. \tag{A12}$$

The intertemporal bond Euler equation is given by

$$1 = \beta \mathrm{E}_t \frac{U_{c,t+1}}{U_{ct}} \frac{R_t}{\pi_{f,t+1}}. \tag{A13}$$

The optimal capital investment decision implies that

$$\frac{1}{q_{kt}} = 1 - \Psi_k\left(\lambda_{I_k,t}\right) - \lambda_{I_k,t}\Psi_k'\left(\lambda_{I_k,t}\right) + \mathrm{E}_t\beta\frac{U_{c,t+1}}{U_{ct}}\frac{q_{k,t+1}}{q_{kt}}\Psi_k'\left(\lambda_{I_k,t+1}\right)\left(\lambda_{I_k,t+1}\right)^2, \tag{A14}$$

where $\lambda_{I_k,t} \equiv \frac{I_{kt}}{I_{k,t-1}}$ denotes the growth rate of capital investment.

The optimal housing investment decision implies that

$$\frac{1}{q_{st}} = 1 - \Psi_s\left(\lambda_{I_s,t}\right) - \lambda_{I_s,t}\Psi_s'\left(\lambda_{I_s,t}\right) + \mathrm{E}_t\beta\frac{U_{c,t+1}}{U_{ct}}\frac{q_{s,t+1}}{q_{st}}\Psi_s'\left(\lambda_{I_s,t+1}\right)\left(\lambda_{I_s,t+1}\right)^2, \tag{A15}$$

where $\lambda_{I_s,t} \equiv \frac{I_{st}}{I_{s,t-1}}$ denotes the growth rate of housing investment.

The capital Euler equation is given by

$$q_{kt} = \mathrm{E_t}\beta\frac{\mathrm{U_{c,t+1}}}{\mathrm{U_{ct}}}\left[(1-\delta_\mathrm{k})\mathrm{q_{k,t+1}} + \mathrm{r_{k,t+1}}\right]. \tag{A16}$$

The housing Euler equation is given by

$$q_{st} = \mathrm{E_t}\beta\frac{\mathrm{U_{c,t+1}}}{\mathrm{U_{ct}}}\left[(1-\delta_\mathrm{s})\mathrm{q_{s,t+1}} + \gamma\mathrm{r_{s,t+1}} + (1-\gamma)\mathrm{r_{h,t+1}}\right]. \tag{A17}$$

The accumulation of capital and housing stocks follows the laws of motion

$$K_t = (1-\delta_k)K_{t-1} + I_{kt}\left[1 - \Psi_k\left(\frac{I_{kt}}{I_{k,t-1}}\right)\right], \tag{A18}$$

$$S_t = (1-\delta_s)S_{t-1} + I_{st}\left[1 - \Psi_s\left(\frac{I_{st}}{I_{s,t-1}}\right)\right], \tag{A19}$$

Goods market clearing implies that

$$X_t = C_t + I_{kt} + I_{st}. \tag{A20}$$

Rental housing market clearing implies that

$$\gamma S_{t-1} = S_{ht}. \tag{A21}$$

The housing services is related to the owner-occupied housing and the rental housing by

$$H_t = (1-\gamma)S_{t-1} + H_{rt}. \tag{A22}$$

The aggregate goods production function is given by

$$G_{ft}X_t = Z_{ft}L_t^{1-\alpha}K_{t-1}^{\alpha}. \tag{A23}$$

The real GDP is given by

$$Y_t = X_t + r_{ht}H_t. \tag{A24}$$

Finally, we rewrite the Taylor rule here for convenience of referencing:

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1-\rho_r)\left[\phi_\pi \left(\gamma_f \hat{\pi}_{ft} + (1-\gamma_f)\hat{\pi}_{ht}\right) + \phi_y \hat{y}_t\right]. \tag{A25}$$

Equations (A1)-(A25) summarize the equilibrium conditions.

A.2. **Optimal pricing rules in recursive forms.** To solve for the equilibrium dynamics, it is convenient to express the optimal pricing rules (22) and (26) in recursive forms.

We begin with the pricing decision (A1) for goods-producing firms. Denote by D_{ft} the denominator and N_{ft} the numerator. That is,

$$P_{ft}^* = \mu_f \frac{N_{ft}}{D_{ft}}, \tag{A26}$$

where $\mu_f \equiv \frac{\theta_f}{\theta_f - 1}$ and

$$N_{ft} \equiv \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha_f \beta)^k U_{c,t+k} X_{t+k} \phi_{t+k} \left(\prod_{i=1}^{k} \pi_{f,t+i}/\bar{\pi}_f \right)^{\theta_f}, \tag{A27}$$

$$D_{ft} \equiv \mathrm{E}_t \sum_{k=0}^{\infty} (\alpha_f \beta)^k U_{c,t+k} X_{t+k} \left(\prod_{i=1}^{k} \pi_{f,t+i}/\bar{\pi}_f \right)^{\theta_f - 1}. \tag{A28}$$

We can then write (A27) and (A28) in recursive forms. In particular, we have

$$N_{ft} = n_{ft} + \alpha_f \beta \mathrm{E}_t N_{f,t+1} \left(\frac{\pi_{f,t+1}}{\bar{\pi}_f} \right)^{\theta_f}, \tag{A29}$$

$$D_{ft} = d_{ft} + \alpha_f \beta \mathrm{E}_t D_{f,t+1} \left(\frac{\pi_{f,t+1}}{\bar{\pi}_f} \right)^{\theta_f - 1}, \tag{A30}$$

where

$$d_{ft} = U_{ct}X_t, \quad n_{ft} = U_{ct}X_t\phi_t. \tag{A31}$$

Similarly, we can rewrite the optimal rental pricing decision (A2) as

$$R^*_{ht} = \mu_h \frac{N_{ht}}{D_{ht}}, \tag{A32}$$

where $\mu_f \equiv \frac{\theta_f}{\theta_f - 1}$ and

$$N_{ht} = n_{ht} + \alpha_h \beta \mathrm{E}_t N_{h,t+1} \left(\frac{\pi_{h,t+1}}{\bar{\pi}_f} \right)^{\theta_h}, \tag{A33}$$

$$D_{ht} = d_{ht} + \alpha_h \beta \mathrm{E}_t D_{h,t+1} \left(\frac{\pi_{h,t+1}}{\bar{\pi}_f} \right)^{\theta_h - 1}, \tag{A34}$$

with

$$d_{ht} = U_{ct} H_{rt} r_{ht}, \quad n_{ht} = U_{ct} H_{rt} \frac{r_{st}}{Z_{ht}}. \tag{A35}$$

We solve the equilibrium dynamics by taking second-order approximations around the steady-state equilibrium.

Table 1. Calibrated Parameters

Parameter	Description	Value
β	Discount factor	0.99
ρ	Expenditure share of housing	0.15
γ	Rental housing share	0.2683
η	Utility weight of leisure	26.99
ξ	Inverse Frisch elasticity	2.00
α	Cost share of capital	0.30
θ_f	Elasticity of substitution between goods	11
θ_h	Elasticity of substitution between rental services	11
δ_k	Non-residential capital depreciation rate	0.0139
δ_s	Residential capital depreciation rate	0.0035
ω_k	Non-residential investment adjustment cost	0.8863
ω_k	Residential investment adjustment cost	6.3380
α_f	Calvo parameter for goods sector	0.75
α_h	Calvo parameter for real estate sector	0.75
ρ_r	Interest-rate smoothing parameter in Taylor rule	0.84
ϕ_π	Inflation coefficient in Taylor rule	2.19
ϕ_y	Output coefficient in Taylor rule	0.30
γ_f	Weight of goods price inflation in Taylor rule	0.70
ρ_f	Persistence of goods sector technology shock	0.95
ρ_h	Persistence of real estate sector technology shock	0.95
σ_f	Standard deviation of goods sector technology shock	0.0041
σ_h	Standard deviation of real estate sector technology shock	0.02

TABLE 2. Welfare losses

Policy rule	Taylor rule coefficients				Welfare loss
	ρ_r	γ_f	ϕ_π	ϕ_y	
Benchmark Taylor rule	0.84	0.70	2.19	0.30	2.126
Optimal Taylor rule	0.00	0.90	7.06	-0.02	0.006
Constant-inflation rule					0.012

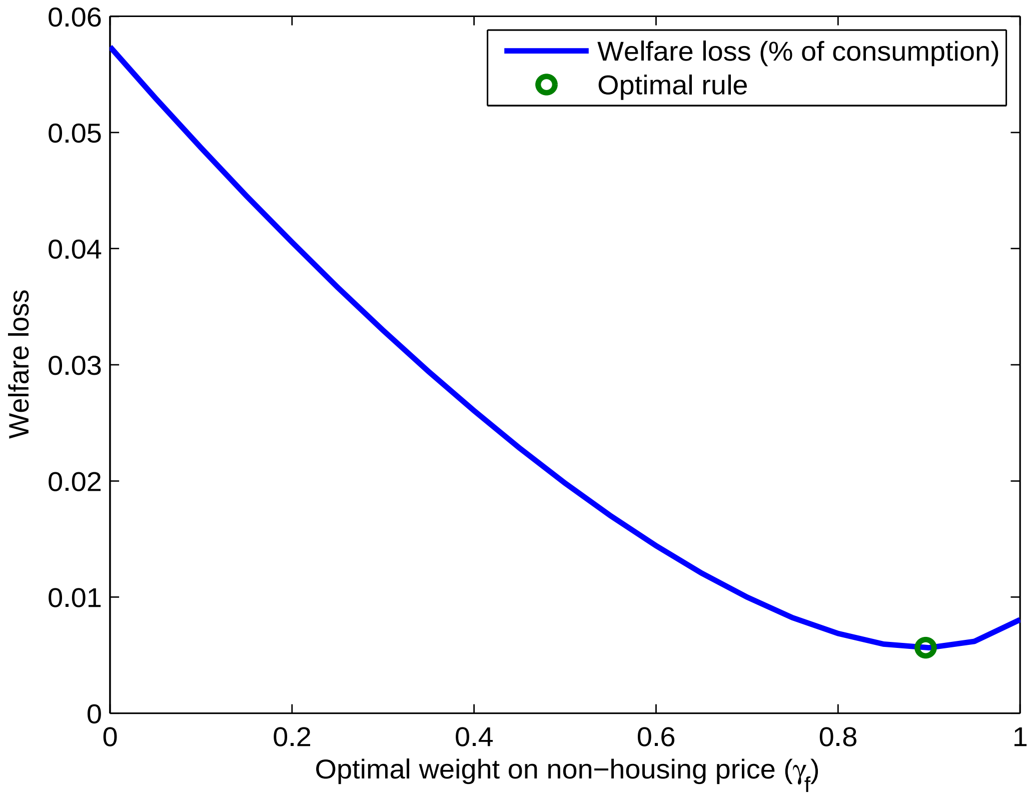

FIGURE 1. Optimal monetary policy trade-off between housing and non-housing inflation rates.

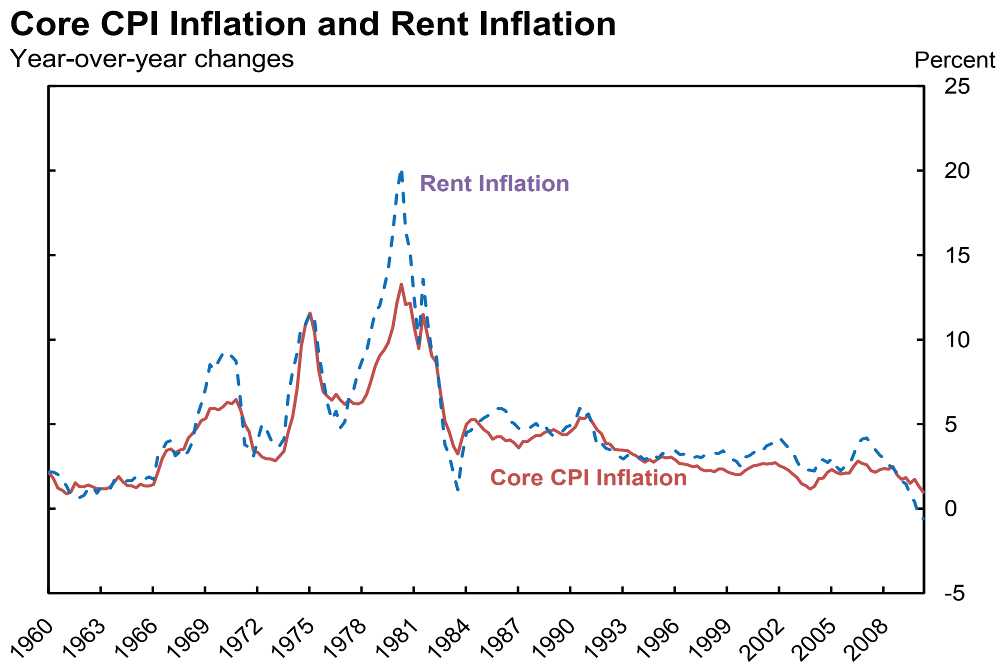

FIGURE 2. Volatile rental price inflation

Note: The solid line is the year-over-year changes in core CPI, that is, the consumer price index for all items excluding food and energy. The dashed line is the year-over-year changes in the rental component of the CPI. Source: Bureau of Labor Statics.

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