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A Macroeconomic Model with Occasional Financial Crises

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November 2018

Working Paper 2017-22
http://www.frbsf.org/economic-research/publications/working-papers/2017/22/

Suggested citation:

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A Macroeconomic Model with Occasional Financial Crises

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Abstract

Financial crises are born out of prolonged and credit-fueled boom periods and, at times, they are initiated by relatively small shocks. Consistent with these empirical observations, this paper extends a standard macroeconomic model to include financial intermediation, long-term loans, and occasional financial crises. Within this framework, intermediaries raise their lending and leverage in good times, thereby building up financial fragility. Crises typically occur at the end of a prolonged boom, initiated by a moderate adverse shock that triggers a liquidation of existing investment, a contraction in lending, and ultimately a deep and persistent recession.

Keywords: Financial Crises, Financial Intermediation, Financial Stability

JEL codes: E32, E44, E52, G1, G01, G21

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I am particularly thankful for detailed comments by Tobias Adrian, Paul Beaudry, Fabrice Collard, Keshav Dogra (discussant), Martin Ellison, Andrea Ferrero, Andrew Foerster, Mark Gertler, Óscar Jordà, Nobuhiro Kiyotaki, Lars Lochstoer (discussant), and Josef Schrotth (discussant). I also thank Joseph Pedtke, Michael Tubbs, and Anita Todd for excellent research & editing assistance and many seminar and conference participants for their insights at Bank of England, Banque de France, CREF, European Central Bank, Federal Reserve Bank of San Francisco, Federal Reserve Board, IMF, the 12th Macro Finance Workshop, New York University, Tilburg University, UC Davis, University of Oxford, the Meeting of the Canadian Macroeconomics Study Group, the European Winter Meetings of the Econometric Society, and the New York Fed - Oxford Monetary Economics Conference. A previous version was circulated with the title "Financial Crises & Debt Rigidities". Financial support by the German Academic Exchange Service, the German National Academic Foundation, and the David Walton Scholarship is gratefully acknowledged. All errors are my own. The views expressed herein are solely those of the author and do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Federal Reserve System.
1 Introduction

The 2007-09 financial crisis revealed the need for macroeconomic models to incorporate connections between the financial sector and the macroeconomy that can amplify economic shocks and lead to occasional deep economic downturns. Over the last years, rapid advances have been made to extend standard macroeconomic models and include financial intermediation to account for episodes of severe financial distress. At the same time, a quickly growing empirical literature has revealed several stylized facts about financial crises.

Crises are rare events that are usually preceded by prolonged boom periods and a buildup of macro-financial imbalances. For example, in the run-up towards crises, credit usually rises rapidly, and credit growth is a robust early-warning indicator of crises (e.g., Schularick and Taylor, 2012; see also Figure 10 in Appendix A.3.2).

Crises are associated with severe recessions that are typically deeper than normal recessions, in particular if they are preceded by a buildup of credit (Jordà, Schularick, and Taylor, 2013). However, the ultimate triggers of crises can be relatively small. With respect to the 2007-09 financial crisis, Gorton and Ordoñez (2014) argue that losses from mortgage-backed securities – the relevant shock for the financial sector around this time – were actually quite modest (see also Ospina and Uhlig, 2018).

These empirical facts about crises pose challenges to current macroeconomic models. Why does financial fragility build up in good times? What is the propagation mechanism that turns shocks that are not particularly large into severe macroeconomic events? In this paper, I develop a model that addresses these questions. In my model, crises are as frequent and severe as in the data, financial fragility endogenously builds up during booms when credit expands, and crises are usually initiated by a moderate adverse shock.

In typical macroeconomic models, two features generally work against a buildup of financial instability in good times. First, agents are risk-averse and therefore prefer to smooth their consumption. When their income temporarily increases, then agents want to save part of it and any prior level of borrowing therefore decreases. Second, in good times, asset prices increase, generally resulting in countercyclical leverage. However, in the model that I consider, these forces are overturned due to agent heterogeneity and limited asset market participation.

At the heart of my model are two types of agents and a firm. Households supply labor to the firm and receive a wage income in return. Financial intermediaries (or banks, for short) invest in the capital stock of the firm and extract a capital income. Both type of agents are risk-averse and consume. However, they differ in their investment opportunities. Households can only trade a short-term bond, and they are restricted from investing in the capital stock of the firm directly. In contrast, banks have access to both asset markets. In addition, agents are also heterogeneous in
their degree of patience as in Kiyotaki and Moore (1997). Intermediaries are more impatient than households and therefore borrow from households in the short-term bond market.

Starting from this basic environment, I consider a series of models to clearly illustrate the key mechanisms. First, I start out with the special case of constant capital and labor, which are inelastically supplied. In this environment, intermediary leverage is countercyclical. However, when allowing for endogenous capital — the second model that I consider — then intermediary leverage can be procyclical. That is because banks’ balance sheets expand more in good times when capital is endogenous and such expansions are largely debt-financed.

Thus, it can be optimal for banks to enlarge their balance sheets and raise their leverage during boom periods. However, by levering up in good times, banks may also increase the risk of funding restrictions by creditors, once an adverse shock hits the economy. The third model considers this trade-off explicitly by introducing long-term loans and occasional financial crises. Similar to Gertler and Kiyotaki (2015), a “run equilibrium” could open up if households were to stop rolling over their short-term funding and the proceeds from banks’ assets were insufficient to cover banks’ outstanding debt. This condition is satisfied whenever banks’ leverage goes above a certain threshold. The increase of leverage in good times moves banks closer to this threshold, thereby building up financial fragility. To avoid a creditor run, banks inefficiently liquidate a fraction of their long-term loans. The early liquidation of loans gives banks additional liquidity and “eliminates” the possible run equilibrium. However, this process is also particularly costly for the economy since ongoing investment projects are stopped and a fraction of capital is lost. In this way, the model is able to account for the sharp contraction of output during financial crises.

Taken together, the model includes standard business cycle dynamics, a realistic representation of the financial sector’s balance sheet, and endogenous financial crises. I calibrate the model to match both the frequency and the severity of crises in the data. In this calibrated version, I find that the typical path leading to a crisis is characterized by a prolonged and credit-fueled boom, followed by a sudden bust that is eventually triggered by a relatively moderate adverse shock.

As in the data, credit growth is a robust predictor of crises (e.g., Schularick and Taylor, 2012). Financial recessions are typically deeper than nonfinancial recessions, in particular if they are preceded by an unusual large buildup of credit, confirming existing empirical evidence (e.g., Jordà et al., 2013). In addition, credit spreads predict well the severity of crises (Krishnamurthy and Muir, 2017). The behavior of the economy around nonfinancial recessions is different, since they are not preceded by an expansion of banks’ balance sheets, a credit boom, or a buildup of leverage. In addition, I show that the model replicates the occurrence of the 2007-09 financial crisis when confronted with a historical series of structural productivity shocks for the U.S. economy.

The mechanisms through which systemic risk builds up in the model and by which the eventual crash is determined are also empirically validated. First, I show that the leveraging behavior
of financial and nonfinancial firms is supported by U.S. data — for both book and market leverage. Second, Chodorow-Reich and Falato (2017) provide empirical evidence for the early liquidation of loans during the 2007-09 financial crisis in the United States. They find that total long-term credit and commitments outstanding contracted by 5.8% in 2008 and 5.9% in 2009 because borrowers had their borrowing limit lowered by an unhealthy lender following a covenant violation. Such a violation gives banks the chance to renegotiate the loan terms or to accelerate repayment, as is the case in this paper. This channel accounts for roughly 2/3 of the total credit reduction by unhealthy banks during the 2007-09 crisis, thus, the dominant channel through which credit contracted.

Last, I also model long-term defaultable debt in a novel and tractable way. Firms invest in long-term projects financed by long-term loans that mature stochastically. Firms’ debt, capital, leverage, and chance of default differ depending on when a loan was initially issued. ¹ This feature implies a rich time dependence and allows the model to match the empirical evidence by Demyanyk and Hemert (2011) and Justiniano et al. (2017), who show that loans that were issued closer to the 2007-09 financial crisis in the United States were of lower quality and had higher default rates ex-post. In addition, the illiquidity and the early liquidation of long-term loans determine the likelihood and the severity of crises in the model.

Related Literature. This paper builds on a vast literature about financial frictions within macroeconomic settings.² Before the 2007-09 financial crisis, financial frictions were mostly considered with respect to the balance sheets of nonfinancial firms (e.g., Bernanke, Gertler, and Gilchrist, 1999). Since then, the literature has quickly progressed. The focus shifted towards modeling financial intermediaries and introducing occasional financial crises explicitly.³ Several papers have shown that aggregate lending and investment can suddenly contract when the financial sector’s net worth or risk-bearing capacity is reduced (e.g., Gertler and Kiyotaki, 2010).

Other contributions have particularly highlighted the nonlinear nature of such mechanisms. An adverse shock can have substantially worse effects if the financial sector is already at or close to its limits on how much funding to raise (e.g., Brunnermeier and Sannikov, 2014). In addition, when the economy enters a recession, borrowing restrictions for households, firms, and financial institutions may bite at the same time. The joint occurrence of these events may again amplify the effects of adverse macroeconomic disturbances, with the different sectors pulling each other down (e.g., Elenev, Landvoigt, and Van Nieuwerburgh, 2017). However, whether these mechanisms provide enough additional amplification to quantitatively match the severity of crises in the data without relying on large shocks remains an open debate.

¹See, for example, Chatterjee and Eyigungor (2012), Gomes, Jermann, and Schmid (2016), Elenev, Landvoigt, and Van Nieuwerburgh (2017), and Greenwald (2018) for alternative ways of modeling long-term and defaultable debt.
²Among the seminal contributions in this field are Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and Fuest (1997), and Bernanke, Gertler, and Gilchrist (1999).
Moreover, only a few papers have attempted to provide a theoretical account of both the buildup of fragilities during the boom period that usually precedes crises and the eventual crash.\(^4\)^\(^5\) One exception is Boissay, Collard, and Smets (2016), in which crises follow credit booms and are initiated by moderate adverse shocks. In their paper, households accumulate bank debt during a boom, giving banks incentives to engage in risky activities, which can result in a collapse of the interbank market.

Gorton and Ordoñez (2014, 2016) build on the idea that debt is informationally insensitive during booms but it can suddenly turn informationally sensitive even after small shocks and therefore lead to a contraction in lending. Boz and Mendoza (2014) show that a model in which agents learn about a new financial environment can replicate the boom-bust pattern observed in the United States around the 2007-09 financial crisis.

Compared with these contributions, my paper presents a novel mechanism for boom-bust cycles to occur. Here, it is the leveraging behavior of intermediaries and their maturity mismatch that occasionally drive the economy into financial crises. In addition, the model is different from the existing literature in that it joint matches the mentioned stylized facts about crises and provides an empirical validation of the mechanisms that give rise to boom-bust-cycles.

The paper shares with Gertler and Kiyotaki (2015) and Gertler, Kiyotaki, and Prestipino (2017) that banks are subject to roll-over crises. Depending on macroeconomic fundamentals, a run equilibrium occasionally arises. In contrast to these papers, however, a run never materializes since banks engage in an early liquidation of their loans to avoid a run. Such liquidations are costly and result in a sharp contraction in aggregate output, capturing the discrete nature of crises.

For the main quantitative analysis, I treat crises as unanticipated events. In an extension, I relax this assumption and consider a slightly modified version that allows for crises to be anticipated. In future work, it would be interesting to take the model with anticipated crises further and consider macro-prudential policy interventions that aim to reduce the likelihood of crises.

Road Map. The next section outlines the basic environment. Starting from this benchmark, Model I considers the case of constant capital and labor. The second one allows capital to be endogenous (Model II). And third, long-term loans and occasional financial crises are added (Model III). This final model is quantitatively analyzed in Sections 5.5–5.9 and Section 6 concludes.

\(^4\)One early contribution that could be interpreted in this way is Lorenzoni (2008). In contrast to his paper, I provide a quantitative analysis of financial crises within a macroeconomic setting and show that financial instability can increase after a sequence of positive shocks. In this regard, the analysis is also different from the so-called “volatility paradox” in Brunnermeier and Sannikov (2014). They show that systemic risk can increase if the volatility of an aggregate shock is reduced. Here, the volatility of the aggregate shock in the model is not altered, but the type of shock realizations are analyzed that move the economy into a crisis.

\(^5\)In Adrian and Boyarchenko (2012), the procyclicality of bank leverage depends on an agency problem between banks and their creditors (a value-at-risk constraint). In contrast, here, it is due to the heterogeneity of agents and their limited asset market participation.
2 The Basic Environment

Firm. A representative firm operates according to a Cobb-Douglas production function
\[ Y_t = A_t^\alpha K_{t-1}^{1-\alpha} \]
combining labor \( H_t \) with aggregate capital \( K_{t-1} \), supplied in period \( t - 1 \), to produce the good \( Y_t \). The only source of aggregate risk in the model enters via the technology level \( A_t \),
\[ A_t = e^{a_t} \]
\[ a_t = \rho_a a_{t-1} + \epsilon_t^a \]
\[ \epsilon_t^a \sim N(0, \sigma_a^2) \]
where \( \epsilon_t^a \) is termed the technology shock. Labor and capital pay their marginal products,
\[ w_t = (1 - \alpha) \frac{Y_t}{H_t} \]
\[ r^K_t = \alpha \frac{Y_t}{K_{t-1}} \]
where \( w_t \) is the wage and \( r^K_t \) is the rental rate per unit of capital.

Households. There is a continuum of measure unity of identical households. Following Greenwood, Hercowitz, and Huffman (1988), a household values consumption \( C_t \) and dislikes labor \( H_t \), captured by the flow utility
\[ U^H(C_t, H_t) = \log \left( C_t - \chi \frac{H_t^{1+\phi}}{1+\phi} \right) \]
where \( \phi \) represents the inverse Frisch elasticity of labor supply. The household chooses contingent plans for consumption, labor supply, and borrowing in the form of short-term and riskless bonds \( B^H_t \), so as to maximize expected lifetime utility. The household discounts the future at the rate \( \beta^H \). Taking wages and interest rates as given, the household maximizes expected lifetime utility,
\[ E_t \left[ \sum_{k=0}^{\infty} (\beta^H)^k U^H(C_{t+k}, H_{t+k}) \right] \]
subject to
\[ C_t + B^H_{t-1} R_t \leq w_t H_t + B^H_t \]
where \( R_t \) is the real interest rate between period \( t - 1 \) and \( t \) on savings in short-term bonds, that is, if \( B^H_t \) is negative as implied by the calibration discussed below. The solution to the above problem
gives the inter- and intratemporal optimality conditions

\[ 1 = E_t [\Lambda_{t,t+1}] R_{t+1} , \]
\[ w_t = \chi^H_{t} , \]  
where

\[ \Lambda_{t,t+1} = \beta^H \left( \frac{C_t - \chi^H_{t+1}}{C_{t+1} - \chi^H_{t+1}} \right) \]

is the household’s stochastic discount factor.

**Financial Intermediaries.** There is a continuum of measure unity of identical financial intermediaries (or banks) that transform short-term and riskless debt \( B^F_t \) into long-term and risky investment in the firm’s capital stock. As in Kiyotaki and Moore (1997), I assume that an intermediary discounts the future at the rate \( \beta^F \) which is lower than the household’s discount rate \( \beta^H \). The intermediary values shareholders’ flow utility of real dividends \( D_t \) according to

\[ U^F(D_t) = \log(D_t) , \]
and chooses new short-term debt \( B^F_t \), capital \( K_t \), and dividends \( D_t \) every period to maximize expected lifetime shareholder utility. Taking prices and interest rates as given, the intermediary maximizes

\[ E_t \left[ \sum_{k=0}^{\infty} \left( \beta^F \right)^k U^F(D_{t+k}) \right] \]

subject to

\[ D_t + Q^K_t K_t + B^F_{t-1} \tilde{R}_t \leq B^F_t + R^K_t Q^K_{t-1} K_{t-1} + T^F_t \]

where \( Q^K_t \) is the price per unit of capital and the return to capital is given by

\[ R^K_t = \frac{Q^K_t (1 - \delta) + a \chi_{t-1}}{Q^K_{t-1}} , \]

where \( \delta \) denotes the rate of depreciation. The interest rate on borrowing in short-term bonds is given by \( \tilde{R}_t \). I assume that \( \tilde{R}_t \) is different from the interest rate on savings \( R_t \). Following Schmitt-Grohe and Uribe (2003), \( \tilde{R}_t \) is debt-elastic with \( \tilde{R}_t = R_t + \psi B^F_{t-1} \) and \( \psi > 0 \), and taken as given by the intermediary. This assumption ensures that the amount of borrowing is uniquely determined in a deterministic steady state and has the sensible implication that the cost of borrowing is positively related to the stock of debt. To ensure that these costs do not affect aggregate resources, I assume that the amount \( \psi \left( B^F_{t-1} \right)^2 \) is rebated to the intermediary as a lump-sum \( T^F_t \) at time \( t \).

Constraint (4) is a budget constraint. The right-hand side states the available resources: the
amount of new debt $B,F_t$, the profits on last period’s capital $R^K_t K_{t-1}$, and the lump sum transfer $T,F_t$. These have to cover the payout of dividends $D_t$, new capital investment $Q^K_t K_t$, and outstanding debt $B,F_{t-1} \tilde{R}_t$. An important variable is bank leverage, which is given by $\frac{B,F_t}{Q^K_t K_t}$. The above problem implies that banks never raise equity but always issue a positive amount of real dividends. The solution to the intermediary’s problem is given by two intertemporal optimality conditions,

\[
\frac{1}{D_t} = \beta^F_E t \left[ \frac{1}{D_{t+1}} \right] \tilde{R}_{t+1}, \quad (6)
\]

\[
\frac{1}{D_t} = \beta^F_E t \left[ \frac{R^K_{t+1}}{D_{t+1}} \right]. \quad (7)
\]

**Capital Good Producer.** A representative capital good producer undertakes real investment. Given the price of capital $Q^K_t$, the capital good producer maximizes profits by choosing the economy-wide units of investment $I_t$

\[
\max_{I_t} \left\{ Q^K_t I_t - \Phi(I_t, K_{t-1}) \right\}
\]

subject to

\[
\Phi(I_t, K_{t-1}) = I_t + \frac{\zeta}{2} \left( \frac{I_t - \delta K_{t-1}}{K_{t-1}} \right)^2 K_{t-1},
\]

implying quadratic adjustment costs that follow the parsimonious specification in He and Krishnamurthy (2014). The above problem gives the intratemporal optimality condition

\[
Q^K_t = 1 + \zeta \left( \frac{I_t}{K_{t-1}} - \delta \right), \quad (8)
\]

and capital evolves according to

\[
K_t = I_t + (1 - \delta) K_{t-1}. \quad (9)
\]

**Resource Constraint.** Total output is divided between household and intermediary consumption, investment, and the costs of adjusting capital,

\[
Y_t = C_t + D_t + \Phi(I_t, K_{t-1}).
\]

Finally, short-term bonds are in zero net supply, such that household saving in bonds ($-B^H_t$) is equal to intermediary borrowing ($B,F_t$).

The next two sections consider two special cases of this basic environment to clearly illustrate what drives bank leverage, the key indicator of financial instability in the final model.
3 Model I: Constant Capital and Labor

Next, I assume that capital and labor are constant, equal to unity in all periods, \( H_t = 1 \) and \( K_t = 1 \) \( \forall t \), and supplied inelastically. Production simplifies to \( Y_t = A_t \), the wage is given by \( w_t = (1 - \alpha)A_t \), and the rental rate of capital is equal to \( r_t^K = \alpha A_t \). Since households supply one unit of labor inelastically in all periods, I set \( \chi = 0 \) and equation (1) is not part of the household’s optimality conditions. The household’s stochastic discount factor simplifies to \( \Lambda_{t,t+1} = \beta^H \left( \frac{C_t}{C_{t+1}} \right) \).

Further, I assume that \( \delta = 0 \) and that there are no capital good producers and no investment. The price of capital \( Q_t^K \) is then determined by the asset pricing equation (7). With these restrictions, households and banks simply receive stochastic endowment streams.

**Calibration.** The model is calibrated to a quarterly frequency for the U.S. economy. The persistence and standard deviation of the technology shock are obtained by estimating an AR(1) process to the linearly detrended logarithm of the total factor productivity (TFP) series by Fernald (2014) for the second half of the post-WWII period (1980 Q1 – 2016 Q4), giving \( \rho_a = 0.93 \) and \( \sigma_a = 0.68\% \). These numbers are close to estimates based on typical business cycle models (e.g., Smets and Wouters, 2007). The capital share \( \alpha \) is set to 0.3. Further, I choose \( \beta^H = 0.99 \), giving an annualized real interest rate of around 4% in steady state. I calibrate \( \beta^F = 0.985 \) to give an annualized excess return of intermediary assets over liabilities of 2% in steady state — in line with the cost of intermediation for the U.S. documented in Philippon (2015). Last, given the calibrated discount factors, I set \( \psi = 0.0005 \) to obtain an asset-to-equity ratio for intermediaries of around 2 in steady state, as implied by the calibrations of the following models and therefore simplifying cross-comparisons.\(^6\)

**Impulse Responses.** The model is solved with a first-order perturbation method around the deterministic steady state. The impulse responses to a positive one-standard-deviation technology shock are shown in Figure (1). Output \( Y_t \), household consumption \( C_t \), and dividends \( D_t \) increase. Both agents would prefer to save part of their additional income. However, in equilibrium, the incentives for banks to save dominate and \( B^F_t \) falls. That is because banks have a lower steady-state level of consumption than households, and consumption smoothing dictates that less resources are used to increase consumption and more of the additional income is saved. In equilibrium, the real interest rate \( R_{t+1} \) declines as well as the premium for borrowing \( \psi B^F_t \). In addition, the value of capital \( Q^K_t \) increases, such that intermediary leverage \( \frac{B^F_t}{Q^K_t} \) unambiguously falls. The correlation between output and leverage is \(-0.5\).

Hence, in this environment with constant capital and labor, bank leverage is **countercyclical**. These results are driven by the differences in discount factors. Appendix A.4.1 shows the response of leverage \( \frac{B^F_t}{Q^K_t} \) for different values of \( \beta^F \), leaving the remaining parameters unchanged. If \( \beta^F > \beta^H \), then intermediary leverage increases after a positive shock for a range of calibrations. If \( \beta^F = \beta^H \), then \( \frac{B^F_t}{Q^K_t} = 0 \) \( \forall t \).

\(^6\)This calibration takes into account that the additional costs of borrowing are in the end rebated to the intermediary.
Next, I show that the cyclicality and the dynamics of leverage depend on whether capital is allowed to change or not. To this end, I endogenize capital but keep labor fixed at unity for simplicity.\footnote{Endogenous labor does not change the results in this section.} Capital now evolves according to (9), the return to capital $R^K_t$ is given by (5), and the value of capital $Q^K_t$ is determined by the capital good producer’s optimality condition (8).

**Calibration.** I calibrate the depreciation rate $\delta$ to the standard value of 0.025 and follow He and Krishnamurthy (2014) in calibrating the capital adjustment cost parameter $\zeta$ to 3. The parameters $\alpha, \rho_a, \sigma_a, \beta^H,$ and $\beta^F$ are kept at their previous values. As it turns out, intermediary leverage may now increase to a positive shock even if banks borrow in steady state (as implied by $\beta^H > \beta^F$ and $\psi > 0$). However, whether that is the case depends on the initial level of leverage and therefore on the calibration of the cost parameter $\psi$, for given calibrated discount factors. The higher the initial leverage, the more likely that leverage falls after a positive shock, since the increase in asset prices will dominate any change in liabilities. To pin down $\psi$, I obtain empirical evidence on the response of intermediary leverage to a technology shock and on the empirical correlation between output and intermediary leverage.\footnote{Alternatively, one could find a calibration for $\psi$ first, and then choose $\beta^H$ and $\beta^F$ to match the evidence.} Based on bank-level data sets for commercial and investment banks, I derive an empirical proxy for the U.S. financial sector’s market leverage (see Appendix...
A.3.1 for details). In the data, the cyclical component of market leverage and a monthly measure of economic activity are mildly positively correlated, with a correlation coefficient of 0.04.9

Further, I use the residual from the estimated AR(1) process of the TFP series as a proxy for the series of structural technology shocks, denoted \( \hat{\epsilon}_a^t \), based on the sample 1980 Q1−2016 Q4. Using local projections (Jordà, 2005) and the generalized method of moments (GMM), I simultaneously estimate the following system10

\[
\log(TFP)_t = \rho_0 \log(TFP)_{t-1} + \hat{\epsilon}_i^a \\
Lev_{t+k-1} - Lev_{t-2} = \beta_0^k + \beta_1^k \hat{\epsilon}_i^a + \epsilon_{t+k} \quad \text{for } k \in \{1, 2, ..., 19\},
\]

where \( Lev_t \) denotes the market leverage of the intermediary sector at time \( t \) and \( \beta_1^k \) gives the reaction of leverage to a technology shock at horizon \( k \). Figure 2 shows the results. Following a positive technology shock, leverage initially declines and then rises over time, turning positive after a few quarters. I find that \( \psi = 0.000625 \) matches well the impulse response and the cyclicality of leverage in the data, giving an asset-to-equity ratio of around 2 for the intermediary. This calibration gives a lower market leverage than found for commercial and investment banks in the collected data, with asset-to-equity ratios above 5 in normal times. However, as explained further below, matching the level of leverage in the data is not essential for the analysis in this paper, what is important is the dynamic behavior of leverage to the aggregate structural shock.11

**Impulse Responses.** Given the new version of the model, the impulse responses to a positive technology shock are shown in Figure 3. Output, consumption, dividends, and the interest rate \( R_{t+1} \) behave similar to the ones above and are shown in Figure 12 in Appendix A.4.1. However, banks’ balance sheets now undergo larger expansions and contractions since capital is endogenous. Following a positive technology shock, the capital stock \( K_t \) and the value of capital \( Q^K_t \) increase. Banks finance the additional capital by acquiring new debt \( B^F_t \). Hence, intermediary leverage \( \frac{B^F_t}{Q^K_t} \) could either increase or decrease. For the chosen calibration, leverage falls initially as asset prices rise immediately. However, over time, the debt-financed balance sheet expansion dominates and the response of leverage turns positive after a few quarters as in the data. Overall, intermediary leverage is mildly procyclical. The correlation between output and leverage is 0.03.

However, due to the lower level of leverage than in the data, the model only generates small quantitative variations in leverage. Nonetheless, it matches well the cyclicity and the impulse response of leverage to a technology shock. These two features matter for the analysis in this paper since they determine how aggregate risk and financial instability interact. Figure 13 in Appendix

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9See Section 5.9 for a detailed analysis on the cyclicality of leverage.
10The system is estimated jointly to avoid a generated regressor problem (Newey and McFadden, 1994). GMM requires the choice of a weighting matrix and instruments. Regarding the weighting matrix, a Newey-West correction for heteroskedasticity and autocorrelation is used. The instruments are the regressors in (10) and (11). I consider impulse responses with respect to leverage at time \( t - 2 \) to account for potential news shocks at time \( t - 1 \).
11I found that one can additionally match the level of leverage when allowing for heterogeneous risk-aversion coefficients. Here, I have assumed log-utility for both agents for simplicity.
A.4.1 shows again the response of leverage for different values of $\beta^F$, leaving the remaining parameters unchanged. The higher $\beta^F$, the more positive the response of leverage to a positive technology shock since the initial increase in asset prices does not lower leverage as much.

Model II shows that it can be optimal for banks to increase their balance sheets and leverage up during expansions. However, by leveraging up, banks may also risk funding restrictions by creditors, once an adverse shock hits the economy. The next section considers this trade-off explicitly by incorporating occasional financial crises. In this third model, the nonlinear nature of crises is replicated since banks inefficiently liquidate existing loans early. What is needed is therefore a distinction between outstanding and newly issued credit. To this end, I introduce long-term loans, giving a realistic representation of the financial sector’s balance sheet and determining the occurrence and the severity of financial crises. Next, I describe the new model additions.
5 Model III: Long-term Loans and Occasional Financial Crises

5.1 Entrepreneurs

To establish a need for loans, I introduce another type of agent — entrepreneurs — that take on long-term debt to finance long-term projects. Instead of investing directly in the economy’s capital stock, intermediaries now lend to entrepreneurs that in turn invest in the aggregate capital stock. An entrepreneur who acquires a loan in period \( t \) is termed a “new entrepreneur” in that period — highlighted by the superscript “new”. There is a unit mass of new entrepreneurs. A new entrepreneur has net worth \( N_t \) available. Additionally, the entrepreneur acquires a long-term, collateralized, and defaultable loan \( Q_t L_{new}^t \) from a bank, where \( Q_t \) denotes the price of long-term loans. The face value of the loan is \( L_{new}^t \) and the underlying collateral are the units of capital \( K_{new}^t \).

Combining \( N_t \) and \( Q_t L_{new}^t \), the entrepreneur purchases \( K_{new}^t \) units of capital, such that

\[
Q_t K_{new}^t = N_t + Q_t L_{new}^t .
\]

**Borrowing Constraint.** In addition, I assume that a new entrepreneur faces a borrowing constraint when taking up a loan. Similar to Kiyotaki and Moore (1997), the constraint demands that the face value of the loan has to be less than or equal to a fraction \( \theta \) of the value of the assets,

\[
L_{new}^t \leq \theta Q_t K_{new}^t ,
\]

with \( 0 < \theta < 1 \). I assume that this borrowing constraint binds continuously for all new entrepreneurs, such that episodes with high capital values allow for larger loan-to-capital ratios on newly issued loans. However, despite the borrowing constraint, a fraction of entrepreneurs may still default on the loan ex-post.

**Stochastic Maturity.** Each loan matures with probability \( 1 - \gamma \) in the next period. An entrepreneur with a nonmaturing loan dating from period \( t \) receives the rental rate \( r_{t+j}^K \) per unit of capital in some future period \( t + j \) from the good producer. These profits are transferred lump-sum to households. The collateral \( K_{new}^t \) of a nonmaturing loan stays constant as households refurbish depreciated capital \( \delta K_{new}^t \).\(^{12}\) Moreover, the face value \( L_{new}^t \) remains unchanged as an entrepreneur does not have to make a payment to a financial intermediary if the loan does not mature.\(^{13}\)

When a loan that was originally issued in period \( t \) matures in period \( t + j \), then the entrepreneur’s project ends as well. In that period, the entrepreneur receives the rental rate \( r_{t+j}^K \) per unit of capital and sells the remaining capital for the price \( Q_{t+j}^K \). Additionally, the profits are hit by an idiosyncratic shock \( \omega \). This shock is drawn from a uniform distribution, \( \omega \sim U[0, 2] \), independent across

\(^{12}\)This assumption ensures that the probability of default of very old entrepreneurs does not approach one since their underlying capital and therefore their profits do not vanish in the long run.

\(^{13}\)This assumption implies no default considerations before a loan matures, which makes the problem tractable and aggregation feasible as shown in Appendix A.1.
time and entrepreneurs, and normalized to have mean and width support unity.\(^ {14}\) The profits are

\[
\omega R^K_{t+j} Q^K_{t+j-1} K^\text{new}_t. \tag{13}
\]

**Default.** When a loan matures, then the entrepreneur can either choose to repay it or to default on this obligation. An entrepreneur defaults if the face value of debt \(L^\text{new}_t\) is larger than the entrepreneur’s profits in (13). Or stated differently, if the idiosyncratic shock \(\omega\) is lower than a particular threshold \(\omega_{t+j|t}\) in period \(t+j\), where

\[
\omega_{t+j|t} = \frac{L^\text{new}_t}{R^K_{t+j} Q^K_{t+j-1} K^\text{new}_t}. \tag{14}
\]

Equation (14) shows that the model incorporates vintage-specific default thresholds \(\omega_{t+j|t}\). These depend on the loan-to-capital ratio \(L^\text{new}_t K^\text{new}_t\) for a loan that is issued in period \(t\) and the aggregate profits to capital \(R^K_{t+j} Q^K_{t+j-1}\) in period \(t+j\). When \(Q^K_t\) is high, the borrowing constraint for entrepreneurs (12) relaxes, allowing for higher loan-to-capital ratios \(L^\text{new}_t K^\text{new}_t\), which in turn results in higher default thresholds \(\omega_{t+j|t}\) in any future period \(t+j\).

When an entrepreneur from period \(t\) defaults in \(t+j\), then the remainder \(\omega R^K_{t+j} Q^K_{t+j-1} K^\text{new}_t\) is recovered by the financial intermediary. The remaining profits \(\omega R^K_{t+j} Q^K_{t+j-1} K^\text{new}_t - L^\text{new}_t\) of all nondefaulting entrepreneurs are equally split among new entrepreneurs in period \(t+j\). The sharing of profits ensures that all new entrepreneurs start with the same amount of net worth. New entrepreneurs consist of all entrepreneurs whose loans matured — whether they defaulted or not — such that the total number stays constant.\(^ {15}\)

### 5.2 Financial Intermediaries

Financial intermediaries now invest in the whole market portfolio of loans \(L_t\), defined recursively and encompassing all outstanding loans according to

\[
L_t = L^\text{new}_t + \gamma L_{t-1}. \tag{16}
\]

An intermediary again maximizes (3) subject to

\[
D_t + Q_t L_t + B_{t-1} \bar{R}_t \leq B_t + R^L_t Q_{t-1} L_{t-1} + T^F_t, \tag{17}
\]

\(^ {14}\)The mean unity ensures that the idiosyncratic shock does not change profits in the aggregate. The width support of one gives a zero lower bound on \(\omega\). This implies that a fraction of loans always defaults in equilibrium as visible from equation (14).

\(^ {15}\)The net worth of a new entrepreneur is given by

\[
N_t = (1 - \gamma) \sum_{j=1}^{\infty} \int_{\omega_{t+j}}^{\infty} \left( \omega R^K_{t+j} Q^K_{t+j-1} K^\text{new}_t - L^\text{new}_{t+j} \right) d\Phi(\omega). \tag{15}
\]

Appendix A.1.2 shows how to derive a simplified formula for this expression.
where \( R_t^L \) is the return per loan in period \( t \). Note that the profits per loan \( R_t^L Q_{t-1} \) account for an infinite number of loan vintages, each with its own vintage-specific default threshold \( \bar{\omega}_{t|t-j} = \frac{L_{t-k}^{new}}{R_t^K Q_{t-1}^{K_{t-1}^{new}}} \) in period \( t \). However, as shown in Appendix A.1.1, it is not necessary to keep track of the whole distribution of loans. Instead, \( R_t^L Q_{t-1} \) follows the simple expression

\[
R_t^L Q_{t-1} = \gamma Q_t + (1 - \gamma) \left( 1 - \frac{\bar{\omega}_t}{4} \right) ,
\]

(18)

where the first term captures profits from nonmatured loans and the second term gives the profits for maturing loans that are either repaid or default.\(^{16}\)

The variable \( \bar{\omega}_t \) is a weighted default threshold across all previous loan vintages, defined as

\[
\bar{\omega}_t = \frac{x_{t-1}}{R_t^K Q_{t-1}^{K_{t-1}}} = \sum_{k=1}^{\infty} \gamma^{k-1} L_{t-k}^{new} \bar{\omega}_{t|t-k} ,
\]

(19)

where

\[
x_{t-1} = L_{t-1}^{new} \cdot R_{t-1}^{new} + \gamma x_{t-2}
\]

(20)

is an auxiliary variable that accounts for the loan-to-capital ratios of all previously issued loans. One can therefore interpret \( x_t \) as a loan risk indicator. Equations (18) and (19) have intuitive interpretations. If the value of outstanding loans \( Q_t \) is low or the weighted default threshold \( \bar{\omega}_t \) is high, then profits per loan \( R_t^L Q_{t-1} \) are low. In turn, \( \bar{\omega}_t \) is high if either current profits to capital \( R_t^K Q_{t-1}^{K_{t-1}} \) are low or \( \frac{x_{t-1}}{L_{t-1}} \) is high.

The distinction between maturing and nonmatured loans plays a key role for the likelihood and the severity of crises as explained below. Moreover, note that the model implies a rich time dependence of the intermediary’s asset portfolio. It takes time to change the overall structure of the financial sector’s balance sheet since a large fraction of loans does not mature each period — credit is rigid.

The solution to the intermediary’s problem is given by two intertemporal optimality conditions,

\[
\frac{1}{D_t} = \beta^F \mathbb{E}_t \left[ \frac{1}{D_{t+1}} \right] \bar{R}_{t+1} ,
\]

(21)

\[
\frac{1}{D_t} = \beta^F \mathbb{E}_t \left[ \frac{R_t^L}{D_{t+1}} \right] ,
\]

(22)

and the intermediary’s leverage is now given by \( \frac{B_t^F}{Q_{t|t-1}}.\)

\(^{16}\)An important assumption that allows for this aggregation is that the idiosyncratic shock \( \omega \) follows a uniform distribution, giving convenient expressions for cumulative distribution functions and partial expectations (see Appendix A.1).
5.3 Occasional Financial Crises

Next, I introduce financial crises into the framework. Similar to Gertler and Kiyotaki (2015), a run equilibrium can emerge if households do not roll over their debt, acquire and sell the banks’ assets, and the revenue is insufficient to cover any outstanding debt, resulting in bank insolvency. However, in contrast to Gertler and Kiyotaki (2015), I assume that banks try to prevent their bankruptcy by inefficiently liquidating a fraction of their long-term loans. The liquidated loans generate additional liquidity for banks and eliminate the possibility of a run. However, they are also costly for the economy, since productive capital is lost due to the early liquidation.\footnote{The liquidation of loans resembles Diamond and Dybvig (1983), but it is not the same. In Diamond and Dybvig (1983), loans are liquidated in the run equilibrium. Here, they are liquidated to avoid such an equilibrium.}

At the beginning of each period, after the realization of the technology shock $\epsilon^a_t$, households decide whether to roll over their lending to banks. If households do not roll over their debt, they acquire the intermediary’s assets. I assume that households are less skilled in handling financial assets and can only sell a fraction $\kappa$ of the nonmatured loans, with $0 < \kappa < 1$. Or put differently, from the point of view of households, long-term loans are illiquid.\footnote{What I have in mind is a situation in which outsiders are taking over a bankrupt bank. The parameter $\kappa$ stands in for the fact that such outsiders know less about the banks’ assets, have to sell them quickly at once, and therefore receive less when selling the assets.} A run equilibrium then opens up if the households’ revenue from selling the intermediary’s assets are smaller than the intermediary’s outstanding debt, that is if

$$B^F_{t-1} R_t > (1 - \gamma) \left(1 - \frac{\overline{\omega}_t}{4}\right) L_{t-1} + \kappa \gamma Q_t L_{t-1},$$

where the first term on the right-hand side denotes profits from maturing loans and the second term gives profits from nonmaturing loans.\footnote{This assumption resembles the resaleability constraint in Kiyotaki and Moore (2012), with the difference that it only applies to households here.} Condition (23) can be rewritten as

$$\text{Lev}^*_t = \frac{B^F_{t-1} R_t - (1 - \gamma) \left(1 - \frac{\overline{\omega}_t}{4}\right) L_{t-1}}{\gamma Q_t L_{t-1}} > \kappa,$$

which shows that a run is possible whenever the intermediary’s leverage at the beginning of a period $\text{Lev}^*_t$ is larger than a threshold $\kappa$. If (24) is satisfied, then it is individually rational for a household not to roll over its debt if it perceives that others will do the same.

I assume that a bank-specific sunspot would determine whether a run equilibrium is realized for an individual bank. However, I assume that this sunspot only picks a very small number of banks to be in a run equilibrium each period. Aggregate prices $Q_t$ and $R^K/Q^K_{t-1}$ (via $\overline{\omega}_t$) in condition (23) can be satisfied even though the intermediary would be solvent if its assets were not liquidated, which is the case if $B^F_{t-1} R_t < R^K/Q^K_{t-1} L_{t-1}$, where the difference is due to the fact that $0 < \kappa < 1$. In the simulation of the model, banks are always solvent.
are therefore the ones that would occur absent a crisis.\textsuperscript{21}

Even though sunspots are idiosyncratic and only choose a small number of banks to be in a run equilibrium each period, they turn out to have aggregate effects. That is because banks are risk-averse which implies an extremely high marginal utility in the bankruptcy state.\textsuperscript{22} To avoid even the small probability of insolvency, all banks therefore try everything possible to avoid a run.

Before the sunspot is realized, banks have the option to liquidate a fraction of their loans by demanding entrepreneurs to repay their outstanding debt early.\textsuperscript{23} If loans are liquidated, then their underlying capital cannot be used in production, resulting in an immediate fall in output. In addition, a fraction \( (1 - \mu) \) of the associated capital is lost due to the early liquidation of projects. The remaining fraction \( \mu \) can be sold and entrepreneurs again receive an idiosyncratic shock \( \omega \sim U[0,2] \) on their revenues.

An entrepreneur from period \( t - j \) that is forced to repay its loan \( L_{t-j}^{new} \) early, defaults whenever the idiosyncratic shock \( \omega \) is lower than a threshold \( \omega^*_t \). Similar to (19), one can define a weighted default threshold across all vintages of loans that are liquidated early,

\[
\omega^*_t = \frac{x_{t-1}}{Q_t^p \mu (1 - \delta) L_{t-1}} = \sum_{j=1}^{\infty} \gamma^{j-1} L_{t-j}^{new} \omega^*_{t-j}.
\]

Following the derivation in Appendix A.1.3, each unit of a liquidated loan then generates \( (1 - \frac{\omega^*_t}{4}) \) as revenue.\textsuperscript{24} To avoid a run equilibrium when (24) is satisfied, all banks liquidate a fraction \( \tau_t \) of their loans at the beginning of a period, where \( \tau_t \) solves

\[
B_{t-1}^p R_t = L_{t-1} \left\{ \tau_t \left( 1 - \frac{\omega^*_t}{4} \right) + (1 - \tau_t) \left( (1 - \gamma) \left( 1 - \frac{\omega^*_t}{4} \right) + \kappa \gamma Q_t \right) \right\},
\]

such that the additional liquidity eliminates the incentives for households to run, but also reduces the overall stock of loans.\textsuperscript{25} A necessary condition for the banks’ decision is that households value the additional liquidity more than long-term illiquid loans, which is verified for a simulation of the calibrated model since households are less skilled in selling assets in the case of bankruptcy.\textsuperscript{26}

\textsuperscript{21}Alternatively, in Diamond and Dybvig (1983) and Gertler and Kiyotaki (2015), a sunspot selects one of two possible aggregate equilibria.

\textsuperscript{22}When banks are insolvent, they cannot pay out dividends. Note that shareholder marginal utility goes towards infinity if dividends approach zero, so all banks choose to avoid insolvency.

\textsuperscript{23}The empirical evidence by Chodorow-Reich and Falato (2017) mentioned in the introduction shows that bank health was mainly transmitted through such a channel during the 2007-09 financial crisis in the United States.

\textsuperscript{24}I assume that the remaining profits are transferred lump-sum to the household.

\textsuperscript{25}There are potentially multiple equilibria in the amount of loans \( \tau_t \) to liquidate, since \( \tau_t \) affects aggregate prices and several combinations of prices and quantities could cover the shortfall from equation (23). To obtain a unique value for \( \tau_t \), I assume that the prices \( Q^X_t, Q_t, \) and \( R^X_t Q_{t-1}^p \) absent a crisis enter (26). Or put differently, when deciding how much of their loans to liquidate, banks do not take into account that other banks will do the same.

\textsuperscript{26}The necessary condition is that \( (1 - \frac{\omega^*_t}{4}) > (1 - \gamma)(1 - \frac{\omega^*_t}{4}) + \kappa \gamma Q_t \), which implies that the liquidation of loans generates additional liquidity and allows to close the gap in (23). In addition, if \( (1 - \frac{\omega^*_t}{4}) < (1 - \gamma)(1 - \frac{\omega^*_t}{4}) + \gamma Q_t \), then banks would prefer not to liquidate their loans unless they had to in order to avoid a run. The difference between the
Thus, there are two key parameters that determine how frequent and severe financial crises are. The parameter $\kappa$ governs how often condition (24) is satisfied and $\mu$ determines the severity of crises by affecting the amount $\tau_t$ that needs to be liquidated.

Next, I analyze this third model in details. In what follows, a crisis is defined as a period during which condition (24) is satisfied and banks liquidate a fraction of their loans early. For the quantitative analysis that follows, I assume that crises are not anticipated. In Section 5.10, I relax this assumption.

The households’, the good producer’s, and the capital producer’s optimality conditions remain equivalent to the ones described in Section 2. All equilibrium conditions of Model III are listed in Appendix A.2.

5.4 Calibration

The calibration of all the structural parameters for Model III is shown in Table 1. The parameters $\rho_a$, $\sigma_a$, $\beta^H$, $\beta^F$, $\delta$, and $\zeta$ are kept at their previous values. The borrowing cost parameter $\psi$ is recalibrated to match the empirical impulse response of intermediary leverage and its cyclicality as discussed in Section 4. Again, the calibration gives a steady-state asset-to-equity ratio for banks of around 2, which is lower than in the data. However, what matters is not necessarily the level of leverage, but how leverage behaves with respect to the technology shock $\epsilon^t_a$. As explained below, the leverage cutoff $\kappa$ that determines the occurrence of financial crises is adjusted such that crises occur as often as in the data.

For a full quantitative analysis of the model, I endogenize households’ labor supply, such that equation (1) is part of the households’ optimality conditions and the household’s stochastic discount factor is given by (2). The inverse Frisch elasticity of labor supply $\phi$ is set to 0.5 and the relative utility weight $\chi$ is chosen to normalize labor supply to one in steady state.

The parameter $\gamma$ determines the maturity mismatch of financial intermediaries. Based on Call Reports for U.S. commercial banks, I find that the average maturity of assets is around 3.49 years, and that of liabilities is around 0.35 years, with a ratio between the two of 9.97 (see Appendix A.3.3 for details). I normalize the maturity of short-term debt to one quarter and choose $\gamma$ to match an average maturity of long-term debt of 9.97 quarters, giving $\gamma = 0.9$. The parameter $\theta$ determines how tight the entrepreneurs’ borrowing constraint (12) is. I pick $\theta$ to give an annualized default

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Note that the households’ budget constraint changes due to the lump-sum transfers from entrepreneurs. However, this does not affect the households’ optimality conditions.

Note that I have assumed log-utility for banks and households for simplicity. The model is therefore not able to replicate typical asset-price moments in the data. Allowing, for example, for Epstein-Zin-preferences could potentially align the model with the data in this respect.
rate of around 3% in steady state (Bernanke et al., 1999).²⁹ Hence, the calibration matches closely the maturity mismatch of intermediaries and their exposure to credit risk in the data.

<table>
<thead>
<tr>
<th>Agents</th>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Target / Source</th>
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<td>St. dev. technology shock</td>
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<td>Persist. technology shock</td>
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<td>Depreciation rate</td>
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<td>Capital adjustment cost</td>
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<td>He and Krishnamurthy (2014)</td>
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<tr>
<td>Households</td>
<td>Discount factor</td>
<td>βᴴ</td>
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</tr>
<tr>
<td></td>
<td>Inv. Frisch elasticity</td>
<td>φ</td>
<td>0.5</td>
<td>Literature</td>
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<td>Frequency Crises: 2.14%−4%</td>
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<td>Recovered capital</td>
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<td>Severity Crises ΔGDP Fin Rec / ΔGDP Ave Rec: 1.31 − 1.56</td>
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<td>Entrepreneurs</td>
<td>Borrowing constraint</td>
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<td>Stochastic maturity</td>
<td>γ</td>
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<td>Mat. Mismatch U.S. Comm. Banks</td>
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</tbody>
</table>

Table 1: Calibration of Structural Parameters for Model III.

Based on the macrohistory data by Jordà et al. (2017) for advanced economies from 1870 to 2013, crises occur around 4% of the time and 2.14% when restricting the sample to the post-WWII period (see Appendix A.3.2 for details). I pick κ such that the frequency of crises lies within this range (2.4%). Based on the data by Jordà et al. (2017) for the two mentioned samples, the change in real GDP from peak to trough is between 31% and 56% larger during financial recessions (−5.31% for 1870-2013 and −3.85% for the post-WWII period) than during average recessions (−4.05% and −2.47%, respectively).³⁰ I choose a conservative calibration for μ, such that the severity between financial and average recessions in a simulation of the model lies around the lower end of these empirical targets (34%).³¹ Below, I also show the results for a lower value of μ.

I obtain a nonlinear global solution of the model using a projection algorithm.³² The stochastic

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²⁹The implied asset-to-equity ratio of entrepreneurs in steady state is 1.2, close to the one in the collected data for nonfinancial firms which is around 1.6 (see A.3.1).

³⁰The model’s implied changes in real GDP are very close to these empirical counterparts: −5.2% with respect to financial recessions and −3.88% for average recessions.

³¹In the model simulation, I determine business cycle peaks and troughs according to the definition that is used by the UK and members of the European Union, among others. A business cycle peak is the quarter before output falls for two consecutive quarters. Following a peak, a trough is reached before output grows again. I further restrict recessions to occur 14.59% of the time in a simulation of the model (as in the macrohistory data by Jordà et al., 2017) by choosing the ones with larger falls in output from peak to trough. If a financial crisis occurs between a peak and a trough, then such an episode is termed a financial recession. All remaining recessions are called nonfinancial recessions.

³²The solution technique is similar to the one described in Appendix A.4.10 for the model with anticipated crises (see Section 5.10), but is adapted to ignore the possibility of a crisis occurring in the future due to the realization of
steady state resulting from this calibration is given in Appendix A.4.2. Before analyzing the behavior of the economy around crises, I gather some intuition on the effects of a technology shock.

### 5.5 Impulse Responses

Figures 4, 5, and 14 (in Appendix A.4.1) show impulse responses to a one-standard-deviation positive technology shock, starting from the stochastic steady state of the model. For these responses, a financial crisis does not occur because intermediary leverage is sufficiently low. As in Model II, following a positive technology shock, output $Y_t$, consumption $C_t$, dividends $D_t$, the capital stock $K_t$, and the price of capital $Q^K_t$ increase (shown in Figure 14 in Appendix A.4.1). Labor $H_t$ is now endogenous and rises to a positive shock.

In addition, profits to capital $R^K_t Q^K_{t-1}$ increase, which lowers the probability of default of all outstanding loans as indicated by the weighted default threshold $\omega_t$ (see equation 19). New entrepreneurs’ net worth $N_t$ rises (see equation 15), allowing them to raise their borrowing $Q_t L_{t}^\text{new}$ in absolute terms. Moreover, a higher price of capital $Q^K_t$ additionally allows entrepreneurs to increase their borrowing relative to their units of collateral $\frac{L_{t}^\text{new}}{K^K_t}$, as determined by their borrowing constraint (see equation 12). The loan risk indicator $x_t$ therefore increases (see equation 20).

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**Figure 4: Impulse Responses.** Impulse responses to a one-standard-deviation positive technology shock, starting at the stochastic steady state of the model, all deviations are multiplied by 100.

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future shocks. The accuracy of the solution is confirmed by analyzing the absolute Euler equation errors (see Judd, 1992).
Following a positive technology shock, intermediaries expand their balance sheets. They take on more debt $B^F_t$ to increase new lending, and the overall stock of loans $L_t$ therefore rises. As in Models I and II, the real interest rate $R_{t+1}$ declines. In addition, the profitability of loans $RL_tQ_{t-1}$ increases, largely due to the rise in the value of all outstanding loans $Q_t$. Intermediary market leverage $B^F_t Q_{t-1}$ initially decreases since the entire loan portfolio rises in value. The auxiliary variable $\text{Lev}^*_t$ as defined in (24) behaves very similarly and the response is omitted for brevity. Over time, intermediaries continue to take on more debt to issue new loans, raising leverage. The response of leverage therefore roughly matches the empirical counterpart in Figure 2.

![Impulse Responses](image)

**Figure 5:** Impulse Responses. Impulse responses to a one-standard-deviation positive technology shock, starting at the stochastic steady state of the model, all deviations are multiplied by 100.

### 5.6 Typical Financial Crises

In principle, a financial crisis as described in Section 5.3 can break out at any time if an adverse technology shock is sufficiently large. Hence, nothing in the model restricts crises to occur out of booms or recessions. However, financial crises are more likely to happen if certain conditions are met. To understand these conditions, I analyze the typical behavior of the model around crises. First, I simulate the model for 500,000 periods. Then, I collect the sequences of endogenous variables and shocks in a window of 30 quarters before and 20 quarters after a crisis. Figures 6, 7, and 8 plot period-by-period the median, 33rd, and 66th percentiles across these sequences for each variable with respect to windows in which only one financial crisis occurs.

In what follows, the median path for each variable is referred to as the “typical” path around
a crisis. The first row in Figure 6 shows the typical behavior of the technology shock $\epsilon_t^a$ and the technology level $a_t$. A buildup period leading to a crisis is characterized by an elevated technology level, which reverses within a one-year window before a crisis. The median shock that triggers a crisis is a negative 1.58 standard deviation shock. Typical financial crises therefore occur out of prolonged productivity booms, followed by subsequent slowdowns, and the model does not need to rely on extremely large adverse shocks to initiate financial crises. The fall in productivity around the outbreak of a crisis is also consistent with the data, as shown in Paul (2018) and replicated in Figure 10 in Appendix A.3.2. However, while the productivity slowdown eventually triggers a crisis, it is the prior productivity boom that is essential for the buildup of risk. In comparison, the probability that a crisis will occur in the next quarter remains very close to its long-run average when starting the economy at its stochastic steady state and feeding it with only the negative productivity shocks that are shown in Figure 6 (using the median series).

Figure 6: Typical Financial Crises. Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median, 33rd, and 66th percentiles are shown.

Output $Y_t$, capital $K_t$, and hours $H_t$ all increase in the buildup period and decrease sharply around the outbreak of a crisis. Financial crises are “credit booms gone bust”. During the boom, more new loans $Q_tL_t^{new}$ are issued and the overall stock of loans $L_t$ therefore increases while falling abruptly during the crisis due to the early liquidation of loans. Based on the simulation of the model, Appendix A.4.6 replicates the baseline regressions in Schularick and Taylor (2012) with the result that credit growth performs well in predicting financial crises. Moreover, in Appendix A.4.7, I replicate the findings by Jordà et al. (2013) that financial crises recessions that are preceded by an unusually large buildup of credit are deeper than the ones without such ex-ante trends.
In Appendix A.4.8, I also show that credit spreads are good predictors of the severity of crises, following the empirical strategy by Krishnamurthy and Muir (2017).

Further, during the boom, loans with higher loan-to-capital ratios \( \frac{L_{\text{new}}}{K_t} \) are issued and the loan risk indicator \( x_t \) increases as a result. The model is therefore able to match the empirical evidence by Demyanyk and Hemert (2011) and Justiniano et al. (2017) who show that loans that were issued closer to the 2007-09 financial crisis had higher default rates ex-post. That is also the case in the model since the default threshold in period \( t \) for vintage \( t-j \) is given by \( \omega_t | t-j = \frac{L_{\text{new}}}{R_t Q_{t-1} K_{t-j}} \) and \( \omega_t | t-j \) is higher for loans with higher \( \frac{L_{\text{new}}}{K_{t-j}} \) ratios.

Banks’ balance sheets and decisions are key to understand how crises arise. During the boom, entrepreneurs are less likely to default, \( \omega_t \) declines, and banks’ profits \( R_t Q_{t-1} \) rise. Due to the higher income, banks pay out more dividends \( D_t \). The issuance of new loans during the boom is financed by taking on additional debt \( B_t^F \), leading to a buildup of leverage \( \frac{B_t^F}{Q_t L_t} \), thereby increasing the risk of a crisis. The rise of leverage during the boom is due to the medium-term response of leverage to positive technological innovations as shown in Figure 5. During a crisis, banks obtain additional liquidity from the early liquidation of loans, leading to a fall in the outstanding debt \( B_t^F \), the interest rate \( R_{t+1} \), and a slight recovery in the price of loans \( Q_t \).
Figure 8: **Typical Financial Crises.** Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median, 33rd, and 66th percentiles are shown.

While the response of leverage to a technology shock determines the typical path towards a crisis, the liquidation of outstanding credit largely governs the severity of crises. To isolate the contribution of this mechanism towards the fall in total credit $L_t$, capital $K_t$, and ultimately output $Y_t$ during the bust, it is useful to compare the behavior of the economy with the same model that is subject to the same series of shocks, but which does not include crises. Such an exercise reveals that the total stock of loans $L_t$ and capital $K_t$ fall approximately by an additional 0.6% relative to the period before a crisis. In turn, output drops on average by around 0.3% more, for some crises by as much as 1.5%. These effects are very persistent. Even 10 years after the crisis, this difference is roughly the same. That is because the liquidation of capital reduces the profits, the starting net worth, and the borrowing capacity of many future generations of entrepreneurs.

The key parameter that determines the severity of crises is $\mu$ — the fraction of capital that can be resold when loans are liquidated early. The lower $\mu$, the more loans need to be liquidated to avoid a run, that is, the larger $\tau_t$. For the calibration in Section 5.4, I considered a value of $\mu$ that gives a severity of crises on the lower end of what the historical data suggests. Next, I recalibrate the model and consider a lower value of $\mu$, setting $\mu = 0.1$. Based on a simulation of the model,

$\mu$ $\tau_t$

$\mu$ $\tau_t$

To put these numbers into perspective, a 0.25-standard-deviation and a 1.2-standard-deviation negative technology shock give similar responses of output on impact.

With this new calibration, banks occasionally get caught in constant liquidations for very long simulations. To ensure a stable simulation, I assume that banks receive a small lump-sum $(\tau + 0.02)B_{t-1}R_t$ as opposed to the revenues from liquidated loans, so that their outstanding debt is sufficiently reduced at the same time.
financial recessions are now 41% deeper compared with average recessions, which lies in about the middle of the historical estimates (31% – 56%). Figures 15-17 in Appendix A.4.3 show the typical behavior around crises based on this new calibration. Loans $L_t$ and capital $K_t$ now fall more sharply during a crisis, on average close to an additional 1% compared with a model without crises. The fall in capital results in an average drop in output of about 0.5% compared with the model without crises, and for some crises by as much as 4%.35

This comparison begs the question of what a reasonable parametrization for $\mu$ would be. The calibrated values may not be deemed excessively low, considering that capital is firesold in a recession and a large fraction of capital that has been previously used in a production process may be firm-specific. However, it is difficult to pin down $\mu$ based on micro-data. Instead, a good orientation is the implied fraction of loans that is liquidated early $\tau_t$. For the baseline calibration, $\tau_t$ is on average 0.3%, with a maximum value of 2.2%. For the calibration with $\mu = 0.1$, $\tau_t$ is on average 1%, with a maximum value of 6%. In comparison, Chodorow-Reich and Falato (2017) find that total long-term credit and commitments outstanding contracted by 5.8% in 2008 and 5.9% in 2009 in the United States because borrowers had their borrowing limit lowered by an unhealthy lender following a covenant violation. Hence, the model’s implied value for $\tau_t$ is generally below the empirical counterparts, such that the model does not rely too heavily on this channel to generate large crises effects.

5.7 Nonfinancial Recessions

To understand how crises differ from “normal” recessions, I repeat the exercise of the last section around nonfinancial recessions (as defined in footnote 31). Figures 18-20 in Appendix A.4.4 plot the typical paths around nonfinancial recessions. Compared with financial crises, the rise and decline in output around nonfinancial recessions is less pronounced. Moreover, nonfinancial recessions are not preceded by strong increases in credit $L_t$, the intermediary’s debt $B^f_t$, market leverage $\frac{B^f_t}{Q_t L_t}$, or the loan risk indicator $x_t$, in contrast to the behavior of these variables in the buildup towards financial crises.

During nonfinancial recessions, the weighted default threshold $\varpi_t$ and the intermediary’s market leverage do not increase as strongly as during financial crises. Moreover, the fall in the intermediary’s funding $B^f_t$, its dividends $D_t$, new lending $Q_t L_t^{new}$, and the value of loans $Q_t$ are stronger during financial crises. In addition, nonfinancial recessions are less severe and persistent compared with financial recessions.36 Overall, this comparison shows that it is the financial sector’s balance sheet that plays the crucial role during the boom-bust cycle around financial crises. In this regard, nonfinancial recessions are different.

35Again, to put these numbers into perspective, a 0.5-standard-deviation and a 3.2-standard-deviation negative technology shock give similar responses of output on impact.

36For example, the median duration of a financial recession is around 153% longer compared with a nonfinancial recession – in the range of the empirical counterpart of 100% in the macrohistory data by Jordà et al. (2017).
5.8  The 2007-09 Financial Crisis

How well does the model perform in replicating the financial crisis of 2007-09? To test the model in this regard, I again use the historical series of structural technology shocks \( \{ \hat{\epsilon}_t \} \) — as discussed in Section 4 — and feed the model with this series. Given this series of shocks, I then check whether the model endogenously generates a crisis around the time when it actually occurred. At the beginning of the sample, I start the simulation from the stochastic steady state of the model. The results are given in Figure 9, showing the path of the technology level \( a_t \), the probability that a financial crisis will occur in the next quarter, the evolution of the intermediary’s market leverage, and the behavior of output.

During the boom period of 2000 to 2005, the technology level increases step-by-step and so does the intermediary’s leverage. This boom period is followed by declines in the technology level. The probability that a crisis will occur in the next quarter jumps to around 25% in 2008, while for the rest of the sample, the crisis probability is close to zero. A crisis endogenously occurs in 2009 when the intermediary’s leverage at the beginning of a period is above the threshold \( \kappa \). Hence, the model performs well in replicating the 2007-09 financial crisis. However, a caveat is that the amount of liquidated loans \( \tau_t \) is relatively small. The impact of the mechanism through which output falls endogenously is therefore limited. However, I find that this result depends on the starting point of the simulation. Choosing a different point than the stochastic steady state can imply higher values for \( \tau_t \).
5.9 Leverage

The behavior of financial and nonfinancial firm leverage is key to the model’s dynamics. Next, I show that this behavior is also consistent with the data. In this regard, it is useful to distinguish between book and market leverage – in the model and in the data. I obtain empirical measures for these two types of leverage for the U.S. financial and nonfinancial sectors by combining equity and balance sheet firm-level data sets (see Appendix A.3.1 for details).\(^{37}\) In the model, the financial sector’s leverage can be defined as

\[
\text{Market Leverage FI} \equiv \frac{B^F_t}{Q_t L_t}, \quad \text{Book Leverage FI} \equiv \frac{B^F_t}{BA^F_t},
\]

where \(BA^F_t\) stands for book assets of the financial sector, defined recursively

\[
BA^F_t = Q_t L^\text{new}_t + \gamma BA^F_{t-1},
\]

such that new loans are recorded on the balance sheet with the value at which they were given out and held constant until they mature. Similarly, define leverage of the nonfinancial sector as

\[
\text{Market Leverage NF} \equiv \frac{L_t}{Q_t K_t}, \quad \text{Book Leverage NF} \equiv \frac{L_t}{BA^\text{NF}_t},
\]

where \(BA^\text{NF}_t\) stands for book assets of the nonfinancial sector, again defined recursively

\[
BA^\text{NF}_t = Q_t K^\text{new}_t + \gamma BA^\text{NF}_{t-1},
\]

such that newly acquired capital is recorded on firms’ balance sheets with the value at which it was acquired and held constant until a firm’s project ends. Equipped with these definitions, one can compare leverage between the model and the data.

Starting with the work of Adrian and Shin (2010), the cyclicality of financial institutions’ leverage has been discussed as a potential indicator of the financial sector’s procyclical risk-taking. Adrian and Shin (2010) show that leverage of certain financial institutions is procyclical based on book value data. More specifically, Adrian and Shin (2010, 2013) term leverage to be procyclical when (percentage) changes in leverage are positively correlated with (percentage) changes in the overall size of banks’ balance sheets.\(^{38}\)

Here, I use a slightly different definition of cyclicality that is in the spirit of Cooley and Prescott (1995) and naturally fits the business cycle setup of the model. Table 2 shows the correlation between the cyclical component of book leverage, differentiated by sectors, and real industrial pro-

\(^{37}\)As a proxy for the U.S. financial sector leverage, I use firm-level data for commercial and investment banks. This data has the disadvantage that it is not possible to net-out interbank loans for the sector as a whole. Krishnamurthy and Vissing-Jorgensen (2015) address this issue, but their approach does not allow to compute market leverage ratios.

\(^{38}\)In Appendix A.4.5, I replicate two key graphs from Adrian and Shin (2013, Figure 3) based on a simulation of the model. As in the data, changes in financial sector book leverage are positively correlated with changes in the size of banks’ balance sheets. Changes in the enterprise value are negatively correlated with changes in enterprise value leverage.
duction as a proxy for economic activity at a monthly frequency.\textsuperscript{39,40} Two samples are considered, one including and one excluding the Great Recession.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial</td>
<td>0.22***</td>
<td>0.16***</td>
</tr>
<tr>
<td>Nonfinancial</td>
<td>-0.06</td>
<td>-0.22***</td>
</tr>
</tbody>
</table>

Table 2: \textit{Cyclicality of Book Leverage}. Correlation between cyclical component of book leverage and real industrial production. Notation: \textit{***} \( p < 0.01 \), \textit{**} \( p < 0.05 \), \textit{*} \( p < 0.1 \).

For the financial sector, book leverage is procyclical and significantly different from zero at the 99% confidence level for both samples. For the nonfinancial sector, book leverage is countercyclical but only mildly based on the sample that ends before the Great Recession. The model’s implied correlation between output \( Y_t \) and book leverage is largely in line with this empirical evidence. For the nonfinancial sector, the correlation is close to zero. That is because of the borrowing constraint (12), which eliminates most of the fluctuations in the book leverage of new entrepreneurs. For the financial sector, the correlation matches the data well.

In Table 3, I repeat the exercise for market leverage. Financial sector market leverage is either mildly procyclical or countercyclical in the data, depending on the sample. Nonfinancial sector market leverage is countercyclical for both samples. The model’s implied cyclicality of market leverage is close to this empirical evidence. For the financial sector, it is very close to the sample that excludes the Great Recession. For the nonfinancial sector, it is slightly more negative than in the data.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial</td>
<td>0.04</td>
<td>-0.27***</td>
</tr>
<tr>
<td>Nonfinancial</td>
<td>-0.14**</td>
<td>-0.38***</td>
</tr>
</tbody>
</table>

Table 3: \textit{Cyclicality of Market Leverage}. Correlation between cyclical component of market leverage and real industrial production. Notation: \textit{***} \( p < 0.01 \), \textit{**} \( p < 0.05 \), \textit{*} \( p < 0.1 \).

Another take-away from this exercise is that the model does not rely on financial sector market

\textsuperscript{39}I use industrial production as opposed to GDP since it is available at a monthly frequency. That is particularly useful when considering the correlation of economic activity with higher-frequency movements in asset prices, as the ones that drive market leverage in Table 3. Nonetheless, I find similar results when using GDP instead.

\textsuperscript{40}The logarithm of real industrial production and leverage are detrended using a Hodrick-Prescott filter (a smoothing parameter of 129,600 for monthly data is applied following Ravn and Uhlig, 2002). The results are not specific to using this filter and its decomposition. In particular, I find that the results are much the same when using a Baxter-King Filter that associates the cycle with frequencies between two months and eight years, i.e. conventional business cycle frequencies (e.g., Comin and Gertler, 2006), or longer-term cycles up to 12 years that may better capture so-called financial cycles.
leverage to be strongly procyclical, so that crises occur out of boom periods. A correlation coefficient that is just slightly positive is already enough. In Appendix A.4.9, I also compare the behavior of leverage around crises. Again, the model’s behavior is largely in line with the data. Thus, overall, the model performs well in matching the empirical behavior of leverage. However, the model’s calibration gives smaller levels of leverage and therefore also smaller quantitative variations.

5.10 Anticipated Crises

So far, I have treated crises as unanticipated events. Next, I relax this assumption. A challenge that arises in this regard is that the liquidation of loans and the selling of liquidated capital adds substantial variation and nonlinearity to the model. To ensure convergence of the solution, I therefore simplify the model and assume that whenever a run equilibrium is possible according to equation (24), banks liquidate a constant fraction of their loans instead of $\tau_t$ being determined endogenously according to (26).\footnote{If a crisis occurs, profits to loans are given by $R^t_Q = \tau(1 - \frac{\omega^t}{\mu}) + (1 - \tau)(\gamma Q_t + (1 - \gamma)(1 - \frac{\omega^t}{\mu})$ and banks anticipate this change depending on the realization of future shocks.} I set $\tau_t = 0.7\%$, in the mid-range of the simulations based on the two calibrations considered in Section 5.6.

Moreover, from the perspective of banks, crises are not necessarily “bad” events. On one hand, for the simulation of the model, it is not optimal to liquidate loans since the payoff of a liquidated loan is always lower than from a loan that is not liquidated, as $\left(1 - \frac{\omega_t}{\mu}\right) < (1 - \gamma) \left(1 - \frac{\omega_t}{\mu}\right) + \gamma Q_t$.\footnote{To ensure the stability of the model, I further assume that crises cannot occur for two consecutive periods.} However, on the other hand, once loans are liquidated, they generate additional liquidity and potentially higher dividend payouts during crises. Intertemporal consumption smoothing therefore does not necessarily imply that banks aim to operate with a lower leverage to stay away from crises regions. In fact, they may be drawn towards them. I find that a lower value of $\mu = 0.05$ balances well the opposing incentives of losses from liquidated loans versus additional resources to pay out dividends. The remaining parameters are kept at their previous values.

The solution technique for the model with anticipated crises is described in Appendix A.4.10. Figures 24-26 in Appendix A.4.11 show how the economy typically behaves around crises. Crises are slightly more frequent (5%) but as severe as in the model with unanticipated crises. Overall, the patterns are very similar to the ones shown in Section 5.6.
6 Conclusion

In recent years, a quickly growing empirical literature has revealed common patterns across a range of historical financial crises. In this paper, I augment a textbook macroeconomic model to include long-term defaultable loans, financial intermediation, and occasional financial crises, and show that this framework is consistent with several stylized facts about financial crises.

Within my model, crises occur out of prolonged booms, during which financial fragility builds up when banks increase their lending and leverage. They also immediately follow low productivity, but it only takes a moderate adverse shock to initiate them. During financial crises, banks face the risk of a creditor run. To avoid a run, banks liquidate a fraction of their loans to generate additional liquidity, thereby sharply reducing aggregate output.

A simulation of the model matches the data in several ways. First, credit growth is a robust predictor of crises. Second, financial recessions are deeper and more persistent than nonfinancial recessions. The model behaves differently around nonfinancial recessions, as they are not preceded by a strong expansion of the financial sector’s balance sheet or a credit boom. And third, the larger the buildup of credit prior to a crisis, the deeper the subsequent downturn in output.

The model is validated in three additional ways. First, when confronted with a historical series of structural shocks, the model replicates the occurrence of the 2007-09 financial crisis. Second, the behavior of leverage of financial and nonfinancial firms that is key to the buildup of risk in good times is consistent with the data.

Third, the quantitative analysis of the model shows that the described mechanism based on the early liquidation of loans to avoid a creditor run can generate substantial endogenous aggregate movements without relying on large exogenous shocks. The recent evidence by Chodorow-Reich and Falato (2017) confirms that this channel is also empirically relevant. An interesting extension of the model would be to allow for creditor runs on some banks as during the 2007-09 financial crisis, likely adding to the endogenous amplification of shocks.

Since the model matches key empirical characteristics of financial crises, it is a suitable laboratory to study policy interventions that aim to reduce the likelihood and severity of financial crises. In future work, it would be interesting to consider such macro-prudential policy interventions.
References


A ONLINE APPENDIX

A.1 Proofs

This section derives the simplified expression for the profits per loan \( R^L_t Q_{t-1} \) shown in equation (18) and the evolution of entrepreneurs’ net worth \( N_t \) shown in equation (15). An important assumption that allows to derive these simplified expressions is that the idiosyncratic shock \( \omega \) follows a uniform distribution, giving convenient expressions for cumulative distribution functions and partial expectations, which are

\[
\Phi(\omega_{t+j|t}) = \frac{\omega_{t+j|t}}{2}, \tag{30}
\]

\[
\int_{\omega_{t+j|t}}^{\omega} \omega d\Phi(\omega) = \left( 1 - \frac{1}{4} \omega_{t+j|t}^2 \right), \tag{31}
\]

for \( \omega \sim U[0, 2] \).

A.1.1 Return on Market Portfolio of Loans

The return on the market portfolio of loans is given by

\[
R^L_t = \gamma Q_t + (1 - \gamma) \frac{1}{L_{t-1}} \left\{ \sum_{j=1}^{\infty} \left[ 1 - \Phi(\omega_{t+j-1|t}) \right] \gamma^{j-1} L_{t-j}^{new} \right\} + \sum_{j=1}^{\infty} \sum_{j=1}^{\infty} \left[ \omega R^K_t Q^K_{t-1} \gamma^{j-1} K^{new}_{t-j} d\Phi(\omega) \right]. \tag{32}
\]

\( R^L_t \) captures both profits from nonmaturing loans \( \gamma Q_t \), as well as from maturing loans across all vintages. The latter consist of repaid loans – the first term in the curly bracket – and the recovery from defaulted loans – the second term in the curly bracket. The two infinite sums in the numerator can be simplified in the following way. First, using the definition of the cumulative distribution function in (30), note that

\[
\sum_{j=1}^{\infty} \left( 1 - \Phi(\omega_{t+j-1|t}) \right) \gamma^{j-1} L_{t-j}^{new} = L_{t-1} - \frac{1}{2} R^K_t Q^K_{t-1} x_{t-1}, \tag{33}
\]

where

\[
L_{t-1} = L_{t-1}^{new} + \gamma L_{t-2},
\]

\[
x_{t-1} = \frac{(L_{t-1}^{new})^2}{K_{t-1}^{new}} + \gamma x_{t-2}.
\]

Second, using the definition of the partial expectation in (31), rewrite

\[
\sum_{j=1}^{\infty} \left[ \omega R^K_t Q^K_{t-1} \gamma^{j-1} K^{new}_{t-j} d\Phi(\omega) \right] = \frac{1}{4} R^K_t Q^K_{t-1} x_{t-1}. \tag{34}
\]
Substituting (33) and (34) in (32) and rearranging gives

\[ R_t^L Q_{t-1} = \gamma Q_t + (1 - \gamma) \left( 1 - \frac{x_{t-1}^Q}{4 R_t^K Q_{t-1} L_{t-1}} \right). \]  

(35)

Next, define

\[ \overline{\omega}_t \equiv \frac{x_{t-1}}{R_t^K Q_{t-1} L_{t-1}}, \]  

(36)

and use (36) to simplify (35) to

\[ R_t^L Q_{t-1} = \gamma Q_t + (1 - \gamma) \left( 1 - \frac{\overline{\omega}_t}{4} \right). \]  

This is the expression that is stated in (18). Q.E.D.

### A.1.2 Entrepreneur’s Net Worth

The evolution of the entrepreneur’s net worth is given by

\[ N_t = (1 - \gamma) \sum_{j=1}^{\infty} \int_{\omega_{t-[j]}}^{\omega_{t-[j]}} \gamma^{j-1} \left( \omega R_t^K Q_{t-1} K_{t-j}^{new} - L_{t-j}^{new} \right) d\Phi(\omega). \]  

(37)

Using expressions (30) and (31), (37) can be written as

\[ N_t = (1 - \gamma) \sum_{j=1}^{\infty} \gamma^{j-1} \left( R_t^K Q_{t-1} K_{t-j}^{new} \left( 1 - \frac{\overline{\omega}_t^2}{4} \right) - L_{t-j}^{new} \left( 1 - \frac{\overline{\omega}_t[1-j]}{2} \right) \right) d\Phi(\omega). \]  

(38)

Noting that

\[ L_{t-j}^{new} = R_t^K Q_{t-1} K_{t-j}^{new} \overline{\omega}_{t-[j]} \]  

for \( j \geq 1 \),

(38) can be written in its simplified form as

\[ N_t = (1 - \gamma) \left\{ R_t^K Q_{t-1} K_{t-1} - L_{t-1} + \frac{1}{4} \frac{x_{t-1}}{R_t^K Q_{t-1}} \right\}, \]  

(39)

where

\[ K_{t-1} = K_{t-1}^{new} + \gamma K_{t-2}, \]  

(40)

\[ L_{t-1} = L_{t-1}^{new} + \gamma L_{t-2}, \]  

(41)

\[ x_{t-1} = (L_{t-1}^{new})^2 + \gamma x_{t-2}, \]  

(42)

are three aggregate state variables.
A.1.3 Recovery on Liquidated Loans

In a crisis, a fraction $\tau_t$ of loans is liquidated early with the payoff

$$
\tau_t \left[ \sum_{j=1}^{\infty} \left( 1 - \Phi(\omega_{t|t-j}) \right) \gamma_j^{j-1} L_{t-j}^{new} + \sum_{j=1}^{\infty} \left( \int_0^\omega Q_t^K \mu(1-\delta) \gamma_j^{j-1} K_{t-j}^{new} d\Phi(\omega) \right) \right],
$$

where the first term in the outer bracket captures repaid loans and the second term the recovery from defaulted loans. The vintage-specific default threshold $\omega_{t|t-j}$ is defined as

$$
\omega_{t|t-j} = \frac{L_{t-j}^{new}}{Q_t^K \mu(1-\delta) K_{t-j}^{new}}.
$$

Again, using the definition of the cumulative distribution function in (30), note that

$$
\tau_t \left[ \sum_{j=1}^{\infty} \left( 1 - \Phi(\omega_{t|t-j}) \right) \gamma_j^{j-1} L_{t-j}^{new} \right] = \tau_t \left[ L_{t-1} - \frac{1}{2} \frac{x_{t-1}}{Q_t^K \mu(1-\delta)} \right],
$$

where $L_{t-1}$ and $x_{t-1}$ are defined in (41) and (42). Again, using the definition of the partial expectation in (31), rewrite

$$
\tau_t \left[ \sum_{j=1}^{\infty} \left( \int_0^\omega Q_t^K \mu(1-\delta) \gamma_j^{j-1} K_{t-j}^{new} d\Phi(\omega) \right) \right] = \tau_t \left[ \frac{1}{4} \frac{x_{t-1}}{Q_t^K \mu(1-\delta)} \right].
$$

Combining (43) and (44) gives

$$
\tau_t L_t \left( 1 - \frac{\omega_t}{4} \right),
$$

where

$$
\omega_t = \frac{x_{t-1}}{Q_t^K \mu(1-\delta) L_{t-1}},
$$

which is the expression stated in (25).
A.2 Model – Equations

Households & Good Producer

\[ E_t [\Lambda_{t,t+1} R_{t+1}] = 1 \]  
(45)

\[ \Lambda_{t,t+1} = \beta^H \left( \frac{C_t \chi_{H_1}^{H_{t+1}^{1+\phi}}}{C_{t+1} \chi_{H_1}^{H_{t+1}^{1+\phi}}} \right) \]  
(46)

\[ (1 - \alpha)Y_t = \chi H_1^{1+\phi} \]  
(47)

\[ Y_t = e^{\alpha t} K_{t-1}^{1-H_{t}^{1-\alpha}} \]  
(48)

\[ a_t = \rho_a a_{t-1} + \epsilon_t^a \]  
(49)

Entrepreneurs

\[ Q_t^K K_t^{new} = N_t + Q_t L_t^{new} \]  
(50)

\[ L_t^{new} = \theta Q_t^K K_t^{new} \]  
(51)

\[ N_t = (1 - \gamma) \left( R_t^K Q_{t-1}^{K} K_{t-1} - L_{t-1} + \frac{1}{4} \frac{x_{t-1}}{R_t^K Q_{t-1}^{K}} \right) \]  
(52)

Banks

\[ D_t + Q_t L_t + B_{t-1}^F R_t = B_t^F + R_t^L Q_{t-1} L_{t-1} \]  
(53)

\[ E_t \left[ \beta^F (R_{t+1} + \psi B_t^F) \right] = E_t \left[ \frac{\beta^F}{D_{t+1}} R_t^{L} \right] = \frac{1}{D_t} \]  
(54)

\[ L_t = L_t^{new} + \gamma L_{t-1} \]  
(55)

\[ K_t = K_t^{new} + \gamma K_{t-1} \]  
(56)

\[ x_t = \frac{(L_t^{new})^2}{K_t^{new}} + \gamma x_{t-1} \]  
(57)

\[ R_t^L Q_{t-1} = \gamma Q_t + (1 - \gamma) \left( 1 - \frac{\omega_t}{4} \right) \]  
(58)

\[ \omega_t = \frac{x_{t-1}}{R_t^K Q_{t-1}^{K} L_{t-1}} \]  
(59)

Capital Producer and Resource Constraint

\[ Q_t^K = 1 + \zeta \left( \frac{I_t}{K_{t-1}} - \delta \right) \]  
(60)

\[ I_t = K_t - (1 - \delta) K_{t-1} \]  
(61)

\[ \Phi_t = I_t + \frac{\zeta}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} \]  
(62)

\[ Y_t = C_t + D_t + \Phi_t \]  
(63)

\[ R_t^{K} Q_{t-1} = \left( Q_t^{K} (1 - \delta) + \alpha \frac{Y_t}{K_{t-1}} \right) \]  
(64)
Occasional Financial Crises

Denote by $K_{t-1}^\text{prev}$, $L_{t-1}^\text{prev}$, and $x_{t-1}^\text{prev}$ last period’s capital stock, loan risk indicator, and overall loans absent a crisis. A crisis is triggered if

$$
\text{Lev}_t^* = \frac{B_{t-1}^F R_t - (1 - \gamma) \left(1 - \frac{\omega_t^*}{4}\right) L_{t-1}^\text{prev}}{\gamma Q_t L_{t-1}^\text{prev}} > \kappa .
$$  \hfill (65)

If a crisis occurs, then the fraction $\tau_t$ of loans is liquidated early, where $\tau_t$ solves

$$
B_{t-1}^F R_t = L_{t-1}^\text{prev} \left\{ \tau_t \left(1 - \frac{\omega_t^*}{4}\right) + (1 - \tau_t) \left((1 - \gamma) \left(1 - \frac{\omega_t^*}{4}\right) + \kappa \gamma Q_t\right) \right\} ,
$$  \hfill (66)

where

$$
\omega_t^* = \frac{x_{t-1}^\text{prev}}{Q_t^K \mu (1 - \delta) L_{t-1}^\text{prev}} .
$$

In (65) and (66), $Q_t, Q^K_t, \omega_t$ are equilibrium values absent a crisis.

If a crisis occurs, the state variables $K_{t-1}^\text{prev}$, $L_{t-1}^\text{prev}$, and $x_{t-1}^\text{prev}$ change to $K_{t-1} = (1 - \tau_t) K_{t-1}^\text{prev}$, $L_{t-1} = (1 - \tau_t) L_{t-1}^\text{prev}$, and $x_{t-1} = (1 - \tau_t) x_{t-1}^\text{prev}$. The amount $\tau_t \mu K_{t-1}^\text{prev}$ is additionally sold on the capital market, affecting the price of capital and aggregate investment, such that

$$
Q_t^K = 1 + \zeta \left(\frac{I_t}{K_{t-1} + \mu \tau_t K_{t-1}^\text{prev}} - \delta\right),
$$

$$
I_t = K_t - (1 - \delta) \left(K_{t-1} + \mu \tau_t K_{t-1}^\text{prev}\right),
$$

$$
\Phi_t = I_t + \frac{\zeta}{2} \left(\frac{I_t}{K_{t-1} + \mu \tau_t K_{t-1}^\text{prev}} - \delta\right)^2 \left(K_{t-1} + \mu \tau_t K_{t-1}^\text{prev}\right) .
$$

In a crisis, banks’ budget constraint changes to

$$
D_t + Q_t L_t + B_{t-1}^F R_t = B_t^L + R_t L_{t-1} + \tau_t L_{t-1}^\text{prev} \left(1 - \frac{\omega_t^*}{4}\right) ,
$$

where

$$
\omega_t^* = \frac{x_{t-1}^\text{prev}}{Q_t^K \mu (1 - \delta) L_{t-1}^\text{prev}} ,
$$

and $Q_t^K$ is the price of capital during a crisis.
A.3 Data

A.3.1 Leverage Data

Measures of the U.S. financial and the nonfinancial sector’s book and market leverage for the period 1980 Q1–2016 Q4 are obtained as follows. Equity and balance sheet data for commercial banks, investment banks, and nonfinancial firms are collected from CRSP and Compustat. Commercial banks are indicated by the SIC Codes 60, 61, and 6712. Nonfinancial firms are the remaining ones, excluding agriculture, forestry, and fishing as well as public administration. From CRSP, monthly time series for the market capitalization of firms are obtained. From Compustat, quarterly total assets and total liabilities are collected. The quarterly data are converted into a monthly frequency using linear interpolation. The two data sets are merged via Cusip and Permno identifiers. I repeat this exercise for the following selected investment banks: Bear Stearns, Citigroup, Credit Suisse, Goldman Sachs, HSBC, JP Morgan, Lehmann Brothers, Merrill Lynch, and Morgan Stanley. Applying these steps gives 502 financial and 3170 nonfinancial firms at the end of 2011.

**Book Leverage.** The measure of book leverage in the analysis is based on the following definition

\[
\text{Sector Book Leverage} \approx \frac{\sum \text{Book Assets}_j}{\sum \text{Book Assets}_j - \sum \text{Book Liabilities}_j},
\]

where the sums are taken over the available observations for a particular sector.

**Market Leverage.** The market leverage of a firm cannot be computed exactly since it requires either data on the market value of assets or the market value of liabilities – both of which are only observed in terms of their book values. To approximate market leverage, one can either assume that

\[
\text{Market Assets} \approx \text{Book Assets}
\]

or

\[
\text{Market Liabilities} \approx \text{Book Liabilities}.
\]

Given one of these assumptions, the market leverage of an industry is then approximated by

\[
\text{Sector Market Leverage} \approx \frac{\sum \text{Book Assets}_j}{\sum \text{Market Equity}_j}
\]

or

\[
\text{Sector Market Leverage} \approx \frac{\sum \text{Market Equity}_j + \sum \text{Book Liabilities}_j}{\sum \text{Market Equity}_j},
\]

where the sums are taken over the observations within an industry. Approximation (68) is likely to be a better one than (67) due to the shorter maturity of liabilities and hence the more frequent updating on a balance sheet. I therefore primarily work with definition (70) and check robustness using definition (69). All of the results in the paper are robust to using definition (69).
A.3.2 Macrohistory Data

The calibration targets for the frequency and the severity of financial crises are obtained from the Jorda-Schularick-Taylor Macrohistory Database (Jordà et al., 2017). This annual data set covers the years 1870–2013 and includes the following advanced economies (and their abbreviations used below): AUS = Australia, BEL = Belgium, CAN = Canada, CHE = Switzerland, DEU = Germany, DK = Denmark, ESP = Spain, FIN = Finland, FRA = France, GBR = Great Britain, ITA = Italy, JPN = Japan, NLD = Netherlands, NOR = Norway, PRT = Portugal, SWE = Sweden, USA = United States. For these countries, the data set contains information on a number of macroeconomic and financial variables and also incorporates indicators of financial crises events. Using a wide variety of sources, financial crises are identified as “events during which a country’s banking sector experiences bank runs, sharp increases in default rates accompanied by large losses of capital that result in public intervention, bankruptcy, or forced merger of financial institutions” (Schularick and Taylor, 2012). Table 4 lists all the financial crises dates. To identify recessions, I follow the same methodology as Jordà et al. (2013). They use the Bry and Boschan (1971) algorithm to determine local minima and maxima in real GDP to distinguish troughs and peaks. If a financial crisis occurs within the neighborhood of a business cycle peak, then the following recession is defined as a “financial recession”. The remaining recessions are termed “nonfinancial recessions”.

<table>
<thead>
<tr>
<th>Country</th>
<th>Year(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>1893 1989</td>
</tr>
<tr>
<td>BEL</td>
<td>1870 1885 1925 1931 1934 1939 2008</td>
</tr>
<tr>
<td>CAN</td>
<td>1907</td>
</tr>
<tr>
<td>DEU</td>
<td>1873 1891 1901 1907 1931 2008</td>
</tr>
<tr>
<td>DNK</td>
<td>1877 1885 1908 1921 1931 1987 2008</td>
</tr>
<tr>
<td>ESP</td>
<td>1883 1890 1913 1920 1924 1931 1977 2008</td>
</tr>
<tr>
<td>FIN</td>
<td>1877 1900 1921 1931 1991</td>
</tr>
<tr>
<td>FRA</td>
<td>1882 1889 1930 2008</td>
</tr>
<tr>
<td>ITA</td>
<td>1873 1887 1893 1907 1921 1930 1935 1990 2008</td>
</tr>
<tr>
<td>JPN</td>
<td>1871 1890 1907 1920 1927 1997</td>
</tr>
<tr>
<td>NLD</td>
<td>1893 1907 1921 1939 2008</td>
</tr>
<tr>
<td>NOR</td>
<td>1899 1922 1931 1988</td>
</tr>
<tr>
<td>PRT</td>
<td>1890 1920 1923 1931 2008</td>
</tr>
<tr>
<td>SWE</td>
<td>1878 1907 1922 1931 1991 2008</td>
</tr>
<tr>
<td>USA</td>
<td>1873 1893 1907 1929 1984 2007</td>
</tr>
</tbody>
</table>

Table 4: Financial Crises Dates.

44Following Jordà et al. (2013), I exclude war periods (WWI and WWII).
A.3.3 Maturity Data

I compute measures of the maturity of assets and liabilities held by U.S. commercial banks using the data that banks report in the Call Reports for the sample 1997 Q2–2016 Q4.45 To this end, I follow the assumptions in English et al. (2014). In the Call Reports, assets and liabilities are categorized by placing them into different buckets depending on their remaining maturity or next repricing date (e.g., remaining maturity between 1 and 3 years). For each interval, English et al. (2014) choose the midpoint as the representative maturity for all assets and liabilities within a particular bucket (e.g., 2 years for the interval 1–3 years). English et al. (2014) also assume that assets reported as having remaining maturity or next repricing date of over 15 years have a repricing/maturity period of 20 years on average (similarly for securities and time deposits with over 3 years that are assumed to have a repricing/maturity period of 5 years). In addition, I assume that mortgage and nonmortgage loans with maturity 5–15 years are assumed to have a maturity of 5 years, mortgage loans with maturity >15 years are assumed to have a maturity of 7 years, and non-mortgage loans with maturity >15 years are assumed to have a maturity of 15 years. Further, I assume that all liabilities apart from time deposits have a zero contractual maturity (e.g., demand, savings, and transaction deposits). Given these assumptions, I find that the average maturity of assets is 3.49 years and of liabilities 0.35 years.

A.4 Additional Results

A.4.1 Impulse Responses

Figure 11: **Impulse Responses — Model I.** Impulse responses to a one-standard-deviation positive technology shock for different values of the intermediary’s discount factor $\beta^F$, holding constant all other calibrated parameters as described in Section 3. Responses are multiplied by 100.

Figure 12: **Impulse Responses — Model II.** Impulse responses to a one-standard-deviation positive technology shock based on the model and its calibration as described in Section 4.
Figure 13: **Impulse Responses – Model II.** Impulse responses to a one-standard-deviation positive technology shock for different values of the intermediary’s discount factor $\beta^F$, holding constant all other calibrated parameters as described in Section 4. Responses are multiplied by 100.

Figure 14: **Impulse Responses – Model III.** Impulse responses to a one-standard-deviation positive technology shock, starting at the stochastic steady state of the model, based on the model described in Section 5 and its calibration stated in Section 5.4.
A.4.2 Stochastic Steady State

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>$C_{SS}$</td>
<td>1.87</td>
<td>Asset-to-Equity-Ratio</td>
<td>$Q_{SS} L_{SS}$</td>
<td>2.16</td>
</tr>
<tr>
<td>Hours</td>
<td>$H_{SS}$</td>
<td>1.00</td>
<td>Dividends</td>
<td>$D_{SS}$</td>
<td>0.02</td>
</tr>
<tr>
<td>Output</td>
<td>$Y_{SS}$</td>
<td>2.27</td>
<td>Return to loans</td>
<td>$R_{SS}^L$</td>
<td>1.015</td>
</tr>
<tr>
<td>Capital</td>
<td>$K_{SS}$</td>
<td>15.34</td>
<td>Loans</td>
<td>$L_{SS}$</td>
<td>2.35</td>
</tr>
<tr>
<td>Return capital</td>
<td>$R_{SS}^K$</td>
<td>1.02</td>
<td>Net worth</td>
<td>$N_{SS}$</td>
<td>1.34</td>
</tr>
<tr>
<td>New loans</td>
<td>$Q_{SS} L_{SS}^{new}$</td>
<td>0.20</td>
<td>Price long-term loans</td>
<td>$Q_{SS}$</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Table 5: Stochastic Steady State. Value of endogenous variables at the stochastic steady state.

A.4.3 Crisis Severity

Figure 15: Typical Financial Crises – Sensitivity to $\mu$. Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median paths are shown.
Figure 16: Typical Financial Crises — Sensitivity to $\mu$. Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median paths are shown.

Figure 17: Typical Financial Crises — Sensitivity to $\mu$. Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median paths are shown.
A.4.4 Typical Nonfinancial Recessions

Figure 18: Typical Nonfinancial Recessions vs. Financial Crises. Event window around nonfinancial recession (red dotted line) or financial crisis (blue solid line) at Quarter = 0. Based on a simulation of 500,000 periods. Median paths are shown.

Figure 19: Typical Nonfinancial Recessions vs. Financial Crises. Event window around nonfinancial recession (red dotted line) or financial crisis (blue solid line) at Quarter = 0. Based on a simulation of 500,000 periods. Median paths are shown.
Figure 20: **Typical Nonfinancial Recessions vs. Financial Crises.** Event window around nonfinancial recession (red dotted line) or financial crisis (blue solid line) at Quarter = 0. Based on a simulation of 500,000 periods. Median paths are shown.
A.4.5 Replicating Adrian & Shin (2013)

Figure 21: Cyclicality of Book Leverage and Enterprise Value Leverage. This Figure replicates Figure 3 in Adrian and Shin (2013) based on a 1,000-period-simulation of the model. The left graph shows a scatter-plot of the percentage change in the financial sector’s book assets (as defined in equation (28)) against the percentage change in the financial sector’s book leverage (as stated in (27)). The right graph shows a scatter-plot of the percentage change in the financial sector’s enterprise value (given by the market value of assets, $Q_tL_t$) against the percentage change in the financial sector’s market leverage (given by $\frac{Q_tL_t}{Q_tL_t-B_t}$).
A.4.6 Replicating Schularick & Taylor (2012)

This section replicates the baseline regressions in Schularick and Taylor (2012). In particular, consider the probabilistic model with the log-odds ratio

$$\log \left( \frac{P[FC_t = 1]}{P[FC_t = 0]} \right) = \alpha + \beta_1 \Delta_4 \log (L_{t-1}) + \beta_2 \Delta_4 \log (Y_{t-1}) + u_t,$$

where $FC_t$ is equal to one if a financial crisis breaks out at time $t$ and zero otherwise. The log-odds ratio of $FC_t$ is assumed to be a function of a constant $\alpha$, the annual change of variable $L$ from period $t - 5$ to $t - 1$ denoted by $\Delta_4 \log (L_{t-1})$, and the annual change of output from period $t - 5$ to $t - 1$, denoted by $\Delta_4 \log (Y_{t-1})$. I consider additional annual lags of each regressor. For $L$, I use either the total stock of loans as defined in (16) or the book value of all loans as defined in (28).

<table>
<thead>
<tr>
<th></th>
<th>Logit</th>
<th>Logit + GDP</th>
<th>Logit</th>
<th>Logit + GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-5.8^{***}$</td>
<td>$-5.8^{***}$</td>
<td>$-5.3^{***}$</td>
<td>$-5.4^{***}$</td>
</tr>
<tr>
<td>$\Delta_4 \log (L_{t-1})$</td>
<td>91.4$^{***}$</td>
<td>287.4$^{***}$</td>
<td>$-23.0^{***}$</td>
<td>159.1$^{***}$</td>
</tr>
<tr>
<td>$\Delta_4 \log (L_{t-5})$</td>
<td>210.5$^{***}$</td>
<td>319.1$^{***}$</td>
<td>66.8$^{***}$</td>
<td>173.9$^{***}$</td>
</tr>
<tr>
<td>$\Delta_4 \log (L_{t-9})$</td>
<td>179.8$^{***}$</td>
<td>241.6$^{***}$</td>
<td>37.2$^{***}$</td>
<td>117.1$^{***}$</td>
</tr>
<tr>
<td>$\Delta_4 \log (L_{t-13})$</td>
<td>76.6$^{***}$</td>
<td>138.1$^{***}$</td>
<td>21.7$^{***}$</td>
<td>64.2$^{***}$</td>
</tr>
<tr>
<td>$\Delta_4 \log (L_{t-17})$</td>
<td>370.5$^{***}$</td>
<td>115.1$^{***}$</td>
<td>85.0$^{***}$</td>
<td>$-30.8^{***}$</td>
</tr>
</tbody>
</table>

|                      |        | X           |        | X           |
| Book Value Loans     |        | X           |        | X           |
| 5 lags GDP growth    |        | X           |        | X           |

Table 6: Notation: $^{***} p<0.01$, $^{**} p<0.05$, $^* p<0.1$. 

50
A.4.7 Replicating Jorda, Schularick & Taylor (2013)

This section replicates the baseline regressions in Jordà et al. (2013). In particular, consider the set of local projections

\[
(\log(Y_{t+k}) - \log(Y_{t-1})) \cdot 100 = \alpha_k + \beta_k (\log(L_{t-1}) - \log(L_{t-9})) \cdot 100 + u_t^k
\]

for \(k = 4, 8, 12, 16, 20\),

where the dependent variable is the percentage change in output from period \(t - 1\) to \(t + k\). The regressors are a horizon-specific constant \(\alpha_k\) and the percentage change of total loans from \(t - 9\) to \(t - 1\), a two-year window. The results are robust to considering a one-year, three-year, four-year, or five-year window for loans instead. The set of regressions is estimated only for periods when a crisis takes place at time \(t\). The coefficient \(\alpha_k\) therefore gives the average percentage change in output from \(t - 1\) to \(t + k\) and \(\beta_k\) captures the additional percentage change in output when credit expanded by one percent more than on average.

<table>
<thead>
<tr>
<th>(k)</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_k)</td>
<td>-1.55***</td>
<td>-1.33***</td>
<td>-1.02***</td>
<td>-0.91***</td>
<td>-0.74***</td>
</tr>
<tr>
<td>(\beta_k)</td>
<td>-1.63***</td>
<td>-2.87***</td>
<td>-3.90***</td>
<td>-4.71***</td>
<td>-5.55***</td>
</tr>
</tbody>
</table>

Table 7: Notation: *** \(p<0.01\), ** \(p<0.05\), * \(p<0.1\).
A.4.8  Replicating Krishnamurthy & Muir (2017)

Given the different maturity of long-term loans and short-term debt, I define credit spreads as

$$Spread_t = \frac{1}{Q_t} - (R_t) \left( \frac{1}{1+\gamma} \right).$$

Moreover, denote the change in output from peak to trough during a financial crisis as $y_{peak, trough_{t+k}}$ and the change in output from $t$ to $t+3$ as $y_{t, t+3}$. Following Krishnamurthy and Muir (2017), I obtain evidence on the predictive power of spreads using the regressions

$$y_{peak, trough_{t+k}} = \alpha + \beta_1 Spread_t + u_t,$$

and

$$y_{t, t+3} = \alpha + \beta_2 \cdot crisis_t \cdot Spread_t + \beta_3 \cdot (1 - crisis_t) \cdot Spread_t + u_t,$$

where $crisis_t$ is a binary financial crisis indicator. I find that $\beta_1$, $\beta_2$, and $\beta_3$ are all estimated to be negative. Hence, higher credit spreads are associated with deeper financial crises and declines in output in the near future, even in non-crisis periods (see also López-Salido, Stein, and Zak rajsek, 2017).
A.4.9 Behavior of Leverage around Crises

In this Section, I consider the behavior of leverage around crises. Figure 22 shows the evolution of the U.S. financial sector’s leverage around the 2007-09 financial crisis, comparing it with the typical behavior of leverage around crises in the model. In both model and data, market leverage increases sharply around a crisis. In contrast, book leverage rises in the run-up and decreases during a crisis. In the data, book leverage barely moves compared to market leverage. Figure 23 repeats the exercise for nonfinancial firm leverage. Book leverage in the model decreases around crises, in contrast to the data, where it barely moves. Nonfinancial sector market leverage shows a stronger increase around the crisis, in line with the data.

![Figure 22: Financial Sector Leverage. Top graphs: Typical behavior of financial sector’s leverage based on simulation of the model as in Section 5.6. Bottom graphs: U.S. financial sector’s leverage around the 2007-09 financial crisis.](image)
Figure 23: **Nonfinancial Sector Leverage.** Top graphs: Typical behavior of nonfinancial sector’s leverage based on simulation of the model as in Section 5.6. Bottom graphs: U.S. nonfinancial sector’s leverage around the 2007-09 financial crisis.
A.4.10 Anticipated Crises - Solution Technique

Given the optimality conditions to the agents’ decision problems and the clearing of goods, labor, and debt markets, the equilibrium conditions of the model for \( t = 0, 1, \ldots, \infty \) are listed in Appendix A.2, given an initial state \( S_0 \). The endogenous variables can be separated into a vector of nonstate variables \( X_t \) and a vector of state variables \( S_t \). \( X_{t+1} \) is unknown in period \( t \) and \( S_t = \{ \bar{S}_t, \hat{S}_t \} \) comprises both exogenous state variables \( \bar{S}_t \) and endogenous state variables \( \hat{S}_t \). \( \hat{S}_t = \{ \rho a_{t-1} + \epsilon^a_t \} \) includes the technology shock \( \epsilon^a_t \) and the probability distribution of this shock is known to all agents. The realization \( \epsilon^a_{t+1} \) is unknown in period \( t \). \( \hat{S}_t = \{ K_{t-1}, L_{t-1}, x_{t-1}, R_{t-1}B_{t-1} \} \) collects the endogenous state variables and \( \hat{S}_{t+1} \) is known in period \( t \). Overall, the model has five state variables. One is linked to the shock, three arise from the loan contracts (\( K_{t-1}, L_{t-1}, \) and \( x_{t-1} \)), and \( R_{t-1}B_{t-1} \) accounts for the outstanding debt of the financial intermediary.

Definition 1. A competitive general equilibrium is a solution of the model which is given by a set of policy functions \( \hat{S}_{t+1} = f_{\hat{S}}(S_t) \) and \( X_t = f_X(S_t) \) that satisfy the model’s equilibrium conditions listed in Appendix A.2 for \( t = 0, 1, \ldots, \infty \) in the relevant state space.

Based on this definition, I obtain the policy functions \( \hat{S}_{t+1} = f_{\hat{S}}(S_t) \) and \( X_t = f_X(S_t) \) using a projection algorithm. Broadly, this involves three choices. First, one has to choose a grid on which the model is solved. Second, a parameterization of the policy functions has to be determined. Third, given an initial parameterization, one has to choose an iteration procedure. These three choices structure the description below.

Grid. The model is solved on a Smolyak sparse grid due to the curse of dimensionality (Bellman, 1961). The construction of the Smolyak sparse grid works as follows. The grid points in the space \([-1, 1]\) for each state variable are obtained by tensor-products of nested sets of Chebyshev extrema and the application of the Smolyak rule for a given level of approximation that controls how many of these tensor-products are included in the grid (see also Malin, Krueger, and Kubler, 2011). I select 4 for the level of approximation, giving 801 grid points for the five state variables. Next, the grid points are transformed from the space \([-1, 1]\) into the relevant space given the model’s calibration. To determine the relevant space, I solve the model first with a third-order perturbation method around a deterministic steady state (ignoring the occasional financial crises), simulate the model for 500,000 periods, and choose the lower and upper bounds for each state variable as the associated ones in the grid (denoted \( L_B \) and \( U_B \)). A linear transformation \( (x + 1) \frac{(U_B - L_B)}{2} + L_B \) is used to transform each grid point \( x \) from \([-1, 1]\) into \([L_B, U_B]\), giving the full set of grid points \( j = 1, \ldots, M \) in the relevant state space.

Parameterization of policy functions. I parameterize several non-state variables using third-order ordinary polynomials. In particular, let \( X^P_t \) be a parameterized variable where

\[
X^P_t \in \{ K^\text{new}_t, \frac{1}{D_t}, R_{t+1} \}
\]
and let $r_t$ be an indicator function determining whether there is a financial crisis in period $t$, in which case $r_t = 1$, or not, such that $r_t = 0$. Then, $X_t^P(S_t)$ is parameterized using the piecewise flexible form with separate coefficients $\beta_{r_t=0}^X$ and $\beta_{r_t=1}^X$.

$$X_t^P(S_t) = (1 - r_t)\beta_{0}^X T(S_t) + r_t\beta_{1}^X T(S_t) ,$$

where $T(S_t)$ is a vector collecting the basis functions. The number of grid points is larger than the number of coefficients. Hence, the outlined solution algorithm does not give an exact solution on the grid points, as opposed to collocation methods (see for example Malin, Krueger, and Kubler, 2011).

**Iteration.** Given an initial guess for the coefficients $\beta_{0}^X$ and $\beta_{1}^X$, the iteration proceeds as follows.

1. Obtain the vector collecting the basis functions $T(S_{t,j})$ for a given grid point $j$. Assume that there is no crisis in period $t$ and calculate the parameterized variables $X_{t,j}^P(S_{t,j};r_t = 0)$ via (71). Substitute $X_{t,j}^P(S_{t,j};r_t = 0)$ into the set of equations summarized in Appendix A.2. Using the equilibrium conditions, solve for the rest of the variables. Check whether the financial intermediary’s leverage $Lev_t^*$ exceeds the threshold $\kappa$. If so, go back to the beginning, set $r_t = 1$, use the parameterization $X_{t,j}^P(S_{t,j};r_t = 1)$ instead, and implement the early liquidation of loans as discussed in Appendix A.2. This separates the grid points into two sets: a set of points for which there is no crisis and a set for which there is one.

2. Having solved for all period $t$ variables at grid point $j$, one obtains next period’s endogenous state variables $\hat{S}_{t+1,j}$. To approximate integrals arising from expectation operators in intertemporal equations, I use five Hermite-Gaussian quadrature nodes and weights for period $t+1$. This gives next period’s exogenous state variables $\overline{S}_{t+1,j}$ and therefore $S_{t+1,j}$ for each node $i \in \{1,2,...,5\}$ in period $t+1$ at each grid point $j$. Assume that there is no crisis in period $t+1$ at node $i$ for grid point $j$. Use the initial parameterization $X_{t+1,j,i}^P(S_{t+1,j,i};r_{t+1} = 0)$ again and solve for the rest of the variables at node $i$ in period $t+1$. If $r_t = 0$, check whether the financial intermediary’s leverage $Lev_{t+1}^*$ in period $t+1$ exceeds threshold $\kappa$. If so, use the parameterization $X_{t+1,j,i}^P(S_{t+1,j,i};r_{t+1} = 1)$ instead, and again implement the liquidation of loans as described in A.2.

3. Having solved for all period $t+1$ variables, compute the expectation integrals in the equilibrium conditions. The intertemporal equations give an estimate $X_{t,j}^P(S_{t,j})$ for each of the initially parameterized variables $X_{t,j}^P(S_{t,j})$ at each grid point $j$. A fixed point is obtained when the initially assumed value $X_{t,j}^P(S_{t,j})$ is equal to $X_{t,j}^P(S_{t,j})$.

4. Until a fixed point is reached, iterate over the coefficients $\beta_{0}^X$ and $\beta_{1}^X$ in the policy functions (71). Collect the vectors of basis functions $T(S_{t,j})$ for all grid points for which there is no crisis in period $t$, combine them in a matrix $T_0(S_t)$ and project these on the obtained estimates.

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46If a crisis occurs, profits to loans are given by $R_t^L Q_{t-1} = \tau(1-\frac{\omega_t}{\tau}) + (1-\tau)(\gamma Q_t + (1-\gamma)(1-\frac{\omega_t}{\tau})$ and banks anticipate this change depending on the realization of future shocks.
\( X_{t,j}^p(S_{t,j}) \), giving
\[
\hat{\beta}_0^X \equiv \left( T_0(S_t)' T_0(S_t) \right)^{-1} T_0(S_t)' X_{t,j=0}^p,
\]
where \( X_{t,j=0}^p \) is a vector collecting the obtained estimates \( X_{t,j}^p(S_{t,j}) \) for which there is no crisis. Repeat the same for all grid points for which there is a crisis in period \( t \), resulting in
\[
\hat{\beta}_1^X \equiv \left( T_1(S_t)' T_1(S_t) \right)^{-1} T_1(S_t)' X_{t,j=1}^p.
\]

5. Compute the coefficients \( \hat{\beta}_0^X' \) and \( \hat{\beta}_1^X' \) that are used for the next iteration via
\[
\hat{\beta}_0^X' = (1 - \xi) \hat{\beta}_0^X + \xi \hat{\beta}_0^X,
\]
\[
\hat{\beta}_1^X' = (1 - \xi) \hat{\beta}_1^X + \xi \hat{\beta}_1^X,
\]
where \( 0 < \xi < 1 \) is a dampening parameter which helps convergence (\( \xi = 0.1 \) is used).

6. After several initial iterations, check for convergence and end iteration if
\[
\frac{1}{3} \sum_{X_{t,j}^p \in A} \frac{1}{M - Mb} \sum_{j=1}^M \frac{|X_{t,j}^p - X_{t,j=0}^p|}{X_{t,j=0}^p} < \eta,
\]
and
\[
\frac{1}{3} \sum_{X_{t,j}^p \in A} \frac{1}{Mb} \sum_{j=1}^M \frac{|X_{t,j=1}^p - X_{t,j=1}^p|}{X_{t,j=1}^p} < \eta,
\]
where \( A = \{ K_1^{\text{new}}, \frac{1}{T_t}, R_{t+1} \} \), \( M \) denotes the number of points in the state space, and \( Mb \) the number of binding grid points. \( \eta = 5^{-04} \) is used.\(^{47}\) The accuracy of the solution is confirmed by analyzing the absolute Euler equation errors (see Judd, 1992).

Given the policy functions \( X_t^p(S_t) \), one can use the model’s equations to obtain the full set of policy functions \( \hat{S}_{t+1} = f_{\hat{S}}(S_t) \) and \( X_t = f_X(S_t) \) for a solution of the model as given in Definition 1.

\(^{47}\)I find that setting \( \eta \) to a lower value does not strongly increase the accuracy of the solution away from the grid points or change any of the results, but results in a significant increase in computational time.
A.4.11 Typical Financial Crises — Anticipated

Figure 24: **Typical Financial Crises.** Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median, 33rd, and 66th percentiles are shown.

Figure 25: **Typical Financial Crises.** Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median, 33rd, and 66th percentiles are shown.
Figure 26: *Typical Financial Crises.* Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median, 33rd, and 66th percentiles are shown.