APPENDIX TO "THE WEAK JOB RECOVERY IN A MACRO MODEL OF SEARCH AND RECRUITING INTENSITY"

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Abstract. This appendix provides a summary of equilibrium conditions in the DSGE model of Leduc and Liu (2017) and presents some additional results with alternative assumptions on the entry-cost distribution.

I. Summary of equilibrium conditions in the DSGE model

A search equilibrium is a system of 18 equations for 18 variables summarized in the vector

$$\left[C_t, \Lambda_t, m_t, q_t^u, q_t^v, N_t, u_t, U_t, Y_t, r_t, v_t, J_t^F, w_t^N, w_t, n_t, a_t, s_t, J_t^V\right].$$

We write the equations in the same order as in the dynare code.

(1) Household's bond Euler equation:

$$1 = \mathrm{E}_t \beta \theta_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} r_t, \tag{B1}$$

(2) Marginal utility of consumption

$$\Lambda_t = \frac{1}{C_t}, \tag{B2}$$

(3) Search intensity

$$h_1 + h_2(s_t - \bar{s}) = \frac{q_t^u}{s_t} \left\{ \frac{b}{1-b}(J_t^F - J_t^V) - \frac{h(s_t)}{1-q_t^u} \right\}, \tag{B3}$$

(4) Matching function

$$m_t = \mu_t (s_t u_t)^{\alpha} (a_t v_t)^{1-\alpha}, \tag{B4}$$

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(5) Job finding rate

$$q_t^u = \frac{m_t}{u_t}, \tag{B5}$$

(6) Vacancy filling rate

$$q_t^v = \frac{m_t}{v_t}, \tag{B6}$$

(7) Employment dynamics:

$$N_t = (1-\delta_t)N_{t-1} + m_t, \tag{B7}$$

(8) Number of searching workers:

$$u_t = 1 - (1-\delta_t)N_{t-1}, \tag{B8}$$

(9) Unemployment:

$$U_t = 1 - N_t, \tag{B9}$$

(10) Law of motion for vacancies:

$$v_t = (1-\rho^o)(1-q_{t-1}^v)v_{t-1} + (\delta_t - \rho^o)N_{t-1} + n_t, \tag{B10}$$

(11) Aggregate production function:

$$Y_t = Z_t N_t \tag{B11}$$

(12) Aggregate Resource constraint:

$$C_t + h(s_t)u_t + \kappa(a_t)v_t + \frac{\xi}{1+\xi}n_t J_t^V = Y_t, \tag{B12}$$

where the search cost function and the recruiting cost function are given by

$$\begin{aligned} h(s_t) &= h_1(s_t - \bar{s}) + \frac{h_2}{2}(s_t - \bar{s})^2 \\ \kappa(a_t) &= \kappa_0 + \kappa_1(a_t - \bar{a}) + \frac{\kappa_2}{2}(a_t - \bar{a})^2 \end{aligned}$$

(13) Value of vacancy:

$$J_t^V = -\kappa(a_t) + q_t^v J_t^F + (1-q_t^v)(1-\rho^o)E_t\frac{\beta\theta_{t+1}\Lambda_{t+1}}{\Lambda_t}J_{t+1}^V, \tag{B13}$$

(14) Recruiting intensity:

$$\kappa_1 + \kappa_2(a_t - \bar{a}) = \frac{q_t^v}{a_t}\left[J_t^F - (1-\rho^o)E_t\frac{\beta\theta_{t+1}\Lambda_{t+1}}{\Lambda_t}J_{t+1}^V\right]. \tag{B14}$$

(15) Match value:

$$J_t^F = Z_t - w_t + E_t\frac{\beta\theta_{t+1}\Lambda_{t+1}}{\Lambda_t}\left\{(1-\delta_{t+1})J_{t+1}^F + \delta_{t+1}Kn_{t+1}\right\}, \tag{B15}$$

(16) Nash bargaining wage:

$$\frac{b}{1-b}(J_t^F - J_t^V) = w_t^N - \phi - \frac{\chi_t}{\Lambda_t} + \frac{h(s_t)}{1-q_t^u} + E_t \frac{\beta\theta_{t+1}\Lambda_{t+1}}{\Lambda_t}\left[(1-\delta_{t+1})\left(1-q_{t+1}^u\right)\frac{b}{1-b}(J_{t+1}^F - J_{t+1}^V)\right]. \tag{B16}$$

(17) Actual real wage (with real wage rigidity)

$$w_t = w_{t-1}^{\gamma}(w_t^N)^{\gamma}, \tag{B17}$$

(18) New vacancy

$$Kn_t = \left(J_t^V\right)^{\xi}. \tag{B18}$$

II. Steady State

(1) Household's bond Euler equation:

$$1 = \beta r, \tag{C1}$$

(2) Marginal utility of consumption

$$\Lambda = \frac{1}{C}, \tag{C2}$$

(3) Search intensity

$$h_1 = \frac{q^u}{s}\frac{b}{1-b}(J^F - J^V), \tag{C3}$$

(4) Matching function

$$m = \mu(\bar{s}u)^{\alpha}(\bar{a}v)^{1-\alpha}, \tag{C4}$$

(5) Job finding rate

$$q^u = \frac{m}{u}, \tag{C5}$$

(6) Vacancy filling rate

$$q^v = \frac{m}{v}, \tag{C6}$$

(7) Employment dynamics:

$$m = \delta N, \tag{C7}$$

(8) Number of searching workers:

$$u = U + m, \tag{C8}$$

(9) Unemployment:

$$U = 1 - N, \tag{C9}$$

(10) Vacancies:

$$[\rho^o + (1-\rho^o)q^v]v = (\delta - \rho^o)N + n, \tag{C10}$$

(11) Aggregate production function:

$$Y = ZN \tag{C11}$$

(12) Aggregate Resource constraint:

$$C + \kappa_0 v + \frac{\xi}{1+\xi} n J^V = Y, \tag{C12}$$

(13) Value of vacancies:

$$q^v J^F - \kappa_0 = \left[1 - \beta(1-q^v)(1-\rho^o)\right] J^V \tag{C13}$$

(14) Recruiting intensity:

$$\kappa_1 \bar{a} = q^v \left[J^F - \beta(1-\rho^o) J^V\right], \tag{C14}$$

(15) Match value:

$$\left[1 - \beta(1-\delta)\right] J^F = Z - w + \beta\delta J^V, \tag{C15}$$

(16) Nash bargaining wage:

$$w^N = \phi + \frac{\chi}{\Lambda} + \frac{b}{1-b}\left[1 - \beta(1-\delta)(1-q^u)\right](J^F - J^V), \tag{C16}$$

(17) Actual real wage

$$w = w^N, \tag{C17}$$

(18) New vacancy

$$Kn = \left(J^V\right)^{\xi} \tag{C18}$$

III. Equilibrium system scaled by steady state (used in dynare)

Denote by $\hat{X}_t \equiv \frac{X_t}{X}$ the scaled value of the variable X_t by its steady-state level. The system of equilibrium conditions can be reduced to the following 18 equations to solve for the 18 endogenous variables summarized in the vector

$$[\hat{C}_t, \hat{\Lambda}_t, \hat{r}_t, \hat{Y}_t, \hat{m}_t, \hat{u}_t, \hat{v}_t, \hat{q}^u_t, \hat{q}^v_t, \hat{N}_t, \hat{U}_t, \hat{J}^F_t, \hat{w}^N_t, \hat{w}_t, \hat{n}_t, \hat{a}_t, \hat{s}_t, \hat{J}^V_t].$$

(1) Household's bond Euler equation:

$$1 = \mathrm{E}_t \exp(\hat{\theta}_{t+1}) \frac{\hat{\Lambda}_{t+1}}{\hat{\Lambda}_t} \hat{r}_t, \tag{D1}$$

(2) Marginal utility of consumption

$$\hat{\Lambda}_t = \frac{1}{\hat{C}_t}, \tag{D2}$$

(3) Search intensity

$$h_1 + h_2 \bar{s}(\hat{s}_t - 1) = \frac{q^u \hat{q}^u_t}{\bar{s} s_t} \left\{ \frac{b}{1-b}(J^F \hat{J}^F_t - J^V \hat{J}^V_t) - \frac{h_1 \bar{s}(\hat{s}_t - 1) + \frac{h_2 \bar{s}^2}{2}(\hat{s}_t - 1)^2}{1 - q^u \hat{q}^u_t} \right\} \tag{D3}$$

(4) Matching function

$$\hat{m}_t = (\hat{s}_t \hat{u}_t)^\alpha (\hat{a}_t \hat{v}_t)^{1-\alpha}, \tag{D4}$$

(5) Job finding rate

$$\hat{q}^u_t = \frac{\hat{m}_t}{\hat{u}_t}, \tag{D5}$$

(6) Vacancy filling rate

$$\hat{q}^v_t = \frac{\hat{m}_t}{\hat{v}_t}, \tag{D6}$$

(7) Employment dynamics:

$$\hat{N}_t = (1 - \delta \exp(\hat{\delta}_t))\hat{N}_{t-1} + \frac{m}{N}\hat{m}_t, \tag{D7}$$

(8) Number of searching workers

$$u\hat{u}_t = 1 - (1 - \delta \exp(\hat{\delta}_t)) N \hat{N}_{t-1}, \tag{D8}$$

(9) Unemployment:

$$U\hat{U}_t = 1 - N\hat{N}_t, \tag{D9}$$

(10) Vacancies:

$$v\hat{v}_t = (1 - \rho^o)(1 - q^v \hat{q}^v_{t-1}) v\hat{v}_{t-1} + (\delta \exp(\hat{\delta}_t) - \rho^o) N\hat{N}_{t-1} + n\hat{n}_t, \tag{D10}$$

(11) Aggregate production function:

$$\hat{Y}_t = \exp(\hat{z}_t)\hat{N}_t \tag{D11}$$

(12) Aggregate Resource constraint:

$$\begin{aligned} \hat{Y}_t &= \left[h_1 \bar{s}(\hat{s}_t - 1) + \frac{h_2 \bar{s}^2}{2}(\hat{s}_t - 1)^2\right] \frac{u}{Y}\hat{u}_t + \left[\kappa_0 + \kappa_1 \bar{a}(\hat{a}_t - 1) + \frac{\kappa_2 \bar{a}^2}{2}(\hat{a}_t - 1)^2\right] \frac{v}{Y}\hat{v}_t \\ &+ \frac{C}{Y}\hat{C}_t + \frac{\xi}{1+\xi}\frac{nJ^V}{Y}\left(\hat{n}_t + \hat{J}^V_t\right) \end{aligned} \tag{D12}$$

(13) Value of vacancy:

$$\begin{aligned} J^V \hat{J}^V_t = &-\left[\kappa_0 + \kappa_1 \bar{a}(\hat{a}_t - 1) + \frac{\kappa_2 \bar{a}^2}{2}(\hat{a}_t - 1)^2\right] + \\ &q^v J^F \hat{q}^v_t \hat{J}^F_t + (1 - q^v \hat{q}^v_t)(1 - \rho^o) \mathrm{E}_t \frac{\beta \exp(\hat{\theta}_{t+1})\hat{\Lambda}_{t+1}}{\hat{\Lambda}_t} J^V \hat{J}^V_{t+1}, \end{aligned} \tag{D13}$$

(14) Recruiting intensity:

$$\kappa_1 + \kappa_2 \bar{a}(\hat{a}_t - 1) = \frac{q^v \hat{q}^v_t}{\bar{a}\hat{a}_t}\left[J^F \hat{J}^F_t - (1 - \rho^o)\mathrm{E}_t \frac{\beta \exp(\hat{\theta}_{t+1})\hat{\Lambda}_{t+1}}{\hat{\Lambda}_t} J^V \hat{J}^V_{t+1}\right]. \tag{D14}$$

(15) Match value:

$$J^F \hat{J}^F_t = \exp(\hat{z}_t) - w\hat{w}_t + \mathrm{E}_t \frac{\beta \exp(\hat{\theta}_{t+1})\hat{\Lambda}_{t+1}}{\hat{\Lambda}_t} \left\{ (1 - \delta \exp(\hat{\delta}_{t+1})) J^F \hat{J}^F_{t+1} + \delta \exp(\hat{\delta}_{t+1}) J^V \hat{J}^V_{t+1} \right\}, \tag{D15}$$

(16) Nash bargaining wage:

$$\frac{b}{1-b}(J^F \hat{J}^F_t - J^V \hat{J}^V_t) = w\hat{w}^N_t - \phi - \frac{\chi}{\Lambda \hat{\Lambda}_t} + \frac{h_1 \bar{s}(\hat{s}_t - 1) + \frac{h_2 \bar{s}^2}{2}(\hat{s}_t - 1)^2}{1 - q^u \hat{q}^u_t}$$
$$+ E_t \frac{\beta \exp(\hat{\theta}_{t+1})\hat{\Lambda}_{t+1}}{\hat{\Lambda}_t} \left[(1 - \delta \exp(\hat{\delta}_{t+1})) \left(1 - q^u \hat{q}^u_{t+1}\right) \frac{b}{1-b}(J^F \hat{J}^F_{t+1} - J^V \hat{J}^V_{t+1}) \right] \tag{D16}$$

(17) Actual real wage (with real wage rigidity)

$$\hat{w}_t = \hat{w}^{\gamma}_{t-1} (\hat{w}^N_t)^{\gamma}, \tag{D17}$$

(18) New vacancies

$$\hat{n}_t = \xi \hat{J}^V_t \tag{D18}$$

(19) Technology shock process

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{zt}, \tag{D19}$$

(20) Discount factor shock process

$$\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \varepsilon_{\theta t}, \tag{D20}$$

(21) Job separation shock process

$$\hat{\delta}_t = \rho_\delta \hat{\delta}_{t-1} + \varepsilon_{\delta t}, \tag{D21}$$

IV. Robustness to alternative distribution assumptions for entry costs

In the baseline model, we follow Fujita and Ramey (2007); Coles and Moghaddasi Kelishomi (2011) and calibrate the elasticity parameter of the vacancy creation condition to $\xi = 1$, corresponding to the case with a uniform distribution of the entry costs. Here, we examine our model's quantitative implications under alternative values of ξ. As we discussed in the paper, a larger value of ξ implies more elastic responses of entry (i.e., vacancy creation) to changes in the value of vacancies. The limiting case with $\xi = \infty$ approximates an environment with free entry, as in the standard DMP model.

To examine the sensitivity of our results to ξ, we consider the counterfactual cases with alternative calibrations of ξ. In particular, we generate smoothed variables from the estimated model, with ξ set to 0.5, 2, or 10 in each of the counterfactual cases. With a larger value of ξ, the model becomes closer to one with free entry, and firms in the model would rely more on variations in the number of vacancies to respond to macroeconomic shocks and less on varying recruiting intensity. Thus, the exercise here also helps evaluate the quantitative importance of cyclical variations in recruiting intensity for labor market dynamics. As in the baseline model, we evaluate the quantitative performance of the model under these alternative ξ values by comparing the model's predictions on the job filling and finding rates and also on the hiring rate to those in the actual data.

We first consider the case with $\xi = 0.5$, so that new vacancies are less responsive to changes in the value of an open vacancy than in the benchmark model with $\xi = 1$. Figure 1 shows the job filling rate and the job finding rate in this counterfactual model with $\xi = 0.5$ (the dashed and dotted lines), along with the actual series in the data (the solid lines) and those implied by the standard matching function (the dashed lines). Evidently, the model with this smaller ξ value still outperforms the standard matching function, and the predictions come close to the data, similar to what we have obtained in the benchmark case. As we show in Table 1, the RMSEs from the counterfactual model with $\xi = 0.5$ are slightly larger but very close to those from the benchmark model (0.0477 vs. 0.466), both are substantially smaller than those implied by the standard matching function (0.1086). However, the correlation between the model-implied hiring and that in the data becomes lower than that in the benchmark case (0.28 vs. 0.55), suggesting that our calibrated value of ξ helps the model to fit the hiring data better.

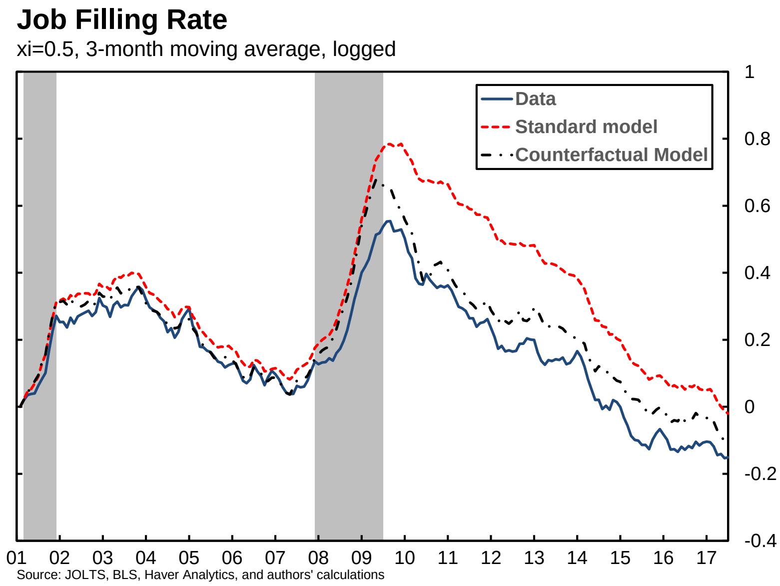
We then consider the case with $\xi = 2$, with the model-implied job filling and finding rates shown in Figure 2. Again, the model with this larger value of calibrated ξ also outperforms the standard matching function, with significantly smaller RMSEs (0.0510 vs. 0.1086). The correlation between the model-implied hiring and that in the data becomes even lower than that in the benchmark case (0.10 vs. 0.55), suggesting that the alternative calibration of a higher value of ξ worsens the model's fit to the hiring data.

Finally, we consider an even larger calibration of $\xi = 10$, so that the model gets even closer to the standard DMP model with free entry. Figure 3 shows the job filling and finding rates implied by the counterfactual model with $\xi = 10$, along with those implied by the standard matching function and those in the actual data. The model appears to fit the data less well than the benchmark model, with higher RMSEs (0.0642 vs. 0.0466). The correlation between model-implied hiring and the actual hiring data turns negative (-0.44), suggesting that the larger value of ξ leads to worse overall fit of the model.

Table 1. Out-of-sample predictions of alternative models: RMSEs

Variable	Standard model	Benchmark model	Counterfactual $\xi = 0.5$	Counterfactual $\xi = 2$	Counterfactual $\xi = 10$
Job filling rate	0.1086	0.0466	0.0477	0.0510	0.0642
Job finding rate	0.1086	0.0466	0.0478	0.0510	0.0642

Note: The numbers in this table are root mean squared errors calculated based on demeaned data and predictions from each model. The job filling rate is the ratio of the hiring rate to the end-of-period job vacancy rate. The job finding rate is the ratio of the hiring rate to the end-of-period unemployment rate.



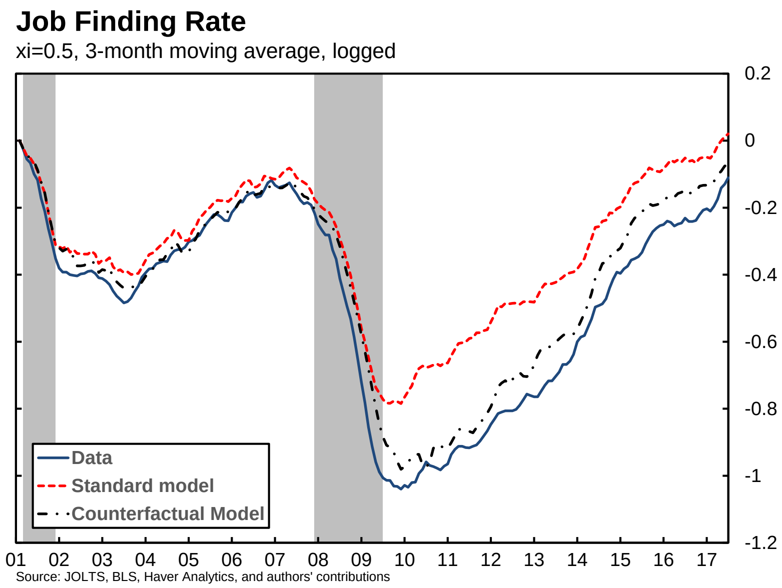
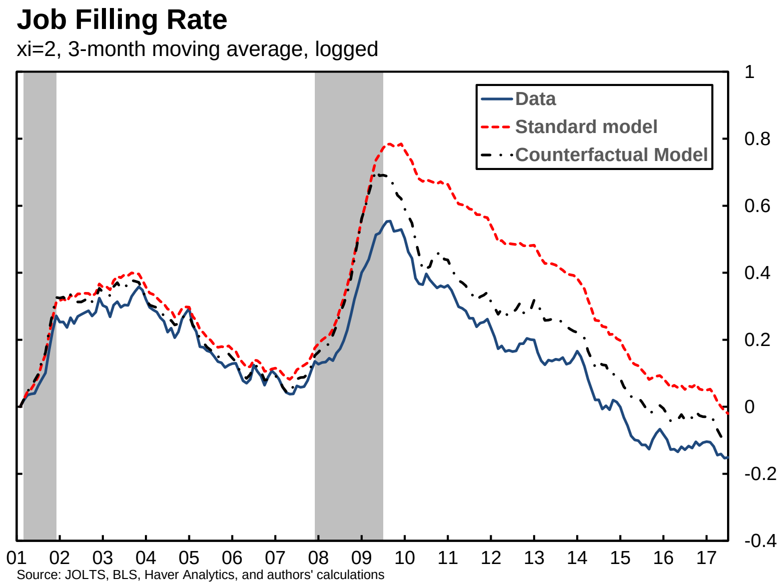


FIGURE 1. Job filling rate and job finding rate: Data, standard model, and counterfactual DSGE model with $\xi = 0.5$.



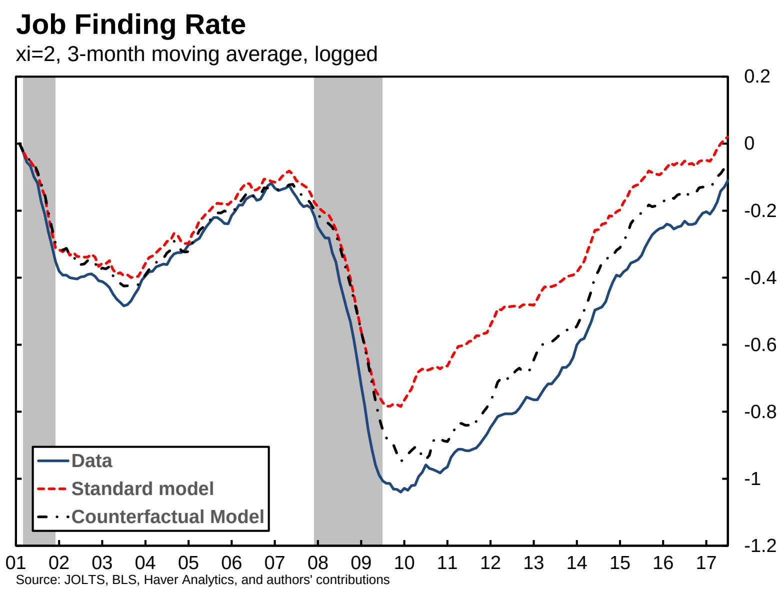
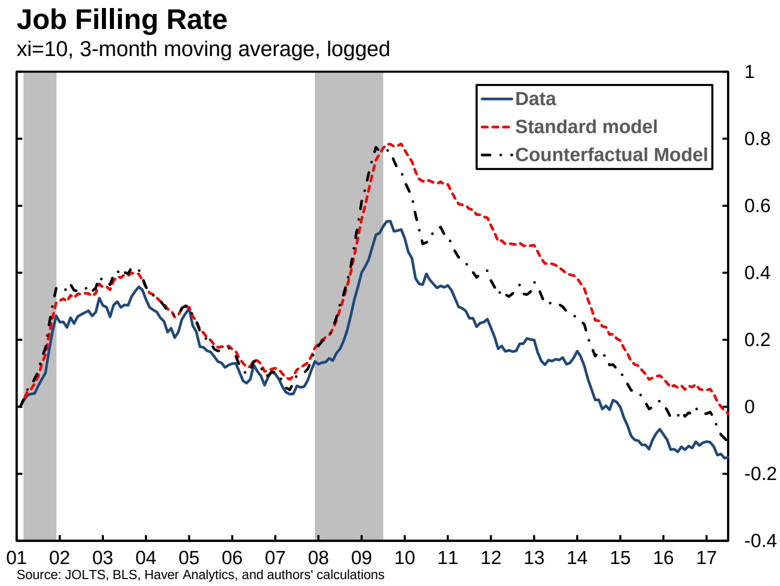


FIGURE 2. Job filling rate and job finding rate: Data, standard model, and counterfactual DSGE model with $\xi = 2$.



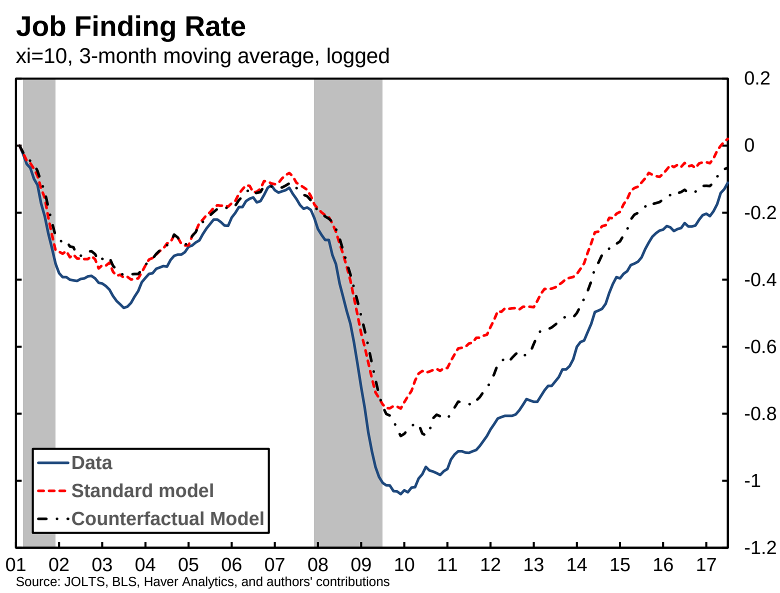


FIGURE 3. Job filling rate and job finding rate: Data, standard model, and counterfactual DSGE model with $\xi = 10$.

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