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Interest Rates Under Falling Stars

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Abstract

While theory predicts that the equilibrium real interest rate, r_t^*, and the perceived trend in inflation, π_t^*, are fundamental determinants of the yield curve, macro-finance models generally treat them as constant. We show that accounting for time-varying macro trends is critical for understanding the empirical dynamics of U.S. Treasury yields and risk pricing. It fundamentally changes estimated risk premiums in long-term bond yields, leads to large gains in predictions of excess bond returns and long-range out-of-sample forecasts of interest rates, and captures a substantial share of interest rate variability at low frequencies.

Keywords: yield curve, macro-finance, inflation trend, equilibrium real interest rate, shifting endpoints, bond risk premiums
JEL Classifications: E43, E44, E47

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1 Introduction

Research in financial economics has made numerous attempts to connect macroeconomic variables to the term structure of interest rates using a variety of approaches ranging from reduced-form no-arbitrage models to fully-fledged dynamic macro models.[1] Despite both theoretical and empirical progress, there is no clear consensus about how macroeconomic information should be incorporated into yield-curve analysis. Notably, two widely cited estimates of the term premium in long-term yields by Kim and Wright (2005) and Adrian et al. (2013) are based on models that include no macroeconomic data. One important link between the macroeconomy and the yield curve that has been largely overlooked is the connection between their long-run trends.[2] Specifically, macroeconomic data and models can provide estimates of the trend in inflation (π_t^*) and the equilibrium real interest rate (r_t^*), and finance theory—from Irving Fisher through modern no-arbitrage models—tells us that such macroeconomic trends must be reflected in interest rates. Of course, as an empirical issue, what matters is whether there is significant variation over time in these long-run trends. Almost all term structure analyses assume that these variables are constant. Instead, in this paper, we document that accounting for the macro-finance link between a time-varying π_t^* and r_t^* and the long-run trend in interest rates is essential for modeling the term structure, estimating bond risk premiums, and forecasting the yield curve.

An illustration of the potential importance of macro trends is provided in Figure 1. The secular decline in the 10-year Treasury yield since the early 1980s reflects a gradual downtrend in the general level of U.S. interest rates. The underlying drivers of this decline and their dynamics remain contentious. In finance, specifically in no-arbitrage term structure models, interest rates are generally modeled as stationary, mean-reverting processes, because over very long historical periods they have always remained range-bound. As a result, low-frequency variation in interest rates is hard to explain in such models, and it is mostly attributed to the residual term premium component, the difference between a long-term interest rate and the model-implied expectations of average future short-term rates. A prominent example is Wright (2011), who concluded that between 1990 and 2010 interest rates fell globally because of declining term premiums that in turn reflected a decrease in inflation uncertainty. However, our estimates of the trends underlying interest rates displayed in Figure 1 suggest a very different explanation. First, our measure of U.S. trend inflation, based on long-horizon

[1] See Ang and Piazzesi (2003), Diebold et al. (2006), Rudebusch and Wu (2008), Bikbov and Chernov (2010), Rudebusch and Swanson (2012), Bansal and Shaliastovich (2013), and Joslin et al. (2014), among many others. For a detailed survey, see Gürkaynak and Wright (2012).

[2] Throughout this paper, we use the Beveridge-Nelson concept of a trend, that is, the expectation for an economic series in the (infinitely) distant future.

inflation survey forecasts, declined by almost six percentage points from the early 1980s to the late 1990s. Hence, expectations about the level of inflation must have played an important role in pushing down nominal yields. Second, over the past two decades as inflation expectations have stabilized, our estimate of the equilibrium real interest rate (which is described in detail below) has exhibited a pronounced decline.[3] This drop implies that the component capturing expectations of future real interest rates helped push interest rates lower as well. The expectations component of nominal yields necessarily contains the sum of both macro trends, $i_t^* = \pi_t^* + r_t^*$, i.e., the equilibrium nominal short rate. As evident in Figure 1, our estimate of i_t^* exhibited similar low-frequency movements as the ten-year Treasury yield. This strongly suggests that the earlier downward trend in π_t^* and the more recent fall in r_t^*—that is, an environment of falling stars—is the main reason for the secular decline in nominal interest rates. Indeed, given the fall in i_t^* there is little room for secular trends in the term premium to account for this decline.

In this paper we quantify the importance of i_t^*, π_t^*, and $r_t^\star$ for the evolution of the yield curve using standard empirical proxies for these macro trends and five different empirical approaches. First, we investigate the link between yields and macro trends using standard time series methods. This analysis reveals that time variation in both π_t^* and r_t^* is responsible for the extremely high persistence of interest rates. The difference between long-term interest rates and i_t^* exhibits quick mean reversion, and tests for unit roots and cointegration clearly indicate that π_t^* and r_t^* account for the trend component in nominal yields. Accounting only for the inflation trend on its own, as in Kozicki and Tinsley (2001) and Cieslak and Povala (2015), leaves a highly persistent component of interest rates unexplained. Accordingly, we show that it is crucial to include r_t^* as well—given the quantitatively important changes in the equilibrium real rate—in order to fully capture the trend component in interest rates. After accounting for shifts in r_t^*, we uncover strong evidence for a long-run Fisher effect in which long-term interest rates and inflation share a common trend. Previous studies have found mixed results on the Fisher effects, because they focus only on a bivariate relationship between yields and inflation (Mishkin, 1992; Wallace and Warner, 1993; Evans and Lewis, 1995). We also document, using a simple error-correction model, that the long-term yields quickly revert back to their underlying macro trend i_t^*.

Second, we estimate predictive regressions for excess bond returns in order to understand

[3] Various underlying fundamental economic forces, such as lower productivity growth and an aging population, appear to have slowly altered global saving and investment and, in turn, pushed down the steady-state real interest rate. Discussions of the decline in r^* include Summers (2014), Rachel and Smith (2015), Hamilton et al. (2016), Holston et al. (2017), Del Negro et al. (2017), and many others. In the macroeconomics literature, r_t^* is often labeled the neutral or natural rate of interest although, as noted below, there are various definitions with subtle differences.

the role of macro trends for bond risk premiums. Accounting for changes in the underlying macro trends fundamentally changes return predictions. Relative to the standard predictive regressions for excess bond returns using current yields, both π_t^* and r_t^* have strong incremental predictive power. Consistent with the intuition from Figure 1, the addition of the equilibrium real rate is crucial later in our sample, when the inflation trend shows less variation. This explains why the fit of the regressions of Cieslak and Povala (2015), who predict bond returns using a moving average of past inflation, has diminished over time. Including i_t^* as a predictor fully captures the relevant information in macro trends, and the predictive gains are economically large: a decline of one percentage point in the trend component predicts an increase in the future annual excess returns by about 7.5 percentage points, as interest rates quickly mean-revert to the lower trend and long-term bond holders benefit, just as they have during the recent period since the Financial Crisis. A parsimonious and effective way to uncover the predictive power in yields is by detrending them, i.e., by focusing on the difference between yields and their underlying macro trend. Our findings extend recent research on predictions of excess bond returns (Cieslak and Povala, 2015; Brooks and Moskowitz, 2017; Garg and Mazzoleni, 2017; Jorgensen, 2017) which documented some gains from including slow-moving averages of past inflation and real GDP or consumption growth as predictors. We show that large gains result from accounting for the underlying macro trends π_t^* and r_t^*, and that the underlying mechanism is mean-reversion of yields to i_t^*. In addition, we provide an explanation why expected returns are not spanned by the yield curve: Because changes in the level of the yield curve can occur due to either movements in i_t^* or level shifts in detrended yields—with very different implications for expectations of future returns—macro trends and yields contain important separate pieces of predictive information.

Third, we turn to out-of-sample forecasting of interest rates. In such forecasting exercises, researchers have found it surprisingly difficult to consistently beat the simple random walk forecast, which predicts future yields with current yields. But we find that simple univariate predictions in which long-term interest rates mean-revert to the shifting endpoint i_t^* leads to substantial forecast gains at medium and long forecast horizons relative to the usual martingale benchmark. These improvements in forecast accuracy are both economically and statistically significant, and they are consistent with the notion of equilibrium correction of yields to their underlying macro trends. Our forecasts also consistently beat long-range projections from the Blue Chip survey of professional forecasters. In related previous work, Dijk et al. (2014) documented some forecast improvements relative to a random walk by including shifting endpoints that are linear projections based on their proxy of π_t^*. We demonstrate that no linear projections are needed and that the right endpoint to use is i_t^*, which importantly includes r_t^*.

Fourth, we investigate the role of macro trends for the term premium and revisit the secular decline in long-term interest rates. We obtain a novel estimate of the term premium using a simple factor model of the yield curve in which three factors of detrended yields follow a first-order vector autoregression (VAR), so that yields revert to a shifting endpoint that is determined by i_t^*. The resulting empirical decomposition of long-term rates into expectations and term premium components starkly contrasts with that from a conventional yield-curve model in which yield factors follow a stationary VAR(1). The conventional decomposition implies an implausibly stable expectations component and attributes most of the secular decline in interest rates to the residual term premium, as discussed in critiques by Kim and Orphanides (2012) and Bauer et al. (2014). Our decomposition instead attributes the majority of the secular decline to the decrease in i_t^*. Consequently, the term premium, instead of exhibiting a dubious secular downtrend, behaves in a predominantly cyclical fashion like other risk premiums in asset prices (Fama and French, 1989). Linking macro trends to the yield curve solves the knife-edge problem of Cochrane (2007), who noted that assuming either stationary or martingale interest rates leads to drastically different implications for the term premium. Assuming a common macro trend, as prescribed by theory, leads to both more accurate forecasts and to more plausible decompositions of long-term rates than either of those previous methods.[4]

As a final avenue of examination, we compare the variance of changes in macro trends to the variance of interest rate changes at different frequencies. Duffee (2016) proposes using the ratio of the variance of inflation news to the variance of yield innovations as a useful metric to assess the importance of inflation in the determination of interest rates. He documents that for one-quarter innovations, this ratio is small for U.S. Treasury yields. We generalize his measure to consider variance ratios for longer h-period innovations, which allows us to compare the size of unexpected changes, over, say, a span of five years, in inflation and in nominal bond yields. For one-quarter changes, we replicate the small inflation variance ratio reported by Duffee. But the inflation variance ratio increases substantially with the horizon, as one would expect if inflation has an important trend component. We also generalize Duffee's measure to incorporate fluctuations in r_t^* and i_t^*. Although confidence intervals are unavoidably wide, our estimates suggest that during the postwar U.S. sample, a large share of the interest rate variability faced by investors over longer holding periods was due to changes in the macroeconomic trend components of nominal yields.

[4] Our analysis of the term premium is related to recent work by Crump et al. (2017), who also allow for slow-moving macroeconomic trends but, in contrast, find that a substantial downward trend in the term premium is the main driver of lower bond yields. The key difference with our approach is their exclusive reliance on survey measures for estimation of i_t^*, which as we discuss below is problematic.

While it has long been recognized that nominal interest rates contain a slow-moving trend component (Nelson and Plosser, 1982; Rose, 1988), our paper is the first empirical work that fully explains this trend by linking it to the macroeconomy. We identify the underlying macroeconomic drivers of i_t^*, and document that these fluctuations are quantitatively important. In previous work, filtering i_t^* from past yield curve data alone has generally proved to be an unsuccessful strategy (Fama, 2006; Dijk et al., 2014; Cieslak and Povala, 2015). Some studies have found a link between the inflation trend and nominal yields (Kozicki and Tinsley, 2001; Dijk et al., 2014; Cieslak and Povala, 2015), but this leaves unexplained the continuing downtrend trend in yields over the last 20 years. We comprehensively document the empirical importance of macro trends for the dynamics of the yield curve, demonstrating the effects of both relevant macro trends, π_t^* and r_t^*. Time variation in r_t^* has so far been largely ignored in finance, which is a substantial oversight given the extensive evidence in the recent macro literature on the equilibrium real interest rate and its structural drivers.

Our work has important implications not only for forecasting of interest rates and bond returns, but also for macro-finance modeling of the yield curve. Existing yield-curve models generally do not account for the crucial link between macro and yield trends. Macro-finance no-arbitrage models of the yield curve (see the references in Footnote 1) generally impose stationary dynamics and do not allow for time-varying macro trends, ruling out the structural, long-run changes which we demonstrate to be empirically important.[5] In light of our findings, it is paramount for yield-curve models to explicitly allow for macroeconomic trends to affect long-run expectations of interest rates.

2 Some theory: macro trends and yields

Absence of arbitrage implies that expectations of future macroeconomic variables are linked to long-term interest rates (Ang and Piazzesi, 2003; Rudebusch and Wu, 2008). Specifically, the yield on a long-term bond is driven by expectations of future inflation and expectations of future real rates, plus a risk premium that depends on the specific asset-pricing model. Here we discuss the implications for yield-curve dynamics if inflation or the real rate contain time-varying trend components.

According to the prevailing consensus in empirical macroeconomics—prominently exemplified by Stock and Watson (2007) and recently surveyed in Faust and Wright (2013)—inflation

[5] Some general-equilibrium macro models allow for changes in the inflation trend that are linked to the yield curve but assume a constant equilibrium real rate (Hördahl et al., 2006; Rudebusch and Wu, 2008). Certain no-arbitrage models developed by Hand Dewachter and coauthors allow for changes in r^* but make strong assumptions such as deterministically linking r_t^* to π_t^* (Dewachter and Lyrio, 2006) or imposing that r_t^* equals trend output growth (Dewachter and Iania, 2011).

is best modeled as an $I(1)$ process if one aims to produce competitive forecasts or accurately capture the evolution of expectations. Hence, a Beveridge-Nelson trend can be defined as

$$\pi_t^* = \lim_{h\to\infty} E_t \pi_{t+h},$$

assuming that inflation does not have a deterministic trend. From a macroeconomic perspective, this time-varying inflation endpoint can be viewed as the perceived inflation target of the central bank. Inflation can thus be modeled as the sum of a (random walk) trend component and a (stationary) cycle component as in this simple formulation:

$$\pi_{t+1} = \pi_t^* + c_t + e_{t+1}, \quad \pi_t^* = \pi_{t-1}^* + \xi_t, \quad c_t = \phi_c c_{t-1} + u_t, \tag{1}$$

where the innovations ξ_t and u_t and the noise component e_t are all *iid*. Expectations at t about future inflation from $t+h$ to $t+h+1$ are given as $E_t\pi_{t+h+1} = \pi_t^* + \phi_c^h c_t$. We will use this specification, which is similar to the one in Duffee (2016), to help illustrate the role of trend and cycle in a bond pricing equation in a no-arbitrage term structure framework.[6]

A similar specification is also relevant for the real interest rate. Structural economic changes, such as changes in the trend rates of productivity and population growth, will affect the equilibrium real rate (see Footnote 3). These provide compelling reasons to allow for the presence of a time-varying trend component in real interest rates, so we also assume that the one-period real rate, r_t, is $I(1)$. We define the equilibrium real rate as the Beveridge-Nelson trend,

$$r_t^* = \lim_{h\to\infty} E_t r_{t+h},$$

which can be understood as the real rate that prevails in the economy after all shocks have died out. We discuss in Section 3 how this definition relates to other empirical and theoretical concepts of what has come to be called "r-star" in the literature. Again a simple parametric specification can best illustrate the implications of the presence of a trend in the real rate:

$$r_t = r_t^* + g_t, \quad r_t^* = r_{t-1}^* + \eta_t, \quad g_t = \phi_g g_{t-1} + v_t, \tag{2}$$

where the cyclical real-rate gap, g_t, captures among other factors variation in the real short rate due to monetary policy (Neiss and Nelson, 2003).

We should stress that the assumption of unit roots in inflation and the real rate is merely

[6] Note that in this specification, the Beveridge-Nelson cycle includes both an AR(1) process (c_t) and measurement error (e_{t+1}). Equation (1) assumes that the shocks ξ_t and u_t affect only expectations of future inflation but not current inflation, which slightly simplifies the bond pricing formulas but has no fundamental significance.

a convenient way to model these very persistent processes. It simplifies the exposition of our model and the arguments regarding trend components, but it is not crucial. Taken literally, a unit root specification is implausible because the forecast error variances of inflation and real rates do not in fact increase linearly with the forecast horizon as predicted by a unit root. Instead, both variables have always remained within certain bounds. However, in finite samples, a stationary process can always be approximated arbitrarily well by a unit root process, and it is well-known that doing so can often be beneficial for forecasting (e.g., Campbell and Perron, 1991). Therefore the unit root assumption is false if taken literally but nevertheless very useful (like all models, according to the famous dictum). The trend components π_t^* and r_t^* can be viewed as highly persistent components of π_t and r_t that capture expectations at the long horizons relevant for investors, even if infinite-horizon expectations are constant. In practice, these relevant time horizons are often in the 5- to 10-year range when cyclical shocks have largely dissipated, as noted by Laubach and Williams (2003) and Summers (2015).

Under the simple parameterization given in equations (1) and (2), and assuming absence of arbitrage, we have the following decomposition for the continuously-compounded nominal yield on a risk-free (government) zero-coupon bond with an n-period maturity:

$$y_t^{(n)} = \underbrace{\pi_t^* + \frac{1-\phi_c^n}{n(1-\phi_c)}c_t}_{\sum_{i=1}^{n} E_t\pi_{t+i}/n} + \underbrace{r_t^* + \frac{1-\phi_g^n}{n(1-\phi_g)}g_t}_{\sum_{i=0}^{n-1} E_t r_{t+i}/n} + CONV^{(n)} + YTP_t^{(n)}, \tag{3}$$

where $CONV^{(n)}$ stands for maturity-specific bond convexity (due to Jensen-inequality terms) and $YTP_t^{(n)}$ is the yield term premium, which in theory captures compensation for duration risk in long-term bonds and the effects of frictions, and in practice is a residual containing all factors other than the expectations component. This equation, which captures our key points, is completely intuitive, but is is also derived in Appendix A from a fully-specified affine term structure model that includes equations (1) and (2) and a specification for the stochastic discount factor and the prices of risk.

The main observation is that because nominal yields reflect expectations of future inflation and real rates, they necessarily share the same trend components. Yields of all maturities contain the trend component $i_t^* = \pi_t^* + r_t^*$, the endpoint for the nominal short rate.[7] As all yields load equally on i_t^* it serves the role of a level factor for the yield curve. Due to the presence of stochastic trends in inflation and the real rate yields are also $I(1)$, whereas

[7]This shifting endpoint i_t^* is the trend component of $i_t = E_t(\pi_{t+1}) + r_t$. In a no-arbitrage model, the nominal short rate in addition to i_t also contains a Jensen inequality term and an inflation risk premium, but both are negligibly small.

detrended yields, $y_t^{(n)} - i_t^*$, are $I(0)$. These detrended yields, or "interest rate cycles" in the parlance of Cieslak and Povala (2015), will play an important role in the empirical analysis below.

The cyclical components c_t and g_t are slope factors as they affect short-term yields more strongly than long-term yields. That the loadings of yields on these factors decline to zero with increasing maturity is particularly easy to see in equation (3) because g_t and c_t follow AR(1) processes, but it is true more generally for stationary yield-curve factors. Since the cycles play a smaller role for long-term yields, we will focus most of our empirical analysis on long-term yields and forward rates, to most clearly see the link between macro trends and the yield curve.

Equation (3) can be viewed as an extended Fisher equation for long-term interest rates. It suggests that loadings on inflation expectations are unity for all maturities, and hence that there is long-run Fisher effect, i.e., that inflation and yields share the same long-run trend. But we have so far focused only on the expectations component of long-term yields, and said little about risk-adjustment and the term premium. Yields are driven by expectations of future short rates under an adjusted, risk-neutral probability measure, and the term premium in (3) captures this adjustment. If pi_t^* directly affected the prices of risk, then $YTP_t^{(n)}$ would systematically vary with changes in π_t^*. In this case, the loadings of long-term yields on π_t^* would not necessarily be unity.[8] The same reasoning of course holds for r_t^*. In other words, there is a clear theoretical prediction about the connection between macro trends and yields through the expectations component, but this could be altered or even partially undone by the term premium. However, we will present evidence that macro trends indeed appear to affect long-term yields one-for-one, suggesting that the any possible links between macro trends and the term premium are not strong enough to appreciably alter the role for macro trends in the yield curve.

While standard theory predicts that the persistent components in inflation and the real interest rate will be reflected in long-term interest rates, the key open question that we consider is whether this link between macro trends and the yield curve matters empirically. How important was variation in i_t^* for the Treasury yield curve? We will demonstrate that accounting for changes in i_t^* substantially alters our interpretation of yield curve movements and our understanding of bond risk premiums.

[8]Technically speaking, we assumed that inflation has a unit root under the real-world probability measure, but this does not necessarily imply that it also has a unit root under the risk-neutral measure. In the model in Appendix A, we additionally assume that the term premium is not systematically affected by macro trends, so that inflation and the real rate consequently also have a unit root under the risk-neutral measure, and yields have unit loadings on macro trends.

3 Data and trend estimates

We now describe the data and the estimates of the macroeconomic trends that we will use in testing the model's predictions. Our data set is quarterly and extends from 1971:Q4 to 2017:Q2. The interest rate data are end-of-quarter zero-coupon Treasury yields from Gürkaynak et al. (2007) with maturities from one to 15 years. We augment these data with three- and six-month Treasury bill rates from the Federal Reserve's H.15 data. In our empirical analysis, we mainly focus on long-term (five-year and ten-year) yields as well as long-term (five-to-ten-year) forward rates to exhibit the importance of r^*_t, π^*_t, and r^*_t, and these are the relevant horizons for our trend measures as well.

For our empirical investigation, we take existing estimates of the macro trends from the literature. Our goal is to assess whether such off-the-shelf measures can provide evidence linking the inflation and real rate trends to the yield curve and risk pricing. An alternative strategy would be to estimate time-varying r^*_t and π^*_t within a no-arbitrage term structure model. We view our approach, which conditions on existing estimates, as an important first step with two important advantages. First, our approach is arguably conservative, because our macro trend estimates have not been fine-tuned to incorporate the information in long-term yields via no-arbitrage restrictions. We avoid using trend estimates from the literature that are derived from long-term yields, such as the estimates of π^*_t by Christensen et al. (2010) or estimates of r^*_t by Johannsen and Mertens (2016), Christensen and Rudebusch (2017), or Del Negro et al. (2017). It would be somewhat tautological to demonstrate a link between long-term bond yields and a trend that was estimated from those yields. Because all of our empirical trend proxies are based only on information in macroeconomic variables, short-term interest rates, and surveys, we avoid any such circularity. Second, the estimation of macro trends, in particular of r^*_t, requires many difficult modeling decisions and, in the case of Bayesian estimation, the choice of priors, all of which have important effects on the properties of the estimated trend series.[9] We prefer to instead use widely-used existing measures of the macro trends and focus on how these trends relate to the yield curve.

Empirical proxies for trend inflation, π^*_t, have been often constructed from surveys, statistical models, or a combination of the two—see, for example, Stock and Watson (2016) and the references therein. We employ a well-known survey-based measure, namely, the Federal Reserve's series on the <u>p</u>erceived inflation <u>t</u>arget <u>r</u>ate, denoted PTR. It measures long-run expectations of inflation in the price index of personal consumption expenditure (PCE), and is widely used in empirical work—see, for example, Clark and McCracken (2013). PTR is

[9] For example, Laubach and Williams (2003) highlight the estimation and specification uncertainty underlying their estimate of r^*_t.

based exclusively on survey expectations since 1979 (i.e., for most of our sample).[10] Figure 1 shows that from the beginning of our sample to the late 1990s, this estimate mostly mirrored the increase and decrease in the ten-year yield. Since then, however, it has been essentially flat at two percent, which is the level of the longer-run inflation goal of the Federal Reserve that was first announced in 2012. Other survey expectations of inflation over the longer run, such as the long-range forecasts in the Blue Chip survey, exhibit a similar pattern.[11]

The recent literature on modeling and estimation of the natural, neutral, or equilibrium real interest rate—commonly referred to as r^*—has grown rapidly. Importantly, there are various closely related definitions and concepts of r^*. Since we require estimates that are consistent with our definition of the equilibrium real rate, it is useful to briefly review these concepts here. In dynamic stochastic general equilibrium models (e.g. Cúrdia et al., 2015), the natural or efficient real rate is the real rate that would prevail in the absence of nominal frictions. This is generally a stationary variable and corresponds to a short-run concept. By contrast, our definition of r^* as the (Beveridge-Nelson) long-run trend component of the real interest rate is a long-run concept. It coincides with the definition used in Lubik and Matthes (2015) who estimate r_t^* as the time-varying mean of the real rate in a time-varying parameter VAR model.[12] Another concept of the natural rate, used by Laubach and Williams (2003) and Kiley (2015), among others, is the real rate at which monetary policy is neither expansionary nor contractionary. In these models the unobserved natural rate is inferred from macroeconomic data using a simple structural specification and the Kalman filter—Laubach and Williams work with a standard IS curve whereas Kiley augments the IS curve with financial conditions. While their r_t^* is a medium-run concept because this neutral policy stance could in principle change over time, it is specified in these models as a random walk, so that the medium-run and long-run concepts coincide and this r_t^* is consistent with our definition. We will therefore use the three model-based estimates of Laubach and Williams (2003), Kiley (2015) and Lubik and Matthes (2015) in our analysis.[13]

Figure 2 plots these three macro estimates of r_t^*, and it shows that since the early 1980s, all

[10]Since 1979, PTR corresponds to long-run inflation expectations from the Survey of Professional Forecasters. Before 1979, PTR is based on estimates from the learning model for expected inflation of Kozicki and Tinsley (2001). For details on the construction of PTR, see Brayton and Tinsley (1996). PTR can be downloaded with the updates of the Federal Reserve's FRB/US large-scale macroeconomic model at https://www.federalreserve.gov/econresdata/frbus/us-models-package.htm.

[11]The inflation trend that Cieslak and Povala (2015) use is a simple weighted moving average of past core inflation, which, as they note, co-moves closely with PTR.

[12]Other estimates of this long-run r_t^* include Johannsen and Mertens (2016) and Del Negro et al. (2017).

[13]Survey-based estimates of r_t^* are problematic for at least two reasons. First, the available time span for interest rate forecasts is limited (the earliest is a biannual Blue Chip Financial Forecasts series that starts in 1986). Second, this would amount to estimating i_t^* as the long-run survey expectations of yields, which leads to inaccurate forecasts as documented in previous studies (e.g., Dijk et al., 2014) and in Section 6.

three have evolved in a broadly similar fashion. A straightforward method to aggregate and smooth the information from these three specific modeling strategies is to take their average, which is the measure of r_t^* we use in our empirical analysis. In the 1970s, 80s, and 90s, this average fluctuated modestly between 2 and 3 percent, which is consistent with the common view of that era that the equilibrium real rate was effectively constant. However, from 2000 to 2017, all of the measures fell, with an average decline of 2.2 percentage points. The equilibrium real rate was likely pushed lower by global structural changes that included slowdowns in trend growth in various countries, increases in desired saving due to global demographic forces and strong precautionary saving flows from emerging market economies, changing demographics, as well as declines in desired investment spending partly reflecting a fall in the relative price of capital goods (Summers, 2015; Rachel and Smith, 2015; Carvalho et al., 2016). A pronounced decline in r_t^* occurred in 2008 during the Financial Crisis, and this decline was followed by almost a decade of sustained low levels of r_t^*, a finding that is common across different models beyond the ones shown here.

Of course, as evident in the original research, there is substantial model and estimation uncertainty underlying the various point estimates of r_t^*. Similarly, our survey-based measure of the long-run inflation trend, π_t^*, is also imprecise. We will show that our measures of the macro trends are closely connected to the yield curve and contain important information for predicting future yields and returns, despite the measurement error that works against finding such links. Classical measurement error would make the coefficients in our regressions both less precise and bias them toward zero. Because our trend proxies are estimates of the true trends using all available information, any measurement error is more likely to be orthogonal to our trend estimates (instead of being orthogonal to the true trend), which would make our estimates noisy but not necessarily biased (Mankiw and Shapiro, 1986; Hyslop and Imbens, 2001). In either case, because of the presence of measurement error our results should be viewed as a lower bound for the tightness of the connection between the yield curve and the *true* underlying macro trends.

Ideally, our trend estimates should reflect information that was available contemporaneously to investors. Having a reasonable alignment of r_t^* and π_t^* to the real-time evolution of investors' information sets is particularly relevant for properly assessing the value of macro trends in predicting future yields and bond returns and determining the term premium in long-term yields (as in Sections 6–7). Since 1979, our survey-based estimate of π_t^* has been available to bond investors at the end of each quarter, when our yields are sampled. Real-time concerns have been more acute for estimates of r_t^* (Clark and Kozicki, 2005). To construct r_t^*, we use filtered (i.e, one-sided) estimates of the equilibrium real rate from the three macroeconomic models cited above. That is, these estimates only use data up to quarter t to infer the unobserved value of r_t^*. While the estimated model parameters are based on the full sample of final revised data, Laubach and Williams (2016) show that truly real-time estimation of their model delivers an estimated series of r_t^* that is close to their final revised estimate over the period that both are available. This suggests that an alternative real-time estimation with real-time empirical trend proxies would likely yield similar results.

Intuitively, our empirical measures of π_t^* and r_t^*—and i_t^*, which is their sum—are consistent with a compelling narrative about the evolution of long-term nominal interest rates, as shown in Figure 1. Starting with the Volcker disinflation of the 1980s, interest rates and inflation trended down together. Around the turn of the millennium, long-run inflation expectations stabilized near 2 percent. However, i_t^* and long-term interest rates continued to decline in part because structural changes in the global economy started pushing down c the equilibrium real rate. The following analysis investigates whether the link between macro trends and the yield curve that underlies this narrative is supported by the empirical evidence, and whether accounting for shifts in i_t^* alters our interpretation of interest rate movements and bond risk premiums.

4 Persistence, unit roots, and cointegration

If the trend components of inflation and the real interest rate play an important role in driving movements in the yield curve, then these macro trends should account for most of the persistence of long-term interest rates. Here we investigate how important empirically i_t^* is for the dynamic properties of yields, consistent with the theoretical discussion in Section 2. A related question is whether changes r_t^* materially contribute to movements in i_t^* and the persistence in bond yields, or whether accounting for π_t^* alone is sufficient. We focus our analysis on the five- and ten-year yields and the five-to-ten-year forward rate as long-term rates give us the cleanest picture of the role of trends.

The key issue is illuminated by considering the raw and detrended interest rate series. Figure 3 shows the ten-year yield, the difference between this yield and π_t^*, and the difference between the yield and i_t^*, where in the first two cases the series is demeaned to enhance the visual comparison. The yield by itself exhibits a clearly trending behavior. Subtracting out π_t^* gives a series that has a less pronounced but still clearly visible downward trend, evident in the substantial decline of about four percentage points from the level prevailing in the 1990s to the end of the sample. Only if we *also* subtract out r_t^*, do we obtain a series that is not obviously trending and has clear mean reversion, i.e., a proper interest rate gap or cycle series. The result is established more formally in the following statistical analysis, which demonstrates that the persistence in long-term rates is very pronounced, that subtracting i_t^* purges most of this persistence, and that the blue line in Figure 3, $y_t^{(10y)} - i_t^*$, is indeed a reasonable measure of the cycle in long-term bond yields.

Table 1 documents the persistence of long-term rates and the macro trends. It reports the standard deviation and two measures of persistence: the estimated first-order autocorrelation coefficient, $\hat{\rho}$, and the half-life, which indicates the number of quarters until half of a given shock has died out and is calculated as $\ln(0.5)/\ln(\hat{\rho})$. The persistence of the interest rates is very high, with first-order autocorrelation coefficient of 0.97 and a half-life between 21 and 25 quarters. The macro trends and i_t^* are even more persistent: Our equilibrium real-rate series has an autocorrelation coefficient of 0.98 and a half-life of about 30 quarters, and the inflation trend and i_t^* have autocorrelation coefficients of 0.99 and half-lives of 86 and 67 quarters, respectively. We also examine the persistence properties of these series by testing the null hypothesis of a unit autoregressive root, and report the following test-statistics in Table 1: the parametric Augmented Dickey-Fuller (ADF) t-statistic, the non-parametric Phillips-Perron (PP) Z_α statistic, and the efficient DF-GLS test statistic of Elliott-Rothenberg-Stock (ERS).[14] All three tests agree that we cannot reject a unit root in these series. In addition, the low-frequency stationarity test of Müller and Watson (2013), for which the p-values are reported in the last column of Table 1, strongly rejects stationarity for each series. In sum, long-term yields and macro trends are highly persistent and can be effectively modeled as $I(1)$ processes.

In light of this evidence, the question naturally arises whether this persistence is driven by the same underlying trend, that is, whether there is a cointegration relationship between long-term rates and the macro trend estimates. A first test considers the cointegration rank r of $Y_t = (y_t, \pi_t^*, r_t^*)'$, where y_t is either the five- or ten-year yield or the five-to-ten-year forward rate. Table 2 reports the results of the Johansen (1991) trace test for the cointegration rank r.[15] For all three rates, the hypothesis $r = 0$ (no cointegration) is strongly rejected against the alternative $r > 0$. The hypothesis that $r = 1$, however, is accepted. These results strongly suggest that there is exactly one cointegration vector among any long-term rate, the inflation trend and the equilibrium real rate.

[14] For the ADF test, we include a constant and k lagged difference in the test regression, where k is determined using the general-to-specific procedure suggested by Ng and Perron (1995). We start with $k = 4$ quarterly lags and reduce the number of lags until the coefficient on the last lag is significant at the ten percent level. For the PP test, we use a Newey-West estimator of the long-run variance with four lags. When the series under consideration is a residual from an estimated cointegration regression, we don't include intercepts in the ADF or PP regression equations and use the critical values provided by Phillips and Ouliaris (1990), which depend on the number of regressors in the cointegration equation. For the ERS test we use four lags.

[15] The test uses two lags of Y_t in the VAR representation, based on information criteria.

We next turn to the nature of the relationship between the macro trends and long-term rates, including the individual roles of π_t^* and r_t^* as well as inference about the cointegration vector β. A natural starting point is a simple regression of yields on the trend components.[16] Table 3 reports the results for such regressions with the three long-term rates as dependent variable. In each case, we estimate two versions of the regressions (with standard errors calculated using the Newey-West estimator with six lags). The first version has only a constant and π_t^* as regressors, which is the same regression that Cieslak and Povala (2015) estimated using their simple moving-average estimate of the inflation trend (see their table 1). These regression results show high R^2's at all maturities and π_t^* coefficients that are just above one and highly significant.[17] Cieslak and Povala (2015) interpret these results as indicating that trend inflation drives the level of yield curve. However, the results for the second regression specification show that incorporating the real rate trend is also important. Indeed, with the addition of r_t^* to the regressions, both the inflation and real rate trends coefficients are highly significant, and the regression R^2's increase a further 7 to 12 percentage points.

Taken at face value, these estimates suggest that changes in r_t^* along with fluctuations in π_t^*, are key sources of variation in long-term interest rates. The interpretation of these results is complicated by the fact that all of the variables in the regressions are very persistent and behave like $I(1)$ variables, as shown in Table 1. If the variables are also cointegrated, as the evidence in Table 2 strongly suggests, it is well-known (e.g., Hamilton, 1994, Chapter 19) that these linear regressions provide (superconsistent) estimates of β and the R^2 converges to one. However, conventional hypothesis tests about the coefficients, such as the Newey-West standard errors we report, are valid only under additional assumptions about the dynamic interactions among the variables. Reliable inference can be obtained using "Dynamic OLS" where leads and lags of first-differences of the regressors are included in the regressions, or using the reduced-rank VAR estimation of Johansen (1991). Both analyses lead to the same conclusions about β as Table 3, though we omit details here for brevity.

All in all, the results suggest that the cointegrating coefficients on both π_t^* and r_t^* are close to one or slightly higher. One important question is whether β can be approximated by $(1, -1, -1)$, which is an intuitively appealing choice that simplifies the detrending of interest rates. The unit coefficients in the cointegration vector is also supported by the theory in

[16] Much empirical work, for example, King et al. (1991), has documented the substantial persistence in nominal interest rates, inflation, and real interest rates. The main difference between our static regressions and the usual cointegration regressions in this context (as in Rose, 1988, for example) is that we use directly observable proxies for the trend components of π_t and r_t.

[17] Our estimated coefficients on π_t^* are somewhat higher than in Cieslak and Povala (2015) because our measure of the inflation trend is less variable, though when r_t^* is added, the estimated coefficients for π_t^* decrease toward one.

Section 2, which predicted that yields are affected one-for-one by changes in π_t^* and r_t^* unless the macro trends interact with the term premium in such a way as to substantially alter the effects through expectations. For the long-term forward rate, we can't reject the null that $\beta = (1, -1, -1)$, suggesting that $f_t^{(5y,10y)} - \pi_t^* - r_t^*$ is not stochastically trending, i.e., stationary. For the ten-year yield, and particularly for the five-year yield, there is some evidence that the coefficients on macro trends are above one, but only slightly so. We will show below that using $\beta = (1, -1, -1)$ works very well in practice for detrending even the five-year and ten-year yields. Importantly, the cointegration relationship involves both macro trends, which in turn implies that a regression of long-term rates on π_t^* alone is misspecified and detrending long-term rates by only π_t^* is insufficient.

How much persistent variation in long-term rates is captured by our measures of π_t^* and r_t^*? To address this question we examine the time series properties of detrended long-term interest rates with one or both of the trend components subtracted out, which are reported in Table 1. We consider four different ways of detrending interest rates: subtracting out either π_t^* or i_t^* or using the residuals from each of the two regressions in Table 3. Several findings stand out: First, detrending with r_t^* as well as π_t^* removes substantially more persistence, typically reducing the half-life by about 40-50%. That is, π_t^* is not the only important driver of interest rate persistence. Second, the detrended series are substantially less variable and less persistent than the original interest rate series. For example, shocks to the ten-year yield have a half-life of about 5-1/2 years, whereas shocks to the difference between the ten-year yield and i_t^* have a half-life of just under one year. Clearly, a very substantial share of the persistence in interest rates is accounted for by i_t^*. Finally, although detrending by calculating residuals generally leads to series that are less persistent than those that are simple differences, if we detrend with both macro trends than the simple differences do almost as well. In particular for the forward rate the two series have very similar properties, because the regression coefficients are already quite close to one. Even detrending with simple differences, i.e., using the cointegrating residual for $\beta = (1, -1, -1)'$, accounts for a large share of the persistence in interest rates, as long as we use i_t^* and not only π_t^*.

Unit root tests provide further evidence supporting detrending with both r_t^* and π_t^*. These tests show strong evidence against the unit root null for the series that are detrended with both π_t^* and r_t^*. By contrast, the unit root null is never rejected at the five percent level for the original interest rate series or for series that are detrended with just π_t^*. When detrending with both π_t^* and r_t^*, the ADF and PP tests, as well as the ERS test in the case of the forward rate, find equally strong or even stronger evidence against a unit root for the simple differences as for the residuals. For the five- and ten-year yields, the ERS test rejects more strongly for the residuals than for the simple differences, but it still rejects the unit root at the ten-percent level for simple differences. Finally, the LFST test supports the view that $y_t - i_t^*$ is stationary for the ten-year yield and the forward rate, and for the five-year yield, it only marginally rejects this hypothesis.

These results have implications for the debate about the long-run Fisher effect, which posits a common trend for inflation and interest rates with a unit coefficient (leaving aside tax considerations). A sizeable literature has tested this hypothesis with mixed results, often estimating an inflation coefficient that is significantly larger than one (see Neely and Rapach (2008)). Our evidence indicates that the series $y_t - \pi_t^* - r_t^*$ is stationary, which provides support for a long-run Fisher effect if shifts in the equilibrium real rate are taken into account. The importance of time variation in r_t^* can explain why past research has generally been unable to find a stable relationship between nominal interest rates and inflation. If yields are regressed only on inflation or an inflation trend, the regression is misspecified as the residual contains the omitted trend r_t^*. Table 3 shows that the coefficients on π_t^* are substantially larger in regressions when r_t^* is excluded, which may explain why it has been difficult to uncover the Fisher effect.

A final question in this context is whether and how quickly yields respond to shifts in the trends. To uncover this dynamic response, we estimate a standard error-correction equation for each of the long-term rate series, using $\beta = (1, -1, -1)$. First-differenced rates, Δy_t are regressed on the error-correction term $(y_{t-1} - \pi_{t-1}^* - r_{t-1}^*)$, an intercept, and four lags of Δy_t, $\Delta \pi_t^*$ and r_t^*. Table 4 shows that the error-correction coefficient is estimated to be significantly negative, indicating that when long-term are high relative to i_t^* they subsequently fall back toward this trend. That is, yields exhibit strong equilibrium correction. As before, the result is particularly strong for the forward rate, which has a highly significant coefficient of -0.22, so a percentage point deviation from the trend is followed by 22 basis points of reversion to the trend within one quarter. A Wald test shows strong evidence against the hypothesis that the macro trends do not Granger-cause interest rates. This evidence further supports the view that interest rates should be jointly modeled with their underlying macro trends, in particular when it comes to forecasting their future evolution.

5 Predicting excess bond returns

The theoretical discussion and evidence above suggests that knowledge of the macroeconomic trends underlying yields—and in particular of the current level of yields *relative to their trend* i_t^*—is important for understanding the evolution of bond yields. We now examine whether such trends can improve predictions of the excess return of long-term bonds over the risk-free interest rate. Expected excess returns capture bond risk premiums and have long been of interest in financial economics (Fama and Bliss, 1987).

The excess return for a holding period of h quarters for a bond with maturity n is

$$rx_{t,t+h}^{(n)} = p_{t+h}^{(n-h)} - p_t^{(n)} - hy_t^{(h)} = -(n-h)y_{t+h}^{(n-h)} + ny_t^{(n)} - hy_t^{(h)}.$$

where $p_t^{(n)}$ denotes the log-price of a zero-coupon bond with maturity of n quarters. We predict the average excess return for all bonds with maturities from two to 15 years, $\overline{rx}_{t,t+h}$, for holding periods of one quarter and four quarters.[18] Since Fama and Bliss (1987) and Campbell and Shiller (1991), it is well-known that the yield curve, and in particular its slope, contains information useful for predicting future excess returns. The key question is whether the current yield curve contains *all* of the information relevant for predicting future returns, that is, whether the *spanning hypothesis* holds. Several studies (including Ludvigson and Ng, 2009; Joslin et al., 2014; Cieslak and Povala, 2015) have documented apparent violations of the spanning hypothesis using various additional predictors. Bauer and Hamilton (2016) demonstrated that inference in these predictive regressions suffers from serious small-sample econometric problems arising from highly persistent predictors, and that accounting for these problems renders most of the proposed predictors insignificant. They found, however, that the proxy for trend inflation of Cieslak and Povala (2015) was a relatively robust predictor. Here we investigate whether including both π_t^* and r_t^* leads to even stronger predictive gains and rejections of the spanning hypothesis, and whether the reversion to i_t^* that was indicated by our error-correction model explains these predictive gains.

Table 5 reports the results for four different predictive regressions: The first is the common baseline specification that includes only a constant and the first three principal components (PCs) of yields.[19] The second specification just adds π_t^*, and the third specification also includes r_t^* in order to simultaneously capture the effects of both macroeconomic trends. The fourth specification includes their sum i_t^* instead of the two separate macro trends. We report conventional asymptotically robust standard errors, as well as small-sample p-values for the macro trends using the parametric bootstrap of Bauer and Hamilton (2016) to avoid the serious size distortions they document in tests of the spanning hypothesis with persistent predictors.[20]

[18]Our long-term bond yields are available only at annual maturities, so we calculate one-quarter returns with the usual approximation $y_{t+1}^{(n-1)} \approx y_{t+1}^{(n)}$.

[19]We scale the PCs such that they correspond to common measures of level, slope and curvature, as in Joslin et al. (2014). For example, the loadings of yields on PC1 add up to one.

[20]For the conventional estimates, we report White's heteroskedasticity-robust standard errors for the case of one-quarter returns and Newey-West standard errors with six lags for the four-quarter returns. For the bootstrap, we simulate 5000 artificial data for yields and predictors under the spanning hypothesis, using

In the full sample, the inclusion of an inflation trend increases the predictive power quite substantially compared to only including yield-curve information: both the inflation trend and the level of yields (PC1) appear highly significant. This parallels the findings of Cieslak and Povala (2015). However, adding r_t^* to the regressions leads to further impressive gains in predictive power. For both one-quarter and four-quarter returns, the R^2 increases substantially, the coefficients and significance for π_t^* and PC1 rise, and the coefficient on r_t^* itself is large and highly significant. Not surprisingly given Figure 1, these results shift in the recent period as the real-rate trend has gained in importance over time relative to the trend in inflation. In the subsample starting in 1985, the inflation trend is not statistically significant when included on its own according to the small-sample p-values.[21] Only with the addition of the equilibrium real rate, do both trends matter for bond risk premiums; the coefficients on π_t^* and PC1 more than double, the R^2 increases substantially, and the coefficients on π_t^* and r_t^* are statistically significant. Altogether, our empirical analysis of long-term interest rates implies that the trend in the real interest rate is as important as, and recently more important than, the trend in inflation.[22]

Furthermore, the values of estimated coefficients have a useful interpretation. First, the similar magnitude of the coefficients on the two individual macro trends suggests that only their sum matters. Indeed, a Wald test does not reject equality of the coefficients for either sample or for any of the holding periods. Applying this restriction and including just the resulting i_t^* in the regression provides similarly strong predictive gains. That is, the key to forecasting excess bond returns is some measure of the overall trend in interest rates. Second, the coefficient on i_t^* has a negative sign and a similar, if slightly larger, absolute magnitude as the positive coefficient on the yield-curve level (PC1). The intuition is that if the trend falls then interest rates also fall in response, producing gains for long-term bond holders. For an annual holding period, a decrease in i_t^* by one percentage point predicts an increase in future excess returns by about 7.5 percentage points. These results document the economic significance of this mechanism and confirm the influence of trends on yields uncovered in Section 4.

separate bias-corrected VAR(1) models for yield factors and predictors. The p-values are the fractions of simulated samples in which the t-statistics of the macro trends are at least as large (in absolute value) as in the actual data.

[21] This result is consistent with Bauer and Hamilton (2016) who in their investigation of the evidence of Cieslak and Povala (2015) also found that in a subsample starting in 1985 the inflation trend is only marginally significant.

[22] In additional, unreported results we have found that the predictive gains from including r_t^* stem mainly from the period since the early 2000s when both r_t^* and long-term interest rates decreased while long-run inflation expectations where anchored close to two percent. Correspondingly, in samples that exclude this later period where r_t^* variation was most pronounced, only π_t^* has significant predictive power.

In the presence of persistent predictors, it is generally difficult to interpret the magnitude of R^2 as a measure of predictive accuracy, because even predictors that are irrelevant in population can substantially increase R^2 in small samples (Bauer and Hamilton, 2016). We can avoid this pitfall by using the bootstrap to generate small-sample distributions of R^2 under the spanning hypothesis and interpret the statistics obtained in the actual data by comparing them to the quantiles of these distributions. Table 6 reports this comparison for predictive regressions of annual excess returns for the four specifications we have considered so far, as well as for two additional ones that will be discussed below. Adding π_t^* to the regression increases R^2 by 20 percentage points, but this is only barely higher than the upper end of the 95%-bootstrap interval, which suggests that under the null hypothesis it would not be too uncommon to observe an increase in R^2 of up to 19 percentage points. In contrast, adding r_t^* increases R^2 to 54%, and the increase relative to the yields-only specification of 31 percentage points is much higher than what is plausible under the null hypothesis. In the post-1985 subsample, the increase in R^2 from only adding π_t^* is not statistically significant, whereas the increase of 29 percentage points from adding both trends is strongly significant. Adding just i_t^* instead of the individual macro trends leads to very similar gains in R^2 which are highly significant.

Our findings so far suggest that the predictive power is really contained in *detrended yields*. We now consider predictive regressions using yields that are detrended by simply subtracting the trend, as motivated by the theory in Section 2 and our evidence in Section 4. One way to assess whether such regressions have similar predictive power is with a hypothesis test of the restriction that is imposed on a regression with three PCs of yields and i_t^* when we instead use the same linear combinations of detrended yields.[23] In the full sample, a Wald test strongly rejects this restriction, while in the post-1985 sample the null is not rejected. To gauge the economic significance, we compare regression R^2 of restricted and unrestricted regressions in Table 6. The bottom two rows provide results for predictions using linear combinations of yields that are detrended either with $y_t^{(n)} - \pi_t^*$ or $y_t^{(n)} - i_t^*$. We first note that using yields that are detrended only by π_t^* leads to increases in R^2 over the yields-only baseline regression that are insignificant, while detrending with i_t^* leads to much larger increases R^2 that are highly significant. The difference is even more striking in the later subsample that starts in 1985: R^2 increases only seven percentage points when detrending with only π_t^* but 28 percentage points when detrending with both π_t^* and r_t^*. In the full sample, regressions with detrended yields give somewhat lower R^2 than regressions with yields and macro trends, but the sampling

[23] The unrestricted regression is $\overline{rx}_{t,t+h} = \beta_0 + \sum_{i=1}^{3} \beta_i w_i' Y_t + \beta_4 i_t^* + u_{t+h}$, where Y_t is a vector with all yields and w_i are the loadings of yields on the i'th PC. The regressors of the restricted regression are $w_i'(Y_t - \iota i_t^*)$ where ι is a vector of ones. Hence the null hypothesis of interest is $\beta_4 = -\sum_{i=1}^{3} \beta_i w_i' \iota$.

variability of these R^2 is large. In the post-1985 subsample, the restricted specifications with detrended yields achieve similar predictive power as the unrestricted specifications.

In sum, accounting for the persistent components of yields is important for understanding return predictability and estimating bond risk premiums. We find that r^*_t has strong incremental predictive power for bond returns, about on par with the importance of π^*_t as a predictor, suggesting that both macro trends need to be accounted for accurate estimation of bond risk premiums. The predictive power in the yield curve is fully revealed if they are detrended, but it is crucial to use i^*_t instead of π^*_t for the detrending. Finally, little is lost if detrending is performed by simply taking the difference between yields and i^*_t.

While these results are strong evidence against the spanning hypothesis, which is implied by essentially all asset pricing models (Duffee, 2013), existing macro-finance models can be readily reconciled with evidence of unspanned predictability. In particular, the addition of very small bond yield measurement errors—with standard errors of just one or two basis points—makes it practically impossible to infer all relevant information from observed yields (i.e., to back out the state variables) (Duffee, 2011b; Cieslak and Povala, 2015; Bauer and Rudebusch, 2017). For the case of macro trends, this problem is particularly acute. There are two level factors with very similar yield loadings: i^*_t on the one hand, and the first principal component of detrended yields on the other hand.[24] Because of measurement error, it is difficult to attribute any observed level shift to i^*_t or to the level of yields relative to i^*_t. Therefore, in practice, yields and macro trends contain separate pieces of important predictive information.

6 Out-of-sample forecasts of interest rates

We now turn to pseudo out-of-sample (OOS) forecasts of long-term interest rates. Despite many advances in yield curve modeling, the random walk model has proven very hard to beat when forecasting bond yields, due to the extreme persistence of interest rates (e.g., Duffee, 2013). But our results so far suggest that one might be able to obtain more accurate forecasts by accounting for the interest rate trends.

In the presence of trends in inflation or the real rate, yields exhibit a "shifting endpoint" (Kozicki and Tinsley, 2001). Specifically, no-arbitrage theory implies, as evident from (3), that

$$y_t^{(n)*} \equiv \lim_{h\to\infty} E_t y_{t+h}^{(n)} = k^{(n)} + \pi^*_t + r^*_t = k^{(n)} + i^*_t,$$

[24]The fact that i^*_t is a level factor is suggested by the coefficients on π^*_t and r^*_t in Table 3, and can be seen most clearly from regressions of yields across all maturities on i^*_t. A principal component analysis of detrended yields, taken either as the residuals of such regressions or as simple differences with i^*_t, reveals another level factor. Results are omitted for the sake of brevity.

where the constant $k^{(n)} = CONV^{(n)} + \overline{YTP}^{(n)}$ captures convexity and the unconditional mean term premium. This implies that long-horizon forecasts of interest rates that incorporate knowledge of i_t^* should be more accurate than forecasts that ignore it. The forecast method we propose uses the endpoint $y_t^{(n)*} = i_t^*$ based on our macro estimates of π_t^* and r_t^*. For parsimony, we set the constant $k^{(n)}$ to zero to avoid introducing additional estimation uncertainty.[25] The other necessary ingredient of our forecast method is a transition path from $y_t^{(n)}$ to $y_t^{(n)*}$, and we simply use a smooth, monotonic path from a fitted first-order autoregression for $y_t^{(n)} - y_t^{(n)*}$.[26] Denoting the (recursively) estimated autoregressive coefficient as $\hat{\rho}_t$, the forecasts are thus constructed as

$$\hat{y}_{t+h}^{(n)} = \hat{\rho}_t^h y_t^{(n)} + (1 - \hat{\rho}_t^h) y_t^{(n)*}. \tag{4}$$

We denote this forecast method as *ME* for macro endpoint.[27]

We compare this model to a driftless random walk as the benchmark, i.e., $\hat{y}_{t+h}^{(n)} = y_t^{(n)}$ for all h (denoted as RW). In addition, we consider shifting-endpoint forecasts that only use the information in π_t^*, in order to assess the importance of incorporating macro estimates of r_t^* in interest-rate forecasts. Specifically, this method uses $y_t^{(n)*} = \pi_t^* + \mu^{(n)}$, where the constant is recursively estimated as the mean of $y_t^{(n)} - \pi_t^*$.[28] We denote these "inflation-only" forecasts as *IO*. Finally we include forecasts from a constant-endpoint model, namely a stationary AR(1) process (AR).

We forecast the five- and ten-year yields and the five-to-ten-year forward rate. At each point in time, starting in 1976:Q1 (at $t = 20$) when five years of data are available, we forecast each interest rate at horizons (h) of 4, 10, 20, 30, and 40 quarters. As indicated above, forecasts are constructed using a recursive scheme, i.e., using all data available up to the forecast date to estimate parameters.[29] Table 7 reports the root-mean-squared errors (RMSEs) and mean-absolute errors (MAEs), in percentage points. We also calculate p-values for tests of equal

[25] We have found that including an estimated constant generally worsens forecast performance.

[26] In a fully-specified model with yields and macro trends—even in the simple no-arbitrage model in Appendix A—the speed of mean reversion to $y_t^{(n)*}$ depends on the importance of the different cyclical factors. Our simple method corresponds to the special case where all cyclical factors have the same speed of mean reversion. Notably, the exact speed of mean reversion affects mainly short-horizon forecasts and is inconsequential for our main results.

[27] Note that this approach could easily be extended to provide joint forecasts of the entire yield curve, for example, extending Diebold and Li (2006) and Dijk et al. (2014) by simply forecasting the Nelson-Siegel level factor in the same fashion.

[28] We found that forecasts which assume that this constant is zero are much less accurate, which is unsurprising since they counterfactually assume that $k^{(n)} + r_t^* = 0$.

[29] A rolling scheme, which uses only a fixed number of observations for parameter estimation, allows for an easier asymptotic justification of tests for predictive ability (Giacomini and White, 2006) but requires a specific choice of the window length. We have also obtained forecasts with such a scheme, using a variety of different window lengths, and found equally strong forecast gains for model *ME* as using a recursive scheme.

finite-sample forecast accuracy using the approach of Diebold and Mariano (1995) (DM).[30] We calculate these DM p-values, using standard normal critical values, for one-sided tests of the null hypothesis that our proposed model *ME* does not improve upon the *RW* and *IO* forecasts. We find that model *ME* achieves substantial and statistically significant gains in forecast accuracy at long horizons. Such gains are evident for both RMSEs and MAEs, but are larger and more strongly significant for absolute-error loss. For example, when forecasting the ten-year yield five years ahead, model *ME* lowers the RMSE by over 25% relative to *RW*, an improvement that is significant at the five-percent level, while the MAE drops by more than 40% and is significant at the one percent level. Model *ME* also improves upon *IO* by a magnitude that is typically large and statistically significant.[31]

These results document that at long horizons one can significantly improve upon random walk interest rate forecasts by incorporating macroeconomic information on i_t^*, the underlying trend in interest rates. In contrast to the random walk forecast, which simply assumes all changes are permanent, using estimates of i_t^* captures the underlying source and share of the highly persistent changes in interest rates with large benefits for forecast accuracy. Furthermore, it is clearly important to incorporate macro estimates of r_t^* in addition to information on π_t^*, because this substantially improves the accuracy of the estimated shifting endpoint in yields.[32] Earlier research by Dijk et al. (2014) found that when forecasting interest rates it is beneficial to link long-run projections of interest rates to long-run expectations of inflation, but this ignores the advantages of recognizing the time variation in r_t^*. In addition, our results also show that there is no need to estimate a constant or to scale the endpoint (as in Dijk et al., 2014) once a macro-based estimate of r_t^* is incorporated into i_t^*.

Finally, we compare the accuracy of our statistical models to that of professional forecasters for predicting the ten-year yield. Since 1988, the Blue Chip Financial Forecasts (*BC*) survey has asked its respondents for long-range forecasts twice a year. The respondents provide their average expectations of the target variable for each of the upcoming five calendar years and for the subsequent five-year period—we will focus on the five annual forecast horizons.[33]

[30] Following common use, we construct the DM test with a rectangular window for the long-run variance and the small-sample adjustment of Harvey et al. (1997). Monte Carlo evidence in Clark and McCracken (2013) indicates that this test has good size in finite samples. However, for very long forecast horizons the long-run variance is estimated with considerable uncertainty as in those cases there are only few non-overlapping observations in our sample.

[31] We have found in additional, unreported analysis—using plots of differences in cumulative sums of forecast errors over time—that the forecast gains of *ME* are not driven by certain unusual sub-periods, but instead are a consistent pattern over most of our sample period.

[32] The source of the forecast gains of model *ME* relative to *IO* is the lower level of r_t^*. Model *IO* uses in effect the (recursively estimated) mean of the difference between yields and π_t^* as its estimate of r_t^*, which over most of the sample period was too high.

[33] For survey dates in the fourth quarter, one calendar year is typically skipped. We use the exact years from

We match the available information sets by using only data up to the quarter preceding the survey date for our model-based forecasts, and we exactly match the forecast horizons with the BC forecasts by taking averages of model-based forecasts over the relevant calendar years. The sample includes 46 forecast dates from March 1988 to December 2010. Table 8 shows the RMSEs and MAEs of the survey forecasts and the four model-based forecasts. Shifting-endpoint forecasts based on i_t^* improve substantially over both RW and BC forecasts in this sample. The gains relative to RW are strongly significant for horizons beyond the first calendar year, and the gains relative to BC are significant at the five- or ten-percent level. The reason for the poor performance of the survey forecasts is that they consistently over-predict future yields at these long horizons: The difference between long-range survey forecasts of the ten-year yield and our (survey-based) estimate of π_t^* is much larger than our macro-based estimate of r_t^* (results not shown). Other studies have documented the poor performance of survey forecasts of interest rates (e.g., Dijk et al., 2014), which contrasts with the very good performance of survey-based inflation forecasts(Ang et al., 2007; Faust and Wright, 2013). Our results suggest that the underlying reason for this poor performance is that professional forecasters have in the past overestimated the trend component in interest rates.

7 The term premium in long-term yields

The term premium is defined as the difference between holding an n-period bond to maturity or facing a sequence of one-period rates over the same period:

$$TP_t^{(n)} = y_t^{(n)} - \frac{1}{n}\sum_{j=0}^{n-1} E_t y_{t+j}^{(1)}.$$

Estimation of the term premium requires expectations of future short-term rates, which are commonly obtained from a stationary VAR that underlies either a no-arbitrage model or a simple factor model. However, our results suggest that the stationarity assumption is problematic for long-horizon forecasts. We now investigate how it affects term premium estimates if we instead allow for a trend component in interest rates based on equation (3) and the underlying macro trends.

Decompositions of long-term interest rates into short-rate expectations and term premiums are commonly obtained from a variety of dynamic models, usually with a factor structure for yields and often with no-arbitrage restrictions on the factor loadings. Our approach will closely follow Cochrane (2007), who estimated simple VAR model for interest rates without no-

the actual survey to line up our forecasts.

arbitrage restrictions. Other examples of this approach include Duffee (2011a) and Joslin et al. (2013), who also showed that imposing no-arbitrage restrictions generally has little effect on the model-implied expectations or forecasts. Like Cochrane, we estimate a first-order annual VAR(1), motivated by the finding of Cochrane and Piazzesi (2005) that some patterns of yield predictability are more evident at the annual frequency. Our data are quarterly, and the VAR includes three PCs of 15 Treasury yields with maturities from one year to 15 years. Our estimation sample is from 1971:Q4 to 2007:Q4; we omit the period of near-zero short-rates beginning in 2008, since the lower bound on nominal interest rates poses problems for linear factor models (Bauer and Rudebusch, 2016).

As a baseline for comparison, we first estimate a model using the PCs of yield levels. This stationary VAR has economic implications that are essentially identical to those of Cochrane's "VAR in levels" model and very similar to the implications of the vast majority of existing no-arbitrage yield-curve models. In the top-left panel of Figure 4, we plot the current one-year yield and model-implied expectations of its future value at different horizons. Paralleling Cochrane's findings, the stationary VAR implies that expectations quickly revert to the unconditional mean of the short rate (which is 6.5 percent). The top-right panel of Figure 4 shows the five-to-ten-year forward rate with its expectations and term premium components. (Results for the ten-year yield are qualitatively similar.) Not surprisingly, in light of the behavior of model-implied forecasts, the expectations component is very stable, hovering around the short-rate mean. Therefore, the term premium, as the residual component, has to account for the trend in the long-term interest rate since the 1980s. As argued by Kim and Orphanides (2012) and Bauer et al. (2014), such behavior of expectations and term premium components appears at odds with observed trends in survey-based expectations (Kozicki and Tinsley, 2001) and the cyclical behavior of risk premiums in asset prices (Fama and French, 1989).

The term structure literature has proposed a number of different remedies to avoid such counterfactual decompositions of long-term rates into expectations and term premium. These modifications generally have the goal of making long-run expectations more variable and hence more plausible, and they include restrictions of risk prices in no-arbitrage models (Cochrane and Piazzesi, 2008; Bauer, 2017), bias correction of interest rate VARs (Bauer et al., 2012), and incorporation of survey-based expectations of future interest rates (Kim and Wright, 2005; Kim and Orphanides, 2012). We propose a different remedy, namely, by incorporating a macroeconomic trend component into forecasts of the yield curve consistent with the theory in Section 2. In our "macro-trend VAR" we impose that yields mean-revert to i^*_t plus a maturity-specific constant, where i_t^* as before is based on our proxies of macro trends.[34] This is easily accomplished by using our VAR model for detrended yields. Specifically, to calculate short-rate forecasts we subtract i_t^* from observed yields, estimate the VAR on three PCs of detrended yields, forecast the detrended one-year yield, and finally add i_t^* back in. This yield-curve model is closely related to the *ME* forecast method in Section 6, with the difference that the model here jointly captures the entire yield curve.

The bottom two panels of Figure 4 show the implications of the macro-trend VAR for expectations and the term premium. As the forecast horizon increases, short-rate expectations approach the trend component i_t^* instead of the unconditional mean of the short rate. Consequently, the expectations component reflects the movements in i_t^* and accounts for the low-frequency movements in the long-term forward rate. The term premium, by contrast, behaves in a cyclical fashion with no discernible trend. The drop in the forward rate from its average during 1980-1982 to its average during 2005-2007 was about 7.5 percentage points. In the conventional (stationary) VAR of interest rates, the estimated term premium accounts for over 80% of this decline. In contrast, the VAR with detrended yields attributes only a quarter of this decline to the term premium and the majority to low-frequency movements in the expectations component, in line with the substantial downward shift in the underlying macro trends. This stark difference demonstrates how accounting for the slow-moving trend component in interest rates fundamentally alters our understanding of the driving forces of long-term interest rates, and bridges the gap between the common wisdom of secular macroeconomic trends and statistical models for the yield curve.

Using a macro-trend VAR solves the knife-edge problem of Cochrane (2007), who pointed out that assuming either stationarity or a random walk for the level of interest rates leads to drastically different implications for expectations and term premiums. While imposing a random walk for the level is known to forecast well, it also leads to the implausible implication that the expectations accounts for essentially all variation in yields, as emphasized by Cochrane. Our results show that for empirical work we can assume that interest rates are I(1) and have a common trend that is estimated from macroeconomic data. Such a formulation avoids both extremes and produces interest rate forecasts that outperform the random walk and term premium estimates that are in line with the common macro-finance priors.

Because of the crucial importance of the macro trend for estimates of the term premium, it is important to first validate that the trend measure indeed captures the low-frequency movements in the yield curve (see Sections 4–6). In recent work that parallels our term premium analysis, Crump et al. (2017) also allow for slow-moving macroeconomic trends in

[34] The constant terms are generally small and could be restricted to zero, in line with our evidence on forecast performance in Section 6.

the yield curve but, in stark contrast to our results, find that "term premiums are the main drivers of bond yields." However, their survey-based measure of i_t^* captures little of the overall downward drift in interest rates, so the residual term premium instead trends down. This highlights the importance of using an accurate, validated estimate of the macro trend underlying bond yields.

8 Variance contributions

Our results suggest that changes in macroeconomic trends and i_t^* should play an important role in accounting for interest rate changes at low frequencies, i.e., over intervals of several years. To complement these results, we use variance ratios to compare the size of fluctuations in r_t^*, π_t^* and i_t^* with those of long-term nominal bond yields. This analysis extends that of Duffee (2016), who considered only quarterly changes in longer-term inflation, by considering movements in both r_t^* and π_t^* and intervals longer than one quarter.

The variance ratio used by Duffee divides the variance of quarterly innovations to average inflation expectations over n periods by the variance of innovations to the bond yield for maturity n. We generalize this measure to allow for innovations to occur not only over one quarter but to accumulate over h quarters:

$$VR_h^{(n)} = \frac{Var\left((E_t - E_{t-h})n^{-1}\sum_{i=1}^{n}\pi_{t+i}\right)}{Var\left((E_t - E_{t-h})y_t^{(n)}\right)}$$

An important result of Duffee's analysis is that even for long-term bonds, $VR_1^{(n)}$ appears to be surprisingly small. That is, one-period changes in expected average future inflation are much less variable than one-period surprises in long-term bond yields. This result can be interpreted using equation (3). One possible explanation is that at a quarterly frequency inflation expectations move much less than the term premium component of long-term interest rates. This interpretation is consistent with additional evidence in Duffee's paper on the role of the term premium and with a large body of evidence on excess volatility of interest rates, going back to Shiller (1979).

But quarterly innovations are in this context a "high-frequency" perspective, given that the goal is to understand the history of U.S. Treasury yields and that bond holders typically invest over much longer periods than one quarter. For a more complete understanding of the link between inflation expectations and bond yields we need to consider horizons longer than $h = 1$. This appears particularly promising for understanding the role of trends, because

$\lim_{h\to\infty} VR_h^{(n)}$ is only affected by changes in trend components.[35] That is, if the inflation trend is an important determinant for yields we should see a clear tendency for variance ratios to rise with h. Of course, the variance ratios become harder to estimate with increasing h as the overlap of observations increases and one loses degrees of freedom and precision. Despite this estimation uncertainty, the profile of variance ratios across horizons can provide evidence about the role of low-frequency movements in macro trends for interest rate dynamics.

Estimation of the these variance ratios requires expectations of inflation and interest rates. Like Duffee, we consider survey-based inflation expectations (in our measure of π_t^*) and martingale interest rate forecasts. But, instead of modeling the inflation process, we approximate inflation news by the change in π_t^*. This allows us to calculate the simplified variance ratio

$$\widetilde{VR}_h^{(n)} = \frac{Var\left(\Delta_h \pi_t^*\right)}{Var\left(\Delta_h y_t^{(n)}\right)}, \qquad \Delta_h z_t = z_t - z_{t-h}.$$

For any given n and h, $\widetilde{VR}_h^{(n)}$ differs from $VR_h^{(n)}$ but, these differences are likely to be small for longer horizons h since

$$\lim_{h\to\infty} \widetilde{VR}_h^{(n)} = \lim_{h\to\infty} VR_h^{(n)} = \frac{Var(\Delta\pi_t^*)}{Var(\Delta i_t^*)}.$$

Thus, for low-frequency movements, the two inflation variance ratios are asymptotically identical and capture the importance of the inflation trend for the overall yield trend.

Figure 5 shows estimates of $\widetilde{VR}_h^{(n)}$ for changes from one quarter to $h = 40$ quarters in the five-year yield, the ten-year yield, and the 5-to-10-year forward rate. Similarly, we also calculate these variance ratios for the contribution of changes in the real-rate trend, r_t^*, and in the overall trend component $i_t^* = \pi_t^* + r_t^*$, simply by replacing the variance in the numerator of $\widetilde{VR}_h^{(n)}$.

For the inflation trend, we find that one-quarter variance ratios are around 0.1. This result is consistent with Duffee's findings and suggests that changes in the inflation trend play a small role for variation in yields at the quarterly frequency. At lower frequencies, however, the relative variability of the inflation trend increases. The point estimates of the π_t^*-variance ratio quickly rise with the horizon, and for $h = 40$ reach a magnitude of around 0.3. This shows that inflation expectations are of substantial importance for movements in bond yields once we shift the focus from month-to-month or quarter-to-quarter variation and look at lower frequencies changes over several years.

[35] This statement assumes that the term premium is stationary, an assumption generally made in yield-curve modeling and supported by our findings in Section 4.

The variance ratio for changes in the real-rate trend, i.e., $Var(\Delta_h r_t^*)/Var(\Delta_h y_t)$, is much lower than for inflation, remaining below 0.1 even at long horizons. This is unsurprising in light of Figure 1, which shows that over the full sample the movements in r_t^* were substantially less pronounced than movements in long-term interest rates and the inflation trend. Of course, this perspective of unconditional variances should not be taken to conclude that changes in the equilibrium real rate are unimportant for interest rate dynamics, given the ample evidence in Sections 4– 6 of the crucial role of r_t^* for modeling and forecasting interest rates. From a conditional perspective, changes in r_t^* have become very important later in our sample, which is not evident in full-sample moments.

To assess the overall importance of the trend components in interest rates, we consider variance ratios for i_t^*, i.e., $Var(\Delta_h i_t^*)/Var(\Delta_h y_t)$. Confidence intervals are obtained using the asymptotic distribution of the sample variances and the delta method. To account for persistence in conditional variances, Newey-West estimates of long-run variances are used.[36] Figure 5 shows that while the sampling uncertainty around the variance ratios for changes in i_t^* is substantial we can be reasonably confident that these variance ratios increase from below 0.15 to a range of around 0.25 to 0.4, depending on the maturity of the interest rate. The highest levels are reached for the 5-to-10-year forward rate—the confidence interval at $h = 40$ extends from about 0.3 to 0.4—which is consistent with the notion that distant forward rates are more strongly affected by the trend components.

While in theory we have $\lim_{h\to\infty} Var(\Delta_h i_t^*)/Var(\Delta_h y_t) = 1$, our estimates top out around 0.4. While this might be due to the fact that we are simply not capturing the trend component with sufficient accuracy, we have provided ample evidence that our trend proxies are closely linked to the yield curve. Hence it appears that the cyclical components of inflation and the real rate together with movements in the term premium still make substantial contributions to interest rate movements at frequencies corresponding to ten-year changes.

To shed further light on this topic, it is useful to consider not only the direct contribution of changes in the trend to changes in interest rates, but also the indirect contribution due to comovement of trend and cycle components. In Table 9, we report the variances of changes in interest rates, in the trend component i_t^*, and in the cycle components $y_t - i_t^*$. The variance of yield changes can be decomposed as follows:

$$Var(\Delta_h y_t) = Var(\Delta_h i_t^*) + Var(\Delta_h y_t - \Delta_h i_t^*) + 2Cov(\Delta_h i_t^*, \Delta_h y_t - \Delta_h i_t^*).$$

[36] We use 12 quarterly lags for all long-run variance estimates as indicated by the automatic lag selection procedure of Newey and West. These confidence intervals may understate the true sampling variability due to the small number of non-overlapping observations, which decreases the reliability of the asymptotic approximations.

The first term captures the direct contribution of trends, whereas the last term captures the their indirect contribution to movements in yields. Table 9 reports all three components. In the data, the contribution of the covariance is small at short horizons, but substantial at long horizons. The two last rightmost columns of Table 9 report the same variance ratio shown in Figure 5 along with a ratio that also includes the covariance contribution—the indirect effects—in the numerator. This second ratio rises to over 0.7 with horizon, which suggests that the cycle component by itself accounts for less than 30% of the variance of interest rate changes at low frequencies. These estimates strengthen our finding that changes in macro trends play an important role in low-frequency movements in interest rates.

9 Conclusion

We have provided much compelling new evidence from a variety of perspectives that interest rates and bond risk premiums are substantially driven by time variation in the trend in inflation and the equilibrium real rate of interest. Our results demonstrate that the links between macroeconomic trends and the yield curve are quantitatively important, and that accounting for these time-varying trend components is crucial for understanding and forecasting long-term interest rates and bond returns. Our paper therefore provides strong support for building yield curve models that allow for slow-moving changes in the long-run means of nominal and real interest rates and inflation instead of the stationary dynamic specifications with constant means that are ubiquitous.[37]

Our analysis established the links between macroeconomic trends and yields by taking as data the off-the-shelf estimates of the trends from surveys and models. While this is a productive exercise, future research should jointly estimate macro trends and yield curve dynamics. One of the many benefits of such analysis would be the ability to investigate the role of model and estimation uncertainty. Another important future research avenue is to to consider potential changes over time in the variability of the trends. For example, it is well known (e.g., Stock and Watson, 2007) that the trend component of inflation was much more variable in the 1970s and 1980s than in more recent decades. This raises the question how our (mostly unconditional) results are affected by taking a conditional perspective. Furthermore, this suggests that incorporating not only macroeconomic trends but also stochastic volatility in these trend components will be useful for term structure modeling.

[37] Conversely, our evidence suggests that long-term yields contain relevant information about r_t^* and should therefore be included in the estimation, and some recent studies take a first step in this direction (Johannsen and Mertens, 2016; Del Negro et al., 2017).

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Appendix

A An illustrative term structure model

Here we describe a stylized affine term structure model for real and nominal yields that demonstrates how, under absence of arbitrage, changes in π_t^* and r_t^*—the inflation trend and the equilibrium real short rate—affect interest rates. This model illustrates the general theoretical points in Section 2 within a specific model setting. Although our framework is very simplified—with few risk factors, no stochastic volatility, and strong risk pricing restrictions—it is sufficient to provide useful insights about the role of macro trends for the yield curve. Our model generalizes the model of Cieslak and Povala (2015) to allow for time variation in r_t^* and more flexible inflation dynamics.

A.1 Model specification

The inflation process is given in (1), and we assume that ξ_t, u_t and e_t are *iid* Gaussian processes with standard deviations σ_ξ, σ_u and σ_e. For simplicity, the shocks are assumed to be mutually independent, but this assumption could easily be relaxed; importantly, it plays no role for our key equation (6).

The real-rate process is given in (2), and we again assume that the shocks are mutually uncorrelated and *iid* normal, with standard deviations σ_η and σ_v.

The final state variable determining interest rates is a risk price factor x_t, which follows an independent autoregressive process:

$$x_t = \mu_x + \phi_x x_{t-1} + w_t,$$

where w_t is *iid* normal with standard deviation σ_w. This way of modeling risk premia can be motivated by the evidence in Cochrane and Piazzesi (2005) for a stationary single factor driving bond risk premia. Also, see the discussion in Cieslak and Povala (2015).

We collect the state variables as $Z_t = (\pi_t^*, c_t, r_t^*, g_t, x_t)'$, so their dynamics can be compactly written as a first-order vector autoregression, a VAR(1):

$$Z_t = \mu + \phi Z_{t-1} + \Sigma \varepsilon_t, \tag{5}$$

where $\mu = (0, 0, 0, 0, \mu_x)'$, $\phi = diag(1, \phi_c, 1, \phi_g, \phi_x)$, $\Sigma = diag(\sigma_\xi, \sigma_u, \sigma_\eta, \sigma_v, \sigma_w)$, and ε_t is a (5×1) *iid* standard normal vector process.

The model is completed by a specification for the log real stochastic discount factor (SDF), m_{t+1}^r, for which we choose the usual essentially affine form of Duffee (2002):

$$m_{t+1}^r = -r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}, \quad \lambda_t = \Sigma^{-1}(\lambda_0 + \lambda_1 Z_t).$$

We only allow x_t to affect the price of risk, so that the first four columns of λ_1 are zero. Furthermore, shocks to x_t are not priced, so that the last element of λ_0 and the last row of λ_1

are zero. The non-zero elements of λ_0 are denoted by $\lambda_{0\pi^*}, \lambda_{0c}, \lambda_{0r^*}$, and λ_{0g}, and those of λ_1 by $\lambda_{\pi^* x}, \lambda_{cx}, \lambda_{r^* x}$, and λ_{gx}.

A.2 Nominal bond yields

The log nominal SDF is $m^n_{t+1} = m^r_{t+1} - \pi_{t+1}$, and the nominal short-term interest rate is

$$i_t = -E_t m^n_{t+1} - \frac{1}{2} Var_t(m^n_{t+1}) = r^*_t + g_t + \pi^*_t + c_t - \frac{1}{2}\sigma^2_e.$$

Due to our timing assumption for the inflation process, and because the noise shocks e_t are not priced ($Cov_t(m^r_{t+1}, \pi_{t+1}) = 0$) there is no inflation risk premium in the nominal short rate (as justified in Cieslak and Povala, 2015), which however does not imply that there is no inflation risk premium in long-term bonds (see below). From a macroeconomic perspective, the nominal short rate equation can be related to the popular Taylor rule for monetary policy in which $r^*_t + \pi^*_t$ represents the neutral/natural level of the nominal policy rate and $g_t + c_t$ captures the cyclical response of the central bank.

Prices of zero-coupon bonds with maturity n, denoted by $P^{(n)}_t$, are easily verified to be exponentially affine, i.e., $\log(P^{(n)}_t) = A_n + B'_n Z_t$, using the pricing equation $P^{(n+1)}_t = E_t(\exp(m^n_{t+1}) P^{(n)}_{t+1})$. The coefficients follow the usual recursions of affine term structure models (e.g., Ang and Piazzesi, 2003):

$$A_{n+1} = A_n + B'_n(\mu - \lambda_0) + C_n, \quad C_n := \frac{1}{2}(\sigma^2_e + B'_n \Sigma\Sigma' B_n), \quad B_{n+1} = -(1,1,1,1,0)' + (\phi - \lambda_1)' B_n,$$

where C_n captures the convexity in bond prices. The initial conditions are $A_0 = 0, B_0 = (0,0,0,0,0)'$. For the individual loadings of bond prices on the risk factors we have

$$B^{\pi^*}_{n+1} = B^{\pi^*}_n - 1, \quad B^c_{n+1} = \phi_c B^c_n - 1, \quad B^{r^*}_{n+1} = B^{r^*}_n - 1, \quad B^g_{n+1} = \phi_g B^g_n - 1,$$

$$B^x_{n+1} = -\lambda_{\pi^* x} B^{\pi^*}_n - \lambda_{cx} B^c_n - \lambda_{r^* x} B^{r^*}_n - \lambda_{gx} B^g_n + \phi_x B^x_n.$$

and the explicit solutions are

$$B^{\pi^*}_n = -n, \quad B^c_n = \frac{\phi^n_c - 1}{1 - \phi_c}, \quad B^{r^*}_n = -n, \quad B^g_n = \frac{\phi^n_g - 1}{1 - \phi_g},$$

$$B^x_n = \frac{\lambda_{\pi^* x} + \lambda_{r^* x}}{1 - \phi_x}\left(n - \frac{1 - \phi^n_x}{1 - \phi_x}\right) + \frac{\lambda_{cx}}{1 - \phi_c}\left(\frac{1 - \phi^n_x}{1 - \phi_x} - \frac{\phi^n_x - \phi^n_c}{\phi_x - \phi_c}\right) + \frac{\lambda_{gx}}{1 - \phi_g}\left(\frac{1 - \phi^n_x}{1 - \phi_x} - \frac{\phi^n_x - \phi^n_g}{\phi_x - \phi_g}\right).$$

For nominal bond yields, the model implies the following decomposition:

$$\begin{aligned} y^{(n)}_t &= -\log(P^{(n)}_t)/n = -A_n/n - B'_n Z_t/n \\ &= \underbrace{\pi^*_t + \frac{1 - \phi^n_c}{n(1 - \phi_c)} c_t}_{\sum_{i=1}^n E_t \pi_{t+i}/n} + \underbrace{r^*_t + \frac{1 - \phi^n_g}{n(1 - \phi_g)} g_t}_{\sum_{i=0}^{n-1} E_t r_{t+i}/n} \quad \underbrace{-A_n/n - B^x_n x_t/n.}_{\text{convexity and yield term premium}} \end{aligned} \tag{6}$$

This equation is identical to equation (3) except for the additional structure on the convexity and term premium terms, and Section 2 explains the role of the individual components. Here we can add some specifics about the risk-premium factor. This factor, x_t, affects long-term yields more strongly than short-term yields: The loadings of yields on x_t start at zero and tend to $\lim_{n\to\infty} - B_x^n/n = -(\lambda_{\pi^* x} + \lambda_{r^* x})/(1-\phi_x)$. Taken together, long-term yields are mostly driven by the trend components π_t^* and r_t^*, as well as by the risk-premium factor x_t.

With the additional assumptions of this model, specifically the assumptions on x_t and λ_t, yields have loadings on the macro trends that are exactly unity at all maturities. This implies, for example, that there is a long-run Fisher effect in this model. It also means that yields, inflation and real rates are cointegrated with a cointegration vector $(1, -1, -1)$.

The fact that macro trends shift all maturities by an equal amount means that in this model risk factors are $I(1)$ not only under the real-world/physical measure but also under the risk-neutral/pricing measure. Such non-stationary dynamics under the risk-neutral measure are inconsistent with absence of arbitrage because the convexity in $-A_n/n$ diverges to minus infinity—see Dybvig et al. (1996) and Campbell et al. (1997, p. 433). However, one could still take such a model to the data, if the maturity of bonds included in the estimation is limited by an upper bound. More generally, the assumption of a unit root—both under the real-world and risk-neutral measures—is simply a convenient and practically useful device for modeling highly persistent processes (see the discussion in Section 2).

A.3 Real yields and the real term premium

Consider prices and yields of real (i.e., inflation-indexed) bonds. Just like prices of nominal bonds, prices of real bonds are exponentially affine in the risk factors, $\log(\hat{P}_t^{(n)}) = \hat{A}_n + \hat{B}_n' Z_t$. Hats denote variables pertaining to real bonds. The loadings are determined by the recursions

$$\hat{A}_{n+1} = \hat{A}_n + \hat{B}_n'(\mu - \lambda_0) + \hat{C}_n, \quad \hat{C}_n := \frac{1}{2}\hat{B}_n'\Sigma\Sigma'\hat{B}_n, \quad \hat{B}_{n+1} = -(0,0,1,1,0)' + (\phi - \lambda_1)'\hat{B}_n,$$

$\hat{C}_n$ captures the convexity in real bonds, and the initial conditions are $\hat{A}_0 = 0$, $\hat{B}_0 = (0,0,0,0,0)'$. Specifically,

$$\hat{B}_n^{\pi^*} = 0, \quad \hat{B}_n^c = 0, \quad \hat{B}_{n+1}^{r^*} = \hat{B}_n^{r^*} - 1, \quad \hat{B}_{n+1}^g = \phi_g \hat{B}_n^g - 1,$$
$$\hat{B}_{n+1}^x = -\lambda_{r^* x}\hat{B}_n^{r^*} - \lambda_{gx}\hat{B}_n^g + \phi_x \hat{B}_n^x.$$

Real yields, $\hat{y}_t^{(n)} = -\log(\hat{P}_t^{(n)})/n$, are affine in the risk factors. It is instructive to consider real forward rates for inflation-indexed borrowing from n to $n+1$, for which we have

$$\begin{aligned}
\hat{f}_t^{(n)} &= \log(\hat{P}_t^{(n)}) - \log(\hat{P}_t^{(n+1)}) = \hat{A}_n - \hat{A}_{n+1} + (\hat{B}_n - \hat{B}_{n+1})'Z_t \\
&= -\hat{B}_n'(\mu - \lambda_0) - \hat{C}_n + r_t^* + \phi_g^n g_t + (\hat{B}_n^x - \hat{B}_{n+1}^x)x_t \\
&= -\hat{C}_n + E_t(r_{t+n}) + \hat{ftp}_t^{(n)}.
\end{aligned}$$

Therefore, changes in r_t^* affect all real forward rates equally and hence act as a level factor. Changes in the real-rate gap g_t affect short-term real rates more strongly than long-term rates, and therefore affect the slope. The last row clarifies that real forward rates can be decomposed into convexity, an expectations component, $E_t(r_{t+n}) = r_t^* + \phi_g^n g_t$, and a real forward term premium, $\hat{ftp}_t^{(n)} = -\hat{B}_n^x \mu_x + \hat{B}_n' \lambda_0 + (\hat{B}_n^x - \hat{B}_{n+1}^x) x_t$.

For real yields we have

$$\begin{aligned} \hat{y}_t^{(n)} &= -\log(\hat{P}_t^{(n)})/n = -\hat{A}_n/n - \hat{B}_n' Z_t/n \\ &= \underbrace{r_t^* + \frac{1-\phi_g^n}{n(1-\phi_g)} g_t}_{\sum_{i=0}^{n-1} E_t r_{t+i}/n} \qquad \underbrace{-\hat{A}_n/n - \hat{B}_n^x x_t/n.}_{\text{convexity and real yield term premium}} \end{aligned}$$

which shows that the equilibrium real rate r_t^* acts as a level factor for the real yield curve, and that the impact of the real-rate gap g_t diminishes with the yield maturity.

To understand the real term premium it is helpful to consider the term premium in the one-period-ahead real forward rate, which is

$$\hat{ftp}_t^{(1)} = Cov_t(m_{t+1}^r, r_{t+1}) = -[\lambda_{0r^*} + \lambda_{0g} + (\lambda_{r^*x} + \lambda_{gx}) x_t].$$

If the real SDF positively correlates with the real rate, then real bonds are risky in the sense that their payoffs are low in times of high marginal utility. In this case, the real term premium is positive to compensate investors for this risk. Note that in this Gaussian model, variation in risk premia are driven by changes in the risk-premium factor x_t, which affects prices of risk, and that quantities of risk are constant due to homoskedasticity of the state variables.

A.4 Nominal forward rates, inflation risk premia, and nominal term premia

Nominal forward rates from n to $n+1$ are:

$$\begin{aligned} f_t^{(n)} &= \log(P_t^{(n)}) - \log(P_t^{(n+1)}) = A_n - A_{n+1} + (B_n - B_{n+1})' Z_t \\ &= \underbrace{-c_n}_{\text{convexity}} + \underbrace{\pi_t^* + \phi_c^n c_t}_{E_t(\pi_{t+n+1})} + \underbrace{r_t^* + \phi_g^n g_t}_{E_t(r_{t+n})} \underbrace{-B_n^x \mu_x + B_n' \lambda_0 + (B_n^x - B_{n+1}^x) x_t}_{\text{forward term premium}} \end{aligned}$$

Naturally, nominal forward rates reflect expectations of future inflation and real rates. Changes in the trend components π_t^* and r_t^* parallel-shift the entire path of these expectations, and therefore affect forward rates at all maturities equally. Distant forward rates are, on the other hand, only minimally affected by changes in c_t and g_t. The loading of forward rates on x_t can be shown to approach $-(\lambda_{\pi^*x} + \lambda_{r^*x})/(1-\phi_x)$ for large n, meaning that x_t affects distant forward rates due to its effect on the prices of risk of π_t^* and r_t^*.

In our empirical analysis we will consider the five-to-ten-year forward rate, i.e.,

$$f_t^{(n_1,n_2)} = (n_2 - n_1)^{-1} \sum_{n=n_1}^{n_2-1} f_t^{(n)}, \quad n_1 = 20, \quad n_2 = 40.$$

Because this interest rate is even less affected by the cyclical components c_t and g_t than, for example, the ten-year yield, it should exhibit a particularly close relationship with the trend components π_t^* and r_t^*.

The term premium in nominal forward rates, $ftp_t^{(n)} = -B_n^x \mu_x + B_n' \lambda_0 + (B_n^x - B_{n+1}^x)x_t$, is composed of the real forward term premium, $\hat{ftp}_t^{(n)}$, and a forward inflation risk premium, $firp_t^{(n)}$. The intuition is again easiest for $n = 1$:

$$firp_t^{(1)} = Cov_t(m_{t+1}^r, E_{t+1}(\pi_{t+2})) = -[\lambda_{0\pi^*} + \lambda_{0c} + (\lambda_{\pi^*x} + \lambda_{cx})x_t].$$

If shocks to inflation expectations are positively correlated with the real SDF, then nominal bonds are more risky than real bonds and require a higher risk premium, i.e., a positive inflation risk premium. Like the real term premium, the inflation risk premium in this model is driven only by changes in x_t.

A.5 Excess bond returns

In this illustrative model, as in Cieslak and Povala (2015)'s model, excess bond returns, $rx_{t+1}^{(n)}$, are driven only by the risk premium factor x_t:

$$rx_{t+1}^{(n)} = p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)} = -\frac{1}{2} B_{n-1}' \Sigma \Sigma' B_{n-1} + B_{n-1}'(\lambda_0 + \lambda_1 \iota_5)x_t + B_{n-1}' \Sigma \varepsilon_{t+1},$$

where $p_t^{(n)}$ denotes the log-price of a zero-coupon bond with maturity of n quarters and ι_5 is a (5×1)-vector of ones.

A.6 Variance ratios

The model implies analytical expressions for the variance ratios in Section 8. For one-period and h-period innovations, respectively, they are

$$\begin{aligned} VR_1^{(n)} &= \frac{\sigma_\xi^2 + a_c(n)\sigma_u^2}{\sigma_\xi^2 + a_c(n)\sigma_u^2 + \sigma_\eta^2 + a_g(n)\sigma_v^2 + \left(\frac{B_n^x}{n}\right)^2 \sigma_w^2}, \\ VR_h^{(n)} &= \frac{h\sigma_\xi^2 + a_c(n)b_c(h)\sigma_u^2}{h\sigma_\xi^2 + a_c(n)b_c(h)\sigma_u^2 + h\sigma_\eta^2 + a_g(n)b_g(h)\sigma_v^2 + \left(\frac{B_n^x}{n}\right)^2 b_x(h)\sigma_w^2}, \\ & a_i(n) = \left(\frac{1-\phi_i^n}{n(1-\phi_i)}\right)^2, \quad b_i(h) = \frac{1-\phi_i^{2h}}{1-\phi_i^2}, \quad i = c, g, x. \end{aligned}$$

These expressions can help elucidate the factors determining the variance ratios. Note that for arbitrarily long maturity

$$\lim_{n \to \infty} VR_1^{(n)} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\eta^2 + \left(\frac{\lambda_{\pi^*x} + \lambda_{r^*x}}{1-\phi_x}\right)^2 \sigma_w^2}.$$

This helps to interpret Duffee's result from the perspective of the model. In particular, this ratio will be small if shocks to the equilibrium real rate (η_t) and to the risk-premium factor (w_t) make more important contributions to yield innovations than shocks to the inflation trend (ξ_t). For any yield maturity n but very long horizons we have:

$$\lim_{h\to\infty} VR_h^{(n)} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\eta^2}.$$

This means that asymptotically, this inflation variance ratio is only affected by changes in the trend components. Changes in term premia become irrelevant at very low frequencies because the risk premium factor x_t and hence the term premium is assumed to be stationary.

For the simplified variance ratio of observe changes we have

$$\lim_{h\to\infty} \widetilde{VR}_h^{(n)} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\eta^2} = \lim_{h\to\infty} VR_h^{(n)}.$$

which shows that our simple variance ratio is likely to be a good approximation of $VR_h^{(n)}$ for large h.

Finally the limits for variance ratios with the real-rate and overall trends are

$$\begin{aligned} \lim_{h\to\infty} \frac{Var\left(\Delta_h r_t^*\right)}{Var\left(\Delta_h y_t^{(n)}\right)} &= \frac{\sigma_\eta^2}{\sigma_\xi^2 + \sigma_\eta^2}, \quad \text{and} \\ \lim_{h\to\infty} \frac{Var\left(\Delta_h i_t^*\right)}{Var\left(\Delta_h y_t^{(n)}\right)} &= 1, \end{aligned}$$

which shows that for very low frequencies the underlying macroeconomic trends are the only drivers of variation in interest rates.

Table 1: Persistence of interest rates and detrended interest rates

Series	SD	$\hat{\rho}$	Half-life	ADF	PP	ERS	LFST
$y_t^{(5y)}$	3.24	0.97	25.0	-1.16	-3.26	-1.74	0.00
$y_t^{(5y)} - \pi_t^*$	1.95	0.93	9.8	-1.84	-9.02	-2.27	0.00
$y_t^{(5y)} - \pi_t^* - r_t^*$	1.28	0.86	4.7	-3.58***	-22.60***	-2.73*	0.04
$y_t^{(5y)} - 1.80\pi_t^*$	1.47	0.88	5.2	-2.60	-19.23*	-2.69*	0.04
$y_t^{(5y)} - 1.26\pi_t^* - 1.63r_t^*$	0.95	0.75	2.4	-4.60***	-46.09***	-3.75***	0.52
$y_t^{(10y)}$	2.92	0.97	24.6	-1.06	-2.94	-1.60	0.00
$y_t^{(10y)} - \pi_t^*$	1.65	0.92	8.8	-2.25	-9.49	-2.12	0.01
$y_t^{(10y)} - \pi_t^* - r_t^*$	1.05	0.84	4.0	-3.88***	-27.15***	-2.77*	0.12
$y_t^{(10y)} - 1.63\pi_t^*$	1.31	0.88	5.3	-2.53	-17.95*	-2.46	0.03
$y_t^{(10y)} - 1.16\pi_t^* - 1.38r_t^*$	0.91	0.79	2.9	-4.73***	-38.89***	-3.53***	0.28
$f_t^{(5y,10y)}$	2.64	0.97	21.0	-1.10	-3.25	-1.55	0.00
$f_t^{(5y,10y)} - \pi_t^*$	1.46	0.91	7.2	-2.43	-11.76*	-2.11	0.01
$f_t^{(5y,10y)} - \pi_t^* - r_t^*$	1.02	0.84	3.9	-3.90***	-28.51***	-2.96**	0.21
$f_t^{(5y,10y)} - 1.45\pi_t^*$	1.27	0.88	5.4	-3.06	-17.59*	-2.36	0.03
$f_t^{(5y,10y)} - 1.07\pi_t^* - 1.12r_t^*$	1.00	0.83	3.8	-4.06**	-30.18**	-3.25**	0.18
π_t^*	1.60	0.99	86.4	-0.64	-1.20	-1.22	0.00
r_t^*	0.87	0.98	29.9	-0.12	-0.30	-1.33	0.00
i_t^*	2.24	0.99	67.1	-0.29	-0.34	-1.23	0.00

Standard deviation (SD); first-order autocorrelation coefficient ($\hat{\rho}$); half-life, calculated as $\ln(0.5)/\ln(\hat{\rho})$; Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Elliott-Rothenberg-Stock (ERS) unit root test statistics and Mueller-Watson low-frequency stationary test p-value (LFST), for interest rates, detrended interest rates, and macro trends, with *,** and *** indicating significance at 10%, 5%, and 1% level. The data are quarterly from 1971:Q4 to 2017:Q2.

Table 2: Tests for cointegration rank

	$y_t^{(5y)}$	$y_t^{(10y)}$	$f_t^{(5y,10y)}$
$H_0 : r = 0, H_1 : r > 0$	42.35***	46.99***	42.38***
$H_0 : r = 1, H_1 : r > 1$	6.03	6.47	6.92
$H_0 : r = 2, H_1 : r > 2$	1.20	1.44	1.66

Johansen cointegration rank test for $Y_t = (y_t, \pi_t^*, r_t^*)'$, where y_t is either the five- or ten-year yield or the five-to-ten-year forward rate: trace test statistic of the null hypothesis that there are $r = r_0$ cointegration relationships, against the alternative that the cointegration rank is higher than r_0. The VAR representation includes two lags of Y_t. The data are quarterly from 1971:Q4 to 2017:Q2.

Table 3: Regressions of long-term interest rates on macroeconomic trends

	$y_t^{(5y)}$		$y_t^{(10y)}$		$f_t^{(5y,10y)}$	
	(1)	(2)	(3)	(4)	(5)	(6)
constant	-0.33	-1.85	0.81	-0.47	1.95	0.90
	(0.67)	(0.29)	(0.58)	(0.31)	(0.53)	(0.41)
π_t^*	1.80	1.26	1.63	1.16	1.45	1.07
	(0.15)	(0.14)	(0.14)	(0.13)	(0.14)	(0.13)
r_t^*		1.63		1.38		1.12
		(0.15)		(0.15)		(0.19)
R^2	0.79	0.91	0.80	0.90	0.77	0.86

Regressions of long-term Treasury yields and forward rates on measures of long-run inflation expectations and the equilibrium real rate, which are described in the text. Numbers in parentheses are Newey-West standard errors with six lags. The data are quarterly from 1971:Q4 to 2017:Q2.

Table 4: Error-correction of yields to shifts in macro trends

	$y_t^{(5y)}$	$y_t^{(10y)}$	$f_t^{(5y,10y)}$
Error-correction coefficient ($\hat{\alpha}$)	-0.113	-0.157	-0.219
	(0.046)	(0.059)	(0.062)
$H_0 : \alpha = 0$	0.014	0.009	0.001
H_0 : macro trends not Granger causal	0.000	0.000	0.000

Estimates of the coefficient α on the error-correction term $(y_{t-1} - \pi_{t-1}^* - r_{t-1}^*)$ in a regression for Δy_t that also includes an intercept and four lags of Δy_t, $\Delta \pi_t^*$ and r_t^*, where y_t is the five- or ten-year yield or the five-to-ten-year forward rate. White standard errors are in parentheses. The first p-value is for a test that the error-correction coefficient is zero. The second p-value is for a test that the coefficients on the error-correction terms and all lags of first-differenced macro trends are zero, i.e., the hypothesis that macro trends do not Granger-cause movements in yields.

Table 5: Predicting excess returns with macro trends

Holding period:	One quarter				Four quarters			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Full sample: 1971:Q4–2017:Q2								
PC1	0.08	0.98	1.96	1.96	0.04	2.73	5.19	5.25
	(0.17)	(0.27)	(0.48)	(0.45)	(0.38)	(0.72)	(0.78)	(0.75)
PC2	0.43	0.48	0.46	0.46	1.62	1.79	1.72	1.71
	(0.18)	(0.17)	(0.17)	(0.16)	(0.38)	(0.30)	(0.28)	(0.28)
PC3	-2.47	-1.82	-2.13	-2.15	-3.91	-1.77	-2.82	-3.04
	(1.37)	(1.30)	(1.20)	(1.27)	(2.07)	(1.78)	(1.58)	(1.62)
π_t^*		-1.95	-2.76			-5.74	-7.74	
		(0.44)	(0.53)			(1.20)	(1.22)	
		[0.00]	[0.00]			[0.02]	[0.00]	
r_t^*			-2.70				-6.97	
			(0.90)				(1.45)	
			[0.02]				[0.01]	
i_t^*				-2.75				-7.58
				(0.55)				(1.10)
				[0.00]				[0.00]
R^2	0.09	0.16	0.21	0.21	0.24	0.43	0.53	0.53
Subsample: 1985:Q1–2017:Q2								
PC1	0.32	0.67	2.13	1.90	0.65	2.03	6.33	5.91
	(0.16)	(0.24)	(0.48)	(0.45)	(0.40)	(0.56)	(1.00)	(0.83)
PC2	0.34	0.44	0.46	0.50	1.18	1.59	1.67	1.76
	(0.14)	(0.15)	(0.14)	(0.14)	(0.37)	(0.32)	(0.27)	(0.26)
PC3	-1.13	-0.97	0.02	-0.17	-2.67	-1.68	1.25	0.97
	(1.08)	(1.07)	(1.17)	(1.10)	(2.11)	(2.31)	(2.03)	(2.08)
π_t^*		-1.05	-2.09			-4.06	-7.17	
		(0.73)	(0.74)			(1.40)	(1.20)	
		[0.38]	[0.08]			[0.16]	[0.01]	
r_t^*			-3.18				-9.38	
			(1.00)				(2.15)	
			[0.02]				[0.03]	
i_t^*				-2.32				-7.66
				(0.69)				(1.11)
				[0.02]				[0.00]
R^2	0.08	0.10	0.16	0.15	0.22	0.30	0.48	0.47

Predictive regressions for quarterly and annual excess bond returns, averaged across two- to 15-year maturities. The predictors are three principal components (PCs) of yields and measures of long-run inflation expectations (π_t^*), the equilibrium real rate (r_t^*), and the long-run nominal short rate ($i_t^* = \pi_t^* + r_t^*$) which are described in the text. Numbers in parentheses are White standard errors for quarterly (non-overlapping) returns, and Newey-West standard errors with 6 lags for annual (overlapping) returns. Numbers in squared brackets are small-sample p-values obtained with the bootstrap method of Bauer and Hamilton (2016). The data are quarterly from 1971:Q4 to 2017:Q2.

Table 6: Predicting excess returns with detrended yields

	Full sample: 1971–2016		Subsample: 1985–2016	
Predictors	R^2	ΔR^2	R^2	ΔR^2
Yields only	0.24		0.22	
	(0.06, 0.40)		(0.10, 0.53)	
Yields and π_t^*	0.43	0.18	0.30	0.08
	(0.11, 0.44)	(0.00, 0.18)	(0.15, 0.56)	(0.00, 0.19)
Yields, π_t^* and r_t^*	0.53	0.29	0.48	0.26
	(0.13, 0.47)	(0.00, 0.22)	(0.18, 0.58)	(0.00, 0.24)
Yields and i_t^*	0.53	0.29	0.47	0.25
	(0.11, 0.44)	(0.00, 0.18)	(0.15, 0.56)	(0.00, 0.19)
Yields detrended by π_t^*	0.33	0.09	0.27	0.05
	(0.08, 0.41)	(-0.09, 0.14)	(0.12, 0.53)	(-0.12, 0.12)
Yields detrended by i_t^*	0.43	0.19	0.45	0.22
	(0.08, 0.42)	(-0.09, 0.13)	(0.11, 0.52)	(-0.16, 0.15)

Predictive power of regressions for annual excess bond returns, averaged across two- to 15-year maturities. The predictors are three principal components (PCs) of yields and measures of long-run inflation expectations (π_t^*) and the equilibrium real rate (r_t^*), which are described in the text. The last two specifications use detrended yields, constructed as either $y_t^{(n)} - \pi_t^*$ or $y_t^{(n)} - i_t^*$. Increase in R^2 (ΔR^2) is reported relative to the first specification with only PCs of yields. Numbers in parentheses are 95%-bootstrap intervals obtained by calculating the same regressions statistics in 5,000 bootstrap data sets generated under the (spanning) null hypothesis that only yields have predictive power for bond returns.

Table 7: Forecasting long-term interest rates

	RMSE					MAE				
Horizon h (quarters):	4	10	20	30	40	4	10	20	30	40
Five-year yield										
RW	1.34	1.87	2.52	2.61	2.90	1.02	1.33	1.92	2.23	2.55
ME	1.36	1.64	1.86	1.57	1.56	1.00	1.12	1.14	1.10	1.36
IO	1.35	1.72	2.27	2.36	2.63	1.02	1.27	1.83	2.10	2.43
AR	1.43	2.14	3.03	3.30	3.76	1.11	1.70	2.68	3.05	3.53
$H_0 : ME \geq RW$	(0.55)	(0.16)	(0.05)	(0.01)	(0.01)	(0.41)	(0.09)	(0.00)	(0.00)	(0.00)
$H_0 : ME \geq IO$	(0.56)	(0.24)	(0.02)	(0.01)	(0.01)	(0.39)	(0.17)	(0.00)	(0.00)	(0.01)
Ten-year yield										
RW	1.51	2.21	2.83	2.95	3.25	1.20	1.74	2.18	2.52	2.80
ME	1.44	1.79	2.12	2.04	2.22	1.09	1.30	1.49	1.68	1.97
IO	1.51	2.01	2.59	2.78	3.10	1.16	1.51	2.16	2.50	2.84
AR	1.65	2.50	3.36	3.70	4.22	1.30	1.98	3.00	3.38	3.88
$H_0 : ME \geq RW$	(0.35)	(0.03)	(0.04)	(0.04)	(0.06)	(0.19)	(0.00)	(0.01)	(0.02)	(0.02)
$H_0 : ME \geq IO$	(0.19)	(0.05)	(0.02)	(0.02)	(0.02)	(0.22)	(0.06)	(0.00)	(0.00)	(0.01)
5-to-10-year forward rate										
RW	1.27	1.72	2.35	2.37	2.64	0.96	1.16	1.77	1.96	2.31
ME	1.35	1.68	1.89	1.46	1.16	1.05	1.25	1.27	0.92	0.94
IO	1.25	1.55	2.01	2.02	2.22	0.93	1.08	1.50	1.69	2.02
AR	1.31	1.93	2.77	2.98	3.38	1.01	1.50	2.39	2.76	3.19
$H_0 : ME \geq RW$	(0.68)	(0.43)	(0.12)	(0.02)	(0.01)	(0.79)	(0.71)	(0.04)	(0.00)	(0.00)
$H_0 : ME \geq IO$	(0.91)	(0.89)	(0.18)	(0.01)	(0.00)	(0.93)	(0.93)	(0.02)	(0.00)	(0.00)

Accuracy of four different forecast methods for long-term interest rates over horizons from 4 to 40 quarters, measured by the root-mean-squared error (RMSE) and the mean-absolute error (MAE) in percentage points. Method RW is a driftless random walk, ME uses the shifting (macro) endpoint $i^*_t = r^*_t + \pi^*_t$, IO uses inflation only for the shifting endpoint, i.e., π^*_t plus a constant, and AR is a stationary AR(1) model and thus has a constant endpoint. Methods ME, IO and AR predict a smooth path from the current interest rate to the endpoint with an autoregressive parameter that is recursively estimated on an expanding window. The data are quarterly from 1971:Q4 to 2017:Q2. The first forecast is made in 1976:Q3 once five years of data are available, and the last forecast is in 2007:Q2, for a total of 124 (overlapping) observations. The last two rows in each panel report one-sided p-values for testing the null hypothesis of equal forecast accuracy against the alternative that method ME is more accurate, using the method of Diebold and Mariano (1995) with small-sample correction.

Table 8: Forecasting the ten-year yield using surveys and models

	RMSE					MAE				
Future calendar year:	1	2	3	4	5	1	2	3	4	5
RW	0.85	1.09	1.22	1.37	1.58	0.70	0.92	1.06	1.17	1.38
BC	1.04	1.36	1.52	1.71	1.96	0.86	1.11	1.28	1.49	1.72
ME	0.64	0.69	0.70	0.69	0.84	0.52	0.59	0.56	0.57	0.66
IO	1.17	1.50	1.69	1.91	2.17	0.92	1.22	1.44	1.71	1.96
AR	1.26	1.78	2.15	2.55	2.94	1.09	1.61	2.04	2.47	2.87
$H_0 : ME \geq RW$	(0.04)	(0.00)	(0.00)	(0.00)	(0.00)	(0.07)	(0.00)	(0.00)	(0.00)	(0.00)
$H_0 : ME \geq BC$	(0.05)	(0.07)	(0.10)	(0.09)	(0.10)	(0.06)	(0.07)	(0.08)	(0.03)	(0.02)

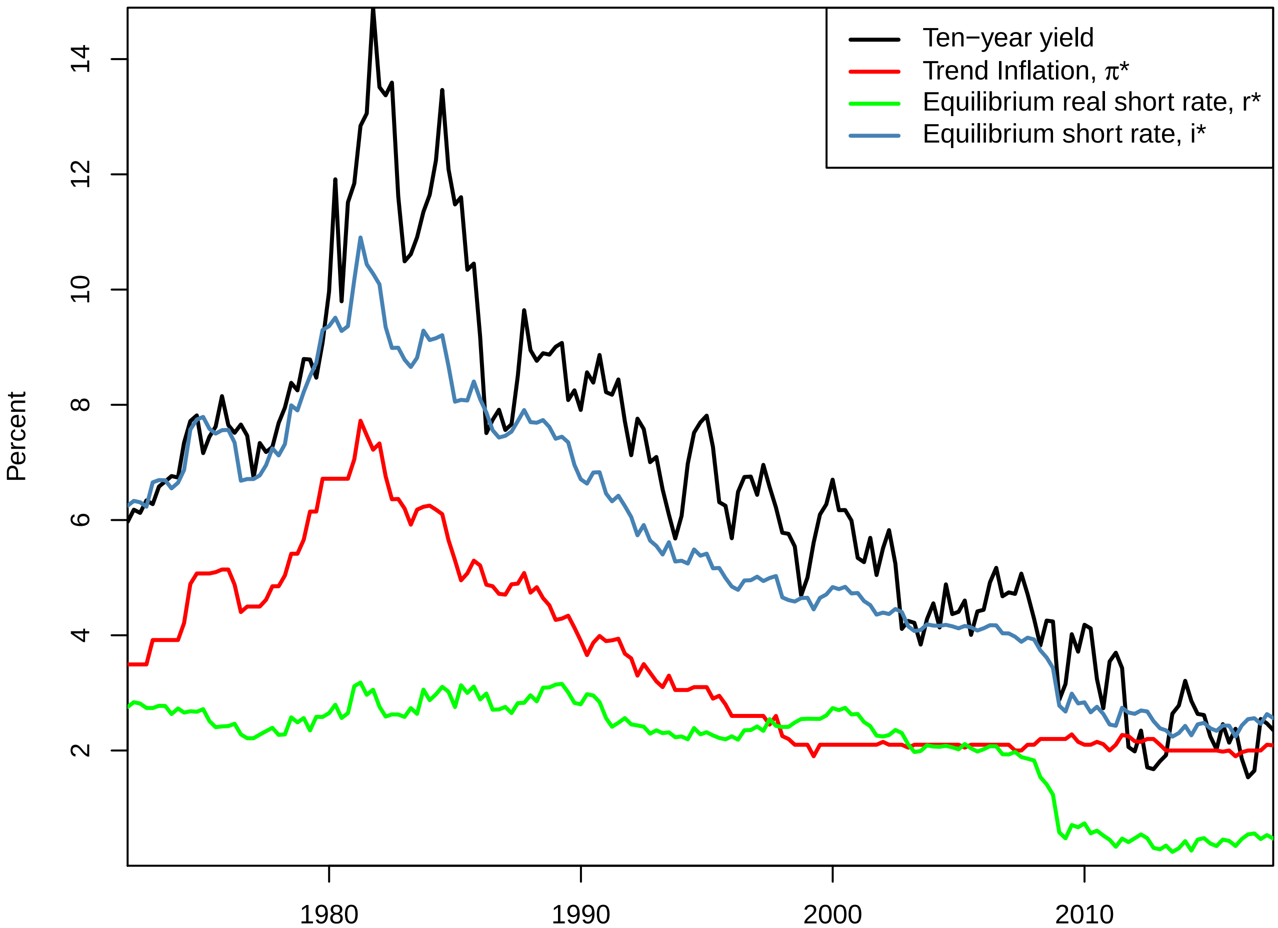
Accuracy of model-based forecasts in comparison to survey forecasts from the Blue Chip Financial Forecasts (BC) for the average ten-year yield over future calendar years. There are 46 forecast dates (mostly semi-annual) between March 1988 and December 2010, corresponding to the release month of the survey. Model-based forecasts are based on data from the quarter preceding the release of the survey, and cover the same forecast horizons as in the survey. For details on the model-based forecasts and the reported statistics see the notes to Table 7.

Table 9: Variance ratios

	Variances and covariances				Variance ratios	
	$\Delta_h y_t$	$\Delta_h i_t^*$	$\Delta_h(y_t - i_t^*)$	$2 \cdot Cov$		
h	(1)	(2)	(3)	(4)	(2)/(1)	[(2)+(4)]/(1)
Five-year yield						
1	0.48	0.05	0.44	-0.01	0.10	0.09
5	2.26	0.35	1.51	0.40	0.16	0.33
10	3.76	0.78	1.95	1.03	0.21	0.48
20	6.84	1.64	2.72	2.48	0.24	0.60
30	8.94	2.69	2.90	3.35	0.30	0.68
40	8.81	2.73	2.75	3.33	0.31	0.69
Ten-year yield						
1	0.38	0.05	0.34	-0.01	0.13	0.10
5	1.79	0.35	1.20	0.24	0.20	0.33
10	2.77	0.78	1.35	0.64	0.28	0.51
20	5.66	1.64	2.07	1.95	0.29	0.63
30	7.74	2.69	2.18	2.87	0.35	0.72
40	7.93	2.73	2.18	3.02	0.34	0.73
5-to-10-year forward rate						
1	0.35	0.05	0.32	-0.02	0.14	0.09
5	1.67	0.35	1.23	0.09	0.21	0.26
10	2.47	0.78	1.45	0.24	0.32	0.41
20	5.21	1.64	2.16	1.42	0.31	0.59
30	7.12	2.69	2.03	2.40	0.38	0.71
40	7.48	2.73	2.04	2.71	0.36	0.73

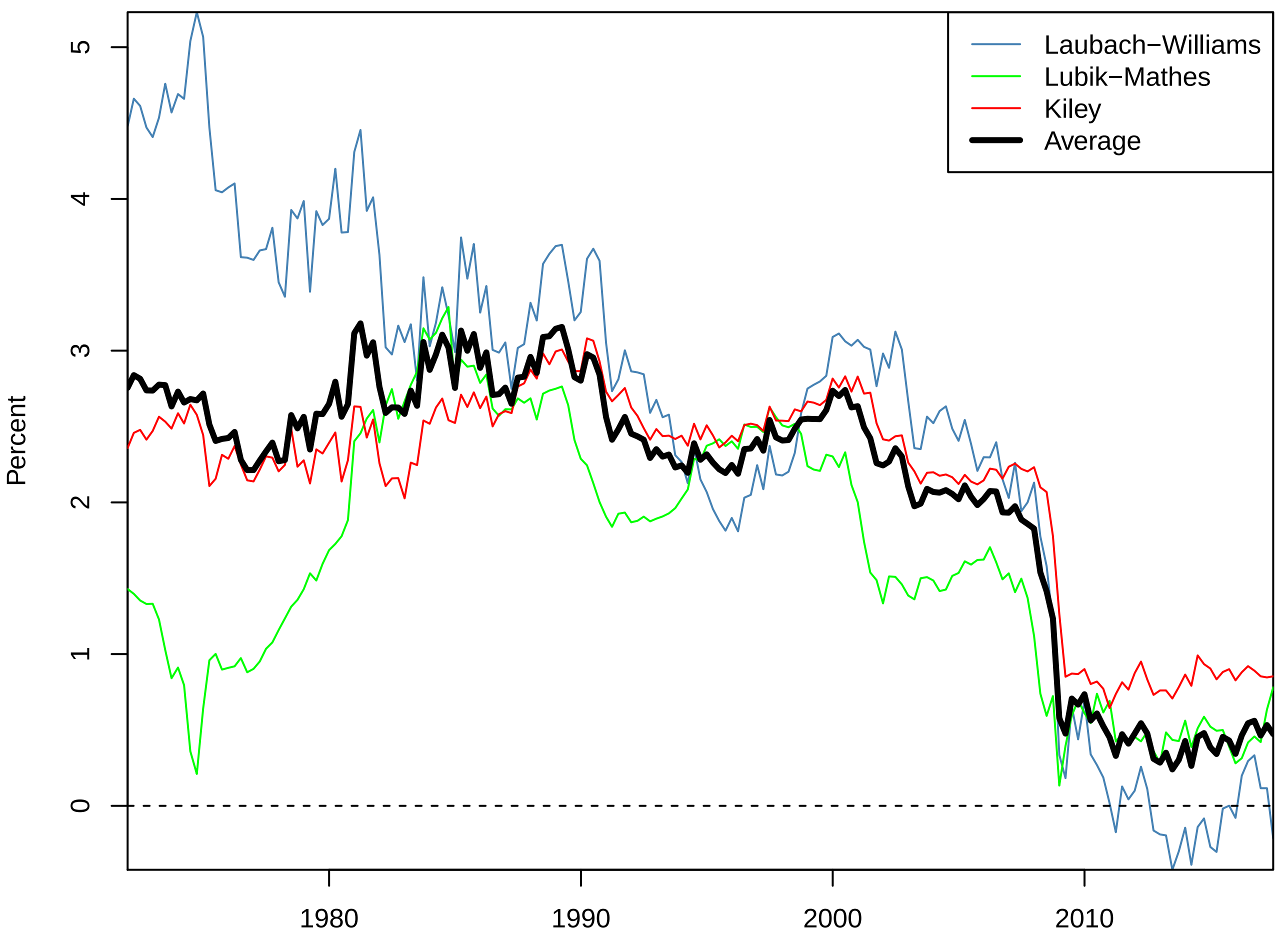
Variances, covariances, and variance ratios for changes in long-term interest rates and the trend component $i_t^* = \pi_t^* + r_t^*$. The first three columns report sample variances for h-quarter changes in the interest rate y_t, the trend component i_t^*, and the cycle component $y_t - i_t^*$. The fourth column reports twice the covariance between changes in the trend component and the cycle component. The last two columns report two different ratios: The first is the ratio of the variance of changes in the trend component relative to the variance of interest rate changes. The second includes the twice the covariance between changes in the trend and cycle components in the numerator, and equals one minus the variance ratio for the cycle component. The data are quarterly from 1971:Q4 to 2017:Q2.

Figure 1: Ten-year yield and macroeconomic trends



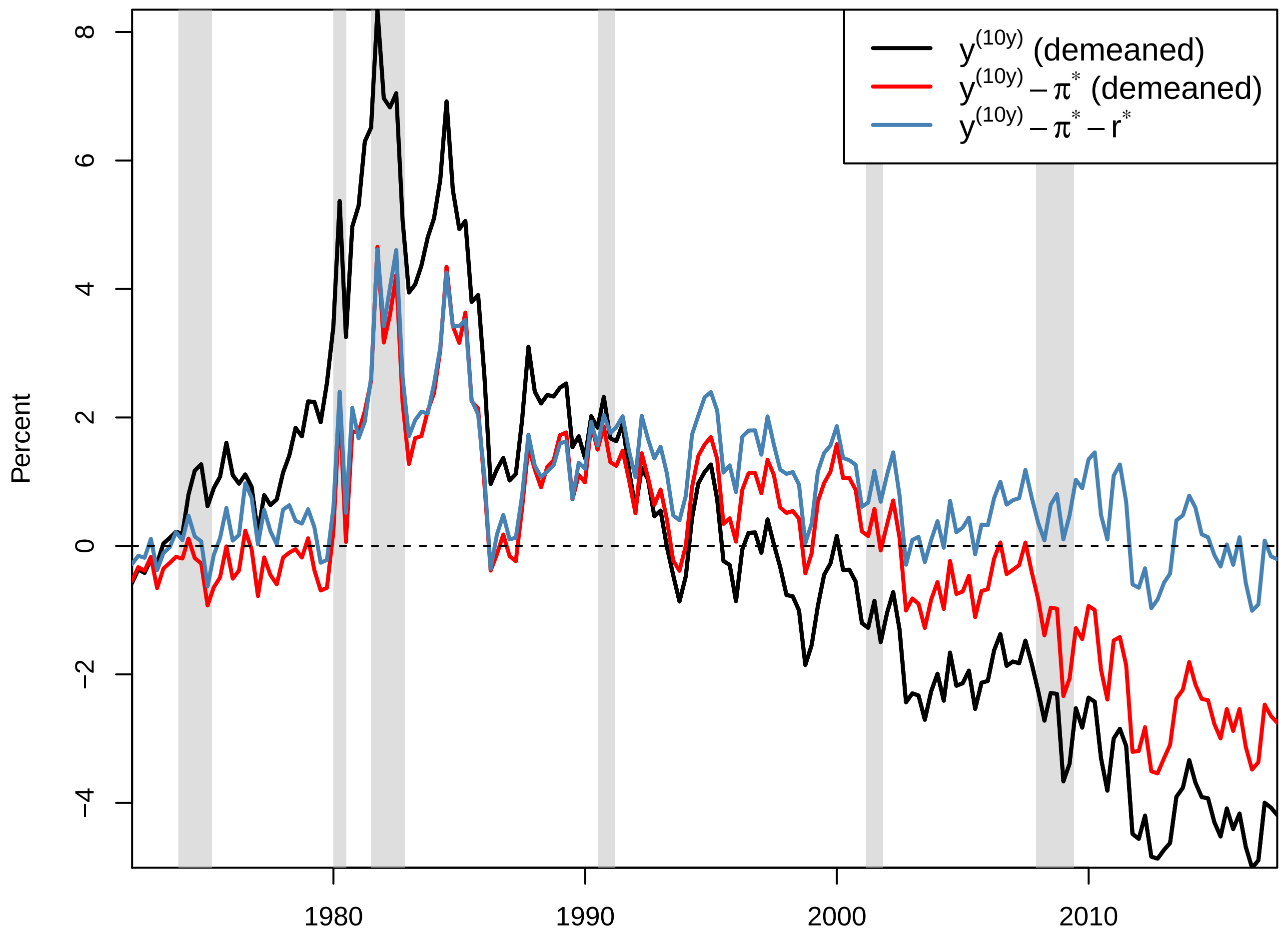
Ten-year Treasury yield and estimates of trend inflation, π^* (the mostly survey-based PTR measure from FRB/US), the equilibrium real rate, r^* (the average of the estimates in Figure 2), and the equilibrium short rate, $i^* = \pi^* + r^*$. The data are quarterly from 1971:Q4 to 2017:Q2.

Figure 2: Measures of the equilibrium real interest rate



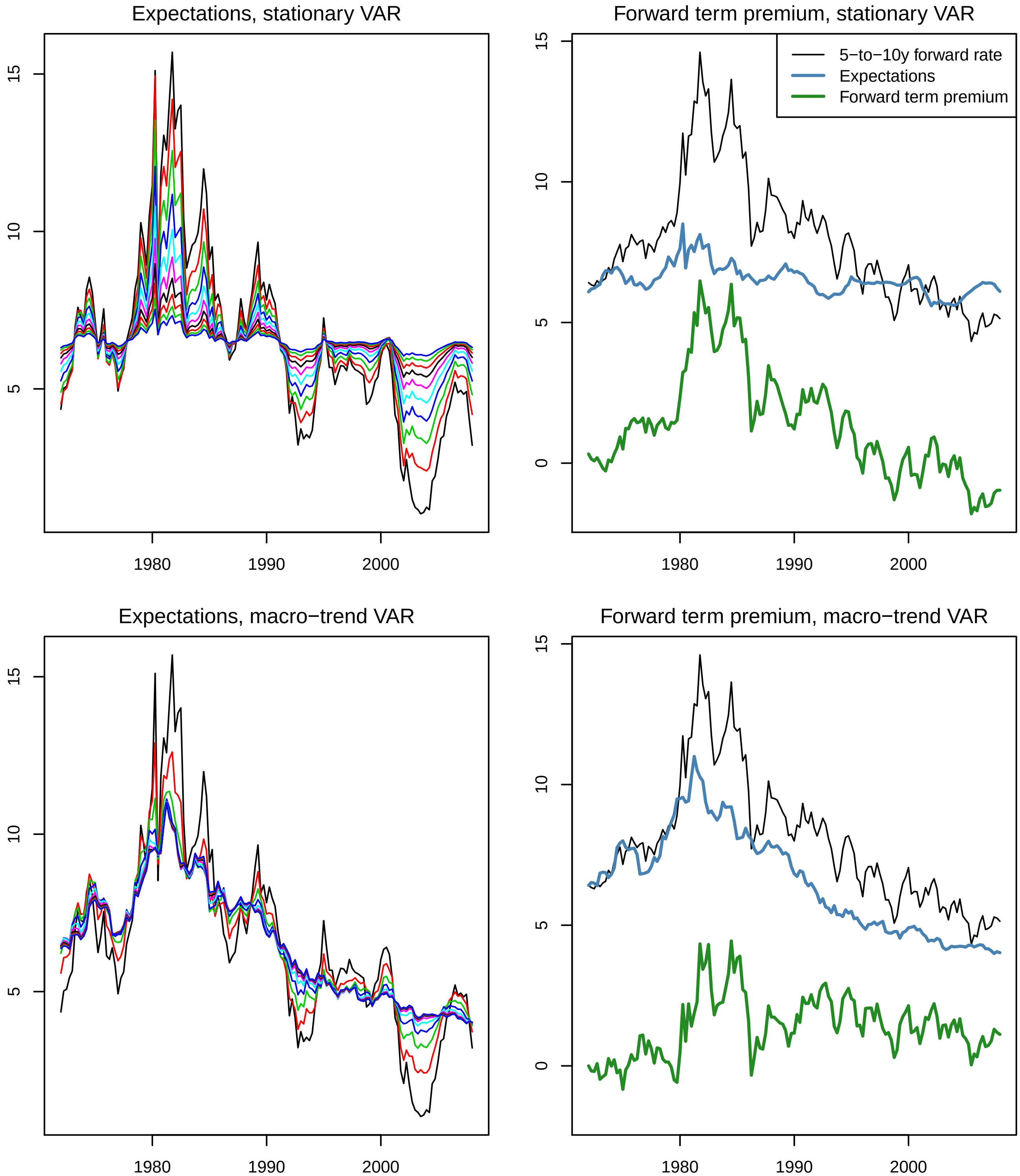
Three macroeconomic estimates of r^* from Laubach and Williams (2003), Lubik and Matthes (2015), and Kiley (2015), as well as the average of these measures. The data are quarterly from 1971:Q4 to 2017:Q2.

Figure 3: Measures of the interest rate cycle



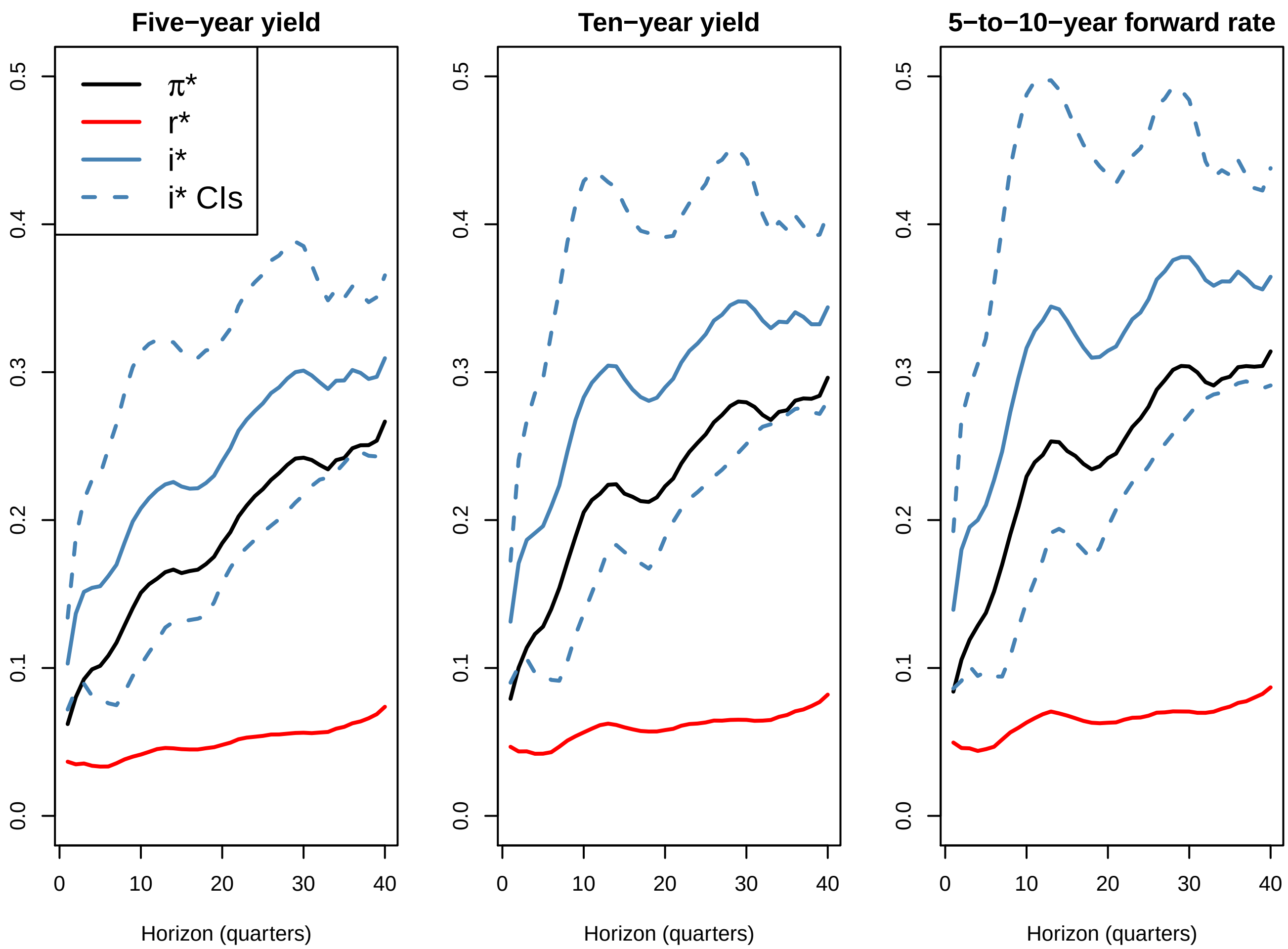
Estimates of the cycle in the level of interest rates. The black line is the demeaned ten-year yield, the red line is the demeaned difference of $y_t^{(10y)} - \pi_t^*$ and the blue line is the difference of $y_t^{(10y)}$ and $i_t^* = \pi_t^* + r_t^*$. Shaded areas are NBER recessions. The data are quarterly from 1971:Q4 to 2017:Q2.

Figure 4: Short-rate expectations and term premium



Left panels: current one-year yield (black line) and expectations of the future one-year yield at horizons from two to 14 years (colored lines) for a stationary VAR (top row) and based on shifting macro trends and a VAR of detrended yields (bottom row). Right panels: five-to-ten-year forward rate with estimated expectations and term premium components. The data are quarterly from 1971:Q4 to 2007:Q4.

Figure 5: Variance ratios



Variance of h-quarter changes in π_t^*, r_t^*, and $i_t^* = \pi_t^* + r_t^*$ relative to variance of h-quarter changes in long-term interest rate. The dashed lines show 95%-confidence intervals for the i_t^*-variance ratio, constructed as described in the text. The data are quarterly from 1971:Q4 to 2017:Q2.