A Appendix to "Foreign stock holdings: the role of information"

A.1 Complementary information on households' portfolios

Tables 11 and 12 complement the information on households' portfolio by summarizing their financial and non-financial asset holdings.[49] Both Tables report the mean holdings for each type of asset as a share of financial and non-financial assets, respectively, showing the evolution of the composition of households' portfolio of assets.

Table 11: Holdings of financial fssets

	Type of Asset[a]:	1998	2001	2004	2007
(1)	Transaction accounts	11.30	11.58	13.18	10.95
(2)	Certificate of deposit	4.27	3.05	3.70	4.03
(3)	Saving bonds	0.67	0.68	0.54	0.44
(4)	Bonds	4.28	4.50	5.28	4.14
(5)	Domestic stocks	15.85	15.12	11.88	11.00
(6)	Foreign stocks	0.86	0.63	0.66	1.14
(7)	Mutual Funds	12.34	12.09	14.64	15.80
(8)	Retirement accounts	27.38	28.64	32.01	34.48
(9)	Cash in life insurance	6.31	5.25	2.96	3.22
(10)	Other managed assets	8.88	10.78	7.98	6.47
(11)	Other financial	1.94	2.11	2.17	2.73
	Total:	100	100	100	100
	Additional stats:				
(12)	Indirect	31.01	34.55	33.65	35.16
(13)	Equity	53.71	55.86	51.18	52.90
(14)	Financial share[b]	40.86	42.55	35.71	34.14

[a] In 2007 dollars (shares of financial assets)
[b] Share of financial assets in total assets

Table 12: Holdings of nonfinancial assets

	Type of Asset[a]:	1998	2001	2004	2007
(1)	Vehicles	6.46	5.99	5.12	4.46
(2)	Real Estate	47.03	47.26	50.31	48.32
(3)	Other real estate	8.73	8.22	10.18	11.20
(4)	Nonresidential RE	7.58	8.22	7.12	5.38
(5)	Business	28.64	28.78	25.79	29.63
(6)	Other non-financial	1.56	1.53	1.48	1.02
	Total	100	100	100	100
	Additional stats:				
(7)	Non-Financial share[b]	59.14	57.45	64.29	65.86

[a]In 2007 dollars (shares of non-financial assets)
[b] Share of non-financial assets in total assets

Each row of Table 11, but the last one, presents the mean value of each type of asset as a share of financial assets. Row (14) shows the mean value of financial assets as a share of total assets. Throughout

[49]These two Tables 11 and 12 closely replicate Tables 4 and 8 of the Federal Reserve Bank Bulletin of February 2009 that follows the data release. Small differences between the tables there reported and the numbers here presented are due to discrepancies between the data publicly available and the full survey data set. In addition, to closely obtain their results, these mean measures are *unconditional* on holding the asset.

the years, after reaching its peak in 2001, households move away from financial assets as a share of total assets. In fact, this decrease over the years verifies for most of the categories reported on the table, with the exception of holdings of retirement accounts and mutual funds (rows (7) and (8)). These last two types of investment are almost steady over the first three trienniums of the Survey and increase between 2004 and 2007, as a share of financial assets.

Figure 5 shows the mean holdings as a share of total financial wealth, conditional on holding the asset by total wealth percentiles. While participation and unconditional shares (unreported) are increasing in wealth percentiles, there is no such clear pattern when looking at conditional shares of financial assets. As Figure 5 shows, instead; it is not true that the shares of each type of asset increase across wealth quartiles; and it is also not true that this share monotonically increases for each percentile over time. For both domestic and foreign direct holdings, it is actually true that in 2001, 2004 and 2007, the shares of these types of assets decrease when moving from the middle percentiles to the highest ones.[50]

Figure 6 reports the mean holdings of each asset class as a share of financial assets, conditional on holding the asset. The hump-shape, again, verifies for equity and indirect holdings, even though the shape is less pronounced than in Figure 3. For direct holdings of domestic and foreign stocks, however, there is no clear pattern across age intervals. These findings are in line with previous documentation of a weak relation between shares of assets held and age. It is interesting to notice that for direct holdings of domestic stocks there is actually an inverse hump-shape in the last three trienniums, suggesting that agents would experiment with those type of assets when young, migrate to other type of investments between late 30s and 60s and go back to these markets when older. No pattern appears to verify when looking at direct holdings of foreign stocks.

For stock holdings, as a share of financial assets, rows (5) and (6) show that households reduce their direct holdings between 2001 and 2004, while increasing their direct holdings of foreign stocks in the last triennium. In addition, to complement the analysis, row (13) of Table 11 present the path followed by equity holdings showing a small increase in such holdings in the last triennium.

Finally, Table 11, row (14), shows that the share of financial assets in total assets held by households has increased between 1998 and 2001, but decreased from the latter year to 2007. This decrease suggests that despite the recent recovery of the financial markets after the crisis in 2001, there is a movement towards non-financial assets on agents' portfolios. As Table 12 indicates, the boom in the housing market appears to be the main driver of the shift towards non-financial assets. There is a continuous increase in real estate asset holdings as a share of non-financial assets until 2004, while in 2007 there is a decline of this same variable.

Table 13 complements the robustness checks of Section 4 in the paper by replicating the *probit* estimations but constraining the sample to households whose asset levels are at the top 10% of the wealth distribution. The table shows that the results do not change substantially indicating that even if the regression variables may be correlated to wealth, this relation is not affecting of biasing the results.

[50] In 1998, the share held in foreign stocks at wealth percentiles as low as 10 showed a spike as a result of some few outliers which are dropped.

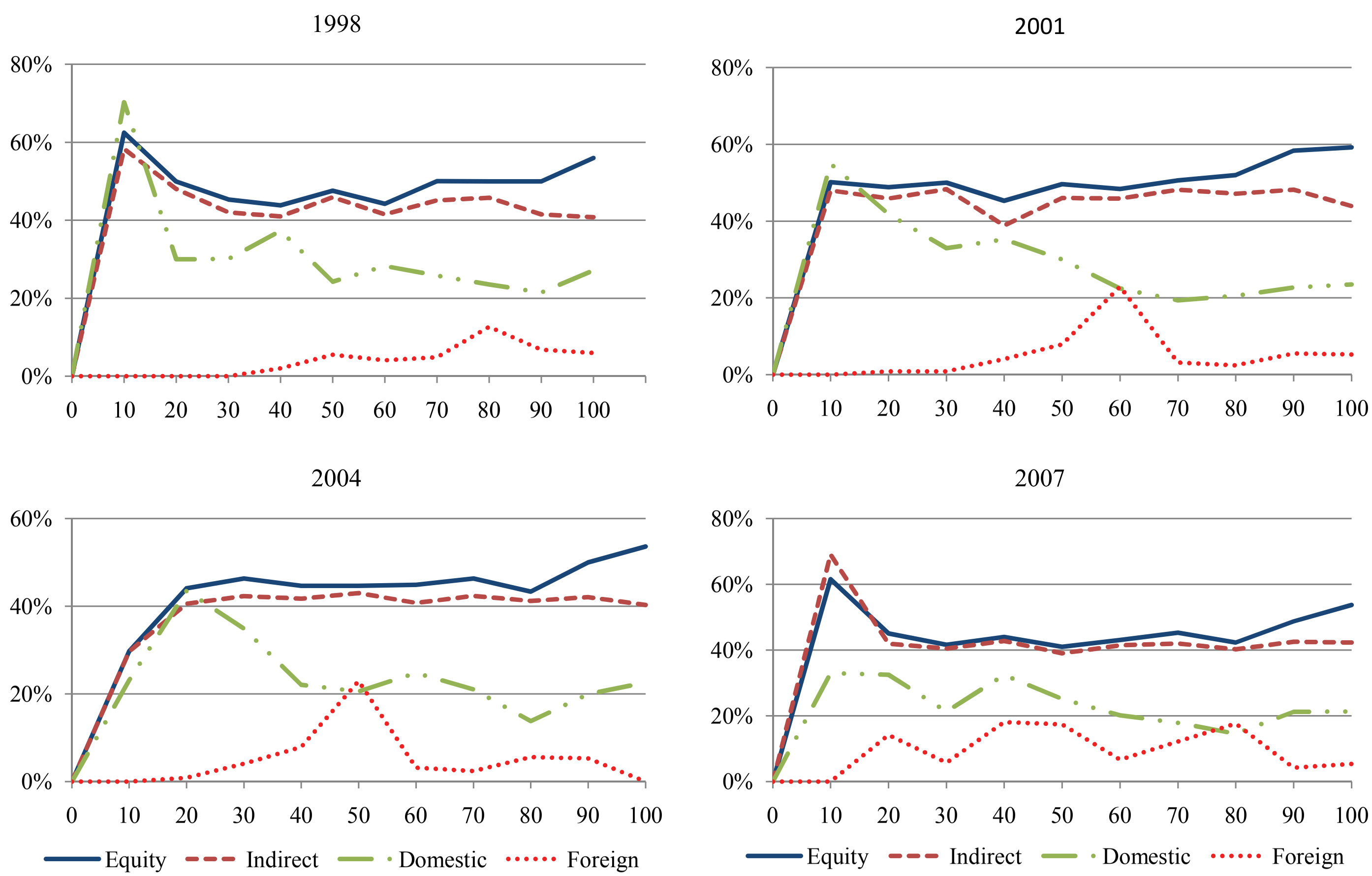

Figure 5: Asset class shares in household portfolios across wealth percentiles

Table 13: Holdings of stocks at the tenth highest wealth percentile

	Foreign Stocks		Domestic Stocks		Indirect Holdings	
Constant	-7.713***	(1.213)	-2.163*	(1.145)	-3.407***	(1.301)
Assets	0.445***	(0.09)	0.16**	(0.076)	0.042	(0.109)
Income	0.092	(0.06)	0.156***	(0.058)	0.325***	(0.072)
Business	-0.129***	(0.031)	-0.059*	(0.031)	-0.128**	(0.058)
Debt	-0.041	(0.039)	-0.025	(0.032)	-0.028	(0.039)
Age	0.008	(0.035)	-0.018	(0.037)	0.095***	(0.037)
Age2	0	(0)	0	(0)	-0.001**	(0)
Married	0.127	(0.178)	-0.106	(0.182)	-0.008	(0.186)
Female	0.62**	(0.298)	-0.125	(0.285)	-0.206	(0.256)
Hispanic	-0.63*	(0.328)	-0.599*	(0.309)	-0.398	(0.388)
No High School	0.675	(0.512)	-0.353	(0.344)	-0.374	(0.351)
College	0.47***	(0.156)	0.314***	(0.121)	0.281**	(0.134)
Self-employed	0.186	(0.137)	-0.174	(0.117)	-0.32*	(0.17)
Retired	0.057	(0.246)	-0.247	(0.239)	-0.412	(0.29)
Icertain	0.386***	(0.14)	-0.368***	(0.1)	0.033	(0.116)
Risk aversion	-0.228***	(0.085)	-0.154**	(0.065)	-0.244***	(0.08)
2001	-0.24	(0.165)	0.031	(0.147)	0.111	(0.182)
2004	-0.323*	(0.171)	-0.097	(0.146)	0.01	(0.184)
2007	0.015	(0.171)	-0.34**	(0.141)	-0.044	(0.169)
Internet	0.316**	(0.125)	0.052	(0.109)	0.224	(0.154)
R-squared	0.147		0.071		0.137	

The table reports coefficients and standard deviation estimates from probit models of ownership of foreign, domestic and indirect holdings of stocks for U.S. households in the 1998, 2001, 2004 and 2007 Surveys of Consumer Finances. Coefficients followed by *** are significant at 1%, ** are significant at 5% level, while coefficients followed by * are significant at 10% level. The Survey of Consumer Finances implements a multiple imputation procedure to correct for missing data, and hence, standard errors are adjusted by averaging the dependent and independent values across implicates. All data is weighted.

Table 14: Stock holders versus nonholders at at the tenth highest wealth percentile - financial characteristics (in 2007 thousand dollars)

	1998			2001		
	Non-holder	Domestic	Foreign	Non-holder	Domestic	Foreign
Income	149.4	269.7	323.7	201.9	351.1	382.9
Fin. Wealth	389.6	1,405.3	2,506.8	524.4	1,757.5	2,940.3
RE Wealth	452.4	417.6	467.7	419.5	576.8	636.4
Bus. Wealth	863.3	694.5	984.8	886.1	738.3	936.2
Debt	196.3	196.7	202.3	147.7	195.9	232.0
	2004			2007		
	Non-holder	Domestic	Foreign	Non-holder	Domestic	Foreign
Income	149.8	305.8	400.5	170.1	384.0	583.7
Fin. Wealth	539.5	1,612.9	2,671.7	272.3	1,831.0	3,272.0
RE Wealth	706.6	760.8	846.3	730.3	930.0	1,047.0
Bus. Wealth	878.7	807.7	910.8	1,400.1	1,094.0	1,595.8
Debt	243.5	277.2	248.1	327.0	334.3	333.3

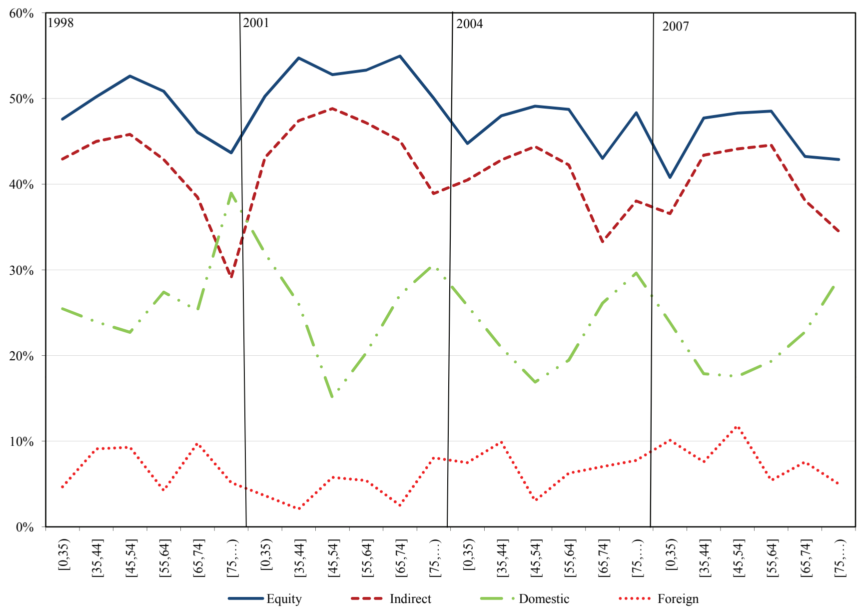

Figure 6: Share of financial assets on household portfolios by age

Table 15: Stock holders versus nonholders at the tenth highest wealth percentile - demographic characteristics

	1998			2001		
	Non-holder	Domestic	Foreign	Non-holder	Domestic	Foreign
Age	58.65	55.12	55.76	56.41	55.96	60.46
Education	14.09	15.59	15.57	14.00	15.65	15.92
Hispanic	0.04	0.01	0.01	0.01	0.01	0.00
Married	0.91	0.83	0.74	0.80	0.85	0.81
Risk Aversion	3.10	2.58	2.31	3.15	2.58	2.69
	2004			2007		
	Non-holder	Domestic	Foreign	Non-holder	Domestic	Foreign
Age	60.52	58.34	60.42	55.49	56.60	57.64
Education	14.45	15.66	15.67	13.74	15.69	16.01
Hispanic	0.07	0.01	0.00	0.12	0.02	0.01
Married	0.83	0.83	0.78	0.74	0.82	0.79
Risk Aversion	3.17	2.65	2.65	3.08	2.63	2.52

Table 16: Regression results – biprobit regression including indirect holdings

	Foreign		Domestic	
Constant	-6.804***	(1.03)	-2.306***	(0.679)
Assets	0.297***	(0.068)	0.281***	(0.05)
Income	0.121*	(0.064)	0.067	(0.053)
Business	-0.079***	(0.028)	-0.072***	(0.024)
Debt	-0.065**	(0.032)	-0.012	(0.026)
Age	0.025	(0.034)	-0.061***	(0.023)
Age2	0	(0)	0.001**	(0)
Married	0.148	(0.21)	-0.064	(0.137)
Female	0.517**	(0.263)	-0.302	(0.191)
Hispanic	-0.742***	(0.249)	-0.663***	(0.232)
No High School	0.236	(0.385)	-0.276	(0.215)
College	0.4***	(0.154)	0.214**	(0.086)
Self-employed	0.218	(0.135)	-0.105	(0.088)
Retired	-0.043	(0.213)	-0.113	(0.078)
Icertain	-0.226***	(0.074)	-0.113	(0.078)
Risk aversion	-0.226***	(0.074)	-0.171***	(0.05)
2001	0.044	(0.151)	0.009	(0.103)
2004	-0.244	(0.152)	-0.113	(0.078)
2007	-0.226***	(0.074)	-0.271**	(0.107)
Internet	0.275**	(0.133)	0.103	(0.086)
Indirect	0.392**	(0.177)	0.367***	(0.092)
Wald test of independent equations:			rho=0: chi2 = 1180.1	
			Prob > chi2 = 0.000	

The table reports coefficients and standard deviation estimates from a bivariate probit model of ownership of foreign and domestic stocks for U.S. households in the 1998, 2001, 2004 and 2007 Surveys of Consumer Finances. Coefficients followed by *** are significant at 1%, ** are significant at 5% level, while coefficients followed by * are significant at 10% level. The Survey of Consumer Finances implements a multiple imputation procedure to correct for missing data, and hence, standard errors are adjusted by averaging the dependent and independent values across implicates. All data is weighted.

Table 17: Additional information variables

	Times		Shop		Paper		Banker	
	Foreign	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign	Domestic
Constant	-7.339***	0.331	-13.567***	-3.924***	-13.697***	-3.891***	-13.451***	-3.948***
	(2.123)	(1.153)	(1.888)	(0.887)	(1.899)	(0.875)	(1.876)	(0.885)
Assets	0.366**	0.127	0.656***	0.388***	0.652***	0.375***	0.657***	0.388***
	(0.157)	(0.083)	(0.13)	(0.062)	(0.13)	(0.062)	(0.132)	(0.061)
Income	0.102	-0.041	0.237**	0.115*	0.237**	0.116*	0.233**	0.116*
	(0.126)	(0.075)	(0.112)	(0.064)	(0.112)	(0.064)	(0.112)	(0.064)
Business	-0.125**	-0.029	-0.191***	-0.105***	-0.186***	-0.099***	-0.187***	-0.105***
	(0.053)	(0.035)	(0.045)	(0.029)	(0.045)	(0.029)	(0.045)	(0.029)
Debt	-0.083	0.025	-0.108*	-0.008	-0.103*	-0.005	-0.109**	-0.008
	(0.065)	(0.041)	(0.056)	(0.034)	(0.056)	(0.034)	(0.055)	(0.034)
Age	0.016	-0.042	0.075	-0.064**	0.076	-0.066**	0.075	-0.064**
	(0.066)	(0.039)	(0.065)	(0.031)	(0.064)	(0.031)	(0.065)	(0.031)
Age2	0	0	-0.001	0.001*	-0.001	0.001*	-0.001	0.001*
	(0.001)	(0)	(0.001)	(0)	(0.001)	(0)	(0.001)	(0)
Married	-0.105	-0.083	0.016	-0.033	0.006	-0.04	0.018	-0.034
	(0.322)	(0.197)	(0.305)	(0.19)	(0.306)	(0.19)	(0.306)	(0.19)
Female	0.878**	-0.973**	0.742*	-0.442	0.747*	-0.451	0.734*	-0.442
	(0.439)	(0.425)	(0.432)	(0.292)	(0.43)	(0.293)	(0.432)	(0.292)
Hispanic	-1.249*	-1.021*	-1.86***	-1.082***	-1.828***	-1.085***	-1.839***	-1.085***
	(0.745)	(0.552)	(0.673)	(0.408)	(0.671)	(0.41)	(0.672)	(0.408)
No High School	1.506*	0.245	0.378	-0.672*	0.409	-0.677*	0.382	-0.675*
	(0.839)	(0.542)	(0.99)	(0.367)	(0.997)	(0.369)	(0.986)	(0.369)
College	1.389***	0.042	0.948***	0.372***	0.913***	0.355***	0.943***	0.374***
	(0.362)	(0.149)	(0.347)	(0.121)	(0.349)	(0.122)	(0.347)	(0.121)
Self-employed	0.151	-0.084	0.279	-0.176	0.278	-0.173	0.278	-0.177
	(0.254)	(0.144)	(0.253)	(0.112)	(0.252)	(0.112)	(0.253)	(0.112)
Retired	0.203	0.014	0.212	0.036	0.246	0.04	0.203	0.039
	(0.423)	(0.263)	(0.399)	(0.246)	(0.4)	(0.246)	(0.396)	(0.246)
Icertain	0.576**	-0.19	0.571**	-0.134	0.562**	-0.138	0.562**	-0.132
	(0.264)	(0.12)	(0.267)	(0.103)	(0.266)	(0.103)	(0.265)	(0.103)
Risk aversion	-0.434***	-0.035	-0.605***	-0.258***	-0.58***	-0.248***	-0.597***	-0.26***
	(0.152)	(0.085)	(0.154)	(0.062)	(0.148)	(0.062)	(0.15)	(0.061)
2001	-0.275	0.097	0.002	0.065	0.028	0.078	0.016	0.062
	(0.304)	(0.16)	(0.311)	(0.136)	(0.301)	(0.136)	(0.31)	(0.136)
2004	-0.256	0.212	-0.48	-0.008	-0.454	0.002	-0.459	-0.01
	(0.318)	(0.166)	(0.299)	(0.141)	(0.293)	(0.139)	(0.298)	(0.14)
2007	0.327	-0.425**	0.092	-0.342**	0.1	-0.337**	0.133	-0.347**
	(0.293)	(0.171)	(0.273)	(0.143)	(0.268)	(0.143)	(0.275)	(0.145)
Information Var.	0.003**	0	0.029	-0.002	0.39*	0.211*	-0.298	0.042
	(0.002)	(0.001)	(0.084)	(0.036)	(0.2)	(0.11)	(0.236)	(0.106)
R-squared	0.119	0.054	0.221	0.129		0.224	0.131	0.223

The table reports coefficients and standard deviation estimates from a cloglog model of ownership of foreign and domestic stocks for U.S. households in the 1998, 2001, 2004 and 2007 Surveys of Consumer Finances. Coefficients followed by *** are significant at 1%, ** are significant at 5% level, while coefficients followed by * are significant at 10% level. The Survey of Consumer Finances implements a multiple imputation procedure to correct for missing data, and hence, standard errors are adjusted by averaging the dependent and independent values across implicates. All data is weighted.

Table 18: Additional information variables - cont.

	Broker		Fin. Planner		Friends	
	Foreign	Domestic	Foreign	Domestic	Foreign	Domestic
Constant	-13.278***	-3.86***	-13.451***	-3.979***	-13.122***	-4.026***
	(1.932)	(0.887)	(1.887)	(0.88)	(1.913)	(0.881)
Assets	0.656***	0.385***	0.658***	0.391***	0.642***	0.391***
	(0.132)	(0.061)	(0.132)	(0.061)	(0.135)	(0.062)
Income	0.224*	0.111*	0.222**	0.12*	0.24**	0.117*
	(0.116)	(0.065)	(0.112)	(0.064)	(0.111)	(0.065)
Business	-0.19***	-0.104***	-0.189***	-0.105***	-0.186***	-0.106***
	(0.046)	(0.029)	(0.045)	(0.029)	(0.046)	(0.029)
Debt	-0.117**	-0.01	-0.108*	-0.009	-0.107*	-0.009
	(0.057)	(0.034)	(0.056)	(0.034)	(0.056)	(0.034)
Age	0.075	-0.063**	0.076	-0.064**	0.071	-0.065**
	(0.065)	(0.031)	(0.064)	(0.031)	(0.064)	(0.031)
Age2	-0.001	0.001*	-0.001	0.001*	-0.001	0.001*
	(0.001)	(0)	(0.001)	(0)	(0.001)	(0)
Married	0.028	-0.028	0.002	-0.039	0.002	-0.027
	(0.304)	(0.189)	(0.308)	(0.189)	(0.303)	(0.191)
Female	0.747*	-0.44	0.694	-0.434	0.768*	-0.45
	(0.432)	(0.292)	(0.438)	(0.291)	(0.427)	(0.293)
Hispanic	-1.852***	-1.076***	-1.82***	-1.081***	-1.842***	-1.089***
	(0.673)	(0.408)	(0.673)	(0.406)	(0.673)	(0.408)
No High School	0.381	-0.662*	0.372	-0.681*	0.336	-0.664*
	(0.983)	(0.368)	(0.999)	(0.368)	(0.984)	(0.367)
College	0.937***	0.366***	0.926***	0.382***	0.97***	0.37***
	(0.345)	(0.121)	(0.351)	(0.121)	(0.349)	(0.121)
Self-employed	0.291	-0.173	0.269	-0.175	0.238	-0.168
	(0.25)	(0.112)	(0.251)	(0.112)	(0.257)	(0.112)
Retired	0.227	0.037	0.193	0.046	0.178	0.038
	(0.401)	(0.245)	(0.4)	(0.245)	(0.411)	(0.247)
Icertain	0.571**	-0.134	0.567**	-0.135	0.578**	-0.141
	(0.267)	(0.103)	(0.267)	(0.103)	(0.268)	(0.103)
Risk aversion	-0.61***	-0.257***	-0.605***	-0.26***	-0.601***	-0.258***
	(0.152)	(0.061)	(0.151)	(0.061)	(0.149)	(0.061)
2001	-0.009	0.061	0.013	0.066	0.026	0.056
	(0.308)	(0.136)	(0.307)	(0.136)	(0.303)	(0.136)
2004	-0.464	-0.007	-0.483	-0.005	-0.47	-0.004
	(0.299)	(0.14)	(0.297)	(0.14)	(0.294)	(0.14)
2007	0.084	-0.344**	0.062	-0.33**	0.134	-0.355**
	(0.271)	(0.143)	(0.276)	(0.145)	(0.269)	(0.144)
Information Var.	0.262	0.115	0.228	-0.109	-0.39*	0.134
	(0.227)	(0.105)	(0.212)	(0.105)	(0.236)	(0.099)
R-squared	0.223	0.13	0.223	0.13	0.225	0.13

The table reports coefficients and standard deviation estimates from a cloglog model of ownership of foreign and domestic stocks for U.S. households in the 1998, 2001, 2004 and 2007 Surveys of Consumer Finances. Coefficients followed by *** are significant at 1%, ** are significant at 5% level, while coefficients followed by * are significant at 10% level. The Survey of Consumer Finances implements a multiple imputation procedure to correct for missing data, and hence, standard errors are adjusted by averaging the dependent and independent values across implicates. All data is weighted.

A.2 The Model

In this Section, I depart from Abel, Eberly and Panageas (2007) in two directions; I first introduce foreign stocks on agents portfolio and then, in line with my previous results, I discuss the role of an entry cost in such market.

Consumer's wealth is held on an investment portfolio and in a riskless liquid asset for transactions. If she decides on entering the stock market, the investment portfolio is composed of a riskless bond and risky stocks, domestic and foreign. The consumer pays a fixed cost to observe their portfolio, proportional to the portfolio' contemporaneous value. Hence, it's optimal for the consumer to check her investment at equally spaced points in time, consuming from a riskless transactions account in the interim. A manager continuously rebalances the portfolio, at each period to guarantee a constant share is held in each type of asset within observation periods.

The consumer maximizes:

$$\begin{aligned} & E_t \int_0^\infty \frac{1}{1-\alpha} c_{t+s}^{1-\alpha} e^{-\rho s} ds, \\ 0 &< \alpha \neq 1 \\ \rho &> 0, \end{aligned}$$

where c stands for consumption, $0 < \alpha \neq 1$ is the inverse of the intertemporal elasticity of substitution and $\rho > 0$ is the intertemporal rate of discount.

The investment portfolio is composed of a riskless bond that pays rate of return $r > 0$, and of non-dividend-paying domestic and foreign stocks with prices D_t and F_t, respectively, with $P_t = \binom{D_t}{F_t}$ following a geometric Brownian motion:

$$\begin{aligned} \frac{dP_t}{P_t} &= \mu dt + \sqrt{\Omega} dZ, \\ \mu &> R, \end{aligned}$$

where:

$$\begin{aligned} \mu &= \begin{pmatrix} \mu_d \\ \mu_f \end{pmatrix}, R = \begin{pmatrix} r \\ r \end{pmatrix} \\ \Omega &= \begin{pmatrix} \sigma_d^2 & \sigma_{df} \\ \sigma_{df} & \sigma_f^2 \end{pmatrix}, \end{aligned}$$

and Z is a Wiener process, μ_d and μ_f are the returns on domestic and foreign stocks, respectively, and Ω is the variance-covariance matrix of stocks returns.

The consumer can observe the investment portfolio by paying a fraction $\theta, 0 \leq \theta < 1$, of the contemporaneous value of the investment portfolio. She can only withdraw funds from the portfolio if she observes the value. She also holds a riskless liquid asset that pays r^L, with $0 \leq r^L < r$, to finance consumption.

Let t_j, $j = 1, 2, 3, ...$, be the times at which consumer observes the value of her portfolio. At time t_j, she chooses: the next "observation" date, $t_{j+1} = t + \tau$; the amount of the riskless liquid asset, $X_{t_j}(\tau)$

to finance consumption from t_j to t_{j+1}; and the fraction ϕ invested in domestic (ϕ_d) and foreign $\left(\phi_f\right)$ stocks:

$$\phi = \begin{pmatrix} \phi_d \\ \phi_f \end{pmatrix}$$

From time t_j to t_{j+1}, the amount of riskless asset to finance consumption is:

$$\phi = \begin{pmatrix} \phi_d \\ \phi_f \end{pmatrix}$$

From time t_j to t_{j+1}, the amount of riskless asset to finance consumption is:

$$X_{t_j}(\tau) = \int_0^{\tau} c_{t_{j+s}} e^{-r^L s} ds,$$

and since $r^L < r$, when "observation" time arrives, the value in the riskless asset will have reached zero, i.e., $X_{t_\tau} = 0$. At this time, the consumer pays the observation cost and the value of her wealth after paying such cost is:

$$W_{t_{j+\tau}} = (1-\theta)' \left(W_{t_j} - X_{t_j}\right) \mathcal{R}\left(t_j, t_j + \tau\right),$$

where $\mathcal{R}(t_j, t_j+\tau)$ is the gross rate of return to investment from time t_j and $t_{j+\tau}$, and $\mathcal{R}(t_j, t_{j+\tau}) = 1$.

Following the Abel et. al. (2007), for simplicity, I also assume that a portfolio manager continuously rebalances the portfolio to maintain fixed the proportion of assets invested in stocks. In this case, the portfolio return then follows a geometric Brownian motion;

$$\frac{d\mathcal{R}(t_j, t_j+s)}{\mathcal{R}(t_j, t_j+s)} = \left[r + \phi'(\mu - R)\right] ds + \phi' \sqrt{\Omega} dZ.$$

To solve the consumer's problem, I divide the problem in four steps: the consumption choice within two consecutive observation periods; the choice of riskless asset and the share invested in stocks; and two final steps that uncover the value function and the optimal observational frequency.

1. Choosing consumption between t_j and $t_j + \tau$, given τ and X_{t_j}

$$\begin{aligned} U_{t_j}(\tau) &\equiv Max \int_0^{\tau} \frac{1}{1-\alpha} c_{t_j+s}^{1-\alpha} e^{-\rho s} ds & (10) \\ st &: \\ X_{t_j}(\tau) &= \int_0^{\tau} c_{t_j+s} e^{-r^L s} ds & (11) \end{aligned}$$

$$\begin{aligned} FOC &: \\ c_{t_j}^{-\alpha} &= \frac{dX_{t_j}}{dc_{t_j}} = \eta \\ c_{t_j+s}^{-\alpha} e^{-\rho s} &= \eta e^{-r^L s} \end{aligned}$$

This implies:

$$c_{t_j+s} = c_{t_j} e^{\frac{-\left(\rho - r^L\right)s}{\alpha}}, \; for \; 0 \leq s < \tau \tag{12}$$

Substituting (12) into (11):

$$\begin{aligned} X_{t_j} &= \int_0^{\tau} c_{t_j} e^{\frac{-\left(\rho - r^L\right)s}{\alpha}} e^{-r^L s} ds \\ &= c_{t_j} h(\tau) \end{aligned} \tag{13}$$

where:

$$\begin{aligned} h(\tau) &= \int_0^{\tau} e^{-\omega s} ds = \frac{1 - e^{-\omega \tau}}{\omega} \\ \omega &= \frac{\left(\rho - (1-\alpha) r^L\right)}{\alpha} \end{aligned} \tag{14}$$

Assume $\omega > 0$.

Substitute (12) into (10) and using (13):

$$\begin{aligned} U_{t_j}(\tau) &\equiv Max \int_0^{\tau} \frac{1}{1-\alpha} \left[c_{t_j+s}^{1-\alpha}\right] e^{-\rho s} ds \\ &= \frac{1}{1-\alpha} \int_0^{\tau} \left[c_{t_j} e^{\frac{-\left(\rho - r^L\right)s}{\alpha}}\right]^{1-\alpha} e^{-\rho s} ds \\ &= \frac{1}{1-\alpha} c_{t_j}^{1-\alpha} \int_0^{\tau} \left[e^{\frac{-\left(\rho - r^L\right)s}{\alpha}}\right]^{1-\alpha} e^{-\rho s} ds \\ &= \frac{1}{1-\alpha} c_{t_j}^{1-\alpha} \int_0^{\tau} e^{\frac{-\left(\rho - (1-\alpha) r^L\right)}{\alpha} s} ds \\ &= \frac{1}{1-\alpha} \left(\frac{X_{t_j}}{h(\tau)}\right)^{1-\alpha} h(\tau) \\ &= \frac{1}{1-\alpha} X_{t_j}^{1-\alpha} h(\tau)^{\alpha} \end{aligned} \tag{15}$$

2. Choosing X_{t_j} and ϕ, given τ

   Given τ, the consumer problem is the same as in classic Samuelson (1969); at times when the consumer observes the portfolio, the value function equals:

$$V\left(W_{t_j}\right) = \max_{X_{t_j}, \phi} U_{t_j}(\tau) + e^{-\rho \tau} E\left\{V\left[(1-\theta)\left(W_{t_j} - X_{t_j}\right) R\left(t_j, t_j + \tau\right)\right]\right\} \tag{16}$$

   Guess that:

$$V\left(W_{t_j}\right) = \frac{1}{1-\alpha} \gamma W_{t_j}^{1-\alpha} \tag{17}$$

   where γ is to be determined.

Replacing (15) and (17) into (16):

$$\begin{aligned}
\frac{1}{1-\alpha}\gamma W_{t_j}^{1-\alpha} &= \max_{X_{t_j},\phi}\frac{1}{1-\alpha}X_{t_j}^{1-\alpha}h(\tau)^{\alpha} \\
&\quad +e^{-\rho\tau}E_{t_j}\left\{\frac{1}{1-\alpha}\gamma\left[(1-\theta)\left(W_{t_j}-X_{t_j}\right)R(t_j,t_j+\tau)\right]^{1-\alpha}\right\} \\
&= \max_{X_{t_j},\phi}\frac{1}{1-\alpha}X_{t_j}^{1-\alpha}h(\tau)^{\alpha} \qquad (18)\\
&\quad +e^{-\rho\tau}\frac{1}{1-\alpha}\gamma(1-\theta)^{1-\alpha}\left(W_{t_j}-X_{t_j}\right)^{1-\alpha}E_{t_j}\left\{\left[R(t_j,t_j+\tau)\right]^{1-\alpha}\right\}
\end{aligned}$$

The optimal allocation of the investment portfolio maximizes:

$$\begin{aligned}
&\frac{1}{1-\alpha}E\left\{\left[R(t_j,t_j+\tau)\right]^{1-\alpha}\right\} \qquad (19)\\
=\ &\frac{1}{1-\alpha}\exp\left\{(1-\alpha)\left(r+\phi'(\mu-R)-\frac{1}{2}\alpha\phi'\Omega\phi\right)\tau\right\}
\end{aligned}$$

Replacing the last equation in the maximization problem (18), the first order constraint with respect to ϕ is:

$$\begin{aligned}
0 &= \gamma(1-\theta)^{1-\alpha}\left(W_{t_j}-X_{t_j}\right)^{1-\alpha}\frac{1}{1-\alpha}\exp\left\{(1-\alpha)\left(r+\phi'(\mu-R)-\frac{1}{2}\alpha\phi'\Omega\phi\right)\tau\right\} * \\
&\quad (1-\alpha)\tau\left[(\mu-R)-\alpha\phi'\Omega\right]
\end{aligned}$$

$$\begin{aligned}
\phi^* &= \frac{1}{\alpha}\Omega^{-1}(\mu-R) \qquad (20)\\
\phi^{*\prime} &= \frac{1}{\alpha}(\mu-R)'\Omega^{-1}
\end{aligned}$$

Substituting (20) into (19):

$$\begin{aligned}
&\max_{\phi}\frac{1}{1-\alpha}\exp(-\rho\tau)E_{t_j}\left\{\left[R(t_j,t_j+\tau)\right]^{1-\alpha}\right\} \\
=\ &\frac{1}{1-\alpha}\exp(-\rho\tau)\exp\left\{(1-\alpha)\left(r+\phi'(\mu-R)-\frac{1}{2}\alpha\phi'\Omega\phi\right)\tau\right\} \\
=\ &\frac{1}{1-\alpha}\exp\left\{-\rho\tau+(1-\alpha)\left(\begin{array}{c} r+\frac{1}{\alpha}(\mu-R)'\Omega^{-1}(\mu-R) \\ -\frac{1}{2}\alpha\frac{1}{\alpha}(\mu-R)'\Omega^{-1}\Omega\frac{1}{\alpha}\Omega^{-1}(\mu-R)\end{array}\right)\tau\right\} \\
=\ &\frac{1}{1-\alpha}\exp\left\{-\rho\tau+(1-\alpha)\left(r+\frac{1}{2}\frac{1}{\alpha}(\mu-R)'\Omega^{-1}(\mu-R)\right)\tau\right\} \\
=\ &\frac{1}{1-\alpha}\exp\left\{-\tau\left[\rho-(1-\alpha)\left(r+\frac{1}{2}\frac{1}{\alpha}(\mu-R)'\Omega^{-1}(\mu-R)\right)\right]\right\}
\end{aligned}$$

Call:

$$\begin{aligned}\Omega(\alpha) &\equiv \left(r+\frac{1}{2}\frac{1}{\alpha}(\mu-R)'\Omega^{-1}(\mu-R)\right)>r^{L}\\ \lambda &= \frac{\rho-(1-\alpha)\Omega(\alpha)}{\alpha}>0\end{aligned}$$

Then, we can rewrite:

$$\begin{aligned}&\max_{\phi}\frac{1}{1-\alpha}\exp(-\rho\tau)E_{t_j}\left\{[R(t_j,t_j+\tau)]^{1-\alpha}\right\}\\ =\ &\frac{1}{1-\alpha}\exp\left\{-\tau\left[\rho-(1-\alpha)\left(r+\frac{1}{2}\frac{1}{\alpha}(\mu-R)'\Omega^{-1}(\mu-R)\right)\right]\right\}\\ =\ &\frac{1}{1-\alpha}\exp\{-\alpha\lambda\tau\}\end{aligned}$$

Substitute this last equation into (18):

$$\begin{aligned}\frac{1}{1-\alpha}\gamma W_{t_j}^{1-\alpha} &= \max_{X_{t_j},\phi}\frac{1}{1-\alpha}X_{t_j}^{1-\alpha}h(\tau)^{\alpha}\\ &\quad+e^{-\rho\tau}\frac{1}{1-\alpha}\gamma(1-\theta)^{1-\alpha}\left(W_{t_j}-X_{t_j}\right)^{1-\alpha}E_{t_j}\left\{[R(t_j,t_j+\tau)]^{1-\alpha}\right\}\\ &= \max_{X_{t_j},\phi}\frac{1}{1-\alpha}X_{t_j}^{1-\alpha}[h(\tau)]^{\alpha}\\ &\quad+\frac{1}{1-\alpha}\gamma(1-\theta)^{\frac{\alpha(1-\alpha)}{\alpha}}\left(W_{t_j}-X_{t_j}\right)^{1-\alpha}\exp\{-\alpha\lambda\tau\}\\ &= \max_{X_{t_j},\phi}\frac{1}{1-\alpha}X_{t_j}^{1-\alpha}[h(\tau)]^{\alpha}+\frac{1}{1-\alpha}\gamma\left(W_{t_j}-X_{t_j}\right)^{1-\alpha}\chi^{\alpha}e^{-\alpha\lambda\tau} \qquad (21)\end{aligned}$$

where $\chi=(1-\theta)^{\frac{(1-\alpha)}{\alpha}}$.

Differentiate the RHS of the above equation with respect to X_{t_j} and set the derivative equal to zero to obtain:

$$\begin{aligned}X_{t_j}^{-\alpha}[h(\tau)]^{\alpha} &= \gamma\left(W_{t_j}-X_{t_j}\right)^{-\alpha}\chi^{\alpha}e^{-\alpha\lambda\tau}\\ X_{t_j} &= h(\tau)\gamma^{-\frac{1}{\alpha}}\left(W_{t_j}-X_{t_j}\right)\chi^{-1}e^{\lambda\tau}\end{aligned}$$

Define

$$A=h(\tau)\gamma^{-\frac{1}{\alpha}}\chi^{-1}e^{\lambda\tau} \qquad (22)$$

Then,

$$\begin{aligned}X_{t_j} &= A\left(W_{t_j}-X_{t_j}\right)\\ (1+A)X_{t_j} &= AW_{t_j}\\ X_{t_j} &= \frac{A}{(1+A)}W_{t_j}\end{aligned}$$

3. Given X_{t_j} and ϕ conditional on τ, the consumer computes the value function on τ

   Replace X_{t_j} into the value function (21), (or (3)) to obtain $\gamma(\tau)$:

$$
\begin{aligned}
\frac{1}{1-\alpha}\gamma W_{t_j}^{1-\alpha} &= \max_{X_{t_j},\phi} \frac{1}{1-\alpha} X_{t_j}^{1-\alpha}\left[h(\tau)\right]^{\alpha} + \frac{1}{1-\alpha}\gamma\left(W_{t_j}-X_{t_j}\right)^{1-\alpha}\chi^{\alpha}e^{-\alpha\lambda\tau} \\
\frac{1}{1-\alpha}\gamma\left[\frac{1+A}{A}X_{t_j}\right]^{1-\alpha} &= \frac{1}{1-\alpha}X_{t_j}^{1-\alpha}\left[h(\tau)\right]^{\alpha} + \frac{1}{1-\alpha}\gamma\left(\frac{X_{t_j}}{A}\right)^{1-\alpha}\chi^{\alpha}e^{-\alpha\lambda\tau} \\
\frac{1}{1-\alpha}\gamma\left(\frac{1+A}{A}\right)^{1-\alpha} &= \frac{1}{1-\alpha}\left[h(\tau)\right]^{\alpha} + \frac{1}{1-\alpha}\gamma\left(\frac{1}{A}\right)^{1-\alpha}\chi^{\alpha}e^{-\alpha\lambda\tau} \\
\gamma(\tau) &= \left(\frac{A}{1+A}\right)^{1-\alpha}\left[h(\tau)\right]^{\alpha} + \gamma(\tau)\left(\frac{1}{1+A}\right)^{1-\alpha}\chi^{\alpha}e^{-\alpha\lambda\tau} \qquad (23)
\end{aligned}
$$

   Equations (22) and (23) are two equations on $\gamma(\tau)$ and A. Using the definition of $h(\tau)$ in (14) and solving this system:

$$
\begin{aligned}
A &= \chi^{-1}e^{\lambda\tau} - 1 \\
\gamma(\tau) &= \left[\frac{1-e^{-\omega\tau}}{1-\chi e^{-\lambda\tau}}\right]^{\alpha}\omega^{-\alpha} \qquad (24)
\end{aligned}
$$

4. The consumer maximizes the value function and sets τ.

   To choose τ, the consumer maximizes (17), that is equivalent to maximizing:

$$
F(\tau) = \frac{\gamma(\tau)}{1-\alpha}
$$

$$
\begin{aligned}
Max_{\tau}\frac{\gamma(\tau)}{1-\alpha} &= \frac{\left[\frac{1-e^{-\omega\tau}}{1-\chi e^{-\lambda\tau}}\right]^{\alpha}\omega^{-\alpha}}{1-\alpha} \\
&\Rightarrow \\
\frac{\partial F(\tau)}{\partial\tau} &= \frac{\left[\frac{1-e^{-\omega\tau}}{1-\chi e^{-\lambda\tau}}\right]^{\alpha-1}\omega^{-\alpha}}{1-\alpha}\left(\frac{\left(\omega e^{-\omega\tau}\right)\left(1-\chi e^{-\lambda\tau}\right)-\left(\chi\lambda e^{-\lambda\tau}\right)\left(1-e^{-\omega\tau}\right)}{\left(1-\chi e^{-\lambda\tau}\right)^{2}}\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
\left(\omega e^{-\omega\tau}\right)\left(1-\chi e^{-\lambda\tau}\right)-\left(\chi\lambda e^{-\lambda\tau}\right)\left(1-e^{-\omega\tau}\right) &= 0 \\
\omega e^{-\omega\tau} - \omega\chi e^{-(\omega+\lambda)\tau} - \chi\lambda e^{-\lambda\tau} + \chi\lambda e^{-(\omega+\lambda)\tau} &= 0 \\
\left(-\omega\chi+\chi\lambda\right)e^{-(\omega+\lambda)\tau} &= -\omega e^{-\omega\tau} + \chi\lambda e^{-\lambda\tau}
\end{aligned}
$$

$$
\left(-\omega\chi+\chi\lambda\right)e^{-(\omega+\lambda)\tau} = -\omega e^{-\omega\tau} + \chi\lambda e^{-\lambda\tau}
$$

Divide by $\omega\chi e^{-\omega\tau}$:

$$\begin{aligned}
\frac{(-\omega+\lambda)}{\omega}e^{-\lambda\tau} &= -\frac{1}{\chi}+\frac{\lambda}{\omega}e^{(\omega-\lambda)\tau} \\
\frac{(\omega-\lambda)}{\omega}e^{-\lambda\tau}+\frac{\lambda}{\omega}e^{(\omega-\lambda)\tau} &= \frac{1}{\chi}
\end{aligned}$$

As in Abel et al. (2007), define:

$$M(\tau)\equiv\frac{(\omega-\lambda)}{\omega}e^{-\lambda\tau}+\frac{\lambda}{\omega}e^{(\omega-\lambda)\tau}$$

In Abel et al. (2007), it is proven that τ^* that maximizes $F(\tau)$ is such that $M(\tau^*)\chi=1$, i.e.:[51]

$$\frac{(\omega-\lambda)}{\omega}e^{-\lambda\tau^*}+\frac{\lambda}{\omega}e^{(\omega-\lambda)\tau^*}-\frac{1}{\chi}=0$$

A second order Taylor expansion to $M(\tau)$ around $\tau=0$ yields:

$$\begin{aligned}
M(\tau) &\equiv \frac{(\omega-\lambda)}{\omega}+\frac{\lambda}{\omega}=1 \\
M'(\tau) &= \frac{-\lambda(\omega-\lambda)}{\omega}e^{-\lambda\tau}+\frac{\lambda(\omega-\lambda)}{\omega}e^{(\omega-\lambda)\tau} \\
M''(\tau) &= \left(\lambda^2\right)\frac{(\omega-\lambda)}{\omega}e^{-\lambda\tau}+\frac{\lambda(\omega-\lambda)^2}{\omega}e^{(\omega-\lambda)\tau}
\end{aligned}$$

$$\begin{aligned}
M(0) &= \frac{(\omega-\lambda)}{\omega}e^{-\lambda\tau}+\frac{\lambda}{\omega}e^{(\omega-\lambda)\tau} \\
M'(0) &= \frac{-\lambda(\omega-\lambda)}{\omega}+\frac{\lambda(\omega-\lambda)}{\omega}=0 \\
M''(0) &= \left(\lambda^2\right)\frac{(\omega-\lambda)}{\omega}+\frac{\lambda(\omega-\lambda)^2}{\omega} \\
&= \lambda(\omega-\lambda)\neq 0
\end{aligned}$$

For any $f(x)$, a second order Taylor expansion gives:

$$f(x)\simeq f(a)+(x-a)f'(a)+\frac{1}{2}(x-a)^2f''(a)$$

For $M(\tau)$, we get:

$$M(\tau)\simeq 1+\tau*0+\frac{1}{2}(\tau)^2\lambda(\omega-\lambda)$$

Let $\hat{\tau}$ be the approximately optimal value of τ. From Abel et al. (2007) it satisfies $M(\hat{\tau})\chi=1$,

[51] For a proof that τ^* is a unique maximum of this function, I refer the reader to Lemma 1 of Abel et al. (2007).

and that implies:

$$\frac{1}{2}(\hat{\tau})^2 = \frac{\chi^{-1}-1}{\lambda(\omega-\lambda)}$$

$$\hat{\tau} = \left(\frac{2\left(\chi^{-1}-1\right)}{(\omega-\lambda)\lambda}\right)^{\frac{1}{2}}$$

A.3 The entry decision

Agents can opt not to enter the stock market. If the consumer decides to never enter this market and hold all its wealth in the riskless liquid asset, her overall rate of return equals r^L. From the definitions of λ and ω, the non-entry decision implies $\lambda = \omega$, and so, $\gamma = \omega^{-\alpha}$.

Hence, for such an agent, her value function equals:

$$V(W_0) = \omega^{-\alpha}\frac{W_0^{1-\alpha}}{1-\alpha} = g$$

I assume agents have to pay a fixed cost, K, out of initial wealth, W_0, at time 0, when entering the stock market. Let's first assume the agent enters the domestic market only and for that, he pays K_d.

If he enters the stock market, she pays K_d and her value function is:

$$V\left(W_0^+\right) = V\left(W_0(1-K_d)\right) = \gamma(\tau)\frac{\left(W_0(1-K_d)\right)^{1-\alpha}}{1-\alpha}$$

$$\gamma(\tau) = \left[\frac{1-e^{-\omega\tau}}{1-\left[(1-\theta)^{\frac{(1-\alpha)}{\alpha}}\right]e^{-\lambda\tau}}\right]^{\alpha}\omega^{-\alpha}$$

$$\tau = \left(\frac{2\left((1-\theta)^{-\frac{(1-\alpha)}{\alpha}}-1\right)}{(\omega-\lambda)\lambda}\right)^{\frac{1}{2}}$$

where:

$$\omega = \frac{\left(\rho-(1-\alpha)r^L\right)}{\alpha}$$

$$\Omega(\alpha) \equiv \left(r+\frac{1}{2}\frac{1}{\alpha}(\mu-R)'\,\Omega^{-1}(\mu-R)\right) > r^L$$

$$\lambda = \frac{\rho-(1-\alpha)\Omega(\alpha)}{\alpha} > 0$$

1. How large K_d has to be to drive agents out of the domestic market, if this is the only available risky asset, as in Abel et al. (2007)?

   I identify the parameters of their model by a subscript d to distinguish from the parameters of the open economy model. For this case, K_d has to be such that equals the value function of consumers that invest and those who don't invest in stocks:

$$V\left(W_0^{+}\right)=V\left(W_0\left(1-K_d\right)\right)=\gamma_d\left(\tau\right)\frac{\left(W_0\left(1-K_d\right)\right)^{1-\alpha}}{1-\alpha}=g=\omega^{-\alpha}\frac{W_0^{1-\alpha}}{1-\alpha}$$

$$\gamma_d\left(\tau\right)\frac{\left(W_0\left(1-K_d\right)\right)^{1-\alpha}}{1-\alpha}=\omega^{-\alpha}\frac{W_0^{1-\alpha}}{1-\alpha}$$

$$\begin{aligned}
1 &= \left[\frac{1-e^{-\omega\tau_d}}{1-\left[\left(1-\theta\right)^{\frac{(1-\alpha)}{\alpha}}\right]e^{-\lambda_d\tau_d}}\right]^{\alpha}\left(1-K_d\right)^{1-\alpha} \\
K_d &= 1-\left[\frac{1-e^{-\omega\tau_d}}{1-\left[\left(1-\theta\right)^{\frac{(1-\alpha)}{\alpha}}\right]e^{-\lambda_d\tau_d}}\right]^{-\frac{\alpha}{1-\alpha}}
\end{aligned}$$

$$K_d=1-\left(\frac{1-e^{-\omega\tau_d}}{1-\left[\left(1-\theta\right)^{\frac{(1-\alpha)}{\alpha}}\right]e^{-\lambda_d\tau_d}}\right)^{\frac{-\alpha}{1-\alpha}}$$

where

$$\begin{aligned}
\phi_d &= \frac{1}{\alpha}\frac{\left(\mu_d-r\right)}{\sigma^2} \\
\Omega_d\left(\alpha\right) &\equiv \left(r+\frac{1}{2}\frac{1}{\alpha}\frac{\left(\mu_d-r\right)^2}{\sigma_d^2}\right)>r^L \\
\lambda_d &= \frac{\rho-\left(1-\alpha\right)\Omega_d\left(\alpha\right)}{\alpha}.
\end{aligned}$$

2. How large K_f has to be to drive agents out of foreign market given they invest in domestic stocks?

   When agents enter only the domestic market, the problem is the same as presented in Abel et. al. (2007), and hence, the two equations to be compared are given by:

$$\begin{aligned}
&\left[\frac{1-e^{-\omega\tau}}{1-\left[\left(1-\theta\right)^{\frac{(1-\alpha)}{\alpha}}\right]e^{-\lambda\tau}}\right]^{\alpha}\omega^{-\alpha}\frac{\left(W_0\left(1-K_f\right)\right)^{1-\alpha}}{1-\alpha} \\
=&\left[\frac{1-e^{-\omega\tau_d}}{1-\left[\left(1-\theta_d\right)^{\frac{(1-\alpha)}{\alpha}}\right]e^{-\lambda_d\tau_d}}\right]^{\alpha}\omega^{-\alpha}\frac{W_0^{1-\alpha}}{1-\alpha}
\end{aligned}$$

$$K_f=1-\left(\frac{\frac{1-e^{-\omega\tau_d}}{1-\left[\left(1-\theta_d\right)^{\frac{(1-\alpha)}{\alpha}}\right]e^{-\lambda_d\tau_d}}}{\frac{1-e^{-\omega\tau}}{1-\left[\left(1-\theta\right)^{\frac{(1-\alpha)}{\alpha}}\right]e^{-\lambda\tau}}}\right)^{\frac{\alpha}{1-\alpha}}$$

A.4 Proofs of Propositions

Proposition 1 The solution to the consumer's problem implies that:

a. The value function is such that:

$$V(W) = \gamma(\tau)\frac{W^{1-\alpha}}{1-\alpha},$$

where:

$$\gamma(\tau) = \left[\frac{1-e^{-\omega\tau}}{1-\chi e^{-\lambda\tau}}\right]^{\alpha}\omega^{-\alpha}$$

b. The optimal shares held in domestic and foreign stocks equal:

$$\phi^{*} = \frac{1}{\alpha}\Omega^{-1}(\mu - R)$$

c. And the consumer optimally chooses to observe and update her portfolio at time τ^{*}, obtained from solving:

$$\frac{(\omega-\lambda)}{\omega}e^{-\lambda\tau^{*}} + \frac{\lambda}{\omega}e^{(\omega-\lambda)\tau^{*}} - \frac{1}{\chi} = 0$$

A second order approximation to this equation yields:

$$\hat{\tau}^{*} = \left(\frac{2\left(\chi^{-1}-1\right)}{(\omega-\lambda)\lambda}\right)^{\frac{1}{2}}$$

where $\chi = (1-\theta)^{\frac{(1-\alpha)}{\alpha}}$, $\omega = \frac{\left(\rho-(1-\alpha)r^{L}\right)}{\alpha}$ and $\lambda = \frac{\rho-(1-\alpha)\left(r+\frac{1}{2}\frac{1}{\alpha}(\mu-R)'\Omega^{-1}(\mu-R)\right)}{\alpha}$.

Proof. Following steps 1-4 of the detailed model derivation described in Subsection A.2 provides the proof for the proposition and extensively describe how to obtain the above equations. ■

Proposition 2 If $\alpha > 1$, the (approximately) optimal level of inattention is smaller once foreign stock holdings is introduced into the model., i.e.:

$$\hat{\tau}^{*} < \tau_{d}^{*}.$$

Proof. We are trying to check if:

$$\left(\frac{2\left(\chi^{-1}-1\right)}{(\omega-\lambda)\lambda}\right)^{\frac{1}{2}} < \left(\frac{2\left(\chi^{-1}-1\right)}{(\omega-\lambda_{d})\lambda_{d}}\right)^{\frac{1}{2}}$$

$$\frac{\left(\chi^{-1}-1\right)}{(\omega-\lambda)\lambda} < \frac{\left(\chi^{-1}-1\right)}{(\omega-\lambda_{d})\lambda_{d}},$$

i.e., if

$$(\omega-\lambda_{d})\lambda_{d} > (\omega-\lambda)\lambda.$$

Recall that because $0 < \theta < 1$, and I assume $\omega < \lambda$ and $\omega < \lambda_{d}$, the above expression is the correct one to be verified.

Recall that:

$$\lambda = \frac{\rho - (1-\alpha)\left(r + \frac{1}{2}\frac{1}{\alpha}(\mu - R)'\,\Omega^{-1}(\mu - R)\right)}{\alpha}$$

$$\lambda_d = \frac{\rho - (1-\alpha)\left(r + \frac{1}{2}\frac{1}{\alpha}\frac{(\mu_d - r)^2}{\sigma_d^2}\right)}{\alpha}$$

One wants to check if $(\omega - \lambda_d)\,\lambda_d > (\omega - \lambda)\,\lambda$. Rewriting this expression:

$$\begin{aligned}
&= (\omega - \lambda_d)\,\lambda_d - (\omega - \lambda)\,\lambda = \\
&= \omega\,(\lambda_d - \lambda) - \left(\lambda_d^2 - \lambda^2\right) \\
&= \omega\,(\lambda_d - \lambda) - (\lambda_d - \lambda)\,(\lambda_d + \lambda) \\
&= (\lambda_d - \lambda)\,(\omega - \lambda_d - \lambda)
\end{aligned}$$

$$\begin{aligned}
&= \left[\begin{array}{c} \frac{\rho-(1-\alpha)\left(r+\frac{1}{2}\frac{1}{\alpha}\frac{(\mu_d-r)^2}{\sigma_d^2}\right)}{\alpha} \\ -\frac{\rho-(1-\alpha)\left(r+\frac{1}{2}\frac{1}{\alpha}(\mu-R)'\Omega^{-1}(\mu-R)\right)}{\alpha} \end{array}\right](\omega - \lambda_d - \lambda) \\
&= \left[\begin{array}{c} \frac{\rho}{\alpha} - \frac{(1-\alpha)}{\alpha}\left(r + \frac{1}{2}\frac{1}{\alpha}\frac{(\mu_d-r)^2}{\sigma_d^2}\right) - \frac{\rho}{\alpha} \\ +\frac{(1-\alpha)}{\alpha}\left(r + \frac{1}{2}\frac{1}{\alpha}(\mu - R)'\,\Omega^{-1}(\mu - R)\right) \end{array}\right](\omega - \lambda_d - \lambda)
\end{aligned}$$

$$\begin{aligned}
&= \frac{(1-\alpha)}{\alpha}\left[-r - \frac{1}{2}\frac{1}{\alpha}\frac{(\mu_d - r)^2}{\sigma_d^2} + r + \frac{1}{2}\frac{1}{\alpha}(\mu - R)'\,\Omega^{-1}(\mu - R)\right](\omega - \lambda_d - \lambda) \\
&= \frac{1}{2}\frac{1}{\alpha}\frac{(1-\alpha)}{\alpha}\left[-\frac{(\mu_d - r)^2}{\sigma_d^2} + (\mu - R)'\,\Omega^{-1}(\mu - R)\right](\omega - \lambda_d - \lambda) \\
&= \frac{1}{2}\frac{1}{\alpha}\frac{(1-\alpha)}{\alpha}\left[\frac{\begin{array}{c}\left(\mu_f - r\right)^2\sigma_d^4 + (\mu_d - r)^2\sigma_{df} \\ -2\,(\mu_d - r)\left(\mu_f - r\right)\sigma_{df}\sigma_d^2\end{array}}{\left(\sigma_d^2\sigma_f^2 - \sigma_{df}^2\right)\sigma_d^2}\right]\left(\frac{\rho - (1-\alpha)\,r^L}{\alpha} - \lambda_d - \lambda\right)
\end{aligned}$$

The denominator of the term in the brackets is positive since it corresponds to the determinant of the variance-covariance matrix. The numerator is such that:

$$\begin{aligned}
&\left(\mu_f - r\right)^2\sigma_d^4 + (\mu_d - r)^2\sigma_{df} - 2\,(\mu_d - r)\left(\mu_f - r\right)\sigma_{df}\sigma_d^2 \\
&= \sigma_d^2\left(\mu_f - r\right)\left(\left(\mu_f - r\right)\sigma_d^2 - (\mu_d - r)\,\sigma_{df}\right) + \sigma_{df}\,(\mu_d - r)\left((\mu_d - r)\,\sigma_{df} - \left(\mu_f - r\right)\sigma_d^2\right) \\
&= \left(\left(\mu_f - r\right)\sigma_d^2 - (\mu_d - r)\,\sigma_{df}\right)\left(\left(\mu_f - r\right)\sigma_d^2 - (\mu_d - r)\,\sigma_{df}\right) \\
&= \left(\left(\mu_f - r\right)\sigma_d^2 - (\mu_d - r)\,\sigma_{df}\right)^2 > 0
\end{aligned}$$

The term in parenthesis equals:

$$
\begin{aligned}
&= \left(\begin{array}{c} \frac{\rho-(1-\alpha)r^L}{\alpha} - \frac{\rho-(1-\alpha)\left(r+\frac{1}{2}\frac{1}{\alpha}\frac{(\mu_d-r)^2}{\sigma_d^2}\right)}{\alpha} \\ -\frac{\rho-(1-\alpha)\left(r+\frac{1}{2}\frac{1}{\alpha}(\mu-R)'\Omega^{-1}(\mu-R)\right)}{\alpha} \end{array} \right) \\
&= -\frac{\rho}{\alpha} + \frac{(1-\alpha)}{\alpha}\left(-r^L + 2r + \frac{1}{2}\frac{1}{\alpha}\left(\frac{(\mu_d - r)^2}{\sigma_d^2} + (\mu - R)' \, \Omega^{-1} (\mu - R)\right)\right) \\
&= -\frac{\rho}{\alpha} - \frac{(1-\alpha) \, r^L}{\alpha} + 2\frac{(1-\alpha) \, r}{\alpha} + \left[\frac{1}{2}\frac{1}{\alpha}\frac{(1-\alpha)}{\alpha}\left(\frac{(\mu_d - r)^2}{\sigma_d^2} + (\mu - R)' \, \Omega^{-1} (\mu - R)\right)\right]
\end{aligned}
$$

For $\alpha > 1$, the above term in brackets is negative. For the remainder of the expression:

$$
\begin{aligned}
& -\frac{\rho}{\alpha} - \frac{(1-\alpha) \, r^L}{\alpha} + 2\frac{(1-\alpha) \, r}{\alpha} \\
= \; & \frac{\rho}{\alpha} - \frac{(1-\alpha) \, r^L}{\alpha} - 2\left(\frac{\rho}{\alpha} - \frac{(1-\alpha) \, r}{\alpha}\right) \\
< \; & \frac{\rho}{\alpha} - \frac{(1-\alpha) \, r^L}{\alpha} - 2\left(\frac{\rho}{\alpha} - \frac{(1-\alpha) \, r^L}{\alpha}\right) \\
= \; & -\left(\frac{\rho}{\alpha} - \frac{(1-\alpha) \, r^L}{\alpha}\right) = -\omega \\
< \; & 0
\end{aligned}
$$

Therefore, $(\omega - \lambda_d) \, \lambda_d - (\omega - \lambda) \, \lambda$ equals:

$$
\begin{aligned}
&= (\lambda_d - \lambda) \, (\omega - \lambda_d - \lambda) \\
&= \frac{1}{2}\frac{1}{\alpha}\underbrace{\frac{(1-\alpha)}{\alpha}}_{<0} \underbrace{\left[\frac{ \begin{array}{c} (\mu f - r)^2 \, \sigma_d^4 + (\mu_d - r)^2 \, \sigma_{df} \\ -2\,(\mu_d - r)\,(\mu_f - r)\,\sigma_{df}\sigma_d^2 \end{array} }{ \left(\sigma_d^2 \sigma_f^2 - \sigma_{df}^2\right) \sigma_d^2 } \right]}_{>0} \underbrace{\left(\frac{\rho - (1-\alpha) \, r^L}{\alpha} - \lambda_d - \lambda\right)}_{<0}
\end{aligned}
$$

Hence:

$$
\begin{aligned}
(\omega - \lambda_d) \, \lambda_d &> (\omega - \lambda) \, \lambda \\
&\Rightarrow \hat{\tau}^* < \tau_d^*
\end{aligned}
$$

■

Proposition As long as $0 < \mu - R < 1$, the total share invested in stocks is positive, even though the share invested in one or the other can be negative.

$$
\phi_d + \phi_f > 0
$$

Proof.

$$\begin{aligned}
& (\mu_d - r)\,\sigma_f^2 - (\mu_d - r)\,\sigma_{df} + \left(\mu_f - r\right)\sigma_d^2 - \left(\mu_f - r\right)\sigma_{df} \\
> \;& (\mu_d - r)\,\sigma_f^2 - (\mu_d - r)\,\sigma_d\sigma_f + \left(\mu_f - r\right)\sigma_d^2 - \left(\mu_f - r\right)\sigma_d\sigma_f \\
> \;& (\mu_d - r)\left(\sigma_f^2 - \sigma_d\sigma_f\right) + \left(\mu_f - r\right)\left(\sigma_d^2 - \sigma_d\sigma_f\right) \\
> \;& (\mu_d - r)\,\sigma_f\left(\sigma_f - \sigma_d\right) + \left(\mu_f - r\right)\sigma_d\left(\sigma_d - \sigma_f\right) \\
= \;& \left(\sigma_f - \sigma_d\right)\left[(\mu_d - r)\,\sigma_f - \left(\mu_f - r\right)\sigma_d\right] \\
> \;& \left(\sigma_f - \sigma_d\right)\left[(\mu_d - r)\,\sigma_f - \sigma_d\right]
\end{aligned}$$

And $0 < (\mu_d - r) < 1$. If $((\mu_d - r)\,\sigma_f - \sigma_d) > 0 \Rightarrow (\sigma_f - \sigma_d) > 0$. If $((\mu_d - r)\,\sigma_f - \sigma_d) < 0 \Rightarrow \sigma_f < \frac{\sigma_d}{(\mu_d - r)} < \sigma_d$. And hence, the above expression is positive. ■

Proposition Regardless the availability of foreign stocks, the following assessments are still true that:

$$\frac{\partial\tau^*}{\partial\theta} > 0, \frac{\partial\tau^*}{\partial r^L} > 0$$

Proof. Following Abel et al. (2007):

$$\begin{aligned}
M(\tau) &\equiv \frac{(\omega - \lambda)}{\omega}e^{-\lambda\tau} + \frac{\lambda}{\omega}e^{(\omega-\lambda)\tau} \\
M(\tau^*)\,\chi &= 1 \\
\chi &= (1-\theta)^{\frac{(1-\alpha)}{\alpha}} \\
\omega &= \frac{\left(\rho - (1-\alpha)\,r^L\right)}{\alpha}
\end{aligned}$$

Total differentiating the above equation:

$$\frac{dM}{d\tau^*}\frac{d\tau^*}{d\theta}\chi + \frac{d\chi}{d\theta}M(\tau^*) = 0$$

$$\begin{aligned}
\frac{d\chi}{d\theta} &= -\frac{(1-\alpha)}{\alpha}(1-\theta)^{\frac{(1-\alpha)}{\alpha}-1} \\
&= -\chi\,(1-\alpha)\left[\alpha\,(1-\theta)\right]^{-1}
\end{aligned}$$

And:

$$\begin{aligned}
\frac{\partial M}{\partial\tau^*} &= -\lambda\frac{(\omega-\lambda)}{\omega}e^{-\lambda\tau} + \frac{(\omega-\lambda)\,\lambda}{\omega}e^{(\omega-\lambda)\tau} \\
&= -\lambda\frac{(\omega-\lambda)}{\omega}e^{-\lambda\tau} + \frac{(\omega-\lambda)\,\lambda}{\omega}e^{\omega\tau}e^{-\lambda\tau} \\
&= \frac{(\omega-\lambda)\,\lambda}{\omega}\left(e^{\omega\tau}-1\right)e^{-\lambda\tau}
\end{aligned}$$

Hence:

$$\begin{aligned}
\frac{d\tau^*}{d\theta} &= \frac{-M(\tau^*)\frac{d\chi}{d\theta}}{\chi\frac{dM}{d\tau^*}} \\
&= \frac{M(\tau^*)\chi(1-\alpha)\left[\alpha(1-\theta)\right]^{-1}}{\chi\frac{(\omega-\lambda)\lambda}{\omega}\left(e^{\omega\tau}-1\right)e^{-\lambda\tau}} \\
&= \frac{\frac{(1-\alpha)}{\alpha}\left[(1-\theta)\right]^{-1}}{\chi\frac{\frac{1-\alpha}{\alpha}\left[\Omega(\alpha)-r^L\right]\lambda}{\omega}\left(e^{\omega\tau}-1\right)e^{-\lambda\tau}} \\
&= \frac{\omega\left[(1-\theta)\right]^{-1}}{\chi\left[\Omega(\alpha)-r^L\right]\lambda\left(e^{\omega\tau}-1\right)e^{-\lambda\tau}} > 0
\end{aligned}$$

That can be obtained by replacing $M(\tau^*)\chi = 1$ and by using the expression as:

$$\omega-\lambda = \frac{1-\alpha}{\alpha}\left[\Omega(\alpha)-r^L\right]$$

Finally, applying the implicit function theorem to the expression for M(τ^*), one obtains:

$$\frac{\partial\tau^*}{\partial r^L} = -\frac{\frac{\partial M}{\partial\omega}\frac{\partial\omega}{\partial r^L}}{\frac{\partial M}{\partial\tau^*}}$$

$$\begin{aligned}
\frac{\partial M}{\partial\omega} &= \left[1-(1-\tau\omega)e^{\tau\omega}\right]\frac{\lambda e^{-\lambda\tau}}{\omega^2} \\
\frac{\partial\omega}{\partial r^L} &= -\frac{(1-\alpha)}{\alpha} \\
\frac{\partial M}{\partial\tau^*} &= \frac{(\omega-\lambda)\lambda}{\omega}\left(e^{\omega\tau}-1\right)e^{-\lambda\tau}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\tau^*}{\partial r^L} &= \frac{\left[1-(1-\tau\omega)e^{\tau\omega}\right]\frac{\lambda e^{-\lambda\tau}}{\omega^2}\frac{(1-\alpha)}{\alpha}}{\frac{1-\alpha}{\alpha}\left[\Omega(\alpha)-r^L\right]\frac{\lambda}{\omega}\left(e^{\omega\tau}-1\right)e^{-\lambda\tau}} \\
&= \frac{1-(1-\tau\omega)e^{\tau\omega}}{\omega\left[\Omega(\alpha)-r^L\right]\left(e^{\omega\tau}-1\right)} > 0 \ for \ \omega\tau > 0
\end{aligned}$$

■

Proposition If $\alpha > 1$, and non-short-selling is assumed, i. e., $\phi_d > 0$ and $\phi_f > 0$, the optimal level of inattention is negatively correlated with mean returns on domestic and foreign stocks, positively correlated to the volatility of those assets' returns, and positively correlated to the covariance of such returns:

$$\begin{aligned}\frac{\partial\tau^*}{\partial\mu_d} &< 0, \frac{\partial\tau^*}{\partial\mu_f} < 0\\ \frac{\partial\tau^*}{\partial\sigma_d^2} &> 0, \frac{\partial\tau^*}{\partial\sigma_f^2} > 0\\ \frac{\partial\tau^*}{\partial\sigma_{df}} &> 0\end{aligned}$$

Proof. First, looking at the returns:

$$\begin{aligned}\frac{(\omega-\lambda)}{\omega}e^{-\lambda\tau^*}+\frac{\lambda}{\omega}e^{(\omega-\lambda)\tau^*}-\frac{1}{\chi} &= 0\\ & \quad \textit{or}\\ M(\tau^*)\chi &= 1\end{aligned}$$

Using the implicit function theorem:

$$\begin{aligned}\frac{\partial\tau^*}{\partial\mu_f} &= -\frac{\frac{\partial M}{\partial\mu_f}}{\frac{\partial M}{\partial\tau^*}} = -\frac{\left(\frac{\partial M}{\partial\lambda}\frac{\partial\lambda}{\partial\Omega(\alpha)}\frac{\partial\Omega(\alpha)}{\partial\mu_f}\right)}{\frac{\partial M}{\partial\tau^*}}\\ \frac{\partial\tau^*}{\partial\mu_d} &= -\frac{\frac{\partial M}{\partial\mu_d}}{\frac{\partial M}{\partial\tau^*}} = -\frac{\left(\frac{\partial M}{\partial\lambda}\frac{\partial\lambda}{\partial\Omega(\alpha)}\frac{\partial\Omega(\alpha)}{\partial\mu_d}\right)}{\frac{\partial M}{\partial\tau^*}}\end{aligned}$$

The common three terms in the previous two derivatives are such that:

$$\frac{\partial M}{\partial\lambda} = \frac{1}{\lambda}M(\tau^*)e^{-\lambda\tau^*}\left[-M(\tau^*)^{-1}+(1-\lambda\tau^*)e^{\lambda\tau^*}\right]$$

Since $M(\tau^*)\chi = 1$, $(1-\lambda\tau^*)e^{\lambda\tau^*} < 1$, for $\lambda\tau > 0$, and $M(\tau^*) > 0$, it is true that $\frac{\partial M}{\partial\lambda} < \frac{1}{\lambda}M(\tau^*)e^{-\lambda\tau^*}[-\chi+1]$.

If $\alpha > 1$, $\chi > 1$, and hence, $\frac{\partial M}{\partial\lambda} < 0$.

In addition, it's easy to obtain that:

$$\begin{aligned}\frac{\partial\lambda}{\partial\Omega(\alpha)} &= -\frac{(1-\alpha)}{\alpha} > 0, \text{ for } \alpha > 1\\ \frac{\partial\Omega(\alpha)}{\partial\mu_f} &= \frac{1}{\alpha}\frac{-(\mu_d-r)\sigma_{df}+\left(\mu_f-r\right)\sigma_d^2}{\sigma_d^2\sigma_f^2-\sigma_{df}^2}\\ \frac{\partial M}{\partial\tau^*} &= \frac{(\omega-\lambda)\lambda}{\omega}\left(e^{\omega\tau}-1\right)e^{-\lambda\tau}\\ &= \frac{1-\alpha}{\alpha}\left[\Omega(\alpha)-r^L\right]\frac{\lambda}{\omega}\left(e^{\omega\tau}-1\right)e^{-\lambda\tau} < 0, \text{ for } \alpha > 1\end{aligned}$$

However, the term for $\frac{\partial\Omega(\alpha)}{\partial\mu_f}$ and $\frac{\partial\Omega(\alpha)}{\partial\mu_d}$ can be positive or negative. Recall that:

$$\begin{aligned}\Omega(\alpha) &\equiv \left(r + \frac{1}{2}\frac{1}{\alpha}(\mu - R)'\,\Omega^{-1}(\mu - R)\right) \\ &= \left(r + \frac{1}{2}\frac{1}{\alpha}\left(\frac{(\mu_d - r)^2\sigma_f^2 - 2(\mu_d - r)(\mu_f - r)\sigma_{df} + (\mu_f - r)^2\sigma_d^2}{\sigma_d^2\sigma_f^2 - \sigma_{df}^2}\right)\right)\end{aligned}$$

$$\begin{aligned}\frac{d\Omega(\alpha)}{d\mu} &= \frac{1}{\alpha}(\mu - R)'\,\Omega^{-1} = \phi^{*\prime} \\ &= \frac{1}{\alpha}\frac{1}{\sigma_d^2\sigma_f^2 - \sigma_{df}^2}\begin{pmatrix}(\mu_d - r)\sigma_f^2 - (\mu_f - r)\sigma_{df} \\ -(\mu_d - r)\sigma_{df} + (\mu_f - r)\sigma_d^2\end{pmatrix}\end{aligned}$$

Hence:

$$\frac{d\Omega(\alpha)}{d\mu_f} = \frac{1}{\alpha}\frac{-(\mu_d - r)\sigma_{df} + (\mu_f - r)\sigma_d^2}{\sigma_d^2\sigma_f^2 - \sigma_{df}^2} \lesseqgtr 0$$

The analogous derivation follows for the domestic stock market:

$$\frac{d\Omega(\alpha)}{d\mu_d} = \frac{1}{\alpha}\frac{(\mu_d - r)\sigma_f^2 - (\mu_f - r)\sigma_{df}}{\sigma_d^2\sigma_f^2 - \sigma_{df}^2} \lesseqgtr 0$$

Although the answers to the last derivative is straightforward for the one-asset only case, the analogous is not true for the case with foreign and domestic stocks. In principle, nothing prevents the investor from short selling, and if this is so, $\frac{d\Omega(\alpha)}{d\mu_f}$ or $\frac{d\Omega(\alpha)}{d\mu_d}$ can attain a negative sign.

Assuming there is no short selling:

$$\begin{aligned}(\mu_d - r)\sigma_f^2 - (\mu_f - r)\sigma_{df} &> 0 \\ -(\mu_d - r)\sigma_{df} + (\mu_f - r)\sigma_d^2 &> 0,\end{aligned}$$

and therefore, $\frac{\partial\Omega(\alpha)}{\partial\mu_f} > 0$ and $\frac{\partial\Omega(\alpha)}{\partial\mu_d} > 0$, implying:

$$\frac{\partial\tau^*}{\partial\mu_f} = -\frac{\frac{\partial M}{\partial\mu_f}}{\frac{\partial M}{\partial\tau^*}} = -\frac{\left(\overbrace{\frac{\partial M}{\partial\lambda}}^{<0}\overbrace{\frac{\partial\lambda}{\partial\Omega(\alpha)}}^{>0}\overbrace{\frac{\partial\Omega(\alpha)}{\partial\mu_f}}^{>0}\right)}{\underbrace{\frac{\partial M}{\partial\tau^*}}_{<0}} < 0$$

$$\frac{\partial \tau^*}{\partial \mu_d} = -\frac{\frac{\partial M}{\partial \mu_f}}{\frac{\partial M}{\partial \tau^*}} = -\frac{\left(\overbrace{\frac{\partial M}{\partial \lambda}}^{<0}\overbrace{\frac{\partial \lambda}{\partial \Omega(\alpha)}}^{>0}\overbrace{\frac{\partial \Omega(\alpha)}{\partial \mu_d}}^{>0}\right)}{\underbrace{\frac{\partial M}{\partial \tau^*}}_{<0}} < 0$$

Now, for the variances:

$$\begin{aligned}
\frac{\partial \tau^*}{\partial \sigma_f^2} &= -\frac{\frac{\partial M}{\partial \sigma_f^2}}{\frac{\partial M}{\partial \tau^*}} = -\frac{\left(\frac{\partial M}{\partial \lambda}\frac{\partial \lambda}{\partial \Omega(\alpha)}\frac{\partial \Omega(\alpha)}{\partial \sigma_f^2}\right)}{\frac{\partial M}{\partial \tau^*}} > 0 \\
\frac{\partial \tau^*}{\partial \sigma_d^2} &= -\frac{\frac{\partial M}{\partial \mu_f}}{\frac{\partial M}{\partial \tau^*}} = -\frac{\left(\frac{\partial M}{\partial \lambda}\frac{\partial \lambda}{\partial \Omega(\alpha)}\frac{\partial \Omega(\alpha)}{\partial \sigma_d^2}\right)}{\frac{\partial M}{\partial \tau^*}} > 0
\end{aligned}$$

Again, the term for $\Omega(\alpha)$ is such that:

$$\Omega(\alpha) \equiv \left(r + \frac{1}{2}\frac{1}{\alpha}\left(\frac{(\mu_d - r)^2 \sigma_f^2 - 2(\mu_d - r)(\mu_f - r)\sigma_{df} + (\mu_f - r)^2 \sigma_d^2}{\sigma_d^2\sigma_f^2 - \sigma_{df}^2}\right)\right)$$

$$\begin{aligned}
\frac{d\Omega(\alpha)}{d\sigma_f^2} &= \frac{1}{2}\frac{1}{\alpha}\left(\frac{2(\mu_d - r)^2 \sigma_f \left(\sigma_d^2\sigma_f^2 - \sigma_{df}^2\right) - \begin{pmatrix} (\mu_d - r)^2 \sigma_f^2 \\ -2(\mu_d - r)(\mu_f - r)\sigma_{df} \\ +(\mu_f - r)^2 \sigma_d^2 \end{pmatrix} 2\sigma_d^2\sigma_f}{\left(\sigma_d^2\sigma_f^2 - \sigma_{df}^2\right)^2}\right) \\
&= \frac{1}{2}\frac{1}{\alpha}\left(\frac{2(\mu_d - r)^2 \sigma_f \left(\sigma_d^2\sigma_f^2 - \sigma_{df}^2\right) - \begin{pmatrix} (\mu_d - r)^2 \sigma_f^2 \\ -2(\mu_d - r)(\mu_f - r)\sigma_{df} \\ +(\mu_f - r)^2 \sigma_d^2 \end{pmatrix} 2\sigma_d^2\sigma_f}{\left(\sigma_d^2\sigma_f^2 - \sigma_{df}^2\right)^2}\right)
\end{aligned}$$

One, hence, has to test the sign of the numerator of the above expression. Rearranging the terms:

$$\begin{aligned}
& (\mu_d - r)^2 \sigma_f \left(\sigma_d^2 \sigma_f^2 - \sigma_{df}^2\right) - \left(\begin{array}{c} \sigma_d^2 \sigma_f (\mu_d - r)^2 \sigma_f^2 - 2\sigma_d^2 \sigma_f (\mu_d - r)(\mu_f - r) \sigma_{df} \\ +\sigma_d^2 \sigma_f (\mu_f - r)^2 \sigma_d^2 \end{array}\right) \\
= & (\mu_d - r) \sigma_f \sigma_{df} \left((\mu_f - r) \sigma_d^2 - (\mu_d - r) \sigma_{df}\right) \\
& + \sigma_d^2 \sigma_f (\mu_f - r) \left((\mu_d - r) \sigma_{df} - (\mu_f - r) \sigma_d^2\right) \\
= & \sigma_f \left((\mu_f - r) \sigma_d^2 - (\mu_d - r) \sigma_{df}\right) \left((\mu_d - r) \sigma_{df} - (\mu_f - r) \sigma_d^2\right) \\
= & -\sigma_f \left((\mu_f - r) \sigma_d^2 - (\mu_d - r) \sigma_{df}\right)^2 \\
< & \; 0
\end{aligned}$$

The analogous follows for the domestic stock:

$$\begin{aligned}
\frac{d\Omega(\alpha)}{d\sigma_d^2} &= \frac{1}{2}\frac{1}{\alpha}\left(\frac{2(\mu_f - r)^2 \sigma_d \left(\sigma_d^2 \sigma_f^2 - \sigma_{df}^2\right) - \left(\begin{array}{c} (\mu_d - r)^2 \sigma_f^2 \\ -2(\mu_d - r)(\mu_f - r) \sigma_{df} \\ +(\mu_f - r)^2 \sigma_d^2 \end{array}\right) 2\sigma_f^2 \sigma_d}{\left(\sigma_d^2 \sigma_f^2 - \sigma_{df}^2\right)^2}\right) \\
&= \frac{1}{2}\frac{1}{\alpha}\left(\frac{2(\mu_f - r)^2 \sigma_d \left(\sigma_d^2 \sigma_f^2 - \sigma_{df}^2\right) - \left(\begin{array}{c} (\mu_d - r)^2 \sigma_f^2 \\ -2(\mu_d - r)(\mu_f - r) \sigma_{df} \\ +(\mu_f - r)^2 \sigma_d^2 \end{array}\right) 2\sigma_f^2 \sigma_d}{\left(\sigma_d^2 \sigma_f^2 - \sigma_{df}^2\right)^2}\right)
\end{aligned}$$

The numerator is such that:

$$\begin{aligned}
& (\mu_f - r)^2 \sigma_d \left(\sigma_d^2 \sigma_f^2 - \sigma_{df}^2\right) - \left((\mu_d - r)^2 \sigma_f^2 - 2(\mu_d - r)(\mu_f - r) \sigma_{df} + (\mu_f - r)^2 \sigma_d^2\right) \sigma_f^2 \sigma_d \\
= & (\mu_f - r)^2 \sigma_d \sigma_d^2 \sigma_f^2 - (\mu_f - r)^2 \sigma_d \sigma_{df}^2 - \sigma_f^2 \sigma_d (\mu_d - r)^2 \sigma_f^2 \\
& + 2\sigma_f^2 \sigma_d (\mu_d - r)(\mu_f - r) \sigma_{df} - \sigma_f^2 \sigma_d (\mu_f - r)^2 \sigma_d^2 \\
= & (\mu_d - r)(\mu_f - r) \sigma_f^2 \sigma_d \sigma_{df} - (\mu_f - r)^2 \sigma_d \sigma_{df}^2 \\
& + (\mu_d - r)(\mu_f - r) \sigma_f^2 \sigma_d \sigma_{df} - (\mu_d - r)^2 \sigma_f^4 \sigma_d \\
= & \sigma_d \left((\mu_d - r) \sigma_f^2 - (\mu_f - r) \sigma_{df}\right) \left((\mu_f - r) \sigma_{df} - (\mu_d - r) \sigma_f^2\right) \\
= & -\sigma_d \left((\mu_d - r) \sigma_f^2 - (\mu_f - r) \sigma_{df}\right)^2 \\
< & \; 0
\end{aligned}$$

Therefore:

$$\frac{\partial \tau^*}{\partial \sigma_f^2} = -\frac{\frac{\partial M}{\partial \sigma_f^2}}{\frac{\partial M}{\partial \tau^*}} = -\frac{\left(\overbrace{\frac{\partial M}{\partial \lambda}}^{<0}\overbrace{\frac{\partial \lambda}{\partial \Omega(\alpha)}}^{>0}\overbrace{\frac{\partial \Omega(\alpha)}{\partial \sigma_f^2}}^{<0}\right)}{\underbrace{\frac{\partial M}{\partial \tau^*}}_{<0}} > 0$$

$$\frac{\partial \tau^*}{\partial \sigma_d^2} = -\frac{\frac{\partial M}{\partial \mu_f}}{\frac{\partial M}{\partial \tau^*}} = -\frac{\left(\overbrace{\frac{\partial M}{\partial \lambda}}^{<0}\overbrace{\frac{\partial \lambda}{\partial \Omega(\alpha)}}^{>0}\overbrace{\frac{\partial \Omega(\alpha)}{\partial \sigma_d^2}}^{<0}\right)}{\underbrace{\frac{\partial M}{\partial \tau^*}}_{<0}} > 0$$

Observe that for the variance case, there is no need for the restriction on non-short-selling. Finally, for the covariance, the optimal level of inattention is increasing in the covariance of the asset returns. Applying the Implicit Function Theorem:

$$\frac{\partial \tau^*}{\partial \sigma_{df}} = -\frac{\frac{\partial M}{\partial \sigma_{df}}}{\frac{\partial M}{\partial \tau^*}} = -\frac{\left(\frac{\partial M}{\partial \lambda}\frac{\partial \lambda}{\partial \Omega(\alpha)}\frac{\partial \Omega(\alpha)}{\partial \sigma_{df}}\right)}{\frac{\partial M}{\partial \tau^*}}$$

As previously obtained:

$$\frac{\partial M}{\partial \lambda} = \frac{1}{\lambda} M(\tau^*) e^{-\lambda \tau^*} \left[-M(\tau^*)^{-1} + (1-\lambda\tau^*) e^{\lambda \tau^*}\right] < 0, \text{ for } \alpha > 1$$

$$\begin{aligned} \frac{\partial \lambda}{\partial \Omega(\alpha)} &= -\frac{(1-\alpha)}{\alpha} \\ \frac{\partial M}{\partial \tau^*} &= \frac{1-\alpha}{\alpha}\left[\Omega(\alpha) - r^L\right]\frac{\lambda}{\omega}\left(e^{\omega\tau} - 1\right) e^{-\lambda\tau} \end{aligned}$$

$$\begin{aligned} \frac{\partial \Omega(\alpha)}{\partial \sigma_{df}} &= \frac{1}{2}\frac{1}{\alpha}\left(\frac{\left[\begin{array}{c} -2(\mu_d - r)(\mu_f - r)\left(\sigma_d^2\sigma_f^2 - \sigma_{df}^2\right) - \\ (-2\sigma_{d,f})\left((\mu_d - r)^2\sigma_f^2 - 2(\mu_d - r)(\mu_f - r)\sigma_{df} + (\mu_f - r)^2\sigma_d^2\right) \end{array}\right]}{\left(\sigma_d^2\sigma_f^2 - \sigma_{df}^2\right)^2}\right) \\ &= \frac{1}{2}\frac{1}{\alpha}\left(\frac{\left[\begin{array}{c} -2(\mu_d - r)(\mu_f - r)\sigma_d^2\sigma_f^2 + 2(\mu_d - r)(\mu_f - r)\sigma_{df}^2 \\ +2\sigma_{df}(\mu_d - r)^2\sigma_f^2 - 4(\mu_d - r)(\mu_f - r)\sigma_{df} + 2\sigma_{df}(\mu_f - r)^2\sigma_d^2 \end{array}\right]}{\left(\sigma_d^2\sigma_f^2 - \sigma_{df}^2\right)^2}\right) \end{aligned}$$

Since the denominator is positive, it lacks to test the numerator:

$$\begin{aligned}
&= -\left(\mu_d - r\right)\left(\mu_f - r\right)\sigma_d^2\sigma_f^2 + \left(\mu_d - r\right)\left(\mu_f - r\right)\sigma_{df}^2 + \sigma_{df}\left(\mu_d - r\right)^2\sigma_f^2 \\
&\quad -2\left(\mu_d - r\right)\left(\mu_f - r\right)\sigma_{df} + \sigma_{df}\left(\mu_f - r\right)^2\sigma_d^2 \\
&< -\left(\mu_d - r\right)\left(\mu_f - r\right)\sigma_d^2\sigma_f^2 + \left(\mu_d - r\right)\left(\mu_f - r\right)\sigma_{df} - 2\left(\mu_d - r\right)\left(\mu_f - r\right)\sigma_{df} \\
&\quad +\sigma_{df}\left(\mu_d - r\right)^2\sigma_f^2 + \sigma_{df}\left(\mu_f - r\right)^2\sigma_d^2 \\
&= -\left(\mu_d - r\right)\left(\mu_f - r\right)\sigma_d^2\sigma_f^2 - \left(\mu_d - r\right)\left(\mu_f - r\right)\sigma_{df} \\
&\quad +\sigma_{df}\left(\mu_d - r\right)^2\sigma_f^2 + \sigma_{df}\left(\mu_f - r\right)^2\sigma_d^2 \\
&= \left(\mu_d - r\right)\sigma_f^2\left(\left(\mu_d - r\right)\sigma_{df} - \left(\mu_f - r\right)\sigma_d^2\right) + \sigma_{df}\left(\mu_f - r\right)\left(\left(\mu_f - r\right)\sigma_d^2 - \left(\mu_d - r\right)\sigma_{df}\right) \\
&= \underbrace{\left(\left(\mu_d - r\right)\sigma_{df} - \left(\mu_f - r\right)\sigma_d^2\right)}_{<0}\underbrace{\left(\left(\mu_d - r\right)\sigma_f^2 - \sigma_{df}\left(\mu_f - r\right)\right)}_{>0} < 0
\end{aligned}$$

and hence, $\frac{d\Omega(\alpha)}{d\sigma_{df}} < 0$. Therefore, one gets:

$$\begin{aligned}
\frac{\partial \tau^*}{\partial \sigma_{df}} &= -\frac{\frac{\partial M}{\partial \sigma_{df}}}{\frac{\partial M}{\partial \tau^*}} = -\frac{\left(\overbrace{\frac{\partial M}{\partial \lambda}}^{<0}\left(-\frac{(1-\alpha)}{\alpha}\right)\overbrace{\frac{\partial \Omega\left(\alpha\right)}{\partial \sigma_{df}}}^{<0}\right)}{\frac{1-\alpha}{\alpha}\left[\Omega\left(\alpha\right) - r^L\right]\frac{\lambda}{\omega}\left(e^{\omega\tau} - 1\right)e^{-\lambda\tau}} \\
&= \frac{\left(\overbrace{\frac{\partial M}{\partial \lambda}}^{<0}\overbrace{\frac{\partial \Omega\left(\alpha\right)}{\partial \sigma_{df}}}^{<0}\right)}{\underbrace{\left[\Omega\left(\alpha\right) - r^L\right]\frac{\lambda}{\omega}\left(e^{\omega\tau} - 1\right)e^{-\lambda\tau}}_{>0}} > 0
\end{aligned}$$

Observe that had I not imposed the non-short-selling condition, the sign of

$$-\left(\left(\mu_f - r\right)\sigma_d^2 - \left(\mu_d - r\right)\sigma_{df}\right)\left(\left(\mu_d - r\right)\sigma_f^2 - \sigma_{df}\left(\mu_f - r\right)\right)$$

would be undetermined, since the two terms in the parameters have to either have opposite signs or be both positive (for the non-short-selling condition case). ■