

# Testing Present Value Models of the Current Account: A Cautionary Note

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*Abstract.* Following Campbell (1987) and Campbell and Shiller (1987), many papers have evaluated the intertemporal approach to the current account by testing restrictions on a Vector Autoregression (VAR). The attractiveness of the Campbell-Shiller methodology is that it is thought to be immune to omitted information. This paper uses results from Hansen and Sargent (1991a) and Quah (1990) to show that this is not true in certain (empirically plausible) situations. In particular, it is shown that if fundamentals are driven by unobserved (to the econometrician) permanent and transitory components, then the theoretical restrictions of a standard Present Value model of the current account might not be testable with a VAR. This is because the theoretical moving average representation can turn out to be noninvertible. This implies that observed data, *including the current account*, do not reveal the underlying shocks to agents' information sets.

These results are potentially relevant given the results of several recent papers which claim that current accounts are 'excessively volatile'. I provide a simple example in which a researcher employing the Campbell-Shiller methodology is tricked into thinking the current account responds excessively to shocks when in fact the data are consistent with the theory.

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## 1 Introduction

The intertemporal approach to the current account has improved our understanding of a wide range of issues in open-economy macroeconomics, including the effects of monetary and fiscal policies, the effects of terms of trade shocks, the effects of trade policies, and the degree of international capital mobility.<sup>3</sup> The basic idea behind the intertemporal approach is that a (small) country operating in a global capital market is like an individual operating in a domestic capital market. This analogy is extremely useful since it makes available all of the machinery that has been developed to study the Permanent Income Hypothesis of consumption. From this perspective, the key prediction of the intertemporal approach is that current accounts should respond to transitory shocks, but not to permanent shocks.<sup>4</sup>

Operationally, the predictions of the intertemporal approach rest on a number of auxiliary assumptions about asset market structure, demographics, and the decomposition of observed time series into various components. Most studies are quite explicit about their asset market and demographic assumptions. In keeping with the Permanent Income Hypothesis, it is usually assumed that international financial markets are incomplete, in the sense that bonds are tradeable but state-contingent claims are not. At the same time, domestic capital markets are usually assumed to be effectively complete, in the sense that a representative agent exists for each country. In addition, most studies assume these representative agents have infinite planning horizons, presumably due to an operative bequest motive.

While the existing literature is usually clear about its asset market and demographic assumptions, it is noticeably less clear about the way decompositions are handled. In fact,

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<sup>3</sup>Obstfeld and Rogoff (1995) survey the intertemporal approach to the current account, and provide an extensive bibliography.

<sup>4</sup>Glick and Rogoff (1995) emphasize another important distinction, i.e., between global and country-specific shocks. They show that under certain conditions the current account should only respond to country-specific (transitory) shocks. Global shocks manifest themselves as changes in the world real interest rate.

one could argue that along this dimension there has been a bit of technological regress. Early empirical studies of the intertemporal approach focused on a single expenditure category, and examined whether the current account responds only to its transitory component. For example, Ahmed (1986) studied the response of Britain's trade balance to government expenditures. Ahmed decomposed government expenditures into permanent and transitory components using a simple univariate linear de-trending procedure. He argued (persuasively) that for this particular case alternative decompositions are likely to produce similar results, since most of the variation in government expenditures were driven by the 'natural experiments' of wartime expenditure, which are widely perceived to be temporary.

More recently, however, empirical testing of the intertemporal approach has shifted to a methodology pioneered by Campbell (1987) and Campbell and Shiller (1987). Campbell and Shiller evaluate the predictions of Present Value models by testing the implied cross-equation restrictions on a Vector Autoregression (VAR). The attractiveness of their methodology rests on its perceived robustness to omitted information. That is, under certain conditions their tests are valid even when the econometrician has access to less information than individual firms and households. Clearly, in most applications this kind of robustness is important. The key insight of Campbell and Shiller is to note that the observed decisions of agents can be *sufficient statistics* for any omitted information. Even if you as econometrician do not observe the underlying shocks to agents' information sets, as long as you observe (ex post) the responses of agents to these shocks you might be able to invert the mapping between the two to infer what the underlying shocks were. Given estimates of the shocks, you can then proceed to test whether the observed responses are in accordance with the theory.

The main contribution of this paper is to point out that the perceived robustness to omitted information is in general illusory. I argue that it rests on very special assumptions about the decomposition of fundamentals into permanent and transitory components. Implicitly, the Campbell-Shiller methodology is based on a Beveridge-Nelson (1981) approach to the decomposition. This approach is based on two identifying restrictions. First, the number of underlying shocks is assumed to be equal to the number of observed time series. This

departs from an older tradition which views each time series as the sum of (orthogonal) unobserved components. Second, Beveridge and Nelson (1981) *define* the permanent component to be the infinite horizon forecast of the series. In the univariate case, these two restrictions combine to produce a decomposition with a pure random walk permanent component, with innovations that are perfectly correlated with innovations to the transitory component.

While it might seem natural to assume that each series is driven by a single shock, from a theoretical perspective this assumption can be problematic. Rational Expectations models impose restrictions across the equations describing the economy's exogenous forcing processes and the equations describing the decision rules of agents. Absent an explicit theory of planning or implementation errors, these decision rules are deterministic functions of the forcing processes. This produces a stochastic singularity in the joint process consisting of the exogenous variables and agents' decision variables. One popular way to avoid a singularity is to appeal to the Unobserved Components idea, and assume that a single observed time series is the sum of distinct underlying components, each of which is observed by agents but not by the econometrician. As pointed out by Quah (1990) in the context of the Permanent Income Hypothesis, unobserved components have profound implications for tests of Present Value models. In particular, the Campbell-Shiller methodology is in general inapplicable, since the theoretical moving average representation can turn out to be noninvertible. This implies that agents' decisions do *not* reveal the underlying shocks.

The remainder of the paper is organized as follows. The next section outlines the standard Present Value model of the current account, and reviews the results of several recent empirical applications. Section 3 shows how unobserved components can produce a noninvertible moving average representation, and hence invalidate the Campbell-Shiller methodology. Section 4 provides a simple yet empirically plausible example in which the Present Value Model of the current account is true, but a researcher using the Campbell-Shiller approach is led to believe it is false. Section 5 concludes by offering suggestions on how to avoid the pitfalls highlighted by this paper.

## 2 The Present Value Model of the Current Account

Consider a small open economy inhabited by an infinitely lived representative agent. This agent has quadratic time-separable preferences with respect to a single nondurable good. This good is the same as the good produced in other countries, so that the only gains from trade derive from intertemporal consumption smoothing. It is assumed that the only way to do this is by trading a non-state contingent bond. The interest rate on this bond is exogenous and constant. Since the country is small I assume the interest rate is equal to the rate of time preference. Finally, in keeping with most of the literature, I adopt a partial equilibrium perspective, and take as given the investment decisions of firms and the expenditure decisions of the government. Given all these assumptions, the objective is to gauge the extent to which observed current account dynamics are consistent with optimal intertemporal consumption smoothing.

The analysis centers on the interaction between three equations: (i) a dynamic forward-looking consumption function, (ii) an economy-wide resource constraint, and (iii) an exogenously specified law of motion for the economy's output net of investment and government spending. The consumption function turns out to be:<sup>5</sup>

$$c_t = r \cdot b_t + (1 - \beta) E_t \sum_{j=0}^{\infty} \beta^j (y_{t+j} - i_{t+j} - g_{t+j}) \quad (1)$$

where  $c_t$  is consumption,  $r$  is the interest rate,  $b_t$  is the stock of foreign assets,  $\beta$  is the consumer's discount rate ( $\beta = (1 + r)^{-1}$ ),  $y_t$  is output,  $i_t$  is investment, and  $g_t$  is government spending.

The economy's resource constraint can be written as follows:

$$CA_t = r \cdot b_t + y_t - c_t - i_t - g_t \quad (2)$$

where  $CA_t$  is the current account balance. Plugging (1) into (2) and simplifying gives us:

$$CA_t = -E_t \sum_{j=1}^{\infty} \beta^j \Delta Q_{t+j} \quad (3)$$

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<sup>5</sup>See, e.g., Obstfeld and Rogoff (1995) for a derivation.

where I have followed the notational convention of defining net output,  $y_t - i_t - g_t$ , by  $Q_t$ .

Equation (3) summarizes the intertemporal approach to the current account. It says that current account surpluses reflect expectations of declining future net output. This makes sense. If net output is high relative to what it is expected to be in the future then the desire to smooth consumption will lead you to save today. This saving shows up as a current account surplus. It is also worth noting that as  $\beta \uparrow 1$  the right-hand side of (3) becomes the Beveridge-Nelson (1981) definition of the transitory component of  $Q_t$ . Again, this accords with the logic of the Permanent Income Hypothesis, i.e., saving responds to transitory (net) income, while consumption responds to permanent income.

To operationalize equation (3) one has to evaluate the expectation that appears on the right-hand side. Simply using past values of  $\Delta Q_t$  to forecast future values could produce misleading results if consumers are in fact basing their forecasts on a larger information set. At first glance, this seems like a fatal problem. How could we ever presume to have access to the thousands of scattered pieces of information that consumers respond to? Campbell and Shiller's insight was to realize that observed choices might be sufficient statistics for this information.<sup>6</sup> If this is true, then by including current and lagged values of the current account into the forecasting equation for  $\Delta Q_t$  we can effectively capture all this inside information.

This suggests the following testing strategy. First, estimate the following first-order VAR:<sup>7</sup>

$$\begin{bmatrix} \Delta Q_t \\ CA_t \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} \Delta Q_{t-1} \\ CA_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \quad (4)$$

Next, use (4) to evaluate the expectations in (3). Letting  $\Psi$  denote the VAR coefficient matrix, this gives us the following current account equation, expressed in terms of observables:

$$\begin{aligned} CA_t &= -(1, 0)\beta\Psi[I - \beta\Psi]^{-1} \begin{bmatrix} \Delta Q_t \\ CA_t \end{bmatrix} \\ &\equiv [\Phi_{\Delta Q}, \Phi_{CA}] \begin{bmatrix} \Delta Q_t \\ CA_t \end{bmatrix} \end{aligned} \quad (5)$$

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<sup>6</sup>This idea also showed up in the earlier work of Hall (1978).

<sup>7</sup>The assumption of a first-order VAR is not restrictive, since higher order systems can always be written as a first-order system with an expanded number of states.

Finally, notice that if the theory is true the coefficients should satisfy the following restriction:

$$[\Phi_{\Delta Q}, \Phi_{CA}] = [0, 1] \tag{6}$$

That is, given the contemporaneous current account, no other variables should be useful in forecasting the discounted sum of future values of  $\Delta Q_t$ . The current account is a sufficient statistic for the expectations in (3).

A number of papers have tested the restrictions in (6), using data from a number of different countries and a number of different time periods. Perhaps the most widely cited are by Ghosh (1995) and Sheffrin and Woo (1990). Ghosh applies the model to five countries – the U.S., Canada, Germany, Japan, and the U.K. – using quarterly data from the period 1960 to 1988. He rejects the restrictions in (6) for all countries except the U.S.. Sheffrin and Woo apply the model to four countries – Canada, Belgium, Denmark, and the U.K. – using annual data from the period 1955 to 1985. They reject the restrictions for all countries except Belgium.<sup>8</sup> More interesting than the statistical rejections of (6) is the reason behind the rejections. In most cases it turns out that the predicted current account is much less volatile than the actual current account. From this, Ghosh concludes that capital is excessively mobile.

In their survey, Obstfeld and Rogoff (1995) offer a potential explanation of these results. They point out that excessively volatile current accounts are the mirror image of excessively smooth consumption. This suggests a link to ‘Deaton’s Paradox’, i.e., if labor income has a unit root component then the Permanent Income Hypothesis predicts that consumption should be *more* volatile than is actually observed. One way to resolve Deaton’s Paradox is to simply deny the presence of a unit root component, and assume all shocks are temporary. This leads Obstfeld and Rogoff to conjecture that if  $Q_t$  is assumed to be trend stationary and the VAR is estimated using the levels of  $Q_t$  then perhaps the model would produce better results.

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<sup>8</sup>Employing a somewhat weaker test, Otto (1992) rejects the model using U.S. and Canadian data from the period 1950 to 1988. As in Ghosh (1995), he concludes that the model performs better for the U.S. than for Canada. More recently, Agenor et. al. (1999) applied the model to data from France for the period 1970 through 1996. In contrast to most previous findings, their results are quite favorable, e.g., they fail to reject the restrictions in (6) and the predicted current account tracks the actual quite closely.

While Obstfeld and Rogoff’s conjecture might indeed provide an explanation of the apparent excess volatility of current accounts, the next section uses results from Quah (1990) to provide another explanation, one that is consistent with the presence of permanent shocks.

### 3 A Caveat to the Campbell-Shiller Methodology

The key implicit assumption of the Campbell-Shiller methodology is that the model’s equilibrium has a VAR representation as in (4), where the residuals represent innovations to agents’ information sets. This section shows that this assumption is generally invalid when the net income process,  $\Delta Q_t$ , is comprised of orthogonal permanent and transitory components, which are observed by the agents in the model but not by the econometrician. Following Quah (1990), I show that when this is the case the model’s moving average representation can easily turn out to be noninvertible. This implies that observations of current and past values of  $\Delta Q_t$  and  $CA_t$  do not reveal the innovations to agents’ information sets. This lack of identification can produce erroneous inferences about current account volatility.

Accordingly, now assume that  $Q_t$  is driven by two orthogonal shocks – a permanent shock,  $\varepsilon_{1t}$ , and a transitory shock,  $\varepsilon_{2t}$ . As in Friedman’s original work, these shocks are assumed to be observed by agents, but not by the econometrician. The econometrician can only observe  $Q_t$ . A general specification of the law of motion for  $\Delta Q_t$  is then:

$$\Delta Q_t = A_1(L)\varepsilon_{1t} + (1 - L)A_2(L)\varepsilon_{2t} \tag{7}$$

where  $A_1(L)$  and  $A_2(L)$  are square-summable polynomials in the lag operator,  $L$ . Note that  $\varepsilon_{1t}$  has a permanent effect on the level of  $Q_t$ , while  $\varepsilon_{2t}$  has only a transitory effect.

Using (7) in (3), along with the Hansen-Sargent prediction formula, delivers the following equilibrium moving average representation (see, e.g., Sargent (1987, p. 304)):

$$\begin{bmatrix} \Delta Q_t \\ CA_t \end{bmatrix} = \begin{bmatrix} A_1(L) & (1 - L)A_2(L) \\ -\beta \left[ \frac{A_1(L) - A_1(\beta)}{L - \beta} \right] & -\beta \left[ \frac{(1 - L)A_2(L) - (1 - \beta)A_2(\beta)}{L - \beta} \right] \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{8}$$

Notice how the the Present Value Model imposes constraints on the moving average polynomials, i.e., the polynomials in the second row are exact functions of the polynomials in the

first row. Also notice, however, that despite these cross-equation constraints the model is not singular, since the econometrician does not observe  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$ .

Now, the Campbell-Shiller methodology is implicitly based on the assumption that the matrix polynomial in (8) has a one-sided inverse in non-negative powers of  $L$ , so that  $[\Delta Q_t, CA_t]$  has a VAR representation with the underlying shocks,  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$ , as innovations. A necessary condition for this to be the case is that its determinant not have any roots inside the unit circle. The determinant is given by the following polynomial:

$$\delta(z) = -\beta \left\{ A_1(z) \frac{(1-z)A_2(z) - (1-\beta)A_2(\beta)}{z-\beta} - (1-z)A_2(z) \frac{A_1(z) - A_1(\beta)}{z-\beta} \right\} \quad (9)$$

If all the roots of  $\delta(z)$  are outside the unit circle then Campbell and Shiller’s approach is valid. If one (or more) roots lie inside the unit circle then Campbell and Shiller’s approach is invalid.<sup>9</sup>

Before studying the roots of  $\delta(z)$ , the following proposition establishes the importance of the Unobserved Components structure of  $Q_t$  to the results in this paper.

**Proposition 1:** *If either component in equation (7) is zero (i.e.,  $A_1(L) = 0$  or  $A_2(L) = 0$ ) then the Campbell-Shiller method is applicable.*

This result follows directly from Hansen and Sargent (1991b), since if either  $A_1(L)$  or  $A_2(L)$  is zero the model becomes an ‘exact’ (i.e., singular) Rational Expectations model. More generally, however, when  $Q_t$  consists of the sum of (orthogonal) permanent and transitory components, there is no guarantee that the Campbell-Shiller methodology will produce valid inferences about current account dynamics. Instead, the validity of the Campbell-Shiller methodology depends on the stochastic properties of the permanent and transitory components, which determine the roots of  $\delta(z)$ . The following proposition provides sufficient conditions under which the Campbell-Shiller methodology is not applicable. These conditions relate to the relative predictability of the two components.

**Proposition 2:** *Let  $|\rho_1|$  denote the absolute value of the correlation between the actual and*

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<sup>9</sup>It might appear as if  $\delta(z)$  has a singularity at  $z = \beta$ . Note, however, that this singularity is canceled (i.e., ‘removed’) by identical zeros in the numerator. Hence,  $\delta(z)$  is one-sided in non-negative powers of  $z$ .

predicted  $\beta$ -discounted sum of future changes in the permanent component of  $Q_t$ . Likewise, let  $|\rho_2|$  denote the absolute value of the correlation between the actual and predicted  $\beta$ -discounted sum of future changes in the transitory component of  $Q_t$ . Then if  $A_1(z)$  and  $A_2(z)$  are invertible, and  $|\rho_2| > 2|\rho_1|$ , the matrix polynomial in (8) will be noninvertible in non-negative powers of  $L$  if the (one-sided) polynomial,  $[(1-z)A_2(z) - (1-\beta)A_2(\beta)]/(z-\beta)$ , has at least one root inside the unit circle.

Proof: Write  $\delta(z)$  as follows:

$$\begin{aligned} \delta(z) &= -A_1(z)A_2(z) \left\{ \left[ \frac{(1-z)A_2(z) - (1-\beta)A_2(\beta)}{z-\beta} \right] \frac{\beta}{A_2(z)} - (1-z) \left[ \frac{A_1(z) - A_1(\beta)}{z-\beta} \right] \frac{\beta}{A_1(z)} \right\} \\ &\equiv -A_1(z)A_2(z)[\gamma_2(z) - (1-z)\gamma_1(z)] \end{aligned}$$

Note that  $|(z-\beta)\gamma_2(z)| = |\rho_2|$  and  $|(z-\beta)\gamma_1(z)| = |\rho_1|$ . Also note that  $|\gamma_2(z)| > |(1-z)\gamma_1(z)|$  on the unit circle,  $\mathcal{C}$ , if and only if  $|(z-\beta)\gamma_2(z)| > |(z-\beta)(1-z)\gamma_1(z)|$ . Finally, note that  $|1-z| \leq 2$  on  $\mathcal{C}$ . Given these facts, the result follows from Rouché's Theorem (Conway (1978, p. 125)), which implies that under the stated conditions  $\gamma_2(z)$  and  $[\gamma_2(z) - (1-z)\gamma_1(z)]$  have the same number of roots inside  $\mathcal{C}$ .  $\square$

Proposition 2 implies that as long as changes in the permanent component are not too predictable (relative to changes in the transitory component) the validity of the Campbell-Shiller methodology is independent of the exact specification of the permanent component,  $A_1(L)$ . Only the properties of the transitory component,  $A_2(L)$ , matter. This is fortuitous, since the results in Quah (1992) suggest that in practice  $A_1(L)$  may be weakly identified.

The next result identifies one simple case where Proposition 2 applies.

**Corollary 1:** *If the permanent component of  $Q_t$  is a random walk, then Proposition 2 applies.*

Proof: If the permanent component is a random walk then  $A_1(z) = 1$ , so that  $|\rho_1| = 0$ .  $\square$

If the permanent component is a random walk then its changes are clearly unpredictable, so that as long as there are some predictable dynamics in  $\Delta Q_t$ , the correlation inequality in Proposition 2 must be satisfied. More generally, however, it may be desirable to allow changes in the permanent component to have some dynamics (see, e.g., Quah (1992)). Verifying the conditions of Proposition 2 then requires more effort.

Even when Proposition 2 applies, it's still not clear whether the Campbell-Shiller methodology applies. One must verify that the roots of  $[(1 - z)A_2(z) - (1 - \beta)A_2(\beta)]/(z - \beta)$  are outside the unit circle. Evidently, this depends on  $A_2(z)$ . The following corollary identifies two simple specifications of  $A_2(L)$  that justify the use of the Campbell-Shiller methodology.

**Corollary 2:** *A sufficient condition for the applicability of the Campbell-Shiller methodology is that the conditions of Proposition 2 are satisfied and the transitory component of  $Q_t$  is either i.i.d. or an AR(1).*

Proof: One can verify by direct substitution that if  $A_2(z) = 1$  or  $A_2(z) = (1 - \alpha z)^{-1}$  then the polynomial identified in Proposition 2 does not have any roots inside the unit circle.  $\square$

Hence, combining Corollary 1 and Corollary 2, we know that the Campbell-Shiller methodology is applicable if the permanent component of  $Q_t$  is a random walk and the transitory component of  $Q_t$  is either i.i.d. or an AR(1). This is heartening, since the implied specification of  $Q_t$  as either an ARIMA(0,1,1) process (when the transitory component is i.i.d.), or an ARIMA(1,1,1) process (when the transitory component is an AR(1)), seems to describe well the observed  $Q_t$  processes of many countries. However, the next section provides an example where the permanent component continues to be a random walk, but the transitory component is specified as an MA(1). This implies  $Q_t$  is an ARIMA(0, 1, 2) process. It is shown that the Campbell-Shiller methodology may not be applicable in this case. This is a problem, since it can be quite difficult to distinguish empirically an ARIMA(0, 1, 2) process from either an ARIMA(0, 1, 1) process or an ARIMA(1, 1, 1) process. Only when the second-order autocorrelation of  $\Delta Q_t$  is reliably estimated as zero, or the third-order autocorrelation of  $\Delta Q_t$  is reliably estimated as nonzero can this distinction be made with any confidence.

## 4 An Example

Suppose the permanent component of  $Q_t$  follows a random walk (i.e.,  $A_1(L) = 1$ ) and the transitory component is an MA(1) (e.g.,  $A_2(L) = 1 + \alpha L$ ). This implies net output is an

ARIMA(0, 1, 2) process. Substituting these into (8) and simplifying yields the following theoretical moving average representation:

$$\begin{bmatrix} \Delta Q_t \\ CA_t \end{bmatrix} = \begin{bmatrix} 1 & (1-L)(1+\alpha L) \\ 0 & -\beta[\alpha(1-L-\beta)-1] \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \quad (10)$$

Notice that the current account does not respond to  $\varepsilon_{1t}$ . From equation (3), the current account only responds to predictable changes in net income, and when the permanent component is a pure random walk, its changes are totally unpredictable. Also notice that the determinant,  $\delta(z)$ , has a single root at  $z = 1 - \beta - 1/\alpha$ . This will be inside the unit circle if,

$$-\beta < \frac{1}{\alpha} < 2 - \beta \quad (11)$$

In what follows, assume the inequality in (11) is satisfied, and let  $\lambda$  denote the root of the determinant (i.e.,  $\lambda = 1 - \beta - \alpha^{-1}$ ).

How likely is this to occur in practice? Perhaps surprisingly, given that an ARIMA(0, 1, 2) provides an adequate (univariate) approximation to the  $\Delta Q_t$  process, *non-invertibility places no further restrictions on the data*. Since there are four parameters (i.e.,  $\sigma_1^2$ ,  $\sigma_2^2$ ,  $\beta$ ,  $\alpha$ ) and three empirical moments to match (i.e., the variance and first two autocovariances of  $\Delta Q_t$ ), the non-invertibility condition in (11) can always be satisfied. One might hope that by further restricting the parameters we could rule out non-invertibility. For example, economic theory suggests that  $.9 < \beta < 1.0$ , the exact value depending on data frequency. Unfortunately, even restricting  $\beta$  to be say,  $.95$ , does not assure an invertible moving average representation. If  $\beta = .95$  then non-invertibility would occur if  $\alpha \geq .96$ . Although this might appear to be unreasonable, since it suggests very large MA roots, this can be offset by reducing the variance of the transitory shock,  $\sigma_2^2$ . Intuitively, the separate variances of the permanent and transitory components gives us an extra degree of freedom in matching a given process to the data. Hence, ruling out non-invertibility requires strong priors about the relative variances of the two components. For example, if we observe relatively small autocovariances of  $\Delta Q_t$  and we *know* that there is some lower bound on the variance of the transitory shock, then the fact that  $\beta > .9$  would tend to rule out non-invertibility.

Now suppose that a researcher applies the Campbell-Shiller methodology to data generated according to (10). Remember, the first step is to fit a VAR to the data. Because of the root inside the unit circle we cannot simply invert the MA representation in (10) to get the VAR representation. Instead, we must first convert (10) to an observationally equivalent ‘fundamental’ moving average representation. To do this, write (10) as  $x_t = \mathcal{A}(L)\epsilon_t$ , with the obvious notational correspondences. Next, following the steps outlined in Hansen and Sargent (1991a), convert this to the observationally equivalent representation,

$$\begin{aligned} x_t &= [\mathcal{A}(L)WB(L)][B(L^{-1})'W'\epsilon_t] \\ &= \mathcal{A}^*(L)\epsilon_t^* \end{aligned} \tag{12}$$

where  $W$  is an orthogonal matrix that zeros out the first column of  $\mathcal{A}(\lambda)$ , and  $B(L)$  is a ‘Blaschke matrix’ that flips the root from  $z = \lambda$  to  $z = \lambda^{-1}$ . These matrices are given by

$$B(z) = \begin{bmatrix} \frac{1-\lambda z}{z-\lambda} & 0 \\ 0 & 1 \end{bmatrix} \quad W = \frac{1}{\sqrt{1+\eta^2}} \begin{bmatrix} -\eta & 1 \\ 1 & \eta \end{bmatrix} \tag{13}$$

where  $\eta = (1 - \lambda)(1 + \alpha\lambda)$ . Notice that  $B(z)B(z^{-1})' = I$  on  $|z| = 1$  and  $WW' = I$ . By construction,  $\mathcal{A}^*(L)$  is invertible, so the observable VAR representation is  $\mathcal{A}^*(L)^{-1}x_t = \epsilon_t^*$ . The key point here is that the VAR residuals,  $\epsilon_t^*$ , are *not* the innovations to agents’ information sets,  $\epsilon_t$ . Instead, what is estimated is the linear combinations defined by  $B(L^{-1})'W'\epsilon_t$ . Although these linear combinations are mutually and serially uncorrelated by construction, they span a strictly smaller information set. Hence, the variance of  $\epsilon_t$  is smaller than the variance of  $\epsilon_t^*$ .

Now suppose our researcher, who has never read Hansen and Sargent (1991a), but has read the RATS manual, proceeds to estimate a VAR and then inverts it to construct a moving average representation. From (12) and (13), this will give him the following result,

$$\begin{bmatrix} \Delta Q_t \\ CA_t \end{bmatrix} = \begin{bmatrix} \frac{1-\lambda L}{L-\lambda} [w_{11} + w_{21}(1-L)(1+\alpha L)] & w_{12} + w_{22}(1-L)(1+\alpha L) \\ -\beta w_{21}(1-\lambda L) & -\beta w_{22}(L-\lambda) \end{bmatrix} \begin{bmatrix} \epsilon_{1t}^* \\ \epsilon_{2t}^* \end{bmatrix} \tag{14}$$

where the  $w_{ij}$  are the elements of the  $W$  matrix in (13). Using this estimated moving average representation, he then computes the usual set of impulse response and variance

decomposition statistics in order to gauge the extent to which they correspond with theory (i.e., eq. (10)).<sup>10</sup>

Comparing (14) to (10), it is clear our researcher is in trouble. For one thing, he will falsely conclude that the current account responds to  $\varepsilon_{1t}^*$ , when in fact the current account does not respond to  $\varepsilon_{1t}$ . This could easily produce an impression of excess volatility. In addition, it is apparent that in general he will reject the Rational Expectations cross-equation restrictions. As noted in the section 2, these kinds of results have been quite common in the literature.

Finally, two remaining observations should be made. First, although this example may seem special and rather contrived, the basic point is far more general. This example was chosen purely for expositional simplicity (although I would argue that its empirical implications are not grossly violated by the data). Certainly, there is nothing special about the assumptions that the permanent component is a random walk and the transitory component is an MA(1). Second, while this paper has shown that one can be misled into falsely rejecting the Present Value model, it is likewise possible to make the opposite error, i.e., falsely accept its validity. To see this, suppose we alter the second row of (10) by adding  $\kappa_1$  to the first element and  $\kappa_2$  to the second element. Hence, contrary to theory, the current account now responds to permanent shocks. Continue to assume, without loss of generality, that there is a single root,  $\lambda$ , inside the unit circle. Performing the matrix multiplication in (12) to get the Wold representation, it becomes apparent that (generically) non-zero values of  $\kappa_1$  and  $\kappa_2$  can be chosen to satisfy the two theoretical cross-equation restrictions linking the first and second rows of (10), thus giving the false impression that the data are consistent with the theory, when in fact they are not. This result is of interest given the recent favorable results of Agenor et. al. (1999).

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<sup>10</sup>In practice, of course, he might simply test the VAR coefficient restrictions in (6). Asymptotically, inferences about the model will be the same either way. Studying the properties of the moving average representation is algebraically more transparent.

## 5 Conclusion

The point made by this paper is not new. It is basically an open-economy analog of Quah (1990), which is itself based on the results in Hansen and Sargent (1991a). Its basic message, as in so many other cases, is that proper tests of the Present Value Model of the current account depend sensitively on the nature of trend specification. If net output is trend stationary, or its permanent and transitory components are perfectly correlated so that a Beveridge-Nelson decomposition is appropriate, then the Campbell-Shiller methodology produces valid inferences about current account dynamics. On the other hand, if net output consists of orthogonal permanent and transitory components then, depending on the stochastic properties of the two components, the Campbell-Shiller methodology may produce erroneous conclusions about current account dynamics. While not new, this is an important message since prior work has often produced such apparently surprising results.

The potential inapplicability of the Campbell-Shiller methodology raises the obvious question – What are you to do if you suspect the model’s moving average representation is indeed noninvertible? Does this mean the Present Value Model of the current account is not testable? Fortunately, in addition to alerting researchers to the dangers of blindly assuming that VAR residuals represent innovations to agents’ information sets, Hansen and Sargent (1991a) also discuss ways around the problem. Perhaps the most straightforward is to simply deal with the moving average representation directly. (See, e.g., Hansen and Sargent (1991a, p. 95).

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