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Taylor Rule Estimation by OLS*

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Abstract

Ordinary Least Squares (OLS) estimation of monetary policy rules produces potentially inconsistent estimates of policy parameters. The reason is that central banks react to variables, such as inflation and the output gap, which are endogenous to monetary policy shocks. Endogeneity implies a correlation between regressors and the error term, and hence, an asymptotic bias. In principle, Instrumental Variables (IV) estimation can solve this endogeneity problem. In practice, IV estimation poses challenges as the validity of potential instruments also depends on other economic relationships. We argue in favor of OLS estimation of monetary policy rules. To that end, we show analytically in the three-equation New Keynesian model that the asymptotic OLS bias is proportional to the fraction of the variance of regressors accounted for by monetary policy shocks. Using Monte Carlo simulation, we then show that this relationship also holds in a quantitative model of the U.S. economy. As monetary policy shocks explain only a small fraction of the variance of regressors typically included in monetary policy rules, the endogeneity bias is small. Using simulations, we show that, for realistic sample sizes, the OLS estimator of monetary policy parameters outperforms IV estimators.

JEL classification codes: E52, E58, E50, E47

Keywords: Taylor rule, OLS, GMM, endogeneity bias, New Keynesian models

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1 Introduction

The macroeconomics literature frequently relies on some version of an interest rate rule, such as the ones introduced in Taylor (1993, 1999), to represent a central bank's reaction function. Such policy rules serve as good representations of how the monetary authority adjusts its policy instrument (typically a short term interest rate) in response to deviations of inflation and/or economic conditions (output or unemployment, for example) from their objectives.

Estimation of the central banks' reaction function poses some challenges, however. Ordinary Least Squares (OLS) estimation of monetary policy rules produces potentially inconsistent estimates of policy parameters. This is so because central banks react to variables that are endogenous to monetary policy shocks. Endogeneity implies a correlation between regressors and the error term, and, hence, yields estimates that are asymptotically biased. In principle, estimation by Instrumental Variables (IV) can solve this endogeneity problem (e.g., Clarida, Gali and Gertler, 2000). In practice, however, finding suitable instruments can be challenging, as their validity depends on details of the economic environment other than the policy rule to be estimated.

In this paper we argue in favor of OLS estimation of policy rules. To do so, we first show analytically in the three-equation New Keynesian model that the asymptotic OLS estimation bias is proportional to the fraction of the variance of regressors accounted for by the monetary policy shock. Since there is wide evidence that monetary policy shocks explain only a small fraction of the variance of regressors typically included in estimation of monetary policy rules (e.g., Leeper, Sims and Zha, 1996 and Christiano, Eichenbaum and Evans, 1999), our analytical finding suggests that the endogeneity bias is small.

Next, we turn to the quantification of estimation biases in well-established models, which we use as laboratories for our experiments. More specifically, we generate Monte Carlo simulations of economies for which we know the "true" (calibrated) policy parameters, and compare them to single-equation OLS and IV estimates based on artificially-generated data. In particular, we calibrate and generate data from the quantitative workhorse DSGE model of Smets and Wouters (2007), as well as from the simple three-equation New Keynesian model. Using each models' simulated data, we estimate their respective interest rate rules and compare the estimates to the models' true parameter values.

Our results suggest that endogeneity does induce some bias in the estimation of interest rate rules by OLS. However, for empirically relevant sample sizes, OLS estimates outperform IV estimates. OLS biases are close to those obtained with IV, but the estimates are much more precise. More importantly, when we look at the economic implications of estimation biases, we find them to be unimportant, in the sense that replacing the true policy rule with the one estimated by OLS does not materially change the dynamics of the model. The impulse response functions obtained under the policy rule estimated by OLS are close to the true ones, and the range of dynamic responses estimated by OLS is narrower than those obtained with IV methods.

Finally, we assess our results by estimating an interest rate rule by OLS and IV using actual data. In particular, we build on the findings of Clarida, Gali and Gertler (2000) and estimate an interest rate rule for subsamples corresponding to different Federal Reserve chairmen. We find that OLS and IV estimated coefficients and the associated IRFs – estimated with the local projection method proposed by Jordà (2005) – are very close to each other.

The literature on Taylor rule estimation is quite large, covering debates about whether monetary policy in the US has changed over time in terms of satisfying the Taylor principle (e.g., Taylor, 1999, Judd and Rudebusch, 1998, Clarida, Gali and Gertler, 2000, Orphanides, 2004), and whether persistence in interest rates stems from monetary policy inertia or persistent monetary policy shocks (e.g., Rudebusch, 2002, and Coibion and Gorodnichenko, 2012), among others.

Our paper does not focus on a particular issue pertaining to Taylor rules, but, rather, sheds light on the costs and benefits of estimation by OLS or IV. Hence, our contribution is closer to papers that focus on issues related to estimation of Taylor rules. Cochrane (2011) argues that Taylor rule parameters are not identified in the baseline New Keynesian model. Sims (2008) shows that Cochrane (2011)'s finding is not a generic implication of New Keynesian models, but is rather the result of a particular assumption on the policy rule. He shows that, under assumptions usually made in the literature, policy parameters are identified. Closest to our paper, de Vries and Li (2013) investigate the magnitude of the estimation bias when monetary shocks are serially correlated and lags of inflation and output gap are endogenous to monetary shocks, and thus, are not valid instruments. They find that the endogeneity problem caused by serial correlation does not cause large bias in the conventional estimation of Taylor rules based on the three-equation New Keynesian model. We focus on OLS estimation, and use IV estimation only as a comparison. We show analytically, in the canonical New Keynesian model, and by simulation in a larger model, that the OLS bias depends on the fraction of the variance of endogenous regressors explained by monetary policy shocks. Because this fraction is small, OLS bias is not a relevant problem.

The paper is organized as follows. Section 2 derives the OLS estimation bias for the policy rule parameters analytically in the three-equation New Keynesian model. Section 3 quantifies estimations biases under OLS and IV methods using simulated data from that model, as well as from the quantitative DSGE model due to Smets and Wouters (2007). Section 4 compares the performance of OLS and IV empirically. Section 5 concludes.

2 The estimation bias in the three-equation model

We are interested in the estimation of interest rate rules such as the ones described in Taylor (1993) and Taylor (1999). In such policy rules, changes in the interest rate are associated with the path of aggregate macroeconomic variables. Estimations of the policy rule, however, lead to potentially biased and inconsistent estimates, as the Taylor rule equation is part of a broad system of equations.

To illustrate and quantify analytically such estimation bias, we begin our analysis by relying on a simple three-equation New Keynesian model. In particular, we focus on a version of the model described in Galí (2008, Chapter 3) that describes output, inflation, and interest rate as a function of technology and monetary shocks.

This simple model consists of: (i) a Phillips curve, equation (1), that relates inflation dynamics, π_t, to the output gap, $\tilde{y}_t$; (ii) an dynamic IS curve, equation (2), that determines the output gap given the paths for the actual real rate, $i_t - E_t(\pi_{t+1})$, and the natural rate of interest, $\hat{r}^n$; and (iii) a simplified policy rule, equation (3), that relates the policy instrument, i_t, to inflation and a monetary shock, v_t. The natural interest rate is determined in equilibrium by the level of output under flexible prices, which is a function of the technology shock, a_t. Technology and monetary shocks follow autoregressive processes. Appendix A provides additional details of the model.

$$\begin{aligned}
\pi_t &= \beta E_t(\pi_{t+1}) + \kappa \tilde{y}_t, && (1)\\
\tilde{y}_t &= E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma}\left(i_t - E_t(\pi_{t+1}) - \hat{r}^n_t\right), && (2)\\
i_t &= \phi_\pi \pi_t + v_t, && (3)
\end{aligned}$$

where $\hat{r}^n_t = \sigma \psi^n_{ya} \Delta a_{t+1}$, $a_t = \rho_a a_{t-1} + \epsilon^a_t$, and $v_t = \rho_v v_{t-1} + \epsilon^v_t$.

The solution to this model takes the form:

$$\begin{aligned}
\pi_t &= \psi_{\pi v} v_t + \psi_{\pi a} \hat{r}^n_t,\\
\tilde{y}_t &= \psi_{yv} v_t + \psi_{ya} \hat{r}^n_t,
\end{aligned}$$

where $\psi_{\pi v}$, $\psi_{\pi a}$, ψ_{yv}, and ψ_{ya} are negative coefficients.[1]

Given this solution, the asymptotic bias obtained from the OLS estimate of ϕ_π equals:

$$plim\hat{\phi}_\pi^{OLS} - \phi_\pi \quad = \quad \frac{cov\,(i,\pi)}{var\,(\pi)} = -\frac{1}{\kappa\Lambda_v}\lambda_v, \tag{4}$$

where:

$$\lambda_v \quad = \quad \frac{(\kappa\Lambda_v)^2\, var\,(v_t)}{(\kappa\Lambda_v)^2\, var\,(v_t) + \left(\sigma\psi_{ya}^n\,(1-\rho_a)\,\kappa\Lambda_a\right)^2 var\,(a_t)}, \tag{5}$$

for $\Lambda_v = \frac{1}{(1-\beta\rho_v)\sigma(1-\rho_v)+\kappa(\phi_\pi-\rho_v)}$ and $\Lambda_a = \frac{1}{(1-\beta\rho_a)\sigma(1-\rho_a)+\kappa(\phi_\pi-\rho_a)}$.

Expression (4) shows that the asymptotic bias is as a function of λ_v, the fraction of the variance of π_t that is accounted for by monetary shocks. The bias also depends on Λ_a, Λ_v, which are determined by model parameters. For typical parameter values, Λ_a, Λ_v, and λ_v are expected to be positive and the asymptotic bias in equation (4) to be negative.

To sidestep endogeneity problems with OLS, a common strategy is to estimate the Taylor rule by instrumental variables implemented using the Generalized Method of Moments (GMM), where lagged state variables are used as instruments. The matrix representation of the simple model above, expression (6), helps motivate the use of lagged variables as instruments:

$$X_t = A\Pi(A'A)^{-1}AX_{t-1} + A\epsilon_t, \tag{6}$$

$$X_t = \begin{pmatrix} y_t \\ \pi_t \end{pmatrix}, \quad A = \begin{pmatrix} \psi_{\pi v} & -\psi_{\pi a}\sigma\psi_{ya}^n(1-\rho_a) \\ \psi_{yv} & -\psi_{ya}\sigma\psi_{ya}^n(1-\rho_a) \end{pmatrix}, \quad \Pi = \begin{pmatrix} \rho_v & 0 \\ 0 & \rho_a \end{pmatrix}, \text{ and } \epsilon_t = \begin{pmatrix} \epsilon_t^v \\ \epsilon_t^a \end{pmatrix}.$$

Relying on GMM is appropriate when the model is identified. In this simple model, the rank condition for identification is satisfied if and only if $\rho_v \neq 0$ and $\rho_a \neq 0$, as the determinant of the matrix $M = A\Pi(A'A)^{-1}A$ equals $\rho_v\rho_a$. Therefore, identification requires persistence in the shocks.

However, the rank condition is not sufficient for reliable estimation and inference, as the estimates may still suffer from weak instruments. Stock, Wright and Yogo (2002) show that weak identification leads to poor parameter identification and asymptotic results become a poor guide to the actual sampling distributions.

The strength of identification implied by the set of instruments can be assessed using a concentration parameter (e.g., Mavroeidis, 2005 and Stock and Yogo, 2002), which measures

[1]See Appendix A for details.

the instruments' signal-to-noise ratio. More precisely, the concentration parameter ($\mathcal{CP}$) is a measure of the variation of the endogenous regressors that is explained by the instrumental variables after controlling for any exogenous regressors, relative to the variance of the residuals of the first-stage regression in a two-stage least squared approach. For the simple model above, the first-stage regression, as described in equation (6), implies a concentration parameter $\Sigma^{1/2}\Pi' Z_t Z_t' \Pi \Sigma^{1/2}$, where $\Sigma^{1/2}$ is the covariance matrix of the vector of errors from the first-stage regression, and Z_t contains the instruments X_{t-1}. The strength of the instruments can be measured by the smallest eigenvalue of that matrix. For a larger model, such as the one we consider below, obtaining the concentration parameter is not as straightforward.

While standard practice in the literature, one additional complication in the use of lagged variables as instruments is that when shocks are persistent, instruments and shocks may be correlated, hampering the asymptotic properties of the GMM estimates.

In short, one faces a few difficulties when estimating interest rate rules. On the one hand, if one uses OLS to estimate Taylor rule parameters, equation (5) indicates the presence of bias due to endogeneity. On the other hand, if one uses GMM to estimate the parameters, it is necessary to check for identification and instruments' strength. Even if the model is identified, weak identification may result in unreliable GMM estimates, and the asymptotic results may be a poor guide to the actual sampling distributions. The simple model described above helps illustrate that one needs shocks to be persistent when relying on lagged state variables as instruments. In addition, if shocks' persistence is not too strong, the estimation may suffer from weak identification, and there is no guarantee that GMM will generate estimates close to true parameters values. However, while one needs persistent shocks for identification, the selection of instruments to implement GMM estimates can be challenging. For persistent shocks, the common practice of relying on lagged dependent variables can imply that the instruments and residuals are correlated.

3 Quantifying the estimation bias

In this section, we rely on New Keynesian economies to quantify the estimation biases from OLS and GMM estimates of the Taylor rule. We start with the simple three-equation New Keynesian economy and turn next to a quantitative dynamic stochastic general equilibrium (DSGE) economy. We use these models as laboratories to quantify the biases in estimating Taylor rules. In particular, we simulate each model to generate artificial data, which we use to estimate the parameters of the Taylor rule by OLS and GMM (for varying parameterizations and sample sizes). We compare those estimates to the calibrated "true" parameters of the Taylor rule to assess the estimation biases.

3.1 Three-equation New Keynesian model

We start by augmenting the simple three-equation New Keynesian model described in equations (1)-(3) by adding (i) an unanticipated persistent inflation shock, $u_t = \rho_u u_{t-1} + \varepsilon_t^u$, which enters in the Phillips curve, and (ii) a fourth term, $\phi_y y_t$, in the Taylor rule. The first modification is necessary in order to generate three independent state variables. The latter modification brings the policy rule closer to the literature (Taylor, 1993, 1999). The resulting augmented model is described in the Appendix A in equations (A.1) to (A.7).

We parameterize the model by assuming log utility, $\sigma = 1$, discount rate at about 4% per year, such that $\beta = 0.99$, Taylor rule parameters $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$, the slope of the Phillips curve at $\kappa = 0.1275$, and $\psi_{ya}^n = 1$.

Equation (5) highlights an important role for the shocks' variance in determining the bias in OLS estimations. To bring this exercise closer to empirical literature, we calibrate the shocks' variances based on the results of Smets and Wouters (2007). The authors estimate a DSGE model and find evidence that monetary policy shocks contribute only a small fraction of the forecast variance decomposition of output and inflation at all horizons.[2] A similar finding is reported in Christiano, Eichenbaum and Evans (2005). Thus, we set the standard deviation of exogenous shocks to 0.08 for monetary shocks, technology shocks to 0.6, and inflation shocks to 0.07 (drawn from a standard normal distribution). These assumptions are set to match the unconditional variance decomposition presented in Smets and Wouters (2007), such that monetary shocks explain a small fraction of the changes in output and inflation, 2.73% and 4.88%, respectively. We later consider alternative values for the shocks' variances.

As discussed before, the persistence of shocks hitting the economy matter a great deal for model identification, particularly when relying on GMM estimators and lagged dependent variables as instruments. Hence, before we turn to the comparison between OLS and GMM estimates, we parameterize the model and assess identification under different levels of shocks' persistence. Note that the augmented model includes three shocks, instead of two, and therefore, the conditions under which identification of GMM holds in this model differ from those in the simple example of the previous section. In the extended version with three shocks, the requirement for identification is such that at least two of the shocks must be persistent.

[2] Smets and Wouters (2007) show that monetary policy shocks played a significantly higher role in the rise of inflation in the 1970s. More recently, price and wage markup shocks are the dominant source inflation fluctuations.

Based on these assumptions, we set the monetary shock persistence to $\rho_v = 0$, and calculate the instruments' strength based on the concentration parameter $\mathcal{CP}$ for different values of ρ_u and ρ_a. We use one lag of inflation and one lag of the output gap as instruments and simulate economies 80 observations ($T = 80$), while ranging technology and inflation shocks persistence parameters from 0 to 0.9.

Figure 1 summarizes the findings by showing the $\mathcal{CP}$ obtained in each simulation. In particular, the figure reports the levels attained by $\mathcal{CP}$ for different combinations of ρ_a and ρ_u. As a rule of thumb, Kleibergen and Mavroeidis (2009) and Stock and Yogo (2002) suggest that in a model with two endogenous variables and two instruments, a concentration parameter of about 10 should suffice. Figure 1 indicates that $\rho_u \geq 0.6$ and $\rho_a \geq 0.6$ imply $\mathcal{CP} \geq 18$. Because we want to step away from identification problems and fairly compare results obtained by OLS and GMM estimators, in what follows, we set $\rho_a = \rho_u = 0.8$, and entertain various values of ρ_v.

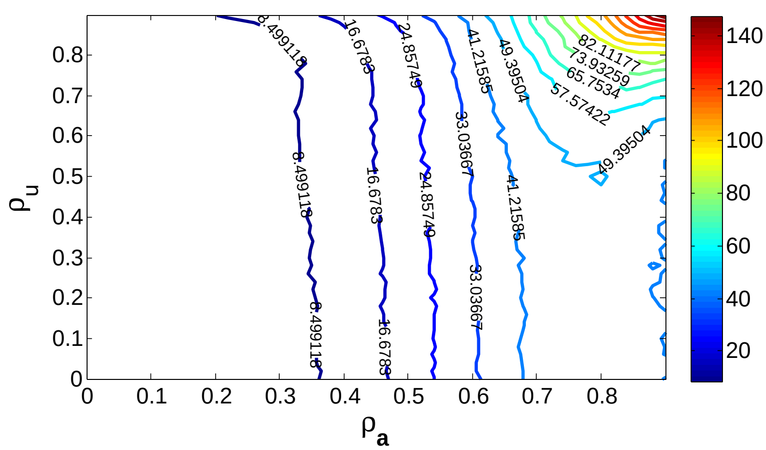

Figure 1: Concentration parameter ($\mathcal{CP}$) for a range of values of ρ_u and ρ_a

To compare the estimation of Taylor rule coefficients by OLS and GMM, we simulate economies with 80, 150, and 500 observations.[3] Estimates by GMM use a *k-step* estimator, which is frequently used in empirical literature (e.g., Clarida, Gali and Gertler, 2000).[4] We exclude simulations that fail Sargan-Hansen's test for the validity of overidentified restrictions (Sargan, 1958). We use two lags of inflation and output gap as instruments, and rely on Newey-West residuals.[5] We repeat this process 1000 times.

Before we turn to the role of the shocks' variance in determining the OLS bias, we consider

[3]These sample sizes are chosen to mimic commonly used samples in the empirical literature (e.g., Clarida, Gali and Gertler, 2000).

[4]Alternatively, we also considered the continuous updating estimator (CUE) proposed by Hansen, Heaton and Yaron (1996), but the method resulted in too many "extreme" estimates. As discussed in that paper, the continuous-updating criterion can make the numerical search for the minimizer difficult. This is so because under weak identification the true parameter does not satisfy the second-order condition for a minimum, resulting in

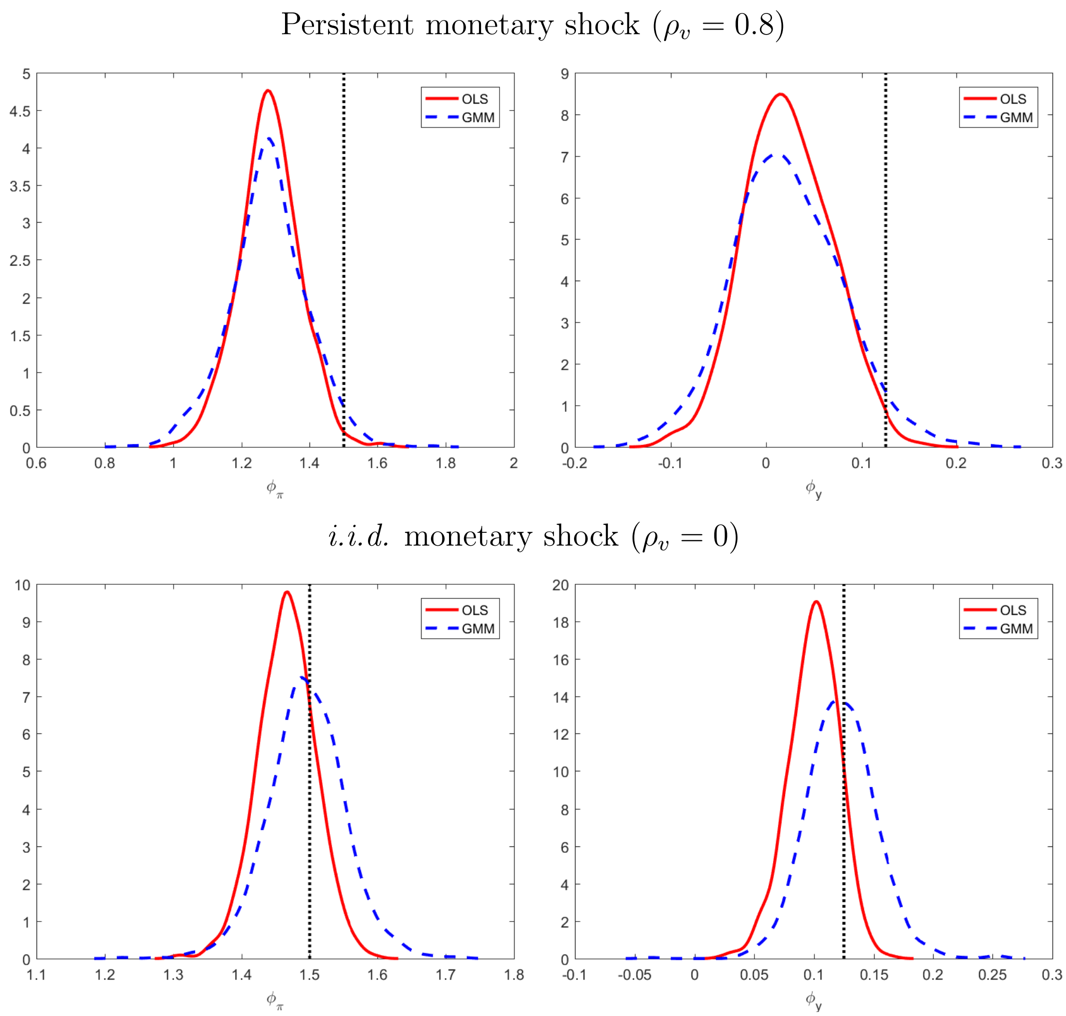

Figure 2: OLS and GMM estimate distributions in the three-equation New Keynesian model.

Note: The blue line reports GMM estimates while the red line reports OLS estimates. The black dotted line corresponds to the true parameter value. Distributions are obtained from 1000 Monte Carlo simulations of the model assuming a with sample size $T = 80$.

the effects of shocks' persistence on estimation biases in this simple three-equation model. The first set of results are reported in Figure 2. The top panel reports the kernel density of the estimated Taylor rule coefficients by OLS (in red) and GMM (in dashed blue) based on the 1000 simulations. The top panels report results when the model is calibrated with persistent monetary shocks, $\rho_v = 0.8$, while the bottom panels report results when the model is calibrated with $i.i.d$ monetary shocks (and $\rho_a = \rho_u = 0.8$). The black dotted line corresponds to the true parameter value.

extreme estimates for true parameters (Sargan, 1958).

[5]HAC allows population moment conditions to be serially correlated. We use Bartlett kernel function with two lags. We note that, while results may differ with the choice of kernel and the bandwidth parameter, they are qualitatively unchanged when we consider additional lags.

The top panels of Figure 2 show that OLS and GMM yield identical mean point estimates at $\hat{\phi}_\pi = 1.29$, and $\hat{\phi}_y = 0.02$.[6] The bias in GMM estimates is not all that surprising. As mentioned before, when monetary shocks are persistent, lagged dependent variables are correlated with the shock, and therefore, are not suitable as instrumental variables. As pointed out by de Vries and Li (2013), the literature on estimating monetary policy rules largely ignores this specific endogeneity problem, resulting estimates by GMM that are as biased as by OLS.

The bottom panels of Figure 2 confirm this assessment by assuming, instead, that $\rho_v = 0$. In this case, OLS does a slightly worse job than GMM, but the estimates are not too far from each other. Note that the set of instruments used to construct the cases depicted in Figure 2 are the same; two lags of inflation and output gap. These are, however, much better instruments when $\rho_v = 0$ as the panels in Figure 2 illustrates. Finally, for all panels estimates the distribution of estimates by OLS is less dispersed than by GMM, indicating better precision from the former.

Figure 3 complements the evidence from Figure 2 by reporting the mean point estimates from the OLS and GMM for ranging levels of ρ_v with samples of 80 observations. In particular, the figure reports ϕ_π and ϕ_y estimated by OLS (in red) and GMM (in blue) when shock persistence ranges from 0 to 0.9. The dashed black lines correspond to the true (calibrated) parameter values.

Figure 3 shows that GMM and OLS estimates are very close to each other and close to the true parameters. The figure also shows that, as the persistence of monetary policy shocks increases, GMM mean point estimates get remarkably close to the OLS estimates. In addition, even for $\rho_v = 0$ and GMM estimates nearly match the true parameter values, estimates by OLS are not too distant from the true parameters either. Finally, both estimation methods yield $\hat{\phi}_\pi > 1$, *i.e.*, the Taylor principle holds in all simulations. These conclusions also hold when we consider larger sample sizes (see Appendix Figure A1).

Our analytical results of Section 2 highlighted the role of the monetary shock volatility in the bias of OLS estimates. To quantify that prediction under this simple model, the panels of Figure 4 consider the effect of artificially increasing the variance of the monetary shock on the estimates of the Taylor rule coefficients by OLS and GMM. In particular, the top panels of Figure 4 assume a persistent monetary shock ($\rho_v = 0.8$), while the bottom panels assume $\rho_v = 0$. The panels report the estimated ϕ_π and ϕ_y under OLS (in red) and GMM (in blue) for ranging values of σ_r. The dotted vertical line corresponds to the true (calibrated) shock volatility, while the horizontal line corresponds to the true (calibrated) Taylor rule coefficients. All other parameters are kept as described before.

[6] The Appendix Table A1 complements this figure by reporting OLS and GMM mean point estimates when $\rho = 0$ and $\rho = 0.8$

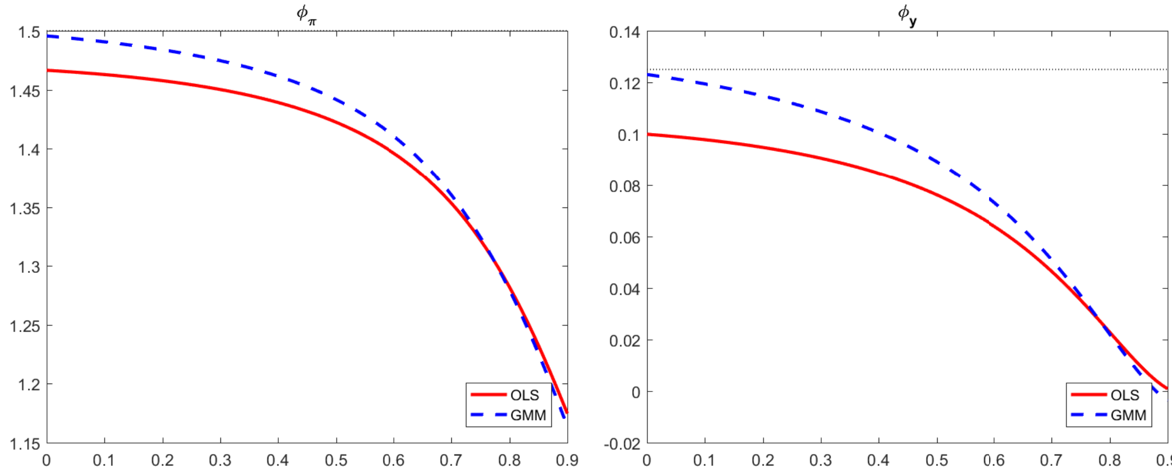

Figure 3: Mean point estimates in the three-equation New Keynesian model for ranging levels of monetary shock persistence.

Note: The blue line reports GMM estimates while the red line reports OLS estimates. The black dotted line corresponds to the true parameter value. Distributions are obtained from 1000 Monte Carlo simulations of the model assuming a with sample size $T = 80$.

The four panels of Figure 4 confirm our findings of Section 2 by showing that as the monetary shock variance artificially increases, the estimation biases also increase. The top panels show that when monetary shocks are persistent, both OLS and GMM yield similar estimation biases for all levels of shock volatility. The bottom panels show that, when shocks are *i.i.d.*, while the estimation biases increase with shock volatility for both estimation methods, the biases are more pronounced for OLS estimates.

The quantitative findings based on this simple three-equation model confirm our analytical predictions and highlight the contribution of shocks volatility in determining the estimation biases by both OLS and GMM. In the next section we consider a similar analysis while relying, instead, on a quantitative workhorse DSGE economy.

3.2 A quantitative DSGE model

The three-equation New Keynesian model considered in Section 3.1 is very simple as a description of an economic environment with only three exogenous shocks to describe all endogenous variables. In a larger-scale model, macroeconomic variables are affected by a larger set of disturbances.

We start by introducing the model and its solution, followed by a discussion on the issues associated with single-equation estimation of the Taylor rule.[7]

[7] We follow Sims (2002) notation.

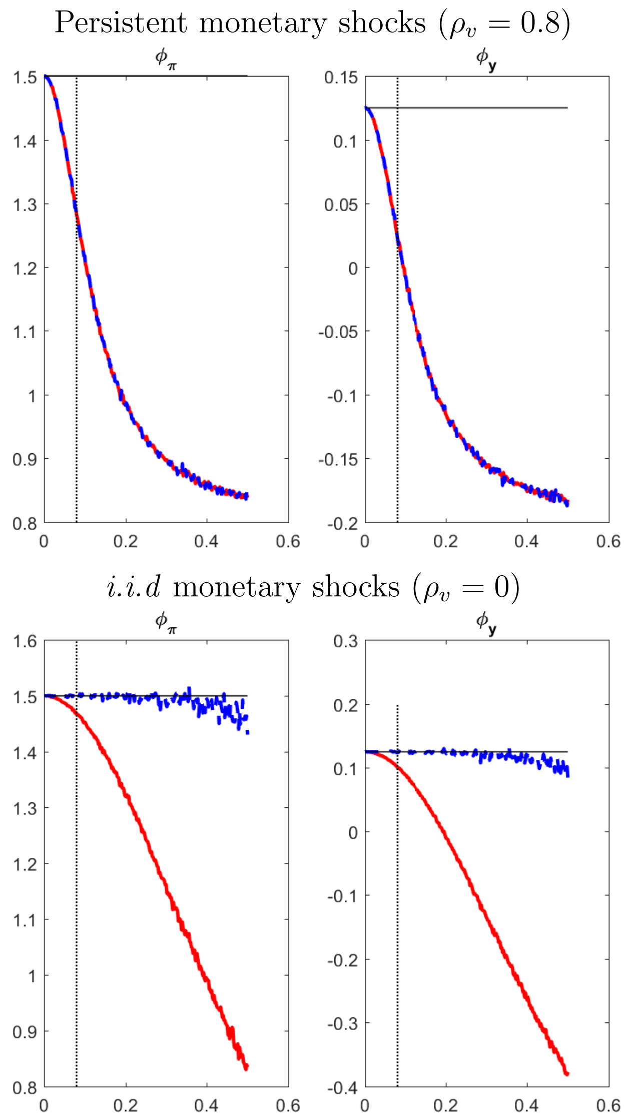

Figure 4: OLS and GMM estimates in the three-equation New Keynesian model with persistent and *i.i.d.* monetary shocks ($\rho_v = 0$) for varying monetary shock volatility.

Note: The blue line reports GMM estimates while the red line reports OLS estimates. The vertical dotted black line corresponds the calibrated standard deviation of the monetary shock, while the horizontal solid black line reports the "true" Taylor rule parameters. Simulations are based on a simple three-equation New Keynesian model and assume $T = 80$.

A large-scale New Keynesian model can be cast in the form:

$$\Gamma_0 y_t = C + \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t,$$

where y_t is a vector of state variables, C is a vector of constants, z_t is an exogenous vector of variables involving normally distributed random disturbance, and η_t is an expectational error, satisfying $E_t \eta_{t+1} = 0$, for all t. The interest rate is included in the vector y_t, while the monetary shock appears in the vector z_t if shocks are independent identically distributed ($i.i.d.$) or in y_t in case of persistence of monetary shocks.

The solution can be stated as a reduced-form of the linear rational expectation model:

$$y_t = A_c(\theta) + A_x(\theta)y_{t-1} + A_z(\theta)z_t, \tag{7}$$

where $\theta \in \Theta$ represents the parametric space of structural parameters. Simulated data from a New Keynesian model will follow the solution described by equation (7). This solution has a vector autoregressive representation (VAR) with the interest of rate as an element of the vector y_t. The solution described in equation (7) shows that all state variables, including the interest rate, can be affected by monetary shocks. Therefore, inflation and output are correlated with monetary shocks raising concerns of endogeneity biases if one estimates the interest rate rule coefficients by OLS. Furthermore, as discussed before, when monetary shocks are persistent, the lagged state variables are correlated with monetary shocks as well, and therefore, they are not proper instruments for estimating Taylor rule under GMM in a single equation approach. The set of permissible instruments will depend on the specification of the policy rule.

To test for the performance of OLS and GMM estimates in such a class of models, we rely on a workhorse model of Smets and Wouters (2007), who estimate a fully-specified model for the U.S. economy that includes several types of real and nominal frictions and structural shocks. The model is estimated using a Bayesian likelihood approach and is used to investigate the relative importance of its various frictions for the U.S. business cycle.[8]

To assess OLS and GMM estimates of the monetary policy rule (in a single equation approach), we perform as in Section 3.1 and generate data from a parameterized version of the Smets and Wouters (2007) model. To parametrize the model, we rely on the authors' own estimates. In particular, the model is parameterized at the mode of the posterior distributions of their estimates. As before, we generate 1000 simulations of the model for different sample sizes (80, 150 and 500 observations), and use the model-simulated data to estimate the model policy rule, which is given by:

$$r_t = \rho r_{t-1} + (1-\rho)\left[\phi_\pi \pi_t + r_y(y_t - y_t^p)\right] + r_\Delta\left[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)\right] + \varepsilon_t^r, \tag{8}$$

where the monetary shock, ε_t^r follows an autoregressive process such that $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$. We, then, estimate the model's policy rule by OLS and GMM, and compare these estimates with

[8] Iskrev (2010) finds that this model is locally identified, which allows for consistent estimation of structural parameters. Local identification also guarantees the usual asymptotic properties of estimators.

the model-calibrated (true) values.

Note that equation (8) can be rewritten as:

$$r_t = \theta_1 r_{t-1} + \theta_2 \pi_t + \theta_3 \tilde{y}_t + \theta_4 \tilde{y}_{t-1} + \epsilon_t^r, \tag{9}$$

where $\theta_1 = \rho$, $\theta_2 = (1-\rho)\phi_\pi$, $\theta_3 = (1-\rho)r_y + r_{\Delta y}$, and $\theta_4 = -r_{\Delta y}$. $\tilde{y}_t = y_t - y_t^p$ is the output gap. Hence, in practice, we estimate equation (9) and recover the structural parameters through the estimates $\{\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4\}$.

Because this model assumes monetary shocks are persistent, we move away from using lagged state variables as instruments, and rely, instead, on current values and three lags of marginal cost and wages as instruments for the GMM estimates.[9] As in the three-equation model, we exclude from our sample "extreme" GMM estimates keeping only cases in which the model is not rejected by Sargan-Hansen's test for the validity of overidentified restrictions (Sargan, 1958).[10] Finally, we rely on Newey-West residuals.

Our analytical results of Section 2 show that the OLS endogeneity bias is a function of the fraction of the variance of Taylor rule determinants accounted for by monetary shocks. Smets and Wouters (2007) find that at the mode of the posterior distribution monetary shocks account for 2.73% and 4.88% of the variance in output and inflation, respectively.

To assess how OLS and GMM estimates would perform if monetary shocks were more important, Figure 5 compares OLS and GMM estimates as the monetary shock volatility, σ_r, ranges from 0 to 0.8. The vertical line in Figure 5 reports the "true" (estimated mode posterior) standard deviation of the monetary shock, $\sigma_r = 0.24$, while the horizontal line reports the "true" Taylor rule parameters. In this exercise, we consider a sample of $T = 80$ observations. Results for large samples are reported in the Appendix Figure A2.

In line with our predictions of Sections 2 and 3.1, OLS estimates become more biased as σ_r increases. Estimates by GMM also become more biased as σ_r increases, albeit by a smaller amount. At the mode posterior estimate from Smets and Wouters (2007), parameters estimates by OLS and GMM show a relatively small bias. Note that while GMM estimates can yield a smaller bias than the OLS ones, the volatility of the former seem to be larger, as we discuss below.

[9] We also considered other sets of instruments, such as lags of output and inflation, but those were less favorable to GMM.

[10] In our model simulations, depending on the covariance between endogenous variables and the set of instruments, GMM may yield extreme values, which is a well-known numerical problem in this method. In what follows, we exclude those extreme values and concentrate on estimates closer to the "true" parameters. Note that this treatment of GMM estimates, once more, biases our results away from our main paper conclusion that estimating by OLS is preferable.

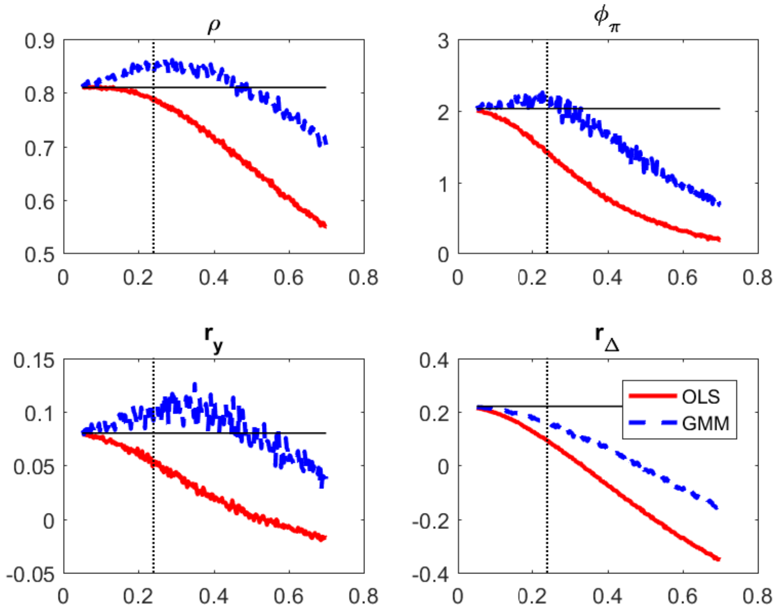

Figure 5: OLS and GMM estimates in a large-scale DSGE model for varying monetary shock volatility.

Note: The blue line reports GMM estimates while the red line reports OLS estimates. The vertical dotted black line corresponds the calibrated standard deviation of the monetary shock, while the horizontal solid black line reports the “true” Taylor rule parameters. Simulations are based on the model and parameterization described in Smets and Wouters (2007) and assume $T = 80$.

To quantify the estimation biases from OLS and GMM, we consider the calibration of the model as described above, *i.e.*, we set all parameter values, including the monetary shock volatility, at the mode of the posterior the distributions estimated by Smets and Wouters (2007). Figure 6 reports the distribution of estimated parameters by OLS (in red) and GMM (in blue) when the sample size equals 80 observations. The horizontal dotted black lines report the true Taylor rule parameters.

Similarly to the findings based on the three-equation model, the OLS and GMM mean point estimates are close to one another and, more importantly, somewhat close to the true parameter values. In addition, the dispersion in estimated coefficients is much larger for GMM than for OLS. These findings also hold when the sample is small but also when we consider a large sample (see Appendix Figure A3).

So far we have focused on properties of individual parameters. This is useful and informative if the focus is on their specific estimated values. Frequently, however, the interest in on the dynamics of the model, which is determined jointly by all estimated parameters. These dynamics are commonly summarized by impulse response functions that reveal the path followed by endogenous variables following an unexpected shock. Hence, we now provide evidence on the the properties of the impulse response functions resulting from estimated by GMM and OLS.

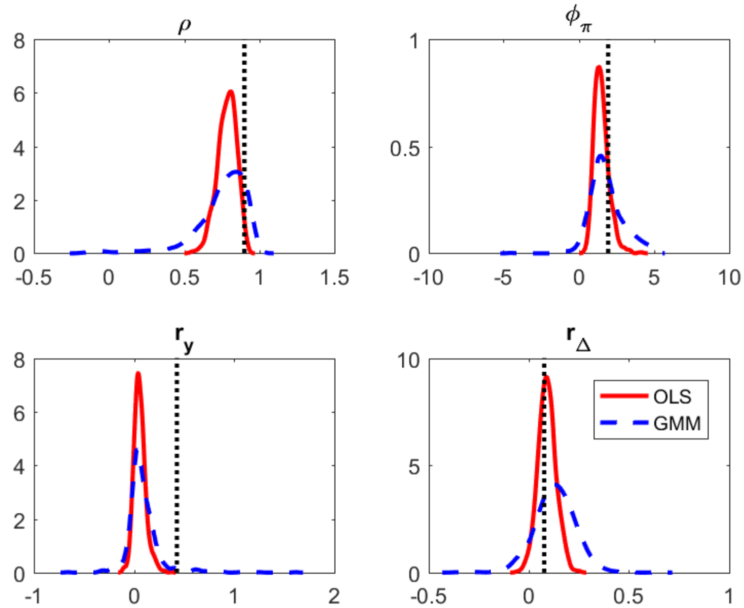

Figure 6: OLS and GMM estimate distributions in a large-scale DSGE model.

Note: The blue line reports GMM estimates while the red line reports OLS estimates. The black dotted line corresponds to the true parameter value. Simulations are based on the model and parameterization described in Smets and Wouters (2007). Distributions are obtained from 1000 Monte Carlo simulations of the model assuming a with sample size $T = 80$.

Figure 7 reports impulse response functions of output and inflation to a monetary policy shock. More specifically, the figure reports, in black, the responses of output and inflation when the Taylor rule is calibrated at the "true" parameters (the mode of the posterior distributions). The figure also reports counterfactual responses obtained when the Taylor rule is parameterized using the mean point estimates from OLS (in red) and GMM (in blue), respectively.[11] Finally, the panels in Figure 7 also include shaded areas that report, for each point in time, the 5^{th} and 90^{th} percentiles of the distribution of IRFs based on estimated coefficients (by OLS and GMM) from the simulated data. In this exercise, we set the sample size to 80 observations. Results for a larger sample are reported in the Appendix Figure A4.

As shown in Figure 7, both the OLS and the GMM responses are very close to the ones obtained from the mode parameterization. In addition, the peak effects coincide in all cases. A comparison between OLS and the true impulse response function suggests that the OLS endogeneity bias does not induce a large distortion in economic variables' response to shocks. In particular, the proximity between the true and the OLS inflation impulse response functions is noteworthy. The shaded areas in Figure 7 reinforce the better precision of OLS estimates. While the mean point estimates from OLS and GMM yield impulse response functions that are

[11] The Appendix Table A2 reports OLS and GMM mean point estimates, as well as true parameter values, used to construct the exhibits in Figure 7.

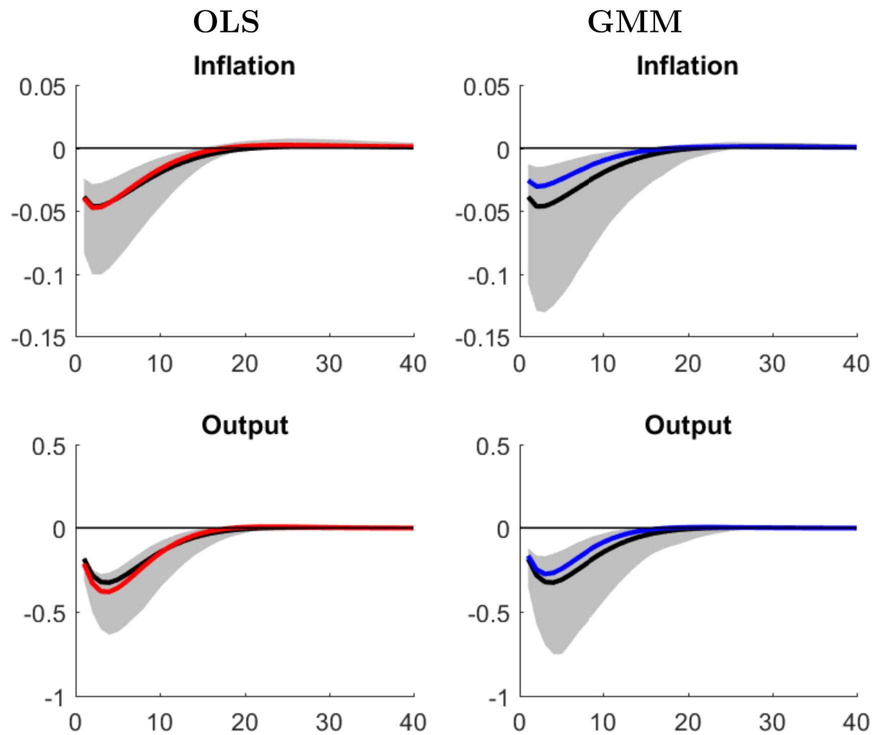

Figure 7: Output and inflation responses to monetary shocks in a large-scale DSGE model with Taylor rule parameters at true versus OLS and GMM estimated values.

Note: The black line shows the estimated responses of the model with parameters at the mode of posterior distributions. The red line shows the response implied by OLS estimates. The blue line reports the response implied by GMM estimates. The shaded areas report, for each point in time, the 5^{th} and 95^{th} percentiles of the distribution of IRFs obtained from estimated coefficients. Simulations are based on the model and parameterization described in Smets and Wouters (2007) and assume $T = 80$.

close to one another and to the true one, the much smaller shaded areas in the left-hand-side panels confirm that the precision of OLS estimates is much higher than the one obtained from GMM.

Our results show that the OLS estimation bias in small. More importantly, estimates by OLS imply model dynamics that are remarkably close to the true ones, with much better precision than dynamics implied by GMM estimates. These findings indicate that, if one is interested in the dynamics of the model, rather than estimating specific coefficients, she can benefit from simplicity and better precision of OLS, and avoid the search for appropriate instruments.

4 An empirical application

In Section 3 we relied on models that we used as laboratories to quantify and study the endogeneity bias attained by different estimation methods. One key advantage of that approach is that, in such an exercise, we know the data-generating processes, the true parameters and the interest rate rule specification to compare our estimates to.

Having made the case for OLS estimation using known data-generating processes, it is only natural that we take our findings to actual data. We compare OLS and GMM estimates of an interest rate rule similar to the one described in Clarida, Gali and Gertler (2000).

Using instrumental variables, Clarida, Gali and Gertler (2000) estimate:

$$E\{[r_t - (1-\rho_1-\rho_2)(rr^* - (\beta-1)\pi^* + \beta\pi_{t,t+1} + \gamma x_t) + \rho_1 r_{t-1} + \rho_2 r_{t-2}]z_t\} = 0, \tag{10}$$

where r_t is the fed funds rate, rr^* is the equilibrium real interest rate, π^* is the target for inflation, $\pi_{t,t+1}$ denotes the percent change in the price level between t and $t+1$, and x_t denotes the average output gap at t. The set of instruments z_t includes four lags of inflation, output gap, the fed funds rate, money growth, the spread between long and short term bond rates, and commodity price inflation.[12]

The authors obtain estimates for $(\pi^*, \beta, \gamma, \rho)$ by setting the equilibrium real rate, $rr^* \equiv r_t - \pi_{t,t+1}$, at its sample average (for each subsample). In their benchmark specification, they consider two sample periods: "Pre-Volcker" which runs from 1960Q1 to 1979Q2, and "Volcker-Greenspan" which runs from 1979Q3 to 1996Q4. For additional details, please refer to Clarida, Gali and Gertler (2000).

Because our goal is to compare OLS and IV estimates, we consider an interest rate rule with current inflation, instead of the forward looking specification of equation (10). In particular, we estimate:

$$r_t = \alpha_{aux} + \beta_{aux}\pi_t + \gamma_{aux}x_t + \rho_{1,aux}r_{t-1} + \rho_{2,aux}r_{t-2} \tag{11}$$

to obtain $\hat{\alpha}_{aux}$, $\hat{\beta}_{aux}$, $\hat{\gamma}_{aux}$, $\hat{\rho}_{1,aux}$, and $\hat{\rho}_{2,aux}$, from which we can back out $\hat{\rho} = \hat{\rho}_{1,aux} + \hat{\rho}_{2,aux}$, $\hat{\gamma} = \frac{\hat{\gamma}_{aux}}{1-\hat{\rho}}$, $\hat{\beta} = \frac{\hat{\beta}_{aux}}{1-\hat{\rho}}$, and $\hat{\pi^*} = \frac{\hat{\alpha}_{aux}-(1-\hat{\rho})rr^*}{(1-\hat{\rho})(1-\hat{\beta})}$.[13]

We rely on the same set of instruments as Clarida, Gali and Gertler (2000), but expand their sample to include more recent data. In particular, we consider 4 subsamples: (1) "Pre-Volcker" from 1960Q1 to 1979Q2, (2) "Volcker-Greenspan" from 1979Q3 to 2005Q4, (3) "Greenspan-Bernanke" from 1987Q3 to 2007Q4, and (4) "Post-Volcker" from 1979Q3 to 2007Q4.

Table 1 reports the results. The top panel shows IV estimates and the bottom panel reports

[12] The data are quarterly running from 1960Q1 to 2007Q4. Interest rate is the federal Funds rate, inflation is the year-on-year rate of change in the core PCE, output gap is constructed using the Congressional Budget Office estimate of potential GDP, money growth is based on M2, commodity prices are a composite of commodity goods (including oil, gold and food items), and long and short term bond rates are the 10-year and 3-month Treasury bill rates, respectively.

[13] We took a similar two-step approach in Section 3.2. Alternatively, one could directly estimate a nonlinear regression similar to equation (10). Results for the latter approach are reported in the Appendix Table A3.

Table 1: Empirical IV and OLS estimates

	IV estimates			
	(1) Pre-Volcker 1960Q1-1979Q2	(2) Volcker-Greenspan 1979Q3-2005Q4	(3) Greenspan-Bernanke 1987Q3-2007Q4	(4) Post-Volcker 1979Q3-2007Q4
β	0.90***	2.24***	1.70***	2.23***
	(0.22)	(0.34)	(0.46)	(0.34)
γ	0.59***	0.97***	0.88***	0.98***
	(0.18)	(0.19)	(0.23)	(0.20)
ρ	0.76***	0.66***	0.88***	0.66***
	(0.08)	(0.07)	(0.03)	(0.06)
π^*	3.36	2.71***	1.95	2.63***
	(7.48)	(0.74)	(1.41)	(0.73)
N	74	106	82	114
	OLS estimates			
	(1) Pre-Volcker 1960Q1-1979Q2	(2) Volcker-Greenspan 1979Q3-2005Q4	(3) Greenspan-Bernanke 1987Q3-2007Q4	(4) Post-Volcker 1979Q3-2007Q4
β	0.89***	2.24***	1.74***	2.23***
	(0.20)	(0.36)	(0.39)	(0.36)
γ	0.58***	0.99***	0.94***	0.99***
	(0.16)	(0.18)	(0.20)	(0.18)
ρ	0.74***	0.68***	0.87***	0.68***
	(0.07)	(0.08)	(0.03)	(0.08)
π^*	3.60	2.72***	1.94*	2.64***
	(5.93)	(0.77)	(1.14)	(0.77)
N	74	106	82	114

Note: The table reports estimates of equation (10) by OLS and IV. The set of instruments includes four lags of inflation, output gap, the fed funds rate, money growth (M2), the spread between long and short term bond rates, and commodity price inflation. Statistical significance at the 90/95/99% confidence level indicated with */**/***, respectively. Robust standard errors are reported in parenthesis.

OLS estimates. While there are some differences in estimated coefficients, the two panels show that, in general, estimates by OLS and IV are not too far from one another.[14]

[14] The estimates reported in columns (1) and (2) of Table 1 do not exactly replicate the findings reported in Clarida, Gali and Gertler (2000, Table II). The reasons for this are fourfold. First, we estimate an interest rate rule with current, instead of future inflation. Second, we rely on core PCE as a measure of inflation, while those authors use, alternatively, GDP deflator or CPI. Third, we expand the Volcker-Greenspan sample up to the last quarter of Greenspan's term, instead of stopping that subsample in 1996Q4 (the latest data point available to those authors). Fourth, there have been significant revisions to the data since their paper was written. When we limit the subsample and use their dataset and interest rate rule specification, our results nearly perfectly match those of Clarida, Gali and Gertler (2000). We thank the authors for kindly sharing their data.

Based on these regression results, we resort to local projections, as introduced by Jordà (2005), to obtain impulse response functions of inflation and output to monetary shocks. If our methods were perfect, the residuals from both OLS and IV estimates would coincide and correspond to monetary shocks. As discussed before, however, both OLS and IV estimates are subject to estimation biases and therefore, in our estimates of the empirical impulse response functions, we allow for that possibility.

More specifically, we extract the residuals from the OLS and IV regressions and estimate:

$$
\begin{aligned}
\pi_{t+h} &= \mu_{\pi,h} + \delta_{\pi,h}\hat{\epsilon}_{m,t} + \lambda_{\pi,i}\mathbf{c}_{t-i}, \\
x_{t+h} &= \mu_{x,h} + \delta_{y,h}\hat{\epsilon}_{m,t} + \lambda_{x,i}\mathbf{c}_{t-i},
\end{aligned}
\tag{12}
$$

where $\hat{\epsilon}_{m,t}$ are the residuals from the OLS and IV regressions from Table 1, *i.e.*, $m = \{OLS, IV\}$. $\mathbf{c}_{t-i}$ is a matrix of control variables that includes four lags ($i = 1, ..., 4$) of r_t, π_t, and x_t. The response of inflation and output are given by the estimated coefficients $\delta_{\pi,h}$ and $\delta_{x,h}$, respectively, for horizons $h = 1, ..., 24$.

Figure 8 reports the resulting impulse response functions of inflation and output gap to a one-standard-deviation shock. We focus on the pre- and post-Volcker periods to achieve reasonable sample sizes.

All four panels of Figure 8 yield a qualitatively similar conclusion that the responses of OLS and IV are close to each other.[15] The finding that the IV- and the OLS-based impulse response functions are close to each other is in line with the patterns observed in impulse responses reported in Figure 7. In addition, the IRFs reported in Figure 8 show patterns that accord with findings of the empirical literature on the effects of monetary policy shocks on inflation and output. In particular, the response of inflation during the pre-Volcker period shows a "price puzzle" (Sims, 1992 and Eichenbaum, 1992), that is absent during the post-Volcker sample (Castelnuovo and Surico, 2010 and Baumeister, Liu and Mumtaz, 2013). Moreover, the response of economic slack supports the evidence that monetary policy has become more stabilizing in the later part of the sample (e.g., Boivin and Giannoni, 2006, Castelnuovo and Surico, 2010 and Baumeister, Liu and Mumtaz, 2013).

[15] Estimates of equation (12) that do not include the control variable matrix $\mathbf{c}_{t-i}$ and those who are obtained through nonlinear methods (as reported in the Appendix Table A3) also yield OLS- and IV-based IRFs that are close to each other.

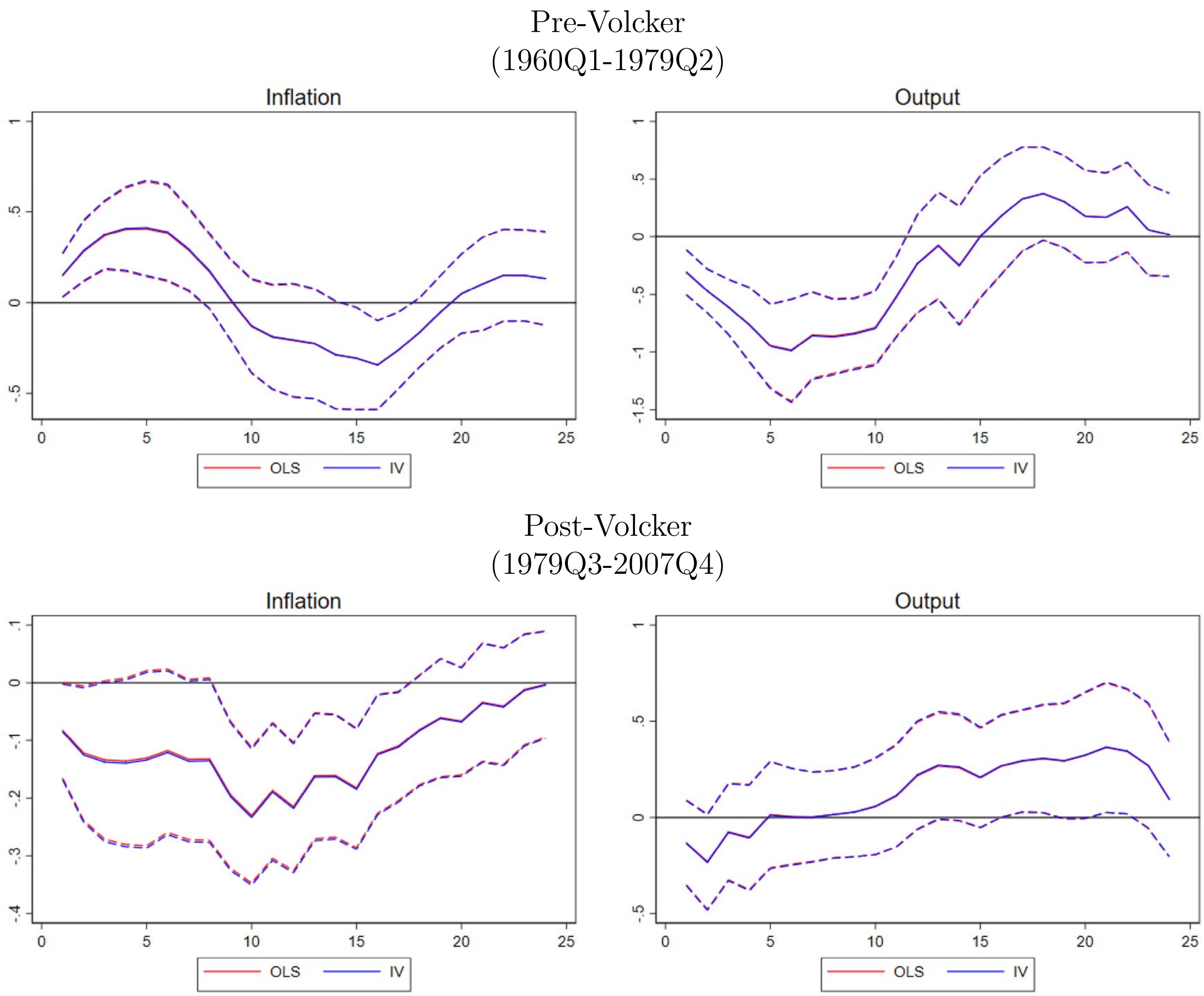

Figure 8: OLS and IV empirical impulse response functions.

Note: The red line shows the response implied by OLS estimates. The blue line reports the response implied by IV estimates. The dashed lines corresponds to 90% confidence bands. IRFs are constructed by local projections of OLS and IV estimated residuals on inflation and output, as described in equation (12).

5 Conclusion

This paper investigates how the estimation of Taylor rules is affected by estimation biases. We show analytically, in the three-equation New Keynesian model, that the OLS asymptotic bias is a function of the fraction of the variance of inflation accounted for by monetary policy shocks. This suggests that the endogeneity bias in OLS estimates is limited, provided that monetary policy shocks explain only a small fraction of endogenous variables to which the monetary authority responds, such as inflation and the output gap.

To quantify the estimation bias, we resort to Monte Carlo simulations of well-established models. In particular, we generate data from the three-equation and the Smets and Wouters (2007) models, and estimate their respective interest rate rules by OLS and GMM.

Our findings show that, for realistic sample sizes, mean point estimates by OLS and GMM are close to one another and close to the true parameter values. The endogeneity bias in OLS estimates is small since monetary policy shocks play a limited role in explaining inflation and output gap variation. OLS estimates are, however, more precise. More importantly, the dynamic properties of the model are essentially unaffected by the OLS estimation bias. More specifically, impulse response functions obtained when using OLS estimates closely mimic the ones obtained under the true parameters.

In sum, the results presented here contribute to the literature by expanding the understanding of endogeneity problems in Taylor rule single-equation estimation. The results show that the OLS endogeneity bias is small for plausible parameterizations and realistic sample sizes. Given the relatively limited role of monetary policy shocks in explaining movements in inflation and output, as indicated by the empirical literature, when estimating Taylor rules one can rely on OLS rather than GMM, and benefit from its simplicity and better precision.

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Appendix

A The three-equation New Keynesian model

This model builds from the basic New Keynesian model described in Galí (2008, chapter 3). The model introduces imperfect competition in the goods market by assuming that firms produce differentiated goods, for which it sets its price. Price adjustment is staggered as in Calvo (1983).

A.1 Households

Assume a representative infinitely-lived household who maximizes:

$$E_o \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

where consumption C_t is given by:

$$C_t \equiv \left(\int_0^1 C_t\left(i\right)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

and $C_t\left(i\right)$ represents the quantity of good i consumed by the household in period t.

The period budget constraint takes the form:

$$\int_0^1 P_t\left(i\right) C_t\left(i\right) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t,$$

for $t = 0, 1, 2, ...$, where $P_t\left(i\right)$ is the price of good i, N_t denotes hours of work, W_t is the nominal wage, B_t represents purchases of one-period bonds at a price Q_t, and T_t is a lump-sum income. A transversality condition $\lim_{T\to\infty} E_t\left\{B_t\right\} \geq 0$ holds for all t.

The solution to this problem yields the household demand equation:

$$C_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t} \right)^{-\varepsilon} C_t,$$

for all $i \in [0, 1]$, where $P_t \equiv \left(\int_0^1 P_t\left(i\right)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is an aggregate price index.

The log-linear optimality condition yields:

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t, \\ c_t &= E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left(\pi_{t+1}\right) - \rho\right), \end{aligned}$$

where $i_t \equiv -logQ_t$ is the short term nominal interest rate and $\rho \equiv -\log \beta$ is the discount rate.

A.2 Firms

Assume a continuum of firms indexed by $i \in [0, 1]$. Firms produce varieties but have identical technology given by:

$$Y_t(i) = A_t N_t(i)^{1-\alpha},$$

where A_t is technology.

Firms face the same household demand equation and take aggregate prices P_t and consumption C_t as given. They adjust prices as in Calvo (1983) and each firm resets its price with probability $(1-\theta)$ in any given period.

Aggregate price dynamics are described by:

$$P_t = \theta P_{t-1} + (1-\theta) P_t^*,$$

where P_t^* is the price set by adjusting forms in period t.

A firm optimizing in period t will choose P_t^* such that it solves:

$$\begin{aligned}
&\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left[P_t^* Y_{t+k/t} - \Psi_{t+k}\left(Y_{t+k/t}\right) \right] \right\} \\
&s.t. \\
Y_{t+k/t} \quad = \quad & \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k}
\end{aligned}$$

for $k = 0, 1, 2, ...$, where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$ is the stochastic discount factor, $\Psi_t(.)$ is the cost function and $Y_{t+k/t}$ denotes output in period $t+k$ for a firm that last adjusted prices in period t.

The log-linear optimality condition yields:

$$p_t^* = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[mc_{t+k/t} + p_{t+k} \right],$$

where $mc_{t+k/t}$ is the real marginal cost at time $t+k$ for a firm that last changed prices at time t, and $-\mu$ is its steady state.

A.3 Equilibrium

Log-linearized market clearing conditions yield:

$$
\begin{aligned}
y_t &= E_t\left\{y_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - E\left\{\pi_{t+1}\right\} - \rho\right) \\
y_t &= a_t + (1-\alpha)\, n_t \\
mc_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \frac{1+\phi}{1-\alpha} a_t - \log\left(1-\alpha\right)
\end{aligned}
$$

Under flexible prices, , the real marginal cost is constant at $mc = -\mu$. Defining the natural level of output y_t^n as the equilibrium level of output under flexible prices:

$$
mc = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - \frac{1+\phi}{1-\alpha} a_t - \log\left(1-\alpha\right).
$$

This implies:

$$
\begin{aligned}
y_t^n &= \psi_{ya}^n a_t + \vartheta_y^n \\
\psi_{ya}^n &\equiv \frac{1+\varphi}{\sigma\left(1-\alpha\right)+\varphi+\alpha} \\
\vartheta_y^n &\equiv -\frac{\left(1-\alpha\right)\left(\mu - \log\left(1-\alpha\right)\right)}{\sigma\left(1-\alpha\right)+\varphi+\alpha}
\end{aligned}
$$

A.4 Three-equation New Keynesian model

The economy above can be summarized in a system of two equations:

$$
\begin{aligned}
\pi_t &= \beta E_t\left\{\pi_{t+1}\right\} + \kappa \tilde{y}_t \\
\tilde{y}_t &= -\frac{1}{\sigma}\left(i_t - E_t\left\{\pi_{t+1}\right\} - r_t^n\right) + E_t\left\{\tilde{y}_{t+1}\right\},
\end{aligned}
$$

where $\tilde{y}_t = y_t - y_t^n$, $r_t^n \equiv \rho + \sigma \psi_{ya}^n E_t\left\{\Delta a_{t+1}\right\}$ and $\kappa \equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$.

We are left with the specification of a policy rule and the shocks hitting the economy.

The policy rule is such that:

$$
i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t,
$$

where v_t is a monetary shock.

This simple New Keynesian economy is summarized by:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t + u_t \tag{A.1}$$

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) + E_t\{\tilde{y}_{t+1}\} \tag{A.2}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t + v_t \tag{A.3}$$

$$\hat{r}_t^n = r_t^n - \rho \equiv \sigma \psi_{ya}^n E_t\{\Delta a_{t+1}\} \tag{A.4}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad \rho_a \in [0,1) \tag{A.5}$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \quad \rho_v \in [0,1) \tag{A.6}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \quad \rho_u \in [0,1) \tag{A.7}$$

The first three equations correspond to the Phillips curve, an IS curve and a policy rule, respectively. The fourth equation defines the natural rate of interest. The last three equations specify the dynamics of the technology, monetary and inflation shocks. In the main text, we consider a further simplified version of this model in which we assume away from inflation shocks ($\epsilon_t^u = 0$ and $u_t = 0$ for all t), and set $\phi_y = 0$.

A.5 Solving the three-equation model by undetermined coefficients

To further simplify the model and attain an analytical solution, we assume $\phi_y = 0$.

In that case, the solution will take the form:

$$\tilde{y}_t = \psi_{yv} v_t + \psi_{ya} \hat{r}_t^n + \psi_{yu} u_t, \tag{A.8}$$

$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} \hat{r}_t^n + \psi_{\pi u} u_t, \tag{A.9}$$

$$i_t = \rho + \phi_\pi \pi_t + v_t`$$

where coefficients ψ_{yv}, ψ_{ya}, ψ_{yu}, $\psi_{\pi v}$, $\psi_{\pi a}$, and $\psi_{\pi u}$ are to be determined.

Because we assume the shocks follow autoregressive processes:

$$\hat{r}_t^n = -\sigma \psi_{ya}^n (1 - \rho_a) a_t \Rightarrow E_t\left(\hat{r}_{t+1}^n\right) = \rho_a \hat{r}_t^n$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \Rightarrow E_t(v_{t+1}) = \rho_v v_t$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \Rightarrow E_t(u_{t+1}) = \rho_u u_t$$

Replacing equations (A.3), (A.8) and (A.9) in equation (A.2) and rearranging:

$$
\begin{aligned}
\tilde{y}_t &= -\frac{1}{\sigma}\left(i_t - E_t\left\{\pi_{t+1}\right\} - r_t^n\right) + E_t\left\{\tilde{y}_{t+1}\right\} \\
&= \left\{-\frac{1}{\sigma}\phi_\pi \psi_{\pi v} + \frac{1}{\sigma}\psi_{\pi v}\rho_v + \psi_{yv}\rho_v - \frac{1}{\sigma}\right\} v_t \\
&\quad + \left\{\frac{1}{\sigma} - \phi_\pi \frac{1}{\sigma}\psi_{\pi a} + \frac{1}{\sigma}\psi_{\pi a}\rho_a + \psi_{ya}\rho_a\right\} \hat{r}_t^n \\
&\quad + \left\{\frac{1}{\sigma} - \frac{1}{\sigma}\phi_\pi \psi_{\pi u} + \frac{1}{\sigma}\psi_{\pi u}\rho_u + \psi_{yu}\rho_u\right\} u_t
\end{aligned}
$$

Replacing equations (A.3), (A.8) and (A.9) in equation (A.1) and rearranging:

$$
\begin{aligned}
\pi_t &= \beta E_t\left\{\pi_{t+1}\right\} + \kappa \tilde{y}_t + u_t \\
&= \left\{\beta \psi_{\pi v}\rho_v + \kappa \psi_{yv}\right\} v_t \\
&\quad + \left\{\beta \psi_{\pi a}\rho_a + \kappa \psi_{ya}\right\} \hat{r}_t^n \\
&\quad + \left\{\beta \psi_{\pi u}\rho_u + \kappa \psi_{yu} + 1\right\} u_t
\end{aligned}
$$

Matching coefficients:

$$
\begin{aligned}
\psi_{yv} &= \left\{-\frac{1}{\sigma}\phi_\pi \psi_{\pi v} + \frac{1}{\sigma}\psi_{\pi v}\rho_v + \psi_{yv}\rho_v - \frac{1}{\sigma}\right\} \\
\psi_{ya} &= \left\{\frac{1}{\sigma} - \phi_\pi \frac{1}{\sigma}\psi_{\pi a} + \frac{1}{\sigma}\psi_{\pi a}\rho_a + \psi_{ya}\rho_a\right\} \\
\psi_{yu} &= \left\{\frac{1}{\sigma} - \frac{1}{\sigma}\phi_\pi \psi_{\pi u} + \frac{1}{\sigma}\psi_{\pi u}\rho_u + \psi_{yu}\rho_u\right\}
\end{aligned}
$$

$$
\begin{aligned}
\psi_{\pi v} &= \left\{\beta \psi_{\pi v}\rho_v + \kappa \psi_{yv}\right\} \\
\psi_{\pi a} &= \left\{\beta \psi_{\pi a}\rho_a + \kappa \psi_{ya}\right\} \\
\psi_{\pi u} &= \left\{\beta \psi_{\pi u}\rho_u + \kappa \psi_{yu} + 1\right\}
\end{aligned}
$$

Solving for the coefficients:

$$
\begin{aligned}
\psi_{yv}\left(1 - \rho_v\right) &= \left(-\frac{1}{\sigma}\phi_\pi + \frac{1}{\sigma}\rho_v\right)\psi_{\pi v} - \frac{1}{\sigma} \\
\psi_{ya}\left(1 - \rho_a\right) &= \left(-\phi_\pi \frac{1}{\sigma} + \frac{1}{\sigma}\rho_a\right)\psi_{\pi a} + \frac{1}{\sigma} \\
\psi_{yu}\left(1 - \rho_u\right) &= \left(-\phi_\pi \frac{1}{\sigma} + \frac{1}{\sigma}\rho_u\right)\psi_{\pi u} + \frac{1}{\sigma}
\end{aligned}
$$

$$
\begin{aligned}
\psi_{\pi v}\left(1-\beta\rho_v\right) &= \kappa\psi_{yv} \\
\psi_{\pi a}\left(1-\beta\rho_a\right) &= \kappa\psi_{ya} \\
\psi_{\pi u}\left(1-\beta\rho_u\right) &= \kappa\psi_{yu}+1
\end{aligned}
$$

$$
\begin{aligned}
\psi_{yv}\left(1-\rho_v\right) &= \left(-\frac{1}{\sigma}\phi_\pi+\frac{1}{\sigma}\rho_v\right)\frac{\kappa\psi_{yv}}{\left(1-\beta\rho_v\right)}-\frac{1}{\sigma} \\
\psi_{yv} &= \frac{-\left(1-\beta\rho_v\right)}{\left[\left(1-\beta\rho_v\right)\sigma\left(1-\rho_v\right)+\kappa\left(\phi_\pi-\rho_v\right)\right]}
\end{aligned}
$$

$$
\begin{aligned}
\psi_{ya}\left(1-\rho_a\right) &= \left(-\phi_\pi\frac{1}{\sigma}+\frac{1}{\sigma}\rho_a\right)\psi_{\pi a}+\frac{1}{\sigma} \\
\psi_{ya} &= \frac{\left(1-\beta\rho_a\right)}{\sigma\left(1-\rho_a\right)\left(1-\beta\rho_a\right)+\kappa\left(\phi_\pi-\rho_a\right)}
\end{aligned}
$$

$$
\begin{aligned}
\psi_{yu}\left(1-\rho_u\right) &= \left(-\phi_\pi\frac{1}{\sigma}+\frac{1}{\sigma}\rho_u\right)\psi_{\pi u}+\frac{1}{\sigma} \\
\psi_{yu} &= \frac{\left(1-\beta\rho_u\right)-\left(\phi_\pi-\rho_u\right)}{\sigma\left(1-\rho_u\right)\left(1-\beta\rho_u\right)+\kappa\left(\phi_\pi-\rho_u\right)}
\end{aligned}
$$

$$
\begin{aligned}
\psi_{\pi u} &= \kappa\frac{\frac{\left(1-\beta\rho_u\right)-\left(\phi_\pi-\rho_u\right)}{\left(1-\beta\rho_u\right)}}{\sigma\left(1-\rho_u\right)\left(1-\beta\rho_u\right)+\kappa\left(\phi_\pi-\rho_u\right)}+\frac{1}{\left(1-\beta\rho_u\right)} \\
&= \frac{\kappa+\sigma\left(1-\rho_u\right)}{\left[\sigma\left(1-\rho_u\right)\left(1-\beta\rho_u\right)+\kappa\left(\phi_\pi-\rho_u\right)\right]}
\end{aligned}
$$

This yields:

$$\begin{aligned}
\psi_{yv} &= \frac{-\left(1-\beta\rho_v\right)}{\left[\left(1-\beta\rho_v\right)\sigma\left(1-\rho_v\right)+\kappa\left(\phi_\pi-\rho_v\right)\right]}, \\
\psi_{ya} &= \frac{\left(1-\beta\rho_a\right)}{\sigma\left(1-\rho_a\right)\left(1-\beta\rho_a\right)+\kappa\left(\phi_\pi-\rho_a\right)}, \\
\psi_{yu} &= \frac{\left(1-\beta\rho_u\right)-\left(\phi_\pi-\rho_u\right)}{\sigma\left(1-\rho_u\right)\left(1-\beta\rho_u\right)+\kappa\left(\phi_\pi-\rho_u\right)}, \\
\psi_{\pi v} &= \frac{-\kappa}{\left[\left(1-\beta\rho_v\right)\sigma\left(1-\rho_v\right)+\kappa\left(\phi_\pi-\rho_v\right)\right]}, \\
\psi_{\pi a} &= \frac{\kappa}{\sigma\left(1-\rho_a\right)\left(1-\beta\rho_a\right)+\kappa\left(\phi_\pi-\rho_a\right)}, \\
\psi_{\pi u} &= \frac{\kappa+\sigma\left(1-\rho_u\right)}{\left[\sigma\left(1-\rho_u\right)\left(1-\beta\rho_u\right)+\kappa\left(\phi_\pi-\rho_u\right)\right]},
\end{aligned}$$

Call:

$$\begin{aligned}
\Lambda_v &= \frac{1}{\left(1-\beta\rho_v\right)\sigma\left(1-\rho_v\right)+\kappa\left(\phi_\pi-\rho_v\right)}, \\
\Lambda_a &= \frac{1}{\left(1-\beta\rho_a\right)\left[\sigma\left(1-\rho_a\right)\right]+\kappa\left(\phi_\pi-\rho_a\right)}, \\
\Lambda_u &= \frac{1}{\sigma\left(1-\rho_u\right)\left(1-\beta\rho_u\right)+\kappa\left(\phi_\pi-\rho_u\right)}.
\end{aligned}$$

And we can rewrite:

$$\begin{aligned}
\psi_{yv} &= -\left(1-\beta\rho_v\right)\Lambda_v, \\
\psi_{ya} &= \left(1-\beta\rho_a\right)\Lambda_a, \\
\psi_{yu} &= \left[\left(1-\beta\rho_u\right)-\left(\phi_\pi-\rho_u\right)\right]\Lambda_u, \\
\psi_{\pi v} &= -\kappa\Lambda_v, \\
\psi_{\pi a} &= \kappa\Lambda_a, \\
\psi_{\pi u} &= \left[\kappa+\sigma\left(1-\rho_u\right)\right]\Lambda_u.
\end{aligned}$$

A.6 Variance decomposition:

Our solution equations yielded:

$$\begin{aligned}
\tilde{y}_t &= \psi_{yv} v_t + \psi_{ya} \hat{r}_t^n + \psi_{yu} u_t, \\
\pi_t &= \psi_{\pi v} v_t + \psi_{\pi a} \hat{r}_t^n + \psi_{\pi u} u_t, \\
i_t &= \rho + \phi_\pi \pi_t + v_t
\end{aligned}$$

and $\hat{r}_t^n = -\sigma \psi_{ya}^n (1 - \rho_a) a_t$.

The solution implies:

$$\pi_t = -\kappa \Lambda_v v_t - \sigma \psi_{ya}^n (1 - \rho_a) \kappa \Lambda_a a_t + [\kappa + \sigma (1 - \rho_u)] \Lambda_u u_t$$

$$\begin{aligned}
plim \hat{\phi}_\pi^{OLS} &= \frac{cov(i_t, \pi_t)}{var(\pi_t)} \\
&= \frac{cov(\rho + \phi_\pi \pi_t + v_t, \pi_t)}{var(\pi_t)} \\
&= \phi_\pi + \frac{cov(v_t, \pi_t)}{var(\pi_t)} \\
&= \phi_\pi + \frac{cov\left(v_t, \left\{-\kappa \Lambda_v v_t - \sigma \psi_{ya}^n (1 - \rho_a) \kappa \Lambda_a a_t + [\kappa + \sigma (1 - \rho_u)] \Lambda_u u_t\right\}\right)}{var\left(\left\{-\kappa \Lambda_v v_t - \sigma \psi_{ya}^n (1 - \rho_a) \kappa \Lambda_a a_t + [\kappa + \sigma (1 - \rho_u)] \Lambda_u u_t\right\}\right)} \\
&= \phi_\pi - \frac{\kappa \Lambda_v var(v_t)}{(\kappa \Lambda_v)^2 var(v_t) + \left(\sigma \psi_{ya}^n (1 - \rho_a) \kappa \Lambda_a\right)^2 var(a_t) + ([\kappa + \sigma (1 - \rho_u)] \Lambda_u)^2 var(u_t)}
\end{aligned}$$

$$plim \hat{\phi}_\pi^{OLS} = \phi_\pi - \frac{1}{\kappa \Lambda_v} \lambda_v,$$

where

$$\lambda_v = \frac{(\kappa \Lambda_v)^2 var(v_t)}{(\kappa \Lambda_v)^2 var(v_t) + \left(\sigma \psi_{ya}^n (1 - \rho_a) \kappa \Lambda_a\right)^2 var(a_t) + ([\kappa + \sigma (1 - \rho_u)] \Lambda_u)^2 var(u_t)}$$

is the fraction of the variance of π_t that is accounted for monetary policy shocks.

A.7 Matrix representation

The matrix representation of the system solution yields:

$$\begin{aligned} X_t &= A\Psi_t \\ \Psi_t &= \Xi\Psi_{t-1} + \varepsilon_t \end{aligned}$$

where:

$$\begin{aligned} X_t &= \begin{pmatrix} \tilde{y}_t \\ \pi_t \end{pmatrix},\ A = \begin{pmatrix} \psi_{yv} & -\sigma\psi^n_{ya}\left(1-\rho_a\right)\psi_{ya} & \psi_{yu} \\ \psi_{\pi v} & -\sigma\psi^n_{ya}\left(1-\rho_a\right)\psi_{\pi a} & \psi_{\pi u} \end{pmatrix},\ \Psi_t = \begin{pmatrix} v_t \\ a_t \\ u_t \end{pmatrix}, \\ \Xi &= \begin{pmatrix} \rho_v & 0 & 0 \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_u \end{pmatrix},\ \text{and } \varepsilon_t = \begin{pmatrix} \varepsilon^v_t \\ \varepsilon^a_t \\ \varepsilon^u_t \end{pmatrix}. \end{aligned}$$

$$\begin{aligned} X_t &= A\Psi_t \Rightarrow A'X_{t-1} = A'A\Psi_{t-1} \Rightarrow \Psi_{t-1} = \left(A'A\right)^{-1}A'X_{t-1} \\ X_t &= A\Psi_t = A\left(\Xi\Psi_{t-1} + \varepsilon_t\right) = A\Xi\Psi_{t-1} + A\varepsilon_t \Rightarrow E_{t-1}\left(X_t\right) = A\Xi\left(A'A\right)^{-1}A'X_{t-1}. \end{aligned}$$

B Additional Results

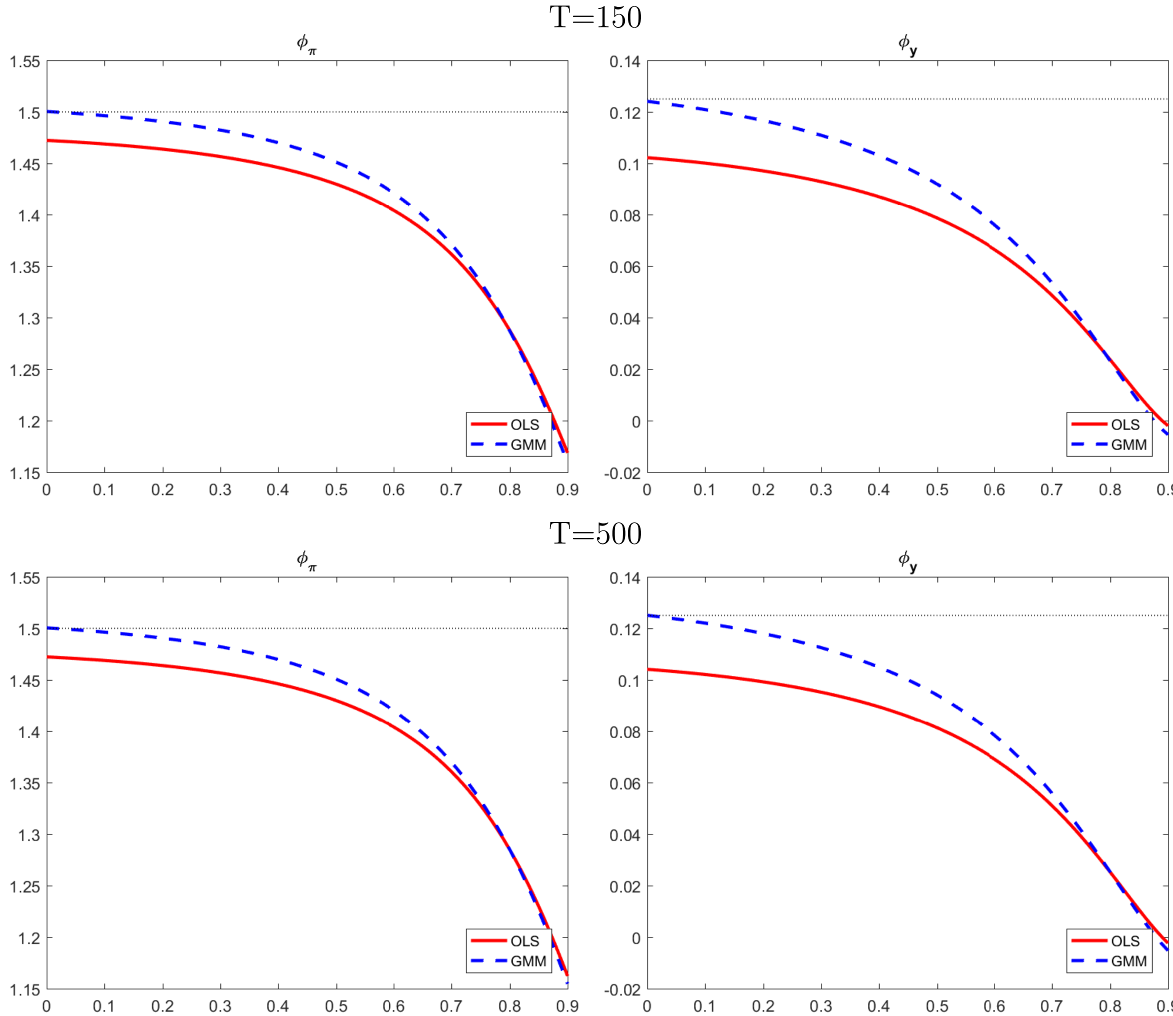

Figure A1: Mean estimates in the three-equation New Keynesian model for ranging levels of monetary shock persistence.

Note: The blue line reports GMM estimates while the red line reports OLS estimates. The black dotted line corresponds to the true parameter value. Distributions are obtained from 1000 Monte Carlo simulations of the model.

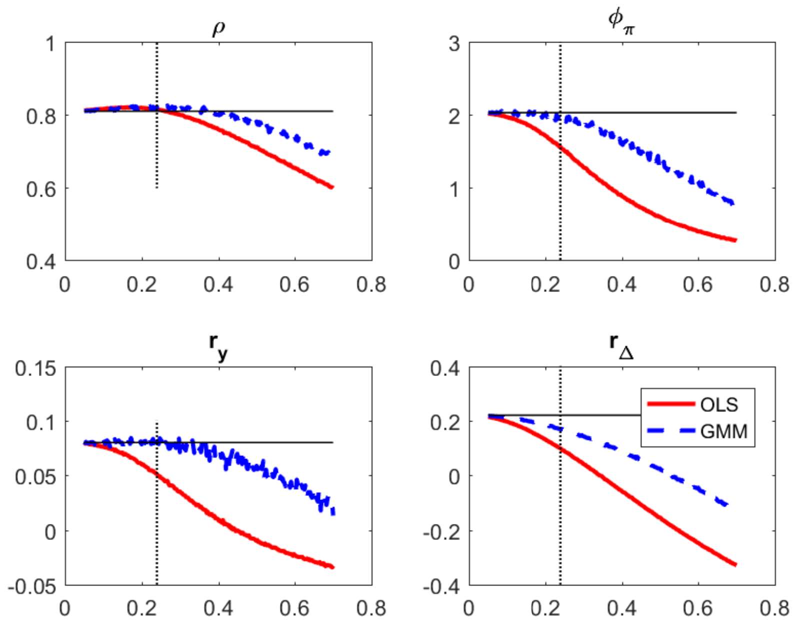

Figure A2: OLS and GMM estimates in a large-scale DSGE model for varying monetary shock volatility.

Note: The blue line reports GMM estimates while the red line reports OLS estimates. The vertical dotted black line corresponds the calibrated standard deviation of the monetary shock, while the horizontal solid black line reports the "true" Taylor rule parameters. Simulations are based on the model and parameterization described in Smets and Wouters (2007) and assume $T = 500$.

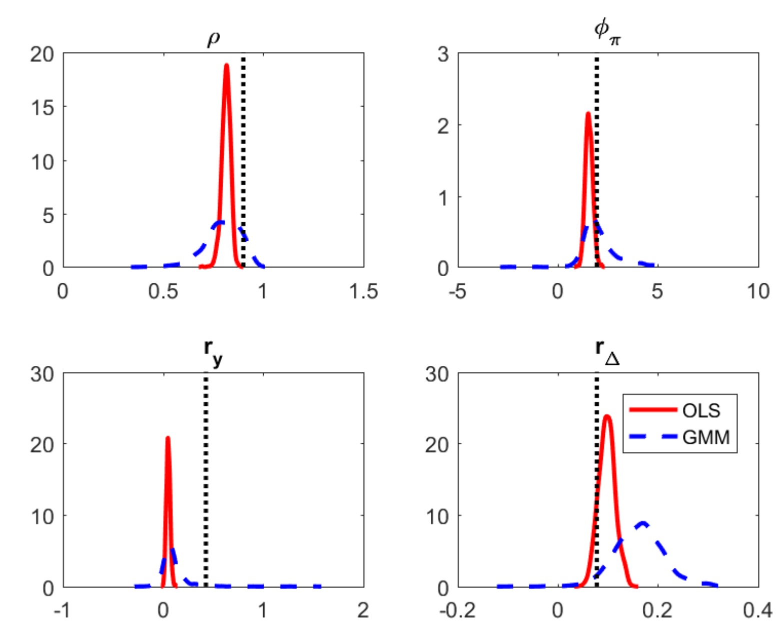

Figure A3: OLS and GMM estimate distributions in a large-scale DSGE model.

Note: The blue line reports GMM estimates while the red line reports OLS estimates. The black dotted line corresponds to the true parameter value. Simulations are based on the model and parameterization described in Smets and Wouters (2007). Distributions are obtained from 1000 Monte Carlo simulations of the model assuming a with sample size $T = 500$.

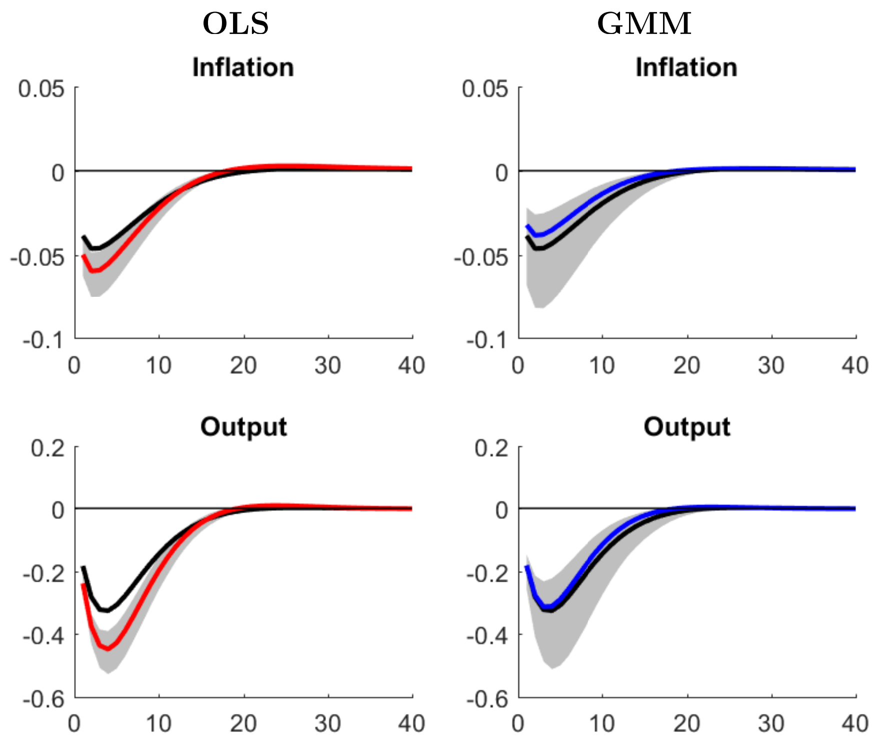

Figure A4: Output and inflation responses to monetary shocks in a large-scale DSGE model with Taylor rule parameters at true versus OLS and GMM estimated values.

Note: The black line shows the estimated responses of the model with parameters at the mode of posterior distributions. The red line shows the response implied by OLS estimates. The blue line reports the response implied by GMM estimates. The shaded areas report, for each point in time, the 5^{th} and 90^{th} percentiles of the distribution of estimated coefficients. Simulations are based on the model and parameterization described in Smets and Wouters (2007) and assume $T = 500$.

Table A1: OLS and GMM mean point estimates in the three-equation model for varying shock persistence

	$\rho_v = 0$						
	True values	OLS	OLS bias	GMM	GMM bias	Relative Bias	Relative MSE
ϕ_π	1.5	1.4665	-0.0335	1.4957	-0.0043	-0.0195	1.0076
ϕ_y	0.125	0.0999	-0.0251	0.1231	-0.0019	-0.1854	1.0081
	$\rho_v = 0.8$						
	True values	OLS	OLS bias	GMM	GMM bias	Relative Bias	Relative MSE
ϕ_π	1.5	1.2822	-0.2178	1.2797	-0.2203	0.0017	0.9987
ϕ_y	0.125	0.0229	-0.1021	0.0221	-0.1029	0.0062	0.9975

Note: The relative bias corresponds to $\frac{|\hat{\beta}^{GMM}-\beta|-|\hat{\beta}^{OLS}-\beta|}{\beta}$, and the relative mean squared error (MSE) equals $\frac{MSE(\hat{\beta}^{GMM})}{MSE(\hat{\beta}^{OLS})}$. A relative bias smaller than zero indicates that GMM outperforms OLS mean estimates. A relative MSE smaller than one indicates GMM is more precise than OLS.

Table A2: OLS and GMM mean point estimates in a large-scale DSGE model for varying sample sizes

	T = 80						
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
ρ	0.81	0.7849	-0.0251	0.7502	-0.0598	0.0427	7.4646
ϕ_π	2.03	1.5084	-0.5216	1.847	-0.183	-0.1668	2.3011
r_y	0.08	0.0576	-0.0224	0.0859	0.0059	-0.2054	6.0016
r_Δ	0.22	0.0924	-0.1276	0.133	-0.087	-0.1846	0.9749
	T = 150						
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
ρ	0.81	0.8027	-0.0073	0.7777	-0.0323	0.0308	9.5275
ϕ_π	2.03	1.5461	-0.4839	1.9784	-0.0516	-0.213	2.6956
r_y	0.08	0.053	-0.027	0.0882	0.0082	-0.2347	8.264
r_Δ	0.22	0.0953	-0.1247	0.1471	-0.0729	-0.2355	0.6741
	T = 500						
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
ρ	0.81	0.8153	0.0053	0.8002	-0.0098	0.0056	20.2849
ϕ_π	2.03	1.5743	-0.4557	2.0682	0.0382	-0.2057	3.0969
r_y	0.08	0.0524	-0.0276	0.1057	0.0257	-0.0242	14.5083
r_Δ	0.22	0.0984	-0.1216	0.1626	-0.0574	-0.2917	0.3608

Note: The relative bias corresponds to $\frac{|\hat{\beta}^{GMM}-\beta|-|\hat{\beta}^{OLS}-\beta|}{\beta}$, and the relative mean squared error (MSE) equals $\frac{MSE(\hat{\beta}^{GMM})}{MSE(\hat{\beta}^{OLS})}$. A relative bias smaller than zero indicates that GMM outperforms OLS mean estimates. A relative MSE smaller than one indicates GMM is more precise than OLS.

Table A3: Empirical IV and OLS nonlinear estimates

	IV estimates			
	(1)	(2)	(3)	(4)
	Pre-Volcker	Volcker-Greenspan	Greenspan-Bernanke	Post-Volcker
	1960Q1-1979Q2	1979Q3-2005Q4	1987Q3-2007Q4	1979Q3-2007Q4
β	0.90***	2.02***	1.87***	1.93***
	(0.14)	(0.25)	(0.34)	(0.26)
γ	0.50***	0.98***	1.01***	0.97***
	(0.14)	(0.13)	(0.19)	(0.13)
ρ_1	1.15***	0.65***	1.44***	0.68***
	(0.09)	(0.09)	(0.06)	(0.10)
ρ_2	-0.37***	0.09	-0.55***	0.07
	(0.09)	(0.08)	(0.06)	(0.09)
π^*	5.52	3.08***	1.94***	2.93***
	(3.97)	(0.22)	(0.32)	(0.23)
N	74	106	82	114
	OLS estimates			
	(1)	(2)	(3)	(4)
	Pre-Volcker	Volcker-Greenspan	Greenspan-Bernanke	Post-Volcker
	1960Q1-1979Q2	1979Q3-2005Q4	1987Q3-2007Q4	1979Q3-2007Q4
β	0.89***	2.24***	1.74***	2.23***
	(0.20)	(0.36)	(0.39)	(0.36)
γ	0.58***	0.99***	0.94***	0.99***
	(0.16)	(0.18)	(0.20)	(0.18)
ρ_1	1.11***	0.44**	1.41***	0.44**
	(0.18)	(0.16)	(0.11)	(0.16)
ρ_2	-0.37*	0.23	-0.54***	0.23
	(0.15)	(0.17)	(0.10)	(0.17)
π^*	3.60	2.72***	1.94***	2.64***
	(3.05)	(0.18)	(0.38)	(0.16)
N	74	106	82	114

Note: The table reports estimates of equation (10) by OLS and IV. The set of instruments includes four lags of inflation, output gap, the fed funds rate, money growth (M2), the spread between long and short term bond rates, and commodity price inflation. Statistical significance at the 90/95/99% confidence level indicated with $^{*/**/***}$, respectively. Robust standard errors are reported in parenthesis.