Bond Flows and Liquidity: Do Foreigners Matter?

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Bond Flows and Liquidity: 
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Abstract

In their search for yield in the current low interest rate environment, many investors have turned to sovereign debt in emerging economies, which has raised concerns about risks to financial stability from these capital flows. To assess this risk, we study the effects of changes in the foreign-held share of Mexican sovereign bonds on their liquidity premiums. We find that recent increases in foreign holdings of these securities have played a significant role in driving up their liquidity premiums. Provided the higher compensation for bearing liquidity risk is commensurate with the chance of a major foreign-led sell-off in the Mexican government bond market, this development may not pose a material risk to its financial stability.

JEL Classification: E43, E44, F36, G12

Keywords: term structure modeling, liquidity risk, financial market frictions, emerging markets, financial stability, foreign holdings

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1 Introduction

In their search for yield in the current low interest rate environment, many investors have turned to sovereign debt in emerging markets. In light of the history of sovereign credit crises in emerging markets, especially in Latin America, significant debt purchases from investors outside the border could pose risks to financial stability. This is particularly true with monetary policy normalization on the horizon in many advanced economies, which could provide an impetus for foreign investments to move elsewhere. Hence, it seems warranted to study the role of foreigners in these markets.

While previous research has explored the ties between debt flows and bond prices (see Mitchell et al. 2007 and Beltran et al. 2013 for examples), the connection between debt flows and market functioning and its potential implications for financial stability have received less attention. To assess the potential financial stability implications of the increased foreign participation in the sovereign debt markets in emerging economies, this paper analyzes the influence of foreign investors on the liquidity premiums of domestic government bond securities in a leading emerging market economy, namely Mexico.

Our focus on Mexican government bonds is motivated by several observations. First, little is known about the magnitudes of liquidity premiums in regular sovereign bond markets. Second, given that Mexico has one of the largest and most important sovereign bond markets among emerging market economies, our analysis can serve as a benchmark for understanding liquidity premiums and financial market frictions in other emerging economies. Finally, the Bank of Mexico maintains a comprehensive database of foreign and domestic holdings for all Mexican government securities that is instrumental to our analysis in order to establish a connection between foreign holdings and bond risk premiums. In short, Mexico offers an ideal setting for studying the question we are interested in.

To estimate the liquidity premiums of Mexican government bonds, we rely on recent research by Andreasen et al. (2018, henceforth ACR), who show how a standard term structure model can be augmented with a liquidity risk factor to accurately measure bond liquidity premiums. Their approach identifies the liquidity risk factor from its unique loading, which mimics the idea that, over time, an increasing fraction of the outstanding notional amount of a given security tends to get locked up in buy-and-hold investors’ portfolios. This raises its sensitivity to variation in the market-wide liquidity captured by the liquidity risk factor. By observing a cross section of securities over time, the liquidity risk factor can be separately identified and distinguished from the fundamental risk factors in the model. We model the fundamental frictionless Mexican yields that would prevail in a world without any frictions to trading using a standard Gaussian model, namely the arbitrage-free Nelson-Siegel (AFNS)
model introduced in Christensen et al. (2011), which we augment with a liquidity risk factor structured as in ACR. We estimate both AFNS models and liquidity-augmented extensions thereof, denoted AFNS-L models, using price information for individual Mexican bonds.²

Our results can be summarized as follows. First, we find that the liquidity-augmented model improves model fit and delivers robust estimates of the risk factors that drive the variation in the frictionless part of the Mexican government bond yield curve. Second, our results show that liquidity premiums in the Mexican government bond market are of considerable size with an average of 0.57 percent and a standard deviation of 0.19 percent. For comparison, the liquidity premium advantage of newly issued ten-year U.S. Treasuries over comparable seasoned securities has averaged less than 0.15 percent the past two decades, see Christensen et al. (2017). Hence, liquidity risk is an important component in the pricing of Mexican government bonds. Furthermore, we see significant variation around a general upward trend during our sample period. The empirical question we are interested in is to what extent this variation can be explained by changes in foreign investor holdings of Mexican government securities. After running regressions with a large number of relevant controlling variables, we find a strongly positive relationship whereby a one percentage point increase in the foreign-held share of Mexican government bonds raises their liquidity premium by roughly 0.75 basis point. Given that the foreign market share has increased more than 40 percentage points between 2010 and 2017, our results suggest that the large increase in foreign holdings during this period could have raised the estimated bond liquidity premiums by as much as 0.3 percent.

What are the financial stability implications from these observations? Provided this increase in the compensation demanded by investors for assuming the liquidity risk of Mexican government bonds matches the risk and expected size of any future market sell-off driven by foreigners leaving the Mexican market, there may not be any major threats to the financial stability of the Mexican government bond market at this point. However, if foreigners turn out to represent a less stable and dedicated source of funding than domestic investors, there does appear to be some risk that the Mexican government could face potentially severe funding challenges down the road if foreigners were to decide to move their money elsewhere. Furthermore, this tilt in the ownership composition of Mexican government bonds could have implications for the monetary policy of the Bank of Mexico as it may be forced to put greater emphasis on foreign developments, which could matter for both macroeconomic outcomes and investors in these markets.

An important caveat to any conclusions, though, is that our sample only covers a period of foreign capital inflows into the Mexican sovereign bond market. Thus, we have not been able to model the dynamics of a potential sudden stop in the foreign supply of funds to the Mexican bond market. More broadly, the analysis presented in this paper should be viewed as a first

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²Only a limited number of papers have estimated dynamic term structure models using Mexican government bond yields; Espada and Ramos-Francia (2008b) and Espada et al. (2008) are examples.
step in connecting capital flows to liquidity premiums and financial stability assessments in emerging markets.

The remainder of the paper is organized as follows. Section 2 reviews existing research on the risks of foreign participation in local bond markets. Section 3 describes the bonos data, while Section 4 details the no-arbitrage term structure models we use and presents the empirical results. Section 5 analyzes the estimated Mexican government bond liquidity premium and evaluates its connections to foreign holdings of Mexican government bonds. Finally, Section 6 concludes. An online appendix contains various robustness checks of our model results and regressions.

2 Risks to Local Bond Markets from Foreign Participation

In general, foreign participation in local currency sovereign bond markets can bring benefits as well as costs, both of which may vary notably across time and complicate assessments of the impact of increases in the share of foreign investors.

As for potential benefits, foreign investors help develop the domestic fixed-income markets through improvements to technology, trading strategies, and related derivative markets and by diversifying the set of financial market participants, all of which could improve market liquidity.

Regarding downside risks, sudden stops in capital inflows are a major concern for emerging market economies integrated into global financial markets. Calvo et al. (2004) define such events as large and unexpected drops in net capital inflows that exceed two standard deviations below prevailing sample means. If aggregate bond spreads are elevated at the same time, they refer to them as systemic sudden stops. As described in Calvo et al. (2004), many emerging market countries have experienced sudden stops caused by a drying-up of capital flows from spikes in global risk aversion or rises in global interest rates. Specifically, they find that, for emerging market economies, systemic sudden stops tend to coincide with large, real currency depreciations (exceeding 20 percent) and output collapses averaging about 10 percent from peak to trough. Furthermore, systemic sudden stops in net capital flows by global investors are associated with greater slowdowns in economic activity and higher currency depreciations and are therefore more concerning than sudden flight events triggered by local investors, which tend to merely cause temporary spikes in gross capital outflows with much less negative impact on the domestic economy, as demonstrated by Rothenberg and Warnock (2011). Thus, while systemic sudden stops may be rare, they are a risk that merits careful monitoring by both policymakers and investors alike. In addition, large foreign inflows may lead to domestic asset price and credit bubbles that would further expose local financial markets and the economy to the risk of a sudden reversal in capital flows. Finally, even absent sudden stops, increased foreign participation can make both the local bond markets and the domestic economy more sensitive to shifts in global financial market sentiment.
In the current environment characterized by low global interest rates (see Holston et al. 2017) and an ongoing gradual normalization of U.S. monetary policy, a potential trigger for a sudden-stop type of event could be deleveraging by international banks in response to contractionary U.S. monetary policy shocks as described in Bruno and Shin (2015). Such spillover effects are also known as the “risk-taking” transmission channel of monetary policy first highlighted by Borio and Zhu (2012). However, in light of the overall deleveraging of international banks since the global financial crisis, such effects may not be bank-led in the future, but rather materialize directly in the markets for debt securities via asset managers and other “buy side” investors as argued by Shin (2013). He therefore encourages careful examination of the bond yields of emerging market debt securities, and our analysis can be viewed as an attempt to meet that objective.

Specific to the concerns about the normalization of U.S. monetary policy, Iacoviello and Navarro (2018) document that GDP in emerging economies tend to drop in response to U.S. monetary policy tightening, particularly when economic and financial vulnerability is high, see also Ammer et al. (2016). As for the international capital flows that are at the heart of sudden stops, Awdjiev et al. (2017) provide evidence of time variation in the drivers of global liquidity and suggest that they are likely to have changed since the global financial crisis.

More broadly, a number of papers have studied the drivers of foreign participation in local currency sovereign bond markets in emerging market economies and their effects on these assets. For example, Burger and Warnock (2006, 2007) examine bond markets in over 40 countries and find that greater foreign participation in local currency debt markets is explained by countries that have stable inflation rates, strong creditor rights, and greater macroeconomic stability. Other papers attribute some of this greater foreign participation in local currency bond markets since the financial crisis to low U.S. Treasury yields (Miyajima et al. 2015). To examine the effects of this increase in foreign participation, Peiris (2010) uses pre-crisis data to show that foreign investors diversify the investor base and increase liquidity of local currency bond markets. Ebeke and Lu (2015) use post-crisis data for 13 emerging market countries to show that increases in the foreign-held share of local currency sovereign bonds tend to be associated with declines in general yield levels but increases in yield volatility. Xiao (2015) and Zhou et al. (2014) analyze mutual fund portfolio flows in Mexico and find that foreign investors are more responsive to global shocks than local investors. The current paper complements this literature by looking directly at the connection between foreign holdings and sovereign bond liquidity premiums.

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3In the latest available annual financial system report released in November 2016 by the Bank of Mexico, a sharp increase in interest rates is listed as a major risk to the Mexican economy thanks to the significant share of foreign holdings of Mexican public debt.
Figure 1: **Overview of the Mexican Bonos Data**
Panel (a) shows the maturity distribution of the Mexican government fixed-coupon bonos considered in the paper. The solid grey rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on June 30, 2007, and limited to bonos prices with more than three months to maturity after issuance. Panel (b) reports the number of outstanding bonos at each point in time.

### 3 Mexican Government Bond Data

This section briefly describes the Mexican government bond price data we use in our model estimations as well as the bond holdings data that serves as the key explanatory variable in our regression analysis.

The set of individual standard Mexican fixed-coupon government bonds, known as bonos, available from Bloomberg at the time of our data pull is illustrated in Figure 1(a). Each bond is represented by a solid black line that starts at its date of issuance with a value equal to its original maturity and ends at zero on its maturity date. These bonds are all marketable non-callable bonds denominated in Mexican pesos that pay a fixed rate of interest semi-annually. In general, the Mexican government has been issuing five-, ten-, twenty- and thirty-year fixed-coupon bonds repeatedly during the shown period. As a consequence, there is a wide variety of bonds with different maturities and coupon rates in the data throughout the considered sample period. It is this variation that provides the foundation for the econometric identification of the factors in the yield curve models we use.

Figure 1(b) shows the distribution across time of the number of bonds included in the sample. We note a gradual increase from six bonds at the start of the sample to fifteen at its end. Combined with the cross sectional dispersion in the maturity dimension observed in Figure 1(a), this implies that we have a very well-balanced panel of Mexican bonos prices.

The contractual characteristics of all 21 bonos securities in our sample are reported in
Table 1: Sample of Mexican Bonos

The table reports the bonos name, type, number of monthly observations in the sample period from June 30, 2007, to December 29, 2017, first issuance date and amount, the total number of auctions, and the total amount issued in millions of Mexican pesos for the considered set of Mexican government fixed-coupon bonos based on the information available as of February 2018.

Table 1. The number of monthly observations for each bond using three-month censoring before maturity is also reported in the table. Although each bond is issued at first with a relatively small outstanding notional amount, they grow quickly thanks to a large number of subsequent reopenings that can raise their outstanding amounts to as much as 350 billion pesos, or close to 20 billion U.S. dollars. Thus, these are large bond series even by international standards.

Figure 2 shows the time series of the yields to maturity implied by the observed Mexican bonos prices downloaded from Bloomberg. We note a persistent decline from the peak of the global financial crisis in the fall of 2008 when bonos yields briefly spiked above ten percent to a trough in the spring of 2013 when long-term bonos yields were below six percent. This was followed by a sharp spike in Mexican medium- and long-term yields during the taper tantrum episode in May 2013. At first, short-term yields remained low, but they started to rapidly move higher as well in the spring of 2016 as the Bank of Mexico tightened the stance of monetary policy to fight elevated levels of price inflation and strengthen the value of the American dollar.

<table>
<thead>
<tr>
<th>Bonos (coupon, maturity)</th>
<th>Bonos type</th>
<th>No. observations</th>
<th>Issuance Date</th>
<th>Amount</th>
<th>Number of auctions</th>
<th>Total notional amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) 9% 12/20/2012</td>
<td>10yr</td>
<td>63</td>
<td>1/9/2003</td>
<td>1,500</td>
<td>40</td>
<td>86,618</td>
</tr>
<tr>
<td>(2) 8% 12/7/2023</td>
<td>20yr</td>
<td>127</td>
<td>10/30/2003</td>
<td>1,000</td>
<td>36</td>
<td>93,495</td>
</tr>
<tr>
<td>(3) 8% 12/17/2015</td>
<td>10yr</td>
<td>99</td>
<td>1/5/2006</td>
<td>3,100</td>
<td>33</td>
<td>106,009</td>
</tr>
<tr>
<td>(4) 10% 11/20/2036</td>
<td>30yr</td>
<td>127</td>
<td>10/26/2006</td>
<td>2,000</td>
<td>37</td>
<td>90,065</td>
</tr>
<tr>
<td>(5) 7.5% 6/3/2027</td>
<td>20yr</td>
<td>127</td>
<td>1/18/2007</td>
<td>4,650</td>
<td>35</td>
<td>179,629</td>
</tr>
<tr>
<td>(6) 7.25% 12/15/2016</td>
<td>10yr</td>
<td>111</td>
<td>2/1/2007</td>
<td>4,800</td>
<td>26</td>
<td>134,687</td>
</tr>
<tr>
<td>(7) 7.75% 12/14/2017</td>
<td>10yr</td>
<td>116</td>
<td>1/31/2008</td>
<td>7,650</td>
<td>25</td>
<td>93,220</td>
</tr>
<tr>
<td>(8) 8.5% 5/31/2029</td>
<td>20yr</td>
<td>108</td>
<td>1/15/2009</td>
<td>2,000</td>
<td>28</td>
<td>102,349</td>
</tr>
<tr>
<td>(9) 8.5% 11/18/2038</td>
<td>30yr</td>
<td>108</td>
<td>1/29/2009</td>
<td>2,000</td>
<td>36</td>
<td>100,072</td>
</tr>
<tr>
<td>(10) 8.5% 12/13/2018</td>
<td>10yr</td>
<td>107</td>
<td>2/12/2009</td>
<td>2,500</td>
<td>29</td>
<td>149,197</td>
</tr>
<tr>
<td>(11) 8% 6/11/2020</td>
<td>10yr</td>
<td>95</td>
<td>2/25/2010</td>
<td>25,000</td>
<td>20</td>
<td>351,000</td>
</tr>
<tr>
<td>(12) 6.5% 6/10/2021</td>
<td>10yr</td>
<td>83</td>
<td>2/3/2011</td>
<td>25,000</td>
<td>29</td>
<td>329,770</td>
</tr>
<tr>
<td>(13) 6.25% 6/16/2016</td>
<td>5yr</td>
<td>56</td>
<td>7/22/2011</td>
<td>25,000</td>
<td>17</td>
<td>167,550</td>
</tr>
<tr>
<td>(14) 7.75% 5/29/2031</td>
<td>20yr</td>
<td>76</td>
<td>9/9/2011</td>
<td>60,500</td>
<td>22</td>
<td>193,000</td>
</tr>
<tr>
<td>(15) 6.5% 6/9/2022</td>
<td>10yr</td>
<td>71</td>
<td>2/15/2012</td>
<td>74,500</td>
<td>22</td>
<td>335,096</td>
</tr>
<tr>
<td>(16) 7.75% 11/13/2042</td>
<td>30yr</td>
<td>69</td>
<td>4/20/2012</td>
<td>33,000</td>
<td>38</td>
<td>228,246</td>
</tr>
<tr>
<td>(17) 5% 6/15/2017</td>
<td>5yr</td>
<td>56</td>
<td>7/19/2012</td>
<td>30,000</td>
<td>11</td>
<td>145,000</td>
</tr>
<tr>
<td>(18) 4.75% 6/14/2018</td>
<td>5yr</td>
<td>53</td>
<td>8/30/2013</td>
<td>211,000</td>
<td>19</td>
<td>339,825</td>
</tr>
<tr>
<td>(19) 7.75% 11/23/2034</td>
<td>20yr</td>
<td>45</td>
<td>4/11/2014</td>
<td>15,000</td>
<td>26</td>
<td>104,377</td>
</tr>
<tr>
<td>(20) 5% 12/11/2019</td>
<td>5yr</td>
<td>38</td>
<td>11/7/2014</td>
<td>15,000</td>
<td>20</td>
<td>242,067</td>
</tr>
<tr>
<td>(21) 5.75% 3/5/2026</td>
<td>10yr</td>
<td>27</td>
<td>10/16/2015</td>
<td>17,000</td>
<td>12</td>
<td>172,711</td>
</tr>
</tbody>
</table>
Figure 2: **Mexican Bonos Yields**
Illustration of the yields to maturity implied by the Mexican government fixed-coupon bonos prices downloaded from Bloomberg. The data are monthly covering the period from June 30, 2007, to December 29, 2017, and censor the last three months for each maturing bond.

Mexican peso. Thus, there has been a fair amount of variation in the general yield level in Mexico during our sample period. Furthermore, as in U.S. Treasury yield data, there is notable variation in the shape of the yield curve. At times, like early in our sample, yields across maturities are relatively compressed. At other times, the yield curve is steep, with long-term bonos trading at yields that are 3 to 4 percent above those of shorter-term securities as in 2015. It is these characteristics that underlie our choice of using a three-factor model for the frictionless part of the Mexican yield curve similar to what is standard for U.S. and U.K. data; see Christensen and Rudebusch (2012).

Finally, regarding the important question of a lower bound, the Bank of Mexico has never been forced to lower its conventional policy rate even close to zero, and the bond yields in the data have remained well above zero throughout the sample period. Thus, there is no need to account for any lower bounds to model these fixed-coupon bond prices, which supports our focus on standard Gaussian yield curve models.
3.1 Mexican Government Debt Holdings

In addition to the bond price data described above, our analysis utilizes data on domestic and foreign holdings of Mexican government debt securities that the Bank of Mexico requires financial intermediaries to report as a way to track market activity in the Mexican sovereign bond markets. These data have been collected since 1978 and are available at daily frequency up to the present. A key strength of the data set is that it covers any change in Mexican government debt holdings by either domestic or foreign investors. For each transaction, the reporting forms also identify the type of Mexican government security. Therefore, we are able to exploit the data reported for holdings of bonos alone and leave other Mexican government securities for future research. Although the data are available at a daily frequency, we use the observations at the end of each month to align them with our bond price data.

Figure 3 shows the monthly level of bonos holdings by domestic residents and foreigners over the period from June 2007 through December 2017. We note that foreigners overtook domestic residents in total holdings by late 2012 and have continued to increase their share quite notably and now exceed those of domestic residents by a wide margin. The empirical question we are interested in is whether this dramatic increase in foreign holdings of Mexican government bonds has affected the liquidity risk in the market for these securities, but before we can address that question we need to introduce and estimate our models.
4 Model Estimation and Results

In this section, we first describe how we model yields in a world without any frictions to trading before we detail the augmented model that accounts for the liquidity premiums in Mexican government bond yields. This is followed by a description of the restrictions imposed to achieve econometric identification of this model and its estimation. We end the section with a brief summary of our estimation results.

4.1 A Frictionless Arbitrage-Free Model

To capture the fundamental or frictionless factors operating the Mexican government bond yield curve, we choose to focus on the tractable affine dynamic term structure model introduced in Christensen et al. (2011).4,5

In this arbitrage-free Nelson-Siegel (AFNS) model, the state vector is denoted by \( X_t = (L_t, S_t, C_t) \), where \( L_t \) is a level factor, \( S_t \) is a slope factor, and \( C_t \) is a curvature factor. The instantaneous risk-free rate is defined as

\[
    r_t = L_t + S_t.
\]

The risk-neutral (or \( Q \)-) dynamics of the state variables are given by the stochastic differential equations\(^6\)

\[
\begin{pmatrix}
    dL_t \\
    dS_t \\
    dC_t
\end{pmatrix} =
\begin{pmatrix}
    0 & 0 & 0 \\
    0 & -\lambda & \lambda \\
    0 & 0 & -\lambda
\end{pmatrix}
\begin{pmatrix}
    L_t \\
    S_t \\
    C_t
\end{pmatrix} dt + \Sigma
\begin{pmatrix}
    dW_t^{L,Q} \\
    dW_t^{S,Q} \\
    dW_t^{C,Q}
\end{pmatrix},
\]

where \( \Sigma \) is the constant covariance (or volatility) matrix.\(^7\) Based on this specification of the \( Q \)-dynamics, zero-coupon bond yields preserve the Nelson and Siegel (1987) factor loading structure as

\[
y_t(\tau) = L_t + \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right) S_t + \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right) C_t - \frac{A(\tau)}{\tau},
\]

\(^4\)To motivate this choice, we note that Espada et al. (2008) show that the first three principal components in their sample of Mexican government bond yields have a level, slope, and curvature pattern in the style of Nelson and Siegel (1987) and account for more than 99 percent of the yield variation.

\(^5\)Although the model is not formulated using the canonical form of affine term structure models introduced by Dai and Singleton (2000), it can be viewed as a restricted version of the canonical Gaussian model, see Christensen et al. (2011) for details.

\(^6\)As discussed in Christensen et al. (2011), with a unit root in the level factor, the model is not arbitrage-free with an unbounded horizon; therefore, as is often done in theoretical discussions, we impose an arbitrary maximum horizon.

\(^7\)As per Christensen et al. (2011), \( \Sigma \) is a diagonal matrix, and \( \theta^Q \) is set to zero without loss of generality.
where the yield-adjustment term is given by

\[
\frac{A(\tau)}{\tau} = \frac{\sigma^2_1}{6} \tau^2 + \sigma^2_2 \left[ \frac{1}{2\lambda^2} \frac{1 - e^{-\lambda\tau}}{\tau} + \frac{1}{4\lambda^3} \frac{1 - e^{-2\lambda\tau}}{\tau} \right] \\
+ \sigma^2_3 \left[ \frac{1}{2\lambda^2} \frac{1 - e^{-\lambda\tau}}{\tau} - \frac{1}{4\lambda^2} e^{-2\lambda\tau} + \frac{3}{4\lambda^3} e^{-2\lambda\tau} + \frac{5}{8\lambda^3} \frac{1 - e^{-2\lambda\tau}}{\tau} - \frac{2}{\lambda^3} \frac{1 - e^{-\lambda\tau}}{\tau} \right].
\]

To complete the description of the model and to implement it empirically, we need to specify the risk premiums that connect these factor dynamics under the Q-measure to the dynamics under the real-world (or physical) P-measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical P-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premiums \( \Gamma_t \) depend on the state variables; that is,

\[
\Gamma_t = \gamma^0 + \gamma^1 X_t,
\]

where \( \gamma^0 \in \mathbb{R}^3 \) and \( \gamma^1 \in \mathbb{R}^{3 \times 3} \) contain unrestricted parameters.

Thus, the resulting unrestricted three-factor AFNS model has P-dynamics given by

\[
\begin{pmatrix}
    dL_t \\
    dS_t \\
    dC_t \\
    dX^\text{liq}_t
\end{pmatrix}
= \begin{pmatrix}
    \kappa^p_{11} & \kappa^p_{12} & \kappa^p_{13} \\
    \kappa^p_{21} & \kappa^p_{22} & \kappa^p_{23} \\
    \kappa^p_{31} & \kappa^p_{32} & \kappa^p_{33}
\end{pmatrix}
\begin{pmatrix}
    \theta^p_1 \\
    \theta^p_2 \\
    \theta^p_3
\end{pmatrix}
\begin{pmatrix}
    L_t \\
    S_t \\
    C_t
\end{pmatrix}
+ \begin{pmatrix}
    dW^L_t \\
    dW^S_t \\
    dW^C_t
\end{pmatrix}.
\]

This is the transition equation in the Kalman filter estimation.

### 4.2 An Arbitrage-Free Model with Liquidity Risk

To augment the AFNS model with a liquidity risk factor to account for the liquidity risk embedded in bonus prices, let \( X_t = (L_t, S_t, C_t, X^\text{liq}_t) \) denote the state vector of this four-factor AFNS-L model. As before, \((L_t, S_t, C_t)\) represent level, slope, and curvature factors, while \(X^\text{liq}_t\) is the added liquidity factor.

As in the AFNS model, we let the frictionless instantaneous risk-free rate be defined by equation (1), while the risk-neutral dynamics of the state variables used for pricing are given by

\[
\begin{pmatrix}
    dL_t \\
    dS_t \\
    dC_t \\
    dX^\text{liq}_t
\end{pmatrix}
= \begin{pmatrix}
    0 & 0 & 0 & 0 \\
    0 & \lambda & -\lambda & 0 \\
    0 & 0 & \lambda & 0 \\
    0 & 0 & 0 & \kappa^Q_{\text{liq}}
\end{pmatrix}
\begin{pmatrix}
    0 \\
    L_t \\
    S_t \\
    C_t
\end{pmatrix}
+ \begin{pmatrix}
    dW^L_t \\
    dW^S_t \\
    dW^C_t \\
    dW^\text{liq}_t
\end{pmatrix},
\]

where \( \Sigma \) continues to be a diagonal matrix. This structure implies that \( X^\text{liq}_t \) is assumed to
be an independent Ornstein-Uhlenbeck process under the pricing measure.

Based on the Q-dynamics above, frictionless Mexican bonos zero-coupon yields preserve the Nelson-Siegel factor loading structure in equation (3). However, due to liquidity risk in the bonos market, bonos prices are sensitive to liquidity pressures and their pricing is therefore performed with a discount function that accounts for the liquidity risk

$$\pi^i_t = r_t + \beta^i(1 - e^{-\lambda^i(t-t^i_0)})X^L_{t} = L_t + S_t + \beta^i(1 - e^{-\lambda^i(t-t^i_0)})X^L_{t},$$

where $t^i_0$ denotes the date of issuance of the bonos in question and $\beta^i$ is its sensitivity to the variation in the liquidity factor with $\lambda^L,i$ being the associated decay parameter. While we could expect the sensitivities to be identical across securities, the results from our subsequent empirical application shows that it is important to allow for the possibility that the sensitivities differ across securities. Furthermore, we allow the decay parameter $\lambda^L,i$ to vary across securities as well. Since $\beta^i$ and $\lambda^L,i$ have a nonlinear relationship in the bond pricing formula, it is possible to identify both empirically. Finally, we stress that equation (4) can be included in any dynamic term structure model to account for security-specific liquidity risks as also emphasized by ACR.

The inclusion of the issuance date $t^i_0$ in the pricing formula is a proxy for the phenomenon that, as time passes, it is typically the case that an increasing fraction of a given security is held by buy-and-hold investors. This limits the amount of the security available for trading and drives up the liquidity premium. Rational and forward-looking investors will take this dynamic pattern into consideration when they determine what they are willing to pay for a security at any given point in time between the date of issuance and the maturity of the bond. This dynamic pattern is built into the model structure we use.

The net present value of one Mexican peso paid by bonos $i$ at time $t + \tau^i$ has the following exponential-affine form

$$P^i_t (t^i_0, \tau^i) = E^Q_t [e^{-\int_{t}^{t+\tau^i} \pi^i(s,t^i_0)ds}] = \exp \left( B^i_1(\tau^i)L_t + B^i_2(\tau^i)S_t + B^i_3(\tau^i)C_t + B^i_4(t^i_0, t, \tau^i)X^L_{t} + A^i(t^i_0, t, \tau^i) \right).$$

This implies that the model belongs to the class of Gaussian affine term structure models. Note also that, by fixing $\beta^i = 0$ for all $i$, we recover the AFNS model.

Now, consider the whole value of the bonos issued at time $t^i_0$ with maturity at $t + \tau^i$ that pays a coupon $C^i$ semi-annually. Its price is given by

$$P^i_t (t^i_0, \tau^i, C^i) = C^i(t_1-t)E^Q_t [e^{-\int_{t}^{t+\tau^i} \pi^i(s,t^i_0)ds}] + \sum_{j=2}^{N} \frac{C^i}{2} E^Q_t [e^{-\int_{t}^{t+\tau^i} \pi^i(s,t^i_0)ds}] + E^Q_t [e^{-\int_{t}^{t+\tau^i} \pi^i(s,t^i_0)ds}].$$

8See Christensen and Rudebusch (2019) for the derivation of this formula.
9This is the clean price that does not account for any accrued interest and maps to our observed bond prices.
Finally, to complete the description of the AFNS-L model, we again specify an essentially affine risk premium structure, which implies that the risk premiums $\Gamma_t$ take the form

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbb{R}^4$ and $\gamma^1 \in \mathbb{R}^{4 \times 4}$ contain unrestricted parameters. Thus, the resulting unrestricted four-factor AFNS-L model has $\mathbb{P}$-dynamics given by

$$
\begin{pmatrix}
    dL_t \\
    dS_t \\
    dC_t \\
    dX_{t}^{liq}
\end{pmatrix} =
\begin{pmatrix}
    \kappa^P_{11} & \kappa^P_{12} & \kappa^P_{13} & \kappa^P_{14} \\
    \kappa^P_{21} & \kappa^P_{22} & \kappa^P_{23} & \kappa^P_{24} \\
    \kappa^P_{31} & \kappa^P_{32} & \kappa^P_{33} & \kappa^P_{34} \\
    \kappa^P_{41} & \kappa^P_{42} & \kappa^P_{43} & \kappa^P_{44}
\end{pmatrix}
\begin{pmatrix}
    \theta^P_1 \\
    \theta^P_2 \\
    \theta^P_3 \\
    \theta^P_4
\end{pmatrix}
- \begin{pmatrix}
    L_t \\
    S_t \\
    C_t \\
    X_{t}^{liq}
\end{pmatrix}
\, dt + \Sigma
\begin{pmatrix}
    dW^L_{t} \\
    dW^{S, P}_{t} \\
    dW^{C, P}_{t} \\
    dW^{liq, P}_{t}
\end{pmatrix}.
$$

This is the transition equation in the extended Kalman filter estimation.

### 4.3 Model Estimation and Econometric Identification

Due to the nonlinearity of the bond pricing formulas, the models cannot be estimated with the standard Kalman filter. Instead, we use the extended Kalman filter as in Kim and Singleton (2012), see Christensen and Rudebusch (2019) for details. To make the fitted errors comparable across bonds of various maturities, we follow ACR and scale each bond price by its duration. Thus, the measurement equation for the bond prices takes the following form:

$$
\frac{P_i(t_0, \tau^i, C^i)}{D_i(t_0, \tau^i, C^i)} = \frac{\hat{P}_i(t_0, \tau^i, C^i)}{D_i(t_0, \tau^i, C^i)} + \varepsilon^i_t,
$$

where $\hat{P}_i(t_0, \tau^i, C^i)$ is the model-implied price of bond $i$ and $D_i(t_0, \tau^i, C^i)$ is its duration, which is fixed and calculated before estimation. In addition, we assume that all bond price measurement equations have i.i.d. fitted errors with zero mean and standard deviation $\sigma_\varepsilon$.

Since the liquidity factor is a latent factor that we do not observe, its level is not identified without additional restrictions. As a consequence, when we include the liquidity factor $X_{t}^{liq}$, we let the first thirty-year bond issued during our sample window have a unit loading on the liquidity factor, that is, bond number (9) in our sample issued on January 29, 2009, with maturity on November 18, 2038, and a coupon rate of 8.5 percent has $\beta^9 = 1$.

Furthermore, we note that the liquidity decay parameters $\lambda^{L,i}$ can be hard to identify if their values are too large or too small. As a consequence, we impose the restriction that they fall within the range from 0.0001 to 10, which is without practical consequences based on the evidence presented in ACR. Also, for numerical stability during the model optimization, we impose the restrictions that the liquidity sensitivity parameters $\beta^i$ fall within the range from 0 to 250, which turns out not to be a binding constraint at the optimum.

---

10The robustness of this formulation of the measurement equation is documented in Andreasen et al. (2019).
Finally, we assume that the state variables are stationary and therefore start the Kalman filter at the unconditional mean and covariance matrix. This assumption is supported by the analysis in Chiquiar et al. (2010), who find that Mexican inflation seems to have become stationary at some point in the early 2000s, while De Pooter et al. (2014) document that measures of long-term inflation expectations from both surveys and the Mexican government bond market have remained anchored close to the 3 percent inflation target of the Bank of Mexico at least since 2003. Assuming real rates and bond risk premiums are stationary, this evidence would imply that Mexican government bond yields should be stationary as well, as also suggested by visual inspection of the individual yield series depicted in Figure 2.

4.4 Estimation Results

This section presents our benchmark estimation results. In the interest of simplicity, we focus on a version of the AFNS-L model where $K_P$ and $\Sigma$ are diagonal matrices. As shown in ACR, these restrictions have hardly any effects on the estimated liquidity premiums, because they are identified from the model’s $Q$-dynamics, which are independent of $K_P$ and display only a weak link to $\Sigma$ through the small convexity adjustment in yields. However, as a robustness check, we relax this assumption by considering alternative specifications of the model’s $P$-dynamics as well as the frequency of the data. These exercises reveal that our results are indeed robust to such changes.\(^{12}\)

The impact of accounting for liquidity risk is apparent in our results. The first two columns in Table 2 show that the bonos pricing errors produced by the AFNS model indicate a reasonable fit, with an overall root mean-squared error (RMSE) of 6.86 basis points. The following two columns reveal a substantial improvement in the pricing errors when correcting for liquidity risk, as the AFNS-L model has a much lower overall RMSE of just 4.16 basis points. Hence, accounting for liquidity risk leads to a notable improvement in the ability of our model to explain bonos market prices. Without exception there is uniform improvement in model fit as measured by RMSE from incorporating the liquidity risk factor into the AFNS model. Note also that neither twenty- nor thirty-year bonds pose any particular challenges for the two models. Thus, both AFNS and AFNS-L models are clearly able to fit those long-term bond yields to a satisfactory accuracy.

The final four columns of Table 2 report the estimates of the specific liquidity parameters associated with each bonos. Except for the last bonos for which we have few observations, all other bonos in our sample are exposed to liquidity risk as their $\beta^i$ parameters are significantly different from zero at the conventional 5 percent level.

Table 3 reports the estimated dynamic parameters. We note that the slope factor is the factor most significantly impacted by adding the liquidity risk factor to the AFNS model.

\(^{11}\)We note that these might be strong assumptions. In the United States, there is evidence of a persistent downward trend in real yields the past two decades; see Christensen and Rudebusch (2019).

\(^{12}\)The results are reported in online appendices A and C.
Table 2: Pricing Errors and Estimated Liquidity Risk Parameters

This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of Mexican government bonds in the AFNS and AFNS-L models estimated with a diagonal specification of $K_P$ and $\Sigma$. The errors are computed as the difference between the bond market price expressed as yield to maturity and the corresponding model-implied yield. All errors are reported in basis points. Standard errors (SE) are not available (n.a.) for the normalized value of $\beta^9$. Asterisk * indicates five-year bonds, dagger † indicates ten-year bonds, plus + indicates twenty-year bonds, and cross × indicates thirty-year bonds.

<table>
<thead>
<tr>
<th>Mexican bonos</th>
<th>Pricing errors</th>
<th>Estimated parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AFNS</td>
<td>AFNS-L</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>RMSE</td>
</tr>
<tr>
<td>(1) 9% 12/20/2012†</td>
<td>-0.26</td>
<td>6.39</td>
</tr>
<tr>
<td>(2) 8% 12/7/2023†</td>
<td>-3.72</td>
<td>7.02</td>
</tr>
<tr>
<td>(3) 8% 12/17/2015†</td>
<td>3.35</td>
<td>7.44</td>
</tr>
<tr>
<td>(4) 10% 11/20/2036×</td>
<td>4.83</td>
<td>6.94</td>
</tr>
<tr>
<td>(5) 7.5% 6/3/2027†</td>
<td>-2.80</td>
<td>6.65</td>
</tr>
<tr>
<td>(6) 7.25% 12/15/2016†</td>
<td>0.93</td>
<td>7.16</td>
</tr>
<tr>
<td>(7) 7.75% 12/14/2017†</td>
<td>-1.44</td>
<td>9.88</td>
</tr>
<tr>
<td>(8) 8.5% 5/31/2029†</td>
<td>0.59</td>
<td>4.37</td>
</tr>
<tr>
<td>(9) 8.5% 11/18/2038×</td>
<td>3.89</td>
<td>6.83</td>
</tr>
<tr>
<td>(10) 8.5% 12/13/2018†</td>
<td>2.04</td>
<td>5.04</td>
</tr>
<tr>
<td>(11) 8% 6/11/2020†</td>
<td>1.46</td>
<td>6.21</td>
</tr>
<tr>
<td>(12) 6.5% 6/10/2021†</td>
<td>3.63</td>
<td>6.78</td>
</tr>
<tr>
<td>(13) 6.25% 6/16/2016*</td>
<td>0.42</td>
<td>8.82</td>
</tr>
<tr>
<td>(14) 7.75% 5/29/2031†</td>
<td>3.02</td>
<td>5.27</td>
</tr>
<tr>
<td>(15) 6.5% 6/9/2022†</td>
<td>3.70</td>
<td>5.57</td>
</tr>
<tr>
<td>(16) 7.75% 11/13/2042×</td>
<td>-2.31</td>
<td>5.55</td>
</tr>
<tr>
<td>(17) 5% 6/15/2017*</td>
<td>-0.96</td>
<td>8.62</td>
</tr>
<tr>
<td>(18) 4.75% 6/14/2018*</td>
<td>5.33</td>
<td>8.84</td>
</tr>
<tr>
<td>(19) 7.75% 11/23/2034†</td>
<td>-0.70</td>
<td>3.95</td>
</tr>
<tr>
<td>(20) 5% 12/11/2019*</td>
<td>2.62</td>
<td>6.82</td>
</tr>
<tr>
<td>(21) 5.75% 3/5/2026†</td>
<td>-2.30</td>
<td>6.43</td>
</tr>
<tr>
<td>All yields</td>
<td>1.01</td>
<td>6.86</td>
</tr>
<tr>
<td>Max $L^{EKF}$</td>
<td>9,352.62</td>
<td>10,036.44</td>
</tr>
</tbody>
</table>

Both its mean-reversion rate and volatility decline significantly, while the changes in these two parameters for the level and curvature factors are much smaller and not statistically significant except for $\sigma_{11}$. Furthermore, the mean parameters for all three frictionless factors change quite markedly, but this is due to the latent nature of our models that allows the filtered state variables to have sizable level differences across the two models (not shown), while maintaining a close fit to the data in both estimations. Finally, there is a slight increase in the estimated value of $\lambda$ as we move from the AFNS model to the AFNS-L model. This implies that in the AFNS-L model the slope factor puts more emphasis on fitting shorter-term bond yields. The fourth factor in the AFNS-L model, the liquidity factor, is a persistent process with a volatility close to that of the level factor. Its mean under the objective $\mathbb{P}$
### Table 3: Estimated Dynamic Parameters

The table shows the estimated dynamic parameters for the AFNS and AFNS-L models estimated with a diagonal specification of $K_P$ and $\Sigma$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AFNS</th>
<th>AFNS-L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>SE</td>
</tr>
<tr>
<td>$\kappa_{11}^P$</td>
<td>0.2846</td>
<td>0.1246</td>
</tr>
<tr>
<td>$\kappa_{22}^P$</td>
<td>6.5806</td>
<td>1.2222</td>
</tr>
<tr>
<td>$\sigma_{11}$</td>
<td>0.0077</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\sigma_{22}$</td>
<td>0.0812</td>
<td>0.0020</td>
</tr>
<tr>
<td>$\sigma_{33}$</td>
<td>0.0282</td>
<td>0.0021</td>
</tr>
<tr>
<td>$\sigma_{44}$</td>
<td>––</td>
<td>––</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.2196</td>
<td>0.0095</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>-0.1803</td>
<td>0.0068</td>
</tr>
<tr>
<td>$\eta_3$</td>
<td>-0.1200</td>
<td>0.0223</td>
</tr>
<tr>
<td>$\eta_4$</td>
<td>––</td>
<td>––</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.1770</td>
<td>0.0036</td>
</tr>
<tr>
<td>$\kappa_{liq}$</td>
<td>––</td>
<td>––</td>
</tr>
<tr>
<td>$\theta_{liq}$</td>
<td>––</td>
<td>––</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.0008</td>
<td>$1.02 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

probability measure is -0.0133, which is close to the average of its filtered path. However, its mean under the risk-neutral $Q$ probability measure used for pricing is 0.0095, which explains why the estimated bonos liquidity premiums described in the next section are strictly positive.

### 5 The Bonos Liquidity Premium

In this section, we analyze the bonos liquidity premium implied by the estimated AFNS-L model described in the previous section. First, we formally define the bonos liquidity premium, study its historical evolution, and assess its robustness before we end the section by relating the estimated liquidity premium to foreign holdings of bonos, while controlling for other relevant factors that could affect the liquidity risk of bonos.

#### 5.1 The Estimated Bonos Liquidity Premium

We use the estimated AFNS-L model to extract the liquidity premium in the bonos market. To compute this premium, we first use the estimated parameters and the filtered states $\{X_{it}\}_{t=1}^T$ to calculate the fitted bonos prices $\{\hat{p}_t\}_{t=1}^T$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\{\hat{y}_{t}^{c,i}\}_{t=1}^T$ by solving
the fixed-point problem

\[ \hat{P}_i = C(t_1 - t) \exp \left\{ -(t_1 - t)\hat{\gamma}^c_{i} \right\} + \sum_{k=2}^{n} \frac{C}{2} \exp \left\{ -(t_k - t)\hat{\gamma}^c_{i} \right\} + \exp \left\{ -(T - t)\hat{\gamma}^c_{i} \right\}, \quad (5) \]

for \( i = 1, 2, ..., N \), meaning that \( \left\{ \hat{\gamma}^c_{i} \right\}_{t=1}^{T} \) is the rate of return on the \( i \)th bonos if held until maturity. To obtain the corresponding yields corrected for liquidity risk, a new set of model-implied bond prices are computed from the estimated AFNS-L model but using only its frictionless part, i.e., using the constraints that \( X_{i|t}^{liq} = 0 \) for all \( t \) as well as \( \sigma_{44} = 0 \) and \( \theta_{liq}^Q = 0 \). These prices are denoted \( \{ \hat{\hat{P}}_i \}_{t=1}^{T} \) and converted into yields to maturity \( \hat{\hat{\gamma}}^c_{i} \) using (5). They represent estimates of the prices that would prevail in a world without any financial frictions. The liquidity premium for the \( i \)th bonos is then defined as

\[ \Psi^i_t \equiv \hat{\gamma}^c_{i} - \hat{\hat{\gamma}}^c_{i}. \quad (6) \]

The average estimated liquidity premium of Mexican bonos implied by the AFNS-L model is shown with a solid black line in Figure 4. We note that the estimated liquidity premium is of considerable size, with an average of 0.57 percent and a standard deviation of 0.19 percent. Hence, liquidity risk is an important component in the pricing of Mexican government bonds. Furthermore, we see significant variation around a general upward trend during our sample period, with notable spikes in the summer of 2011 and spring of 2014 and a persistent decline in the fall of 2016.

Next, we are interested in understanding the determinants of the bonos liquidity premium series and its ties to the increase in foreign holdings of Mexican government bonds described in Section 3.1. Therefore, in Figure 4, we also show the market share of foreigners defined as foreign net holdings divided by total public holdings (solid gray line) where we note a high positive correlation (70 percent) between it and the average estimated bonos liquidity premium. The key empirical question is to what extent variation in the estimated bonos liquidity premium can be explained by the foreign-held share of the Mexican bonos market.

5.2 Regression Analysis

To explain the variation of the bonos liquidity premiums, we run standard regressions with the liquidity premium series as the dependent variable and the share of foreign holdings of bonos as the explanatory factor.\(^{13}\) In addition, we include a number of controls that are thought to matter for bonos market liquidity specifically or bond market liquidity more broadly as described in the following.\(^{14}\)

---

\(^{13}\)Our analysis is inspired by Hancock and Passmore (2015), who use the U.S. Federal Reserve’s holdings as a share of the U.S. Treasury and mortgage backed securities (MBS) markets as explanatory variables to determine their effect on MBS yields and mortgage rates.

\(^{14}\)The full details of all control variables are provided in online appendix B.
Figure 4: Estimated Bonos Liquidity Premium and Foreign Share of Bonos Market
Illustration of the average estimated liquidity premium of Mexican bonos for each observation date implied by the AFNS-L model estimated with a diagonal specification of $K^P$ and $\Sigma$. The Mexican bonos liquidity premiums are measured as the estimated yield difference between the fitted yield to maturity of individual Mexican bonos and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. Also shown is the share of the bonos market held by foreigners at the end of each month. Both series cover the period from June 30, 2007, to December 29, 2017.

In a core set of controls, we first consider the Mexican peso-U.S. dollar exchange rate. Presumably foreign flows to and from the Mexican bonos market would be sensitive to exchange rate developments. Second, to control for factors that affect emerging market sovereign bonds more broadly, we include the J.P. Morgan Emerging Market Bond Index (EMBI). The third variable is the West Texas Intermediate (WTI) Cushing crude oil price. As a major oil producing country, the revenue and bond issuance of the Mexican government are affected by changes in oil prices, which could play a role for the liquidity in the Mexican government bond market. Our final three core controls are specific to Mexico, namely the year-over-year change in the Mexican consumer price index (CPI), the public debt-to-GDP ratio as measured by the OECD, and the average bid-ask spread in the bonos market. Combined the listed six variables represent our core set of control variables.

In an extended group of controls, we add the one-month cetes rate to proxy for the opportunity cost of holding money and the associated liquidity convenience premiums of bonos, as explained in Nagel (2016). Furthermore, we include the average bonos age and the one-month realized volatility of the ten-year bonos yield as additional proxies for bond liquidity following the work of Houweling et al. (2005). Inspired by the analysis of Hu et
al. (2013), we also include a noise measure of bonos prices to control for variation in the amount of arbitrage capital available in this market. In addition, we use the five-year credit default swap (CDS) rate for Mexico and the monthly return of the MSCI Mexico stock index as two other measures of general developments in the Mexican economy of importance to investors in the bonos market.\textsuperscript{15} We also add the VIX, which represents near-term uncertainty about the general stock market as reflected in options on the Standard & Poor’s 500 stock price index and is widely used as a gauge of investor fear and risk aversion. Furthermore, we include the yield difference between seasoned (off-the-run) U.S. Treasury securities and the most recently issued (on-the-run) U.S. Treasury security of the same ten-year maturity mentioned earlier. This on-the-run (OTR) premium is a frequently used measure of financial frictions in the U.S. Treasury market. The final variable is the U.S. TED spread, which is calculated as the difference between the three-month U.S. LIBOR and the three-month U.S. T-bill interest rate. This spread represents a measure of the perceived general credit risk in global financial markets that could affect the pricing and trading of Mexican bonos.

To begin, we run regressions with each explanatory variable in isolation. The results are reported in the last two columns of Table 4. The foreign-held share of the bonos market and the average bonos age series have the largest individual explanatory power followed by the debt-to-GDP ratio, the peso-U.S. dollar exchange rate, and the WTI oil price, while the financial variables (the EMBI, the CDS rate, the return of the MSCI index, the VIX, the on-the-run premium, and the TED spread) and CPI inflation only have a weak link with the bonos liquidity premium as measured by the adjusted $R^2$. The same holds for our proxies of bonos market liquidity and frictions (bonos bid-ask spread, yield volatility, and noise measure). Finally, the one-month cetes rate and the associated opportunity cost of holding cash has a negative relationship with our estimated bonos liquidity premium series. This is consistent with the findings of Nagel (2016) as increases in the convenience yield of holding bonos, as measured by the cetes rate, should put downward pressure on the illiquidity discount of bonos.

The columns labeled (1) and (2) in Table 4 show the results of our preferred joint regression with our core set of variables and the full joint regression with all explanatory variables included, respectively. Three things stand out. First, both regressions produce about the same adjusted $R^2$ (roughly 73 percent). Thus, the preferred regression yields about as much explanatory power as possible given our fifteen control variables. Second, the foreign-held share has an estimated coefficient that is close to 0.75 and statistically significant, that is, we find a positive relationship whereby a 1 percentage point increase in the foreign-held share of Mexican government bonds tends to raise their liquidity premium by about 0.75 basis point. Third, the WTI, the Mexican CPI inflation, and the average bonos bid-ask spread all have a stable relationship with the bonos liquidity premium series in these joint regressions and

\textsuperscript{15}The MSCI index is a free-float weighted equity index designed to measure the performance of the large and mid cap segments of the Mexican stock market. The index is reported in U.S. dollars.
## Individual regressions

<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>$\hat{\beta}$</th>
<th>adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign share</td>
<td>0.76**</td>
<td>0.70*</td>
<td></td>
<td>0.79**</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.35)</td>
<td></td>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>Peso/USD exchange rate</td>
<td>1.30</td>
<td>0.69</td>
<td>0.17</td>
<td>4.67**</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(1.58)</td>
<td>(2.24)</td>
<td>(2.39)</td>
<td>(0.85)</td>
<td></td>
</tr>
<tr>
<td>EMBI</td>
<td>6.56*</td>
<td>4.44</td>
<td>2.91</td>
<td>-7.40**</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(2.77)</td>
<td>(3.32)</td>
<td>(3.24)</td>
<td>(2.62)</td>
<td></td>
</tr>
<tr>
<td>WTI</td>
<td>-0.32**</td>
<td>-0.28*</td>
<td>-0.25*</td>
<td>-0.44**</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>CPI inflation</td>
<td>-5.12**</td>
<td>-4.22*</td>
<td>-3.43</td>
<td>-5.30</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(1.10)</td>
<td>(1.73)</td>
<td>(1.96)</td>
<td>(3.05)</td>
<td></td>
</tr>
<tr>
<td>Debt-to-GDP ratio</td>
<td>-0.54</td>
<td>0.59</td>
<td>0.93</td>
<td>1.59**</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(1.41)</td>
<td>(1.80)</td>
<td>(0.30)</td>
<td></td>
</tr>
<tr>
<td>Bonos bid-ask spread</td>
<td>-6.52**</td>
<td>-5.59**</td>
<td>-6.26**</td>
<td>-5.32</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(1.89)</td>
<td>(2.04)</td>
<td>(2.22)</td>
<td>(2.95)</td>
<td></td>
</tr>
<tr>
<td>One-month cetes rate</td>
<td>-0.11</td>
<td>-1.90</td>
<td></td>
<td>-3.06*</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(2.64)</td>
<td>(2.62)</td>
<td></td>
<td>(1.52)</td>
<td></td>
</tr>
<tr>
<td>Avg. bonos age</td>
<td>-2.61</td>
<td>1.84</td>
<td></td>
<td>9.18**</td>
<td>0.50</td>
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<tr>
<td></td>
<td>(5.56)</td>
<td>(5.73)</td>
<td></td>
<td>(1.47)</td>
<td></td>
</tr>
<tr>
<td>One-month bonos yield vol.</td>
<td>-0.13</td>
<td>-0.04</td>
<td></td>
<td>-0.12</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.11)</td>
<td></td>
<td>(0.08)</td>
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</tr>
<tr>
<td>Bonos noise measure</td>
<td>0.46</td>
<td>-0.16</td>
<td></td>
<td>0.08</td>
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<td></td>
<td>(0.80)</td>
<td>(0.87)</td>
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<td>(0.95)</td>
<td></td>
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<tr>
<td>CDS rate</td>
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<td>0.01</td>
<td></td>
<td>0.00</td>
<td>-0.01</td>
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<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
<td></td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>MSCI one-month return</td>
<td>-0.10</td>
<td>-0.18</td>
<td></td>
<td>-0.29</td>
<td>0.00</td>
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<tr>
<td></td>
<td>(0.15)</td>
<td>(0.17)</td>
<td></td>
<td>(0.25)</td>
<td></td>
</tr>
<tr>
<td>VIX</td>
<td>-0.02</td>
<td>-0.33</td>
<td></td>
<td>-0.73**</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
<td>(0.40)</td>
<td></td>
<td>(0.21)</td>
<td></td>
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<tr>
<td>OTR premium</td>
<td>0.18</td>
<td>-0.10</td>
<td></td>
<td>-0.52**</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.26)</td>
<td></td>
<td>(0.17)</td>
<td></td>
</tr>
<tr>
<td>TED spread</td>
<td>0.10</td>
<td>0.16</td>
<td></td>
<td>-0.04</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.08)</td>
<td></td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>64.90*</td>
<td>43.70</td>
<td>62.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(26.62)</td>
<td>(42.69)</td>
<td>(49.27)</td>
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<td></td>
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<tr>
<td>Adjusted $R^2$</td>
<td>0.73</td>
<td>0.74</td>
<td>0.71</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: **Regression Results**

The table reports the results of regressions with the average estimated bonos liquidity premium as the dependent variable and seven explanatory variables described in the main text. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

Contribute significant explanatory power on their own as evidenced by the regression results in the column labeled (3), which omits the foreign-held share as an explanatory variable. However, as this latter regression only generates an adjusted $R^2$ of 71 percent, it is clear that
the foreign-held share is a key determining factor in explaining the variation in the bonos liquidity premium series. Still, regression (3) reveals that our controls are relevant and able to explain a significant part of the variation in the bonos liquidity premium series.

Furthermore, as reported in online appendix D, we also run our regressions in first differences without obtaining any significant results. This suggests that our results are driven by stock effects from foreigners holding the bonds rather than by flow effects from their bond purchases and sales.

To summarize, the regression results reveal that the increase in the share of foreign holdings of Mexican bonos is significantly positively correlated with the change in the bonos liquidity premiums, both on its own and after including a large number of control variables. In terms of magnitude, the results imply that a 1 percentage point increase in the foreign share raises the liquidity premium by about 0.75 basis point. Given that the foreign market share has increased by more than 40 percentage points between 2010 and 2017, our results suggest that the large increase in foreign holdings during our sample period has played a significant role for the upward trend in the liquidity premiums in the Mexican bonos market since then and raised them by as much as 0.3 percent.

A potential explanation for our findings is tied to the fact that our measure of liquidity premiums is forward-looking and not determined by the current trading conditions in the market, which indeed have improved as measured by transaction costs such as our average bid-ask spread series (see online appendix B). Hence, investors may expect that, as foreign holdings of Mexican government debt increase, the probability of a large sell-off has increased as well—particularly with the normalization of U.S. monetary policy in progress. As a consequence, investors may demand higher liquidity premiums. If so, our results would imply that investors are indeed being compensated for the added liquidity risk in case these flows were to reverse in coming years. Provided the increased compensation is commensurate with the risk of such events, the expanded role of foreigners in the Mexican government bond market may not pose a material risk to its financial stability currently.

However, as already noted in the introduction, there is an important caveat to any conclusions in that our sample only covers a period with an increasing foreign share in the Mexican bonos market. As a consequence, the estimated model may not accurately represent the dynamics surrounding a sudden stop in the foreign supply of funds to the Mexican bond market.

6 Conclusion

In this paper, we analyze the relationship between foreign holdings of Mexican government bonos and the premiums investors demand for assuming their liquidity risk. Our results show that increases in foreign holdings tend to put upward pressure on liquidity premiums in the bonos market. Although foreign holdings of Mexican bonos have increased significantly in
recent years and likely contributed to the upward trend in their liquidity premiums, this may not pose a risk to financial stability provided the uptick in liquidity premium compensation is adequate relative to the underlying risk of any major sell-off in coming years. More broadly, this type of research may shed light on the important role that foreign investors play for the stability of financial markets in emerging economies.

Finally, we feel compelled to stress the versatility of our empirical approach. For one, it is straightforward to cast the considered model as a shadow-rate model that respects a lower bound for bond yields using formulas provided in Christensen and Rudebusch (2015) in case that is needed. Also, it is feasible to allow for stochastic volatility using the generalized AFNS models developed in Christensen et al. (2014). Finally, the presented model can be expanded with macroeconomic variables as in Espada and Ramos-Francia (2008a), with expectations from surveys as in Kim and Orphanides (2012), or with real yields as in ACR. Thus, there are many ways to enrich the analysis. However, we leave it for future research to explore those avenues.
References


Online Appendix

Bond Flows and Liquidity:
Do Foreigners Matter?

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The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

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## Contents

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D  Regressions in First Differences
A  Sensitivity of Liquidity Premium to Data Frequency

In this appendix, we study the sensitivity of the estimated bonos liquidity premiums to the frequency of the data used in the model estimation. Specifically, we repeat the estimation using data at daily, weekly (Fridays), and monthly (end-of-month) frequencies. The averages of the estimated bonos liquidity premiums for each observation date from the three estimations are shown in Figure 1. We note the closeness and very high positive correlation across all three series based on which we conclude that the estimated bonos liquidity premium series is robust to varying the data frequency used in the model estimation.

![Figure 1: Sensitivity of Estimated Bonos Liquidity Premiums to Data Frequency](image)

Illustration of the average estimated liquidity premium of Mexican bonos for each observation date implied by the AFNS-L model estimated with three different data frequencies. Note that $K^p$ and $\Sigma$ have diagonal specifications in all estimations. The Mexican bonos liquidity premiums are measured as the estimated yield difference between the fitted yield to maturity of individual Mexican bonos and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The full daily data cover the period from June 1, 2007, to December 29, 2017.

B  Description of Regression Variables

In this appendix, we provide a detailed description of the variables used in the regression analysis in the paper starting with our core set of control variables followed by details about
the extended set of control variables also used in the analysis.

B.1 Core Control Variables

For a start, Figure 2 shows the Mexican peso per U.S. dollar exchange rate that we use in the regressions. We include it in levels because we are interested in controlling for the effects of persistent trends in the exchange rate on the bonos liquidity premium series.

The J.P. Morgan Emerging Market Bond Index (EMBI) is shown in Figure 3. This series
is used to control for factors that affect emerging market sovereign bonds more broadly.

Figure 4: **West Texas Intermediate Oil Price**

The third variable in our core set of controls is the West Texas Intermediate (WTI) Cushing crude oil price shown in Figure 4. As a major oil producing country, the revenue and bond issuance of the Mexican government are affected by changes in oil prices, which could play a role for the liquidity in the Mexican government bond market.

Figure 5: **Mexican Consumer Price Index Inflation**

Figure 5 shows the year-over-year change in the Mexican consumer price index (CPI). Since this is the price index targeted by the Bank of Mexico in its operation of monetary
policy, its variation is likely to matter for the pricing and market conditions of Mexican bonos.

Figure 6: **Mexican Public Debt-To-GDP Ratio**
Illustration of the Mexican public debt-to-GDP ratio. The data series is quarterly and therefore linearly interpolated to produce the monthly series used in the regression analysis.

Figure 6 shows the Mexican public debt-to-GDP ratio, which serves as an important control for changes in the supply of Mexican bonos. Note that this series is quarterly. Hence, we use linear interpolation to convert it into a monthly series that can be used in the regression analysis.

As a classic measure of current market liquidity, we include the average bid-ask spread in the bonos market shown in Figure 7. This series is constructed by first taking the average of the bid-ask spreads of the individual bonos observed on a given day, then we calculate the four-week moving average of the daily average bid-ask spread series. Finally, we take out the observation at the end of each month, which is the series shown in the figure and used in our regressions. We note that bid-ask spreads in the bonos market are orders of magnitude above the corresponding spreads in the U.S. Treasury market, which is consistent with the sizable liquidity premiums we observe in our estimation results.

### B.2 Additional Control Variables

In this section, we describe the additional series included in our extended set of control variables.

Figure 8 shows the one-month cetes rate, which serves as a proxy for the leading policy
Figure 7: Mexican Bonos Bid-Ask Spreads

Figure 8: Mexican Interest Rates
Illustration of the fitted ten-year Mexican bonos yield implied by the AFNS model along with the one-month cetes rate downloaded from Bloomberg.
rate of the Bank of Mexico. We use this series as a measure of the opportunity costs of holding money, which has been shown by Nagel (2016) to be a good proxy of the liquidity premiums in U.S. Treasury bills, and we want to control for similar effects in the pricing of Mexican bonos. The figure also shows the ten-year Mexican zero-coupon yield implied by the AFNS model estimated with a one-step approach as recommended by Andreasen et al. (2019). This series is used to construct the realized yield volatility series discussed later on.

![Figure 9: Average Mexican Bonos Age and Time to Maturity](image)

Illustration of the average age and time to maturity of the Mexican bonos included in the sample, which covers the period from June 30, 2007, to December 29, 2017 and censors each bond’s price when it has less than three months to maturity.

Following the work of Houweling et al. (2005), we include the average bonos age as a proxy for bond liquidity. This series is shown in Figure 9 along with the average remaining time to maturity. We note that the age of the available universe of bonos has trended up during the sample period and is at a sample-high at the end of our sample with a value of 7.27 years. On the other hand, the remaining time to maturity of the available set of bonos has in general been trending down and stood at 9.58 years at the end of our sample. This still indicates that our sample is dominated by medium- and long-term bonos.

Houweling et al. (2005) also recommend to use yield volatility as a proxy for bond liquidity. To that end, we use a standard measure of realized volatility based on daily data. First, we estimate the AFNS model using the daily sample of bonos prices considered in Appendix A. This gives us daily fitted Mexican zero-coupon yields at all relevant maturities. We then generate the realized standard deviation of daily changes in interest rates for the past 31-day
period on a rolling basis. The realized variance measure is used by Andersen and Benzoni (2010), Collin-Dufresne et al. (2009), as well as Jacobs and Karoui (2009) in their assessments of stochastic volatility models. This measure is fully nonparametric and has been shown to converge to the underlying realization of the conditional variance as the sampling frequency increases; see Andersen et al. (2003) for details. The square root of this measure retains these properties. For each observation date $t$ we determine the number of trading days $N$ during the past 31-day time window (where $N$ is most often 21 or 22). We then generate the realized standard deviation as

$$RV^{STD}_{t,\tau} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \Delta y_{t+n/N}(\tau)},$$

where $\Delta y_{t+n/N}(\tau)$ is the change in yield $y(\tau)$ from trading day $(n-1)$ to trading day $n$.\(^1\)

---

**Figure 10: Realized Yield Volatility**

Illustration of the one-month realized volatility of yields with three different maturities, all constructed from the AFNS model estimated with our daily sample of Mexican bonos prices.

Figure 10 shows that the realized yield volatility series used in the regression analysis is not sensitive to the choice of the yield maturity considered. Furthermore, given that the average time to maturity of the available Mexican bonos is close to ten years for much of our sample period as demonstrated in Figure 9, we choose to use the one-month realized volatility of the ten-year yield in our regressions, but we stress that our results are clearly robust to alternative choices in terms of the maturity considered.

\(^1\)Note that other measures of realized volatility have been used in the literature, such as the realized mean absolute deviation measure as well as fitted GARCH estimates. Collin-Dufresne et al. (2009) also use option-implied volatility as a measure of realized volatility.
Figure 11: **Noise Measures**
Illustration of the Mexican noise measures constructed based on the fitted errors implied by the estimated AFNS model using the Mexican bonos covering the period from June 1, 2007, to December 29, 2017 with a comparison to the U.S. noise measure constructed by HPW.

Inspired by the analysis of Hu et al. (2013, henceforth HPW), we also include a noise measure of Mexican bonos prices to control for variation in the amount of arbitrage capital available in this market. In principle, this could be constructed using any yield curve model. However, to be consistent with the rest of the analysis, we choose to focus on the AFNS model already used and estimated with the one-step approach recommended by Andreasen et al. (2019). The resulting noise measure defined as the average absolute fitted errors of the yield to maturity across all available bonds at each observation date is shown in Figure 11 with a comparison to the U.S. noise measure constructed by HPW and accessed from Jun Pan’s personal website with data through the end of 2018.\(^2\) We note that the Mexican noise measure is larger and more variable than the U.S. noise measure, which is consistent with the wider bid-ask spreads in this market and the sizable liquidity premiums we observe in our model estimations.

Figure 12 shows the five-year credit default swap (CDS) rate for Mexico. We include this measure to control for any relationships between the bonos liquidity premiums and investors’ perceptions about the credit risk of holding bonos securities. Except for a sharp short-lived spike around the peak of the financial crisis, we note that the credit risk premium of the Mexican government has fluctuated around a fairly stable level close to 100 basis points.

\(^2\)See the link: http://en.saif.sjtu.edu.cn/junpan/
Figure 12: Five-Year Mexican CDS Rate

Figure 13: Monthly Return of the MSCI Mexico Stock Index
Figure 13 shows the monthly return of the MSCI Mexico stock index. This index is a free-float weighted equity index designed to measure the performance of the large and mid cap segments of the Mexican stock market. With 25 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in Mexico. The MSCI Mexico Index was launched on January 1, 2001. The index is reported in U.S. dollars.

Figure 14: VIX and U.S. Treasury On-the-Run Premium

Figure 14 shows the CBOE’s volatility index (VIX) along with the U.S. Treasury ten-year on-the-run premium derived from the difference between par-coupon yields of seasoned ten-year Treasury bonds (as per Gürkaynak et al. (2007)) and the yields on newly issued ten-year Treasury bonds (as reported in the Federal Reserve’s H.15 series). We note the high positive correlation (65%) between these two measures of financial market uncertainty and risk aversion.

The final variable in our extended set of controls is the U.S. TED spread, which is calculated as the difference between the three-month U.S. LIBOR and the three-month U.S. T-bill interest rate and shown in Figure 15. This spread represents a measure of the perceived general credit risk in global financial markets that could affect the pricing and trading of Mexican bonos.

\[ \text{Accessed as of October 29, 2019 at the link: } \text{https://www.msci.com/documents/10199/abfcf377-7c15-47c7-9204-a6405eb0cd34} \]
Figure 15: Three-Month U.S. TED Spread

C Bonos Term Premium Analysis

In this appendix, we analyze the term premium estimates implied by the AFNS-L model.

To begin, we define the term premiums as the difference in expected return between a buy and hold strategy for a \( \tau \)-year bond and an instantaneous rollover strategy at the risk-free short rate \( r_t \)

\[
TP_t(\tau) = y_t(\tau) - \frac{1}{\tau} \int_t^{t+\tau} E^{\rho}_t(r_s)ds.
\]  

(1)

Equation (1) highlights that the decomposition of yields into expectations and risk premium components can be distorted if the observed yields are biased by liquidity effects. Therefore, by adjusting the bond prices for liquidity effects, we indeed obtain estimates of the ideal or frictionless yields that feature in the yield decomposition in equation (1).

For estimation of short rate expectations and associated term premiums, the specification of the mean-reversion matrix \( K^P \) is critical, unlike the estimation of the liquidity premiums, which is relatively insensitive to the choice of \( K^P \). To select the best fitting specification of the AFNS-L model’s real-world dynamics, we use a general-to-specific modeling strategy in which the least significant off-diagonal parameter of \( K^P \) is restricted to zero and the model is re-estimated. This strategy of eliminating the least significant coefficient is carried out down to the most parsimonious specification, which has a diagonal \( K^P \) matrix. The final specification choice is based on the value of the Bayesian information criterion as in Christensen et al. (2014).\(^4\)

\(^4\)The Bayesian information criterion is defined as \( \text{BIC} = -2 \log L + k \log T \), where \( T \) is the number of data
Table 1: Evaluation of Alternative Specifications of the AFNS-L Model

There are thirteen alternative estimated specifications of the AFNS-L model of Mexican bonos prices. Each specification is listed with its maximum log likelihood (log $L$), number of parameters ($k$), the $p$-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the Bayesian information criterion (BIC). The period analyzed covers monthly data from June 29, 2007, to December 29, 2017.

The summary statistics of the model selection process are reported in Table 1. The Bayesian information criterion (BIC) is minimized by specification (7) with a $K^P$ matrix given by

$$K^P_{BIC} = \begin{pmatrix}
\kappa_1^P & 0 & 0 & \kappa_4^P \\
0 & \kappa_2^P & 0 & 0 \\
\kappa_3^P & \kappa_3^P & \kappa_3^P & \kappa_3^P \\
\kappa_4^P & \kappa_4^P & 0 & \kappa_4^P 
\end{pmatrix}.
$$

The estimated dynamic parameters for this preferred specification are reported in Table 2. We note two interesting observations. First, the liquidity factor matters for the expected excess return of bonos through the significant off-diagonal parameters tied to this factor. Second, the level factor is the important interaction point between the liquidity factor and the frictionless factors in the model given that both $\kappa_{14}^P$ and $\kappa_{41}^P$ are different from zero.

Now, we briefly detail the steps involved in our decomposition of the ten-year bonos yield shown in Figure 16. The starting point for the decomposition is the fitted ten-year yield from the AFNS model, which represents a simple smoothing of the observed bond price data without any adjustments and depicted with a solid red line. In addition, the estimated ten-observations, which is 127 in our case.
Table 2: Estimated Parameters in the Preferred AFNS-L Model

The estimated parameters for the mean-reversion matrix $K^P$, the mean vector $\theta^P$, and the volatility matrix $\Sigma$ in the AFNS-L model preferred according to the BIC. The Q-related parameter is estimated at $\lambda = 0.2810$ (0.0107), $\kappa^{Qliq} = 1.7911$ (0.2700), and $\theta^{Qliq} = 0.0098$ (0.0019). The maximum log likelihood value is 10,057.62. The numbers in parentheses are the estimated standard deviations.

<table>
<thead>
<tr>
<th>$K^P$</th>
<th>$K^P_{11}$</th>
<th>$K^P_{12}$</th>
<th>$K^P_{13}$</th>
<th>$K^P_{14}$</th>
<th>$\theta^P$</th>
<th>$\Sigma^P$</th>
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<td>$K_{11}$</td>
<td>1.1506 (0.5502)</td>
<td>0</td>
<td>0</td>
<td>0.4534 (0.2941)</td>
<td>0.0974 (0.0179)</td>
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<td>0</td>
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<tr>
<td>$K_{33}$</td>
<td>4.2311 (1.4208)</td>
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<td>0.6939 (0.4328)</td>
<td>1.4803 (0.8396)</td>
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<td>$\Sigma_{33}$ (0.0208)</td>
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<tr>
<td>$K_{44}$</td>
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<td>1.6044 (0.6920)</td>
<td>0</td>
<td>1.4747 (0.6913)</td>
<td>-0.0175 (0.0356)</td>
<td>$\Sigma_{44}$ (0.0171)</td>
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</tbody>
</table>

year frictionless yield from the AFNS-L model produced by inserting the estimated level, slope, and curvature factors into the frictionless yield function in equation (3) in the paper is shown with a solid black line. The difference between the former and the latter ten-year yield represents a measure of the liquidity premium in ten-year bonos yields and indicated with yellow shading. Finally, the frictionless ten-year yield can be decomposed into the short-rate expectations component over the next ten years and the residual ten-year term premium as detailed in equation (1) and shown with solid green and blue lines, respectively. We note that, although the liquidity premium is sizable and important, it is of modest magnitude relative to both the short rate expectations component and the term premium. Furthermore, we note that the ten-year term premium is mostly positive even though it did turn negative briefly between mid-2012 and mid-2013 before the taper tantrum in May and June of 2013 caused it to spike back up.

Next, we are interested in understanding the determinants of the ten-year bonos term premium series and its ties to the increase in foreign holdings of Mexican government bonds described in Section 2.1 in the paper. Therefore, we run a set of regressions similar to the ones reported in Section 4.2 in the paper. Specifically, we use the ten-year bonos term premium as the dependent variable instead of the bonos liquidity premium and consider the same sixteen explanatory variables since they should matter for bonos term premiums in much the same way as they should for the bonos liquidity premiums.

The results are reported in Table 3, and for robustness we repeat the exercise using the five-year bonos term premium with results reported in Table 4. Since the two term premium series are highly positively correlated (93%), it is not surprising that the regression results in
the two tables are very similar. One key takeaway is that the share of foreign holdings in the bonos market is insignificant in the joint regressions and hence does not affect the bonos term premiums. Thus, its role really runs through its impact on the bonos liquidity premiums. Instead, we note that increases in the EMBI and oil prices tend to put upward pressure on the bonos term premiums, while increases in the one-month cetes rate is highly statistically correlated with declines in the bonos term premiums. This latter finding suggests that the opportunity cost of holding money discussed in Nagel (2016) appears to operate through the standard term premiums in the Mexican bonos market rather than through the bonos liquidity premiums.

As a final exercise, we assess the robustness of both our liquidity premium estimates and the term premium estimates by studying their sensitivity to the choice of dynamic specification within the AFNS-L model. First, we compare the different bonos liquidity premium series we get from each of the specifications considered in Table 1. These 13 different liquidity premium series are shown in Figure 17 with the one generated by the most parsimonious AFNS-L model with diagonal $K^P$ and $\Sigma$ matrices considered in the main text highlighted with a thick solid black line. We note a mild tendency for larger dispersion across specifications early in our sample period when we observe prices for only a small set of bonds. However, overall our results confirm the findings of ACR that liquidity premiums estimated with the ACR approach are extracted primarily from the cross sectional information on each observation date with
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Table 3: Regression Results for the Ten-Year Term Premium
The table reports the results of regressions with the ten-year term premium from the preferred AFNS-L model as the dependent variable and sixteen explanatory variables described in the main text. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

relatively little sensitivity to the specification of the time series dynamics. Therefore, based on the evidence presented in this appendix, we limit the analysis in the main text to the most parsimonious specification with diagonal $K^P$ and $\Sigma$ matrices as it is clearly representative of
Table 4: Regression Results for the Five-Year Term Premium
The table reports the results of regressions with the five-year term premium from the preferred AFNS-L model as the dependent variable and sixteen explanatory variables described in the main text. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

the liquidity premiums one would estimate with other more flexible specifications.

In Figure 18, we repeat the above exercise for the ten-year term premium, that is, we compare the estimated ten-year bonos term premiums from each of the specifications considered in
Figure 17: Sensitivity of Estimated Bonos Liquidity Premiums to Model Specification
Illustration of the average estimated liquidity premium of Mexican bonos for each observation date implied by the AFNS-L model estimated with the 13 different specifications of $K^P$ considered in Table 1. Note that $\Sigma$ has a diagonal specification in all estimations. The Mexican bonos liquidity premiums are measured as the estimated yield difference between the fitted yield to maturity of individual Mexican bonos and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The data cover the period from June 29, 2007, to December 29, 2017.

Table 1. We note some dispersion across specifications, which underscores the importance of identifying appropriate specifications of the model’s $P$-dynamics. We consider our approach based on the Bayesian information criterion to be robust and offer a sensible tradeoff between flexibility and model fit, and we note that more parsimonious specifications yield term premium estimates, which are fairly close to those produced by our preferred specification.

D Regressions in First Differences
In this appendix, we repeat the regression exercises in Section 4.2 of the paper with all variables measured in monthly changes. The purpose is to analyze whether the dynamic relationship between the average bonos liquidity premium and the foreign-held share is a stock effect as modeled in Section 4.2 or is due to flow effects from monthly changes in the share held by foreigners.

The results are reported in Table 5. First, all estimated coefficients are insignificant. Still, it remains the case that a monthly increase in the foreign-held share of bonos is weakly associated with an increase in the estimated bonos liquidity premium. We take this evidence
Figure 18: **Sensitivity of Ten-Year Bonos Term Premiums to Model Specification**

Illustration of the estimated ten-year term premium of Mexican bonos for each observation date implied by the AFNS-L model estimated with the 13 different specifications of $K^P$ considered in Table 1. Note that $\Sigma$ has a diagonal specification in all estimations. The data cover the period from June 29, 2007, to December 29, 2017.

to suggest that our findings reflect stock effects rather than flow effects. Thus, it is the persistent increase in foreign holdings over time that matter rather than the month-to-month variation.
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Table 5: **Results for Regressions in First Differences**

The table reports the results of regressions in first differences with the average estimated bonos liquidity premium as the dependent variable and sixteen explanatory variables described in the main text. Standard errors computed by the Newey-West estimator (with 3 lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.
References


