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A Probability-Based Stress Test of Federal Reserve Assets and Income

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Abstract

To support the economy, the Federal Reserve amassed a large portfolio of long-term bonds. We assess the Fed's associated interest rate risk—including potential losses to its Treasury securities holdings and declines in remittances to the Treasury. Unlike past examinations of this interest rate risk, we attach probabilities to alternative interest rate scenarios. These probabilities are obtained from a dynamic term structure model that respects the zero lower bound on yields. The resulting probability-based stress test finds that the Fed's losses are unlikely to be large and remittances are unlikely to exhibit more than a brief cessation.

JEL Classification: G12, E43, E52, E58.

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1 Introduction

In late 2008, in response to a severe financial crisis and recession, the Federal Reserve reduced its target for a key policy rate—the overnight federal funds rate—to a range between 0 and 25 basis points. To provide additional monetary stimulus to spur economic growth and avoid deflation, the Fed then conducted several rounds of large-scale asset purchases—commonly referred to as quantitative easing (QE). These actions left its portfolio of longer-term securities several times larger than its pre-crisis level. Although the Fed's securities portfolio carries essentially no credit risk, its market value can vary over time, and the greater size of the Fed's portfolio potentially exposes it to unusually large financial gains and losses from interest rate fluctuations. Furthermore, the Fed's purchases have shifted the composition of the portfolio toward longer-maturity securities, which increases the sensitivity of its market value to interest rate changes. This larger exposure to interest rate risk has raised certain policy concerns. For example, former Fed Governor Frederic Mishkin (2010) has argued that "major holdings of long-term securities expose the Fed's balance sheet to potentially large losses if interest rates rise. Such losses would result in severe criticism of the Fed and a weakening of its independence."[1]

In fact, there are two types of interest rate risk that the Fed faces. First, there is the risk that increases in *longer-term* interest rates will erode the market value of the Fed's portfolio—that is, balance sheet risk. Such declines in market value constitutes unrealized capital losses, which would become realized only if the securities were sold. Second, there is also the risk that increases in *short-term* interest rates, notably the interest rate that the Fed pays on bank reserves, will greatly increase the funding cost of the Fed's securities portfolio—that is, income risk. Because the Fed's interest income is generated from fixed coupon payments on longer-maturity securities, rising short-term interest rates and increased payments would reduce the Fed's net interest income, which in turn would lower the Fed's remittances to the U.S. Treasury. Under extreme circumstances, the remittances could fall to zero. While the Fed's ability to conduct monetary policy operations under such adverse conditions would not be directly impeded, concerns have been raised about the attendant political fallout from large capital losses (realized or unrealized) or a cessation in remittances.[2] See Rudebusch (2011) and Dudley (2013). For example, such concerns were noted in the minutes of the March 20, 2013 Federal Open Market Committee meeting, which stated that "[s]ome participants

[1]Such fears of political fallout have affected other central banks. The Bank of Japan has at times limited its bond purchases from a fear that capital losses could tarnish its credibility, while the Bank of England obtained an explicit indemnity from the British Treasury in advance for capital losses stemming from QE.

[2]Importantly, regardless of its portfolio losses or income expenses, the Fed still has operational control of short-term interest rates because its ability to pay interest on bank reserves allows it to conduct monetary policy independently of the size of its balance sheet. Unrealized portfolio losses do not affect the Fed's reported balance sheet, which is not marked to market. While realized portfolio losses are recorded, any resulting negative net income would not diminish the Fed's capital; instead, the Fed would maintain its capital by promising future remittances to the Treasury via creation of a deferred asset.

were concerned that a substantial decline in remittances might lead to an adverse public reaction or potentially undermine Federal Reserve credibility or effectiveness."

To understand and assess the Fed's balance sheet and income risks, it is crucial to quantify them. Two recent papers—Carpenter et al. (2013) and Greenlaw et al. (2013), henceforth GHHM—have made great progress in doing so. Both studies generated detailed projections of the market value and cash flow of the Fed's assets and liabilities under a few specific interest rate scenarios. In essence, their projections are akin to the "stress tests" that large financial institutions undergo to gauge whether they have enough capital to endure adverse economic scenarios.[3] As is common, these stress tests do not place probabilities on the alternative interest rate scenarios but simply consider, say, shifting the level of the entire yield curve up or down from its baseline projection by 100 basis points. Clearly, it is also of great interest to know what probabilities should be attached to the range of considered outcomes.[4] Attaching likelihoods to the alternative scenarios—or more generally, looking at the entire distributional forecast—results in what we term "probability-based" stress tests. The additional information about the probability distribution of interest rate scenarios allows us to provide new assessments of the likelihood of certain interest rate risk events. In this paper, we illustrate such a probabilistic methodology by examining potential mark-to-market losses on the Fed's Treasury holdings and the potential cessation of its remittances to the Treasury. Importantly, adding information on probability distributions enables us to examine the likelihood of certain events, such as the possibility that losses on the Fed's securities portfolio will exceed a certain threshold or that net interest income will fall negative for more than one year.

A key component of our probability-based stress-testing methodology is a dynamic term structure model that generates yield curve projections consistent with historical interest rate variation. Since nominal yields on Treasury debt are near their zero lower bound (ZLB), we use the shadow-rate, arbitrage-free Nelson-Siegel (AFNS) model class developed by Christensen and Rudebusch (2013a,b) to generate the requisite, potentially asymmetric, distributional interest rate forecasts. Shadow-rate models are latent-factor models in which the state variables have standard Gaussian dynamics, but the standard short rate is replaced by a shadow short rate that may be negative, as in the spirit of Black (1995). The model-generated observed short rate and yield forecasts thus respect the ZLB. Despite its inherent nonlinearity, shadow-rate AFNS models remain as flexible and empirically tractable as standard AFNS models. Critically for our purposes, we demonstrate that these models are able to accurately price the Fed's portfolio of Treasury securities.

To assess the Fed's balance sheet risk, we examine the distributional forecast of the value of the

[3] Stress testing financial institutions, and the financial system more broadly, has taken on great importance in the wake of the financial crisis; see Schuermann (2013) and Borio et al. (2013). Our analysis is directly related to interest rate risk stress testing, which is discussed by Drehmann et al. (2010) and Abdymomunov and Gerlach (2013).

[4] Berkowitz (2000) and Pritsker (2011) make a similar point regarding bank stress tests. In contrast, Borio et al. (2013) express the standard view that stress tests should focus only on a few hand-picked scenarios.

Fed's Treasury securities across $10,000$ yield curve simulations. We focus just on nominal Treasury securities because in 2013 they represent almost 60 percent of the securities held outright and the largest share of the Fed's assets. The next largest share is composed of agency mortgage-backed securities (MBS).[5] However, the valuation of MBS is very difficult because it requires assessing the refinancing and prepayment probabilities for a wide range of mortagages in alternative interest rate environments.[6] Our focus on Treasury securities should provide an important milestone for future, more detailed probability-based stress tests.

For our empirical assessment of the Fed's balance sheet risk, we use two simulation-based methods for generating Treasury yield curve projections. The first approach is based on the shadow-rate AFNS model favored by Christensen and Rudebusch (2013b, henceforth CR)in their analysis of U.S. Treasury yields near the ZLB. The second uses an approach based on the historical dynamics of the state variables extracted via the favored shadow-rate model. Despite the difference in methodology, both sets of results indicate that potential losses on the Fed's Treasury securities holdings should be small and manageable. In particular, our simulation results show that the projected median value of the Fed's Treasury holdings does not fall below face value over the three-year horizon of our exercise.

To assess the Fed's income risk, we use the model-based yield curve projections to generate distributional projections of the Fed's remittances to the Treasury up to seven years forward. In nearly 90 percent of the simulations, no remittance shortfalls are projected over the seven-year horizon. In fact, even at the lower fifth percentile of the distribution of outcomes, the cumulative remittance shortfall (i.e., the Fed's deferred asset) peaks at less than \$11.0 billion in 2017. Accordingly, our probability-based stress-testing methodology suggests that the risk of a long or substantial cessation of remittances to the Treasury is remote. Our approach also allows us to assess the distribution of cumulative remittances to the Treasury resulting from the Fed's QE program. Our results suggest that the Treasury in all likelihood (i.e., greater than 98% probability) will receive more remittances in total because of the Fed's QE securities purchases than it would have received if remittances had simply grown at their historical trend without the QE program.

Finally, an important caveat to our analysis should be noted. We are not conducting a comprehensive assessment of the costs and benefits of the Fed's program of QE, as discussed by Rudebusch (2011). Indeed, our probability-based stress test captures only part of the financial consequences of the Fed's securities purchases and, notably, excludes two key fiscal benefits accruing to the Treasury as longer-term interest rates were pushed lower by the Fed's securities purchases.[7] First, the lower interest rates likely resulted in higher output and household income, which boosted federal tax rev-

[5]The other securities in the Fed's asset portfolio are foreign assets and other claims, Treasury inflation protected securities (TIPS), and agency debt.

[6]Carpenter et al. (2013) and GHHM provide initial attempts at this effort.

[7]On this effect, see Gagnon et al. (2011), Christensen and Rudebusch (2012), and Bauer and Rudebusch (2013) among many others.

enue and reduced federal outlays. Second, the lower interest rates associated with QE helped lower the Treasury's borrowing costs for issuing new debt. Furthermore, it is important to stress that any kind of financial or fiscal accounting of the type we are conducting is ancillary to the Fed's mission. The Fed's statutory goal for setting monetary policy is to promote maximum employment and price stability, and these macroeconomic goals are the fundamental metrics for judging monetary policy. Financial considerations—even potentially large capital losses—are secondary.

The rest of the paper is structured as follows. Section 2 describes the evolution of the Fed's securities portfolio since the onset of the financial crisis and our data sample. Section 3 describes the shadow-rate AFNS model, while Section 4 presents our specific empirical representation. Section 5 explains the yield curve projections and their conversion into probability-based stress tests of the Fed's Treasury security holdings. Section 6 details our probability-based stress test of the Fed's remittances to the U.S. Treasury. Section 7 concludes.

2 The Fed's Securities Portfolio

Figure 1 shows the evolution of the assets of the Federal Reserve System at a weekly frequency since the start of 2008. In the early stages of the financial crisis, the Fed's balance sheet was expanded through various emergency lending facilities, most notably the Term Auction Facility (TAF).[8] In the figure, this lending appears in the "Other Assets" category, which as of January 2013 represented less than 10 percent of the Fed's assets. The "Non-Treasury Securities" category is composed of agency MBS almost exclusively, much of which was purchased during the Fed's first large-scale asset purchase program (QE1), which ran from late 2008 to early 2010.[9] Later, in its third purchase program (QE3), the Fed purchased additional MBS, and at the start of 2013, the MBS portfolio totaled $927 billion and represented 34.7 percent of the securities held outright. This MBS portfolio is made up of many small, heterogeneous, difficult-to-value securities. For example, about 5.5 percent of the portfolio was spread across 22,164 securities, each with a holding of less than $10 million. Another 9.2 percent was held in 3,932 securities with a maximum face value of $50 million.

The "Treasury Securities" category experienced a large expansion during the second purchase program (QE2), which operated from November 2010 through June 2011.[10] At the start of 2013, the Fed's nominal Treasury portfolio totaled $1.58 trillion and was spread across 241 different securities. It is this portfolio—about 59 percent of the Fed's securities held outright—that we focus on in our analysis of the Fed's balance sheet risk. As noted earlier, the long duration of this portfolio is also

[8] See Christensen et al. (2013) for details on the functioning and effectiveness of the TAF.

[9] In early 2013, the Non-Treasury Securities category also contained about $77 billion of federal agency debt issued by Fannie Mae and Freddie Mac.

[10] A small share of these purchases were inflation-indexed, Treasury inflation protected securities (TIPS). These totaled $75 billion in principal and another $11 billion in accrued inflation compensation as of January 2, 2013. Christensen and Gillan (2013) analyze the effects of the TIPS purchases included in QE2.

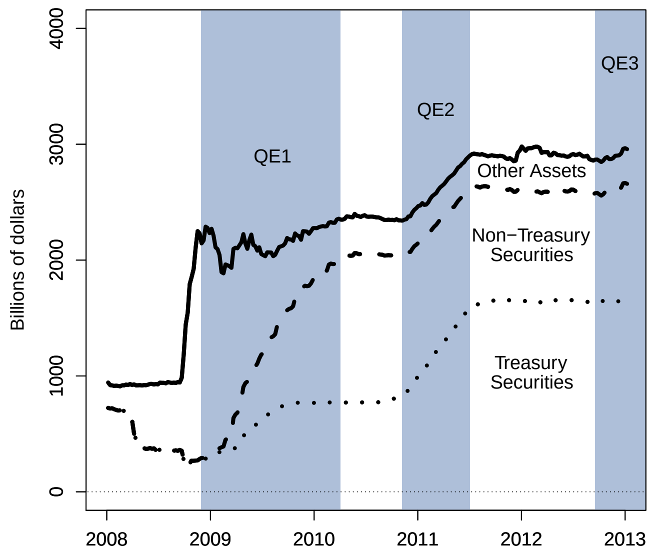

Figure 1: **Assets of the Federal Reserve System.**
Illustration of the total assets of the Federal Reserve System broken down into Treasury securities, non-Treasury securities, and other assets. The data are weekly covering the period from January 2, 2008, to January 2, 2013.

relevant for assessing balance sheet risk. From September 2011 through the end of 2012, the Fed conducted a Maturity Extension Program that sold Treasury securities with remaining maturities of three years or less and purchased a similar amount of Treasury securities with remaining maturities of six to thirty years. As a result of this policy, the Fed sold almost all of its short-term Treasury securities, so Treasuries with less than three years to maturity represented only 0.25 percent of its Treasury securities holdings at year-end 2012.

As noted in the introduction, this enlarged portfolio of longer-term securities greatly increases the Fed's interest rate risk. To model the market value of the Fed's Treasury holdings, we use the data set of zero-coupon Treasury yields described in Gürkaynak et al. (2007).[11] We use daily yields from January 2, 1986, to January 2, 2013, for the following 11 maturities: 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 15-year, 20-year, and 30-year.[12] As shown in Figure 2, Treasury yields were historically low at all maturities towards the end of our sample and near, if not

[11]For each business day, a zero-coupon yield curve is fitted to price a large pool of underlying off-the-run Treasury bonds. The Federal Reserve Board of Governors frequently updates the factors and parameters of this method; see the related website http://www.federalreserve.gov/pubs/feds/2006/index.html

[12]The longest maturity Treasury yields are not available prior to November 25, 1985. Also, between October 2001 and February 2006 the U.S. Treasury did not issue any 30-year bonds, but this absence has only a minuscule effect on our estimation results, which are primarily determined by the yields with 10 years or less to maturity.

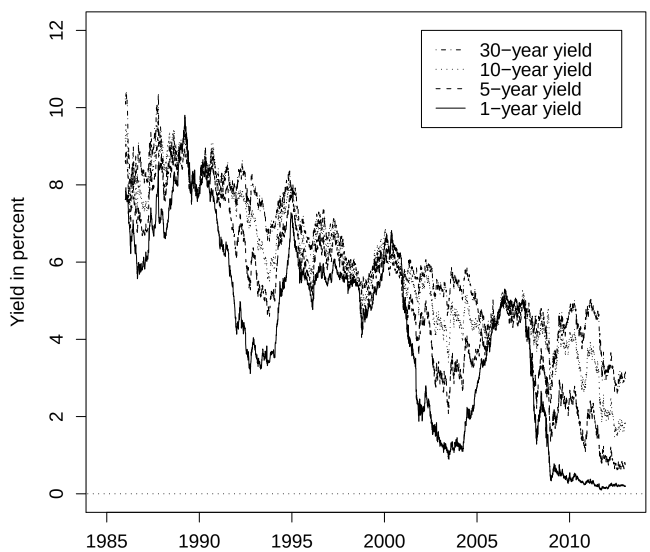

Figure 2: **Time Series of Treasury Bond Yields.**
Illustration of the daily Treasury zero-coupon bond yields covering the period from January 2, 1986, to January 2, 2013. The yields shown have maturities in 1 year, 5 years, 10 years, and 30 years, respectively.

at, the effective ZLB on nominal yields.

3 A Shadow-Rate Model of U.S. Treasury Yields

A key ingredient for our probability-based stress test is a data-generating process for the Treasury yield curve, and in this section, we describe the term structure model we use for this purpose. Because short-term interest rates have been near zero since 2009, the proximity of the ZLB affects the pricing of Treasuries and induces a notable asymmetry into the distributional forecasts of future yields. To respect the ZLB and account for its effects, we employ a shadow-rate term structure model.

3.1 The Option-Based Approach to the Shadow-Rate Model

The concept of a shadow interest rate as a modeling tool to account for the ZLB can be attributed to Black (1995). He noted that the observed nominal short rate will be nonnegative because currency is a readily available asset to investors that carries a nominal interest rate of zero. Therefore, the existence of currency sets a zero lower bound on yields. To account for this, Black postulated using a shadow short rate, s_t, which is unconstrained by the ZLB, as a modeling tool. The usual observed instantaneous risk-free rate, r_t, which is used for discounting cash flows when valuing securities, is then given by the greater of the shadow rate or zero:

$$r_t = \max\{0, s_t\}. \tag{1}$$

Accordingly, as s_t falls below zero, the observed r_t simply remains at the zero bound.

While Black (1995) described circumstances under which the zero bound on nominal yields might be relevant, he did not provide specifics for implementation. The small set of empirical research on shadow-rate models has relied on numerical methods for pricing.[13] To overcome the computational burden of numerical-based estimation that limits the use of shadow-rate models, Krippner (2012) suggested an alternative option-based approach that makes shadow-rate models almost as easy to estimate as the standard model. To illustrate this approach, consider two bond-pricing situations: one without currency as an alternative asset, and the other that has a currency in circulation with a constant nominal value and no transaction costs. In the world without currency, the price of a shadow-rate zero-coupon bond, $P_t(\tau)$, may trade above par; that is, its risk-neutral expected instantaneous return equals the risk-free shadow short rate, which may be negative. In contrast, in the world with currency, the price at time t for a zero-coupon bond that pays \$1 when it matures in τ years is given by $\underline{P}_t(\tau)$. This price will never rise above par, so nonnegative yields will never be observed.

Now consider the relationship between the two bond prices at time t for the shortest (say, overnight) maturity available, δ. In the presence of currency, investors can either buy the zero-coupon bond at price $P_t(\delta)$ and receive one unit of currency the following day or just hold the currency. As a consequence, this bond price, which would equal the shadow bond price, must be capped at 1:

$$\begin{aligned} \underline{P}_t(\delta) &= \min\{1, P_t(\delta)\} \\ &= P_t(\delta) - \max\{P_t(\delta) - 1, 0\}. \end{aligned}$$

That is, the availability of currency implies that the overnight claim has a value equal to the zero-coupon shadow bond price minus the value of a call option on the zero-coupon shadow bond with a strike price of 1. More generally, we can express the price of a bond in the presence of currency as the price of a shadow bond minus the call option on values of the bond above par:

$$\underline{P}_t(\tau) = P_t(\tau) - C_t^A(\tau, \tau; 1), \tag{2}$$

where $C_t^A(\tau, \tau; 1)$ is the value of an American call option at time t with maturity in τ years and strike

[13] For example, Kim and Singleton (2012) and Bomfim (2003) use finite-difference methods to calculate bond prices, while Ichiue and Ueno (2007) employ interest rate lattices.

price 1 written on the shadow bond maturing in τ years. In essence, in a world with currency, the bond investor has had to forgo any possible gain from the bond rising above par at any time prior to maturity.

Unfortunately, analytically valuing this American option is complicated by the difficulty in determining the early exercise premium. However, Krippner (2012) argues that there is an analytically close approximation based on tractable European options. Specifically, Krippner (2012) shows that the ZLB instantaneous forward rate, $\underline{f}_t(\tau)$, is

$$\underline{f}_t(\tau) = f_t(\tau) + z_t(\tau),$$

where $f_t(\tau)$ is the instantaneous forward rate on the shadow bond, which may go negative, while $z_t(\tau)$ is an add-on term given by

$$z_t(\tau) = \lim_{\delta \to 0} \Big[\frac{\partial}{\partial \delta} \frac{C_t^E(\tau, \tau+\delta; 1)}{P_t(\tau+\delta)} \Big],$$

where $C_t^E(\tau, \tau+\delta; 1)$ is the value of a European call option at time t with maturity $t+\tau$ and strike price 1 written on the shadow discount bond maturing at $t+\tau+\delta$. Thus, the observed yield-to-maturity is

$$\begin{aligned}
\underline{y}_t(\tau) &= \frac{1}{\tau}\int_t^{t+\tau} \underline{f}_t(s)ds \\
&= \frac{1}{\tau}\int_t^{t+\tau} f_t(s)ds + \frac{1}{\tau}\int_t^{t+\tau} \lim_{\delta \to 0} \Big[\frac{\partial}{\partial \delta} \frac{C_t^E(s, s+\delta; 1)}{P_t(s+\delta)} \Big] ds \\
&= y_t(\tau) + \frac{1}{\tau}\int_t^{t+\tau} \lim_{\delta \to 0} \Big[\frac{\partial}{\partial \delta} \frac{C_t^E(s, s+\delta; 1)}{P_t(s+\delta)} \Big] ds.
\end{aligned}$$

Thus, bond yields constrained at the ZLB can be viewed as the sum of the yield on the unconstrained shadow bond, denoted $y_t(\tau)$, which is modeled using standard tools, and an add-on correction term derived from the price formula for the option written on the shadow bond that provides an upward push to deliver the higher nonnegative yields actually observed.

As highlighted by Christensen and Rudebusch (2013a,b), the Krippner (2012) framework should be viewed as not fully internally consistent and simply an approximation to an arbitrage-free model.[14] Of course, away from the ZLB, with a negligible call option, the model will match the standard arbitrage-free term structure representation. In addition, the size of the approximation error near the ZLB has been determined via simulation in Christensen and Rudebusch (2013a,b) to be quite modest.[15]

[14] In particular, there is no explicit partial differential equation (PDE) that bond prices must satisfy, including boundary conditions, for the absence of arbitrage as in Kim and Singleton (2012).

[15] Christensen and Rudebusch (2013a,b) analyze how closely the option-based bond pricing from their estimated

3.2 The Shadow-Rate AFNS Model

In theory, the option-based shadow-rate result is quite general and applies to any assumptions about the dynamics of the shadow-rate process. However, as implementation requires the calculation of the limit term under the integral, option-based shadow-rate models are limited practically to the Gaussian model class where option prices are available in analytical form. The arbitrage-free Nelson-Siegel (AFNS) representation developed by Christensen et al. (2011, henceforth CDR) is well suited for this extension.[16] Its three factors correspond to the level, slope, and curvature factors commonly observed for Treasury yields and are denoted L_t, S_t, and C_t, respectively. The state vector is thus defined as $X_t = (L_t, S_t, C_t)$.[17]

In the shadow-rate AFNS model, the instantaneous risk-free rate is the nonnegative constrained process of the shadow risk-free rate, which is defined as the sum of level and slope as in the original AFNS model class:

$$r_t = \max\{0, s_t\}, s_t = L_t + S_t. \tag{3}$$

Also, the dynamics of the state variables used for pricing under the Q-measure remain as in the regular AFNS model:

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,Q} \\ dW_t^{S,Q} \\ dW_t^{C,Q} \end{pmatrix}, \tag{4}$$

where Σ is the constant covariance (or volatility) matrix.[18]

Based on this specification of the Q-dynamics, the yield on the shadow discount bond maintains the popular Nelson and Siegel (1987) factor loading structure

$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A(\tau)}{\tau}, \tag{5}$$

where $A(\tau)/\tau$ is a maturity-dependent yield-adjustment term. The corresponding instantaneous

shadow-rate AFNS models matches an arbitrage-free bond pricing that is obtained from the same models using Black's (1995) approach based on Monte Carlo simulations. They consider bonds of maturities out to 10 years. We extended these simulation results to consider bond maturities of 30 years (needed for pricing the longest bonds in the Fed's portfolio). At the 30-year maturity, the approximation errors are understandably larger but still do not exceed 6 basis points, which are notably smaller than the model's fitted errors.

[16] For details of this derivation, see Christensen and Rudebusch (2013a). For general discussion of the AFNS model, see Diebold and Rudebusch (2013)

[17] Note that this factor structure fits U.S. supervisory guidance on stress testing depository institution interest rate risk quite well. As summarized in Supervision and Regulation Letter SR 10-1 (2010), firms are instructed to examine large changes in the level, slope, and shape of the yield curve.

[18] As per CDR, Σ is a diagonal matrix, and θ^Q is set to zero without loss of generality.

shadow forward rate is given by

$$f_t(\tau) = -\frac{\partial}{\partial \tau} \ln P_t(\tau) = L_t + e^{-\lambda\tau} S_t + \lambda\tau e^{-\lambda\tau} C_t + A^f(\tau), \tag{6}$$

where the final term is another maturity-dependent yield-adjustment term.

Christensen and Rudebusch (2013a) show that, within the shadow-rate AFNS model, the zero-coupon bond yields that observe the zero lower bound, denoted $\underline{y}_t(\tau)$, are readily calculated as

$$\underline{y}_t(\tau) = \frac{1}{\tau} \int_t^{t+\tau} \Big[f_t(s)\Phi\Big(\frac{f_t(s)}{\omega(s)}\Big) + \omega(s)\frac{1}{\sqrt{2\pi}} \exp\Big(-\frac{1}{2}\Big[\frac{f_t(s)}{\omega(s)}\Big]^2\Big)\Big] ds, \tag{7}$$

where $\Phi(\cdot)$ is the cumulative probability function for the standard normal distribution, $f_t(\tau)$ is the shadow forward rate, and $\omega(\tau)$ takes the following simple form

$$\omega(\tau)^2 = \sigma_{11}^2 \tau + \sigma_{22}^2 \Big(\frac{1-e^{-2\lambda\tau}}{2\lambda}\Big) + \sigma_{33}^2 \Big(\frac{1-e^{-2\lambda\tau}}{4\lambda} - \frac{1}{2}\tau e^{-2\lambda\tau} - \frac{1}{2}\lambda\tau^2 e^{-2\lambda\tau}\Big),$$

when the volatility matrix Σ is assumed diagonal.

As in the affine AFNS model, the shadow-rate AFNS model is completed by specifying the price of risk using the essentially affine risk premium specification introduced by Duffee (2002), so the risk premium Γ_t is defined by the measure change

$$dW_t^Q = dW_t^P + \Gamma_t dt,$$

with $\Gamma_t = \gamma^0 + \gamma^1 X_t$, $\gamma^0 \in \mathbf{R}^3$, and $\gamma^1 \in \mathbf{R}^{3\times 3}$. Therefore, the real-world dynamics of the state variables can be expressed as

$$dX_t = K^P(\theta^P - X_t)dt + \Sigma dW_t^P. \tag{8}$$

In the unrestricted case, both K^P and θ^P are allowed to vary freely relative to their counterparts under the Q-measure just as in the original AFNS model.

Finally, we note that, due to the nonlinear measurement equation for the yields in the shadow-rate AFNS models, their estimation is based on the extended Kalman filter as described in Christensen and Rudebusch (2013a).

4 Model Estimation and Yield Curve Fit

In this section, we describe the in-sample estimation results for our preferred model, its fit to the yield curve, and its ability to price the Treasury securities in the Fed's portfolio.

K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	θ^P		Σ
$K^P_{1,\cdot}$	10^{-7}	0	0	0	σ_{11}	0.0043
						(0.0000)
$K^P_{2,\cdot}$	0.4240	0.3914	-0.4799	0.0386	σ_{22}	0.0086
	(0.1695)	(0.1182)	(0.1059)	(0.0355)		(0.0001)
$K^P_{3,\cdot}$	0	0	0.4249	-0.0296	σ_{33}	0.0264
			(0.1661)	(0.0119)		(0.0002)

Table 1: **Parameter Estimates for the B-CR Model.**
The estimated parameters of the K^P matrix, θ^P vector, and diagonal Σ matrix are shown for the B-CR model. The estimated value of λ is 0.4868 (0.0010). The numbers in parentheses are the estimated parameter standard deviations.

4.1 Estimation

In this subsection, we briefly describe the shadow-rate AFNS model used here, which is the shadow-rate equivalent of the AFNS model preferred by Christensen and Rudebusch (2012). Using both in-sample and out-of-sample performance measures, the authors determined that the zero-value restrictions on the K^P matrix in the following dynamic system for the P-dynamics were empirically warranted; i.e.,

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 10^{-7} & 0 & 0 \\ \kappa^P_{21} & \kappa^P_{22} & \kappa^P_{23} \\ 0 & 0 & \kappa^P_{33} \end{pmatrix} \left(\begin{pmatrix} 0 \\ \theta^P_2 \\ \theta^P_3 \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW^{L,P}_t \\ dW^{S,P}_t \\ dW^{C,P}_t \end{pmatrix}, \tag{9}$$

where the covariance matrix Σ is assumed diagonal and constant. Throughout, we refer to the shadow-rate AFNS model given by equations (3), (4), and (9) as the B-CR model.[19]

Note that the level factor is restricted to be an independent unit-root process under both probability measures.[20] As discussed in Christensen and Rudebusch (2012), this restriction helps improve forecast performance independent of the specification of the remaining elements of K^P.[21] Second, we test the significance of the four parameter restrictions imposed on K^P in the model relative to the corresponding model with an unrestricted K^P matrix.[22] The test results show that the four parameter restrictions are either statistically insignificant or at most borderline significant throughout our sample period. Thus, the B-CR model is flexible enough to capture the relevant information in the

[19]Following Kim and Singleton (2012), the prefix "B-" refers to a shadow-rate model in the spirit of Black (1995).

[20]Due to the unit-root property of the first factor, we can arbitrarily fix its mean at $\theta^P_1 = 0$. We note that, in the model estimation, we handle the nonstationarity of the factor dynamics in equation (9) in the way described in Christensen and Rudebusch (2012).

[21]As described in detail in Bauer et al. (2012), bias-corrected K^P estimates are typically very close to a unit-root process, so we view the imposition of the unit-root restriction as a simple shortcut to overcome small-sample estimation bias.

[22]That is, we test the hypotheses $\kappa^P_{12} = \kappa^P_{13} = \kappa^P_{31} = \kappa^P_{32} = 0$ jointly using a standard likelihood ratio test.

Maturity in months	B-CR model		
	Mean	RMSE	$\widehat{\sigma}_{\varepsilon}(\tau_i)$
3	-2.62	10.28	10.31
6	-0.04	0.17	0.68
12	2.80	6.63	6.64
24	2.94	5.21	5.29
36	0.01	0.74	1.46
60	-5.35	8.18	8.28
84	-6.61	11.05	11.08
120	-3.66	9.32	9.30
180	1.74	4.70	4.69
240	1.49	11.19	11.22
360	-10.23	33.69	33.73
Max log L	417,381.9		

Table 2: **Summary Statistics of the Fitted Errors.**
The mean and root mean squared fitted errors (RMSE) as well as the estimated yield error standard deviations for the B-CR model are shown. All numbers are measured in basis points. The data covers the period from January 2, 1986, to January 2, 2013.

data. Third, and more importantly, CR extend the analysis of Christensen and Rudebusch (2012) to encompass the most recent period with yields near the ZLB. They document that the B-CR model outperforms its standard AFNS model equivalent in that period in terms of forecasting future policy rates and matching the compression in yield volatility. To summarize, the B-CR model has desirable dynamic properties in the current yield environment in addition to enforcing the ZLB.

The estimated model parameters are reported in Table 1. The summary statistics of the model fit in Table 2 indicate a very good fit to the entire maturity range up to 20 years with some deterioration in the fit for the 30-year yield. The good fit of the B-CR model is also apparent in Figure 3, which shows the fitted yield curve as of January 2, 2013, with a comparison to the 11 observed yields on that day. The main weakness of the model on this particular day is a tendency to underestimate the 15- and 20-year yields, which will convert, as discussed below, into a slight overestimation of the market value of the Fed's portfolio.

4.2 Market Value of the Fed's Treasury Securities

To further validate the performance of the B-CR model, we calculate the model-implied value of the 241 Treasury securities that were in the Fed's portfolio as of January 2, 2013 and compare the result to the bond prices downloaded from Bloomberg on that same day.[23] Table 3 reports the total

[23] The Fed's portfolio holdings are available at: http://www.newyorkfed.org/markets/soma/sysopen_accholdings.html. Our calculations use the face value of the Fed's holdings as a straightforward benchmark for comparison to their market value. Alternatively, one could use their historical cost, but taking account of the purchase premiums and

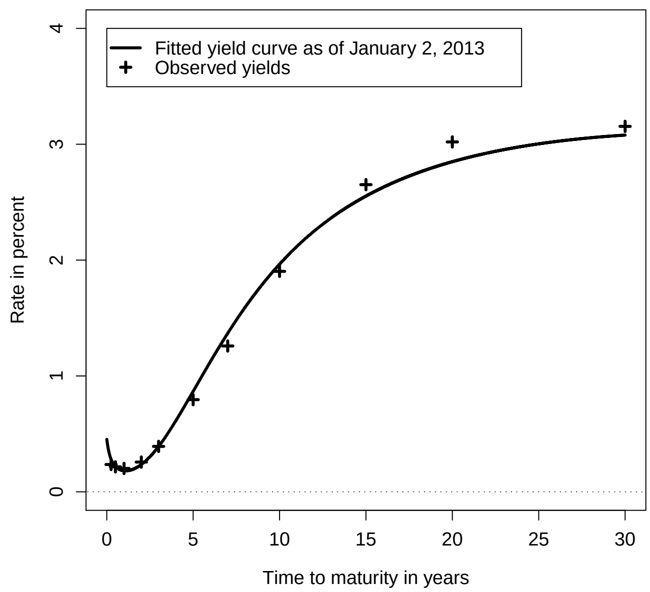

Figure 3: **Fitted Yield Curve from the B-CR Model.**
Illustration of the fitted yield curve as of January 2, 2013, based on the B-CR model. Included are the 11 observed Treasury yields on that day. The data used in the model estimation cover the period from January 2, 1986, to January 2, 2013.

Maturity	#	Official account		Market value as of January 2, 2013			
		Face value	Percent	Bloomberg	Percent	B-CR model	Percent
All	241	1,580	100.00	1,847	100.00	1,871	100.00
3 years or less	93	4	0.25	4	0.24	5	0.24
4-6 years	72	630	39.87	699	37.82	706	37.74
7-10 years	38	577	36.53	663	35.87	667	35.67
11 or more years	38	369	23.35	482	26.07	493	26.34

Table 3: **Value of the Fed's Treasury Securities Portfolio.**
The table reports the distribution of the 241 Treasury securities in the Fed's portfolio as of January 2, 2013 across maturity buckets, using three different valuation methods. The first method is the official account based on the bonds' principal values. The second method is to calculate the market value based on bond prices from Bloomberg. The third method is to calculate the market value based on the estimated B-CR model. The reported bond values are measured in billions of dollars.

value of the Fed's Treasury securities portfolio and its distribution across maturity buckets. The first column shows the number of securities in each maturity bucket. The second and third columns report the official account based on the face value of the securities. The following two columns reflect the market value of the portfolio based on bond prices from Bloomberg. The last two columns contain

discounts (appropriately amortized) for all the Fed's historical acquisitions of Treasuries is a topic for future research.

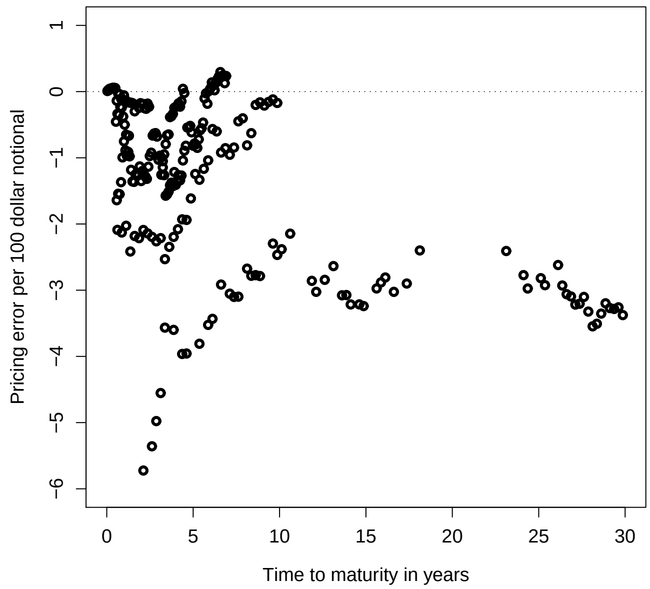

Figure 4: **Pricing Errors across Maturities.**
Illustration of the pricing error in dollars per $100 notional based on the B-CR model relative to Bloomberg data for the 241 Treasury securities in the Fed's portfolio as of January 2, 2013. The pricing errors are sorted by time to maturity in years along the x-axis.

the market value of the portfolio implied by the B-CR model as of January 2, 2013. Please note that the Fed's Treasury securities holdings have a market value that is more than $250 billion above their face value according to both pricing methods.

Contrasting the two pricing methods more directly, the B-CR model's implied portfolio value was $22 billion (or 1.27%) greater than the market value reported by Bloomberg. Overall, we consider this valuation difference to be acceptable for our purposes. Figure 4 presents the distribution of pricing errors across bond maturities. We note that nearly one-third of the bonds (i.e., 69 of the 241 bonds) have fitted prices that are within a quarter-dollar of the Bloomberg price. These bonds were either close to maturity or had been issued fairly recently. The model does slightly overvalue long-maturity bonds on this date, as the fitted long-term yields are a bit too low, as shown in Figure 3. The largest pricing errors are also associated with bonds with high coupon rates, and these tend to very seasoned and illiquid securities. Despite these pricing errors, we emphasize that this exercise is a strong test of the B-CR model as it has to match raw bond price data not directly used in the model estimation. Overall, it appears that the model is able to price medium- and long-term bonds quite accurately.

5 Stress Testing the Fed's Portfolio of Treasuries

In this section, we describe a probability-based stress test of the Fed's portfolio of Treasury securities using distributional forecasts of the yield curve. Our focus is on the likely evolution of the market value of the Fed's portfolio of Treasury securities during a three-year horizon.[24] We use two different approaches to illustrate a range of possible probability-based stress tests. The first approach is based on yield curve projections from the B-CR model. A model-based approach captures the important dynamics of the term structure in a parsimonious, theoretically consistent framework, but relies heavily on the chosen model specification and Gaussian distributional assumption. The second approach is based on the historical distribution of Treasury yield changes as filtered through the B-CR model. The greater reliance on the empirical distribution of yield curve changes provides an alternative set of interest rate scenarios.

5.1 Projections Based on the B-CR Model of the Treasury Yield Curve

Treasury yield curve projections based on the estimated B-CR model allow us to assign probabilities to specific yield curve outcomes. As the yield function in equation (7) is nonlinear in the state variables, we use Monte Carlo simulations to generate the yield curve projections. Specifically, we simulate 10,000 sample paths of the state variables up to three years ahead, each starting from the filtered values at the end of our sample, denoted $\widehat{X}_t = (\widehat{L}_t, \widehat{S}_t, \widehat{C}_t)$. For each projection N months ahead, the simulated state variables are converted into a full yield curve, and we calculate the corresponding portfolio values.

Figure 5 presents a summary of the short rate (i.e., overnight federal funds rate) from the simulated yield curves. In particular, the figure shows the median and the 5th and 95th percentile values for the B-CR model's implied short rate over the first 10 years of the forecast horizon.[25] For the short rate, the median simulated yield remains at the ZLB for the first two years of the forecast horizon and gradually rises to 3 percent at the 10-year horizon. The upper 95th percentile rises more rapidly and reaches 7 percent at the ten-year horizon, while the lower 5th percentile remains at the ZLB throughout. The long-term projections of the federal funds rate from the Blue Chip forecasters are also presented in the figure for the median (or consensus) forecast as well as the averages of the top and bottom 10 forecasts at each horizon. These Blue Chip forecasts fit well within the

[24] To be clear, the Fed values its securities at acquisition cost and registers capital gains and losses only when securities are sold. Such historical-cost accounting is considered appropriate given the Fed's macroeconomic policy objectives and is consistent with the buy-and-hold securities strategy the Fed has traditionally followed. However, the Fed also does report unrealized capital gains and losses on its securities portfolio, which mimics private-sector mark-to-market accounting on holdings of longer-term securities. For an example, see the unaudited financial report of the Federal Reserve Banks for the second quarter of 2013, p. 8, available at: http://www.federalreserve.gov/monetarypolicy/files/quarterly-report-20130630.pdf.

[25] Note that the lines do not represent yield curves from a single simulation run over the forecast horizon; instead, they delineate the distribution of all simulation outcomes at a given point in time.

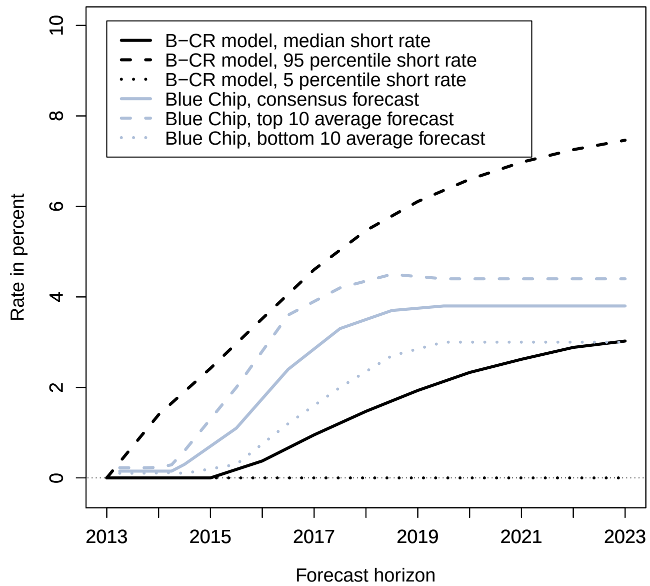

Figure 5: **Comparison of Short Rate Projections.**
The graph presents the median and $[5\%, 95\%]$ range of the fed funds rate from the B-CR model's simulated interest rate scenarios as of January 2, 2013. The graph also shows the consensus federal funds rate forecast as well as the averages of the top and bottom ten forecasts from the Blue Chip Financial Forecasts survey released on December 1, 2012.

range of our simulated projections, which provides evidence that our analysis encompasses current rate expectations. A similar pattern is observed for other maturities generated by the Blue Chip forecasters.

Figure 6 presents the lower percentiles and the median of the projected portfolio value over the next three years. These results clearly show that the projected value of the Fed's Treasury holdings is not likely to fall below face value over the forecast horizon. The median value remains above the face value through 2016, and at most, losses are expected to occur only with a 1 percent probability by 2015.

As shown in Figure 5, the upward trending short rate projection over the forecast horizon explains why the median portfolio value in our projections trend lower as the forecast horizon is increased. Still, when the portfolio is kept fixed as in this exercise, extending the forecast horizon has two effects that go in opposite directions. On one hand, with a longer projection horizon, a wider range of outcomes is likely, and the potential yield changes are larger, in particular in the tail of the distribution. On the other hand, there is a mechanical reduction in the time to maturity on all securities in the portfolio. Bonds with a shorter time to maturity have lower duration and, as a

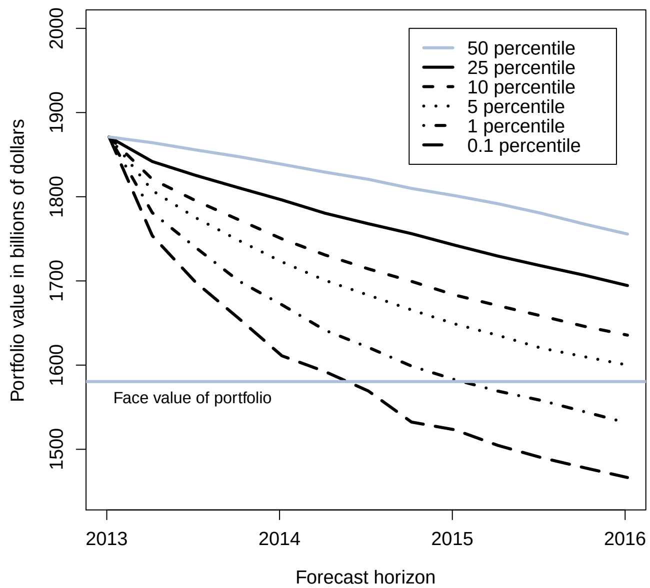

Figure 6: **Model-Based Projected Market Value of the Fed's Treasury Securities.** Illustration of the percentiles ranging from 0.1% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 36 months ahead based on $N = 10{,}000$ Monte Carlo simulations of the B-CR model as described in the text.

consequence, their prices are less sensitive to changes in the interest rate environment. For this exercise, the results in Figure 6 indicate that the former effect dominates at all forecast horizons considered.

To provide a sense of what kind of yield changes it would take to observe the lower tail outcomes for these portfolio values, Figure 7 shows the projected yield curves that produce the 1st, 5th, and 50th (i.e., median) percentiles of portfolio values as of the end of 2015. From the figure, it is clear that the projected yield curve that pushes the value of the Fed's Treasury portfolio below its face value at the 1st percentile is associated with a federal funds rate above 6 percent and a corresponding dramatic increase in the entire yield curve from its level as of January 2, 2013, as shown in Figure 3. This simulated yield curve represents a very different state of monetary policy actions and corresponding economic conditions. On the other hand, it is also clear that the simulated yield curve generating the median outcome reflects only a slight change from the yield curve as of January 2, 2013; i.e., the simulated curve matches the compression in yield volatilities near the ZLB, which works to reduce the magnitude of the yield curve changes and the variation in the model's projected market valuations. Finally, to put the three shown yield curves in the context of the entire distribution of projected yield curves, Figure 7 also shows the band between the 5th and 95th percentile values for the B-CR

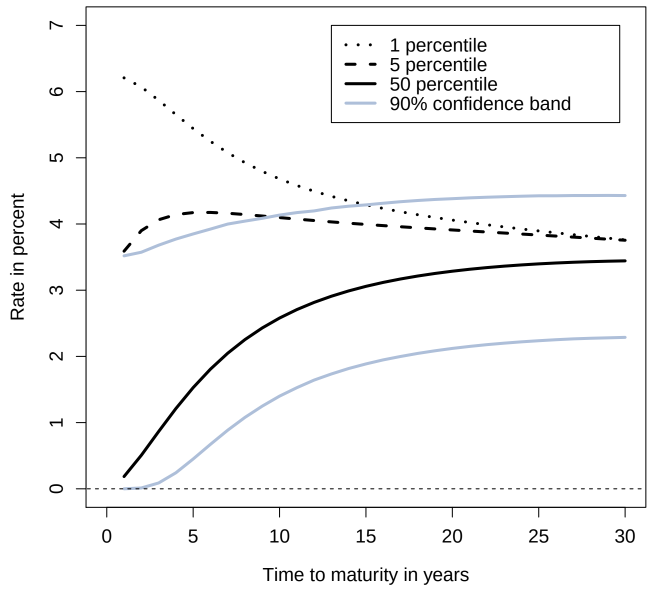

Figure 7: **Projected Yield Curves at the End of 2015.**
Illustration of the projected yield curves that produce the 1 percentile, 5 percentile, and 50 percentile (median) portfolio values as of the end of 2015. Also shown is the $[5\%, 95\%]$ range of yields for each maturity from the B-CR model's simulated interest rate scenarios as of the end of 2015.

model's yield curve simulations as of the end of 2015 across all maturities. Clearly, the tail outcomes for the portfolio values at the end of 2015 are associated with short- and medium-term yields outside of the 90 percent band of simulated outcomes, even though these curves move with the band at longer maturities.

5.2 Projections Based on Filtered Treasury Yield Curve Changes

In this section, we use the filtered yield curve changes in our sample of daily Treasury yields since 1986 to generate Treasury yield curve projections up to three years ahead. Our goal is to use a more empirically-driven distributional assumption for our yield curve projections. One avenue for doing so is simply to generate simulated yield curves using the historical yield curve changes observed for the eleven maturities in the data and their corresponding covariance matrix. However, aside from standard computational concerns, this approach presents very direct challenges to the standard assumption of a symmetric distribution of yield changes, which is less likely in the downward-trending yield environment in our data (as shown in Figure 2), and the ZLB.[26] For our analysis here, we chose

[26]Historical simulation approaches have well-known challenges, as described by Pritsker (2006). Jamshidian and Zhu (1997) present an alternative approach based on principle components analysis and known as "scenario simulation".

to filter the observed yield curve changes through the B-CR model and use the subsequent historical changes in the model's three state variables in the simulation exercise.

The basic idea is to use an empirical distribution of yield changes, instead of a parametric one, to simulate what might happen N months ahead in the current situation. The first step is to estimate the B-CR model on the full sample that ends on January 2, 2013, which filters the data through a consistent model that respects the ZLB. In the second step, for the N-month projection, we use the in-sample changes in the three state variables from the B-CR model to generate all the needed N-month yield curve changes.[27] We denote the total number of such yield curve or, equivalently, state variable changes by m_N. In the third step, we take the estimated state variables as of January 2, 2013, denoted $\widehat{X}_t = (\widehat{L}_t, \widehat{S}_t, \widehat{C}_t)$, and add each of the $i = 1, \dots, m_N$ factor shock constellations, identified in the previous step. This gives us m_N new state variable constellations, each of which has the property that the yield curve shocks happened once before. In the fourth step, we convert each new state variable constellation into a full yield curve using the estimated yield function in equation (7) and then calculate the value of the Fed's Treasury portfolio. In the fifth step, we rank all the estimated portfolio values and focus on the lowest percentiles as well as the median. Finally, this process is repeated for the forecast horizons $N \in \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36\}$ measured in months.[28]

Figure 8 provides the lower percentiles and the median of the projected distribution of the market value of the Fed's Treasury portfolio over the next three years using the filtered empirical distributions. Based on this approach, the declines in the median and the lower 25th percentile are much more modest as the forecast horizon lengthens. In particular, projected portfolio values do not dip below face value even at the 0.1% tail. This milder outcome is due to the fact that the filtered empirical distributions contain realizations that do not extend as far into the tail as the model's assumed distribution.

A key difference between these two approaches to generating yield curve projections is that the latter approach reflects only yield curve changes observed in the past, but does not take the current, very unusual conditions into consideration. The advantage of the model-based approach is that it does condition its projections on where the interest rate environment was at the end of the sample period, even if it still has no more experience with exiting a ZLB period than what is reflected in the data from past tightening episodes.

Our approach takes steps in this direction.

[27] In the exercise, one month is defined as 21 observation dates (or trading days), which is close to the average for the entire sample. Thus, for example, the six-month projections are based on the observed yield curve changes 126 observation dates apart.

[28] The total number of projections for each forecast horizon ranges from 6,675 at the 3-month horizon to 5,989 at the 36-month horizon. This decline reflects a fewer number of available N-month periods as the horizon increases.

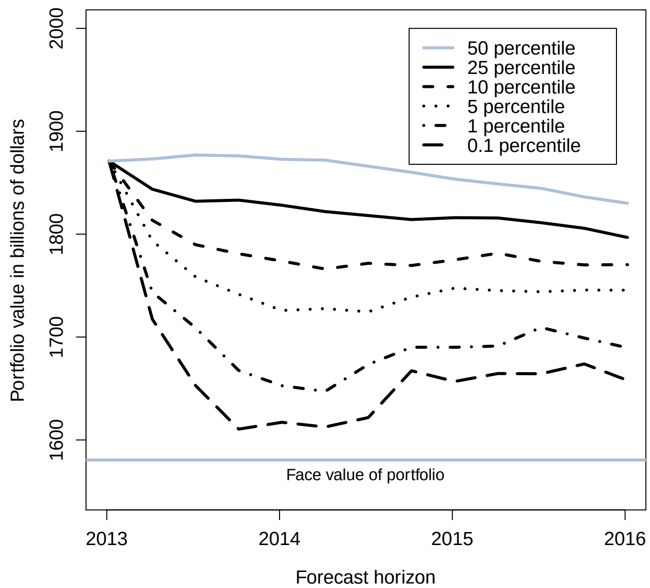

Figure 8: **Projected Market Value of the Fed's Treasury Securities Based on Filtered Yield Changes.**
Illustration of the percentiles ranging from 0.1% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 36 months ahead based on the filtered empirical distributions of yield curve changes as described in the text.

6 Stress Testing the Fed's Income

A key contribution of this paper is to introduce a probability-based approach to policy questions associated with the Federal Reserve's balance sheet, particularly with respect to stress testing scenarios that provide insight regarding the range of plausible adverse outcomes. As discussed earlier, the approach can be applied to questions of balance sheet risk, particularly involving the potential future value of the Fed's Treasury holdings. The approach can also be applied to questions regarding the Fed's income risk; that is, the sensitivity of its net income to alternative interest rate scenarios. Again, the primary concern is that certain interest rate outcomes could lead the Federal Reserve's net income to decline sufficiently that the Fed would halt its remittances to the Treasury Department.

Carpenter et al. (2013) and GHHM directly consider the question of whether the Fed's remittances to the Treasury (i.e., payments of excess interest income beyond expenses) would remain positive under several scenarios. In this section, we address this policy question using our probabilistic, model-based approach to generate yield curve distributions. To translate these projections

Variable	GHHM assumptions (p. 64)	Our assumptions
1. Asset purchases	Continue at current pace through December 2013, then slow to maintenance levels and end in 2014.	Purchases through 2014 match Primary Dealer Survey as of June 2013 and end in 2014.
2. Asset sales	No Treasury sales. MBS sales start in late 2015 and are completed in 2019.	No Treasury or MBS sales.
3. MBS prepayment	Calibrated to current market expectations.	Same.
4. Liabilities	Currency grows at 7% annual rate (2 percentage points above Blue Chip forecast for nominal GDP growth per historical experience); required reserves grow at 4% annual rate.	Same.
5. Interest rates	Driven by Blue Chip forecasts.	B-CR model projections.
6. Fed capital	Grows at 10% annual rate per historical average.	Same.
7. Operating expenses	Grow on historical trend.	Same.

Table 4: **Assumptions Underlying Balance Sheet and Income Projections.**

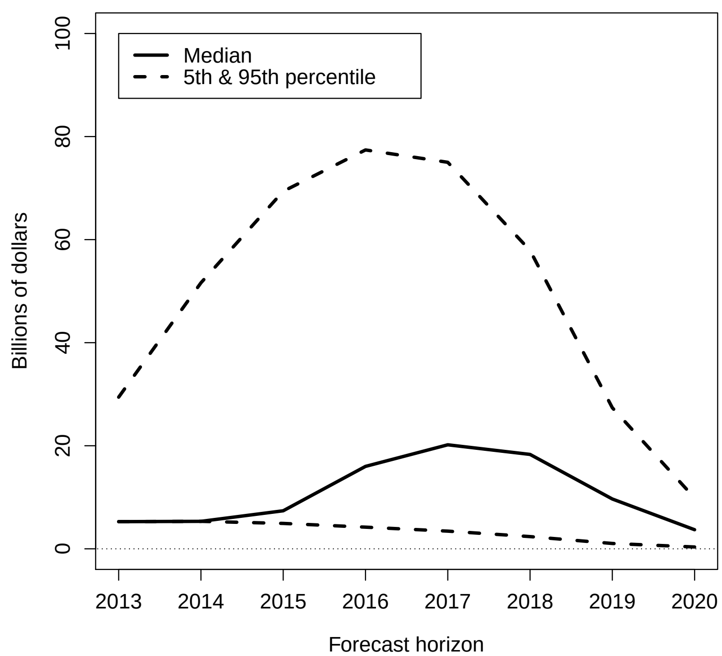

Figure 9: **Projections of Annual Fed Interest Expenses.**
Illustration of the median and the 5th and 95th percentiles of the projected interest expenses based on the CLR baseline scenario combined with $N = 10{,}000$ Monte Carlo simulations of the B-CR model.

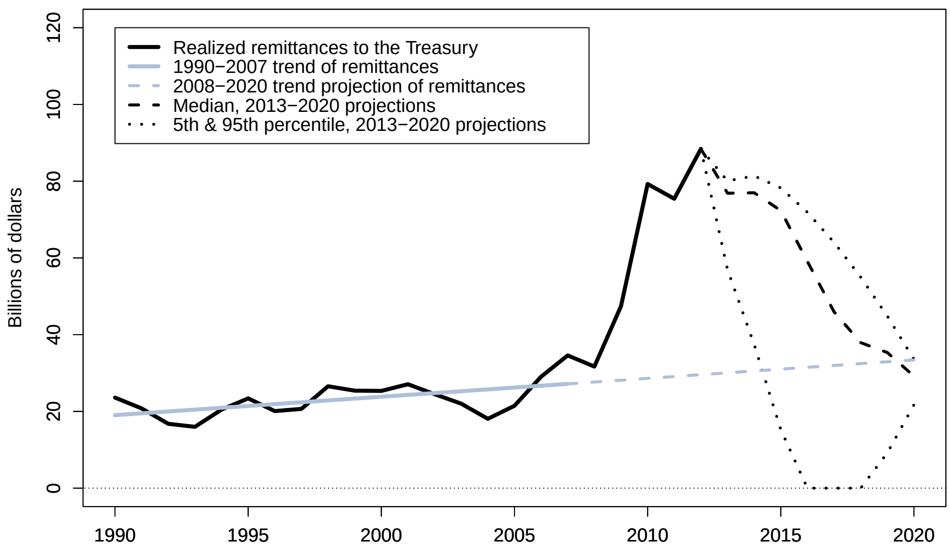

Figure 10: **Actual and Projected Remittances to the U.S. Treasury.**
The solid black line shows the realized remittances to the U.S. Treasury over the period from 1990 to 2012. The solid gray line indicates the simple linear trend of the remittances from 1990 to 2007, while the dashed gray line shows the extrapolation of that trend to the 2008-2020 period. Also shown with are the median (dashed black line) and the 5th and 95th percentiles (dotted black lines) of the projected payments to the U.S. Treasury over the 2013-2020 period based on the CLR baseline scenario combined with N = 10,000 Monte Carlo simulations of the B-CR model.

into remittances, we use the accounting framework of GHHM.[29] In particular, as shown in Table 4, we adopt the GHHM assumptions regarding future MBS prepayment, currency (or liability) growth, capital accretion, operating expenses, and asset re-investment. However, we do update and alter several other assumptions. First, we link the expected path of Federal Reserve asset purchases through 2014 directly to the publicly announced results of the New York Fed Primary Dealer Survey as of June 2013.[30] This path is similar to that assumed in the GHHM baseline, with purchases ending at year-end 2014. Second, we assume that the Fed does not sell any securities through 2020. We make this change in light of the consensus at the June 2013 FOMC meeting not to sell MBS as part of the policy normalization process.[31] A final modification is that we set the path for the interest on excess reserves (IOER) interest rate (i.e., the rate the Fed pays on the reserves that banks hold and its

[29]We greatly appreciate the authors' sharing of their spreadsheets with us for the purposes of this analysis.
[30]The survey is available at: http://www.newyorkfed.org/markets/survey/2013/June_result.pdf
[31]See page 2 of the FOMC meeting minutes at
http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20130619.pdf

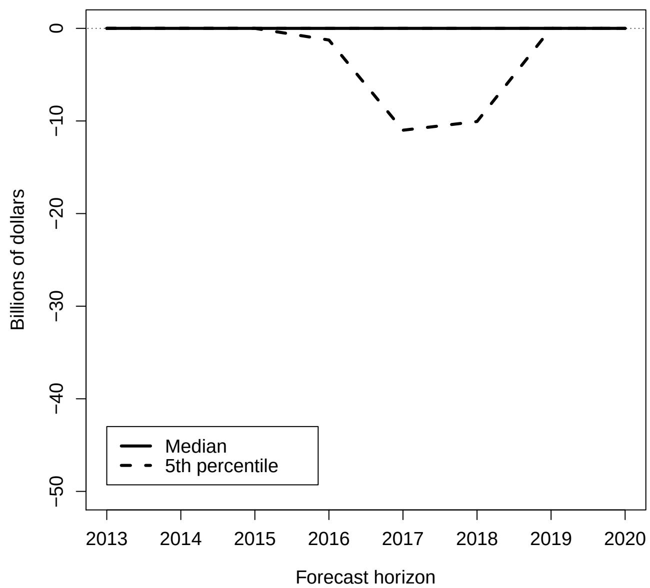

Figure 11: **Projected Cumulative Negative Remittances (i.e., the Deferred Asset).** Illustration of the median and the 5th percentile of projections of the Fed's deferred asset based on the CLR baseline scenario combined with $N = 10{,}000$ Monte Carlo simulations of the B-CR model.

main interest expense) equal to the overnight rate as implied by our yield curve simulations.[32] Given the variation in our simulated short rates, this results in important variation in the Fed's interest expenses going forward. Indeed, as shown in Figure 9, a 90 percent confidence interval for the Fed's annual interest expenses in 2016 ranges from $4 billion to almost $80 billion.

Figure 10 and 11 present the key results of our simulation-based approach using this set of assumptions. Figure 10 presents the projected range of the Fed's positive remittances to the Treasury over the period from 2013 to 2020. The median value (represented as the black, dashed line) declines over the period both as interest income declines based on assets maturing and as interest rate expenses rise due to interest rates increasing from their current low values near the ZLB. However, these declines by the end of the forecast horizon are back in line with the linearly projected remittances (the gray dashed line) based on data from 1990 to 2007. Remittances are projected to end with a 5% probability in the period from 2016 through 2018. Figure 11 shows the corresponding projected range of cumulative negative remittances or, in accounting terms, the deferred asset.[33] As before,

[32]In this exercise, the overnight rate is approximated by an instantaneous short rate given by $r_t = \max\{0.25\%, s_t\}$; i.e., we impose a minimum of 25 basis points for the IOER rate consistent with current practice.

[33]The Fed's policy is to remit all net income to the U.S. Treasury—after expenses, dividends, and additions to capital. If earnings are insufficient to cover these costs, the Fed creates new reserves against a deferred asset, which represents a claim on future earnings and remittances to the Treasury.

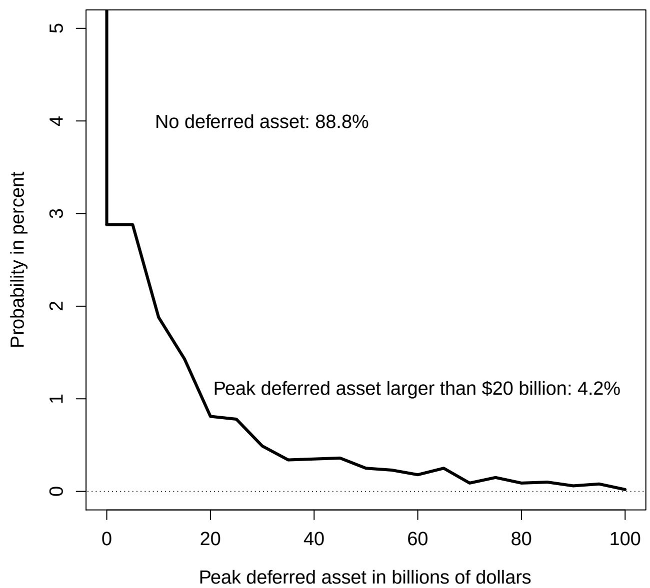

Figure 12: **Simulated Distribution of Maximum Deferred Asset over Forecast Horizon.** The graph depicts the simulated probability distribution function for the maximum deferred asset over the seven-year forecast horizon. Nearly 90 percent of the values are zero.

our results imply zero remittances for only the lower 5th percentile of outcomes. In these cases, the low point is only from 2016 to 2018, and the deferred asset does not exceed $11 billion. Thus, as of year-end 2013, our probabilistic results suggest that the Federal Reserve is unlikely to stop earning net interest income and making Treasury remittances over the next seven years under reasonable assumptions.

To provide further insight into the probability that the Fed may need to use a deferred asset, Figure 12 shows our simulated probability distribution of the maximum deferred asset amount over the forecast horizon up through 2020. The results are heavily left-skewed with 89 percent of the probability mass at zero—that is, no cessation of remittances. For the remainder of the distribution, the probability of observing a maximum deferred asset in the range between $0 and $20 billion is 7 percent, and the probability of a maximum greater than $20 billion is just 4 percent.

As a final exercise, we try to assess the cumulative remittances to the Treasury from the Fed's expansion of its balance sheet starting in 2008. Figure 13 shows our simulated probability distribution for the cumulative remittances from 2008 to 2020 net of the projected linear trend based on remittances from 1990 to 2007. The trend totals nearly $400 billion in cumulative remittances during the 2008-2020 period—about $30 billion per year. This amount, which is a benchmark for the absence of any QE programs or balance sheet expansion by the Fed, is then deducted from the

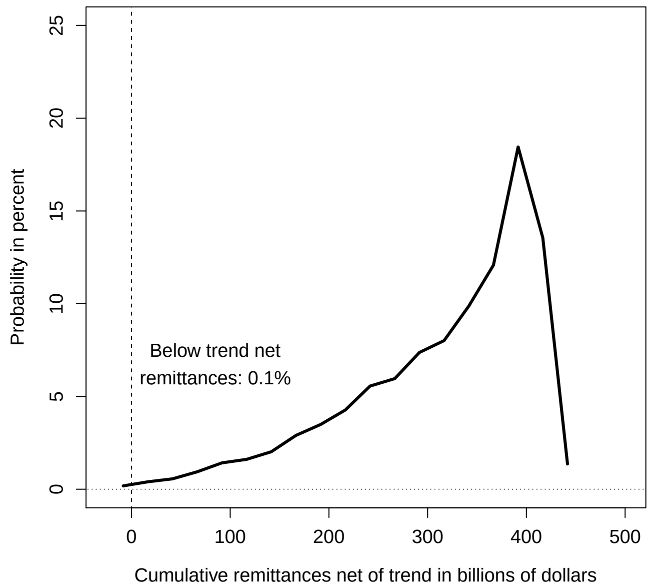

Figure 13: **Simulated Distribution of Cumulative Remittances Net of Trend.**
The graph depicts the simulated probability distribution function for the cumulative remittances by the Fed to the U.S. Treasury from 2008 to 2020 net of the projected trend of remittances from the 1990-2007 period. About 0.1% of the values are below zero.

sum of projected remittances (including the known 2008-2012 remittances of $322.23 billion) in each of the 10,000 simulation runs to produce the distribution. There are two things to note in the figure. First, with a large probability, the expansion of the Fed's balance sheet is likely to generate hundreds of billions of dollars in excess remittances to the U.S. Treasury over the entire 2008-2020 period. Thus, the extraordinary monetary policy initiatives most likely will have provided a direct financial benefit to the Treasury, in addition to any indirect benefits from improved economic outcomes noted in the introduction. Second, the chance that these policies will ultimately produce below-trend net remittances is less than 0.1 percent, as shown in the small amount of probability to the left of the vertical axis.

7 Conclusion

Financial stress tests, including those that have examined the Fed's financial position, are generally based on only a few hand-picked, ad hoc scenarios. This selection introduces a substantial degree of arbitrariness and makes it difficult to judge the plausibility of the results. Our methodological contribution is to introduce a probabilistic structure into a stress testing framework of the Fed's balance sheet and income risks. We argue that attaching likelihoods to adverse outcomes based on interest rate fluctuations is a crucially important addition to the policy debate.

In terms of substantive results, we use a completely model-based approach and a filtered historical approach to generate Treasury yield curve projections. In both cases, the results indicate that in all likelihood the potential losses to the Fed's Treasury securities holdings over the next several years are quite moderate. We also generate more comprehensive projections of the Fed's future income and find a small chance of a temporary halt of the remittances to the Treasury; furthermore, the magnitude of the deferred asset created during this period likely would be modest. In addition, cumulative remittances to the Treasury over the period from 2008 to 2020 are almost surely to be greater than in a counterfactual scenario in which remittances were a linear projection of what they were from 1990 through 2007. In summary, our probability-based scenario analysis provides generally reassuring results regarding questions related to the financial costs of the Fed's balance sheet policy.

Of course, our analysis leaves room for further research. For example, the analysis relies on historical data to estimate forecast distributions, and these may not be completely appropriate for assessing all future circumstances. Also, we do not consider distributional projections of all possible relevant conditioning factors—such as the inflation path or possible asset sales by the Fed. Finally, as noted earlier, unlike for a stress test of a commercial enterprise, it is the political consequences of the financial costs that are of key concern to the Fed. It may be that in those states of the world in which the Fed bears large losses owing to higher long rates, economic growth is also likely to be strong, which may mitigate the political risks. Alternatively, in scenarios with very small Fed remittances to the Treasury because of a high IOER rate, substantial payments of interest on reserves would be paid to large commercial banks, likely boosting political risk. Further research can expand and refine our probabilistic structure in these directions.

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