Efficiency in Sequential Labor and Goods Markets

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Abstract

This paper studies the optimal sharing of value added between consumers, producers, and labor. We first define a constrained optimum. We then compare it with the decentralized allocation. They coincide when the price maximizes the expected marginal revenue of the firm in the goods market, an outcome of the competitive search equilibrium, and when the wage exactly offsets the congestion externality of firm entry in the labor market, which is the traditional Hosios condition. Under price and wage bargaining, this allocation is achieved under a double Hosios condition combining the logic of competitive search and Hosios efficiency. The consumer receives a share of the goods market trading surplus equal to the amount of externality occasioned by its search activity and the worker receives a share of the labor match surplus to offset the externality of firm entry in the matching process. A calibration of the model to the US economy indicates that the labor market is near efficient, and free-entry of consumers leads to excess consumer market power in setting prices. Restoring efficiency leads to a modest change in welfare.

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1 Introduction

In competitive markets products are sold at marginal cost, the consumption surplus is driven to zero, labor is paid its marginal product and factors are fully utilized. However modern economies are frictional, experience persistent unemployment, production does not take place at full capacity, wages are below marginal products and firms price goods at a markup. We study here the optimal sharing of value added between consumers, producers, and labor when markets are frictional and participants have market power. The trading frictions take the form of search in labor and goods market, which lead to the existence of positive surplus to transactions. Market power in bargaining over the rents from trade determine whether a good is sold near its marginal cost or closer to its marginal utility, and whether labor is paid its marginal product or nearer its reservation wage.

Production in the model requires a worker, and once a consumer is found the product is sold until either the consumer changes tastes or the producer loses its worker. In the labor market, firms search for workers expecting a certain profit from selling goods and paying wages. In the goods market, consumers decide how actively to search for products to add to their consumption basket depending the ease of finding a product and the consumption surplus after paying the price of the good. In a decentralized allocation labor and goods market interact in general equilibrium. In particular, the payoff to entry in the labor market is a function of the equilibrium in the goods market. As such, an increase in the ability of a firm of extract a larger share of the consumption surplus, or its market power, has two, opposing, effects on the equilibrium in the labor market. First, firms obtain a higher price. This increases the marginal revenue in the goods market and the incentive to hire labor. Second, the higher price dissuades consumers from actively searching in the goods market and increases the difficulty for all firms to find new customers. This reduces the expected revenue in the goods market and reduces the incentives to create jobs. There thus exists a level of market power in the goods market that maximizes job creation and tightness in the labor market.

The constrained efficient allocation balances trading externalities of search frictions in labor and goods markets: the congestion of firms entering the labor market, and that of consumers searching in the goods market. This second externality reduces the economic surplus of the labor match, a general equilibrium feedback absent in standard models of equilibrium unemployment. Constrained efficiency in the goods market also leads to the highest possible surplus to a match in the labor market. This allocation in the decentralized economy is achieved for once price that maximizes a firm’s expected goods market marginal revenue. In the labor market, the is one wage such to achive the socially efficient job creation which balanace turnover cost of job creation, and the second best allocation is achieve when both the price in the goods market and wage in the labor market internalize the congestion in the meeting rates from entry that agents do not take in account when making their private decisions.

The efficiency conditions in each market are exactly to give a share of the surplus equal to the congestion externality caused by the other side of the market’s entry. In the goods market, constrained efficiency is achieved if the share of the consumption surplus going the consumer is exactly equal to the elasticity of goods market matching with respect to consumer search. In the labor market, constrained efficiency is achieved if the share of the surplus going to labor is
equal to the elasticity of matching with respect to unemployment, and, importantly, if constrained efficiency in the goods market holds. That is, one recovers a standard but double Hosios condition in each market where efficiency in the labor market requires having efficiency in the goods market as well.

The efficient price and wages are shown to follow simple linear rules in market tightness. If a price setting mechanism in a decentralization can conform to these rules under parameter restrictions, that the decentralized allocation can be constrained efficient. Our benchmark environment assumes the bargained wage follows that same rule when the firm is currently searching for a customer or selling its product. This changes, under sequential bargaining, with or without strategic interactions between markets, where wage setters internalize their impact on price determination. In this setting we obtain that the bargaining powers that achieves efficiency in one market are not independent of the market power and the equilibrium in the other market. It remains that, in the labor market, constrained efficiency is achieved if the share of the surplus going to labor is equal to the elasticity of matching with respect to unemployment, and if constrained efficiency in the goods market holds. In the goods market, the share of the surplus to the consumer to ensure constrained efficiency depends both on the elasticity of goods market matching to consumer search and the division of rents in the labor market as part of the cost of producing - the wage - is passed on to the consumer.

Four regimes, characterized by markets being either too slack or too tight relative to the constrained efficient, arise as we move away from the Hosios conditions in the goods and labor markets. Whenever consumers receive an inefficiently low share of the consumption surplus, the goods market is too slack. Whenever workers receive a share of the surplus above the elasticity of the matching function, the labor market is too slack. However, below this share in the labor market the decentralized allocation may have too little or too much job creation, contrary to the standard model of equilibrium unemployment in which there is unambiguously too much job creation. Moving away from the constrained efficient tightness of the goods market reduces the economic surplus to forming a labor match. There is thus less firm entry in the labor market for any given degree of rent sharing with labor as we depart from Hosios in the goods market. As such, the constrained efficient tightness of the labor market can be achieve away from Hosios in the goods market by giving a larger share of the labor match surplus to the firm.

The mapping of tightness regimes to prices and wages, however, is not direct. That is, for instance, equilibria with prices in the goods market above those desired by a social planner can be associated with either too much of too little job creation in the labor market depending on the share of the surplus accruing to labor in wage setting. Similarly, equilibria with excessive job creation can exist along side a goods market in which prices are either too high or too low depending of the degree of consumer market power in price setting. While wages are always too low in equilibria with excessive job creation, equilibria with insufficient job creation can occur with both wages that are too high and wages that are too low. This depends on the distance away from Hosios in the goods market, when the distortion in the goods market has sufficiently reduced the surplus to any match in the labor market.

[UPDATE Numbers and welfare]Taking benchmark environment to the US data, calibrating to
a set of moments in the goods and labor markets, we find that labor’s share in bargaining is too low, and consumers have too much goods market power. Labor receives 43 percent of the labor match surplus and consumers receive 30 percent of the consumption match surplus. Both share are below the corresponding market matching elasticities. This compares to constrained efficient shares of 50 and 13 percent, respectively. Restoring efficiency would lower markups in the goods market, by 2 percentage points, and lead to an increase in the long run rate of unemployment of almost 1 percentage point.

A recent strand of research has revived the original ideas of the search literature where consumers needed to search in the goods market in order to consume (e.g., Diamond, 1971, 1982). An early attempt to integrate goods and labor frictions with money is in Shi [1998]. Trading frictions in product markets studied in new monetarist models (see Lagos et al. 2017 for a review), usually in conjunction with credit market frictions, establish a similar mechanism through which product market frictions affect the marginal revenue of firms and the equilibrium in the labor market. In Berentsen et al. [2011] it is inflation, reducing the real income of consumers, that affects the real demand faced by firms, their mark ups, and demand for labor.1 More recently, Bethune et al. [2016] study retail trade markets with trading frictions in a mixture of directed and random search in a New Monetarist framework. The model, in which there is a role for credit captures observations of dispersion in price and quality, as well as markups. They study the effects of changing inflation and credit conditions in the equilibrium allocations.

Our results can be related to the recent work of Mangin and Julien [2018] who describe a generalized Hosios condition in a variety of environments. They show a dual condition on the classical congestion externality and what they term the output externality, which is when the surplus of a match depends on entry by at least one of the agents. In our environment, a related externality arises from the interaction of multiple markets in general equilibrium. Efficiency in the labor market in our environment requires efficiency in the goods market as the labor match surplus is a function of the tightness in the goods market.

In this latter sense our work relates to the model of Blanchard and Giavazzi [2003] in which the rents available to workers when bargaining wages with firms depend on the size of the rents in the goods market. In their environment goods market rents arise from monopolistic competition, and the effect of moving towards competitive pricing is a monotonic decrease in unemployment. This is in contrast with our environment in which the efficient price in the goods market is in between these two extremes, and is a price that leads to the lowest rate of unemployment.

Petrosky-Nadeau and Wasmer [2015] study the business cycle implication of extending the search model of equilibrium unemployment to a search frictional product market. Their main finding is an added factor of amplification and persistence of shocks stemming from the feedback from product to labor markets. Bai et al. [2011], in a directed search environment explore the implications for the measurement of aggregate productivity. Goods market frictions lead to under utilized capacity and a time varying wedge between aggregate productivity and the Solow residual. Similar findings arise in Michaillat and Saez [2014] who argue that changes in goods market frictions must account for a large share of fluctuations in the labor market.

1See also Branch et al. [2014].
The rest of the paper is organized as follows. Section 2 sets out the environment, the trading frictions agents face in the labor and goods markets and solves for the constrained efficient allocation. Section 3 studies the decentralized equilibrium and section 4 derives the constrained efficiency conditions in the labor and goods market under price and wage bargaining. Section 5 takes the model to the data. Section 6 investigates the role of alternative bargaining assumptions.

2 Efficiency in an economy with frictions in goods and labor markets

2.1 Environment

Matching and separation

In the labor market, workers are either unemployed or employed. Let 1 be the total labor force, and \( U \) is the number of unemployed workers. Firms post a number \( V \) of vacancies. These two inputs combine through an increasing and constant returns to scale function \( M_L(V,U) \) into a number of matches in the labor market. Workers separate from these employment relations at constant rate \( s^L \). The law of motion of unemployment is therefore:

\[
\dot{U} = s^L (1 - U) - M_L(U,V)
\]  

In the goods market, consumers supply \( D_U \) search units to find products to add to a stock comprising their current consumption bundle \( D_M \). The number of firms searching for consumers, \( \mathcal{N}_G \), is predetermined by past transitions. These two inputs combine into a number of matches in the goods market through an increasing and constant returns to scale function \( M_G(D_U, \mathcal{N}_G) \). This flow adds to the number of matched firms in the goods market, \( \mathcal{N}_\pi \), and the number of matched consumers \( D_M \). The measure \( D_U \) can be thought of as unmatched consumption demand, while \( D_M \) as matched demand. A match with a seller continues until one of two events occurs. First, a consumer can quit a particular consumption relationship. This arrives at a constant rate \( s^G \). Second, consumer-firm relationships are also terminated when the producer, i.e., the firm, loses its worker. This occurs at rate \( s^L \), and we denote the sum both separation rates by \( s = s^G + s^L \). Therefore, the laws of motions in the goods market are:

\[
\begin{align*}
D_M &= M_G(D_U, \mathcal{N}_G) - s D_M \quad (2) \\
\mathcal{N}_\pi &= M_G(D_U, \mathcal{N}_G) - s \mathcal{N}_\pi \quad (3) \\
\mathcal{N}_G &= M_L(U,V) + s^G \mathcal{N}_\pi - M_G(D_U, \mathcal{N}_G) - s^L \mathcal{N}_G \quad (4)
\end{align*}
\]

The inflows into the mass of firms searching in the goods market, in equation (4), stem from two sources. First there is the inflow of new hires in the labor market. Second, the inflow from firms that have lost a consumer, \( s^G \mathcal{N}_\pi \). Likewise, there are two sources of outflows: new matches with consumers in the goods market, and separation from workers \( (s^L \mathcal{N}_G) \).
Identities and tightness in goods and labor markets

The total number of filled jobs corresponds to the total number of employed workers, that is:

$$1 - \mathcal{U} = \mathcal{N}_G + \mathcal{N}_\pi$$ (5)

The total number of consumers matched with a producer of the search good corresponds to the number of firms supplying to consumers, namely that:

$$\mathcal{D}_M = \mathcal{N}_\pi$$ (6)

The combination of (5) and (6) delivers a constraint that the number of unemployment workers, matched consumers and employed workers at firms without a goods market match sum to one:

$$1 = \mathcal{U} + \mathcal{D}_M + \mathcal{N}_G$$ (7)

Tightness in both markets is defined from the perspective of the buyer: $\theta = \mathcal{V}/\mathcal{U}$ in the labor market and $\xi = \mathcal{D}_U/\mathcal{N}_G$ in the goods market. Transitions are quicker in a tight market for the seller. The job finding rate $f(\theta) = M_L/\mathcal{U}$ is increasing labor market tightness. Greater tightness $\xi$ in the goods market reflects more consumer search for a good relative to number of firms searching for consumers. The firm’s meeting rate in the goods market $\lambda(\xi) = M_G/\mathcal{N}_G$ is increasing in $\xi$, while the consumer’s meeting rate $\psi(\xi) = M_G/\mathcal{D}_U$ is decreasing in $\xi$. The elasticity of matching in the labor market with respect to the unemployed is denoted $\eta_L(\theta)$, while the elasticity of matching in the goods market with respect to consumer demand $\mathcal{D}_U$ is denote by $\eta_G(\xi)$. This is summarized below:

<table>
<thead>
<tr>
<th>Labor market matching frictions:</th>
<th>Goods market matching frictions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = \mathcal{V}/\mathcal{U}$</td>
<td>$\xi = \mathcal{D}_U/\mathcal{N}_G$</td>
</tr>
<tr>
<td>$f(\theta) = M_L/\mathcal{U}$ with $f' &gt; 0$</td>
<td>$\psi(\xi) = M_G/\mathcal{D}_U = \lambda(\xi)/\xi$ with $\psi' &lt; 0$</td>
</tr>
<tr>
<td>$q(\theta) = M_L/\mathcal{V}$ with $q' &lt; 0$</td>
<td>$\lambda(\xi) = M_G/\mathcal{N}_G$ with $\lambda' &gt; 0$</td>
</tr>
<tr>
<td>$\eta_L(\theta) = -\theta q'(\theta)/q(\theta) \in (0,1)$</td>
<td>$\eta_G(\xi) = \xi \lambda'(\xi)/\lambda(\xi) \in (0,1)$</td>
</tr>
</tbody>
</table>

2.2 Social planner’s problem

Each job produces 1 unit of the search good, sold at at unit price $\mathcal{P}$ in the decentralized economy. The good is valued by the consumer at the margin by $\Phi > 0$. There is a flow cost to searching in the goods market $\sigma > 0$. The non-employed have flow utility $z > 0$, and there is a flow cost to job vacancies $\gamma > 0$. The social planner balances the utility of consumption and these costs of production. It seeks to maximize the value of consumption of the search good and non-employment utility, net of labor and goods market search costs:

$$\Omega = \int_0^\infty e^{-rt} [D_M\Phi + z\mathcal{U} - \gamma\mathcal{V} - \sigma\mathcal{D}_U] \, dt$$

5
Equations (1) to (4) are dynamic frictional constraints. They apply to both the decentralized equilibrium and the social planner’s program. Equations (5) and (6) are also common to the decentralized equilibrium and the social planner’s program. Using these constraints, and in particular constraint (7) to replace \( N_G \) by a function of \( U \) and \( D_M \), we have:

\[
\Omega = \max_{D_U, V, U, D_M} e^{-rt} \left[ D_M \Phi + zU - \gamma V - \sigma D_U \right] + \Psi_U \left[ M_L(U, V) - s^L (1 - U) \right] + \Psi_{DM} \left[ (s^G + s^L) D_M - M_G(D_U, 1 - U - D_M) \right]
\]

where the planner chooses consumer search units \( D_U \) and job vacancies \( V \), subject to the laws of motion of the co-states of unemployment \( U \) and matched consumers \( D_M \). The associated multipliers are \( \Psi_U \) and \( \Psi_{DM} \), respectively.

The social planner’s allocation is a pair of efficient goods and labor market tightness \((\bar{\xi}_{o p t}, \bar{\theta}_{o p t})\) that solve the pair of conditions for optimal consumer entry and optimal labor market entry, denoted by \( CE^{OPT} \) and \( LE^{OPT} \) respectively:\(^2\)

\[
CE^{OPT}: \quad (r + s) \frac{\sigma}{\psi(\bar{\xi})} = \eta_G \Phi - (1 - \eta_G) \sigma \bar{\xi}
\]

\[
LE^{OPT}: \quad (r + s^L) \frac{\gamma}{q(\bar{\theta})} = (1 - \eta_L) \left[ \Phi - z - \frac{\sigma (r + s)}{\psi(\bar{\xi})} \frac{1}{\eta_G} \right] - \eta_L \gamma \bar{\theta}
\]

The constrained efficient level of tightness in the goods market \( \bar{\xi}_{o p t} \) is uniquely determined by equation (8). It is independent of equilibrium labor market tightness. It is set at a level equating the costs of forming a consumption match, to the social benefit of the marginal consumption match. The latter allows for an optimal balancing of the benefits of consumptions (with weight \( \eta_G \)) and the opportunity costs of searching the goods for consumers (with the complement weight). The condition is represented as the vertical line in \((\bar{\xi}, \bar{\theta})\) space of Figure 1.

\(^2\)See appendix A for details of the derivations.
The equilibrium condition (9) describes a decreasing relation between labor and goods market tightness. It is represented as the decreasing solid curve in Figure 1. The equation is similar to that obtained in model of equilibrium unemployment with perfect product markets, e.g. in Pissarides [2000]. It is identical when goods market frictions disappear, when either \( \sigma = 0 \) or \( \psi(\xi) \) tends to infinity. The tightness in the labor market achieve under perfect goods markets, when \( \sigma / \psi \) tends toward zero, denoted \( \bar{\theta} \) is represented as the horizontal dashed line in Figure 1. Goods market frictions unambiguously reduce labor market tightness relative to what the social planner could achieve in their absence. Forming a match in the goods market is costly, reducing the net surplus to creating employment.

3 Decentralized Equilibrium

3.1 Value functions of agents: consumer, sellers and workers

In the goods market, consumers set the privately optimal level of search units \( D_U \) in order to exhaust all consumption opportunities given a cost of search in the goods market \( \sigma \). The consumer Bellman equations in a steady-state are

\[
\begin{align*}
    rW_{D_U} &= -\sigma + \psi (W_{D_M} - W_{D_U}) \\
    rW_{D_M} &= (\Phi - P) + s^G (W_{D_U} - W_{D_M}) + s^L (W_{D_U} - W_{D_M})
\end{align*}
\]

where \( W_{D_M} \) and \( W_{D_U} \) denote the value functions of matched and searching consumers, respectively. Consumers find goods at rate \( \psi \) and enjoy the consumption surplus \( (W_{D_M} - W_{D_U}) \) in equation (10). The stream of net flow utility from consuming the search good \( (\Phi - P) \) ends following either a change in tastes, which arrives at rate \( s^G \), or following a labor turnover event at the firm selling the good. Indeed, the last term \( s^L (W_{D_U} - W_{D_M}) \) captures the fact that if the firm from which the good is purchased loses its worker there is an interruption the flow of surplus as the good is no longer produced.

Implicitly, any resource not spent on the search good is used as a numeraire and spent into search activities.\(^3\) We do not report the income of the consumer as it scales up both value functions in the same way, dropping out of the consumption surplus. A discussion of this point can be found in (Petrosky-Nadeau and Wasmer, 2017, Chapter 7).\(^4\)

Denote the wage paid to labor when the firm is searching in the goods market by \( w_g \), and by

\(^3\)Alternatively, it can be spent on a perfectly supplied numeraire good with no incidence on the results.

\(^4\)In this previous work, we had studied the case where both the number of consumers and of searching firms is predetermined. The number of consumers was assumed to be equal to the number of employees (the unemployed could not consume the search good) and therefore the tightness in the goods market was determined by the identities above and the stock flow equations, independently of the optimal behavior of consumers and firms. Here, we study the case where one of the two inputs in \( M_G \) is endogenous to workers decisions, namely the consumer side. The symmetric assumption \( (D_U \) predetermined and \( N_g \) as a control variable of the firm) could also be studied, as well as both variables as control variables of the agents.
when the firm is selling its product. The firm’s Bellman equations are:

\[ r_{Jg} = -w_g + \lambda (J\pi - J_g) + s^L (J_v - J_g) \]  
\[ r_{J\pi} = \mathcal{P} - w_{\pi} + s^G (J_g - J\pi) + s^L (J_v - J\pi) \]  

(12)

(13)

The firm matches with a consumer at rate \( \lambda \), and gains the goods market surplus \((J\pi - J_g)\). However, it faces the risk of a labor turnover shock in both stages \( g \) and \( \pi \), and a loss \((J_v - J\pi)\) and \((J_v - J_g)\), respectively. The profit flow \( \mathcal{P} - w_{\pi} \) from selling the good may also be interrupted following a loss of the consumer, at rate \( s^G \). In this event the worker remains with the firm and the capital loss is \((J_g - J\pi)\).

In the labor market, the value of a vacancy to a firm, which is the sum of an outflow cost \( \gamma \) and a capital gain \( J_g - J_v \) following a match with a worker at rate \( q(\theta) \), can be written as:

\[ r_{Jv} = -\gamma + q(\theta) (J_g - J_v) \]  

(14)

The Bellman equations for workers distinguish employment at a firm that is matched with a consumer and selling its goods, and one that is not. The respective asset values \( W_{e\pi} \) and \( W_{eg} \) are:

\[ rW_{e\pi} = w_{\pi} + s^L (W_u - W_{e\pi}) + s^G (W_{eg} - W_{e\pi}) \]  
\[ rW_{eg} = w_g + s^L (W_u - W_{eg}) + \lambda (W_{e\pi} - W_{eg}) \]  
\[ rW_u = z + f(\theta) (W_{eg} - W_u) \]  

(15)

(16)

(17)

while \( W_u \) is the asset value of an unemployed worker with flow utility \( z \) and job finding rate \( f(\theta) \).

3.2 Entry in the market for goods and labor

We are now in a position to see how the market can achieve the second best efficient allocation described above. This will depend on how prices and wages are determined by agents.

Goods market entry

Free entry of consumer search in the goods market leads to \( W_{Du} = 0 \) in equilibrium. Applying this to equations (10) and (11) we have a goods market entry condition:

\[ \frac{\sigma}{\psi(\xi)} = \frac{\Phi - \mathcal{P}}{r + s} \]  

(18)

under which entry occurs until the average cost of finding a search good, \( \sigma/\psi \), equals the consumer’s consumption surplus \( W_{Du} - W_{Du} = (\Phi - \mathcal{P})/(r + s) \), which is simply the discounted present value of the net utility flow from consumption \((\Phi - \mathcal{P})\). The entry condition describes a negative relation between the price and tightness of the goods market (see Figure ??). The lower the price, the more consumers expend search effort \( D_{Du} \), leading to a tighter market. An equilibrium with positive goods market tightness can exist as long as the price is below the marginal utility \( \Phi \).
Consumer entry decisions, in response to the price, determine the incentives for firms to enter the labor market in order to produce. The marginal revenue from selling in the goods market, from the perspective of an entering firm, is a function of the price and tightness of the goods market, and we denote it by $\pi(\xi, P)$ with

$$\pi(\xi, P) = \mu(\xi) \times P$$

It is the product of the price $P$ at which the good will be sold and a discounting factor $\mu(\xi)$ that reflects the time after entry it will take to find a customer and be able to sell. This discount factor is defined as:

$$\mu(\xi) = \frac{\lambda(\xi)}{r + s + \lambda(\xi)} \in (0, 1)$$

and discounts for the firm’s the likelihood of finding ($\lambda(\xi)$) and retaining ($s$) its demand in the goods market. The factor $\mu(\xi)$ is declining in the price $P$ as goods market tightness, and hence $\lambda(\xi)$, is declining in $P$. As a result, the expected revenue from selling the good $\pi(\xi, P)$ is increasing and concave in the price, equal to 0 when $P = 0$, and also when $P = \Phi$ as such a price results in $\lambda(0) = 0$ (see Figure 2b). This factor may also be interpreted as a discounted rate of capacity utilization, an interpretation also found in Petrosky-Nadeau and Wasmer [2017], that converges to 1 as goods market frictions disappear.

**Labor market entry and job creation**

Firms are assumed to freely enter the labor market, resulting in an equilibrium with $J_v = 0$ and, combining (12), (13), and (14), we have:

$$\frac{\gamma}{q(\theta)} = \frac{\pi(\xi, P) - \omega}{r + s L}$$

where $\omega = \frac{\lambda w + (r+s)w}{r + s + \lambda}$ is an expected wage payment from the perspective of a firm entering the labor market. This equation is a generalization of the classical entry equation in the labor market, the job creation condition, now that firms anticipate the effects of goods market frictions on the marginal revenue $\pi(\xi, P)$. An equilibrium with positive labor market tightness can exist if $\pi(\xi, P) > \omega$. As seen above, $\pi(\xi, P)$ is increasing and concave in the price, directly through the price of the good $P$ and indirectly through unused capacity $\mu(\xi)$. As such, there is an increasing and concave relation between the price and the tightness of the labor market for a given wage $\omega$, a function $\theta(P; \omega)$ with $\partial \theta(P; \omega) / \partial P > 0$ and $\partial^2 \theta(P; \omega) / \partial P^2 < 0$. The function is drawn in Figure 2c. It crosses 0 when there is no firm entry at $\omega = \mu(\xi)P$. This occurs for two price levels: a low price level with a high likelihood of finding a consumer, and a high price level and a slack goods market in which it is difficult to find a consumer (see again 2b).

Finally, and this is an important property: there exists a price $P^\theta$ that maximizes the expected revenue $\pi(\xi, P)$, and thus labor market tightness, for any given expected wage at entry $\omega$. This results is the consequence of the free-entry of consumers. Their demand adjusts to price changes. Raising the price reduces their numbers, and affects adversely the firms. Reducing the price reduces profit per consumer but raises the number of consumers, benefiting firms. At the optimum, firms...
Figure 2: Prices, expected revenue at entry and goods and labor market tightness
maximize their profits and attract the optimal number of consumers. This is the logic of wage posting in the labor market, and in particular the result of competitive search equilibrium inspired by the work of Moen [1997]: when firms can create a new labor market segment and attract as many workers as they can with the appropriate wage policy, firm’s profit maximization leads to the constrained optimum.

### Set of equilibria with positive goods and labor market tightness

As long as the price $\mathcal{P}$ is below the marginal utility $\Phi$ there can exist an equilibrium with positive goods market tightness $\xi$. In the labor market, the worker’s reservation wage $z$ places a lower bound on $\omega$ for an equilibrium with positive labor market tightness. It also defines the lowest and highest price for an equilibrium with $\theta > 0$ as $z/\mu(\mathcal{P})$. The largest feasible market tightness $\theta$ is defined by the set of wages and prices where $\omega$ is included in between $z$ and $\omega = \pi(\mathcal{P})$, and $\mathcal{P}$ is within the two bounds defined by $z/\mu(\mathcal{P})$. The firm entry equation (20) expressed as the function $\omega = \pi(\xi, \mathcal{P}) - (r + s^L) \gamma/q(\theta)$ is an iso-labor market tightness condition in wage and price space.

The set of equilibrium with positive labor and goods market tightness is drawn as the shaded area under the curve of this iso-labor market tightness function evaluated at $\theta = 0$ in Figure 3. Achieving greater equilibrium labor market tightness $\theta$ shifts the curve down to a lower wage at all price levels.

### 3.3 Constrained efficient prices and wages in the market for goods and labor

We can now determine the price and the wage that would lead the decentralized allocation to be constrained efficient, before investigating under which price and wage determination mechanism such price would arise in equilibrium.
There is a unique price for which the decentralized goods market entry condition (18) results in the constrained efficient tightness of the goods market $\xi^{opt}$. It is the price at which the consumer’s consumption surplus $(\Phi - P) / (r + s)$ equals the marginal social benefit of a goods market match, the right hand side of the planners goods market condition $(\eta_G \Phi - (1 - \eta_G)) / (r + s)$. Denote this price $P^{opt}$. When $P > P^{opt}$ there is excess consumer entry and tightness in the goods market. When $P < P^{opt}$ the good market is too slack with insufficient consumer entry.

By combining the entry condition (18) with the social planner condition for goods market tightness (8), we find that if the price follows the rule (plotted in Figure 2a)

$$P = (1 - \eta_G) (\Phi + \sigma \xi)$$

then the decentralized equilibrium tightness of the goods market $\xi^*$ is the constrained efficient $\xi^{opt}$. This price rule implementing constrained efficiency in the goods market does not depend on equilibrium in the labor market.

Likewise, in the labor market, for a given price $P$ there exist a wage $\omega^{opt}(P)$ such that the decentralized tightness of the labor market equals the constrained efficient $\theta^{opt}$. From the labor market entry condition (20) this wage follows $\omega^{opt}(P) = \pi(\xi, P) - (r + s_L) \gamma / q(\theta^{opt})$. If the prevailing wage firms expect to pay $\omega < \omega^{opt}$, there will excess firm entry and tightness in the labor market. If $\omega > \omega^{opt}$ there is too little firm entry and the labor market is too slack. Moreover, by combining the labor market entry condition (20) and the social planner’s condition for labor market tightness (9), the decentralized allocation equals the constrained efficient if the expected wage payment $\omega$ satisfies the condition:

$$\omega^{opt}(P^{opt}) = (1 - \eta_L) z + \eta_L [\gamma \theta + \mu(\xi^{opt}) P^{opt}]$$

### 3.4 Slackness regimes

Any decentralized equilibrium with a price below $P^{opt}$ induces excess consumer entry and a goods market that is too tight, or $\xi^* > \xi^{opt}$, and the reciprocal is true. This corresponds to all regions with the right of the vertical solid line in Figure 3 at $P^{opt}$. Likewise, the labor market entry condition (20) describes pairs of price and wage for a given equilibrium market tightness. The increasing and concave curve in Figure 3 describes the combinations of price and wage that achieve the constrained efficient tightness of the labor market. That is, $\theta^*$ can be constrained efficient away from the optimal price in the goods market if the wage paid to labor is below $\omega^{opt}(P^{opt})$. Any deviation from the efficient tightness in the goods market results in a lower expect revenue $\mu P$. The firm must thus be compensated with a lower wage in order to induce the same amount of entry and job creation in the labor market.

### 4 Market power, market tightness and efficiency

We now study allocations where prices in the goods an labor market are determined through bargaining, and whether restrictions can be place on bargaining power to achieve a constrained
efficient allocation. Each consumer-producer and worker-firm pair engage in bilateral Nash Bargaining. The share of the match surplus a particular party obtains in bargaining is its market power, and we relate market power to the efficiency of the resulting decentralized allocations. In our baseline prices and wages are bargained independently, and the wage is bargained to apply both when the firm is searching and matched in the goods market.

There are, however, several alternatives that take into account the sequential nature of meetings and strategic interactions. One could allow, in particular, for different outside options in the negotiation, and one could consider various strategic interactions in bargaining across different matches the labor and the goods market. That is, wages could be negotiated in each stage separately, and could take into account future effects on prices in the goods market. Several such extensions are studied in section 6.

4.1 Price setting in the goods market

The price in the goods market $P$ divides the total consumption surplus, $(W_{Dm} - W_{Du}) + (J_\pi - J_g)$, by solving the Nash problem:

$$P = \arg\max (W_{Dm} - W_{Du})^{\alpha_G} (J_\pi - J_g)^{1-\alpha_G}$$

where where $\alpha_G \in (0, 1)$ is the bargaining strength of a consumer. The slope of $W_{Dm}$ in equation (11) and the slope of $J_\pi$ in equation (13) are opposite but identical in absolute value. Therefore the price must satisfy the sharing rule $(1 - \alpha_G) (W_{Dm} - W_{Du}) = \alpha_G (J_\pi - J_g)$, dividing the surplus into shares $\alpha_G$ for the consumer and its complement $(1 - \alpha_G)$ for the producers. As such, $\alpha_G$ is the consumer’s market power, and $(1 - \alpha_G)$ the seller’s market power. The sharing condition leads to a simple rule for determining prices:

$$P = (1 - \alpha_G) (\Phi + \xi \sigma) + \alpha_G (w_\pi - w_g)$$

The detailed derivations are available in Appendix B.1. The price increases in the market power of the firm, $1 - \alpha_G$, the marginal utility of consumers $\Phi$ and their relative threat point, captured by $\xi \sigma$. The latter term has the interpretation that higher search costs and a tighter search market for goods push the negotiated price up as the seller has a relatively better outside option. The last element of the price rule is the wage differential between the search and selling stages in the goods market, $(w_\pi - w_g)$. When the difference is positive, that is, when the firm must pay labor a higher wage when selling, part of this increase in cost is passed on to the consumer as a higher price. The positive effect of $w_\pi$ is a markup effect: firms pass on to prices current production costs. The negative effect of $w_g$ is a surplus effect from the outside option of the firm. The firm has more to lose from a breakdown in bargaining with the consumer when $w_g$ is higher, and thus is ready to sacrifice profits by reducing the price in bargaining.
4.2 Wage setting in the labor market

We focus here on the simplest wage bargaining process as little generality is lost in this case, a benchmark assumption of a constant wages rule. In particular, assuming that \( w_\pi = w_g = w \) implies that the price does not depend on wages, as evident from the price equation (24). Thus workers and firms cannot, when they bargain over wages, anticipate or strategically exploit this mechanism (section 6 relaxes this assumption). The constant wage rule also implies that the value of employment is \( rW_e = rW_{e\pi} = rW_c = w + s^L (W_u - W_e) \), and that the expected wage payment in (20) \( \omega = w \).

The wage splits the labor match surplus \((W_e - W_u) + (J_g - J_v)\) by solving the Nash problem

\[
w = \argmax (W_e - W_u)^{\alpha_L} (J_g - J_v)^{1-\alpha_L}
\]

where \( \alpha_L \in (0, 1) \) is the bargaining strength of a worker. As each side’s respective match surplus is of same absolute slope with respect to the wage, the solution is a wage that satisfies the surplus sharing rule \((1 - \alpha_L) (W_e - W_u) = \alpha_L (J_g - J_v)\), dividing the total match surplus into shares \( \alpha_L \) for the worker, and its complement \((1 - \alpha_L)\) for the firm. The resulting wage rule

\[
w = (1 - \alpha_L) z + \alpha_L [\gamma \theta + \pi(\xi, P)]
\]

increases with non-employment utility \( z \) and labor market tightness. It also reflects the outcome of the equilibrium in the goods market along the two dimensions of the expected marginal revenue \( \pi(\xi, P) \), the meeting rate \( \lambda(\xi) \) and the price \( P \).

4.3 Equilibrium and a double Hosios condition

To compare with the social planner’s allocation, eliminate prices and wages in the consumer entry condition (18) and in the firm entry condition (20). The decentralized equilibrium is now a pair \((\xi, \theta)\) that solves the two entry conditions in the goods and labor markets. Denote these by \( CE^* \) and \( LE^* \), respectively, and by analogy with the social planner’s conditions.\(^5\)

For the goods market

\[
CE^*: \quad (r + s) \frac{\sigma}{\psi(\xi)} = \alpha_G \Phi - (1 - \alpha_G) \xi \sigma
\]

For the labor market

\[
LE^*: \quad (r + s^L) \frac{\gamma}{q(\theta)} = (1 - \alpha_L) \left[ \Phi - z - \frac{(r + s) \sigma}{\psi(\xi)} \left( \frac{1}{\alpha_G} \right) \right] - \alpha_L \gamma \theta
\]

Comparing the equilibrium conditions (27) in the decentralized economy to the social planner’s allocation in (8), tightness in the goods market is constrained efficient if \( \alpha_G = \eta_G \). Turning to the labor market, comparing equations (28) and (9), the decentralized equilibrium in the labor market corresponds to the social planner’s allocation if \( \alpha_L = \eta_L \) and \( \alpha_G = \eta_G \). At this point, the search externalities in the goods and labor markets are internalized and the decentralized agents reach an optimal allocation. This leads to a double Hosios condition:

\(^5\)See appendix B for details of the derivation.
Proposition 1 - The decentralized allocation in an economy with search and bargaining in goods and labor markets, and in which wages are not bargained conditional on whether the firms is selling its product \( w_{\pi} = w_{g} = w \), is constrained efficient if and only if \( \alpha_{L} = \eta_{L} \) and \( \alpha_{G} = \eta_{G} \).

The Hosios conditions can also be seen through the bargained wage and price rules. The wage in (26) coincides with the wage delivering the constrained efficient tightness of the labor market \( \omega^{opt} \) in (22) if the worker’s bargaining weight \( \alpha_{L} = \eta_{L} \) and the goods market is at the constrained efficient allocation with \( P^{opt} \). The goods market price (23) corresponds to the optimal price in (21) when the wage rule applies is constant across goods market stages and the consumer’s share of the surplus \( \alpha_{G} = \eta_{G} \).

This result is a generalization of our finding Petrosky-Nadeau and Wasmer [2017] (chapter 7) where the social planner was prevented from affecting goods market tightness as consumer search effort was inelastic. Therefore, only the labor market could be constrained efficient, with the condition \( \alpha_{L} = \eta_{L} \) for constant value of goods market tightness. Here, the endogenous entry of consumers leads an entry condition in the goods market. Thus two conditions instead of one are required to reach efficiency, in order to the internalize the trading externalities on both the labor and the goods market.

4.4 Market power, market tightness and efficiency

The consumer entry condition (18) describes a decreasing relation between the price and the tightness in the goods market. The price rule (23) is an increasing function of goods market tightness. As long as consumer market power \( \alpha_{G} \) is strictly positive there is a unique equilibrium with positive goods market tightness.\(^6\) Moreover, goods market tightness \( \xi^{*} \) is strictly increasing in the consumer’s bargaining power \( \alpha_{G} \). An increase in the goods market power of consumers, \( \alpha_{G} \), tilts and shifts the price curve outwards. The decentralized equilibrium shifts down the entry curve to a higher level of tightness \( \xi^{*} \) and lower price \( P^{*} \). Consumers receive more surplus in the goods market, through a lower price, and are induced to greater entry. Thus if \( \alpha_{G} > \eta_{G} \) there is excess entry of consumers and \( \xi^{*} > \xi^{opt} \). If \( \alpha_{G} < \eta_{G} \) there is insufficient entry of consumers and \( \xi^{*} < \xi^{opt} \), as seen in the first panel of Figure 4.

The effect of goods market bargaining power on tightness in the labor market, in contrast, is non-monotonic (see the second panel of Figure 4). An increase in \( \alpha_{G} \) always reduces the price, which reduces the incentives for firms to enter the labor market. However, the increase in goods market tightness accelerates firm’s meeting rate \( \lambda(\xi) \), which increases the expected payoff from hiring labor. Indeed, \( \frac{\partial \pi(\xi,P)}{\partial \xi} > 0 \) and \( \frac{\partial^{2} \pi(\xi,P)}{\partial \xi^{2}} < 0 \), and the expected revenue \( \pi(\xi,P) \) has a maximum at \( \alpha_{G} = \eta_{G} \). In the region \((0, \eta_{G})\) greater bargaining power for consumers leads to greater equilibrium labor market tightness. The trading effect dominates the (declining) price\(^6\)

\(^6\)The equilibrium in the goods market is represented in \((P, \xi)\) space in the top left panel of Appendix Figure 4. The right hand panel of the same figure represents the equilibrium in the labor market in \((P, \theta)\). The job creation condition (20) describes a strictly increasing relation between the price in the goods market and tightness of the labor market. The equilibrium price in the goods market is independent of labor market tightness and, as long as the expected revenue for the firm \( \pi(\xi,P) \) is greater than the flow value of non-employment \( z \), there exists a unique equilibrium with positive labor market tightness. Additional comparative statics beyond those discuss here are Appendix section X.
Figure 4: Equilibrium effects of changes in goods market bargaining strength $\alpha_G$

Figure 5: Equilibrium effects of changes in labor market bargaining strength $\alpha_L$
effect. Beyond this point, when the seller’s market power $(1 - \alpha_G)$ is less than the elasticity of the goods market matching function $\eta_G$, further increases in market power reduce equilibrium labor market tightness. While goods market tightness $\xi^*$ is strictly increasing in $\alpha_G$, in the labor market the equilibrium effect of changes in goods market power $\alpha_G$ depends on whether the price or the trading effect dominates. This is summarized in the following proposition:

**Proposition 2** - The decentralized equilibrium goods market tightness $\xi^*$ is strictly increasing in the consumer’s bargaining power $\alpha_G$. The decentralized equilibrium tightness of the labor market $\theta^*$ is an increasing and concave function of $\alpha_G$, maximized when the consumer’s share of the goods market surplus equals $\alpha_G = \eta_G$.

Next we consider the equilibrium effects of changes in the worker’s bargaining weight $\alpha_L$ in wage setting. Changes in $\alpha_L$ have no incidence on the equilibrium in the goods market, while labor market tightness is strictly decreasing in the share of the labor match surplus accruing to labor (see Figure 5). However, relative to an environment with perfect goods markets, it is possible to achieve the constrained efficient $\theta^{opt}$ in the decentralized allocation away from the Hosios condition in the labor market $\alpha_L = \eta_L$. Indeed, this can be achieved by allow deviations from constrained efficiency in the goods market. In establishing Proposition 2 we saw that the expected revenue from hiring labor $\pi(\xi^*, P)$ is maximized at $\alpha_G = \eta_G$. It is strictly lower for any $\alpha_G \neq \eta_G$. Thus, moving away from the Hosios condition in goods market it is possible to maintain $\theta^* = \theta^{opt}$ by giving more of the labor match surplus to the firm, i.e., with an $\alpha_L < \eta_L$. In fact, we can define...
\( \tilde{\alpha}_L(\alpha_G) \) as the share of the surplus to labor in wage bargaining that ensure \( \theta^* = \theta^{opt} \) for a given consumer price bargaining weight \( \alpha_G \). This \( \tilde{\alpha}_L \) is implicitly defined by the function

\[
\tilde{\alpha}_L : \quad \left( r + s_L \right) \frac{\gamma}{q(\theta^{opt})} = (1 - \tilde{\alpha}_L) \left[ \Phi - z - \frac{\sigma(r + s)}{\psi(\tilde{\xi}^*)} \frac{1}{\alpha_G} \right] - \tilde{\alpha}_L \gamma \theta^{opt}
\]

(29)
in which \( \theta^{opt} \) is taken as given, and \( \tilde{\xi}^* \) is the decentralized goods market tightness that solves (27). This \( \tilde{\alpha}_L(\alpha_G) \) function is plotted as the increasing and concave curve in Figure 6. All location below the curve correspond to a decentralized labor market allocation with \( \theta^* > \theta^{opt} \). All locations above the curve correspond to decentralized allocations with \( \theta^* < \theta^{opt} \).

**Proposition 3** - The decentralized equilibrium goods market tightness \( \tilde{\xi}^* = \tilde{\xi}^{opt} \) if \( \alpha_G = \eta_G \), while labor market tightness \( \theta^* = \theta^{opt} \) if \( \alpha_L = \tilde{\alpha}_L(\alpha_G) \), where \( \tilde{\alpha}_L(\alpha_G) \) is given by equation (29).

The condition for constrained efficiency in the goods market, \( \alpha_G = \eta_G \), is plotted vertical solid line in Figure 6. To the right of the curve the goods market is too tight. To the left of the curve the goods market is too slack. The intersection of both curves, which corresponds to \( \alpha_L = \eta_L \) and \( \alpha_G = \eta_G \), is the constrained efficient allocation in which \( \tilde{\xi}^* = \tilde{\xi}^{opt} \) and \( \theta^* = \theta^{opt} \). Thus all possible deviations of allocations away from the constrained efficient can be represented in \( (\alpha_G, \alpha_L) \) space. We thus have the four regimes, depending on which market is either too tight or too slack relative to the socially efficient, as a function of bargaining power in each market.

### 4.5 Efficiency of prices, wages and markups

We can now address whether prices, wages, and their ratio are inefficiently high or low, and how this maps to market power in goods and labor market. In Figure 7 we superimpose on the four tightness regimes of Figure 6 regions where an outcome of interest is above (red) or below (blue) the same outcome in the social planner’s allocation implementing the constrained efficient allocation for a given pair of market power \( \alpha_G \) and \( \alpha_L \).

All decentralized allocations that result in a goods market tightness \( \tilde{\xi}^* < \tilde{\xi}^{opt} \) have a price \( P^* \) that is inefficiently high, and conversely when \( \tilde{\xi}^* > \tilde{\xi}^{opt} \) the decentralized allocation price is too low (see Figure 7a). The labor market, however, can be either too tight or too slack when the price in the goods market deviates from \( P^{opt} \), depending on whether the share of the surplus accruing to the worker in wage setting \( \alpha_L \) is above or below \( \tilde{\alpha}_L(\alpha_G) \). For instance, for a given market power in the goods market \( \alpha_G < \eta_G \), we have a decentralized equilibrium with prices that are too high along with too little job creation when \( \alpha_L \) is above the value \( \tilde{\alpha}_L(\alpha_G) \) that ensures \( \theta^* = \theta^{opt} \).

In all four tightness regimes identified earlier, the price in the goods market is either too high or too low but never both. This is not the case for wages, as seen in Figure 7b. Allocation with insufficient job creation, \( \theta^* < \theta^{opt} \), can arise when the wage is either too high or too low. In the standard model of equilibrium unemployment this only occurs when the wage is too high. In the presence of product market frictions this can occur even with a low wage the further the equilibrium in the goods market departs from the constrained efficient allocation. This is because the
Figure 7: Efficiency of prices, wages, and markups in goods and labor market power space. Regions in which the decentralized equilibrium features an inefficiently high outcome of interest are in red. Inefficiently low decentralized values are in blue. The loci of bargaining strengths for which $\xi^* = \xi^{opt}$ and $\theta^* = \theta^{opt}$ are represented by the dashed lines.
expected revenue $\pi(\xi, P)$ is at a maximum at $\xi^* = \xi^{opt}$. Allocations with excessive job creation, $\theta^* > \theta^{opt}$, only arise with wages that are too low.

The last panel, Figure 7c, presents the complementary information of the mark up of price over wage. Tight labor markets, with excessive seller market power, always have a markup that is inefficiently high. The markup is inefficiently low in the mirror regime of a slack labor market with too much market power for consumers and a tight goods market.

5 Evaluating costs of frictions in the goods and labor markets

In what follows we calibrate the model to the US economy in order to recover the key parameters and evaluate the costs of frictions in labor and goods markets.

5.1 Calibrating to the US economy

The basic unit of time is a month. The rate of time preference is set to match an average return on 3-month Treasury bill of 2.5 percent at an annualized rate. The calibration of labor market and goods market parameters are described next, and summarized in Table 1.

We begin with the set of parameters in the labor market $s^L, \eta_L, \chi_L, \alpha_L, z,$ and $\gamma$. The rate of job separation $s^L$ is set to 0.032 corresponding to estimates from the Job Openings and Labor Turnover Survey (JOLTS) over the period January 2001 to December 2016. The matching function in the labor market is assumed to be a Cobb-Douglas $M_L = \chi_L U^{\eta_L} V^{1-\eta_L}$. Petrongolo and Pissarides [2001] provide an extensive survey of empirical estimates of the labor market matching function. The most frequent finding is an elasticity of the matching function $\eta_L = 0.5$, which we adopt here. The average vacancy rate in the US over the period spanning January 2001 to December 2016 was 4 percent, which pins down the value of the level parameter in the matching function $\chi_L$. The average unemployment rate over the same period, 5 percent, serves as calibration target for the worker’s share of the labor surplus, $\alpha_L$. The flow value of non-employment relative to the wage is set to $z/w = 0.50$, as suggested in Mulligan [2012]. Finally, the cost of an open vacancy $\gamma$ is tied to the entry of firms. It is set to ensure a product entry rate that leads to a value of the consumer goods market meeting rate $\psi$ of 0.22 at an annualized rate. This corresponds to the empirical estimates of entry rates into consumer consumption baskets found in Broda and Weinstein [2010].

Next we address the set of parameters in the goods market $s^G, \eta_G, \chi_G, \alpha_G, \Phi,$ and $\sigma$. The marginal utility of the search goods $\Phi$ is normalized to 1. We use estimates of product exit rates from household consumption baskets provided in Broda and Weinstein [2010] to set $s^G = 0.001$, as in Petrosky-Nadeau and Wasmer [2015]. The matching function in the goods market is assumed to be a Cobb-Douglas $M_G = \chi_G D_U^{\eta_G} N_s^{1-\eta_G}$, where $\eta_G$ is the elasticity with respect to searching consumers $D_U$, and $\chi_G$ is a level parameter. The model implies a price elasticity of demand\(^7\)

\[
\frac{dD_M}{dP} \times P / D_M = -\frac{\eta_G}{1-\eta_G} \frac{P}{P - \Phi}
\]

\(^7\)See appendix ?? for the derivation. In particular, the elasticity keeps the supply of goods constant.
The goods market matching function elasticity $\eta_G$ is calibrated to this price elasticity. Our baseline targets an elasticity of -2, which is at the low end of estimates such as Hottman et al (2016). However, this is an aggregate elasticity rather than the more micro (industry or sectoral) estimates found in the latter. We provide a sensitivity of the results to a range of target elasticities below. The rate of capacity utilization, defined in the model as $\lambda = (N_G + N_\pi)/N_\pi$, helps pin down the level parameter of the matching function $\chi_G$. While the US manufacturing sector rate of capacity utilization averaged 75 percent over the period spanning January 2001 to December 2016, it is higher in the service sector. As such we calibrate to a value of 0.85. As in Petrosky-Nadeau and Wasmer [2015], we use estimates from the American Time Use Survey of the time spent shopping for goods and services to calibrate the search cost parameter $\sigma$. That is, $\sigma$ is such that the value of consumer search $\sigma D_U$ equals 5 percent of labor earnings. Finally, we target a markup over the wage $P/w$ to calibrate the value of the goods market bargaining power $\alpha_G$. Our baseline results target a 20% markup over the wage. We present the results for a range of values further below as there exists a range of estimates of the markup.

Table 1: Calibration targets and parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Target or reference:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time discount rate</td>
<td>$r = e^{(2.5/1200)} - 1$</td>
</tr>
<tr>
<td>3-month Treasury bill</td>
<td></td>
</tr>
</tbody>
</table>

**Labor market:**
- Worker bargaining weight $\alpha_L = 0.34$
- Elasticity of matching function $\eta_L = 0.50$ [Petrongolo and Pissarides, 2001]
- Level of matching function $\chi_L = 0.68$
- Job-separation rate $s_L = 0.032$ [JOLTS]
- Vacancy cost $\gamma = 0.87$
- Non-employment value $z = 0.37$ [Mulligan, 2012]
- Unemployment rate $U = 0.05$
- Job vacancy rate $V = 0.04$
- Product entry rate $\psi = 0.015$
- $\frac{1}{\lambda + 3} = 0.85$
- $\frac{\sigma D_U}{\sigma D_U} = 0.05$

**Goods market:**
- Consumer bargaining weight $\alpha_G = 0.33$
- Price markup over wage $P/w = 1.25$
- Price elasticity of demand $\frac{dP}{dP_w} P_w = -2$
- Rate of capacity utilization $\lambda = (N_G + N_\pi)/N_\pi = 0.85$
- Product exit rate
- American Time Use Survey $\sigma D_U = 0.05$
- Normalization

### 5.2 Estimating the costs of market frictions

The calibration of the model to moments from the US economy implies bargaining shares in goods and labor market that deviate from the Hosios conditions. The worker’s wage bargaining weight $\alpha_L$ of 0.34 is below the elasticity of the labor market matching function of $\eta_L = 0.5$. The consumer’s share of the goods market match $\alpha_G$ is found to be 0.33, above the elasticity of the goods market matching function of $\eta_G = 0.14$. Bai et al. [2011], using a different approach based on the elasticity
of shopping time to income to infer this elasticity, find a value of 0.23. Gourio and Rudanko [2013], on the other hand find value of 0.11 in a model where firms acquire customers as capital through search and calibrated to firm level data on advertising expenditure.  

Both the labor and the goods market in the calibrated US economy are too tight. In order to evaluate the economic magnitude of these departures from efficiency, we calculate the allocation restoring the Hosios conditions in both markets, keeping all other parameters constant. That is, we set $\alpha_L = \eta_L$ and $\alpha_G = \eta_G$. The first column of results in Table 2 presents the values of outcomes of interest in the baseline calibration. The second column presents their values at the constrained efficient allocation. The constrained efficient rate of unemployment is 6.71 percent, 1.71 percentage points above the calibration target. The rate of capacity utilization would be a little lower, 83 compared to 85 percent, and the constrained efficient market is 2 percent higher. In the last two columns of Table 2 we report the effects of imposing a Hosios condition in one of the markets alone. The results emphasize that the greatest welfare gains, the change in social value $\Omega$, arise from removing the distortion in the goods market. Overall, however, the U.S. appear relatively close to the constrained efficient allocation.

6 Extensions and robustness: alternative price and wage setting

Our baseline assumption was a case in which wages are not bargained separately in stages $g$ and $\pi$, an assumption we relax in the section 6.1. 9 The first extension does not allow for strategic effect in wage bargaining. Agents do not anticipate how wages may affect the goods market match

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8The sensitivity of the results to changes the calibration targets are presented in detail in appendix ??.

9In addition, we explicitly allow for a different bargaining power of workers in stage $\pi$, and denote it by $\alpha'_L$ in appendix X. Assuming $\alpha'_L = \alpha_L$ the wage rules are identical whether the outside option is a break up of the labor match in wage bargaining during stage $\pi$, giving the value $J_v$ for the firm and $W_U$ to the worker.
surplus and prices. This is explored separately in section 6.2.

In all cases considered the outside option in negotiating the wage in stage \( g \) is assumed to be the dissolution of the labor match. That is,
\[
w_g = \arg\max (W_{eg} - W_u)^{\alpha_L} (J_g - J_v)^{1-\alpha_L}
\]  
(30)

Disagreement when bargaining over wage \( w_\pi \) leads each side to revert to its value in stage \( g \) instead of a full dissolution of the match. That is, the wage solves the bargaining problem
\[
w_\pi = \arg\max (W_{e\pi} - W_{eg})^{\alpha_L} (J_\pi - J_g)^{1-\alpha_L}
\]  
(31)

### 6.1 Wages bargained sequentially: no strategic interactions

This first extension does not allow for strategic interactions in bargaining, and assumes that the wage in the entry stage \( w_g \) does not affect the subsequent wage \( w_\pi \). Let \( \tilde{\alpha}_G \) be given by the expression:
\[
\tilde{\alpha}_G(a_L, \alpha_G) = \alpha_G \times \left( \frac{1 - \alpha_L}{1 - \alpha_L \alpha_G} \right)
\]  
(32)

This is an effective consumer share of the goods market match surplus, after the worker has been paid the bargained wage. If \( \alpha_L = 0 \), meaning the firm in stage \( \pi \) receives the entire surplus of the match with the worker, none of the economic surplus from production is dissipated or transferred to labor. The consumer receives a share corresponding to its bargaining power, \( \tilde{\alpha}_G = \alpha_G \). If instead \( \alpha_L = 1 \), the firm in stage \( \pi \) had no surplus from the match with labor. The effect consumer bargaining weight is then \( \tilde{\alpha}_G = 0 \) as the consumer receives no consumption surplus irrespective of its goods market power. All of the surplus from production has accrued to labor.

The effective consumer share \( \tilde{\alpha}_G \) plays an important role in the equilibrium allocation.\(^{10}\) After one eliminates the price and wage from the equilibrium entry conditions in the goods and labor markets, the decentralized equilibrium is a pair \((\xi, \theta)\) that solves:
\[
(r + s) \frac{\sigma}{\psi(\xi)} = \tilde{\alpha}_G \Phi - (1 - \tilde{\alpha}_G) \tilde{\xi} \sigma
\]  
(33)
\[
(r + sL) \frac{\gamma}{q(\theta)} = (1 - \alpha'_L) \left[ \Phi - \frac{(r + s) \sigma}{\psi(\xi)} \left( \frac{1}{\tilde{\alpha}_G} \right) \right] - (1 - \alpha_L) z - \alpha_L \gamma \theta
\]  
(34)

It is then easy to derive that:

**Proposition 3** - When wages are set separately at firms searching in the goods market and matched in the goods market, the decentralized allocation in an economy with search and bargaining in goods and labor markets is constrained efficient if and only if \( \alpha_L = \eta_L \) and \( \tilde{\alpha}_G(a_L, \alpha_G) = \eta_G \), where the second equality is equivalent to \( \alpha_G = \eta_G \times \frac{1}{1 - \alpha_L (1 - \eta_G)} \geq \eta_G \).

\(^{10}\)The wage rules for this extension are derived in the appendix and presented as equations (??) and (??), and lead to the price equation (??).
Figure 8: Efficiency of tightness in the goods and labor markets with sequential wage bargaining

As in Section 3.4, the possible deviations of allocations away from the constrained efficient can be represented in \((\alpha_G, \alpha_L)\) space. The condition for constrained efficiency in the goods market, \(\alpha_G = \frac{\eta_G}{1-\alpha_L(1-\eta_G)}\), is plotted as the upward slope curve in the range \(\alpha_G \in [\eta_G, 1]\). The curve represents all the pairs of bargaining weights in the goods and labor market that deliver \(\xi^* = \xi^{opt}\).

To the right of the curve the goods market is too tight. To the left of the curve the goods market is too slack. In particular, it states that for greater worker bargaining strength \(\alpha_L\) consumers need to receive a greater share of the surplus in the goods market to ensure the efficient tightness of the goods market. If not, if effective consumer market power is too weak, there is an insufficient entry of consumer in the goods market relative to goods as the decentralized price \(P\) is too high.

In the limit, as the worker share \(\alpha_L\) tends to 1, the consumer needs to receive a share of the goods market surplus \(\alpha_G\) that also tends to 1. At the other end of rent sharing, when the worker receives its reservation wage \((\alpha_L = 0)\), efficient goods market tightness is achieved for \(\alpha_G = \eta_G\). Below \(\alpha_G = \eta_G\) it is not possible to restore constrained efficiency in the goods market by varying market power in the labor market.\(^\text{11}\)

The second (increasing and concave) curve separating the bargaining weight space is defined by the combinations of bargaining weights \(\alpha_L\) and \(\alpha_G\) which deliver a decentralized labor market tightness \(\theta^*\) exactly equal to the constrained efficient \(\theta^{opt}\), as earlier.\(^\text{12}\) All locations below the curve correspond to a decentralized labor market allocation with \(\theta^* < \theta^{opt}\). All locations above the curve correspond to decentralized allocations with \(\theta^* > \theta^{opt}\). The intersection of both curves, which corresponds to \(\alpha_L = \eta_L\) and \(\alpha_G = \eta_G / [1 - \alpha_L(1 - \eta_G)]\), is the constrained efficient allocation in which \(\xi^* = \xi^{opt}\) and \(\theta^* = \theta^{opt}\).

\(^{11}\)In addition, the \(\alpha_L\) for constrained efficiency is always strictly less than \(\alpha_G\) when \(\eta_G \geq 0.5\). For any \(0 < \eta_G < 0.5\), there a crossing at \(\tilde{\alpha}_G\) with \(\alpha_L < \alpha_G\) when \(\alpha_G < \tilde{\alpha}_G\), and \(\alpha_L > \alpha_G\) when \(\alpha_G > \tilde{\alpha}_G\).

\(^{12}\)The curve is defined by the function \((r + s^L)\frac{\tilde{\xi}^{opt}}{\theta^{opt}} = (1 - \tilde{\alpha}_L)\left(\frac{\tilde{\xi}^{opt}}{\theta^{opt}}\right) - \tilde{\alpha}_L\gamma\theta^{opt}\) in which \(\theta^{opt}\) is taken as given, and \(\tilde{\xi}^{opt}\) is the decentralized goods market tightness that solves (27).
6.2 Wages bargained sequentially: workers and firms in stage π strategically anticipate the effect on prices

Suppose now that the worker and the firm in negotiating the wage $w_π$ internalize the effect on the consumption match surplus in the goods market. That is, they know $P = (1 - α_G) (Φ + ξ σ) + α_G (w_π - w_g)$. In this case the sharing rule derived from the Nash maximand must take into account that $\frac{dW_π}{dw_π} = \frac{1}{r + s}$ and that $\frac{dJ_π}{dw_π} = -\frac{(1 - a_G)}{r + s}$. The wage must now satisfy a more complex sharing rule, $α_L (J_π - J_π^*) = (1 - α_L) (1 - α_G) (W_π - W_π^*)$, giving a larger share to the worker. This share, $α_L / [1 - α_G (1 - α_L)]$ tends to $α_L$ when the consumer has no bargaining power ($α_G = 0$), and tends to 1 when $α_G = 1$.

The implication of this strategic context for the goods market is a new effective bargaining share of the consumer:

$$\hat{α}_G = α_G (1 - α_L)$$

More bargaining power to labor reduces the effective share of the consumption surplus for the consumer.

The equilibrium decentralized tightness of the goods and labor market $(\tilde{ξ}, θ)$ solve:

$$\frac{σ (r + s)}{q(\tilde{ξ})} = \hat{α}_G Φ - (1 - \hat{α}_G) \tilde{ξ} σ$$

$$(r + s)^L \frac{γ}{q(θ)} = (1 - α_L') \left[ Φ - \frac{σ (r + s)}{q(\tilde{ξ})} \left( \frac{1}{\hat{α}_G} \right) \right] - (1 - α_L) z - α_L γ θ$$

from which we have the following proposition:

**Proposition 4** - When wages are set separately at firms searching in the goods market and matched in the goods market, and wage bargaining at selling firm internalizes the effect on the goods market match surplus, the decentralized allocation in an economy with search and bargaining in goods and labor markets is constrained efficient if and only if $α_L = η_L$ and $\hat{α}_G (α_L, α_G) = η_G$.

The conditions for constrained efficiency remain similar in nature in this extension as well. The worker’s bargaining weight must equal the elasticity of the labor matching function with respect to unemployment, and the effective bargaining weight of the consumer in the goods market $\hat{α}_G$ must equal the elasticity of the goods matching function with respect to consumer entry $η_G$. However, the second equality, which is equivalent to $α_G = η_G \times \frac{1}{α_L} ≥ η_G$ has the following additional implication. If $η_G < 1 - η_L$, then the $α_G$ that implements the constrained efficient allocation is less than 1. In this case deviations away from the constrained efficient fall once again in four tightness regimes. If $η_G = 1 - η_L$, then the $α_G$ that implements the constrained efficient allocation is equals 1. In this case the regime with $θ^* < θ^{opt}$ and $ξ^* > ξ^{opt}$ does not exist. It is plotted in Figure 8b. Finally, If $η_G > 1 - η_L$, then there exist no $α_G$ that can implement the constrained efficient allocation.
7 Conclusion

When frictions affect several markets, welfare implications are rich and complex. Inefficiency on one market may affect the efficient outcome in the other markets. This is a well known result in second best economics. In our case, the optimal pricing rule is the one that maximizes the revenue of firms as a result of free entry of consumers in the goods market, and this is in fact independent of the state of the labor market. This is a version of the well known competitive search equilibrium in the labor market, as stated by Moen [1997]. The reciprocal is not true: the optimal sharing of the value added generated by labor between the workers and the firms involves the revenue of the firm and the pricing rule, and here two situations may arise. If the price is ineffeciently high and there is too little consumer entry, the low expected revenue of the firm can be compensated with lower wage to achieve efficient employment. If the price is ineffeciently low and there is too much consumer entry, once again the expected revenue of the firm is inefficiently low and can be compensated with a lower wage to achieve efficient employment. Empirically, we find that the US economy is close to efficient in the labor market but that, consumers appear to have excess market power on on pricing.
References

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A Social planner’s problem

This section provides the detailed derivations for section 2.2. Recall the social planner’s program:

\[
H = \max_{D_U, V, U, D_M} e^{-rt}[D_M \Phi + zU - \gamma V - \sigma D_U] \\
+ \Psi_{D_M} \left[(s^G + s^L)D_M - M_G(D_U, 1 - U - D_M)\right] \\
+ \Psi_U \left[M_L(U, V) - s^L(1 - U)\right]
\]

using \( \frac{\partial M_G}{\partial t} = \frac{\partial M_G}{\partial N_g} \frac{\partial N_g}{\partial t} \), \( \frac{\partial M_G}{\partial D_U} = \frac{\partial M_G}{\partial N_g} \frac{\partial N_g}{\partial D_M} \), and the following properties of the matching functions: \( \frac{\partial M_L}{\partial U} = \eta_L f(\theta); \frac{\partial M_G}{\partial D_U} = \eta_G \psi; \frac{\partial M_L}{\partial V} = (1 - \eta_L)q(\theta); \frac{\partial M_G}{\partial N_g} = (1 - \eta_G)\lambda(\theta) \). The first order conditions are:

\[
\frac{\partial H}{\partial D_U} = 0 \rightarrow e^{-rt}(-\sigma) - \Psi_{D_M} \frac{\partial M_G}{\partial D_U} = 0 \quad (A.1)
\]

\[
\frac{\partial H}{\partial V} = 0 \rightarrow e^{-rt}(-\gamma) + \Psi_U (1 - \eta_L)q = 0 \quad (A.2)
\]

\[
\frac{\partial H}{\partial D_M} = -\Psi_{D_M} \rightarrow e^{-rt}\Phi + \Psi_{D_M} \left[(s^G + s^L) + (1 - \eta_G)\lambda(\theta)\right] = -\Psi_{D_M} \quad (A.3)
\]

\[
\frac{\partial H}{\partial U} = -\Psi_U \rightarrow e^{-rt}z + \Psi_U \left[\eta_L f + s^L\right] + \Psi_{D_M} (1 - \eta_G)\lambda(\theta) = -\Psi_U \quad (A.4)
\]

A.1 Constrained efficient goods market tightness

To obtain the efficient goods market tightness, begin with (A.1):

\[
\Psi_{D_M} = -\frac{e^{-rt}\sigma}{\eta_G \psi} \quad \text{and} \quad \Psi_{D_M} = \frac{re^{-rt}\sigma}{\eta_G \psi}
\]

Combining with (A.3) we have:

\[
e^{-rt}\Phi + \Psi_{D_M} \left[(s^G + s^L) + (1 - \eta_G)\lambda(\theta)\right] = -\Psi_{D_M}
\]

\[
e^{-rt}\Phi - \frac{e^{-rt}\sigma}{\eta_G \psi} \left[(s^G + s^L) + (1 - \eta_G)\lambda\right] = \frac{re^{-rt}\sigma}{\eta_G \psi}
\]

\[
\eta_G \Phi - \sigma \zeta (1 - \eta_G) = \frac{\sigma}{\psi} (r + s)
\]

A.2 Constrained efficient labor market tightness

Solving for \( \Psi_U \) in (A.2) we obtain:

\[
\Psi_U = \frac{e^{-rt}\gamma}{q(1 - \eta_L)} \quad \text{and} \quad \Psi_U = \frac{-re^{-rt}\gamma}{q(1 - \eta_L)}
\]
and in combination with (A.4), we have:

\[ -\Psi_U = e^{-rt}z + \Psi_U \left( \eta_L f + s^L \right) + \Psi_{DM} (1 - \eta_G) \lambda \]

\[ -re^{-rt} \gamma \frac{q}{(1 - \eta_L)} = e^{-rt}z + \frac{e^{-rt} \gamma}{q (1 - \eta_L)} \left( \eta_L f + s^L \right) - \frac{e^{-rt} \sigma}{\eta_G \psi} (1 - \eta_G) \lambda \]

\[ -r \gamma \frac{q}{(1 - \eta_L)} = z + \frac{\gamma}{q (1 - \eta_L)} \left( \eta_L f + s^L \right) - \frac{\sigma}{\eta_G \psi} (1 - \eta_G) \lambda \]

\[ (r + s^L) \gamma \frac{q}{(1 - \eta_L)} = \frac{\sigma}{\eta_G \psi} (1 - \eta_G) \lambda - z - \frac{\eta_L \gamma f}{q (1 - \eta_L)} \]

\[ (r + s^L) \gamma \frac{q}{(1 - \eta_L)} = (1 - \eta_L) \left[ \frac{\sigma}{\eta_G \psi} (1 - \eta_G) \lambda - z \right] - \eta_L \gamma \theta \]

\[ z = \frac{\sigma \xi}{(1 - \eta_G)} \left( \frac{1}{\eta_G} \right) - \eta_L \gamma \theta \]

Note that from the equilibrium condition for goods market tightness we have that \( \sigma \xi \frac{(1 - \eta_G)}{\eta_G} = \Phi - \frac{\sigma \eta_G \psi}{\eta_G} (r + s) \). This delivers the social planner’s job creation condition (9) in the text.

**B Decentralized equilibrium**

The consumer, worker, and firm Bellman equations are reproduced here for convenience:

\[ rW_{Du} = -\sigma + \psi (W_{DM} - W_{Du}) \quad (A.5) \]

\[ rW_{Dm} = (\Phi - \mathcal{P}) + s^G (W_{Du} - W_{Dm}) + s^L (W_{Du} - W_{Dm}) \quad (A.6) \]

\[ rW_{e\pi} = w_{\pi} + s^L (W_u - W_{e\pi}) + s^G (W_{eg} - W_{e\pi}) \quad (A.7) \]

\[ rW_{eg} = w_{e} + s^L (W_u - W_{eg}) + \lambda (W_{e\pi} - W_{eg}) \quad (A.8) \]

\[ rW_u = z + f(\theta) (W_{eg} - W_u) \quad (A.9) \]

\[ rJ_v = -\gamma + q(\theta) (J_g - J_v) \quad (A.10) \]

\[ rJ_g = -w_g + \lambda (J_{\pi} - J_g) + s^L (J_v - J_g) \quad (A.11) \]

\[ rJ_{\pi} = \mathcal{P} - w_{\pi} + s^G (J_g - J_{\pi}) + s^L (J_v - J_{\pi}) \quad (A.12) \]

The above Bellman equations lead to expressions for the consumer and firm private surpluses in a goods market match:

\[ W_{DM} - W_{Du} = \frac{(\Phi - \mathcal{P} - rW_{Du})}{r + s} \quad (A.13) \]

\[ J_{\pi} - J_g = \frac{\mathcal{P} - w_{\pi} - (r + s^L) J_g + s^L J_v}{r + s} \quad (A.14) \]

A worker’s and firm’s surplus generated from forming a match in the labor market are, respectively:
\[ W_{c\pi} - W_u = \frac{w_\pi - w_g + \lambda (W_{c\pi} - W_{c\bar{G}}) - rW_u}{r + sL} \]

\[ J_\pi - J_g = \frac{-w_g + \lambda (J_\pi - J_g)}{r + sL} \]

The worker’s surplus is the present discounted value of the wage \( w_g \), plus the change in match surplus \((W_{c\pi} - W_{c\bar{G}})\) when a consumer is found, net of the worker’s outside option \( rW_u \). The firm’s surplus is the present discounted value of the difference between the wage paid to the worker and the expected gain once a customer has been found, \((J_\pi - J_g)\).

The match surpluses between a worker and firm, once a match in the goods market is formed and the firm moves the profit state, gain respectively:

\[ W_{c\pi} - W_{c\bar{G}} = \frac{w_\pi - w_g}{r + s + \lambda} \quad (A.15) \]

\[ J_\pi - J_g = \frac{\mathcal{P} - (w_\pi - w_g)}{r + s + \lambda} \quad (A.16) \]

A worker gains the present value of any change in the bargained wage. A firm gains the present value of the revenue from selling its good net of any change in the cost of labor.

**B.1 Price and wage determination: benchmark assumptions**

**Price bargaining**

Starting from the sharing rule

\[ (1 - \alpha_G) (W_{DM} - W_{DU}) = \alpha_G (J_\pi - J_g) \quad (A.17) \]

the left hand side can be replaced with equation (A.13), and the right hand side with (A.14):

\[ (1 - \alpha_G) (\Phi - \mathcal{P}) = \alpha_G \left[ \mathcal{P} - w_\pi + sL (J_\pi - J_g) \right] \]

Re-arranging, we obtain an expression for the price:

\[ \mathcal{P} = (1 - \alpha_G) \Phi + \alpha_G \left[ w_\pi + sL (J_g - J_\pi) \right] \]

We further have, using (12),

\[ sL (J_g - J_\pi) = -w_g + \lambda (J_\pi - J_g) = -w_g + \lambda \frac{1 - \alpha_G}{\alpha_G} (W_{DM} - W_{DU}) \]

\[ = -w_g + \lambda \frac{1 - \alpha_G}{\alpha_G} \frac{\sigma \xi}{\psi(\xi)} = -w_g + \frac{1 - \alpha_G}{\alpha_G} \sigma \xi \]

such that one obtains:

\[ \mathcal{P} = (1 - \alpha_G) [\Phi + \sigma \xi] + \alpha_G (w_\pi - w_g) \]
Wage determination with constant wage profile

Under the baseline assumption that the wage in both stages $g$ and $\pi$ are identical, that is $w = w_g = w_{\pi}$, we have that $W_{e\pi} = W_{eg} = W$. In addition, $J_{\pi} - J_g$ does not depend on the wage. Hence:

$$\frac{\partial (W_e - W_u)}{\partial w} = \frac{1}{r + s^L} = -\frac{\partial (J_g - J_v)}{\partial w_g}$$

and the simple Nash-bargaining rule applies:

$$(1 - \alpha_L) (W_e - W_u) = \alpha_L (J_g - J_v) \quad (A.18)$$

The wage is therefore the solution to

$$(1 - \alpha_L) \left[ w - rW_u + \lambda (W_{e\pi} - W_{eg}) \right] = \alpha_L \left[ -w + \lambda (J_{\pi} - J_g) \right]$$

$$w = (1 - \alpha_L) rW_u + \alpha_L \lambda (J_{\pi} - J_g)$$

using $W_{e\pi} - W_{eg} = 0$. From the wage bargaining sharing rule and the firm’s free-entry we have $rW_u = z + f(\theta) (W_e - W_u) = z + \gamma \theta \alpha_L / (1 - \alpha_L)$. Moreover, the firm’s surplus can be expressed as $J_{\pi} - J_g = P / (r + s + \lambda)$, such that we have the wage rule (with $\mu = \lambda / (r + s + \lambda)$):

$$w = (1 - \alpha_L) z + \alpha_L [\gamma \theta + \mu P]$$

B.2 Equilibrium conditions

Inserting the rule wage into the free-entry condition of firms leads to the labor market equilibrium presented in the text:

$$\left( r + s^L \right) \frac{\gamma}{q} = -w_g + \lambda (J_{\pi} - J_g)$$

$$\left( r + s^L \right) \frac{\gamma}{q} = -w_g + \lambda \left( \frac{P - (w_{\pi} - w_g)}{r + s + \lambda} \right)$$

$$\left( r + s^L \right) \frac{\gamma}{q} = \lambda P - \lambda (w_{\pi} - w_g) - (r + s + \lambda) w_g$$

$$\left( r + s^L \right) \frac{\gamma}{q} = \lambda (P - w_{\pi}) - (r + s + \lambda) w_g$$

$$\left( r + s^L \right) \frac{\gamma}{q} = \lambda P - (\lambda w_{\pi} + (r + s) w_g)$$

$$\left( r + s^L \right) \frac{\gamma}{q} = \mu P - \omega$$

$$\left( r + s^L \right) \frac{\gamma}{q(\theta)} = (1 - \alpha_L) [\mu P - z] - \alpha_L \gamma \theta$$

since $\omega = w$ in the constant wage profile case.
Next we show that the expected revenue $\pi(\xi, P) = \mu P$ can be re-expressed in two ways.\textsuperscript{13}

First,
\[
\frac{\sigma}{\psi} = \frac{\alpha_G}{1 - \alpha_G} (J_\pi - I_\xi) \\
\frac{\sigma}{\lambda \xi} = \frac{\alpha_G}{1 - \alpha_G} P \\
\frac{\mu P}{\alpha_G} = \frac{1 - \alpha_G}{\sigma \xi}
\]

and, second,
\[
\frac{\sigma}{\psi} = \frac{\alpha_G \Phi - (1 - \alpha_G) \sigma \xi}{r + s} \\
(\alpha_G \psi) = \Phi \frac{1 - \alpha_G}{\sigma \xi} \\
\frac{1 - \alpha_G}{\alpha_G} \frac{\sigma \xi}{\psi} = \Phi - \frac{\sigma (r + s)}{\psi} \frac{1}{\alpha_G} = \mu P
\]

Summarizing, the equilibrium conditions are:
\[
\frac{\sigma}{\psi} = \frac{\alpha_G \Phi - (1 - \alpha_G) \xi \sigma}{r + s} \tag{A.19} \\
\left( r + s \right) \frac{\gamma}{q(\theta)} = (1 - \alpha_L) \left[ \Phi - z - \frac{\sigma}{\alpha_G \psi(\xi)} (r + s) \right] - \alpha_L \gamma \theta \tag{A.20}
\]

\textsuperscript{13}The following derivations make use of $\frac{\sigma}{\psi} = W_{D_M} - W_{D_H}; W_{D_M} - W_{D_H} = \frac{\alpha_G}{1 - \alpha_G} (J_\pi - I_\xi); (J_\pi - I_\xi) = \frac{p - (w_\pi - w_\xi)}{r + s + \lambda}$; $w_\pi = \alpha_L P + w_\xi; P = (1 - \alpha_G) (\Phi + \sigma \xi) + \alpha_G (w_\pi - w_\xi) = (1 - \alpha_G) (\Phi + \sigma \xi) + \alpha_G \alpha_L P$. 

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