The Weak Job Recovery in a Macro Model of Search and Recruiting Intensity

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THE WEAK JOB RECOVERY IN A MACRO MODEL OF SEARCH AND RECRUITING INTENSITY

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Abstract. An estimated model with labor search frictions and endogenous variations in search intensity and recruiting intensity does well in explaining the deep recession and weak recovery of the U.S. labor market during and after the Great Recession. The model features a sunk cost of vacancy creation, under which firms rely on adjusting both the number of vacancies and recruiting intensity to respond to aggregate shocks. This stands in contrast to the textbook model with free entry, which implies constant recruiting intensity. Our estimation suggests that fluctuations in search and recruiting intensity driven by productivity and discount factor shocks help substantially bridge the gap between the actual and model-predicted job filling and finding rates.

I. Introduction

The U.S. labor market has improved substantially since the Great Recession. The unemployment rate has declined steadily from its peak of about 10 percent in 2009 to less than 4.5 percent in 2017, accompanied by a steady increase in the job openings rate. However, the recovery of in hiring has been much more subdued in comparison.

These patterns present a puzzle for the standard labor search model. In the standard model, hiring is related to unemployment and job vacancies through a matching function. The matching function implies that the job filling rate—defined as new hires per job vacancy—is inversely related to labor market tightness measured by the vacancy-unemployment

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(v-u) ratio. It also implies that the job finding rate—defined as new job matches per unemployed worker—is positively related to labor market tightness. Thus, when the vacancy rate increases and the unemployment rate falls, as has been the case during the recent recovery, the v-u ratio rises, pushing the job finding rate up and the job filling rate down.

The standard theory fails to predict the deep labor market downturn and the subsequent weak job recovery. As shown in Figure 1, the theory’s predicted paths for the job filling rate and the job finding rate tracked the data fairly well up to early 2009. Since then, the theory-implied series both lie significantly above those observed in the data. The reason for these discrepancies is that the actual hiring rate fell more during the Great Recession and, during the recovery, it did not increase as much as predicted by the theory with the standard matching function.1,2

To understand the forces behind this weak job recovery, we develop and estimate a DSGE framework that incorporates endogenous variations in two additional margins of labor-market adjustment: search intensity and recruiting intensity. We examine the quantitative importance of cyclical fluctuations in search and recruiting intensity for the job filling and finding rates in our estimated general equilibrium model.

We make three contributions to the literature. First, we develop a DSGE model that allows for endogenous variations in recruiting intensity because vacancy creation incurs a sunk cost. In the textbook model with recruiting intensity (Pissarides, 2000), vacancy creation

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1 The standard matching function takes the form $m_t = \mu u_t^\alpha v_t^{1-\alpha}$, where $m_t$ denotes new job matches, $u_t$ and $v_t$ denote unemployment and job vacancies, respectively, $\alpha$ measures the elasticity of matching with respect to unemployment, and $\mu$ is a scale parameter that captures the average matching efficiency. With this matching function, the job filling rate is given by $q^v_t = \frac{m_t}{v_t} = \mu \left( \frac{u_t}{v_t} \right)^{-\alpha}$ and the job finding rate is given by $q^u_t = \frac{m_t}{u_t} = \mu \left( \frac{u_t}{v_t} \right)^{1-\alpha}$. The job filling and finding rates implied by the standard matching function shown in Figure 1 are calculated by using the observed data on job openings (from JOLTS) and the unemployment rate (from BLS), with $\alpha = 0.5$. By construction, the job filling rate and the job finding rate implied by the standard matching function are perfectly negatively correlated. To highlight the changes of the job filling and finding rates implied by the model relative to those in the data, we transform each series into log units and normalize each series by setting the first observation to zero (so that all subsequent observations are log-deviations from the first observation). Under this normalization, the scale parameter $\mu$ in the matching function becomes irrelevant.

2 Alternative measures of the job finding rate are also used in the literature. One such alternative is based on the transition rate from unemployment to employment reported in the Current Population Survey (CPS). This CPS measure is highly correlated with our measure, with a correlation of about 0.96 over the JOLTS sample period from December 2000 to July 2017. Another measure of the job finding rate considered by Shimer (2005) is calculated based on the level and duration of unemployment. However, the Shimer measure is not well-suited for the post-2008 period because of large swings in the distribution of unemployment duration (Rothstein, 2011; Elsby et al., 2011).
is costless (i.e., there is free entry). When macroeconomic conditions change, firms vary the number of vacancies—which are costless to create or destroy—to meet new hiring needs and choose the level of recruiting intensity to minimize the cost of posting each vacancy. The free-entry assumption implies counterfactually that vacancies are a flow variable that can be adjusted continuously. Furthermore, as shown by Pissarides (2000), free-entry also implies that recruiting intensity is independent of macroeconomic fluctuations. However, in our model where vacancy creation incurs a sunk cost, vacancies become a slow-moving state variable, and firms adjust both the number of new vacancies and recruiting intensity in response to aggregate shocks. Thus, our model generates not only plausible vacancy dynamics, but also business-cycle variations in recruiting intensity.\(^3\)

In our model, the cyclical properties of recruiting intensity are a priori ambiguous. Optimal recruiting intensity results from a tradeoff between the marginal costs of recruiting efforts and the marginal benefit of raising the probability of filling a job opening, thus obtaining the net value of a filled position. Although by filling a position the firm gains the value of an employment match, it also loses the value of an open vacancy, which is non-zero in equilibrium because of costly entry. Because the match value and the vacancy value both decline in a recession, the net value of filling a vacancy is ambiguous. Depending on model parameters, recruiting intensity may be pro- or counter-cyclical.

Our second contribution is to quantitatively examine the importance of cyclical fluctuations in search and recruiting intensity for the job filling and finding rates. We do so by estimating the model using Bayesian methods, fitting three monthly time series data of the U.S. labor market: the unemployment rate, the job vacancy rate, and a measure of search intensity.

The model estimation shows that recruiting intensity is procyclical and positively correlated with aggregate hiring; and it interacts with cyclical variations in search intensity to amplify labor market dynamics. With sharp declines in both search intensity and recruiting intensity during the Great Recession and with weak recoveries in these intensive margins following the recession, our model predicts a sharp downturn and weak recovery in hiring, and thus much lower job filling and finding rates than those implied by the standard matching function without these intensive margins. Furthermore, our model’s predicted recovery paths of the job filling and finding rates are roughly in line with the data, as shown in Figure 2.\(^3\)

We are not the first to introduce fixed costs of vacancy creation. Elsby et al. (2015) consider a labor search model with fixed vacancy creation costs to examine the qualitative implications of recruiting intensity for shifts in the Beveridge curve. Fujita and Ramey (2007) introduce a fixed cost of creating vacancies in a search model to account for the sluggish responses of employment and the v-u ratio following productivity shocks, although they do not model recruiting intensity. See Coles and Moghaddasi Kelishomi (2011) for a detailed discussion of the implications of costly entry for the labor market dynamics.
Remarkably, the periods for which our DSGE model outperforms the standard model coincide with those during which the predicted paths of the job filling and finding rates from the standard matching function diverged from those in the data. In contrast, the standard matching function does equally well as our DSGE model prior to the depth of the Great Recession in 2009, as shown in Figure 2. The improvement in the fit of our model to the data stems from fluctuations in the search and recruiting intensity. As Figure 3 shows, our measure of search intensity is procyclical and falls sharply during the Great Recession. It has rebounded sharply after the recession, but it has not fully recovered to the pre-recession level. The sharp decline in search intensity during the Great Recession and its weak recovery contributed to the sharp downturn in hiring in the recession and the subdued recovery. In addition, shocks in our model that led to declines in search intensity also led to declines in recruiting intensity, which further depress hiring, contribution to the weak job recovery.

In our estimated model, forecast error variance decompositions show that the labor market dynamics are driven mainly by a technology shock and a discount factor shock, and to a lesser extent, also by a job separation shock. As we discuss below, our estimated persistence and volatility of these structural shocks are in line with the calibration in the literature, such as the studies by Shimer (2005) and Hall (2017).

Our model shares the Shimer (2005) puzzle of the standard DMP model: unless real wages are rigid, it is difficult to generate the observed large volatilities in unemployment and vacancies. Unsurprisingly, with real wage rigidities assumed in our model, technology shocks are an important source of labor market fluctuations. It is also unsurprising that a job separation shock plays a relatively minor role in the model, because it generates counterfactually positive correlations between unemployment and vacancies (Shimer, 2005). What is new is that a discount factor shock plays a quantitatively important role in driving labor market fluctuations, mainly through its impact on the present values of a job match and an open vacancy for firms, and also on the employment surplus for job seekers. By driving changes in these present values, a discount factor shock contributes to a significant fluctuations in unemployment, vacancies, search and recruiting intensity, and hiring. This finding is in line with the literature (Hall, 2017).  

Our third contribution is to show that, despite our macro approach, the aggregate correlation between hiring and recruiting intensity obtained from our estimated DSGE model is positive and relatively high, broadly in line with estimates derived from micro data. In particular, Davis et al. (2013) construct a measure of recruiting intensity based on the Job

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4 Albuquerque et al. (2016) argue that discount factor shocks are important for asset pricing models because they give rise to a valuation risk that allows the model to account for the volatile asset price fluctuations and the weak correlations between stock returns and fundamentals.
Openings and Labor Turnover Survey (JOLTS) at the establishment level. They present evidence that employers rely not only on the number of vacancies, but also heavily on other instruments for hiring, which they call recruiting intensity. Under the assumption that their microeconomic estimates hold at the macro level, Davis et al. (2013) document a strong correlation between recruiting intensity and hiring at the aggregate level. Our work complements theirs by providing a macro perspective on the positive relationship between recruiting intensity and hiring.

Despite a better fit to the data along several important dimensions, our model has several shortcomings. First, our model misses the timing of the trough in hiring during the Great Recession by about one year, although the depth of the predicted fall is roughly in line with that observed in the data. This lagged response is partly driven by the persistent declines in search intensity and recruiting intensity during the downturn. Second, we emphasize that the model’s improved performance is partly driven by the procyclicality of the search intensity measure that we use in estimating the model. When we estimate the model without using the search intensity series, the predicted job filling and job finding rates deviate more from the data and come closer to the predictions from the standard matching function.

II. OTHER RELATED LITERATURE

Our paper contributes to the recent theoretical literature on cyclical variations in recruiting intensity. For example, Kaas and Kircher (2015) study a competitive search environment with heterogeneous firms facing a recruiting cost function that is convex in the number of open vacancies. In their model, since the marginal cost of recruiting increases with the number of vacancies, growing firms do not rely solely on vacancy posting to attract workers; they also rely on varying their posted wage offers. Gavazza et al. (2014) assume a recruiting cost function similar to that in Kaas and Kircher (2015) and study the importance of financial shocks for shifting the Beveridge curve through their impact on firms’ recruiting intensity. We add to this literature by introducing an alternative departure from the textbook search model. In particular, we relax the free entry condition to allow for business cycle fluctuations in recruiting intensity. The resulting tractability of our framework has the added advantage of making it straightforward to estimate the model to fit time-series data using standard techniques.

Motivated by the observed patterns in labor adjustments at the establishment level, Cooper et al. (2007) estimate a labor search model with non-convexities in vacancy posting costs and firing costs using simulated methods of moments to match aggregate unemployment, vacancies, and hours. Our work is also motivated by micro-level facts about search intensity and recruiting intensity. We use these micro-level facts to discipline an aggregate
DSGE model and we estimate the model to understand aggregate fluctuations in the labor market.

Lubik (2009) estimate a macro model with the standard labor search frictions, and he finds that the model relies heavily on exogenous shocks to matching efficiency to fit time series data of unemployment and vacancies. Our model enriches the standard model with search and recruiting intensity and thus relies on endogenous responses of search and recruiting intensity (instead of exogenous variations in matching efficiency) to explain the observed labor market dynamics.

Our paper is also related to recent work on screening, an implicit form of recruiting intensity. For instance, Ravenna and Walsh (2012) examine the effects of screening on the magnitude and persistence of unemployment following adverse technology shocks in a search model with heterogeneous workers and endogenous job destruction. Relatedly, Sedláček (2014) empirically studies the fluctuations in matching efficiency and proposes countercyclical changes in hiring standards as an underlying force.

By examining the interaction between search and recruiting intensity, our work also complements the analysis of Gomme and Lkhagvasuren (2015), who study how the addition of search intensity and directed search can amplify the responses of the unemployment and vacancy rates following productivity shocks, although their model is not estimated to fit time-series data.

III. THE MODEL WITH SEARCH AND RECRUITING INTENSITY

In this section, we present a DSGE model with search frictions in the labor market. To study the underlying forces behind the weak job recovery from the Great Recession, we introduce endogenous intensive-margin adjustments in the matching technology. First, we introduce recruiting intensity as an additional margin of adjustments for firms. Second, we introduce sunk costs for vacancy creation. In the standard textbook search model, recruiting intensity does not depend on macroeconomic conditions because free-entry implies that an unfilled vacancy has zero value, so that firms rely on varying the number of job vacancies to respond to shocks instead of adjusting recruiting intensity (Pissarides, 2000). With sunk costs for vacancy creation, as we show, firms respond to shocks by adjusting both the number of vacancies (i.e., the extensive margin) and recruiting intensity (i.e., the intensive margin). In addition, having sunk costs in the model generate more interesting dynamics for job vacancies, as shown by Fujita and Ramey (2007); Coles and Moghaddasi Kelishomi (2011); Elsby et al. (2015). Third, we also introduce search intensity as an additional adjustment margin for unemployed workers.
The economy is populated by a continuum of infinitely lived and identical households with a unit measure. The representative household consists of a continuum of worker members. The household owns a continuum of firms, each of which uses one worker to produce a consumption good. In each period, a fraction of the workers are unemployed and they search for a job. Searching workers also choose optimally the levels of search effort. New vacancy creation incurs an entry cost. Posting existing vacancies also incurs a per-period fixed cost. The number of successful matches are produced with a matching technology that transforms efficiency units of searching workers and vacancies into an employment relation. Job matches are exogenously separated each period. Real wages are determined by Nash bargaining between a searching worker and a hiring firm. The government finances transfer payments to unemployed workers by lump-sum taxes.

III.1. The Labor Market. In the beginning of period $t$, there are $N_{t-1}$ employed workers. A fraction $\delta_t$ of job matches are separated in each period. We assume that the job separation rate $\delta_t$ is stochastic and follows the stationary process

$$\ln \delta_t = (1 - \rho_\delta) \ln \bar{\delta} + \rho_\delta \ln \delta_{t-1} + \varepsilon_{\delta t}. \quad (1)$$

In this shock process, $\rho_\delta$ is the persistence parameter and the term $\varepsilon_{\delta t}$ is an i.i.d. normal process with a mean of zero and a standard deviation of $\sigma_\delta$. The term $\bar{\delta}$ denoted the steady state rate of job separation.

Workers in a separated match go into the unemployment pool. Following Blanchard and Galí (2010), we assume full labor force participation, with the size of the labor force normalized on one. Thus, the number of unemployed workers searching for jobs is given by

$$u_t = 1 - (1 - \delta_t)N_{t-1}. \quad (2)$$

After observing aggregate shocks, new vacancies are created. Following Fujita and Ramey (2007) and Coles and Moghaddasi Kelishomi (2011), we assume that creating new vacancies incurs a sunk cost. Newly created vacancies add to the existing stock of vacancies carried over from the previous period. We follow Fujita and Ramey (2007) and assume that a vacant position becomes obsolete at a constant rate of $\rho^o$. A fraction of the open vacancies in the previous period are filled with job matches, and those filled vacancies subtract from the stock of vacancies carried over into the current period provided that they are not obsolete. In addition, newly separated jobs also add to the stock of vacancies if those positions are not obsolete.

Denote by $q_t^v$ the job probability of filling a vacancy in period $t$, and by $n_t$ the number of newly created vacancies. The law of motion for the stock of job vacancies $v_t$ is described by

$$v_t = (1 - q_{t-1}^v)(1 - \rho^o)v_{t-1} + (\delta_t - \rho^o)N_{t-1} + n_t. \quad (3)$$
The searching workers and firms with job vacancies form new job matches based on the matching function

\[ m_t = \mu (s_t u_t)^\alpha (a_t v_{t-1})^{1-\alpha}, \tag{4} \]

where \( m_t \) denotes the number of successful matches, \( s_t \) denotes search intensity, \( a_t \) denotes recruiting intensity (or advertising), the parameter \( \mu \) represents the scale of matching efficiency, and the parameter \( \alpha \in (0, 1) \) is the elasticity of job matches with respect to efficiency units of searching workers.

The probability that an open vacancy is filled with a searching worker is given by

\[ q^v_t = \frac{m_t}{v_t}. \tag{5} \]

The probability that an unemployed and searching worker finds a job is given by

\[ q^u_t = \frac{m_t}{u_t}. \tag{6} \]

New job matches add to the employment pool so that aggregate employment evolves according to the law of motion

\[ N_t = (1 - \delta_t) N_{t-1} + m_t. \tag{7} \]

At the end of the period \( t \), the searching workers who failed to find a job match remains unemployed. The unemployment rate is given by

\[ U_t = u_t - m_t = 1 - N_t. \tag{8} \]

III.2. The households. There is a continuum of infinitely lived and identical households with a unit measure. The representative household has a utility function given by

\[ E \sum_{t=0}^{\infty} \beta^t \Theta_t (\ln C_t - \chi N_t), \tag{9} \]

where \( E [\cdot] \) is an expectation operator, \( C_t \) denotes consumption, and \( N_t \) denotes the fraction of household members who are employed. The parameter \( \beta \in (0, 1) \) denotes the subjective discount factor, and the term \( \Theta_t \) denotes an exogenous shifter to the subjective discount factor.

We assume that the discount factor shock \( \theta_t \equiv \frac{\Theta_t}{\Theta_{t-1}} \) follows the stationary stochastic process

\[ \ln \theta_t = \rho_{\theta} \ln \theta_{t-1} + \varepsilon_{\theta_t}. \tag{10} \]

In this shock process, \( \rho_{\theta} \) is the persistence parameter and the term \( \varepsilon_{\theta_t} \) is an i.i.d. normal process with a mean of zero and a standard deviation of \( \sigma_{\theta} \). Here, we have implicitly assumed that the mean value of \( \theta \) is one.
The representative household chooses consumption $C_t$, saving $B_t$, and search intensity $s_t$ to maximize the utility function in (9) subject to the sequence of budget constraints

$$C_t + \frac{B_t}{r_t} = B_{t-1} + w_t N_t + \phi(1 - N_t) - u_t h(s_t) + d_t - T_t, \quad \forall t \geq 0, \tag{11}$$

where $B_t$ denotes the household’s holdings of a risk-free bond, $r_t$ denotes the gross real interest rate, $w_t$ denotes the real wage rate, $h(s_t)$ denotes the resource cost of search efforts, $d_t$ denotes the household’s share of firm profits, and $T_t$ denotes lump-sum taxes. The parameter $\phi$ measures the flow benefits of unemployment.

We follow Pissarides (2000) and assume that the cost of searching is an increasing and convex function of the level of search effort $s_i$ for an individual unemployed worker $i$. In particular, the search cost function satisfies the conditions

$$h_{it} = h(s_{it}), \quad h'(s_{it}) > 0, h''(s_{it}) \geq 0, \tag{12}$$

where $h_{it}$ is the search cost in consumption units and applies only for unemployed members of the household.

Raising search intensity, while costly, may increase the job finding probability. For each efficiency unit of searching workers supplied, there will be $m/(su)$ new matches formed. For a worker who supplies $s_{it}$ units of search effort, the probability of finding a job is

$$q_{it} = \frac{s_{it}}{s_t u_t} m_t, \tag{13}$$

where $s$ (without the subscript $i$) denotes the average search intensity. The household takes the economy-wide variables $s$, $u$, and $m$ as given when choosing the level of search intensity $s_i$. A marginal effect of raising search intensity on the job finding probability is given by

$$\frac{\partial q_{it}}{\partial s_i} = \frac{m_t}{s_t u_t} = \frac{q_{it}^u}{s_t}, \tag{14}$$

which depends only on aggregate economic conditions.

As we show in the Appendix B, the household’s optimal search intensity decision (in a symmetric equilibrium) is given by

$$h'(s_t) = \frac{q_{it}^u}{s_t} \left[ w_t - \phi - \frac{\chi}{\Lambda_t} + \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \delta_{t+1})(1 - q_{t+1}^u S_{t+1}^H) \right], \tag{15}$$

where $S_t^H$ is the household’s surplus of employment (relative to unemployment). Thus, at the optimal level of search intensity, the marginal cost of searching equals the marginal benefit, which is the increased odds of finding a job multiplied by the net benefit of employment, including both the contemporaneous net flow benefits and the continuation value of employment.
The employment surplus $S^H_t$ itself, as we show in the appendix, satisfies the Bellman equation

$$S^H_t = w_t - \phi - \frac{\chi}{\Lambda_t} + \frac{h(s_t)}{1 - q^u_t} + E_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \delta_{t+1})(1 - q^u_{t+1}) S^H_{t+1}, \tag{16}$$

where $\Lambda_t = \frac{1}{C_t}$ denotes the marginal utility of consumption.

The employment surplus has a straightforward economic interpretation. If the household adds a new worker in period $t$, then the current-period gain would be wage income net of the opportunity costs of working, including unemployment compensations and the disutility of working. The contemporaneous benefit also includes saved search cost because it reduces the pool of job seekers, the measure of which is $1 - q^u_t$ at the end of period $t$. In addition, the household also enjoys the continuation value of employment if the employment relation continues. Having an extra worker today adds to the employment pool tomorrow (if the employment relation survives job separation); however, adding a worker today would also reduce the pool of searching workers tomorrow, a fraction $q^u_{t+1}$ of whom would be able to find jobs. Thus, the marginal effect of adding a new worker in period $t$ on employment in period $t$ is given by $(1 - \delta_{t+1})(1 - q^u_{t+1})$, resulting in the effective continuation value of employment shown in equation (16).

We also show in the appendix that the household’s optimizing consumption/saving decision implies the intertemporal Euler equation

$$1 = E_t \beta \theta_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} r_t. \tag{17}$$

III.3. The firms. A firm can produce the final consumption goods only if it successfully matches with a worker. The production function for firm $j$ with one worker is given by

$$y_{jt} = Z_t,$$

where $y_{jt}$ is output and $Z_t$ is an aggregate technology shock. The technology shock follows the stochastic process

$$\ln Z_t = (1 - \rho_z) \ln \bar{Z} + \rho_z \ln Z_{t-1} + \varepsilon_{zt}. \tag{18}$$

The parameter $\rho_z \in (-1, 1)$ measures the persistence of the technology shock. The term $\varepsilon_{zt}$ is an i.i.d. normal process with a zero mean and a finite variance of $\sigma^2_z$. The term $\bar{Z}$ is the steady-state level of the technology shock.\(^5\)

If a firm $j$ finds a match, it obtains a flow profit in the current period after paying the worker. In the next period, if the match survives (with probability $1 - \delta_{t+1}$), the firm

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\(^5\)The model can be easily extended to allow for trend growth. We do not present that version of the model to simplify presentation.
continues; if the match breaks down, the firm posts a new job vacancy at a flow cost of $\kappa_{jt}$, with the value $J_{j,t+1}^V$. The firm’s match value therefore satisfies the Bellman equation

$$J_{jt}^F = Z_t - w_t + E_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} \left\{ (1 - \delta_{t+1}) J_{j,t+1}^F + (1 - \rho^o) \delta_{t+1} J_{j,t+1}^V \right\}. \quad (19)$$

Here, the value function is discounted by the representative household’s marginal utility because all firms are owned by the household.

Following Coles and Moghaddasi Kelishomi (2011), we assume that vacancy creation incurs a non-negative entry cost of $x$ drawn from an i.i.d. distribution $F(\cdot)$. A new vacancy is created if and only if $x \leq J_t^V$, or equivalently, if and only if its net value is non-negative. Thus, the number of new vacancies $n_t$ equal to $F(J_t^V)$—the cumulative density of entry costs at the value of a vacancy. With appropriate assumptions about the functional form of the distribution function $F(\cdot)$, the number of new vacancies created is related to the value of vacancies through the equation

$$n_t = \eta (J_t^V)^{\xi}, \quad (20)$$

where $\eta$ is a scale parameter and $\xi$ measures the elasticity of new vacancies with respect to the value of the vacancy. The special case with $\xi = \infty$ corresponds to the standard DMP model with free entry (i.e., $J_t^V = 0$). In general, a smaller value of $\xi$ would imply a less elastic response of new vacancies to changes in aggregate conditions (through changes in the value of vacancies).

The flow cost of posting a vacancy is an increasing and convex function of the level of advertising. In particular, we follow Pissarides (2000) and assume that

$$\kappa_{jt} = \kappa(a_{jt}), \quad \kappa'(\cdot) > 0, \quad \kappa''(\cdot) \geq 0, \quad (21)$$

where $a_{jt}$ is firm $j$’s level of advertising.

Advertising efforts also affect the probability of filling a vacancy. For each efficiency unit of vacancy supplied, there will be $m/av$ new matches formed. Thus, for a firm that supplies $a_{jt}$ units of advertising efforts, the probability of filling a vacancy is

$$q^v(a_{jt}) = \frac{a_{jt}}{a_t v_t} m_t, \quad (22)$$

where $a_t$ is the average advertising efforts by firms.

If the vacancy is filled (with probability $q^v_{jt}$), the firm obtains the value of a match $J_{jt}^F$. If the vacancy remains unfilled, then the firm goes into the next period and obtains the continuation value of the vacancy. Thus, the value of an open vacancy is given by

$$J_{jt}^V = -\kappa(a_{jt}) + q^v(a_{jt}) J_{jt}^F + (1 - \rho^o)(1 - q^v(a_{jt})) E_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} J_{j,t+1}^V. \quad (23)$$
The firm chooses advertising efforts \( a_{jt} \) to maximize the value of vacancy \( J_{jt}^V \). The optimal level of advertising is given by the first order condition

\[
\kappa'(a_{jt}) = \left( J_{jt}^F - (1 - \rho^o) E_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} J_{jt+1}^V \right),
\]

(24)

where, from (22), we have

\[
\frac{\partial q^v(a_{jt})}{\partial a_{jt}} = m_t a_t v_t = \frac{q_t^v}{a_t}.
\]

(25)

We concentrate on a symmetric equilibrium in which all firms make identical choices of the level of advertising. Thus, in equilibrium, we have \( a_{jt} = a_t \). In such a symmetric equilibrium, the optimizing advertising decision (24) can be written as

\[
\kappa'(a_t) = \frac{q_t^v}{a_t} \left[ J_t^F - (1 - \rho^o) E_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} J_{jt+1}^V \right].
\]

(26)

If the firm raises advertising effort, it incurs a marginal cost of \( \kappa'(a_t) \). The marginal benefit of raising advertising efforts is that, by increasing the probability of forming a job match, the firm obtains the match value \( J_t^F \), although it loses the continuation value of the vacancy, which represents the opportunity cost of filling the vacancy.

The optimizing recruiting intensity (advertising) decision equation (26) reveals that the cyclical properties of recruiting intensity are a priori ambiguous. In a recession, the job filling rate rises, and firms respond by exerting more recruiting efforts. However, in a recession, the match value \( J_t^F \) and the vacancy value \( J_t^V \) both decline, so that changes in the net value of filling a vacancy—the difference between \( J_t^F \) and \( J_t^V \)—are in general ambiguous. Depending on model parameters, recruiting intensity can be pro- or counter-cyclical.

In the special case with free entry, the value of vacancy would be driven down to zero. Thus, equation (23) reduces to

\[
\kappa(a_t) = q_t^v J_t^F.
\]

(27)

Furthermore, the optimal advertising choice (26) reduces to

\[
\kappa'(a_t) = \frac{q_t^v}{a_t} J_t^F.
\]

(28)

These two equations together imply that

\[
\frac{\kappa'(a_t) a_t}{\kappa(a_t)} = 1.
\]

(29)

In this case, the level of advertising is chosen such that the elasticity of the cost of advertising equals 1 and it thus is invariant to macroeconomic conditions, as in the textbook model of Pissarides (2000).

This special case highlights the importance of incorporating sunk costs of vacancy creation. Absent any vacancy creation cost, as in the textbook models, firms can freely adjust vacancies to respond to changes in macroeconomic conditions and choose the level of advertising to
minimize the cost of each vacancy. In this case, the optimal level of advertising is independent of market variables. In contrast, if vacancy creation is costly, as we assume in our model, firms would rely on adjusting both the level of advertising and the number of vacancies to respond to changes in macroeconomic conditions.

III.4. The Nash bargaining wage. Firms and workers bargain over wages. The Nash bargaining problem is given by

$$\max_{w_t} \left( S^H_t \right)^b (J^F_t - J^V_t)^{1-b},$$  \hspace{1cm} (30)

where $b \in (0,1)$ represents the bargaining weight for workers. The first-order condition implies that

$$b (J^F_t - J^V_t) \frac{\partial S^H_t}{\partial w_t} + (1-b) S^H_t \frac{\partial (J^F_t - J^V_t)}{\partial w_t} = 0,$$  \hspace{1cm} (31)

where, from the household surplus equation (16), we have $\frac{\partial S^H_t}{\partial w_t} = 1$; and from the firm’s value function (19), we have $\frac{\partial (J^F_t - J^V_t)}{\partial w_t} = -1$.

Define the total surplus as

$$S_t = J^F_t - J^V_t + S^H_t.$$  \hspace{1cm} (32)

Then the bargaining solution is given by

$$J^F_t - J^V_t = (1-b)S_t, \quad S^H_t = bS_t.$$  \hspace{1cm} (33)

The bargaining outcome implies that firm surplus is a constant fraction $1-b$ of the total surplus $S_t$ and the household surplus is a fraction $b$ of the total surplus.

The bargaining solution (33) and the expression for household surplus in equation (16) together imply that the Nash bargaining wage $w^N_t$ satisfies the Bellman equation

$$\frac{b}{1-b} (J^F_t - J^V_t) = w^N_t - \phi - \frac{h_t(s_t)}{\lambda_t} + E_{t+1} \frac{\beta_1 (J^F_{t+1} - J^V_{t+1})}{\lambda_{t+1}} \left[ (1 - \delta_{t+1}) (1 - q^u_{t+1}) \frac{b}{1-b} (J^F_{t+1} - J^V_{t+1}) \right].$$  \hspace{1cm} (34)

III.5. Wage Rigidity. In general, however, equilibrium real wage may be different from the Nash bargaining solution. Hall (2005a) and Shimer (2005) point out that real wage rigidities are important for generating empirically plausible volatilities of vacancies and unemployment relative to the volatility of labor productivity.\footnote{The recent literature identifies several sources of real wage rigidities. For example, Christiano et al. (2015) report that an estimated DSGE model with wages determined by an alternating offer bargaining game in the spirit of Hall and Milgrom (2008) fits the data better than the standard model with Nash bargaining. Liu et al. (2016) show that, in an estimated DSGE model with labor search frictions and collateral constraints, endogenous real wage inertia can be obtained conditional on a housing demand shock even if wages are determined from the standard Nash bargaining game.} We follow the literature and consider real

...
wage rigidity. We assume that the real wage is a geometrically weighted average of the Nash bargaining wage and the realized wage rate in the previous period. That is,

$$w_t = w_{t-1}^\gamma (w_t^N)^{1-\gamma}, \quad (35)$$

where $\gamma \in (0, 1)$ represents the degree of real wage rigidity.\(^7\)

III.6. **Government policy.** The government finances unemployment benefit payments $\phi$ for unemployed workers through lump-sum taxes. We assume that the government balances the budget in each period so that

$$\phi(1 - N_t) = T_t. \quad (36)$$

III.7. **Search equilibrium.** In a search equilibrium, the markets for bonds and goods all clear. Since the aggregate supply of bond is zero, the bond market-clearing condition implies that

$$B_t = 0. \quad (37)$$

Aggregate output $Y_t$ is related to employment through the aggregate production function

$$Y_t = Z_t N_t. \quad (38)$$

Goods market clearing requires that real spendings on consumption, search efforts, recruiting efforts, and vacancy creation equal to aggregate output. This requirement yields that the aggregate resource constraint

$$C_t + h(s_t)u_t + \kappa(a_t)v_t + \int_0^{J_V} xdF(x) = Y_t, \quad (39)$$

where the last term on the left-hand side of the equation corresponds to the aggregate cost of creating job vacancies. Under our distribution assumption of the vacancy creation cost, the cumulative density function of $x$ is given by $F(x) = \eta x^\xi$. Thus, the aggregate cost of vacancy creation is given by $\int_0^{J_V} xdF(x) = \frac{\eta^\xi}{1+\xi} (J_V)^{1+\xi}$. Using the relation between the number of job vacancies and the value of an open vacancy in equation (20), the aggregate resource cost for vacancy creation can be written as $\frac{\xi}{1+\xi} n_t J_V^\xi$.

\(^7\)We have examined other wage rules as those in Blanchard and Galí (2010) and we find that our results do not depend on the particular form of the wage rule.
IV. Empirical strategies

We solve the DSGE model by log-linearizing the equilibrium conditions around the deterministic steady state.\(^8\) We calibrate a subset of the parameters to match steady-state observations and estimate the remaining structural parameters and shock processes to fit the U.S. time series data.

We begin with parameterizing the vacancy cost function \(\kappa(a)\) and search cost function \(h(s)\). We assume that these cost functions are both quadratic and take the forms

\[
\kappa(a_t) = \kappa_0 + \kappa_1(a_t - \bar{a}) + \frac{\kappa_2}{2}(a_t - \bar{a})^2, \\
h(s_t) = h_1(s_t - \bar{s}) + \frac{h_2}{2}(s_t - \bar{s})^2,
\]

where we normalize the steady-state levels of recruiting intensity and search intensity so that \(\bar{a} = 1\) and \(\bar{s} = 1\).\(^9\) We also assume that the search cost is zero in the steady state.

We first calibrate a subset of model parameters using steady-state restrictions. These parameters include \(\beta\), the subjective discount factor; \(\chi\), the dis-utility of working; \(\alpha\), the elasticity of matching with respect to searching workers; \(\mu\), the matching efficiency; \(\delta\), the average job separation rate; \(\rho^\phi\), the vacancy obsolescence rate; \(\phi\), the flow unemployment benefits; \(b\), the Nash bargaining weight; \(\kappa_0\) and \(\kappa_1\), the intercept and the slope of the vacancy cost function; \(h_1\), the slope parameter of the search cost function; \(\gamma\), the parameter that measures real wage rigidities; and \(\xi\), the elasticity parameter of vacancy creation.

We estimate the remaining structural and shock parameters using Bayesian methods to fit the time-series data of unemployment, vacancies, and search intensity. The structural parameters to be estimated include \(K \equiv \frac{1}{\eta}\), the scale of the vacancy-creation cost function; \(\kappa_2\), the curvature of the vacancy-posting cost function; and \(h_2\), the curvature of the search cost function. The shock parameters include \(\rho_z\) and \(\sigma_z\), the persistence and the standard deviation of the technology shock; \(\rho_\theta\) and \(\sigma_\theta\), the persistence and the standard deviation of the discount factor shock, and \(\rho_\delta\) and \(\sigma_\delta\), the persistence and the standard deviation of the job separation shock.

IV.1. Calibration. The calibrated values of the model parameters are summarized in Table 1.

\(^8\)Details of the equilibrium conditions, the steady state, and the log-linearized system are available in the online appendix at http://www.frbsf.org/economic-research/files/wp2016-09_appendix.pdf.

\(^9\)The quadratic form of the search cost function is supported by the empirical evidence provided by Yashiv (2000) and Christensen et al. (2005).
We consider a monthly model. Thus, we set $\beta = 0.9967$, so that the model implies a steady-state annualized real interest rate of about 4 percent. We set $\alpha = 0.5$ following the literature (Blanchard and Gali, 2010; Gertler and Trigari, 2009). We set the steady-state job separation rate to $\bar{\delta} = 0.034$ per month, consistent with the JOLTS data for the period from December 2000 to April 2015. Following Hall and Milgrom (2008), we set $\phi = 0.25$ so that the unemployment benefit is about 25 percent of normal earnings. We set $b = 0.5$ following the literature. In our baseline experiment, we follow the literature and set $\xi = 1$, as in Fujita and Ramey (2007) and Coles and Moghaddasi Kelishomi (2011).\footnote{When we fix $\xi$ at 0.5 or 2, the quantitative results are similar to our benchmark model. However, if $\xi$ is fixed at a larger value of 10, the model performs less well, because in that case, the model becomes closer to one with free entry. For details, see the online appendix available at \url{http://www.frbsf.org/economic-research/files/wp2016-09_appendix.pdf}.}

We set a value for the steady-state level of vacancy cost $\kappa_0$ so that the total cost of posting vacancies is about 1 percent of gross output. To assign a value of $\kappa_0$ then requires knowledge of the steady-state number of vacancies $v$ and the steady-state level of output $Y$.

Given the job separation rate of $\bar{\delta} = 0.034$ and our calibrated steady-state unemployment rate of $U = 0.055$, we obtain the steady-state hiring rate of $m = \bar{\delta}(1 - U) = 0.0321$. We then calibrate the steady-state value of $v$ such that the model’s steady-state job filling rate $q^v = \frac{m}{v}$ matches that in the data. In particular, we match the daily job filling rate of 0.05 estimated by Davis et al. (2013) using establishment-level JOLTS data. This implies a monthly job filling rate of $q^v = 0.6415$.$^{11}$

Given the calibrated values of $m$ and $q^v$, we obtain the steady-state vacancy rate of $v = \frac{m}{q^v} = 0.05$ To obtain a value for $Y$, we use the aggregate production function that $Y = ZN$ and normalize the level of technology such that $Z = 1$. This procedure yields a calibrated value of $\kappa_0 = 0.1887$. We set $\kappa_1 = 0.2072$ so that the steady-state recruiting intensity is $\bar{a} = 1$. We set $h_1 = 0.1123$ so that the steady-state search intensity is $\bar{s} = 1$.

Given the steady-state values of $m$, $u$, and $v$, we use the matching function to obtain an average matching efficiency of $\mu = 0.4864$. We calibrate the vacancy obsolescence rate to $\rho_o = 0.0353$, so that the steady-state ratio of newly created vacancies to employment in the model equals 0.036, the same ratio as that estimated by Davis et al. (2013) based on establishment-level JOLTS data.

To obtain a value for $\chi$, we solve the steady-state system so that $\chi$ is consistent with an unemployment rate of 5.5 percent. The process results in $\chi = 0.666$. Finally, as in the standard DMP model, our model relies on real wage rigidities to generate the observed large

\footnote{Assuming that one month consists of 20 business days. We can then infer the monthly job filling rate $q^v$ from the daily rate $f = 0.05$ by using the relation $q^v = f + f(1 - f) + f(1 - f)^2 + \cdots + f(1 - f)^{19} = 1 - (1 - f)^{20} = 0.6415$.}
fluctuations in labor market variables (Shimer, 2005). We set the wage rigidity parameter to $\gamma = 0.95$, which lies at the high end of the literature (Hall, 2005b).

IV.2. Estimation. We now describe our data and estimation approach.

IV.2.1. Data and measurement. We fit the DSGE model to three monthly time-series data of the U.S. labor market: the unemployment rate, the job vacancy rate, and a measure of search intensity. The sample covers the period from July 1967 to July 2017.

The unemployment rate in the data (denoted by $U_t^{\text{data}}$) corresponds to the end-of-period unemployment rate in the model $U_t$. We demean the unemployment rate data (in log units) and relate it to our model variable according to

$$\ln(U_t^{\text{data}}) - \ln(U_t^{\text{data}}) = \hat{U}_t,$$

(42)

where $U_t^{\text{data}}$ denotes the sample average of the unemployment rate in the data and $\hat{U}_t$ denotes the log-deviations of the unemployment rate in the model from its steady-state value.

Similarly, we relate the demeaned vacancy rate data (also in log units) and relate it to the model variable according to the relation

$$\ln(v_t^{\text{data}}) - \ln(v_t^{\text{data}}) = \hat{v}_t,$$

(43)

where $v_t^{\text{data}}$ denotes the sample average of the vacancy rate data and $\hat{v}_t$ denotes the log-deviations of the vacancy rate in the model from its steady-state value.

Our measure of search intensity is constructed by Davis (2011). He combines mean unemployment spells from the Current Population Survey (CPS) and regression results from Krueger and Mueller (2011), who find that search intensity declines as the duration of unemployment increases in high-frequency longitudinal data. In particular, Davis (2011) postulates that

$$s_t = A - Bd_t,$$

(44)

where $s_t$ is search intensity and $d_t$ is the mean unemployment duration (in weeks). He then constructs the search intensity index by setting $A = 122.30$ and $B = 0.90$ after adjusting for some potential biases in the regression results obtained by Krueger and Mueller (2011).

We follow the same methodology as Davis (2011) in constructing a search intensity series, with the exception that we use the median unemployment duration in weeks instead of the mean.\footnote{See the discussions of this methodology in Davis (2011), p.66.}

\footnote{In the Great Recession period, some workers experienced extremely long spells of unemployment, contributing to the sharp increase in the mean duration of unemployment for this period. For this reason, we use the median unemployment duration to construct our search intensity series, and we believe this median measure better reflects the underlying factors that influence an individual job seeker’s search efforts than...}
Figure 3 displays this measure of aggregate search intensity. Clearly, search intensity is procyclical, rising in booms and falling in recessions. In the Great Recession and its aftermath, search intensity declined substantially, as the duration of unemployment lengthened. We discuss in Section V.3.2 the importance of using the time series data of search intensity to discipline the estimation of our DSGE model and to bring the model’s predictions of labor market variables closer to those in the data.

IV.2.2. Prior distributions and posterior estimates. The prior and posterior distributions of the estimated parameters from our benchmark model are displayed in Table 2 (Panel A).

The priors of the structural parameters $K$, $\kappa_2$, and $h_2$ each follows the gamma distribution. We assume that the prior mean of $K$ is 5 with a standard deviation of 1. The prior distribution of $\kappa_2$ and $h_2$ each has a mean of 1 and a standard deviation of 0.1. For the shock parameters, we follow the literature and assume that the priors of $\rho_z$, $\rho_\theta$, and $\rho_\delta$ each follows the beta distribution and the priors of $\sigma_z$, $\sigma_\theta$, and $\sigma_\delta$ each follows an inverse gamma distribution.

The posterior estimates and the 90% confidence interval for the posterior distributions are displayed in the last three columns of Table 2. The posterior mean estimate of vacancy creation cost parameter is $K = 5.29$, implying a modest steady-state share of vacancy creation costs of about 0.32 percent of aggregate output. The posterior mean estimates of curvature parameters of the vacancy posting cost function and of the search cost functions are $\kappa_2 = 1.42$ and $h_2 = 1.47$, respectively. The 90% confidence intervals suggest that these curvature parameters are significantly different from their priors and thus, the data are informative on these structural parameters.

Our estimation of the shock parameters suggests that technology shocks are less persistent and less volatile than discount factor shocks and job separation shocks. The AR(1) coefficient of the technology shock (0.64) is also somewhat smaller than that calibrated in the literature (e.g., Shimer (2005)). The standard deviation of the innovation to the technology shock (0.0156) is more in line with the calibration literature.

The discount factor shock is estimated to be persistent, with a monthly AR(1) parameter of 0.92 and a standard deviation of the innovation of 0.0157. These parameters are broadly in line with the literature. For example, Hall (2017) estimates a Markov-chain process of the discount factor shock using the U.S. stock market data. His estimation implies a monthly persistence parameter of 0.99 and a standard deviation of the innovation of about 0.02.\textsuperscript{14}

\textsuperscript{14}These values are calculated based on Figure 5 in Hall (2017) and the descriptions in Section V.A. Under his estimation, a one standard deviation shock leads to an increase in the discount rate from 8.3 to 10.2 on the mean. The time series of median unemployment durations from the BLS is available from July 1967 and on.
The job separation shock is also estimated to be persistent, with a monthly AR(1) coefficient of 0.93. It’s standard deviation is larger than the other shocks, at 0.078. These estimated values are also broadly in line with the standard calibration in the literature. For example, Shimer (2005) calibrates the quarterly persistence parameter of the job separation shock to 0.733, corresponding to a monthly value of 0.733^{1/3} = 0.90. He also calibrates the standard deviation of the job separation shock to 0.075, which is remarkably similar to our estimation.

V. Economic implications

We now discuss the economic mechanism through which search and recruiting intensity help amplify and propagate the impact of the shocks on labor market dynamics in our model. We also examine the estimated model’s quantitative performance for explaining the sharp labor-market downturns during the Great Recession and the subsequent weak job recovery.

V.1. The model’s transmission mechanism. To help understand the role of different shocks in driving labor market dynamics in our model and the model’s propagation mechanism, we examine forecast error variance decompositions and impulse response functions.

V.1.1. Forecast error variance decompositions. Table 3 displays the conditional forecast error variance decompositions for several key labor market variables and aggregate output. We focus on the time horizons between 12 months and 48 months, which capture cyclical fluctuations in these aggregate variables.

Consistent with the intuition provided by Shimer (2005), real wage rigidities in our model allow technology shocks to play an important role in driving labor market fluctuations. The variance decomposition results suggest that technology shocks account for 60-80% of the cyclical fluctuations in unemployment, vacancies, hiring, search intensity, and the job filling and finding rates, accounting for 60-80% of the variances of these variables. Technology shocks are also the primarily driving force of recruiting intensity, accounting for about 90% of its fluctuations.

A discount factor shock can directly affect the present values of a job match, an open vacancy, and the employment surplus for a job seeker. Thus, it is also potentially important for explaining the observed labor market fluctuations (Hall, 2017). Quantitatively, our variance decomposition shows that a discount factor shock contributes to a significant fraction—impact, which gradually returns to the ergodic mean of 8.3 in about 48 months. The monthly persistence parameter \( \rho_\theta = 0.99 \) is inferred from this information, since \( \theta_{48} = \rho_\theta^4 \theta_1 \), where \( \theta_{48} = 8.3 \) and \( \theta_1 = 10.2 \). The initial size of the increase in the discount rate from 8.3 to 10.2 represents a one standard deviation increase in \( \theta_1 \), so we have \( std(\theta) = (10.2 - 8.3)/8.3 = 0.2289 \). Given the AR(1) coefficient of 0.9956, the standard deviation of the innovation is given by \( \sigma_\theta = std(\theta) \sqrt{1 - \rho_\theta^2} = 0.0214 \).
about 20-30\%—of fluctuations in unemployment, vacancies, search intensity, and the job filling and finding rates. It also contributes to fluctuations in recruiting intensity and hiring, albeit to a lesser extent (about 10-15\%).

A job separations shock also contributes modestly to fluctuations in unemployment, vacancies, and hiring, accounting for about 10-20\% of their variances. The shock is unimportant for the other variables. As noted by Shimer (2005), a job separation shock generates a counterfactually positive correlation between unemployment and vacancies. Accordingly, in our estimated model, this shock plays a relatively minor role.

V.1.2. *Impulse responses.* To further understand the model’s transmission mechanism for each shock, we examine impulse responses. Figure 5 shows the impulse responses of several key labor market variables to a one standard deviation negative technology shock. The decline in productivity reduces the value of new job matches. Firms respond by reducing hiring and vacancy postings. These responses lead to a drop in workers’ job finding rate and an increase in the unemployment rate.

Unlike the standard model with free entry, our model with costly vacancy creation implies that the value of an unfilled vacancy is non-zero. Thus, as we have alluded to in the introduction, vacancies become a state variable that evolves slowly over time according to the law of motion in Equation (3). This gives rise to persistent dynamics in vacancies, as shown in Figure 5. The initial drop in the stock of vacancies is attributable to declines in newly created vacancies: since the shock reduces the value of an open vacancy \( J^V_t \), firms have less incentive to create new vacancies.

The figure also shows that a contractionary technology shock reduces both search intensity and recruiting intensity. The household’s optimizing decisions for search intensity (Eq. (15)) show that search intensity increases with the job finding probability and the employment value, which is proportional to the match surplus from Nash bargaining. Since the technology shock lowers both the job finding rate and the match surplus, it reduces search intensity as well.

Recruiting intensity falls following the negative technology shock partly because the expected value of a job match declines. This can be seen from the optimizing decision for recruiting intensity in Equation (26), which shows that recruiting intensity increases with both the job filling probability and the value of a new job match \( J^F \) relative to the value of an unfilled vacancy \( J^V \). However, because the technology shock reduces both \( J^F \) and \( J^V \), the net effect on recruiting intensity can be ambiguous. Under our estimated parameters, the net surplus falls following a contractionary technology shock. Thus, recruiting intensity falls as well.
Declines in search and recruiting intensity counteract the effects of the rise in unemployment on hiring and reinforce the effects of the drop in vacancies. The net effect leads to a fall in hiring. With both hiring and vacancies declining following the negative technology shock, the response of the job filling rate—which is the ratio of hires to vacancies—can be ambiguous. Under our estimation, the job filling rate rises initially and then falls below steady state before returning to it.

As search intensity and recruiting intensity both decline, the technology shock leads to a decline in the measured matching efficiency and thus an outward shift of the Beveridge curve. The measured matching efficiency here is defined as

$$
\Omega_t = \mu_s \alpha t^{\alpha - 1}.
$$

(45)

Although there are no exogenous changes in true matching efficiency (i.e., if $\mu$ is constant), measured matching efficiency ($\Omega$) in our model still fluctuates with endogenous variations in search and recruiting intensity.

The variance decompositions in Table 3 suggest that, in addition to technology shocks, shocks to the discount factor and the job separation rate in our model also contribute to the observed labor market fluctuations. We now turn to discussing the impulse responses following those shocks.

Figures 6 shows the impulse responses of labor market variables following a one standard deviation negative discount factor shock. The shock lowers the continuation value of a job match, leading to declines in hiring. Since the shock lowers the employment surplus, search intensity declines. Recruiting intensity also falls because the decline in the present value of a new job match outweighs that in the value of an open vacancy. Firms respond to the drop in vacancy value by creating and posting fewer vacancies. The job filling rate increases, as the decline in vacancies outweighs the fall in hiring.

Figures 7 shows the impulse responses following a positive shock to the job separation rate. With a higher rate of job separation, the unemployment rate rises. At the same time, separated jobs add to the stock of vacancies, so that the vacancy rate rises as well. Since both $u$ and $v$ increase, the hiring rate also rises. Equilibrium adjustments of hiring also depend on the responses of the intensive margins. The job separation shock reduces both the match value $J^F$ and the vacancy value $J^V$, rendering the net effect on recruiting intensity ambiguous. Under our estimation, recruiting intensity edges down following the separation shock. On the other hand, since firms reduce the number of newly created vacancies in response to the shock, households choose to reduce search intensity slightly. On net, the job separation shock leads to only small responses of these intensive margins. It follows that the job filling and finding rates are both driven mostly by the labor market tightness ($v - u$
ratio). Since the shock leads to an increase in both $v$ and $u$, its impact on the job filling and finding rates are small.

V.2. The Great Recession and the weak job recovery. The impulse responses show that search and recruiting intensity are both procyclical in our estimated model. We now show that procyclical fluctuations in these intensive margins significantly improve the model’s ability to quantitatively capture the observed labor market dynamics during the Great Recession and the subsequent recovery.

Figure 1 shows that, in the data (the blue solid lines), the job filling rate rose sharply and the job finding rate declined sharply during the Great Recession, and they both recovered gradually to pre-recession levels. These patterns are consistent with a sharp downturn in hiring in the recession and a relatively subdued recovery thereafter.

The standard model without search and recruiting intensity has difficulties in replicating these observations. As Figure 1 shows (the red dashed lines), the predicted job filling rate and the job finding rate from the standard matching function diverged from the data since early 2009, and they both stayed persistently above the data after the recession. Thus, according to the standard matching function, hiring should not have declined as sharply as that actually occurred in the recession, and the recovery should have been stronger than observed.

In comparison, our model with endogenous search intensity and recruiting intensity does not share these counterfactual predictions from the standard model. As shown in Figure 2 (the black dashed and dotted lines), the predicted job filling and finding rates from our benchmark model track the actual data closely throughout the recession and recovery periods.

To get a quantitative sense of the goodness of fit of our model relative to the standard matching function, we consider the root mean squared errors (RMSE) for the variables of interest in each model. We calculate the RMSEs based on demeaned series in the data and in each model from the sample period starting from 2001. As shown in Table 4, the standard matching function’s predicted job filling rate has an RMSE of 0.1086 relative to the data. In contrast, the RMSE of the job filling rate predicted from our estimated benchmark model is 0.0466, less than half of that implied by the standard matching function. Our model with search and recruiting intensity also improves the fit for the job finding rate relative to the standard matching function, with a similar magnitude of improvement.

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15The series shown in Figure 2 are normalized based on the demeaned series, so that the first observation of each series is indexed to 0. We do not calculate the RMSEs based on the normalized series, but instead, we use the “raw” demeaned series.
It is important to also note that the improved model performance is not trivially attributable to the introduction of search intensity in the matching function. For instance, Table 4 reports the RMSEs of the standard matching function modified to also include our measure of search intensity. The table shows that augmenting the standard matching function with the observed search intensity by itself does not improve the fit to the data significantly; the RMSEs from this augmented matching function are marginally smaller than those implied by the standard matching function (0.0914 vs. 0.1086), both substantially exceeding those predicted from our DSGE model (0.0466).

Thus, search intensity in our model entails important interactions with recruiting intensity that help better account for the movements in the job filling and job finding rates since 2007.

One concern is that the improvements in fit during the Great Recession and its aftermath may come at the cost of a worsening performance in periods prior to the Great Recession. Figure 4 shows that this is not the case. While we focus on the post-2001 period in Figure 2 because of the availability of JOLTS data, we still estimate our model over a longer period (starting from July 1967) that captures several business cycles. This allows a comparison of the fluctuations in the job filling and job finding rates from our benchmark model and from the standard matching function for the periods prior to 2001. In contrast to the Great Recession and its aftermath, Figure 4 shows that the two models’ predictions track each other remarkably well before the Great Recession.

The improved ability of our benchmark model to track the observed paths of the job filling and finding rates since the Great Recession suggests that the model may also be able to account for the observed weak recovery in hiring. This is only partly true, however. Figure 9 shows the paths of hiring predicted from our benchmark model and that in the actual data, with the sample starting in December 2000, which is the beginning of the JOLTS sample. The figure shows that, while the magnitude of declines in hiring predicted from our model during the Great Recession is roughly in line with that in the data, our model misses the timing of the trough by about one year. Nonetheless, the correlation between the model’s predicted hiring rate and the actual hiring rate is still positive and large, at about 0.55, which is remarkable given that our model is not fitted to the observed time-series data of hiring.

Our model also implies that recruiting intensity is positively correlated with the hiring rate, as found by Davis et al. (2013), despite clear differences in the approaches. Davis et al. (2013) construct a measure of recruiting intensity based on establishment-level data. They show that recruiting intensity delivers a better-fitting Beveridge curve and accounts for a

\[ \mu \left( \frac{m_s}{v} \right)^\alpha \]

and the job finding rate is

\[ \mu_s \left( \frac{m}{w} \right)^{1-\alpha}, \]

where the variables and the parameters are defined earlier.
large share of fluctuations in aggregate hires. They further impute an aggregate relation between recruiting intensity and the hiring rate based on their estimated microeconomic relations. They show that this aggregate measure of recruiting intensity is highly correlated with the aggregate hiring rate, with a sample correlation of about 0.82.

We have followed a very different approach to obtaining an empirical measure of recruiting intensity \( (a_t) \) in our estimated macro model. To assess the cyclical behaviors of our measure of recruiting intensity, we calculated the sample correlation between the model-based time series of recruiting intensity and the hiring rate. We obtained a correlation of 0.48, which is lower than that reported by Davis et al. (2013), though still significantly positive. We view our result as strengthening the argument by Davis et al. (2013) that recruiting intensity is procyclical and it plays an important role in explaining cyclical fluctuations in aggregate hires.

V.3. **The importance of search and recruiting intensity.** To understand the importance of cyclical variations in search and recruiting intensity, we conduct two sets of counterfactual experiments. In the first experiment, we consider a counterfactual model with both search intensity and recruiting intensity held constant. In the second experiment, we re-estimate the benchmark model to fit time-series data on unemployment and vacancies only, without using data on search intensity.

V.3.1. *A counterfactual model with constant search and recruiting intensity.* We first consider a counterfactual model with no cyclical fluctuations in search and recruiting intensity. In particular, we keep the parameters and the shock processes the same as in the estimated benchmark model, but we force the search intensity and recruiting intensity to stay constant at their steady-state levels. We calculate the impulse responses in this counterfactual model and compare them to those obtained in our benchmark model. We focus on the effects of a negative technology shock, partly because our variance decompositions show that technology shocks are the most important shock that drive the labor market dynamics in our model.

Figure 8 shows the impulse responses to a negative technology shock in the counterfactual model with constant search and recruiting intensity (the blue dashed lines), along with those in the benchmark model (the black solid lines).

The figure highlights that a negative technology shock leads to a more muted decline in hiring and a smaller increase in unemployment in the counterfactual model than those predicted by the benchmark model with endogenous intensive-margin adjustments. Without variations in search and recruiting intensity, hiring is solely determined by the number of

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\[17\] The qualitatively results under a discount factor shock or a job separation shock are similar. See the online appendix for details.
job-seekers and the number of job vacancies according to the matching function. Since the number of job seekers entering the matching function is predetermined and the stock of job vacancies is a slow-moving state variable, the responses of hiring in the counterfactual model are more muted than in the benchmark model. With a smaller response of hiring, declines in vacancies imply a stronger increase in the job filling rate following the negative technology shock. The counterfactual model also generates smaller declines in the job finding rate than does the benchmark model.

Overall, Figure 8 shows that endogenous adjustments in search and recruiting intensity to changes in macroeconomic conditions help amplify the responses of the labor market variables.

V.3.2. The importance of using information from search intensity data. In estimating our benchmark model, we have used three time series data: the unemployment rate, the job vacancy rate, and a measure of search intensity. We followed Davis (2011) and constructed a time series of search intensity based on the median unemployment duration. The resulting search intensity series is procyclical, as shown in Figure 3. The procyclical behavior of search intensity is consistent with the textbook model (Pissarides, 2000).18

Yet, the empirical literature is not conclusive about whether search intensity is procyclical. For example, in an influential study, Shimer (2004) argues that search intensity is countercyclical based on cross-sectional data of the average number of search methods used by job seekers observed in the Current Population Survey (CPS). Mukoyama et al. (2014) combine information from the CPS data and the American Time Use Survey (ATUS) and obtain similar results.

On the other side of the debate, Tumen (2014) criticizes the interpretation of the cyclical behavior of search intensity measured by cross-sectional average number of search methods in the CPS. He emphasizes that these cross-sectional measures are likely to suffer from a composition bias if a job seeker with stronger labor-market attachment also uses more search methods, since the share of job seekers with stronger labor-market attachment increases during a recession. When this composition bias is corrected, Tumen (2014) finds that search intensity is procyclical. Gomme and Lkhagvasuren (2015) make a similar argument about the composition bias. They use merged data from the ATUS and the CPS to study cyclical

---

18The measure of search intensity that we use, which is the same measure used by Davis (2011), has an advantage in that it is constructed based on estimated parameters using longitudinal data that track unemployed workers' amount of time spent for job searching as well as the number of weeks they have been unemployed. A drawback of this method is that it is based on answers from interviews conducted over a 24-week period during the fall of 2009 and winter of 2010, so the estimation uses a relatively short time-series dimension.
variations in search intensity. They find that, when the composition bias is corrected, the evidence suggests procyclical search intensity.\footnote{Mueller (2017) shows that the pool of unemployed shifts to high-wage workers in recessions. If high-wage workers search more intensely, this can lead to a substantial composition bias. Faberman and Kudlyak (2016) also discuss the implications of composition bias in measuring search intensity. They report that long-term unemployed job seekers tend to exert more efforts throughout their search process, reflecting that long-term unemployed individuals have stronger labor-market attachment.}

Given this debate, we assess the robustness of our findings by fitting our model to the observed unemployment rate and the vacancy rate only. Under this alternative estimation, we do not use information of search intensity in the data. In this alternative estimation, we keep the priors of the parameters the same as in our benchmark estimation.

The posterior estimation results are shown in Table 2 (Panel B). Compared to the benchmark estimation, this alternative estimation without using information from search intensity data results in a few notable changes in the posterior distributions of the structural parameters and the shock parameters. In particular, the technology and discount factor shocks are now much less persistent. Specifically, the posterior mean of the autoregressive coefficients of the technology shock, $\rho_z$, and of the discount factor shock, $\rho_\theta$, are now significantly lower at 0.24 and 0.17, respectively, compared to 0.64 and 0.92 in the benchmark estimation. In addition, the standard deviations of both shocks also decline.

Omitting information from search intensity leads to significantly poorer fit of our model to the data, as shown in Figure 10. The job filling and finding rates predicted from our model estimated without using search intensity data still track the data more closely than those from the standard matching function, but the improvements in fitting are substantially smaller than those obtained from our benchmark estimation, in which we use search intensity data. Table 4 shows that the RMSEs for the job filling rate and the job finding rate predicted from our model under this alternative estimation are only marginally lower than those implied by the standard matching function, and significantly worse than our benchmark estimation.

In addition, when we estimate the model without fitting to the search intensity series, the hiring rate implied by the model displays a much weaker correlation with that in the data than that obtained from our benchmark estimation. Specifically, the correlation between model-implied hiring and actual hiring turns negative (-0.17), whereas the benchmark model displays a sizable positive correlation (0.55). These results suggest that fluctuations in search intensity are important to account for fluctuations in hiring.

Furthermore, cyclical fluctuations in search intensity also help amplify cyclical fluctuations in recruiting intensity. When we do not use information from search intensity to estimate the model, the correlation between recruiting intensity and hiring becomes negative (-0.17),

$\rho_z$ and $\rho_\theta$ are the autoregressive coefficients of the technology shock and the discount factor shock, respectively.

RMSE stands for Root Mean Square Error, a statistical measure of the differences between values predicted by a model or an estimator and the values actually observed.

Cyclical fluctuations refer to the changes that occur in economic variables over the business cycle, i.e., expansions and recessions.
compared to the positive correlation in the benchmark model (0.48) and in Davis et al. (2013).

Overall, these exercises suggest that using our measure of search intensity in estimating the DSGE model helps discipline the estimation. It also suggests that there are important general equilibrium interactions between search intensity and recruiting intensity that amplify the impact of the shocks on labor market variables. Furthermore, procyclical fluctuations in search intensity and recruiting intensity help bridge the gap between the model’s predicted job filling and job finding rates and those in the data.

VI. Conclusion

The sharp downturn in hiring during the Great Recession and the weak recovery presented a challenge for the standard model of labor search and matching. We have developed and estimated a DSGE model that generalizes the standard model to incorporate cyclical fluctuations of search and recruiting intensity. We find that these intensive margins of labor-market adjustments are quantitatively important. In the depth of the recession and during the recovery period, the job filling rate and the job finding rate predicted from our estimated model are much closer to the actual time-series data than those implied by the standard model without search and recruiting intensity. Our model suggests that the observed deep labor market recession and the weak recovery stem to a large extent from procyclical fluctuations in search and recruiting intensity.

To allow for aggregate fluctuations in recruiting intensity, we modify the standard model by assuming that firms need to pay a fixed cost to create a new job vacancy. This simple modification facilitates tractability and makes it straightforward to estimate the model to fit time-series data using standard techniques. Interestingly, our macro emphasis nonetheless yields predictions of the cyclical movements in recruiting intensity that are very much in line with those postulated in Davis et al. (2013) based on establishment-level data. In particular, both approaches highlight a procyclical recruiting intensity. In addition, our empirical findings also point to an important interaction between search and recruiting intensity that helps account for the observed behavior of the job filling and job finding rates since the end of the Great Recession.

To better highlight our model’s mechanism, we focus on three particular sources of business cycle fluctuations: technology shocks, discount factor shocks, and job separation shocks. All these shocks are arguably reduced-form representations of some microeconomic frictions or policy distortions that are not considered in our model. For example, the discount factor shock in our model that represents stochastic changes in households’ intertemporal preferences may reflect factors outside of the model that drive asset market fluctuations (Hall,
Our model also assumes that job separations vary exogenously, while in reality, job separations occur endogenously in response to the state of the economy.

Our model also restricts the labor force participation rate to be constant. Relaxing this assumption can have important implications for labor market dynamics. For example, Diamond (2013) argues that incorporating flows into and out of the labor force helps better understand the shifts of the Beveridge curve after the Great Recession. Kudlyak and Schwartzman (2012) show that persistent declines in labor force participation contributed to the large increases in unemployment during the Great Recession and also to the subsequent slow decline in unemployment. Future research should extend our framework to incorporate endogenous job separations and labor force participation to study their potential interactions with search and recruiting intensity. Such a framework should be useful for better understanding labor market fluctuations and for policy designs. Our work provides a step forward for this promising research agenda.
Table 1. Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.9967</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Unemployment benefit</td>
<td>0.25</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Elasticity of matching function</td>
<td>0.50</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Matching efficiency</td>
<td>0.4864</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Job separation rate</td>
<td>0.034</td>
</tr>
<tr>
<td>$\rho^o$</td>
<td>Vacancy obsolescence rate</td>
<td>0.0353</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>Steady-state advertising cost</td>
<td>0.1887</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>Slope of vacancy posting cost</td>
<td>0.2072</td>
</tr>
<tr>
<td>$h_1$</td>
<td>Slope of search cost</td>
<td>0.1123</td>
</tr>
<tr>
<td>$b$</td>
<td>Nash bargaining weight</td>
<td>0.50</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Real wage rigidity</td>
<td>0.95</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Elasticity of vacancy creation</td>
<td>1</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Mean value of preference shock</td>
<td>0.666</td>
</tr>
<tr>
<td>$\bar{Z}$</td>
<td>Mean value of technology shock</td>
<td>1</td>
</tr>
</tbody>
</table>
### Table 2. Estimated parameters

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Priors</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type [mean, std]</td>
<td>Mean 5% 95%</td>
</tr>
<tr>
<td>A. Benchmark estimation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K$ scale of vacancy creation</td>
<td>gamma [5, 1]</td>
<td>5.2907 3.5410 6.5215</td>
</tr>
<tr>
<td>$\kappa$ curvature of vacancy posting</td>
<td>gamma [1, 0.1]</td>
<td>1.4223 1.2748 1.5346</td>
</tr>
<tr>
<td>$h$ curvature of search cost</td>
<td>gamma [1, 0.1]</td>
<td>1.4749 1.4146 1.5337</td>
</tr>
<tr>
<td>$\rho_z$ AR(1) of technology shock</td>
<td>beta [0.3333, 0.2357]</td>
<td>0.6445 0.6073 0.6832</td>
</tr>
<tr>
<td>$\rho_\theta$ AR(1) of discount factor shock</td>
<td>beta [0.3333, 0.2357]</td>
<td>0.9231 0.9108 0.9429</td>
</tr>
<tr>
<td>$\rho_\delta$ AR(1) of job separation shock</td>
<td>beta [0.3333, 0.2357]</td>
<td>0.9333 0.9050 0.9527</td>
</tr>
<tr>
<td>$\sigma_z$ std of technology shock</td>
<td>inv gamma [0.01, 1]</td>
<td>0.0156 0.0147 0.0165</td>
</tr>
<tr>
<td>$\sigma_\theta$ std of preference shock</td>
<td>inv gamma [0.01, 1]</td>
<td>0.0157 0.0145 0.0170</td>
</tr>
<tr>
<td>$\sigma_\delta$ std of separation shock</td>
<td>inv gamma [0.01, 1]</td>
<td>0.0778 0.0739 0.0815</td>
</tr>
<tr>
<td>B. Alternative estimation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(without using search intensity data)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K$ scale of vacancy creation</td>
<td>gamma [5, 1]</td>
<td>5.2373 4.1283 6.4467</td>
</tr>
<tr>
<td>$\kappa$ curvature of vacancy posting</td>
<td>gamma [1, 0.1]</td>
<td>1.6005 1.4803 1.7406</td>
</tr>
<tr>
<td>$h$ curvature of search cost</td>
<td>gamma [1, 0.1]</td>
<td>1.2878 1.1554 1.4740</td>
</tr>
<tr>
<td>$\rho_z$ AR(1) of technology shock</td>
<td>beta [0.3333, 0.2357]</td>
<td>0.2394 0.1362 0.3092</td>
</tr>
<tr>
<td>$\rho_\theta$ AR(1) of discount factor shock</td>
<td>beta [0.3333, 0.2357]</td>
<td>0.1679 0.0094 0.3646</td>
</tr>
<tr>
<td>$\rho_\delta$ AR(1) of job separation shock</td>
<td>beta [0.3333, 0.2357]</td>
<td>0.8957 0.8726 0.9186</td>
</tr>
<tr>
<td>$\sigma_z$ std of technology shock</td>
<td>inv gamma [0.01, 1]</td>
<td>0.0089 0.0084 0.0094</td>
</tr>
<tr>
<td>$\sigma_\theta$ std of preference shock</td>
<td>inv gamma [0.01, 1]</td>
<td>0.0051 0.0024 0.0076</td>
</tr>
<tr>
<td>$\sigma_\delta$ std of separation shock</td>
<td>inv gamma [0.01, 1]</td>
<td>0.0688 0.0657 0.0716</td>
</tr>
</tbody>
</table>
Table 3. Forecasting Error Variance Decomposition

<table>
<thead>
<tr>
<th>Variables</th>
<th>Productivity shock</th>
<th>Discount factor shock</th>
<th>Job separation shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecasting horizon: 12 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>79.16</td>
<td>17.71</td>
<td>3.13</td>
</tr>
<tr>
<td>Vacancy</td>
<td>78.73</td>
<td>20.85</td>
<td>0.42</td>
</tr>
<tr>
<td>Search intensity</td>
<td>84.75</td>
<td>15.25</td>
<td>0.01</td>
</tr>
<tr>
<td>Recruiting intensity</td>
<td>91.52</td>
<td>8.45</td>
<td>0.02</td>
</tr>
<tr>
<td>Hiring</td>
<td>80.95</td>
<td>18.11</td>
<td>0.94</td>
</tr>
<tr>
<td>Job filling</td>
<td>73.55</td>
<td>26.45</td>
<td>0.00</td>
</tr>
<tr>
<td>Job finding</td>
<td>81.72</td>
<td>18.28</td>
<td>0.00</td>
</tr>
<tr>
<td>Aggregate output</td>
<td>96.77</td>
<td>2.75</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>Forecasting horizon: 24 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>61.18</td>
<td>28.56</td>
<td>10.26</td>
</tr>
<tr>
<td>Vacancy</td>
<td>72.09</td>
<td>20.59</td>
<td>7.33</td>
</tr>
<tr>
<td>Search intensity</td>
<td>78.59</td>
<td>21.27</td>
<td>0.14</td>
</tr>
<tr>
<td>Recruiting intensity</td>
<td>90.18</td>
<td>9.73</td>
<td>0.09</td>
</tr>
<tr>
<td>Hiring</td>
<td>68.59</td>
<td>15.64</td>
<td>15.76</td>
</tr>
<tr>
<td>Job filling</td>
<td>69.38</td>
<td>30.40</td>
<td>0.23</td>
</tr>
<tr>
<td>Job finding</td>
<td>74.94</td>
<td>24.89</td>
<td>0.17</td>
</tr>
<tr>
<td>Aggregate output</td>
<td>81.94</td>
<td>13.29</td>
<td>4.77</td>
</tr>
<tr>
<td></td>
<td>Forecasting horizon: 48 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>60.24</td>
<td>27.88</td>
<td>11.88</td>
</tr>
<tr>
<td>Vacancy</td>
<td>70.21</td>
<td>19.91</td>
<td>9.87</td>
</tr>
<tr>
<td>Search intensity</td>
<td>78.48</td>
<td>21.27</td>
<td>0.25</td>
</tr>
<tr>
<td>Recruiting intensity</td>
<td>90.16</td>
<td>9.75</td>
<td>0.09</td>
</tr>
<tr>
<td>Hiring</td>
<td>65.61</td>
<td>14.71</td>
<td>19.68</td>
</tr>
<tr>
<td>Job filling</td>
<td>69.20</td>
<td>30.32</td>
<td>0.48</td>
</tr>
<tr>
<td>Job finding</td>
<td>74.80</td>
<td>24.87</td>
<td>0.33</td>
</tr>
<tr>
<td>Aggregate output</td>
<td>81.21</td>
<td>13.18</td>
<td>5.61</td>
</tr>
</tbody>
</table>

Note: Columns 2 to 4 report the contributions (in percentage terms) of each of the three shocks to the forecast error variances in the estimated benchmark model at the forecasting horizons of 12 months, 24 months, and 48 months.
Table 4. Out-of-sample predictions of alternative models: RMSEs

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard matching function</th>
<th>Standard matching function w/ search intensity</th>
<th>Benchmark model</th>
<th>Alternative estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job filling rate</td>
<td>0.1086</td>
<td>0.0914</td>
<td>0.0466</td>
<td>0.0939</td>
</tr>
<tr>
<td>Job finding rate</td>
<td>0.1086</td>
<td>0.0914</td>
<td>0.0466</td>
<td>0.0939</td>
</tr>
</tbody>
</table>

Note: The numbers in this table are root mean squared errors calculated based on demeaned data and predictions from each model. The job filling rate is the ratio of the hiring rate to the end-of-period job vacancy rate. The job finding rate is the ratio of the hiring rate to the end-of-period unemployment rate.
Figure 1. Job filling rate and job finding rate: Data vs. standard model. The shaded areas indicate the NBER recession dates. In the data, the job filling rate is the ratio of the hiring rate to the end-of-period job vacancy rate and the job finding rate is the ratio of the hiring rate to the end-of-period unemployment rate. For constructions of these variables in the standard model, see Footnote 1. All series are in log terms and normalized relative to the February 2001 observation, corresponding to the starting point of the 3-month moving averages of the JOLTS data.
Figure 2. Job filling rate and job finding rate: Data, standard model, and benchmark DSGE model. The shaded areas indicate the NBER recession dates. For explanations of the variable constructions in the data and the standard model, see Figure 1. The variables from the benchmark model are the Kalman smoothed series implied by the DSGE estimation.
Figure 3. Time series of search intensity. The shaded areas indicate recession dates. The search intensity series is imputed from the median duration of unemployment (weeks) based on the regression analysis of Davis (2011).
Figure 4. Job filling rate and job finding rate: Data, standard model, and benchmark DSGE model, in the full sample. The shaded areas indicate the NBER recession dates. All series are in log terms and normalized relative to the February 2001 observation, corresponding to the starting point of the 3-month moving averages of the JOLTS data. The definitions of these variables in the data, the standard model, and the benchmark model are the same as in Figure 2.
Figure 5. Impulse responses to a negative technology shock
FIGURE 6. Impulse responses to a positive discount factor shock
Figure 7. Impulse responses to a positive job separation shock
Figure 8. Impulse responses to a negative technology shock: Benchmark model vs. counterfactual model with constant search and recruiting intensity.
**Figure 9.** The hiring rates: Data vs. benchmark model. The hiring rate in the data is the ratio of total hires to total employment. The hiring rate in the model corresponds to the new job matches ($m_t$). The shaded areas indicate the NBER recession dates. Both series are in log terms and normalized relative to the February 2001 observation, corresponding to the starting point of the 3-month moving averages of the JOLTS data.
Figure 10. Job filling rate and job finding rate: Data, standard model, and DSGE model estimated without using search intensity series. The shaded areas indicate the NBER recession dates. All series are in log terms and normalized relative to the February 2001 observation, corresponding to the starting point of the 3-month moving averages of the JOLTS data.
Appendix A. Data

We fit the DSGE model to three monthly time-series data of the U.S. labor market: the unemployment rate, job vacancies, and a measure of search intensity. We also use monthly time-series data of hires to construct our measures of the job filling and finding rates.

1. Unemployment: Civilian unemployment rate (16 years and over) from the Bureau of Labor Statistics (LR@USECON in Haver), seasonally adjusted monthly series.

2. Job vacancies: For the period from December 2000 and on, we use the seasonally adjusted job opening rate series from JOLTS (LJJTPA@USECON in Haver). For the period prior to December 2000, we use the measure of the job vacancy rate constructed by Barnichon (2010) based on Help Wanted Index.

3. Search intensity: constructed based on the approach in Davis (2011), using the following empirical relation between search intensity \( s_t \) and the duration of unemployment \( d_t \)

\[
s_t = 122.30 - 0.90d_t,
\]

where \( d_t \) is measured by the seasonally adjusted monthly series of the median duration of unemployment (in weeks) reported in the Current Population Survey (WAMED@EMPL in Haver).

4. Hires: Total hires rate from JOLTS (LJHTPA@USECON in Haver), seasonally adjusted monthly series.

The sample range for the unemployment rate, the job vacancy rate, and the measure of search intensity covers the period from July 1967 to July 2017. The sample for the hiring rate from the JOLTS ranges from December 2000 to July 2017.

The job filling rate and the job finding rate in the actual data are constructed based on the data for hires, unemployment, and vacancies. In particular, the job filling rate in the data is the ratio of the hiring rate to the end-of-period job vacancy rate. The job finding rate is the ratio of the hiring rate to the end-of-period unemployment rate.

Appendix B. Derivations of household’s optimizing conditions

Our approach to incorporating search intensity in the DSGE model builds on the textbook treatment by Pissarides (2000). The basic idea is that the representative household can choose the effort level that is devoted to searching for those members who are unemployed. Increasing search effort incurs some resource costs, but it also creates the benefits of increasing the individual searching worker’s job finding rate.

We now derive the optimal search intensity decision from the first principle. To economize notations, we do not carry around the individual index \( i \) in describing the household’s
optimizing problem. Keep in mind that, in choosing the individual search intensity and employment, the household takes the economy-wide variables as given. In a symmetric equilibrium, the individual optimal choices coincide with the aggregate optimal choices.

Denote by $V_t(B_{t-1}, N_{t-1})$ the value function for the representative household. The household’s optimizing problem can be written in the recursive form

$$V_t(B_{t-1}, N_{t-1}) \equiv \max \ln C_t - \chi N_t + \beta \Theta_t E_t V_{t+1}(B_t, N_t),$$  

(A2)

where $\Theta_t$ is an exogenous preference shifter. The household’s utility-maximizing decision is subject to the budget constraint

$$C_t + \frac{B_t}{r_t} = B_{t-1} + w_t N_t + \phi(1 - N_t) - u_t h(s_t) + d_t - T_t,$$

(A3)

and the law of motion for employment

$$N_t = (1 - \delta_t) N_{t-1} + q^u(s_t) u_t, \quad \text{(A4)}$$

where the measure of job seekers is given by

$$u_t = 1 - (1 - \delta_t) N_{t-1}.$$  

(A5)

The household chooses $C_t$, $B_t$, $N_t$, and $s_t$, taking prices and the average job finding rate as given.

Denote by $\Lambda_t$ the Lagrangian multiplier for the budget constraint (A3). The first-order condition with respect to consumption implies that

$$\Lambda_t = \frac{1}{C_t}.$$  

(A6)

The optimizing decision for $B_t$ implies that

$$\frac{\Lambda_t}{r_t} = \beta E_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial B_t},$$  

(A7)

where $\theta_t \equiv \frac{\Theta_t}{\Theta_{t-1}}$ denotes the discount factor shock. Combining equation (A7) with the envelope condition with respect to $B_{t-1}$, we obtain the intertemporal Euler equation

$$1 = E_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} r_t,$$

(A8)

which is equation (17) in the text.

Optimal choice of search intensity $s_t$ implies that

$$h'(s_t) = \frac{q^u}{s_t} \left[ w_t - \phi - \frac{\chi_t}{\Lambda_t} + \beta E_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial N_t} \frac{1}{\Lambda_t} \right],$$  

(A9)
where we have used equation (14) to replace the term $\frac{\partial q^u(s_t)}{\partial s_t}$ by $\frac{q^u}{s_t}$. The envelope condition implies that

$$\frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_{t-1}} = \left[ \Lambda_t(w_t - \phi) - \chi_t + \beta E_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial N_t} \right] \frac{\partial N_t}{\partial N_{t-1}} - \Lambda_t h_t \frac{\partial u_t}{\partial N_{t-1}} \tag{A10}$$

Equations (A4) and (A5) imply that

$$\frac{\partial N_t}{\partial N_{t-1}} = (1 - \delta_t)(1 - q^u(s_t)) \tag{A11}$$

and that

$$\frac{\partial u_t}{\partial N_{t-1}} = -(1 - \delta_t). \tag{A12}$$

Define the employment surplus (i.e., the value of employment relative to unemployment) as

$$S_t^H = \frac{1}{\Lambda_t} \frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_t} = \frac{1}{\Lambda_t} \frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_{t-1}} \frac{\partial N_t}{\partial N_{t-1}} = \frac{1}{\Lambda_t} \frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_{t-1}} \frac{1}{(1 - \delta_t)(1 - q^u(s_t))}. \tag{A13}$$

Thus, $S_t^H$ is the value for the household to send an additional worker to work in period $t$. Then the envelope condition (A10) implies that

$$S_t^H = w_t - \phi - \frac{\chi_t}{1 - q^u} + E_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \delta_{t+1})(1 - q^u_{t+1})S_{t+1}^H. \tag{A14}$$

The employment surplus $S_t^H$ derived here corresponds to equation (16) in the text and it is the relevant surplus for the household in the Nash bargaining problem.

Given the definition of employment surplus in equation (A13), the optimal search intensity decision (A9) can be rewritten as

$$h'(s_t) = \frac{q^u}{s_t} \left[ w_t - \phi - \frac{\chi_t}{1 - q^u} + E_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \delta_{t+1})(1 - q^u_{t+1})S_{t+1}^H \right]. \tag{A15}$$

Thus, at the optimum, the marginal cost of search intensity equals the marginal benefit, where the benefit derives from the increased job finding rate and the net value of employment. This last equation corresponds to equation (15) in the text.
REFERENCES


