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Examining the Sources of Excess Return Predictability: Stochastic Volatility or Market Inefficiency?*

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Abstract

We use a consumption based asset pricing model to show that the predictability of excess returns on risky assets can arise from only two sources: (1) stochastic volatility of model variables, or (2) departures from rational expectations that give rise to predictable investor forecast errors and market inefficiency. From an empirical perspective, we investigate whether 1-month ahead excess returns on stocks can be predicted using measures of consumer sentiment and excess return momentum, while controlling directly and indirectly for the presence of stochastic volatility. A variable that interacts the 12-month sentiment change with recent return momentum is a robust predictor of excess stock returns both in-sample and out-of-sample. The predictive power of this variable derives mainly from periods when sentiment has been declining and return momentum is negative, forecasting a further decline in the excess stock return. We show that the sentiment-momentum variable is positively correlated with fluctuations in Google searches for the term "stock market," suggesting that the sentiment-momentum variable helps to predict excess returns because it captures shifts in investor attention, particularly during stock market declines.

Keywords: *Equity Premium, Excess Volatility, Return Predictability, Market Sentiment, Time Series Momentum*

JEL Classification: E44, G12.

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1 Introduction

A vast literature, pioneered by Fama and French (1988), examines the so-called "predictability" of excess returns on risky assets. Predictability is typically measured by the size of a slope coefficient and the adjusted R-squared statistic in forecasting regressions over various time horizons. This paper examines the predictability question from both a theoretical and empirical perspective.

Our theoretical approach employs a standard consumption based asset pricing model. We show that the predictability of excess returns on risky assets can arise from only two sources: (1) stochastic volatility of model variables, or (2) departures from rational expectations that give rise to predictable investor forecast errors and market inefficiency. Specifically, we show that excess returns on risky assets can be represented by an additive combination of conditional variance terms and investor forecast errors. This result holds for any stochastic discount factor, any consumption or dividend process, and any stream of bond coupon payments. The conditional variance terms can be a source of predictability if one or more of the model's fundamental state variables exhibit *exogenous* stochastic volatility or if some nonlinear feature of the model gives rise to *endogenous* stochastic volatility. Investor forecast errors can be a source of predictability if the representative investor's subjective forecast rule is misspecified in some way. We provide some analytical examples to illustrate each of these possibilities.

Our empirical approach examines whether 1-month ahead excess returns on stocks relative to the risk free rate can be predicted using measures of consumer sentiment and excess return momentum, while controlling directly and indirectly for the presence of stochastic volatility. The predictor variables that control for stochastic volatility are the price-dividend ratio, the 3-month moving average of the variance risk premium (the difference between the implied and realized variance of stock returns), and the 12-month change in the federal funds rate. These predictor variables are almost always statistically significant, regardless of the regression specification or the sample period. The predictor variables that are designed to detect departures from market efficiency are the 12-month change in the University of Michigan's consumer sentiment index and a measure of return momentum given by the trailing 1-month change in the excess stock return. As an additional predictor variable, we interact the 12-month sentiment change with our measure of return momentum.

While the regression coefficients on sentiment and return momentum are individually almost never significant, the sentiment-momentum interaction variable is almost always significant. The sentiment-momentum variable enters the regression equation with a negative sign, regardless of whether sentiment has been rising or declining or whether return momentum is positive or negative. Periods of rising sentiment and positive return momentum tend to be followed by reversal in the excess return while periods of declining sentiment and negative return momentum tend to be followed by further downward drift in the excess return. The statistically significant predictive power of the sentiment-momentum variable derives mainly from periods of declining sentiment and negative return momentum, forecasting a further decline in the excess stock return. Our full-sample predictability regression for the period from 1990.M3 to 2017.M12 yields an adjusted R-squared statistic of 17.3%. If we omit the sentiment-momentum variable, the adjusted R-squared statistic drops to 12.9%. For the more-recent sample period from 2004.M1 to 2017.M12, our predictability regression yields an adjusted R-squared statistic of 24.7%. In out-of-sample tests, including the sentiment-momentum variable serves to markedly increase the out-of-sample R-squared statistic. In split-sample regressions, including the sentiment-momentum variable increases the out-of-sample R-squared statistic to 15.9% versus 8.8% without this variable. In 10-year rolling window regressions, including the sentiment-momentum variable increases the out-of-sample R-squared statistic to 16.1% versus 10.5% without this variable.

We show that the sentiment-momentum variable is positively correlated with monthly changes in the volume of Google searches for the term "stock market," which is available from 2004.M1 onwards. This pattern suggests that our sentiment-momentum variable helps to predict excess returns because it captures shifts in investor attention, particularly during stock market declines. Indeed, augmenting our baseline regression equation with a variable that measures momentum in the Google search index yields a highly significant negative regression coefficient and raises the adjusted R-squared statistic to 34.2% versus around 25% without this variable.

The sentiment-momentum variable and the Google search variable both help to predict episodes of sequential declines in excess stock returns, even after controlling for the presence of stochastic volatility. Both variables appear to serve as a type of investor pessimism indicator that presages investors' decisions to sell stocks. Investors' decisions to sell stocks puts further downward pressure on stock prices and contributes to a lower excess stock return over the next month. Overall, we interpret our empirical results as providing evidence that the predictability of excess stock returns is coming from both of the two sources identified by the theory.

In Section 2, we derive general expressions for excess returns on stocks and long-term bonds in a standard consumption-based asset pricing model. Section 3 shows how predictable excess returns can arise under rational expectations if the model exhibits stochastic volatility. Section 4 shows how a departure from rational expectations can be an independent source of predictable excess returns. Section 5 presents the results of predictability regressions using monthly data. Section 6 concludes. An appendix provides the details for all derivations, the sources and methods used to construct the data, and bootstrapped critical values of the t-statistics for each of our primary predictor variables.

1.1 Related literature

Theories that ascribe a causal role to sentiment or momentum in driving observed movements in stock prices have a long history in economics. Keynes (1936, p. 156) likened the stock market to a "beauty contest" where participants devote their efforts not to judging the underlying concept of beauty, but instead to "anticipating what average opinion expects the average opinion to be." Shiller (2005) describes a simple and intuitive feedback model of stock price movements. If prices start to rise, the success of some investors can attract public attention that fuels enthusiasm for the market. New (and often less sophisticated) investors enter the market and help bid up prices. Upward price motion begets expectations of further upward motion to the point where "irrational exuberance " may cause prices to exceed levels that can be justified by fundamentals. But if prices begin to sag, pessimism can take hold, causing some investors to exit the market. Downward price motion begets expectations of further downward motion, and so on, until a bottom is eventually reached. More recently, Shiller (2017) argues that investors' optimistic or pessimistic beliefs about the stock market are similar to fads that can spread throughout the popular culture like an infectious disease.

Our empirical results are broadly consistent with other studies that examine the effects of sentiment and momentum on aggregate stock market returns. Fischer and Statman (2003) and Brown and Cliff (2004) find that measures of sentiment alone have little predictive power over short (one -week or one-month) horizons. But Brown and Cliff (2005) find that higher levels of sentiment forecast negative returns over longer horizons. Lemmon and Portniaguina (2006) find that higher levels of sentiment forecast lower future returns on value stocks but not growth stocks. Schmeling (2009) finds that higher levels of consumer confidence negatively forecast aggregate stock returns across countries on average at both short and long horizons. Huang, et al. (2014) show that a refined version of the investor sentiment index originally constructed by Baker and Wurgler (2007) is a robust negative predictor of 1-month ahead excess stock returns. Our sentiment variable has no predictive power by itself, but it does help to predict 1-month ahead excess stock returns when interacted with return momentum. We purposely do not perform long-horizon predictability regressions in order to sidestep the econometric problems created by overlapping return observations, as discussed in detail by Boudoukh, Richardson, and Whitelaw (2008) and Bauer and Hamilton (2017).

Our empirical results contribute to a significant body of evidence showing that investors appear to react asymmetrically to gains versus losses. This idea can be traced back to Roy (1952) and Markowitz (1952). The asymmetric treatment of gains versus losses is a central concept in the "prospect theory" of asset pricing (Kahneman and Tversky 1979, Barberis 2013). Tetlock (2007) finds that a measure of media pessimism constructed from the "Abreast the Market" column in the Wall Street Journal is a significant negative predictor of daily returns on the Dow Jones Industrial Average (DJIA). His predictability regressions control for the lagged volatility of returns and return momentum. In a follow up study, García (2013) finds that a sentiment measure constructed using counts of positive versus negative words in financial columns of the New York Times help to predict daily DJIA returns—a result that is concentrated during recessions. Vlastakis and Markellos (2012) find that Google searches for the term "S&P 500" are positively correlated with the VIX index. Both measures tend to spike upwards during stock market declines. Klemola, Nikkinen and Peltomäki (2016) find that weekly changes in the volume of Google searches for the terms "market crash" and "bear market" are significant negative predictors of 1-week ahead percentage changes in the S&P 500 stock index. But in contrast to our approach, their predictability regressions do not control for the presence of stochastic volatility. Neely et al. (2014) find that various trend-following "technical indicators" commonly employed by investment practitioners are mostly helpful for predicting declines in excess stock returns following business cycle peaks. Cujean and Hasler (2017) find that time series momentum in excess stock returns is strongest in "bad times," defined as periods of low dividend growth.

With regard to individual traded securities, Frank and Sanati (2018) show that individual stocks exhibit over-reaction to good news on the upside, followed by reversal, but under-reaction to bad news on the downside, followed by drift. This is similar to the pattern we find for aggregate excess stock returns in response to movements in the sentiment-momentum variable. Da, Engelberg, and Gao (2011) show that an increase in the Google search intensity for individual stocks tends to predict a short-term (2-week) price increase followed by a price reversal, suggestive of over-reaction on the upside. Moskowitz, Ooi, and Pedersen (2012) find that lagged excess returns on futures contracts (a measure of momentum) predict higher excess returns in the near-term but lower excess returns at longer horizons. Asness, et al. (2015) review the considerable empirical evidence in favor of momentum-based investment strategies. Shen, Yu, and Zhao (2017) find that higher levels of investor sentiment tend to predict lower excess returns when comparing high-risk stock portfolios to low-risk portfolios.

Our empirical results are also in line with other studies that link the predictability of excess returns to evidence of departures from rational expectations. Bacchetta, Mertens, and van Wincoop (2009) find that financial markets which exhibit predictable excess returns also exhibit predictable forecast errors of returns from surveys, arguing against full rationality of the survey forecasts. Also using survey data, Casella and Gulen (2018) show that the ability of the dividend yield (inverse of the price-dividend ratio) to forecast 12-month ahead excess returns is contingent on a variable that measures the degree to which investors extrapolate past stock returns. Piazzesi, Salomao, and Schneider (2015) find evidence of departures from rational expectations in expected excess bond returns from surveys. Cieslik (2018) shows that investors' real time forecast errors about the short-term real interest rate help to account for predictability in the bond risk premium. Inflation illusion represents a particular type of departure from full rationality. A study by Katz, Lustig, and Nielsen (2017) finds that lagged inflation (a proxy for expected inflation) helps to predict lower real stock returns, suggesting a form of sticky information in stock investors' inflation forecasts.

Studies by Fischer and Statman (2002), Vissing-Jorgenson (2004), Amromin and Sharpe (2014), and Frydman and Stillwagon (2018) all find evidence of extrapolative or procyclical expected returns among stock investors. Greenwood and Shleifer (2014) and Adam, Marcet, and Beutel (2017) show that measures of investor expectations about future stock returns are strongly correlated with past stock returns and the price-dividend ratio.[1] Koijen, Schmeling, and Vrugt (2015) find similar evidence in other assets classes, including global equities, currencies, and global fixed income investments. Interestingly, even though a higher price-dividend ratio in the data empirically predicts lower *realized* stock returns (Cochrane 2008), the survey evidence shows that investors fail to take this relationship into account; instead they continue to forecast high future returns on stocks following a sustained run-up in the price-dividend ratio.

With regard to macroeconomic variables (inflation, output growth, the unemployment rate, and housing starts), Coibion and Gorodnichenko (2015) find strong evidence of predictability in the mean ex post forecast errors of professional forecasters—a feature that is not consistent with rational expectations. A follow-up study by Bordolo et al. (2018) finds that individual forecasters tend to over-react to news that causes them to revise their own forecasts.

2 Excess returns in a consumption-based model

The framework for our theoretical analysis is a standard consumption-based asset pricing model. For any type of purchased asset and any specification of investor preferences, the first-order condition of the representative investor's optimal saving choice yields

$$1 = \widehat{E}_t \left[M_{t+1} R^i_{t+1} \right], \tag{1}$$

where M_{t+1} is the investor's stochastic discount factor and R^i_{t+1} is the gross holding period return on asset type i from period t to $t+1$. The symbol $\widehat{E}_t$ represents the investor's subjective expectation, conditional on information available at time t. Under rational expectations, $\widehat{E}_t$ corresponds to the mathematical expectation operator E_t evaluated using the objective distribution of all shocks, which are assumed known to the rational investor.

For a dividend-paying stock, we have $R^s_{t+1} = \left(d_{t+1} + p^s_{t+1}\right)/p^s_t$, where p^s_t is the ex-dividend stock price and d_{t+1} is the dividend received in period $t+1$. For a default-free bond that pays a stream of coupon payments (measured in consumption units) we have $R^b_t = \left(1 + \delta p^b_{t+1}\right)/p^b_t$,

[1] We confirm this finding in Figure 9 using data from the University of Michigan survey.

where p_t^b is the ex-coupon bond price and δ is a parameter that governs the decay rate of the coupon payments. A bond purchased in period t yields a coupon stream of 1, δ, δ^2... starting in period $t+1$. When $\delta = 0$, we have a one period discount bond that delivers a single coupon payment of one consumption unit in period $t+1$. In this case, $R_{t+1}^f \equiv 1/p_t^b$ is the risk-free rate of return which is known with certainty in period t. When $\delta = 1$, we have a consol bond that delivers a perpetual stream of coupon payments, each equal to one consumption unit. More generally, the value of δ can be calibrated to achieve a target value for the Macaulay duration of the bond, i.e., the present-value weighted average maturity of the bond's cash flows.[2]

With time-separable constant relative risk aversion (CRRA) preferences, we have $M_{t+1} = \beta\left(c_{t+1}/c_t\right)^{-\alpha}$, where β is the subjective time discount factor, c_t is the investor's real consumption, and α is the risk aversion coefficient. An exponential utility function, which delivers constant absolute risk aversion (CARA), implies $M_{t+1} = \beta \exp\left[-\alpha\left(c_{t+1} - c_t\right)\right]$. With recursive preferences along the lines of Epstein and Zin (1989), we have $M_{t+1} = \beta^{\omega}\left(c_{t+1}/c_t\right)^{-\omega/\psi}\left(R_{t+1}^c\right)^{\omega-1}$, where $R_{t+1}^c \equiv \left(c_{t+1} + p_{t+1}^c\right)/p_t^c$ is the gross return on an asset that delivers a claim to consumption c_{t+1} in period $t+1$, ψ is the elasticity of intertemporal substitution (EIS), and $\omega \equiv (1-\alpha)/\left(1-\psi^{-1}\right)$. In the special case when $\alpha = \psi^{-1}$, we have $\omega = 1$ such that Epstein-Zin preferences coincide with CRRA preferences. With external habit formation preferences along the lines of Campbell and Cochrane (1999), we have $M_{t+1} = \beta\left[s_{t+1}c_{t+1}/\left(s_t c_t\right)\right]^{-\alpha}$, where $s_t \equiv 1 - x_t/c_t$ is the surplus consumption ratio, x_t is the external habit level, and α is a curvature parameter that governs the steady state level of risk aversion.

For stocks, equation (1) can be rewritten as

$$p_t^s/d_t \;=\; \widehat{E}_t\left[M_{t+1}\frac{d_{t+1}}{d_t}\left(1 + p_{t+1}^s/d_{t+1}\right)\right], \tag{2}$$

where p_t^s/d_t is the price-dividend ratio and d_{t+1}/d_t is the gross growth rate of dividends. At this point, it is convenient to define the following nonlinear change of variables:

$$z_t^s \;\equiv\; M_t\,\frac{d_t}{d_{t-1}}\left(1 + p_t^s/d_t\right), \tag{3}$$

where z_t^s represents a composite variable that depends on the stochastic discount factor, the growth rate of dividends, and the price-dividend ratio.[3] The investor's first-order condition (2) becomes

$$p_t^s/d_t = \widehat{E}_t z_{t+1}^s, \tag{4}$$

which shows that the equilibrium price-dividend ratio is simply the investor's conditional forecast of the composite variable z_{t+1}^s. Substituting $p_t^s/d_t = \widehat{E}_t z_{t+1}^s$ into the definition (3)

[2] See, for example, Lansing (2015).

[3] This nonlinear change of variables technique is also employed by Lansing (2010, 2016) and Lansing and LeRoy (2014).

yields the following transformed version of the investor's first-order condition

$$z_t^s = M_t \frac{d_t}{d_{t-1}}(1 + \widehat{E}_t z_{t+1}^s). \tag{5}$$

The gross stock return can now be written as

$$\begin{aligned} R_{t+1}^s &= \frac{d_{t+1} + p_{t+1}^s}{p_t^s} = \left(\frac{1 + p_{t+1}^s / d_{t+1}}{p_t^s / d_t} \right) \frac{d_{t+1}}{d_t} \\ &= \left(\frac{z_{t+1}^s}{\widehat{E}_t z_{t+1}^s} \right) \frac{1}{M_{t+1}}, \end{aligned} \tag{6}$$

where we have eliminated p_t^s / d_t using equation (4) and eliminated $p_{t+1}^s / d_{t+1} + 1$ using the definitional relationship (3) evaluated at time $t + 1$.

Starting again from equation (1) and proceeding in a similar fashion yields the following transformed first-order condition for bonds:

$$z_t^b = M_t (1 + \delta \widehat{E}_t z_{t+1}^b), \tag{7}$$

where $z_t^b \equiv M_t \left(1 + \delta p_t^b\right)$ and $p_t^b = \widehat{E}_t z_{t+1}^b$. The gross bond return can now be written as

$$\begin{aligned} R_{t+1}^b &= \frac{1 + \delta p_{t+1}^b}{p_t^b} \\ &= \left(\frac{z_{t+1}^b}{\widehat{E}_t z_{t+1}^b} \right) \frac{1}{M_{t+1}}. \end{aligned} \tag{8}$$

When $\delta = 0$ we have $z_{t+1}^b = M_{t+1}$ and the above expression simplifies to $R_{t+1}^b = R_{t+1}^f = 1/(\widehat{E}_t M_{t+1})$.

Combining equations (6) and (8) yields the following ratio of the gross stock return to the gross bond return:

$$\frac{R_{t+1}^s}{R_{t+1}^b} = \frac{z_{t+1}^s}{\widehat{E}_t z_{t+1}^s} \frac{\widehat{E}_t z_{t+1}^b}{z_{t+1}^b}. \tag{9}$$

Taking logs of both sides of equation (9) yields the following compact expression for the excess stock return, i.e., the realized equity premium:

$$\log\left(R_{t+1}^s\right) - \log\left(R_{t+1}^b\right) = \log\left[z_{t+1}^s / (\widehat{E}_t z_{t+1}^s)\right] - \log\left[z_{t+1}^b / (\widehat{E}_t z_{t+1}^b)\right], \tag{10}$$

where the second term on the right side simplifies to $-\log[M_{t+1}/(\widehat{E}_t M_{t+1})]$ when $\delta = 0$.

Similarly, we can compute the excess bond return which compares the return on a longer-term bond ($\delta > 0$) to the risk free rate ($\delta = 0$). In this case, we have

$$\log\left(R_{t+1}^b\right) - \log(R_{t+1}^f) = \log\left[z_{t+1}^b / (\widehat{E}_t z_{t+1}^b)\right] - \log\left[M_{t+1} / (\widehat{E}_t M_{t+1})\right]. \tag{11}$$

Equations (10) and (11) are striking. If we apply the approximation $\log(A/B) \simeq (A-B)/B$ to the terms that appear on the right sides of equations (10) and (11), then $A-B$ would represent the investor's forecast error. Imposing rational expectations such that $\widehat{E}_t = E_t$ might therefore seem to imply that $\log(A/B)$ should be wholly unpredictable. However, as we show below, predictability can arise under rational expectations if the model exhibits stochastic volatility. Nonetheless, the intuition of $\log(A/B) \simeq (A-B)/B$ helps to explain why is it very difficult for consumption-based asset pricing models to generate significant predictability of excess returns under rational expectations. The same intuition also helps to explain why these same models struggle to produce a sizeable mean equity premium, except in cases where there is a high degree of curvature in investor preferences. The high degree of curvature serves to invalidate the approximation $\log(A/B) \simeq (A-B)/B$.

3 Predictability from stochastic volatility

In the special case of CRRA utility, normally and independently distributed consumption growth, and $c_t = d_t$, the equilibrium price-dividend ratio is constant. The realized equity premium relative to the risk free rate is $\log(R^s_{t+1}/R^f_{t+1}) = \varepsilon_{t+1} + (\alpha - 0.5)\,\sigma^2_\varepsilon$, where ε_{t+1} is the innovation to consumption growth and σ^2_ε is the associated variance.[4] In this special case, excess returns at time $t+1$ are not predictable using variables dated time t or earlier. But as we show below, models that exhibit stochastic volatility from either exogenous or endogenous sources can generate predictability under rational expectations.

When solving consumption-based asset pricing models, it is common to employ approximation methods that deliver conditional log-normality of the relevant variables. If a random variable q_t is conditionally log-normal, then

$$\log(E_t q_{t+1}) \;=\; E_t\left[\log(q_{t+1})\right] \;+\; \tfrac{1}{2} Var_t\left[\log(q_{t+1})\right], \tag{12}$$

where Var_t is the mathematical variance operator conditional on information available to the investor at time t.

Starting from equation (10) and imposing rational expectations such that $\widehat{E}_t = E_t$, we make the assumption that the composite variables z^s_{t+1} and z^b_{t+1} are both conditionally log-normal. Making use of equation (12) to eliminate $\log\left(E_t z^s_{t+1}\right)$ and $\log\left(E_t z^b_{t+1}\right)$ yields the following alternate expression for the excess stock return

$$\begin{aligned}\log(R^s_{t+1}) - \log(R^b_{t+1}) \;&=\; \left[\log\left(z^s_{t+1}\right) - E_t \log\left(z^s_{t+1}\right)\right] \;-\; \left[\log(z^b_{t+1}) - E_t \log(z^b_{t+1})\right] \\ &\quad -\tfrac{1}{2} Var_t\left[\log\left(z^s_{t+1}\right)\right] + \tfrac{1}{2} Var_t\left[\log(z^b_{t+1})\right] \end{aligned} \tag{13}$$

[4] For the derivation, see Lansing and LeRoy (2014), Appendix B. Note that in the risk neutral case with $\alpha = 0$, we have the result that $E[R^s_{t+1}/R^f_{t+1}] = E[\exp(\varepsilon_{t+1} - 0.5\sigma^2_\varepsilon)] = 1$.

where $z^b_{t+1} = M_{t+1}$ for a 1-period discount bond with $\delta = 0$. Notice that the first two terms in equation (13) are the investor's forecast errors for $\log\left(z^s_{t+1}\right)$ and $\log\left(z^b_{t+1}\right)$, respectively. These forecast errors cannot be a source of predictability under rational expectations. However, the last two terms in equation (13) show that predictability can arise under rational expectations if the laws of motion for the endogenous variables $\log\left(z^s_{t+1}\right)$ and $\log(z^b_{t+1})$ exhibit stochastic volatility. This is because the conditional variance terms at time t but would partly determine the realized excess return at time $t+1$.

Specializing equation (13) to the case where $\delta = 0$ such that $R^b_{t+1} = R^f_{t+1}$ and $z^b_{t+1} = M_{t+1}$, we have

$$\begin{aligned} \log(R^s_{t+1}) - \log(R^f_{t+1}) \;&=\; \left[\log\left(z^s_{t+1}\right) - E_t \log\left(z^s_{t+1}\right)\right] \;-\; \left[\log(M_{t+1}) - E_t \log(M_{t+1})\right] \\ &\quad -\tfrac{1}{2} Var_t[\log \underbrace{(M_{t+1} R^s_{t+1} p^s_t / d_t)}_{=z^s_{t+1}}] + \tfrac{1}{2} Var_t\left[\log(M_{t+1})\right], \end{aligned} \tag{14}$$

where the last line exploits the definition of z^s_{t+1}. Equation (14) implies that the rational expected excess return on stocks is given by

$$E_t[\log\left(R^s_{t+1}\right)] - \log(R^f_{t+1}) = -\tfrac{1}{2} Var_t[\log(M_{t+1} R^s_{t+1} p^s_t / d_t)] + \tfrac{1}{2} Var_t\left[\log(M_{t+1})\right], \tag{15}$$

where R^f_{t+1} is known at time t.

Following Campbell (2014), an alternative expression for the rational expected excess return on stocks can be derived by decomposing the conditional rational expectation in equation (1) as follows

$$\underbrace{E_t\left[M_{t+1} R^s_{t+1}\right]}_{=1} \;=\; \underbrace{E_t M_{t+1}}_{=1/R^f_{t+1}} E_t R^s_{t+1} \;+\; Cov_t\left[M_{t+1},\; R^s_{t+1}\right]. \tag{16}$$

Solving the above expression for $E_t\left(R^s_{t+1}\right)/R^f_{t+1}$ and then taking logs yields

$$\log\left(E_t R^s_{t+1}\right) - \log(R^f_{t+1}) \;=\; \log\left\{1 - Cov_t\left[M_{t+1},\; R^s_{t+1}\right]\right\}, \tag{17}$$

$$E_t\left[\log\left(R^s_{t+1}\right)\right] - \log(R^f_{t+1}) \;=\; \log\left\{1 - Cov_t\left[M_{t+1},\; R^s_{t+1}\right]\right\} - \tfrac{1}{2} Var_t\left[\log R^s_{t+1}\right], \tag{18}$$

where, in going from equation (17) to (18), we have assumed conditional log-normality of the gross stock return R^s_{t+1}. The above expression shows that the rational expected excess return on stocks will be predictable if $Cov_t\left[M_{t+1},\; R^s_{t+1}\right]$ or $Var_t\left[\log R^s_{t+1}\right]$ are time-varying. Attanasio (1991) undertakes a derivation similar to equation (18) and concludes (p. 481): "predictability of excess returns constitutes direct evidence against the joint hypothesis that markets are efficient and second moments are constant." While our derivation of equation (14) delivers a similar conclusion, it helps to focus attention on investor forecast errors as an alternative source of predictable excess returns when expectations are not fully rational.

3.1 Exogenous stochastic volatility

Here we provide an analytical example to show how exogenous stochastic volatility in the law of motion for consumption growth can generate predictable excess returns under rational expectations. Suppose the investor's stochastic discount factor is given by

$$\begin{aligned} M_{t+1} &= \beta\left(c_{t+1}/c_t\right)^{-\alpha} = \beta \exp\left(-\alpha x_{t+1}^c\right), && (19)\\ x_{t+1}^c &= \overline{x} + \rho\left(x_t^c - \overline{x}\right) + \sigma_t \varepsilon_{t+1}, \quad |\rho| < 1, \quad \varepsilon_t \sim NID\left(0,1\right), && (20)\\ \sigma_{t+1}^2 &= \overline{\sigma}^2 + \gamma\left(\sigma_t^2 - \overline{\sigma}^2\right) + u_{t+1}, \quad |\gamma| < 1, \quad u_t \sim NID\left(0, \sigma_u^2\right), && (21) \end{aligned}$$

where $x_{t+1}^c \equiv \log\left(c_{t+1}/c_t\right)$ is real consumption growth that evolves as an AR(1) process with mean $\overline{x}$ and persistence parameter ρ. The innovation ε_{t+1} is normally and independently distributed (NID) with mean zero and variance of one. We allow for exogenous stochastic volatility along the lines of Bansal and Yaron (2004), where γ governs the persistence of volatility and u_{t+1} is the innovation to volatility.[5] Real dividend growth $x_{t+1}^d \equiv \log\left(d_{t+1}/d_t\right)$ is given by

$$x_{t+1}^d = x_{t+1}^c + v_{t+1}, \quad v_t \sim NID\left(0, \sigma_v^2\right), \tag{22}$$

where v_{t+1} is an innovation with mean zero and variance σ_v^2.

Under rational expectations, we have

$$R_{t+1}^f = 1/(E_t M_{t+1}) = \beta^{-1} \exp\left[\alpha\overline{x} + \alpha\rho\left(x_t^c - \overline{x}\right) - \tfrac{1}{2}\alpha^2\sigma_t^2\right], \tag{23}$$

$$\log\left[M_{t+1}/(E_t M_{t+1})\right] = -\alpha\sigma_t\, \varepsilon_{t+1} - \tfrac{1}{2}\alpha^2\sigma_t^2. \tag{24}$$

The left side of equation (24) will be predictable only when σ_t^2 is time-varying, i.e., when $\sigma_u^2 > 0$. Appendix A provides an approximate analytical solution for the composite variable z_{t+1}^s that appears in the excess stock return equation (10). Under rational expectations, the approximate solution implies the following expression:

$$\log\left[z_{t+1}^s/(E_t z_{t+1}^s)\right] = a_1\sigma_t\, \varepsilon_{t+1} + a_2 u_{t+1} + v_{t+1} - \tfrac{1}{2}\left(a_1\right)^2\sigma_t^2 - \tfrac{1}{2}\left(a_2\right)^2\sigma_u^2 - \tfrac{1}{2}\sigma_v^2, \tag{25}$$

where a_1 and a_2 are Taylor series coefficients that depend on the model parameters. Substituting equations (24) and (25) into the excess stock return equation (10) and imposing $\delta = 0$ such that $R_{t+1}^b = R_{t+1}^f$ yields

$$\begin{aligned} \log\left(R_{t+1}^s\right) - \log(R_{t+1}^f) &= \left(a_1 + \alpha\right)\sigma_t\, \varepsilon_{t+1} + a_2 u_{t+1} + v_{t+1} \\ &\quad + \tfrac{1}{2}\left[\alpha^2 - \left(a_1\right)^2\right]\sigma_t^2 - \tfrac{1}{2}\left(a_2\right)^2\sigma_u^2 - \tfrac{1}{2}\sigma_v^2, && (26) \end{aligned}$$

[5] When simulating their model, Bansal and Yaron (2004) ensure that σ_t^2 remains positive by replacing any negative realizations with a very small number, which happens in about 5% of the realizations.

which shows that excess stock returns will be predictable only when σ_t^2 is time-varying, provided that $\alpha^2 - (a_1)^2 \neq 0$. In the special case when $\rho = 0$, the first Taylor series coefficient becomes $a_1 = 1 - \alpha$ and the coefficient on σ_t^2 in equation (26) becomes $\alpha - 0.5$, which is increasing in the value of the risk aversion coefficient α.

It is important to note that the mere presence of the state variable σ_t^2 in equation (26) does not guarantee that the observed amount of excess return predictability will be statistically significant. Depending on the model calibration, the fundamental shock innovations ε_{t+1}, u_{t+1} and v_{t+1} may end up being the main drivers of fluctuations in *realized* excess returns, thus washing out the influence of the state variable σ_t^2 which is sole driver of fluctuations in *expected* excess returns. This washing out effect appears to be present in most of the leading consumption based asset pricing models.

In the rational external habit model of Campbell and Cochrane (1999), stochastic volatility is achieved via a nonlinear sensitivity function that determines how innovations to consumption growth influence the logarithm of the surplus consumption ratio. In the rational long-run risks model of Bansal and Yaron (2004), stochastic volatility is achieved by assuming an AR(1) law of motion for the volatility of innovations to consumption growth and dividend growth, along the lines of equation (21). Despite these features, subsequent analysis has shown that these fully-rational models fail to deliver predictability results that resemble those found in the data. Li (2001) extends the model of Campbell and Cochrane (1999) to allow for AR(1) consumption growth. He finds (p. 895) "The fraction of stock [excess] return variance that can be explained by surplus consumption is economically small."

Kirby (1998) had previously shown that the rational habit model of Abel (1990) and the rational recursive preferences model of Epstein and Zin (1989, 1991) both fail to generate significant predictability in excess stock returns. Chen and Hwang (2018) extend Kirby's analysis to the rational models of Campbell Cochrane (1999) and Bansal and Yaron (2004) and find that neither model can generate any significant predictable excess returns. Using simulated data, Beeler and Campbell (2012) show that the rational long-run risk models of Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2012) both fail to match the predictability patterns observed in the data.

3.2 Endogenous stochastic volatility

Endogenous stochastic volatility can arise from the nonlinear nature of the model's functional forms. Consider the time-separable exponential utility function $u(c_t) = 1 - \exp(-\alpha c_t)$ which exhibits constant absolute risk aversion such that $-u''(c_t)/u'(c_t) = \alpha$. The investor's stochastic discount factor is given by

$$M_{t+1} = \beta \exp\left[-\alpha\left(c_{t+1}-c_t\right)\right] = \beta \exp\left(-\alpha c_t x_{t+1}^c\right), \tag{27}$$

$$x_{t+1}^c = \overline{x} + \rho\left(x_t^c - \overline{x}\right) + \varepsilon_{t+1}, \qquad |\rho| < 1, \qquad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right), \tag{28}$$

where $x_{t+1}^c \equiv \left(c_{t+1} - c_t\right)/c_t$ is real consumption growth that evolves as an AR(1) process with constant innovation variance σ_ε^2.

Under rational expectations, we have

$$R_{t+1}^f = 1/(E_t M_{t+1}) = \beta^{-1} \exp\left\{c_t\left[\alpha \overline{x} + \alpha\rho\left(x_t^c - \overline{x}\right)\right] - \tfrac{1}{2}\alpha^2\sigma_\varepsilon^2 c_t^2\right\}, \tag{29}$$

$$\log\left[M_{t+1}/(E_t M_{t+1})\right] = -\alpha c_t\, \varepsilon_{t+1} - \tfrac{1}{2}\alpha^2\sigma_\varepsilon^2 c_t^2, \tag{30}$$

which shows that the left side of equation (30) will be predictable because c_t^2 is time-varying and helps to partly determine the realized excess stock return at time $t+1$. Similarly, the term $\log\left[z_{t+1}^s/(E_t z_{t+1}^s)\right]$ that appears in the excess stock return equation (10) will also be predictable.

4 Predictability from market inefficiency

The failure of leading rational asset pricing models to produce empirically realistic predictability of excess stock returns lends support to considering a second possible source of predictability, namely, departures from rational expectations that give rise to predictable investor forecast errors. Here we provide two simple examples to illustrate this idea.

4.1 Random walk forecast

For the first example, suppose that the representative investor employs a naive random-walk forecast such that $\widehat{E}_t M_{t+1} = M_t$, where the stochastic discount factor is governed by equations (19) through (21). In this case, we have

$$\begin{aligned}\log[M_{t+1}/(\widehat{E}_t M_{t+1})] &= \log\left[\frac{\beta \exp\left(-\alpha x_{t+1}^c\right)}{\beta \exp\left(-\alpha x_t^c\right)}\right], \\ &= -\alpha\sigma_t\, \varepsilon_{t+1} + \alpha\left(1-\rho\right)\left(x_t^c - \overline{x}\right),\end{aligned} \tag{31}$$

which shows that $\log[M_{t+1}/(\widehat{E}_t M_{t+1})]$ will be predictable due to the term involving $x_t^c - \overline{x}$. The expression for investor's subjective forecast error $\log\left(M_{t+1}\right) - \widehat{E}_t \log\left(M_{t+1}\right)$ also includes the term $\alpha\left(1-\rho\right)\left(x_t^c - \overline{x}\right)$ and is therefore predictable using the previous period's forecast error. Similarly, the investor's use of the random-walk forecast $\widehat{E}_t\, z_{t+1}^s = z_t^s$ would introduce a term involving $x_t^c - \overline{x}$ into the equilibrium expression for $\log[(z_{t+1}^s/(\widehat{E}_t z_{t+1}^s)]$ that appears in the excess stock return equation (10).

4.2 Sticky information

For the second example, consider an environment with "sticky information" along the lines described by Mankiw and Reis (2002) and Carroll (2003). In this case, only a fraction $\lambda \in (0, 1]$ of investors in the market update their conditional forecast to reflect current information each period. The consensus market forecast in any given period is thus an exponentially-weighted moving average of current and past vintages of rational forecasts. With sticky information, the consensus market forecast for the stochastic discount factor would take the recursive form

$$\begin{aligned} \widehat{E}_t M_{t+1} &= (E_t M_{t+1})^{\lambda} (\widehat{E}_{t-1} M_t)^{1-\lambda} \\ &= \prod_{j=0}^{\infty} (E_{t-j} M_{t-j+1})^{\lambda(1-\lambda)^j}, \end{aligned} \tag{32}$$

where the second line is obtained by repeated substitution of $\widehat{E}_{t-j} M_{t-j+1}$ for $j = 1, 2, 3...$ Starting from equations (19) through (21), sticky information yields the result

$$\begin{aligned} \log[M_{t+1}/(\widehat{E}_t M_{t+1})] \;\; = \;\; & -\alpha\sigma_t \, \varepsilon_{t+1} \; - \; \lambda \tfrac{1}{2} \alpha^2 \sigma_t^2 \\ & - (1-\lambda) \left[\log\beta \; + \; \alpha\overline{x} + \alpha\rho\, (x_t^c - \overline{x}) + \log(\widehat{E}_{t-1} M_t) \right], \end{aligned} \tag{33}$$

which collapses to equation (24) when $\lambda = 1$. The above expression shows there are now three sources of predictability for $\log[M_{t+1}/(\widehat{E}_t M_{t+1})]$. In addition to the stochastic volatility term involving σ_t^2, the terms involving $x_t^c - \overline{x}$ and $\log(\widehat{E}_{t-1} M_t)$ would represent additional sources of predictability when $\lambda < 1$. Similarly, a sticky information environment would introduce additional terms involving $x_t^c - \overline{x}$ and $\log(\widehat{E}_{t-1} z_t^s)$ into the equilibrium expression for $\log[(z_{t+1}^s/(\widehat{E}_t z_{t+1}^s)]$ that appears in the excess stock return equation (10). Equation (33) helps to motivate our empirical strategy (described in the next section) which controls for the presence of stochastic volatility in order to identify possible alternative sources of predictable excess stock returns that are linked to market inefficiency.

A more elaborate sticky information model would allow the updating fraction λ to vary over time in response to recent market developments. Along these lines, we present evidence in Section 5.3 that investors pay more attention to the stock market during periods when stock prices and consumer sentiment are both declining. Abel, Eberly, Panageas (2013) develop a model of "optimal inattention to the stock market." In some configurations, their model can generate a state contingent time between agents' decision to update their information.

5 Predictability regressions

In this section we describe: (1) our motivation for the choice of predictor variables, (2) properties of the data, and (3) the results of 1-month ahead predictability regressions.

5.1 Choice of predictor variables

Our predictability regressions take the following form:

$$\begin{aligned} \mathbf{ersf}_{t+1} \;\; &= \;\; c_0 + c_1\,\mathbf{pd} + c_2\,\mathbf{vrp3} + c_3\,\Delta\mathbf{ff12} \\ &\quad + c_4\,\Delta\mathbf{sent12} + c_5\,\Delta\mathbf{ersf} + c_6\,\Delta\mathbf{sent12} \times \Delta\mathbf{ersf}, \end{aligned} \tag{34}$$

where $\mathbf{ersf}_{t+1} \equiv \log(R^s_{t+1}/R^f_{t+1})$ is the realized excess return on stocks relative to the risk free rate in month $t+1$. The return on stocks R^s_{t+1} is measured by the 1-month nominal return on the S&P 500 stock index. The risk free rate R^f_{t+1} is measured by the 1-month nominal return on a 3-month Treasury Bill. The predictor variables on the right side of equation (34) are all dated month t. We do not perform long-horizon predictability regressions because the empirical reliability of such results have been called into question by Boudoukh, Richardson, and Whitelaw (2008) and Bauer and Hamilton (2017).

The variable **pd** is the price-dividend ratio for the S&P 500 stock index— a standard predictor variable defined as the end-of-month nominal closing value of the index divided by cumulative nominal dividends over the past 12 months. Any consumption-based asset pricing model with rational expectations implies that the price-dividend ratio will depend on the model's fundamental state variables, including any that would give rise to the conditional variance terms in equation (13). We illustrate this idea in Appendix A with a rational asset pricing model that exhibits stochastic volatility of consumption growth along the lines of the long-run risk model of Bansal and Yaron (2004). Cochrane (2017) shows that the price-dividend ratio in U.S. data exhibits strong co-movement with a measure of "surplus consumption" constructed from the data using the parameters of Campbell and Cochrane (1999) habit formation model. Hence, including **pd** as a regressor is a way to control indirectly for the presence of stochastic volatility when the state variables that drive stochastic volatility are not directly observable.

The variable **vrp3** is the 3-month moving average of the "variance risk premium" originally defined by Bollerslev, Tauchen, and Zhou (2009) as the difference between the implied volatility from options on the S&P 500 index and the realized volatility of the S&P 500 stock index. Numerous studies find that variance risk premium is a useful predictor of excess stock returns.[6] Including **vrp3** as a regressor is a way to control directly for the presence of stochastic volatility since **vrp3** represents a time-varying measure of stock return variance. The variance risk premium can be quite volatile from one month to the next. Our preliminary investigations

[6] See, for example, Drechsler and Yaron (2011), Bollerslev, et al. (2014), and Zhou (2018).

revealed that the 3-month moving average of the variance risk premium is a better predictor of monthly excess stock returns than the variance risk premium measured over the most recent month. We also experimented with regression equations that included implied volatility and realized volatility as separate predictor variables, but the resulting fit was not improved. Christensen and Prabhala (1998) show that past implied volatility and past realized volatility are both useful for predicting future realized volatility. Other studies, such as Attanasio (1991), Guo (2006), and Welch and Goyal (2008), have employed measures of realized stock return volatility as predictor variables.

The variable $\mathbf{\Delta ff12}$ is the 12-month change the federal funds rate. This variable bears some resemblance to the "stochastically detrended nominal risk free rate" employed by Guo (2006) as a predictor variable. Along similar lines, Campbell and Yogo (2006) and Ang and Bekaert (2007) employ the nominal 3-month Treasury bill yield as a predictor variable. A study by Miranda-Agrippino and Rey (2018) finds that a single global factor, partly driven by U.S. monetary policy, helps to explains a significant share of the variance of equity and bond returns around the world.[7] From a rational asset pricing perspective, equations (23) and (29) show that changes in the risk free rate would capture changes in the variables that drive stochastic volatility. Indeed, sample periods when the variable $\mathbf{\Delta ff12}$ is declining roughly correspond to sample periods when the 12-month rolling variance of the federal funds rate is increasing. Welch and Goyal (2008) employ the Treasury term spread as predictor variable. Faria and Verona (2018) show that the low-frequency component of the Treasury term spread is a better predictor of excess stock returns than the Treasury term spread itself. From 1990.M3 to 2017.M12, the correlation coefficient between $\mathbf{\Delta ff12}$ and the 12-month change in the Treasury term spread (nominal yield difference between 10-year Treasury bond and 3-month Treasury bill) is -0.82. Similar to **pd,** we view the inclusion of $\mathbf{\Delta ff12}$ as a way to control indirectly for the presence of stochastic volatility.

Although **pd, vrp3**, and $\mathbf{\Delta ff12}$ are intended to control for stochastic volatility, these controls are imperfect. Departures from rational expectations could affect the price-dividend ratio and the variance of stock returns. Numerous empirical studies starting with Shiller (1981) and LeRoy and Porter (1981) have shown that stock prices appear to exhibit excess volatility when compared to fundamentals, as measured by the discounted stream of ex post realized dividends.[8] A recent study by Greenwood, Shleifer, and You (2017) using stock returns for various U.S. industries finds that stock valuation ratios and stock return volatility both increase substantially during the 24 months preceding what they define as "bubble peaks." Movements in stock prices that are linked to market inefficiency could influence $\mathbf{\Delta ff12}$ if Federal Reserve

[7]Similarly, Luo and Ma (2017) find that a global factor is an important driver of house price movements around the world.

[8]Lansing and LeRoy (2014) provide a recent update on this literature.

monetary policy reacts to the stock market. As noted by Brav and Heaton (2002), it is often difficult to distinguish behavioral versus rational explanations of financial market phenomena. Nevertheless, in our empirical analysis, we treat **pd, vrp3**, and **Δff12** as controls for stochastic volatility and look for evidence of market inefficiency using other predictor variables.[9]

As reviewed in the introduction, numerous empirical studies find that measures of sentiment and momentum are often helpful in predicting aggregate stock market returns or individual security returns. The variable **Δsent12** is the 12-month change in the University of Michigan's consumer sentiment index—a gauge of investor optimism or pessimism. We experimented we higher frequency changes in the sentiment index, but the resulting fit was not improved. The variable **Δersf** is the 1-month change in the excess stock return—a measure of return momentum. In a recent comprehensive study of excess return predictability, Gu, Kelly, and Xiu (2018) find that "allowing for (potentially complex) interactions among the baseline predictors" can substantially improve forecasting performance. Motivated by this finding, we interact the sentiment and momentum variables to obtain **Δsent12×Δersf** as an additional predictor variable. The three "behavioral" predictor variables are intended to detect market inefficiency that may manifest itself in the form of excessive optimism or pessimism, extrapolation, or over/under reaction to news.

5.2 Data

We use monthly data for the period from 1990.M3 to 2017.M12. The starting date for the sample is governed by the availability of data for **vrp3** which makes use of the VIX index. The sources and methods used to construct the data are described in Appendix B.

Table 1 reports summary statistics of excess stock returns and the six predictor variables. The average monthly excess return on stocks relative to the risk free rate is 0.57%. The summary statistics show that excess stock returns exhibit negative skewness and excess kurtosis. Interestingly, four out of the six predictor variables also exhibit negative skewness and excess kurtosis, namely, **vrp3, Δff12**, **Δsent12,** and **Δsent12×Δersf**.

The four predictor variables **pd, vrp3, Δff12**, and **Δsent12** are each highly persistent. The other two predictor variables **Δersf,** and **Δsent12×Δersf** exhibit negative autocorrelation statistics. In Appendix C, we use a bootstrap procedure to gauge the quantitative impact of persistent regressors on the critical values of the standard *t*-statistic. The bootstrapped critical values are not substantially different from the asymptotic ones, but there are noticeable shifts in either direction for some of the persistent predictor variables.

The strongest correlation amongst the predictor variables is between **Δff12** and **Δsent12.**

[9] We experimented with including additional controls for stochastic volatility in the form of volatility measures for consumption growth or dividend growth, computed using rolling data windows of various lengths. None of these measures were found to be statistically significant.

This pair exhibits a correlation coefficient of 0.35. The interaction variable $\mathbf{\Delta sent12}\times\mathbf{\Delta ersf}$ exhibits a quantitatively small correlation coefficient with each of the other five predictor variables, supporting its inclusion as additional regressor.

5.3 Predictive regressions

The results of our predictability regressions are summarized in Tables 2 through 5 and Figures 1 through 7. The t-statistics for the estimated coefficients are computed using Newey-West HAC corrected standard errors. Bold entries in the tables indicate that the predictor variable is significant at the 5% level using the bootstrapped critical values. Adjusted R-squared values are shown at the bottom of each regression specification.

Figure 1 shows scatter plots for each of the six predictor variables in month t versus the excess return on stocks in month $t+1$. The slope of the univariate regression lines show that higher levels of **pd** (upper left panel) and $\mathbf{\Delta sent12}\times\mathbf{\Delta ersf}$ (lower right panel) tend to forecast a lower excess stock return while higher levels of the other four predictor variables **vrp3,** $\mathbf{\Delta ff12}$, $\mathbf{\Delta sent12}$**,** and $\mathbf{\Delta ersf}$ tend to forecast a higher excess return. Extreme values for the data points are labeled, many of which occurred during the global financial crisis of 2008 and 2009. Our main results are robust to sample periods that do not include the crisis.

Table 2 shows the full-sample regression results. Specification 1 includes **pd, vrp3** and $\mathbf{\Delta ff12}$ which are the predictor variables that control for stochastic volatility. Recall that stochastic volatility is the only source of predictability under rational expectations. Regardless of the regression specification, the estimated coefficient on **pd** is always negative and statistically significant. This robust result is consistent with numerous previous studies which find that a higher price-dividend ratio predicts a lower excess stock return. The estimated coefficient on **vrp3** is positive and statistically significant, also consistent with previous studies. The literature has interpreted the variance risk premium as a proxy for macroeconomic uncertainty. The positive coefficient on **vrp3** implies that higher uncertainty in month t induces investors to demand a higher excess stock return in month $t+1$. The estimated coefficient on $\mathbf{\Delta ff12}$ is positive and statistically significant. As shown in Appendix C, the relevant bootstrapped critical values for **pd, vrp3,** and $\mathbf{\Delta ff12}$ are -2.473, 1.933, and 2.120, respectively. If we use the variance risk premium measured over the most recent month in place of **vrp3,** the regression coefficient remains positive and strongly significant, but the adjusted R-squared statistic for Specification 1 drops from 13.0% to 10.1%.

The positive and statistically significant coefficient on $\mathbf{\Delta ff12}$ does not have a direct counterpart with previous results in the literature but, as we shall see, it is very robust across different regression specifications and sample periods. Guo (2006) reports a negative and statistically significant coefficient on the stochastically detrended nominal risk free rate (the risk free rate minus its past 12-month moving average) using quarterly data. Campbell and Yogo (2006) report a negative and statistically significant coefficient on the nominal 3-month Treasury bill yield using quarterly and monthly data. Ang and Bekaert (2007) report a negative and statistically significant coefficient on the nominal 3-month Treasury bill yield using annual data. If we replace **Δff12** with either the federal funds rate itself or its 12-month moving average, then we obtain a negative coefficient, but one that is not statistically significant. If we replace **Δff12** with the detrended federal funds rate (the funds rate minus its 12-month moving average), then we once again obtain a statistically significant positive coefficient, but the adjusted R-squared statistic is somewhat reduced. Since **Δff12** captures changes in monetary policy over the medium-term, the positive coefficient implies that a more contractionary (expansionary) monetary policy induces investors to demand a higher (lower) excess stock return. Along these lines, Bekaert, Hoerova, and Lo Duca (2013) find that a more contractionary monetary policy increases risk aversion in the future, implying a higher expected excess return on stocks.

Specification 2 in Table 2 adds the two behavioral predictor variables **Δsent12** and **Δersf** while Specification 3 goes a step further and adds the interaction variable **Δsent12**×**Δersf.** The estimated coefficients on **Δsent12** and **Δersf** are not statistically significant. A finding of non-significance for these two variables is a typical result across all of our regression specifications. However, the estimated coefficient on **Δsent12**×**Δersf** is negative and strongly significant, exhibiting a t-statistic of -4.377. The bootstrapped critical value from Appendix C is -2.049. The fact that **Δsent12**×**Δersf** is significant while neither **Δsent12** or **Δersf** are significant individually argues against the interpretation that either one of these variables is actually measuring rational investors' time-varying risk aversion. Specification 3 delivers an adjusted R-squared statistic of 17.3% versus 13.0% for Specification 1 and 12.9% for Specification 2. The full-sample fitted values from Specification 3 are plotted in Figure 2.

At first glance, the negative coefficient on **Δsent12**×**Δersf** in Specification 3 is suggestive of over-reaction of excess stock returns on the upside followed by reversal in the excess return (when **Δsent12** and **Δersf** are both positive) combined with under-reaction of excess stock returns on the downside followed by further downward drift in the excess return (when **Δsent12** and **Δersf** are both negative). Specification 4 explores this idea further using a set of four dummy variables to classify the four possible sign combinations of **Δsent12** and **Δersf.** The symbol Δ^+ represents a positive change in the predictor variable while Δ^- represents a negative change. Specification 4 shows that the estimated coefficient on the sentiment-momentum variable is negative for all four sign combinations. However, the statistical significance of this variable derives mainly from periods of declining sentiment and negative return momentum, forecasting a further decline in the excess stock return.[10] We will return to this point in

[10] The frequencies of occurrence for the four possible sign combinations are as follows: 27% ($\Delta^+\Delta^+$), 29%

more detail below when we link movements in the sentiment-momentum variable to an index of Google searches for the term "stock market." Search volume for this term tends to spike during pronounced stock market declines.

We can also offer some interpretation of the negative estimated coefficients on the sentiment-momentum variable for the two cases when this variable is negative. This interpretation is speculative, however, given that the estimated coefficients for these two cases are not statistically significant. When $\mathbf{\Delta sent12} < 0$ and $\mathbf{\Delta ersf} > 0$, positive return momentum may provide a short-term bullish signal for stocks in a bear market where sentiment has been declining over the past year, thus forecasting a higher excess stock return over the next month. When $\mathbf{\Delta sent12} > 0$ and $\mathbf{\Delta ersf} < 0$, negative return momentum may represent a temporary correction in a bull market where sentiment has been rising over the past year. This event may represent a "buy-the-dip" opportunity for stocks, forecasting a higher excess stock return over the next month.

Table 3 shows split-sample regression results. The first split-sample runs from 1990.M3 to 2003.M12 while the second runs from 2004.M1 to 2017.M12. The regression results for the first split-sample are similar to the full-sample results, with the exception that the adjusted R-squared statistics are now somewhat lower. These results confirm that our basic findings are robust to the exclusion of data associated with the global financial crisis of 2008 and 2009. The results for the second split-sample show much higher adjusted R-squared statistics—some in the vicinity of 25%. Notice that the regression coefficients on **pd** and $\mathbf{\Delta ff12}$ are much larger in magnitude in the second split sample. This is because both variables exhibit lower average values from 2004 onwards. In Specification 3, the variable $\mathbf{\Delta sent12} \times \mathbf{\Delta ersf}$ is statistically significant in both split-samples. The bootstrapped critical value from Appendix C is -2.049. In Specification 4, the estimated coefficient on the sentiment-momentum variable is once again negative for all four sign combinations, regardless of the sample period. However, the reduced number of observations for each particular sign combination now serves to dilute the statistical significance.

Figure 3 shows the results of rolling regressions using Specification 3, where each regression employs a 10-year (120-month) moving window of data. The rolling regression coefficients on **pd, vrp3** and $\mathbf{\Delta ff12}$ (top panels) exhibit consistent signs and are almost always significant from the early 2000s onwards. The rolling regression coefficient on $\mathbf{\Delta sent12}$ (bottom left panel) is rarely significant while the rolling regression coefficient on $\mathbf{\Delta ersf}$ (bottom middle panel) is never significant. However, similar to the results for **pd, vrp3** and $\mathbf{\Delta ff12}$, the rolling regression coefficient on $\mathbf{\Delta sent12} \times \mathbf{\Delta ersf}$ (bottom right panel) exhibits a consistent sign and is almost always significant from the early 2000s onwards. These results show that

($\Delta^{+}\Delta^{-}$), 23% ($\Delta^{-}\Delta^{+}$), and 21% ($\Delta^{-}\Delta^{-}$).

the sentiment-momentum variable is a robust predictor of excess stock returns.

Table 4 compares goodness-of-fit statistics for predictive regressions that include the variable $\boldsymbol{\Delta}\mathbf{sent12}\times\boldsymbol{\Delta}\mathbf{ersf}$ versus otherwise similar regressions that omit this variable. An asterisk (*) indicates the superior goodness-of-fit statistic for the two regressions being compared. The goodness of fit statistics are: (1) the root mean squared forecast error ($RMSFE$), (2) the mean absolute forecast error ($MAFE$), the correlation coefficient between the forecasted excess return and the realized excess return ($Corr$), and (4) either the adjusted R-squared statistic (for in-sample forecasts) or the out-of-sample R-squared statistic (for out-of-sample forecasts). The out-of-sample R-squared statistic compares the performance of the predictive regression to a benchmark forecast model that assumes constant excess stock returns. The statistic is defined as one minus the ratio of summed squared residuals from the predictive regression to summed squared deviations of realized excess returns from the mean excess return of the estimation sample.

The top panel of Table 4 shows the results for in-sample regressions. The middle panel shows the results for split out-of-sample regressions, where the regression equation is estimated for the period from 1990.M3 to 2003.M12 and then used to forecast excess stock returns for the period from 2004.M1 to 2017.M12. The bottom panel shows the results for rolling out-of-sample regressions, where each regression employs a 10-year (120-month) moving window of data. The regression equation estimated for a given window of data is used to forecast the excess stock return for the subsequent month, without re-estimation of the equation.

In all cases in Table 4, including $\boldsymbol{\Delta}\mathbf{sent12}\times\boldsymbol{\Delta}\mathbf{ersf}$ in the predictive regression serves to improve forecast performance as measured by the goodness-of-fit statistic. When including $\boldsymbol{\Delta}\mathbf{sent12}\times\boldsymbol{\Delta}\mathbf{ersf}$, the out-of-sample R-squared statistics are 15.9% and 16.1% for the split and rolling out-of-sample regressions, respectively. When omitting $\boldsymbol{\Delta}\mathbf{sent12}\times\boldsymbol{\Delta}\mathbf{ersf}$**,** the corresponding statistics are substantially lower at 8.82% and 10.5%. Figures 4, 5 and 6 show scatter plots of realized versus predicted excess returns for each of the various regression pairings in Table 4. A perfect forecast in any given month would lie directly on the 45-degree line.

Having established that the sentiment-momentum variable is a robust predictor of excess stock returns, we wish to explore the behavioral implications of this result for investors. The left panel of Figure 7 shows that the variable $\boldsymbol{\Delta}\mathbf{sent12}\times\boldsymbol{\Delta}\mathbf{ersf}$ is positively correlated with the variable $\boldsymbol{\Delta}\mathbf{SVI}$**,** defined as the 1-month change in the Google Search Volume Index (SVI) for the term "stock market."[11] The correlation coefficient between the two variables is around 0.3. In the right panel of Figure 7 we plot the momentum in SVI in month t**,** defined as the 1-month change in $\boldsymbol{\Delta}\mathbf{SVI}$ (i.e., $\boldsymbol{\Delta}^2\mathbf{SVI}$), versus the excess return on stocks in month $t+1$.

[11] The Google SVI data are available from 2004.M1 onwards and can be downloaded from https://trends.google.com/trends/?geo=US.

The univariate regression line shows that positive SVI momentum tends to predict lower excess stock returns. This result, together with the positive correlation between Δ**SVI** and Δ**sent12**$\times\Delta$**ersf,** suggests that our sentiment-momentum variable helps to predict excess returns because it captures shifts in investor attention to recent stock market movements. These movements, in turn, appear to influence investors' decisions to buy or sell stocks, resulting in upward or downward pressure on stock prices.

When do investors pay more attention to the stock market? To help answer this question, Figure 8 plots the 12-month change in the SVI versus: (1) the 12-month percentage change in the S&P 500 stock index and (2) the 12-month change in the University of Michigan consumer sentiment index. Differencing each series over a 12-month period allows the broader comovements in the data to emerge. Figure 8 shows that Google searches for the term "stock market" tend to increase sharply during periods when stock prices and consumer sentiment are both declining. This pattern is particularly evident during the height of the global financial crisis in October 2008 (the month following the Lehman Brothers bankruptcy) when the SVI reaches its all-time high. The correlation coefficient between the 12-month change in the SVI and the 12-month percentage change in the S&P 500 stock index is -0.26. The correlation coefficient between the 12-month change in the SVI and the 12-month change in the University of Michigan consumer sentiment index is -0.25.

An illustrative example is the recent month of October 2018. During this month, the S&P 500 stock index declined by about 7%, the 12-month change in the consumer sentiment index turned negative, and the Google SVI for "stock market" spiked upwards to its third highest level ever. Although not plotted, the correlation coefficient between Google SVI for "stock market" and the Google SVI for "stock market crash" is 0.83. Along these lines, a recent study by Fisher, Martineau, Sheng (2018) finds that bad news about macroeconomic fundamentals raises media attention (as measured by Wall Street Journal and New York Times article counts) by more than good news.

While Figure 8 is suggestive, we wish to formally examine whether movements in the Google SVI can help to predict excess stock returns. Table 5 augments our baseline regression equation (34) with one additional predictor variable at a time, either **SVI,** Δ**SVI,** or Δ^2**SVI.** The estimated coefficient on the additional predictor variable is negative in each case, but is statistically significant only for Δ**SVI** and Δ^2**SVI.** Including Δ**SVI** in the regression equation serves to raise the adjusted R-squared statistic to 29.1% versus 25.4% for the baseline regression. Including Δ^2**SVI** in the regression equation serves to raise the adjusted R-squared statistic to 34.2%. The fitted values from the specification that includes Δ^2**SVI** are plotted in Figure 9.

Figure 10 provides evidence that the degree of investor optimism or pessimism about the stock market is strongly linked to recent movements in stock prices. Specifically, we plot the results of a University of Michigan survey that asks people to assign a probability that stock prices will increase over the next year.[12] The correlation coefficient between the mean probability response from the survey and the trailing 12-month percentage change in the S&P 500 stock index is 0.61. Together with the Google SVI data, this pattern shows that a recent drop in stock prices contributes to both an increase in investor attention and a more pessimistic outlook for stocks.

An increase in investor attention would be expected to increase investors' information about fundamentals. The theoretical links between investor information and stock price movements are complex. Using rational expectations models, Veronesi (2000) and Lansing and LeRoy (2014) show that an increase in investor information can either increase or decrease the variance of excess stock returns, depending on risk aversion and other parameter values. Andrei and Hasler (2015) develop a rational model with exogenous time-varying attention to the stock market. Their model predicts that an increase in investor attention leads to a higher excess stock return and a higher stock return variance. In contrast, the regressions in Table 5 show that an increase in investor attention (as measured by $\Delta\mathbf{SVI}$ or $\Delta^2\mathbf{SVI}$) predicts a lower excess stock return while controlling for changes in stock return variance.

In summary, our results show that investors pay more attention to the stock market during periods when stock prices and consumer sentiment are both declining. The resulting pessimism appears to motivate many investors to sell stocks, putting further downward pressure on stock prices which contributes to a lower excess return on stocks over the next month. It is difficult to justify this source of excess return predictability as being driven by stochastic volatility (as would be required under rational expectations) because we have controlled for this source of predictability with the variables **pd, vrp3** and $\Delta\mathbf{ff12}$. Rather, it seems far more likely that the statistical significance of the predictor variables $\Delta\mathbf{sent12}\times\Delta\mathbf{ersf}$, $\Delta\mathbf{SVI}$, and $\Delta^2\mathbf{SVI}$ represents evidence of market inefficiency that is linked to shifts in investor attention. More specifically, it would appear that each of these "behavioral" variables serves as a type of investor pessimism indicator that helps to predict episodes of sequential declines in excess stock returns.

6 Conclusion

This paper shows that realized excess returns on risky assets can be represented by an additive combination of conditional variance terms and investor forecast errors. As a result,

[12] The data is available from June 2002 onwards from https://data.sca.isr.umich.edu/tables.php. The survey question reads: "Suppose that tomorrow someone were to invest one thousand dollars in a type of mutual fund known as a diversified stock fund. What do you think is the percent chance that this one thousand dollar investment will increase in value in the year ahead, so that it is worth more than one thousand dollars one year from now?"

predictability of realized excess returns can arise from only two sources: (1) stochastic volatility of model variables, or (2) departures from rational expectations that give rise to predictable investor forecast errors. This is a general result that holds for any stochastic discount factor, any consumption or dividend process, and any stream of bond coupon payments.

From an empirical perspective, we find that a variable that interacts the 12-month consumer sentiment change with recent return momentum is a robust predictor of excess stock returns even after controlling for the presence of stochastic volatility. Specifically, the estimated regression coefficient on the sentiment-momentum variable remains stable and statistically significant over various sample periods. Inclusion of the sentiment-momentum variable consistently helps to predict excess stock returns in out-of-sample forecasting tests. The predictive power of the sentiment-momentum variable derives mainly from periods when sentiment has been declining and return momentum is negative, forecasting a further decline in the excess stock return. We show that the sentiment-momentum variable is positively correlated with fluctuations in Google searches for the term "stock market," which tend to spike during pronounced stock market declines. Overall, we interpret our empirical results as providing evidence that the predictability of excess stock returns is coming from both of the two sources identified by the theory.

A Appendix: Rational solution with stochastic volatility

This appendix derives an approximate analytical solution to the rational asset pricing model employed in Section 3. Gelain and Lansing (2014) employ similar methods to derive an approximate analytical solution to a rational asset pricing model for housing that exhibits stochastic volatility in fundamental rent growth.[13] Substituting the functional forms for M_t and d_t/d_{t-1} into the transformed first-order condition for stocks (5) yields

$$z_t^s \;=\; \beta \exp\left[(1-\alpha)\, x_t^c + v_t\right]\left(1 + E_t z_{t+1}^s\right), \tag{A.1}$$

where $x_t^c \equiv \log\left(c_t/c_{t-1}\right)$. A conjectured solution to (A.1) takes the form

$$z_t^s \;=\; a_0 \exp\left[a_1\left(x_t^c - \overline{x}\right) + a_2\left(\sigma_t^2 - \overline{\sigma}^2\right) + a_3 v_t\right]. \tag{A.2}$$

Iterating ahead the conjectured law of motion for z_t^s and then taking the conditional expectation yields

$$E_t z_{t+1}^s \;=\; a_0 \exp\left[a_1 \rho\left(x_t^c - \overline{x}\right) + \tfrac{1}{2}\left(a_1\right)^2 \sigma_t^2 + a_2 \gamma\left(\sigma_t^2 - \overline{\sigma}^2\right) + \tfrac{1}{2}\left(a_2\right)^2 \sigma_u^2 + \tfrac{1}{2}\left(a_3\right)^2 \sigma_v^2\right], \tag{A.3}$$

where $p_t^s/d_t = E_t z_{t+1}^s$ from equation (4). The above expression shows that p_t^s/d_t is a function of the fundamental state variable σ_t^2 that drives the stochastic volatility of consumption and dividend growth. This analytical result motivates the inclusion of the price-dividend ratio as a right side variable in the predictability regressions of Section 5.

Substituting the conditional forecast (A.3) into the transformed first order condition (A.1) and then taking logarithms yields

$$\begin{aligned}
\log\left(z_t\right) \;&=\; F\left(x_t^c,\, \sigma_t^2,\, v_t\right) = \log\left(\beta\right) \;+\; \left(1-\alpha\right) x_t^c \;+\; v_t \\
&\quad +\; \log\Big\{1 + a_0 \exp\Big[a_1 \rho\left(x_t^c - \overline{x}\right) + \tfrac{1}{2}\left(a_1\right)^2 \sigma_t^2 + a_2 \gamma\left(\sigma_t^2 - \overline{\sigma}^2\right) \\
&\qquad\qquad +\; \tfrac{1}{2}\left(a_2\right)^2 \sigma_u^2 \;+\; \tfrac{1}{2}\left(a_3\right)^2 \sigma_v^2\Big]\Big\}, \\
&\simeq\; \log\left(a_0\right) + a_1\left(x_t^c - \overline{x}\right) + a_2\left(\sigma_t^2 - \overline{\sigma}^2\right) + a_3 v_t,
\end{aligned} \tag{A.4}$$

where $a_0 \equiv \exp\left\{E\left[\log\left(z_t\right)\right]\right\}$, a_1, a_2, and a_3 are Taylor-series coefficients. After some manipulation, it can be shown that the Taylor series coefficients must satisfy the following system

[13] Lansing (2010) demonstrates the accuracy of this solution method for the level of the price-dividend ratio by comparing the approximate analytical solution to the exact theoretical solution derived by Burnside (1998) for the version of the model without stochastic volatility, i.e., $\sigma_u^2 = 0$.

of nonlinear equations

$$a_0 = \exp\left[F\left(\overline{x},\, \overline{\sigma}^2,\, 0\right)\right] = \frac{\beta \exp[(1-\alpha)\overline{x}]}{1-\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2\right]}, \tag{A.5}$$

$$a_1 = \left.\frac{\partial F}{\partial x_t^c}\right|_{\overline{x},\, \overline{\sigma}^2,\, 0} = \frac{(1-\alpha)}{1-\rho\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2\right]}, \tag{A.6}$$

$$a_2 = \left.\frac{\partial F}{\partial \sigma_t^2}\right|_{\overline{x},\, \overline{\sigma}^2,\, 0} = \frac{\left[(a_1)^2/2\right]\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2\right]}{1-\gamma\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2\right]}, \tag{A.7}$$

$$a_3 = \left.\frac{\partial F}{\partial v_t}\right|_{\overline{x},\, \overline{\sigma}^2,\, 0} = 1, \tag{A.8}$$

provided that $\beta \exp\left[(1-\alpha)\,\overline{x} + (a_1)^2\,\overline{\sigma}^2/2 + (a_2)^2\,\sigma_u^2/2 + (a_3)^2\,\sigma_v^2/2\right] < 1$. From equations (A.2) and (A.3), we can compute $\log[z_{t+1}^s/(E_t z_{t+1}^s)]$, yielding equation (25) in the text where we have inserted $a_3 = 1$.

B Appendix: Data sources

Monthly data on the end-of-month nominal S&P 500 stock index, nominal dividends, and the nominal risk free rate of return are from Welch and Goyal (2008). Updated data through the end of 2017 are available from Amit Goyal's website.[14] The gross nominal return on the S&P 500 stock index in month t is defined as $(P_t + D_t/12)\,/P_{t-1}$, where P_t is the end-of-month closing value of the index and D_t is cumulative nominal dividends over the past 12 months. The price-dividend ratio in month t is defined as P_t/D_t. Data on the variance risk premium are from Zhou (2018). Updated monthly data through the end of 2017 are available from Hao Zhou's website.[15] The variance risk premium is defined as the difference between implied variance as measured by the end-of-month VIX-squared, de-annualized (i.e., $\text{VIX}^2/12$) and realized variance as measured by the sum of squared 5-minute log returns of the S&P 500 stock index over the month. Both variance measures are expressed in percentage-squared terms and are available in real time at the end of the observation month. The federal funds rate is the monthly average value in percent from the FRED database of the Federal Reserve Bank of St. Louis. The University of Michigan consumer sentiment index is from www.sca.isr.umich.edu/tables.html.

[14] www.hec.unil.ch/agoyal/.
[15] https://sites.google.com/site/haozhouspersonalhomepage/.

C Appendix: Bootstrapped critical values

The literature on return predictability has raised an important issue about the potential size distortion of the standard test, such as the t-statistic, in finite samples when the regression equation includes persistent regressors. Table 1 shows that the predictor variables **pd**, **vrp3,** **Δff12**, and **Δsent12** are highly persistent. We address this issue using a bootstrap procedure to gauge the quantitative impact of persistent regressors for our specific application.

Stambaugh (1999) and Mankiw and Shapiro (1986) show that the highly persistent price-dividend ratio leads to a finite-sample bias in the estimated slope coefficient and its associated t-statistic when one regresses stock returns (or excess stock returns) on the lagged price-dividend ratio. More recently, Bauer and Hamilton (2017) evaluate the impact of persistent regressors on standard tests in long-horizon predictability regressions for excess bond returns that involve overlapping return observations. Consider a system of the type studied in Stambaugh (1999) and Mankiw and Shapiro (1986):

$$\log(R^s_{t+1}/R^f_{t+1}) = \alpha_0 + \alpha_1(p^s_t/d_t) + u_{t+1}, \qquad u_t \sim NID\left(0, \sigma^2_u\right), \tag{C.1}$$

$$p^s_{t+1}/d_{t+1} = \beta_0 + \beta_1(p^s_t/d_t) + v_{t+1}, \qquad v_t \sim NID\left(0, \sigma^2_v\right). \tag{C.2}$$

Stambaugh (1999) shows that the bias in the least squares estimate of α_1 depends on the contemporaneous correlation between the two innovations u_t and v_t, and is proportional to the bias in the estimate of the AR(1) coefficient β_1. The expression for the bias in the estimate of α_1 is

$$E\left(\widehat{\alpha}_1 - \alpha_1\right) = \left[Cov\left(u_t, v_t\right)/\sigma^2_v\right]\, E(\widehat{\beta}_1 - \beta_1). \tag{C.3}$$

Upward movements in the stock price tend to drive up the price-dividend ratio and the excess stock return simultaneously, implying that $Cov\left(u_t, v_t\right) > 0$. Indeed, Table 1 shows that there is a small positive correlation between **pd** and the excess stock return **ersf**. The AR(1) parameter estimate $\widehat{\beta}_1$ has a downward bias such that $E(\widehat{\beta}_1 - \beta_1) < 0$, as shown originally by Kendall (1954). He also derives an expression for the estimation bias, which is given by $-(1 + 3\beta_1)/N$, where N is the sample size. Therefore, the downward bias becomes larger as β_1 increases, implying a more persistent price-dividend ratio. The upshot is that the least squares estimate of α_1 and its t-statistic tend to have a non-trivial downward bias when the regressor is highly persistent and there is a positive correlation between the two shocks.

It is important to note, however, some important differences between our regression exercises and those in the previous literature. Although we include some highly persistent regressors (**pd**, **vrp3**, **Δff12**, and **Δsent12**), our primary focus relates to the variable **Δsent12$\times\Delta$ersf** which is not persistent. The sentiment-momentum variable exhibits an autocorrelation statistic of -0.22 and a correlation with **ersf** of -0.12.

Nevertheless, we still wish to gauge the magnitude of the potential size distortion of the standard t-statistic for our specific application. We follow Nelson and Kim (1993), Mark (1995), and Rapach and Wohar (2006) to implement a bootstrapping procedure to provide some guidance for our discussion of the regression results using the actual data. In the bootstrap, we postulate that the data are generated by the following system under the null hypothesis:

$$\log(R^s_{t+1}/R^f_{t+1}) = a_0 + \varepsilon_{1t+1}, \tag{C.4}$$

$$p^s_{t+1}/d_{t+1} = b_0 + b_1\,(p^s_t/d_t) + ... + \; b_j(p^s_{t-j+1}/d_{t-j+1}) + \varepsilon_{2t+1}, \tag{C.5}$$

where the two innovations are distributed as $NID(0,\Sigma)$. To obtain the parameters for bootstrapping, we first use the actual data sample to estimate these two equations using ordinary least squares (OLS). The number of lags in equation (C.5) is determined using the AIC (with a maximum order of four). Given the parameter estimates, we compute and store the residuals $(\hat{\varepsilon}_{1t},\ \hat{\varepsilon}_{2t})$. Next, we take random draws (with replacement) of the actual data from these OLS residuals in tandem, preserving the contemporaneous correlation between these disturbances in the original sample. For each simulation, we obtain a bootstrapped data of sample size $N * (1 + 25\%)$, where $N = 333$ is the sample length of monthly U.S. data from 1990.M3 to 2017.M12. We drop the first 25% of the bootstrapped data to remove any potential impact of the initial values, thus keeping the length of the pseudo-sample equal to the length of the U.S. data sample. Following Shaman and Stine (1988), we also implement a bias correction procedure for the estimated AR coefficients in equation (C.5). We use the bias-corrected parameter values and the randomly-drawn residuals to generate bootstrapped data from equations (C.4) and (C.5). For each bootstrapped sample, we compute and store the t-statistics for the slope coefficient α_1 in equation (C.1). The t-statistics are computed using Newey-West HAC corrected standard errors. We repeat the process 1000 times and obtain an empirical distribution of the bootstrapped t-statistics. We report the 2.5% and 97.5% percentiles of the empirical distribution as the empirical critical values corresponding to the 5% size level. See Rapach and Wohar (2006) for additional details of the bootstrapping procedure.

We carry out the bootstrap procedure using two types of regressions. For the first type, we run a univariate regression by regressing excess stock returns in month $t+1$ on a constant and **pd** in month t, as in equation (C.1). In the second type of regression, we regress excess stock returns in month $t+1$ on a constant, **pd** in month t, and one additional predictor variable in month t. The additional predictor variables that we test, one at a time, are **vrp3, Δff12, Δsent12, Δersf,** and **Δsent12**×**Δersf**. This procedure results in a three-variable system consisting of equation (C.4) and two equations similar to (C.5), one for the price-dividend ratio and one for the additional predictor variable. This three-variable system is used to generate the pseudo-sample. The reason to include both **pd** and the additional predictor variable in the second type of regression is to gauge the size of potential impacts of the interdependence between the two predictor variables on the test statistic. We implement this bootstrap procedure for the full sample. The bootstrapping results are reported in Table C.1

The two-sided 5% asymptotic critical values of a t-statistic that adheres to a standard normal distribution are -1.96 and $+1.96$. The bootstrapped critical values in Table C.1 are not substantially different from the asymptotic ones, but there are some noticeable shifts in either direction for the persistent predictor variables, depending upon the direction of the underlying correlation between the innovations.

For example, the 2.5% percentile of the bootstrapped t-statistic for **pd** is -2.473. This value is larger in absolute value than the asymptotic value of -1.96, thus raising the bar for one to reject the null hypothesis of a zero coefficient in favor of a negative coefficient. At the same time, the 97.5% percentile of the bootstrapped t-statistic for **pd** is 1.445, less than the asymptotic value of 1.96. This left-skewed distribution of the test statistics results from the positive correlation between the innovations in equations (C.4) and (C.5) which gives rise to a downward bias in the slope coefficient and the associated t-statistic.

The distributions of the t-statistic for the other predictor variables in the second-round regressions all appear less skewed and closer to the standard normal or student-t distribution. For example, although the 97.5% percentile of the bootstrapped t-statistic for **vrp3** is 1.933, slightly smaller in magnitude than the asymptotic value of 1.96, its 2.5% percentile is also slightly smaller in absolute value than the asymptotic value of -1.96, leading to a more or less symmetric distribution. The resulting distribution of the bootstrapped t-statistic for **Δff12** is also quite symmetric despite the highly persistent nature of **Δff12.** The bootstrapped critical values for **Δersf** and **Δsent12×Δersf** are close to the asymptotic critical values. This result is to be expected because these predictor variables exhibit very little persistence, resulting in minimal estimation bias.

Table C.1: Bootstrapped Critical Values

Variable	2.5% percentile	97.5% percentile
pd	−2.473	1.445
vrp3	−1.953	1.933
Δff12	−2.014	2.120
Δsent12	−2.037	1.769
Δersf	−1.970	1.910
Δsent12×Δersf	−2.049	1.837

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Table 1: Summary Statistics: 1990.M3 to 2017.M12

Variable	Mean	Std. Dev.	Skewness	Kurtosis	Min.	Max.	Autocorr.
ersf	0.57	4.13	−0.79	4.83	−18.4	10.5	0.05
pd	52.4	14.2	0.54	3.09	25.5	92.2	0.98
vrp3	16.2	14.6	−1.10	15.1	−73.9	74.9	0.81
Δff12	−0.29	1.41	−0.65	3.50	−4.58	2.67	0.98
Δsent12	0.18	10.1	−0.64	3.52	−30.0	22.8	0.85
Δersf	0.002	5.69	0.43	4.10	−14.6	21.8	−0.47
Δsent12 × **Δersf**	−3.57	58.4	−0.21	10.4	−292.7	290.3	−0.22

Contemporaneous Cross Correlations

	ersf	**pd**	**vrp3**	**Δff12**	**Δsent12**	**Δersf**	**Δsent12** × **Δersf**
ersf	1.00						
pd	0.03	1.00					
vrp3	0.16	0.17	1.00				
Δff12	0.14	0.20	−0.12	1.00			
Δsent12	0.18	0.03	−0.03	0.35	1.00		
Δersf	0.69	0.00	0.03	0.00	−0.06	1.00	
Δsent12 × **Δersf**	−0.12	0.03	−0.10	0.04	0.05	−0.20	1.00

Notes: **ersf** = excess return on S&P 500 stock index relative to the risk free rate in percent, as measured by the return on 3-month Treasury bills, **pd** = price-dividend ratio for the S&P 500 index defined as the end-of-month nominal closing value of the index divided by cumulative nominal dividends over the past 12 months, **vrp3** = 3-month moving average of variance risk premium for the S&P 500 stock index, defined as the difference between the implied variance in percent-squared from options and the realized variance in percent-squared measured using 5-minute return intervals over the month, **Δff12** = 12-month change in the federal funds rate in percent, **Δsent12** = 12-month change in the University of Michigan's consumer sentiment index, **Δersf** = excess return momentum, defined as the 1-month change in **ersf**.

Table 2: Predicting Excess Returns on Stocks: Full Sample Results

1990.M3 to 2017.M12	1	2	3	4
pd	**−0.063**	**−0.062**	**−0.059**	**−0.059**
	(−3.626)	(−3.687)	(−3.510)	(−3.539)
vrp3	**0.089**	**0.088**	**0.082**	**0.081**
	(5.792)	(5.721)	(6.047)	(5.479)
Δff12	**0.671**	**0.616**	**0.621**	**0.583**
	(4.590)	(4.502)	(4.727)	(4.126)
Δsent12		0.021	0.024	0.011
		(1.212)	(1.387)	(0.414)
Δersf		0.030	−0.001	−0.037
		(0.627)	(−0.025)	(−0.731)
Δsent12 × **Δersf**			**−0.016**	
			(−4.377)	
Δ^+**sent12** × Δ^+**ersf**				−0.010
				(−0.783)
Δ^+**sent12** × Δ^-**ersf**				−0.007
				(−0.553)
Δ^-**sent12** × Δ^+**ersf**				−0.014
				(−1.954)
Δ^-**sent12** × Δ^-**ersf**				**−0.025**
				(−3.049)
Adj. R^2	13.0%	12.9%	17.3%	16.9%

Notes: All regressions include a constant term with regressors dated month t. Dependent variable is **ersf** for month $t+1$. Newey-West HAC corrected t-statistics in parentheses. Boldface indicates significant at the 5% level using the boostrapped critical values shown in Appendix C. The symbol Δ^+ represents a positive change in the corresponding variable while Δ^- represents a negative change. See Table 1 for variable definitions.

Table 3: Predicting Excess Returns on Stocks: Split Sample Results

1990.M3 to 2003.M12	1	2	3	4
pd	**−0.056**	**−0.057**	**−0.053**	**−0.052**
	(−3.037)	(−3.151)	(−2.939)	(−2.907)
vrp3	**0.107**	**0.116**	**0.102**	**0.105**
	(4.736)	(5.236)	(4.494)	(4.642)
Δff12	**0.528**	**0.489**	**0.461**	**0.459**
	(3.339)	(2.793)	(2.647)	(2.652)
Δsent12		0.024	0.035	0.020
		(0.764)	(1.150)	(0.511)
Δersf		−0.042	−0.058	−0.067
		(−0.809)	(−1.426)	(−1.119)
Δsent12 × Δersf			**−0.016**	
			(−3.725)	
Δ^{+}**sent12** × Δ^{+}**ersf**				−0.019
				(−1.080)
Δ^{+}**sent12** × Δ^{-}**ersf**				−0.009
				(−0.509)
Δ^{-}**sent12** × Δ^{+}**ersf**				−0.011
				(−1.196)
Δ^{-}**sent12** × Δ^{-}**ersf**				**−0.026**
				(−3.755)
Adj. R^2	10.4%	9.93%	13.8%	12.5%
2004.M1 to 2017.M12	**1**	**2**	**3**	**4**
pd	**−0.208**	**−0.206**	**−0.196**	**−0.198**
	(−3.861)	(−4.247)	(−4.378)	(−4.264)
vrp3	**0.079**	**0.076**	**0.076**	**0.075**
	(3.897)	(4.099)	(4.549)	(4.258)
Δff12	**1.355**	**1.267**	**1.268**	**1.190**
	(3.575)	(3.858)	(4.049)	(3.026)
Δsent12		0.031	0.024	0.013
		(1.419)	(1.130)	(0.307)
Δersf		0.125	0.084	0.045
		(1.661)	(1.394)	(0.433)
Δsent12 × Δersf			**−0.013**	
			(−2.528)	
Δ^{+}**sent12** × Δ^{+}**ersf**				−0.006
				(−0.361)
Δ^{+}**sent12** × Δ^{-}**ersf**				−0.006
				(−0.331)
Δ^{-}**sent12** × Δ^{+}**ersf**				−0.011
				(−1.021)
Δ^{-}**sent12** × Δ^{-}**ersf**				−0.021
				(−1.159)
Adj. R^2	19.4%	21.7%	24.7%	23.5%

Notes: Same as Table 2.

Table 4: Goodness-of Fit Statistics

1-month ahead forecast	$RMSFE$	$MAFE$	$Corr$	Adj. R^2	OOS R^2
In-sample with **Δsent12** × **Δersf**	3.72%*	2.84%*	0.43*	17.3%*	
In-sample without **Δsent12** × **Δersf**	3.82%	2.87%	0.38	12.9%	
Split out-of-sample with **Δsent12** × **Δersf**	3.59%*	2.79%*	0.45*	–	15.9%*
Split out-of-sample without **Δsent12** × **Δersf**	3.74%	2.85%	0.38	–	8.82%
Rolling out-of-sample with **Δsent12** × **Δersf**	3.91%*	2.98%*	0.41*	–	16.1%*
Rolling out-of-sample without **Δsent12** × **Δersf**	4.03%	3.05%	0.35	–	10.5%

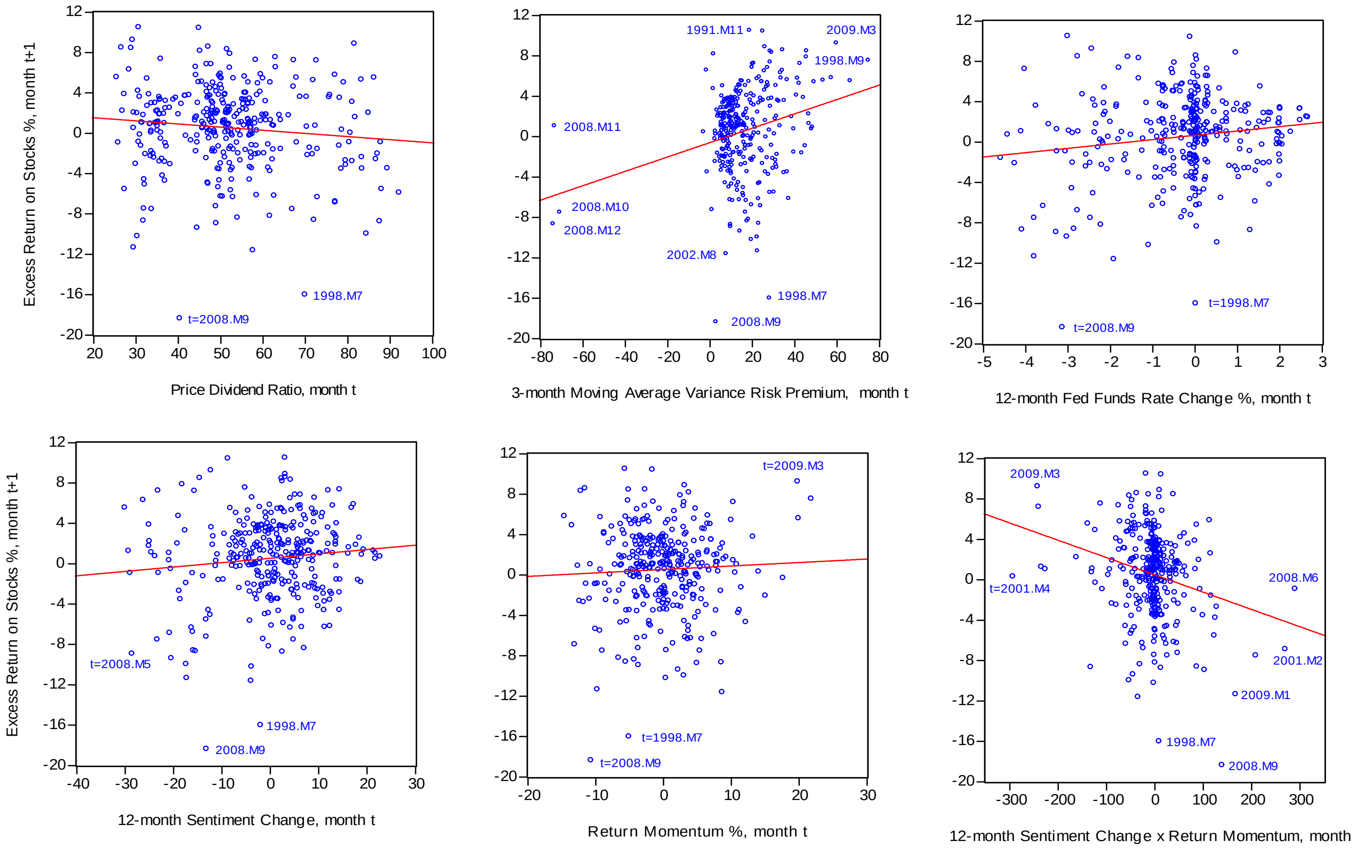
Notes: $RMSFE$ = Root mean squared forecast error, $MAFE$ = Mean absolute forecast error, $Corr$ = correlation coefficient between realized excess return and and forecasted excess return, Adj. R^2 = Adjusted R-squared statistic for in-sample regressions, OOS R^2 = Out-of-sample R-squared statistic defined as $1-SSR/SST$, where SSR is the sum of the squared residuals from the predictive regression and SST is the sum of the squared deviations of realized excess returns from the mean excess return of the estimation sample. The in-sample regressions cover the period from 1990.M3 to 2017.M12. For the split out-of-sample regressions, the regression equation is estimated for the period from 1990.M3 to 2003.M12 and then used to forecast excess stock returns for the period from 2004.M1 to 2007.M12. The rolling out-of-sample regressions each employ a 10-year (120-month) moving window of data. The regression equation estimated for a given window of data is used to forecast the excess stock return for the subsequent month, without re-estimation of the equation. An asterisk $*$ indicates the superior goodness-of-fit statistic for the two regressions being compared.

Table 5: Predicting Excess Returns on Stocks: Alternative Specifications

2004.M3 to 2017.M12	Baseline	SVI 1	SVI 2	SVI 3
pd	**−0.221**	**−0.231**	**−0.224**	**−0.213**
	(−4.799)	(−3.851)	(−4.649)	(−4.836)
vrp3	**0.075**	**0.072**	**0.078**	**0.083**
	(4.683)	(3.891)	(4.650)	(7.756)
Δff12	1.**380**	**1.392**	**1.401**	1.**351**
	(4.261)	(4.097)	(4.340)	(4.381)
Δsent12	0.019	0.018	0.019	0.022
	(0.850)	(0.826)	(0.894)	(1.024)
Δersf	0.082	0.081	0.058	0.065
	(1.343)	(1.333)	(1.004)	(1.217)
Δsent12 × **Δersf**	**−0.012**	**−0.012**	−0.009	**−0.009**
	(−2.455)	(−2.292)	(−1.725)	(−2.074)
SVI		−0.013		
		(−0.401)		
ΔSVI			−**0.087**	
			(−2.264)	
Δ²SVI				**−0.082**
				(−3.294)
Adj. R^2	25.4%	24.9%	29.1%	34.2%

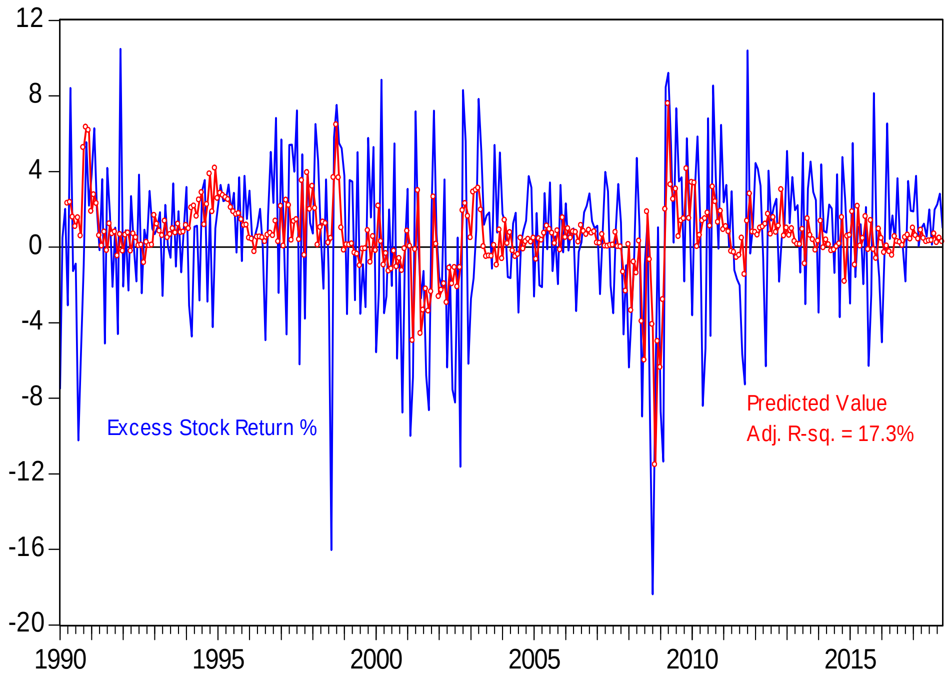
Notes: All regressions include a constant term with regressors dated month t. Dependent variable is **ersf** for month $t+1$. Newey-West HAC corrected t-statistics in parentheses. Boldface indicates significant at the 5% level. **SVI** = Google search volume index for the term "stock market," **ΔSVI** = 1-month change in **SVI**, Δ^2**SVI** = Momentum in **SVI** defined as the 1-month change in **ΔSVI.** See Table 1 for other variable definitions.

Figure 1: *Predictor Variables versus 1-Month Ahead Excess Returns on Stocks*



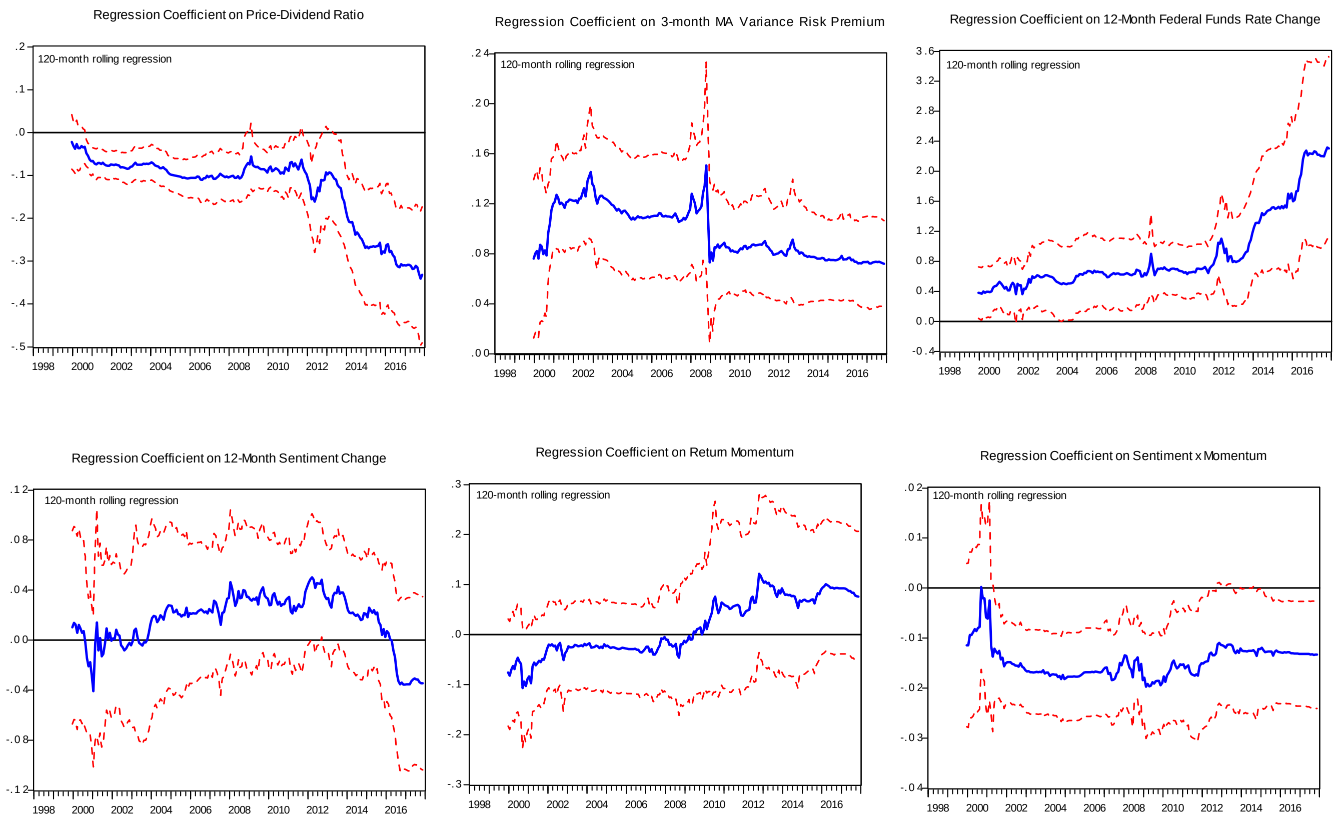
Notes: The scatter plots show the relationships between each of the six predictor variables and the 1-month ahead excess return on stocks. The slope of the line indicates the sign of the regression coefficient in a univariate predictive regression for the period from 1990.M3 to 2017.M12.

Figure 2: *Realized versus Predicted Excess Returns on Stocks*



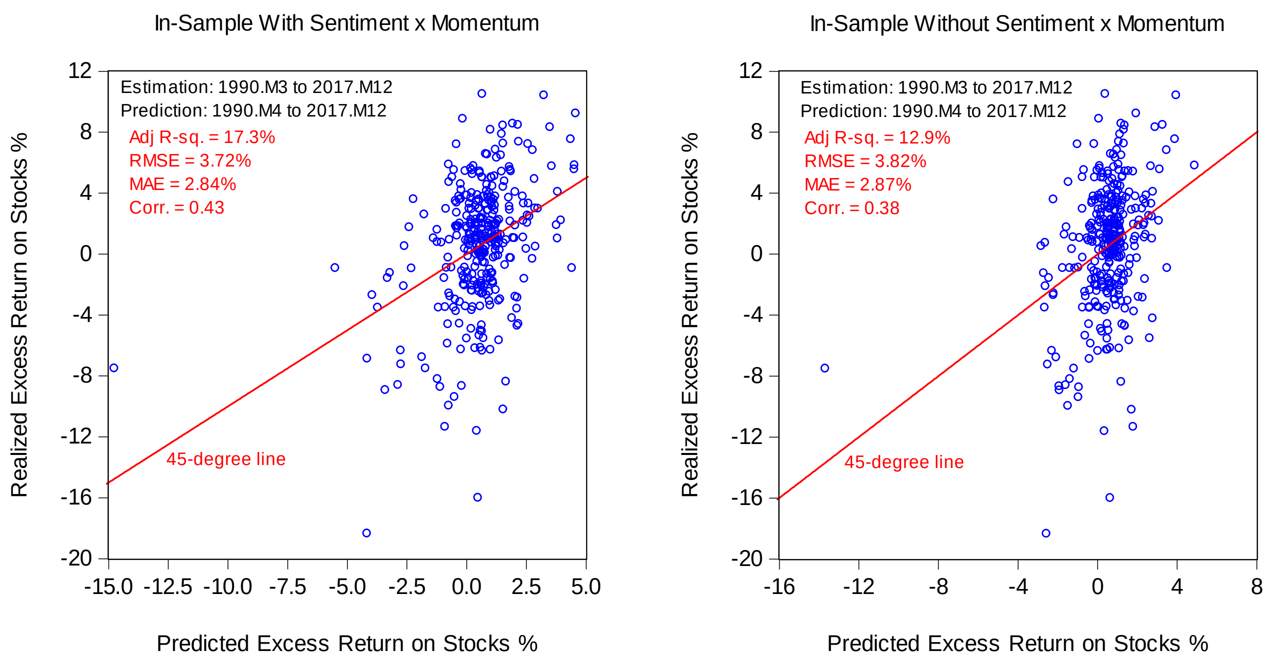
Notes: Monthly excess stock returns are characterized by positive means, high standard deviations, negative skewness, excess kurtosis, very low autocorrelation, and time-varying volatility. A predictive regression estimated over the period from 1990.M3 to 2017.M12 using all six predictor variables (Specification 3 in Table 2) exhibits an adjusted R^2 statistic of 13.9%.

Figure 3: *Rolling Regression Coeffficients*



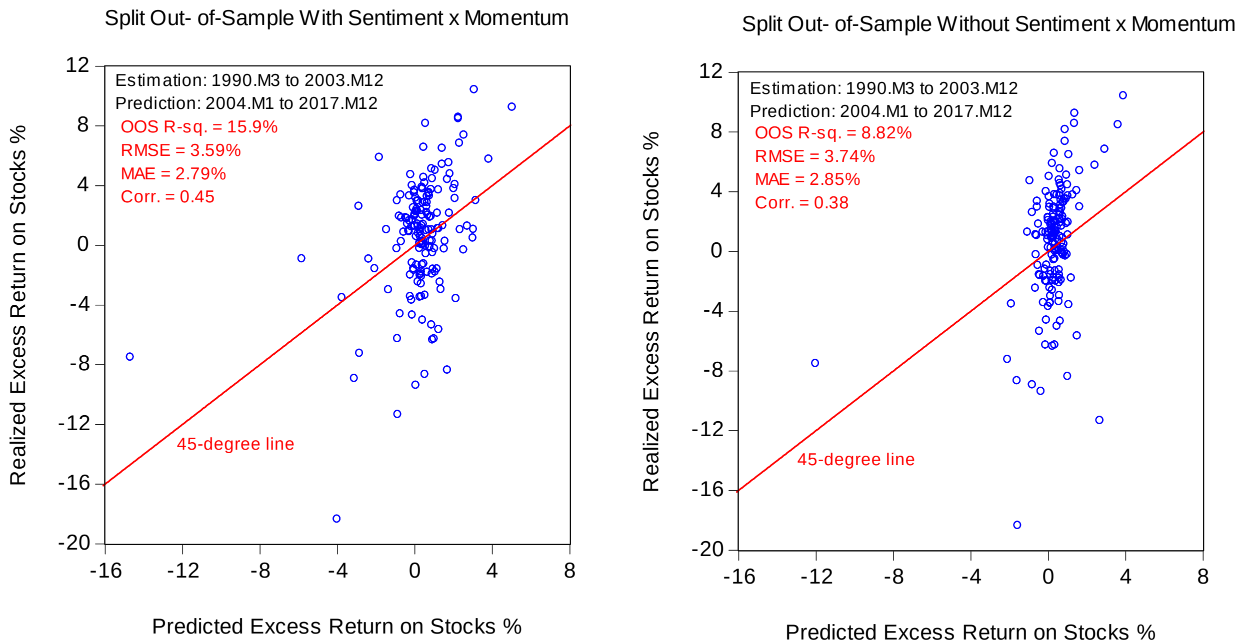
Notes: The rolling regression coefficients on **pd, vrp3** and **Δff12** (top panels) exhibit consistent signs and are mostly significant from the early 2000s onwards. The rolling regression coefficient on **Δsent12** (lower left panel) is rarely significant while the rolling regression coefficient on **Δersf** (lower middle panel) is never significant. Similar to the results for **pd, vrp3** and **Δff12**, the rolling regression coefficient on **Δsent12**×**Δersf** (lower right panel) exhibits a consistent sign and is mostly significant from the early 2000s onwards.

Figure 4: *In-Sample Predictive Regression Results*



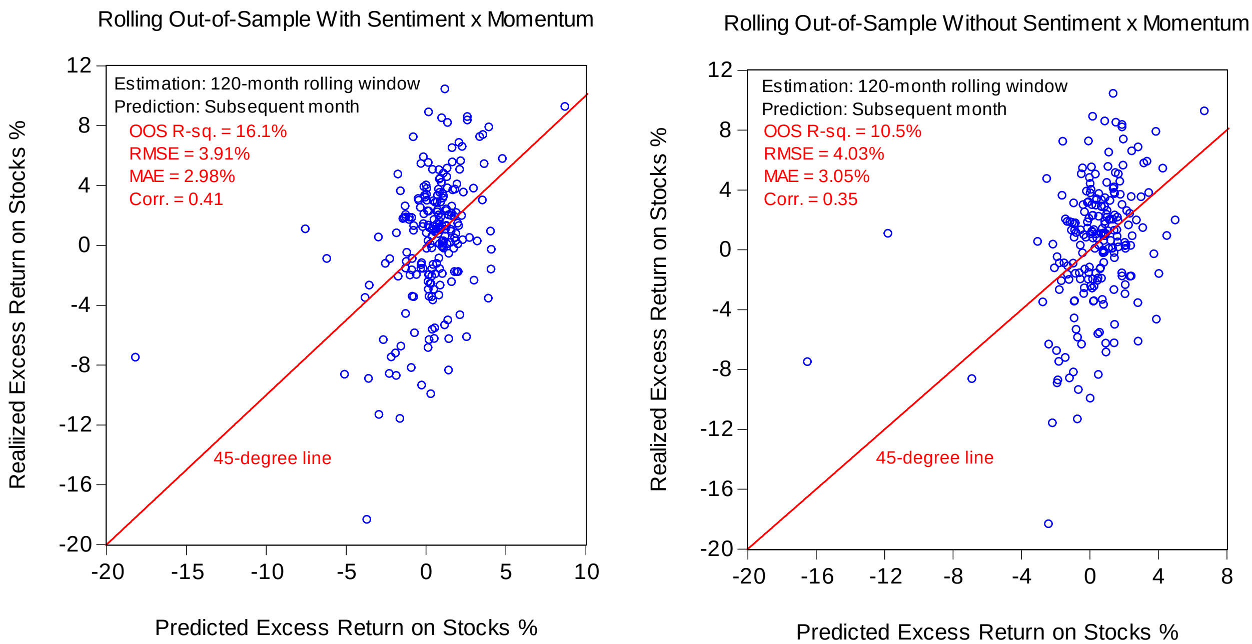
Notes: An in-sample regression that includes $\mathbf{\Delta sent12} \times \mathbf{\Delta ersf}$ as a predictor variable outperforms an otherwise similar regression that omits $\mathbf{\Delta sent12} \times \mathbf{\Delta ersf}$.

Figure 5: *Out-of-Sample Predictive Regression Results*



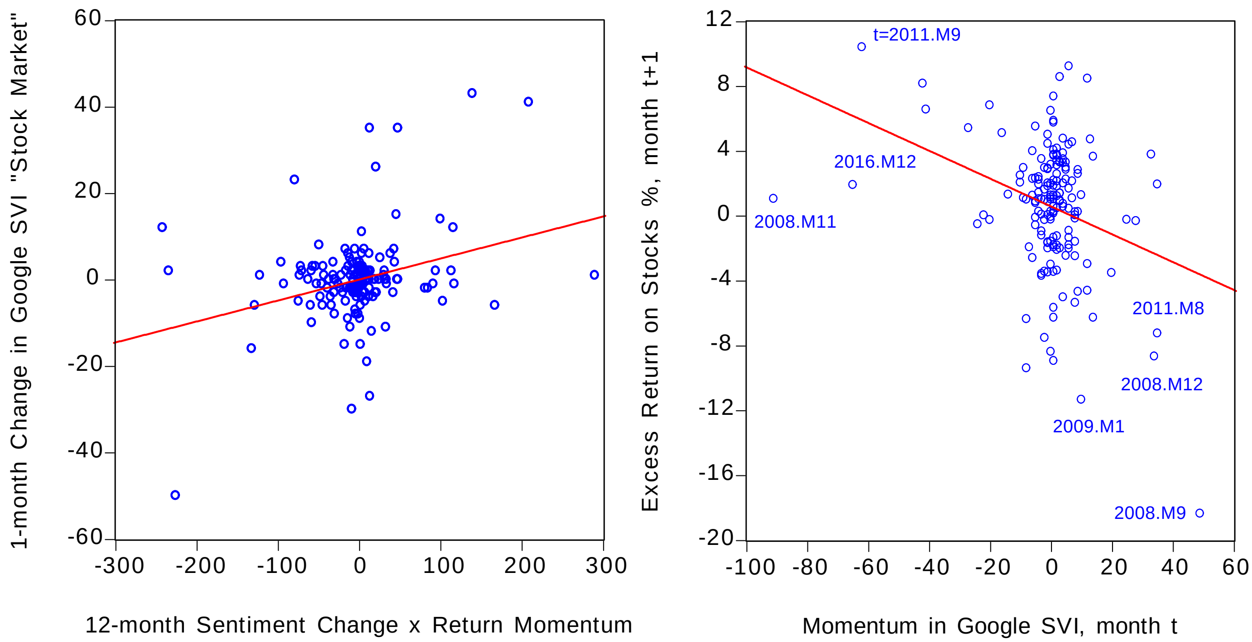
Notes: A split out-of-sample regression that includes $\boldsymbol{\Delta}$**sent12**$\times\boldsymbol{\Delta}$**ersf** as a predictor variable outperforms an otherwise similar regression that omits $\boldsymbol{\Delta}$**sent12**$\times\boldsymbol{\Delta}$**ersf**.

Figure 6: *Rolling Out-of-Sample Predictive Regression Results*



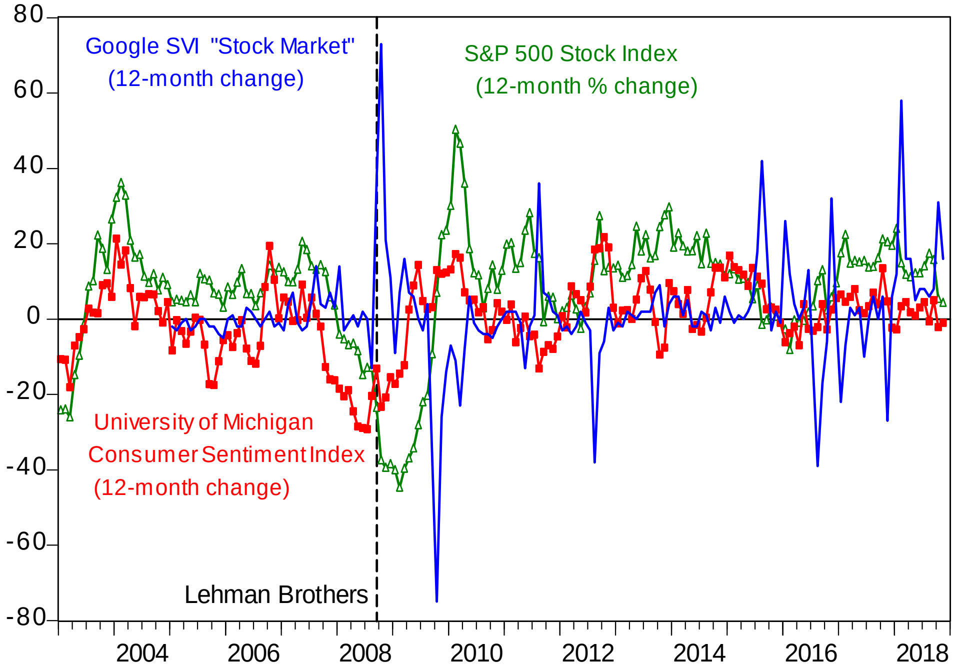
Notes: A rolling out-of-sample regression that includes $\boldsymbol{\Delta}$**sent12**$\times\boldsymbol{\Delta}$**ersf** as a predictor variable outperforms an otherwise similar regression that omits $\boldsymbol{\Delta}$**sent12**$\times\boldsymbol{\Delta}$**ersf**.

Figure 7: *Positive Momentum in Google SVI Predicts Lower Excess Returns*



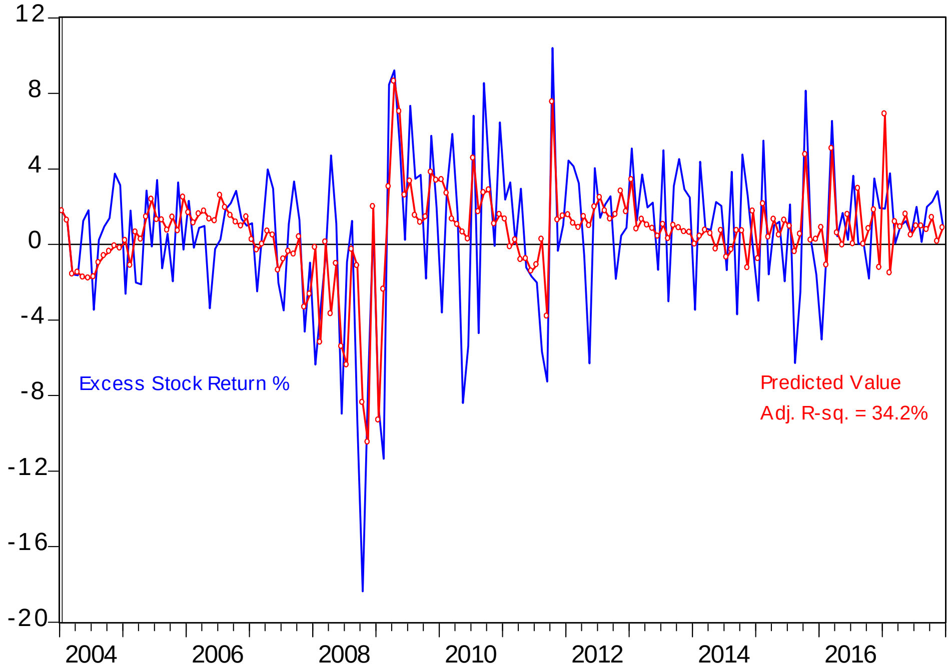
Notes: The predictor variable $\mathbf{\Delta sent12}\times\mathbf{\Delta ersf}$ is positively correlated with changes in the Google Search Volume Index (SVI) for the term "stock market," suggesting that $\mathbf{\Delta sent12}\times\mathbf{\Delta ersf}$ helps to predict excess stock returns because it captures shifts in investor attention. Higher momentum in the Google SVI for month t predicts a lower excess stock return in month $t+1$.

Figure 8: *Declines in Stock Prices and Sentiment Spur Increased Investor Attention*



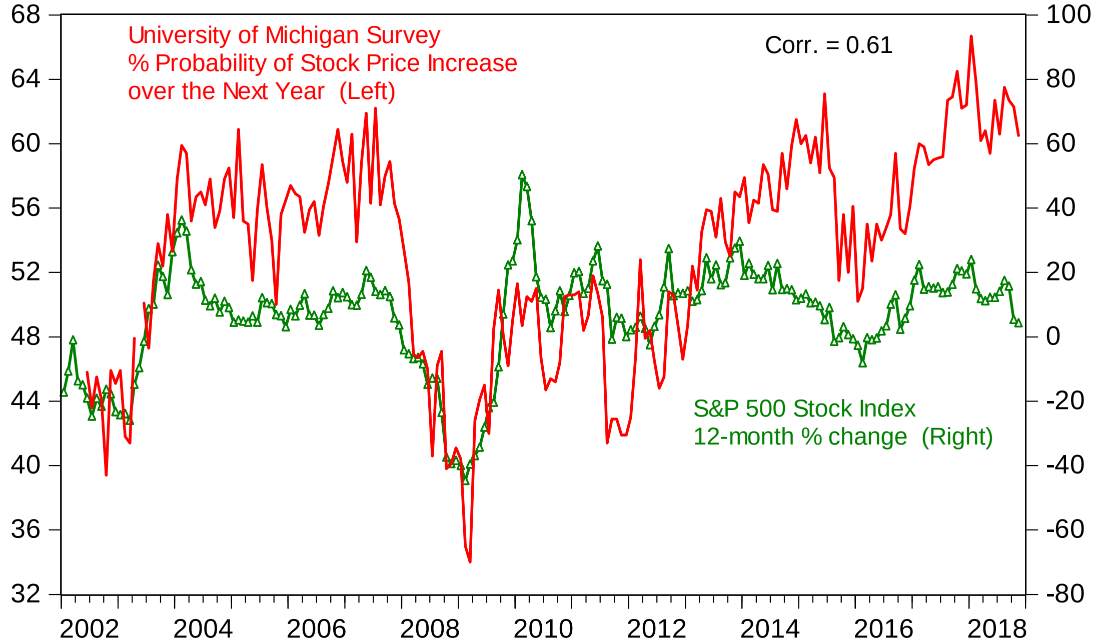
Notes: Google searches for the term "stock market" tend to increase sharply during periods when stock prices and consumer sentiment are both declining. For the sample period from 2005.M1 to 2018.M11, the correlation coefficient between the 12-month change in the SVI and the 12-month percentage change in the S&P 500 stock index is -0.26. The correlation coefficient between the 12-month change in the SVI and the 12-month change in the University of Michigan consumer sentiment index is -0.25.

Figure 9: *Realized versus Predicted Excess Returns on Stocks*



Notes: A predictive regression estimated over the period from 2004.M3 to 2017.M12 using all six predictor variables plus Δ^2**SVI** (Specification SVI 3 in Table 5) exhibits an adjusted R^2 statistic of 34.2%.

Figure 10: *Optimism or Pessimsim About Stocks is Strongly Linked to Recent Price Movements*



Notes: The degree of investor optimism or pessimism about the stock market is strongly linked to recent movements in stock prices. Together with the Google SVI data, this pattern shows that a recent drop in stock prices contributes to an increase in investor attention and a more pessimistic outlook for stocks.