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# Aggregate Implications of Changing Sectoral Trends\*

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## Abstract

We find disparate trend variation in TFP and labor growth across major U.S. production sectors that together imply secular declines in aggregate labor and TFP growth. We study the implications of this trend variation given linkages between sectors including, crucially, in the production of investment goods. We estimate that the annual trend rate of growth of GDP has declined by more than 2 percentage points since 1950, and that this decline has been primarily shaped by sector-specific rather than aggregate factors. Sustained contractions in growth specific to Construction, Nondurable Goods, and Professional and Business Services by themselves are responsible for close to sixty percent of the estimated trend decrease in GDP growth. The importance of idiosyncratic trend shocks arises in part because, even absent non-linearities in production, the endogenous response of capital to trend changes in TFP and labor amplifies the knock-on effects of production linkages. Capital deepening contributed around 0.5 percentage points to growth throughout most of the post-war period, surged in the early 1990's but reversed course shortly thereafter, and has now been negative for most of the past two decades. Finally, because trend shocks propagate endogenously through capital accumulation, we estimate that trend GDP growth will continue to decline for the next 10 years absent new shocks that persistently raise TFP and labor growth.

Keywords: trend growth, sectoral linkages, investment network

JEL Codes: C32, E23, O41

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# 1 Introduction

Following the so-called Great Recession of 2008-2009, U.S. GDP recovered only very gradually which resulted in a conspicuously low average growth rate in the ensuing decade. [Fernald, Hall, Stock, and Watson \(2017\)](#) found this weak recovery stemmed mainly from slow growth in total factor productivity (TFP) and a fall in labor input, and note that these adverse forces preceded the Great Recession. [Antolin-Diaz, Drechsel, and Petrella \(2017\)](#) likewise document a slowdown in output growth that predates the Great Recession.<sup>1</sup> This paper investigates what has in fact been a steady decline in trend GDP growth over the entire post-war period, 1950 – 2016. We explore the implications of TFP and labor inputs in accounting for this secular decline, but we do so at a disaggregated sectoral level. We estimate an empirical model where, in each sector, TFP growth and labor growth have unobserved persistent and transitory components, and where each component can itself stem from either aggregate or idiosyncratic forces. We then study the implications of our empirical findings for trend growth within a multi-sector framework with production linkages that mimic those of the US economy including, crucially, in the production of investment goods.

We find that trends in TFP and labor growth have steadily decreased across a majority of U.S. sectors since 1950. Interestingly, more than 3/4 of the secular decline in aggregate TFP growth results from the combination of sector-specific disturbances, thus leaving only an ancillary role for the aggregate component. Therefore, if technical progress in general purpose technologies has helped drive trends in sectoral TFP growth, this thrust has not been shared widely enough across sectors to generate comovement in TFP growth. Similarly to TFP growth, trend labor growth has generally been dominated by sector-specific factors, especially after 1980. The decline in trend labor growth is especially large in the Durable Goods sector, though it never becomes negative. In general, secular changes in TFP and labor growth have been mostly driven by sector-specific rather than common components. These findings, therefore, highlight the quantitative importance of idiosyncratic shocks not only for business cycle fluctuations, as in [Gabaix \(2011\)](#) or [Foerster, Sarte, and Watson \(2011\)](#), but also for variations in trends.

Conditional on the behavior of trend growth in sectoral TFP and labor, we derive balanced growth accounting equations in a dynamic multi-sector framework where a sector uses not only materials but also investment goods produced in other sectors. We then use these new growth accounting equations to assess the aggregate effects of observed sectoral changes in the trend growth rates of labor and TFP. Our analysis, therefore, extends the work of [Greenwood, Hercowitz, and Krusell \(1997\)](#) to a multi-sector environment.<sup>2</sup> At the same time, its focus on estimating sector-specific trends, and what their aggregate implications are for the long-run, differentiates it from the literature building on [Greenwood et al. \(1997\)](#). Specifically,

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<sup>1</sup>[Cette, Fernald, and Mojon \(2016\)](#) suggest that a slowdown in productivity growth that began prior to the Great Recession reflects in part the fading gains from the Information Technology (IT) revolution. This view is consistent with the long lags associated with the productivity effects of IT adoption found by [Basu and Fernald \(2001\)](#), and the collapse of the dot-com boom in the early 2000s. [Decker, Haltiwanger, Jarmin, and Miranda \(2016\)](#) point to a decline in business dynamism that began in the 1980s as an additional force underlying slowing economic activity.

<sup>2</sup>[Ngai and Pissarides \(2007\)](#) provide a seminal study of balanced growth in a multi-sector environment. They consider both multiple intermediates and multiple capital-producing sectors but not at the same time. More importantly, the analysis abstracts from pairwise linkages in both intermediates and capital-producing sectors that play a key role in this paper.

Fisher (2006), Justiniano, Primiceri, and Tambalotti (2010, 2011) and Basu, Fernald, Fisher, and Kimball (2013) are primarily concerned with the business cycle implications of sectoral shocks, and in particular, investment-specific shocks. More recently, vom Lehn and Winberry (2019) show that the input-output network of investment goods is critical in accounting for shifts in the cyclicality and relative volatilities of aggregate time series since the 1980s.

We show that sectors that produce a majority of capital goods in the economy are associated with quantitatively large sectoral multipliers. In other words, these sectors markedly amplify the role of idiosyncratic trend shocks on GDP growth. Sectoral multipliers in our setting reflect the knock-on effects induced by production linkages in the presence of capital, and their size depends in part on a sector's importance as a supplier of investment goods to other sectors. Given observed U.S. production linkages, we find that the influence of individual sectors on GDP growth may be as large as 3 times their share in the economy.

Ultimately, we estimate that changing sectoral trends, translated through the economy's production network, have on net lowered trend GDP growth by roughly 2.2 percentage points since 1950. Because capital amplifies the knock-on effects of production linkages, slowing idiosyncratic trends in just a few sectors explain most of the trend decline in GDP growth. Construction, which accounts for the largest component of investment spending in several industries, stands out more than any other sector for its contribution to the trend decline in aggregate growth since 1950, generating around 30 percent of this decline. Shocks to investment producing sectors, therefore, matter not just at cyclical frequencies but also in the longer run. Changes in Professional and Business Services and Nondurable Goods together account for another 30 percent.

We find that about 1/2 of trend GDP growth arises from the direct contributions of input growth, TFP and labor, with the other 1/2 resulting from indirect contributions of those inputs via induced capital growth. Capital deepening, defined as deviations of actual long-run GDP growth from its model implied trend, has contributed around 0.5 percentage points to long-run GDP growth over most of the post-war period. Our time-varying estimate of capital deepening shows it increasing in the early 1990's but reversing course shortly thereafter and, consistent with Alexander and Eberly (2018), collapsing in the late 1990's, well before the Great Recession. Remarkably, the contributions of capital deepening to long-run growth have been negative for most of the past two decades.<sup>3</sup>

Our paper also falls within the literature on equilibrium multi-sector models first developed by Long and Plosser (1983) and later Horvath (1998, 2000) and Dupor (1999). Since then, a large body of work has explored important features of those models for generating aggregate fluctuations from idiosyncratic shocks. We maintain the original assumptions of competitive input and product markets as well as constant-returns-to-scale technologies. Even absent non-linearities in production, and beyond the role of idiosyncratic shocks in explaining aggregate cyclical variations, the analysis reveals that sector-specific disturbances also dominate

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<sup>3</sup>Early work by Mankiw, Romer, and Weil (1992), and Mankiw (1995), found a relatively large role of capital deepening in explaining long-run average growth rates while Klenow and Rodríguez-Clare (1997), and more recently Fernald and Jones (2014), find a larger role for exogenous sources of growth. Importantly, while findings in this literature vary, to this point they also all abstract from the consideration that trends can vary over time.

trend variations in U.S. GDP growth.<sup>4</sup>

This paper is organized as follows. Section 2 gives an overview of the behavior of trend GDP growth over the past 60 years. Section 3 provides an empirical description of TFP and labor growth by industry that allows for persistent and transitory components, where each component itself may be driven by idiosyncratic or aggregate forces. Section 4 develops the implications of these changes at the sector level in the context of a dynamic multi-sector model with production linkages in materials and investment. This model serves as the balanced growth accounting framework that we use to determine the aggregate implications of changes in the sectoral trend growth rates of labor and TFP. Section 5 presents our quantitative findings. Section 6 concludes and discusses possible directions for future research. An online [Technical Appendix](#) contains a detailed description of the data, statistical methods, economic model, discussions of departures from our benchmark assumptions, and includes additional figures and tables referenced in the text.

## 2 The Long-Run Decline in U.S. GDP Growth

Figure 1 shows the behavior of U.S. GDP growth over the post-WWII period. Here, annual GDP growth is measured as the share-weighted value added growth from 16 sectors comprising the private U.S. economy; details are provided in the next section.

Panel A shows aggregate private-sector growth rates computed by chain-weighting the sectors, and by using three alternative sets of fixed sectoral shares computed as averages over the entire sample (1950–2016), over the first fifteen years of the sample (1950–1964) and over the final fifteen years (2002–2016). Panel A shows large variation in GDP growth rates – the standard deviation is 2.5 percent over the period 1950–2016 – but much of this variation is relatively short-lived and is associated with business cycles and other relatively transitory phenomena. Moreover, to the extent that sectoral shares have changed slowly over time, these share shifts have little effect in Panel A. In other words, changes in aggregate growth largely stem from changes within sectors rather than between them. Our interest, however, is in longer-run variation.

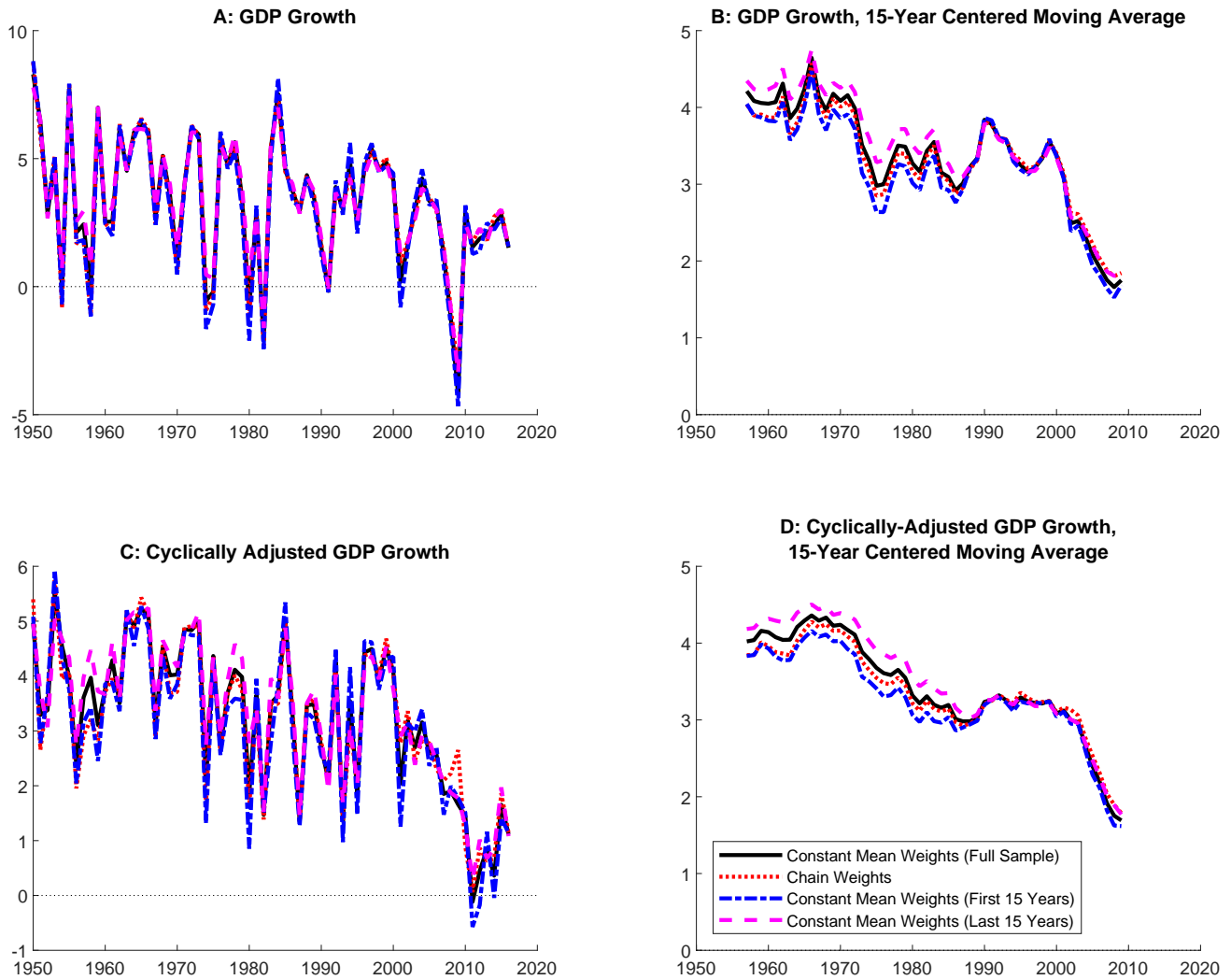
Panel B, therefore, plots centered 15–year moving averages of the annual growth rates. Here too there is variability. In the 1950s and early 1960s average annual growth exceeded 4 percent. This fell to 3 percent in the 1970s, rebounded to nearly 4 percent in the 1990s, but plummeted to less than 2 percent in the 2000s (See Table 1). At these lower frequencies, the effects of slowly shifting shares over the sample become more visible but remain small. Early in the sample, the difference in GDP growth computed with shares averaged over the first 15 years differs from that with shares averaged over the last 15 years by at most 0.3 percentage points. This difference disappears towards the end of the sample as some sectors with initially low shares grew faster than average.

Panels C and D refine these calculations by eliminating the cyclical variation using an Okun’s law

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<sup>4</sup>See [Gabaix \(2011\)](#) and [Atalay \(2017\)](#) for assessments of the importance of idiosyncratic shocks in driving business cycle fluctuations. We build on [Acemoglu et al. \(2012\)](#), [Baqaae and Farhi \(2019\)](#) and [Miranda-Pinto \(2019\)](#) by considering an explicitly dynamic framework with a focus on the empirical estimation of processes characterizing the behavior of sectoral inputs, as well as their implications for the observed behavior of the growth rates of sectoral value added and GDP since 1950.

Figure 1: US GDP Growth Rates 1950-2016  
(Percentage points at an annual rate)



Notes: Growth rates are share-weighted value added growth rates from 16 sectors making up the private U.S. economy. Cyclical adjustment uses a regression on leads and lags of the first-difference in the unemployment rate.

regression in GDP growth rates as in [Fernald et al. \(2017\)](#).<sup>5</sup> Thus, panel C plots the residuals from a regression of GDP growth rates onto a short distributed lead and lag of changes in the unemployment rate ( $\Delta u_{t+1}, \Delta u_t, \Delta u_{t-1}$ ). This cyclical adjustment eliminates much of the cyclical variability evident in

<sup>5</sup>Compared to other measures of cyclical slack or resource utilization, [Fernald et al. \(2017\)](#) point out that the civilian unemployment rate has two key advantages. First, it has been measured using essentially the same survey instrument since 1948. Second, changes in the unemployment rate have nearly a mean of zero over long periods.

Table 1: 15-Year Averages of GDP Growth Rates

Dates	Const Mean Weights		Time-Varying Weights		Const Weights (First 15 Years)		Const Weights (Last 15 Years)	
	Growth rates	Cyc-adj rates	Growth rates	Cyc-adj rates	Growth rates	Cyc-adj rates	Growth rates	Cyc-adj rates
1950-2016	3.3	3.2	3.3	3.2	3.2	3.1	3.4	3.4
1950-1965	4.3	4.1	4.2	3.9	4.2	3.9	4.5	4.2
1966-1982	3.1	3.7	3.0	3.6	2.8	3.5	3.4	4.0
1983-1999	3.9	3.3	3.9	3.3	3.9	3.3	3.9	3.4
2000-2016	1.8	1.9	1.9	2.0	1.7	1.8	1.9	2.0

Notes: The values shown are averages of the series plotted in Figure 1, panels (A) and (C), over the periods shown.

panel A. In addition, the 15-year moving average in Panel D now produces a more focused picture of the trend variation in the growth rate of private GDP. Again, time-varying share weights have discernable, but relatively small effect on the aggregate growth rate or its 15-year moving average.

The numbers reported in Table 1 frame the key question of this paper: why did the average growth rate of GDP fall from 4 percent per year in the 1950s to just over 3 percent in the 1980s and 1990s, and then further decline precipitously in the 2000s? As the different columns of the table make clear, this question arises regardless of the shares used in constructing GDP. We look to inputs – specifically TFP and labor at the sectoral level – for the answer. That is, interpreting long-run variations of the data as a time-varying balanced growth path, changes in trend GDP growth are primarily determined by changes in the trend growth rates of those two inputs. However, as the analysis in Section 4 makes clear, not all sectoral inputs are created equal. Sectors not only differ in their size, that is, their value-added share in GDP, but also in the share of materials or capital that they provide to other sectors. Put another way, input variation across sectors is a particularly important driver of low frequency movements in aggregate GDP growth.

Before investigating these input-output interactions, we begin by describing the sectoral data, both how these data are measured and how sectoral value-added as well as labor and TFP inputs have evolved over the post-WWII period. In much our analysis we construct aggregates using the constant weights computed using full-sample averages. As Figure 1 and Table 1 suggest, results using these constant shares are robust to alternative weighting schemes.

### 3 An Empirical Description of Trend Growth in TFP and Labor

We begin by estimating an empirical model of TFP and labor growth for different sectors of the U.S. economy. As a benchmark, our paper applies the insights of [Hulten \(1978\)](#) on the interpretation of aggregate productivity (TFP) changes as a weighted average of sector-specific value-added TFP changes. In particular,



under constant-returns-to-scale and perfect competition in product and input markets, the sectors' weights are the ratios of their valued added to GDP.<sup>6</sup> We calculate standard TFP growth rates at the sectoral level following [Jorgenson et al. \(2017\)](#) among others, and estimate permanent and transitory components in these growth rates.

### 3.1 Data

Sectoral TFP growth rates are calculated using the KLEMS dataset constructed by Jorgenson and his collaborators, as well as its recently updated version in the form of the BEA's Integrated Industry-Level Production Accounts (ILPA). These datasets are attractive for our purposes because they provide a unified approach to the construction of gross output, the primary inputs capital and labor, as well as intermediate inputs ('materials') for a large number of industries. The KLEMS data are based on U.S. National Income and Product Accounts (NIPA) and consistently integrate industry data with Input-Output tables and Fixed Asset tables. The KLEMS dataset contains quantity and price indices for inputs and outputs across 65 industries. The growth rate of any one industry's aggregate is defined as a Divisia index given by the value-share weighted average of its disaggregated component growth rates. Labor input is differentiated by gender, age, education, and labor status. Labor input growth is then defined as a weighted average of growth in annual hours worked across all labor types using labor compensation shares of each type as weights. Similarly, intermediate input growth reflects a weighted average of the growth rate of all intermediate inputs averaged using payments to those inputs as weights. Finally, capital input growth reflects a weighted average of growth rates across 53 capital types using payments to each type of capital as weights. Capital payments are based on implicit rental rates consistent with a user-cost-of-capital approach. Total payments to capital are the residuals after deducting payments to labor and intermediate inputs from the value of production. Put another way, there are no economic profits.

An industry's TFP growth rate is defined in terms of its Solow residual, specifically output growth less the revenue-share weighted average of input growth rates. This calculation is consistent with the canonical theoretical framework we adopt in Section 4 where all markets operate under perfect competition and production is constant-returns-to-scale. For earlier versions of Jorgenson's KLEMS data up to 1990, [Basu and Fernald \(1997, 2001\)](#) compute total payments to capital as the sum of rental rates implied by the user-cost-of-capital and find small industry profits on average that amount at most to three percent of gross output. In the presence of close to zero profits, elasticities to scale and markups are equivalent. More recently, an active debate has emerged on the extent to which the competitive environment has changed in the U.S. over the last two decades. On the one hand, [Barkai \(2017\)](#), also applying the user-cost-of-capital framework but using post 1990 data, finds substantial profit shares over that period. On the other hand, [Karabarbounis and Neiman \(2018\)](#) argue that the user-cost-of-capital framework, to the extent that it implies

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<sup>6</sup>In the absence of constant-returns-to-scale or perfect competition, [Basu and Fernald \(1997, 2001\)](#) and [Baqae and Farhi \(2018\)](#) show that aggregate TFP changes also incorporate reallocation effects. These effects reflect the movement of inputs between low and high returns to scale sectors stemming from changes in relative sectoral TFP.



high profit shares starting in the 1990s, also implies unreasonably high profit shares in the 1950s.<sup>7</sup> In this paper, we maintain the assumptions of competitive markets and constant-returns-to-scale as a benchmark from which to study the aggregate implications of sectoral changes in labor and TFP inputs.

Our calculations rely on the 2017 version of the Jorgenson KLEMS dataset which covers the period 1946 – 2014, and the ILPA KLEMS dataset which covers the period 1987 – 2016.<sup>8</sup> For ease of presentation, and in order to consider the role of structural change in individual sectors separately, we carry out the empirical analysis using private industries at the two-digit level. In particular, we aggregate the 65 industries included in the two KLEMS datasets into 16 two-digit private industries following the procedure in [Hulten \(1978\)](#).<sup>9</sup> Another advantage of the aggregation into two-digit industries is that any differences between the two KLEMS datasets are attenuated and we feel comfortable splicing the levels of the two datasets in 1987.<sup>10</sup> That is, we use the growth rates calculated using the Jorgenson KLEMS data before 1987 and using ILPA data after that date.

Table 2 lists the 16 sectors we consider. For each sector, the table shows average cyclically adjusted growth rates of value added, labor, and TFP over 1950 – 2016, and it also shows their average shares in aggregate value added and labor input. The aggregate growth rates in the bottom row are the value-weighted averages of the sectoral growth rates with average value added and labor shares used as fixed weights.

Clearly sectors grow at different rates and this disparity is hidden in studies that only consider aggregates. Average real value added growth rates range from 1.1 percent in Mining to 4.7 percent in Wholesale Trade, bracketing the aggregate value added growth rate of 3.2 percent. With the exception of the Durable Goods sector, most sectors with growth rates that exceed the aggregate growth rate provide services. Similarly, labor input growth rates range from a negative 1.4 percent in Agriculture to 3.3 percent in Professional and Business Services, bracketing the average aggregate growth rate of 1.5 percent. Again, most sectors with labor input growth rates that exceed the aggregate growth rate provide services. Finally, TFP growth rates range from -0.7 percent Utilities to 3.2 percent in Agriculture, bracketing the average aggregate TFP growth rate of 0.7 percent. Sectoral TFP growth rates are less aligned with either value added or labor input growth rates. There are three sectors with notable TFP declines, namely Utilities, Other Services,

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<sup>7</sup>In addition, [De Loecker and Eeckhout \(2017\)](#), estimating industry production functions from corporate balance sheets, present evidence of rising markups and returns to scale since the 1980s. However, [Traina \(2018\)](#) argues that the evidence on rising markups from corporate balance sheets depends crucially on the measurement of variable costs and weights in aggregation. Similarly, [Rossi-Hansberg et al. \(2018\)](#) show that while sales concentration has unambiguously risen at the national level since the 1980s, concentration has steadily declined at the Core-Based Statistical Area, county, and ZIP code levels over the same period. While these facts can seem conflicting, the authors present evidence that large firms have become bigger through the opening of more establishments or stores in new local markets, but this process has lowered concentration in those markets.

<sup>8</sup>The Jorgenson dataset is downloaded from <http://www.worldklems.net/data.htm> and the ILPA is downloaded from <https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems>. A detailed description of the Jorgenson data can be found in [Jorgenson et al. \(2014\)](#), as well as [Jorgenson et al. \(2017\)](#), and the BEA dataset is described in [Fleck et al. \(2014\)](#).

<sup>9</sup>The procedure is described in detail in Section 1 of the online-only [Technical Appendix](#) to this paper, [Foerster et al. \(2019\)](#).

<sup>10</sup>While the ILPA builds on the Jorgenson KLEMS data, the two datasets are not exactly identical for the time period in which they overlap. Since both datasets are constructed to be consistent with the BEA’s input-output tables, they mostly agree on industry details and both cover the same 65 private industries. Nevertheless, there remain differences but these are reflected mostly in the levels of variables and not their growth rates.

Table 2: 16 Sector Decomposition of the U.S. Private Economy  
(1950-2016)

Sectors	Average growth rate Cyclically adjusted data (Percentage points at an annual rate)			Average share (Percentage points)	
	Value Added	Labor	TFP	Value Added	Labor
1 Agriculture	2.63	-1.42	3.16	2.70	3.25
2 Mining	1.06	0.27	-0.06	2.15	1.66
3 Utilities	1.87	0.98	-0.74	2.42	1.05
4 Construction	1.67	1.60	-0.10	5.04	7.64
5 Durable goods	3.36	0.42	1.88	13.48	15.59
6 Nondurable goods	2.29	0.07	0.79	9.31	8.84
7 Wholesale trade	4.65	1.68	1.13	6.92	6.50
8 Retail trade	3.16	1.08	1.07	8.44	9.86
9 Trans. & Ware.	2.43	0.85	1.35	4.19	4.91
10 Information	4.58	1.30	0.86	4.90	3.68
11 FIRE (x-Housing)	3.93	2.78	-0.22	10.03	7.41
12 PBS	4.36	3.26	0.30	8.59	10.88
13 Educ. & Health	3.32	2.75	0.04	6.01	9.28
14 Arts, Ent., & Food svc.	2.44	2.00	0.22	3.70	4.67
15 Other services (x-Gov)	2.02	2.38	-0.60	2.98	4.50
16 Housing	3.51	1.68	0.16	9.15	0.25
Aggregate	3.24	1.49	0.67	100	100

Notes: The values shown are average annual growth rates for the 16 sectors. The row labelled “Aggregate” is the share-weighted average of the 16 sectors.

and Construction, as well as a number of sectors with stagnant TFP levels. Negative TFP growth rates are a counter-intuitive but well known feature of disaggregated industry data. To the degree that they occur in service industries, they are in part attributed to measurement issues with respect to output.

To a first approximation, the contributions of the different sectors to aggregate outcomes are given by the nominal value added and labor input shares in the last two columns. Two notable contributors to value added and TFP are Durable Goods and FIRE excluding Housing. The two largest contributors to labor payments are Durable Goods and Professional and Business Services. Over time, the shares of goods-producing sectors has declined while the shares of services-producing sectors has increased. However,

despite these changes, aggregating sectoral outputs and inputs using constant mean shares, as opposed to time-varying shares, has little effect on the measurement of aggregate outputs and inputs (Figure 1A).

### 3.2 Empirical Framework

Let  $\Delta \ln \tilde{x}_{j,t}$  denote the growth rate ( $100 \times$  the first difference of the logarithm) of annual measurements of labor or TFP in sector  $j$  at time  $t$ . These sectoral growth rates are volatile and, in many sectors, much of the variability is associated with the business cycle. Our interest is in trend (i.e., low-frequency) variation, which is more easily measured after cyclically adjusting the raw growth rates. Thus, as with the cyclically adjusted measure of GDP shown in Figure 1, we follow Fernald et al. (2017) and cyclically adjust these growth rates using the change in the unemployment rate,  $\Delta u_t$ , as a measure of cyclical resource utilization. That is, we estimate

$$\Delta \ln \tilde{x}_{j,t} = \mu_j + \beta_j(L)\Delta u_t + e_{j,t}, \tag{1}$$

where  $\beta_j(L) = \beta_{j,1}L + \beta_{j,0} + \beta_{j,-1}L^{-1}$  and the leads and lags of  $\Delta u_t$  captures much of the business-cycle variability in the data. Throughout the remainder of the paper, we use  $\Delta \ln x_{j,t} = \Delta \ln \tilde{x}_{j,t} - \hat{\beta}_j(L)\Delta u_t$ , where  $\hat{\beta}_j(L)$  denotes the OLS estimator, and where  $x_{j,t}$  represents the implied cyclically adjusted value of sector TFP (denoted  $z_{j,t}$ ) or labor input (denoted  $\ell_{j,t}$ ) growth rates.<sup>11</sup>

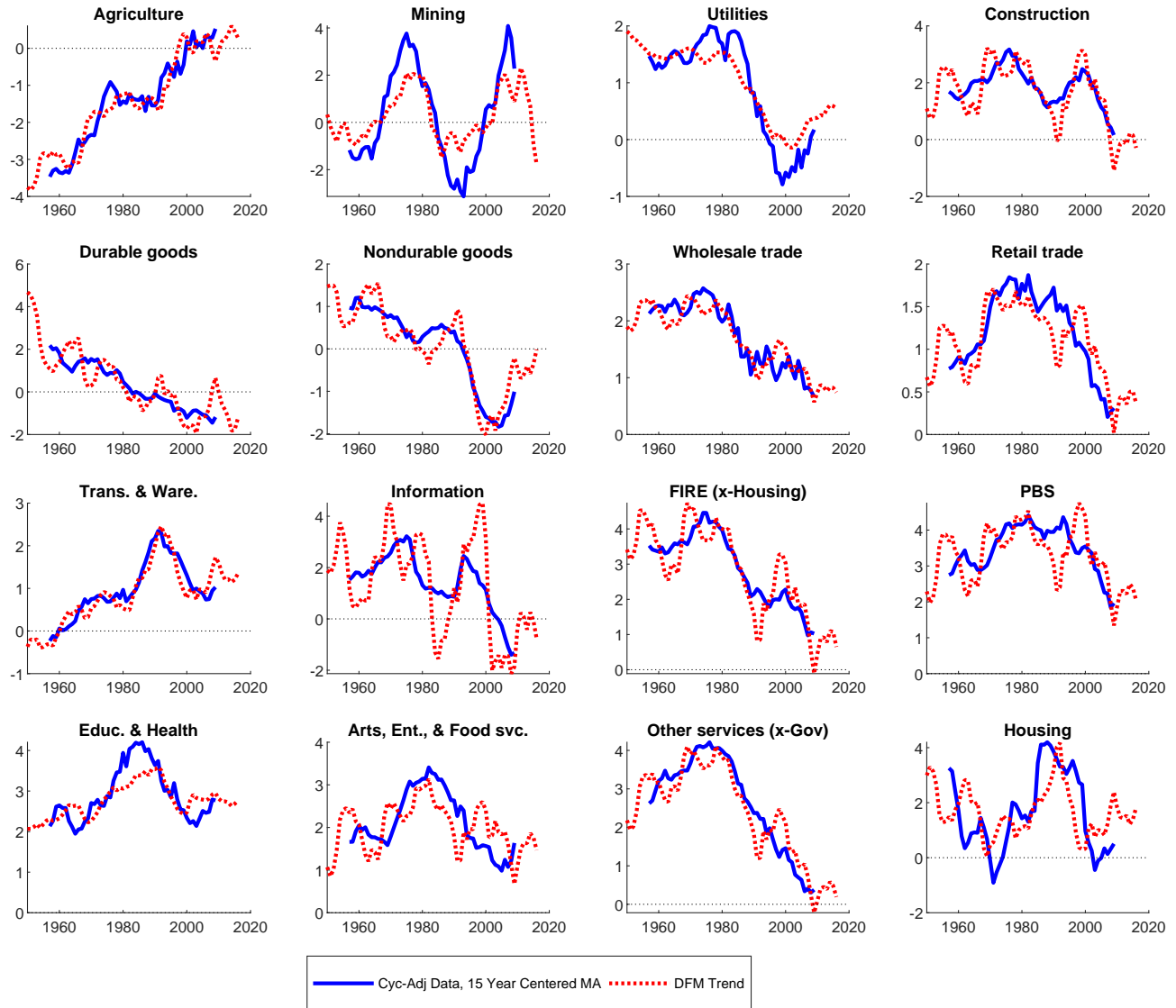
Figures 2 and 3 plot centered 15-year moving averages of the cyclically adjusted growth rates of labor and TFP. These are shown as the thick blue lines in the figures (ignore the thin dotted red line for now). The disparity in experiences across different sectors stands out. In particular, the moving averages show large sector-specific variation through time. For example, labor input was contracting at nearly 4 percent per year in agriculture in the 1950s, but stabilized near the end of the sample. In contrast, labor input in the Durables and Nondurable goods sectors was increasing in the 1950s, but has been contracting since the mid-1980s. At the same time, the rate of growth of labor in several service sectors are shown to exhibit large ups and downs over the sample. Looking at TFP, there are important differences across sectors as well. In Sections 4 and 5, we quantify the aggregate implications of these sectoral variations in labor and TFP inputs.

For the purpose of growth accounting, we condition on the values  $z_{j,t}$  and  $\ell_{j,t}$  in the economic model presented in Section 4. We then study the implied values of output and value-added that arise from realizations of the  $(z, \ell)$  process. We do so in a dynamic model that features input-output and capital flow linkages between the sectors. The model includes forward-looking agents, so its solution requires expectations of future values of  $z_{j,t}$  and  $\ell_{j,t}$ . For this purpose, we use a reduced-form statistical model that captures the salient cross-sectional and dynamic correlations in the data. A useful by-product of this exercise is that it provides a statistical four-way decomposition of variations in the data into persistent and non-persistent components, and into common and sector-specific components.

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<sup>11</sup>Other measures of variable utilization are estimated in Kimball et al. (2006). These are generally found to be stationary so that any differences across measures of utilization will likely affect the transitory components of TFP or labor rather than their trends.

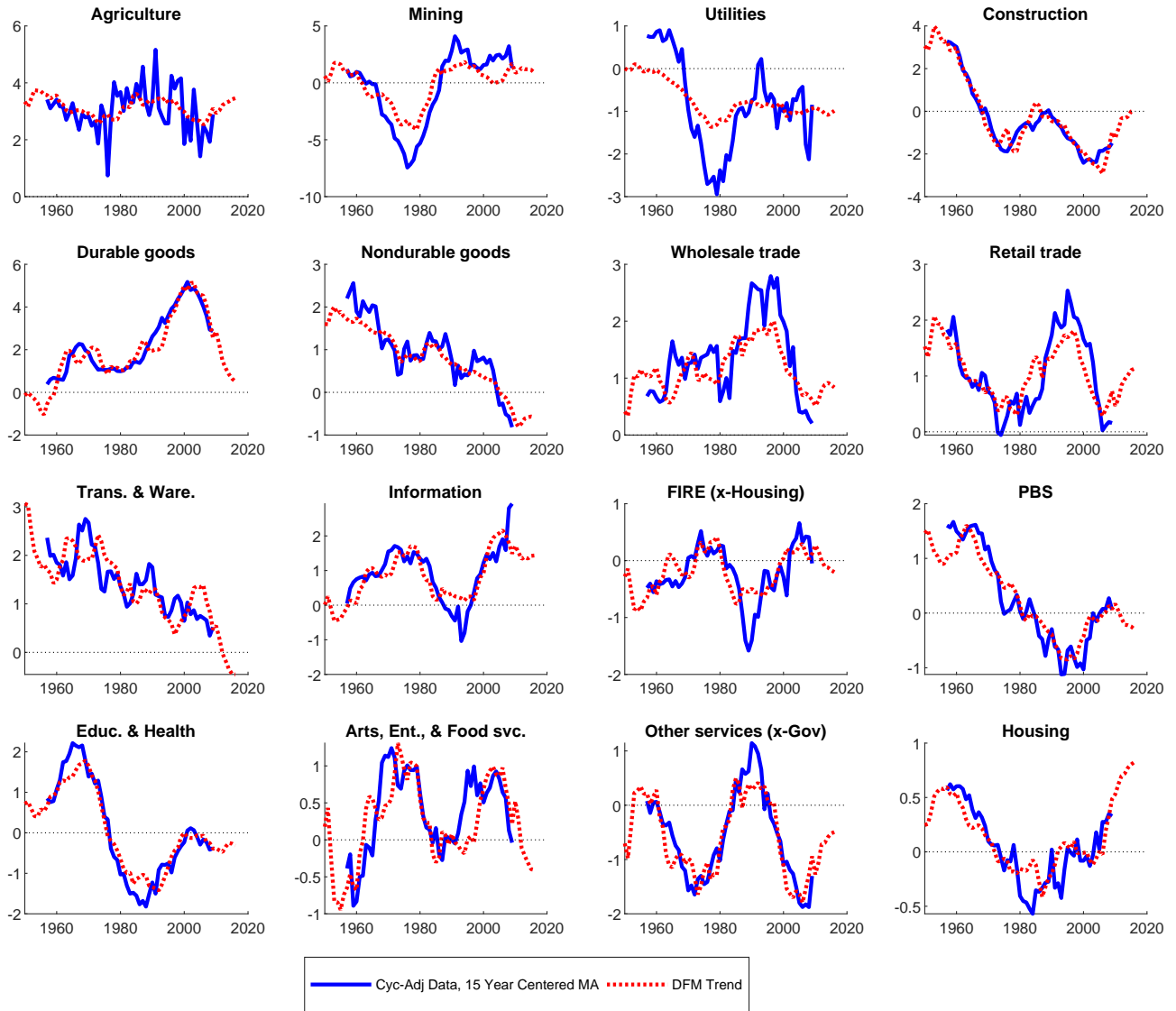
Figure 2: Trend Growth Rate in Labor by Sector  
(Percentage points at an annual rate)



Notes: The solid line is the centered 15-year moving average of the annual rate of growth of labor in each of the sectors shown. The dotted line is the estimated trend component (common + idiosyncratic) for sectoral growth rates estimated from the DFM.

The statistical model is specified so it can track the trend variation in sectoral growth rates evident in Figures 2 and 3, but also to capture other important second-moment characteristics of the data. Long-run cross-correlations calculated using the methods developed in Müller and Watson (2018), and summarized in

Figure 3: Trend Growth Rate in TFP by Sector  
(Percentage points at annual rate)



Notes: See notes to Figure 2.

Section 2.1 of the online [Technical Appendix](#), suggest a reduced-form model with three features. First, while trend variation is important, both  $\Delta \ln \ell_{j,t}$  and  $\Delta \ln z_{j,t}$  exhibit substantial year-to-year variation around their slowly varying levels. Second, the sectoral growth rates of labor ( $\Delta \ell_{j,t}$ ) are somewhat correlated across sectors over the long-run; there is also cross-sector long-run correlation in the sectoral TFP growth rates

( $\Delta z_{j,t}$ ). Third, there is little evidence of long-run correlation between ( $\Delta \ell_{j,t}$ ) and ( $\Delta z_{j,t}$ ) across or within sectors, that is  $cor(\Delta \ell_{j,t}, \Delta z_{k,t})$  is small for all  $j$  and  $k$ . This lack of correlation between sectoral labor and TFP is an interesting puzzle, but we do not attempt to solve it in this paper.<sup>12</sup>

Rather, as mentioned, our analysis develops a growth accounting framework *conditional* on the values of these inputs to trace out the implications for trend variation in sectoral and aggregate value added.

These features lead us to describe  $\Delta \ln \ell_{j,t}$  and  $\Delta \ln z_{j,t}$  using independent stochastic processes, where the processes have a structure that includes factors common to all sectors together with sector-specific factors, and where these factors include both slowly-varying level terms (modeled as martingales) and terms capturing more transitory variation (modeled as white noise). Specifically, we consider a dynamic factor model (DFM) of the form,

$$\Delta \ln x_{j,t} = \lambda_{j,\tau}^x \tau_{c,t}^x + \lambda_{j,\varepsilon}^x \varepsilon_{c,t}^x + \tau_{j,t}^x + \varepsilon_{j,t}^x, \quad (2)$$

where  $x = z$  or  $\ell$  and  $(\Delta \tau_{c,t}^x, \varepsilon_{c,t}^x, \{\Delta \tau_{j,t}^x, \varepsilon_{j,t}^x\}_{j=1}^n)$  are *i.i.d.* Gaussian random variables with mean zero and variable-specific variances.

The  $\tau$ -terms are random walks and describe the slowly varying (or ‘local’) levels in the growth rate of  $x_{j,t}$ . Some of this variation is common, through  $\tau_{c,t}^x$ , and some is sector-specific, through  $\tau_{j,t}^x$ . Deviations of the data from their local levels are represented by the  $\varepsilon$ -terms, part of which is common,  $\varepsilon_{c,t}^x$ , and part of which is sector-specific,  $\varepsilon_{j,t}^x$ .<sup>13</sup>

The sectoral model produces aggregate versions of  $\ell_t$  and  $z_t$  that also have random-walk-plus-white-noise representations. In particular, let  $s_j$  denote the (time-invariant) share weight of sector  $j$ . The aggregate value of  $x_t$  then satisfies  $\Delta \ln x_t = \sum_{j=1}^n s_j \Delta \ln x_{j,t} = \tau_t + \varepsilon_t$ . Here,  $\tau_t = \tau_{c,t}^x \sum_{j=1}^n s_j \lambda_{j,\tau}^x + \sum_{j=1}^n s_j \tau_{j,t}^x$  is a random walk that represents the ‘local’ level of the aggregate growth rate and  $\varepsilon_t$ , defined analogously, is white noise.

While the empirical model has a simple dynamic and cross-sectional structure, it fits the sectoral labor and TFP data well (details are provided in Section 2 of the [Technical Appendix](#)) and versions of the model have proved useful in describing sectoral data in other contexts (cf. [Stock and Watson \(2016\)](#)). The dynamic factor model is estimated using Bayesian methods together with a Gaussian likelihood for the various shocks. Priors are standard (normal priors for the  $\lambda$ -coefficients and inverse gamma priors for the variances). The scale of the common factors and the  $\lambda$  coefficients are not separately identified; we impose a normalization where  $\Delta \tau_{c,t}^x$  and  $\varepsilon_{c,t}^x$  have unit variance and the average value of  $\lambda$  is non-negative. The priors for the

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<sup>12</sup>To the degree that labor is reallocated across sectors as a result of idiosyncratic shocks, we would expect the implied pairwise correlations in labor between different sectors to be negative on average. If instead sectoral labor were primarily driven by aggregate shocks, we would expect these pairwise correlations to be positive. We find no systematic pattern to the long-run cross-correlations between labor and TFP. Specifically, 32 percent of the pairwise long-run correlations for labor and TFP are statistically significantly different from zero at the 33 percent level, while only 24 percent of the labor-TFP cross correlations are statistically significant. The point estimates are also consistent with small correlations: the average estimated labor-TFP long-run cross-correlation is -0.03. See Section 2.1 of the [Technical Appendix](#) for detailed results.

<sup>13</sup>There is an apparent tension between the DFM model, which includes the random walk  $\tau$  terms and the model presented in the next section, which requires stationarity of the growth rates. The tension is resolved by assuming the  $\tau$  terms follow highly persistent, but stationary AR(1) models with AR roots local to unity. These processes are approximated as random walks in the DFM.

variance of the idiosyncratic terms are reasonably uninformative but we use more informative priors for  $\lambda$ . Details are provided in Appendix A.

### 3.3 Estimated Sectoral and Aggregate Trend Growth Rates in Labor and TFP

Appendix A and Section 2 of the [Technical Appendix](#) contain details of the estimation method and results for the empirical models. For our purposes, the key results are summarized in three figures and a table. Figures 2 and 3, introduced earlier, show the composite estimated trend component (posterior mean)  $(\lambda_{j,\tau}^x \tau_{c,t}^x + \tau_{j,t}^x)$  as the red dotted line along with the 15-year moving averages of the cyclically adjusted growth rates. While the estimated trends from the dynamic factor model closely track the 15-year moving averages for most of the sectors, these trends now also allow for a decomposition into common and sector-specific components. Table 3 shows the changes in trend growth for labor and TFP over different periods, as well as the decomposition of these changes into various components. In the table, common and sector-specific changes in trend growth rates add up to the aggregate change.

Figure 4 plots the aggregate values of the growth rates of labor and TFP along with their estimated trends from the sectoral empirical model. Panels A and D show the growth rates and the estimates of  $\tau$ ; panels B and E show the 15-year moving averages of the data along with the estimate of  $\tau$  and associated 68 percent credible intervals; and panels C and F decompose the estimate of  $\tau$  into its common component,  $\sum_{j=1}^n s_j \lambda_{j,\tau}^x \times \tau_{c,t}^x$ , and its sector-specific component,  $\sum_{j=1}^n s_j \tau_{j,t}^x$ .<sup>14</sup> As with the sectoral data, the implied aggregate trends estimated from the dynamic factor model closely track the low-frequency movements in the aggregate data.

Panels A and B include error bands (68 percent posterior credible intervals) computed from the dynamic factor model. The width of these error bands (approximately 0.50 percentage points) highlights the inherent uncertainty in estimating the ‘level’ of time series from noisy observations. This uncertainty carries over to the structural exercise in Section 4 and is amplified by uncertainty concerning the economic model postulated in that exercise, its calibrated parameters, as well as the quality of the data. However, to the degree that our estimates of sectoral trends mimic the behavior of 15-year moving averages, the economic model with parameters informed by BEA measures of the production structure traces out how these trends propagate to the rest of the economy.

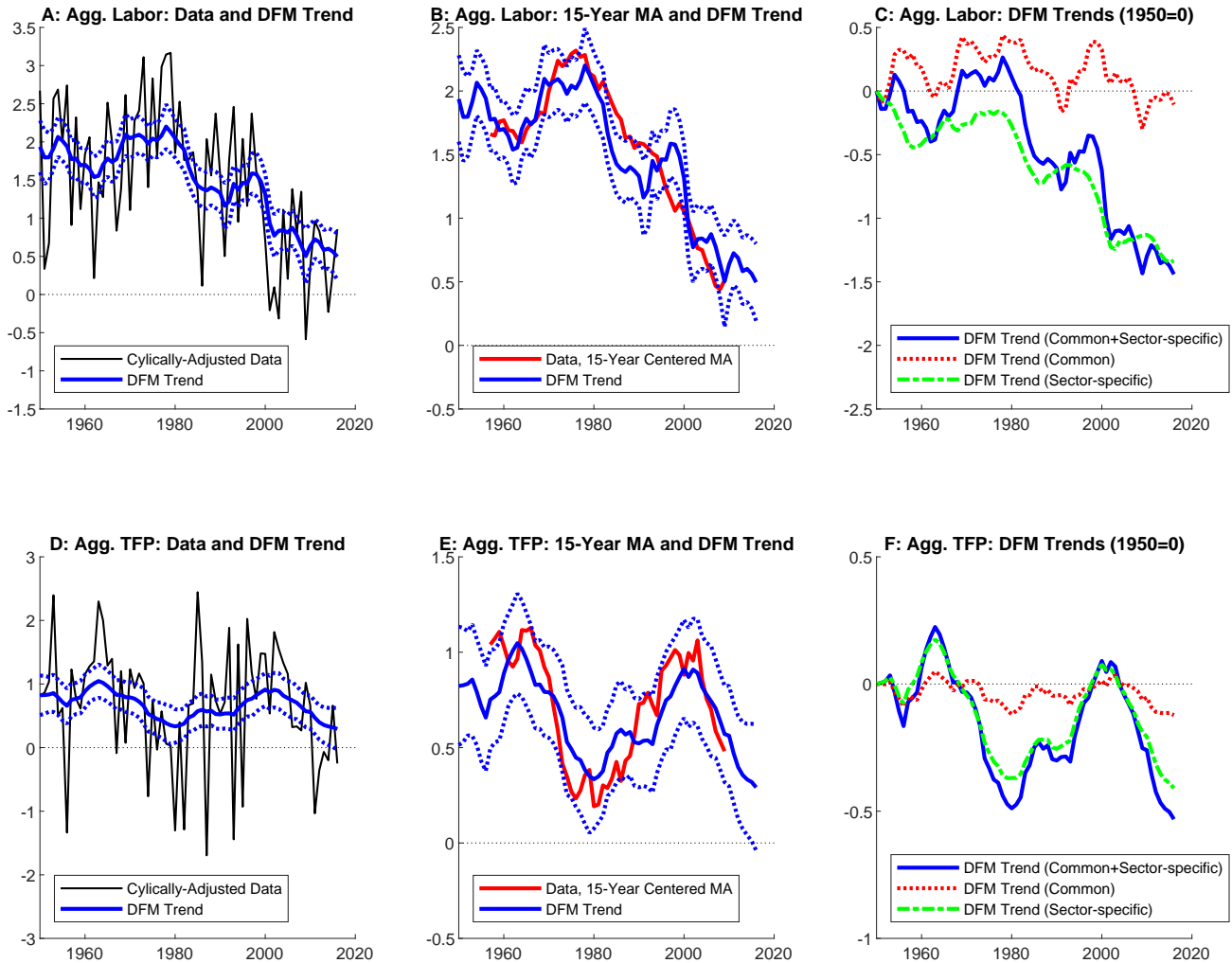
Panel C shows that aggregate trend labor growth fell by around 1.4 percentage points between 1950 and 2016. It also shows considerable variation over this period, and almost all of that decline took place after 1980. Panel F displays considerable swings in the trend of aggregate TFP growth between 1950 and 2016, with long stretches of rising and falling growth over different decades. It also shows that through these swings, trend TFP growth has fallen by approximately 0.5 percentage points in the last 65 years. Panels C and F suggest that much of the low-frequency variation in aggregate labor and TFP, as identified by the

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<sup>14</sup>Share weights for labor and TFP use labor compensation and value added weights respectively. The initial values for the common and sector-specific trend values are not separately identified - the data is only informative about their sum - so that Panels C and F normalize the initial values in 1950 to be zero.



Figure 4: Aggregate Trend Growth Rate in Labor and TFP  
(Percentage points at annual rate)



Notes: The dotted lines in panels (A), (B), (D), and (E) are 68 percent credible intervals for the DFM trends. In panels (C) and (F) the estimated DFM trends are normalized to 0 in 1950.

dynamic factor model, is associated with sector-specific rather than common trends. In particular, less than 20 percent of the trend decline in TFP growth is the result of shocks common to all sectors, and only about 10 percent of the trend decline in labor is attributable to common shocks.

Figures 5 and 6 illustrate the estimated common and sector-specific trends in the growth rates of labor and TFP for each sector. Table 3 highlights selected values of the changes in the trends plotted in the figures. From the beginning of the sample in 1950 until the end of the sample in 2016, the annual trend growth rate

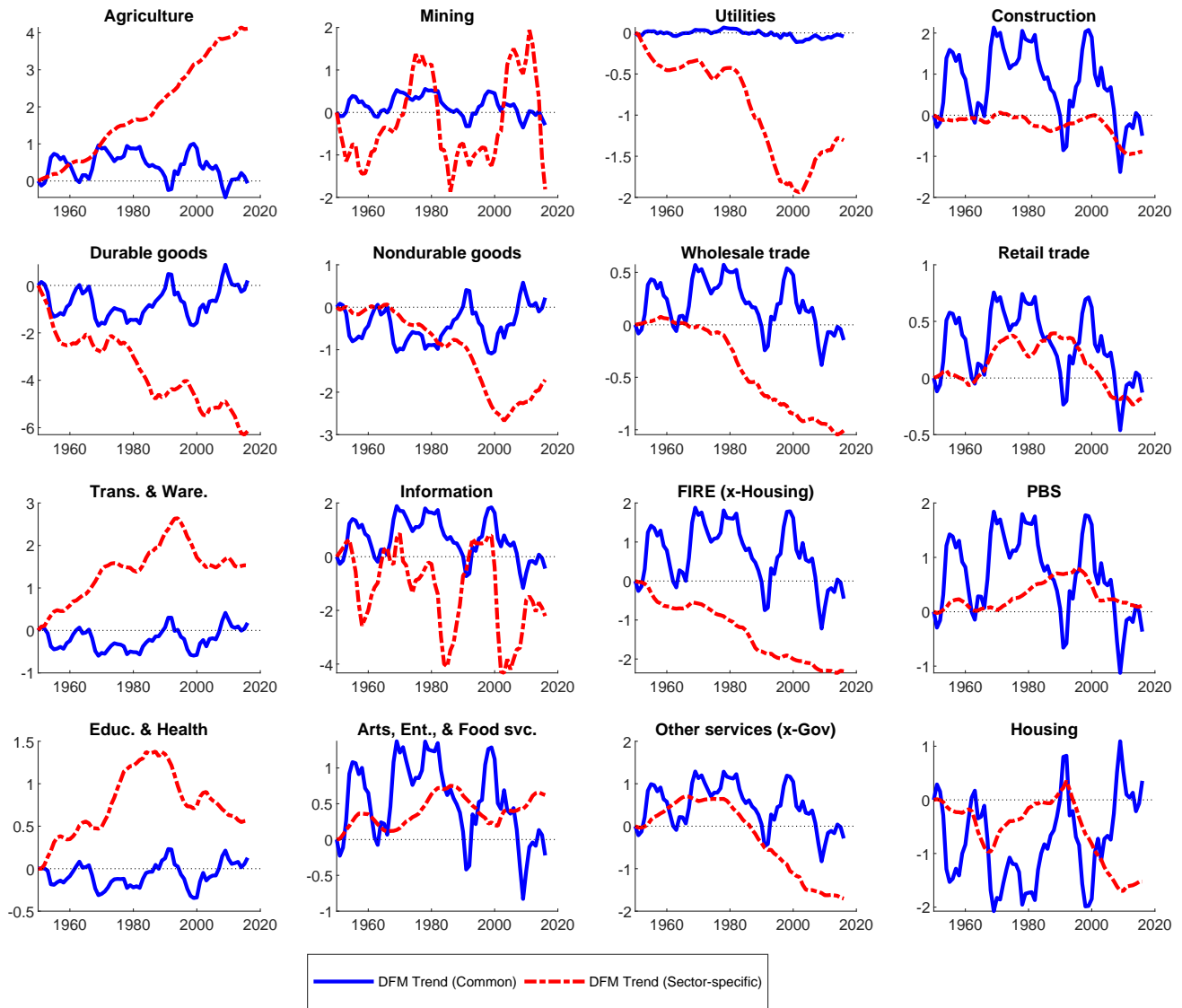
Table 3: Changes in Trend Value of Labor and TFP Growth Rates

	1950 – 2016		1950 – 1982		1982 – 1999		1999 – 2016	
	Labor	TFP	Labor	TFP	Labor	TFP	Labor	TFP
Aggregate	-1.44	-0.53	-0.04	-0.45	-0.41	0.49	-0.99	-0.58
Common	-0.11	-0.12	0.40	-0.09	-0.02	0.09	-0.49	-0.12
Sector specific (total)	-1.33	-0.41	-0.44	-0.35	-0.39	0.40	-0.50	-0.46
<i>Sector specific (by sector)</i>								
Agriculture	0.13	-0.00	0.05	-0.00	0.05	-0.00	0.03	0.00
Mining	-0.03	0.00	0.00	-0.04	-0.02	0.06	-0.01	-0.01
Utilities	-0.01	-0.02	-0.00	-0.02	-0.01	0.00	0.01	-0.00
Construction	-0.07	-0.16	-0.02	-0.17	0.02	-0.05	-0.07	0.06
Durable goods	-0.96	0.16	-0.55	0.19	-0.16	0.41	-0.25	-0.44
Nondurable goods	-0.15	-0.19	-0.08	-0.05	-0.13	-0.06	0.05	-0.08
Wholesale trade	-0.07	0.02	-0.02	0.05	-0.03	0.01	-0.01	-0.04
Retail trade	-0.02	-0.04	0.02	-0.05	-0.01	0.04	-0.03	-0.04
Trans. & Ware.	0.08	-0.12	0.07	-0.06	0.02	-0.03	-0.02	-0.03
Information	-0.08	0.07	-0.05	0.03	0.08	0.00	-0.11	0.04
FIRE (x-Housing)	-0.17	0.02	-0.08	-0.00	-0.06	0.02	-0.03	-0.00
PBS	0.01	-0.13	0.05	-0.12	0.01	-0.05	-0.05	0.04
Educ. & Health	0.05	-0.05	0.12	-0.10	-0.05	0.04	-0.01	0.01
Arts, Ent., & Food svc.	0.03	-0.00	0.03	0.01	-0.02	0.00	0.02	-0.01
Oth. serv. (x-Gov)	-0.08	-0.00	0.02	0.01	-0.06	-0.01	-0.03	-0.00
Housing	-0.00	0.04	-0.00	-0.04	-0.00	0.02	-0.00	0.06

Notes: This table shows the change in the DFM trend growth rates over the period shown. For example, the first column shows the DFM estimates of  $\tau_{2016} - \tau_{1950}$ . The first row shows the results for the share-weighted aggregate; the following two rows decompose this aggregate change into the component associated with the common  $\tau$  and the sector-specific  $\tau$ 's, which is further decomposed by sector in the remaining rows.

of aggregate labor fell from 1.92 percent to 0.48 percent, a decline of 1.44 percentage points. Much of this decline (1 percentage point) occurred between 1999 and the end of the sample. The dynamic factor model attributes most of the full-sample decline to sector-specific factors (1.33 percentage points) that themselves primarily reflect labor growth declines in the Durable Goods sector (0.96 percentage points). As shown in Figure 5, the secular decline in labor growth in Durable Goods has been large (over 6 percentage points relative to 1950) and steady throughout the period. Overall, variations in sector-specific trends,  $\tau_{j,t}^{\ell}$ , tend to

Figure 5: Sector-Specific and Common Growth Rate Trends for Labor from the DFM

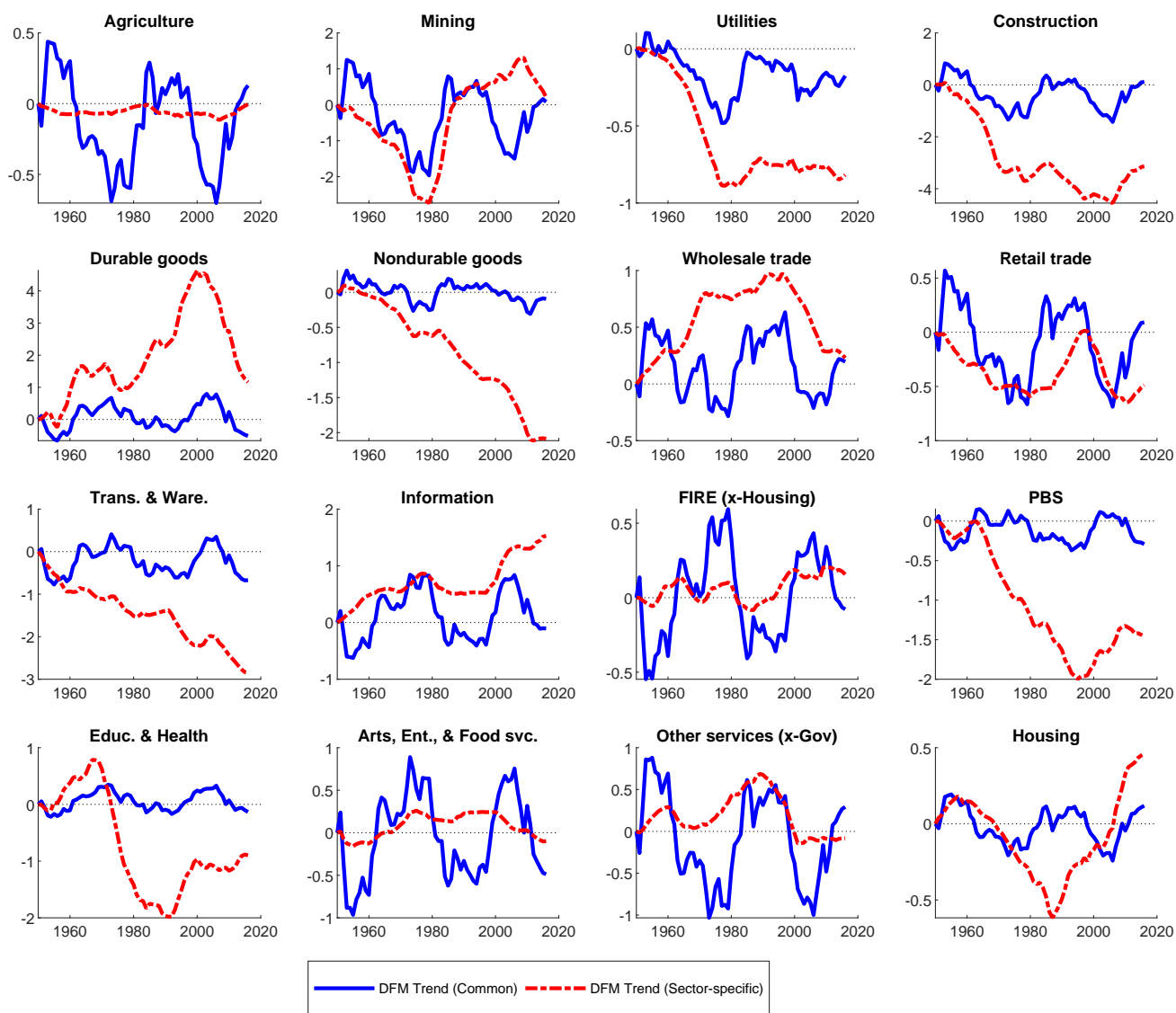


Notes: The figure plots the DFM common and sector-specific trend estimates for each sector. The trends are normalized to equal 0 in 1950.

be larger than those in common trends. This result underscores a diversity of sectoral experiences at secular frequencies that stays otherwise hidden in analyses of long-run trends that rely solely on aggregate data.

Similarly, the annual trend growth rate of aggregate TFP fell from 0.82 percent to 0.29 percent over the course of the sample, a decline of 0.53 percentage points. As we saw in Figure 4, and further underscored in Table 3, this decline was not monotonic: trend annual growth fell by half a percentage point over the period

Figure 6: Sector-Specific and Common Growth Rate Trends for TFP from the DFM



Notes: See notes to Figure 5.

1950 – 1982, then rebounded over the period 1982 – 1999, before falling again from 1999 to 2016. Over the entire post-war period, only a fifth of the decline is common to all sectors, and the largest sector-specific contributions to the declines were in Construction (0.16 percentage points, primarily in the first half of the sample) and Nondurable Goods (a near steady decline of 0.19 percentage points over the entire sample

period). Remarkably, Figure 6 shows that in the period from 1950 – 1999, sector-specific trend TFP growth in Durables led to an increase in aggregate TFP growth (0.60 percentage point in Table 3) that largely offset the decrease in several other sectors including Construction, Nondurable Goods, Transportation and Warehousing, as well as Professional and Business Services.<sup>15</sup> However, since 1999, trend TFP growth in Durable Goods has fallen over 4 percentage points, and by itself contributed 0.44 percentage points to the decline in aggregate TFP. Nevertheless, for the full sample the positive contribution of the Durable Goods sector to aggregate TFP growth is of the same magnitude as the negative contribution of the Construction sector.<sup>16</sup>

To assess the implications of the sectoral changes highlighted in this section for the secular behavior of GDP growth, one needs to be explicit about how secular change in one sector potentially impacts other sectors. Put another way, one needs to account for the fact that sectors interact through various input-output and capital flow relationships. We show in the next section that, in the presence of capital accumulation, production linkages between sectors can significantly amplify the effects of structural change in a sector on GDP growth.

## 4 Changing Sectoral Trends and the Aggregate Economy

This section explores how sectoral changes, here captured by the behavior of sectoral TFP and labor growth, shapes the behavior of GDP growth. Consistent with our TFP calculations in Section 3, we consider a canonical multi-sector growth model with competitive product and input markets. Each sector uses materials and capital produced in other sectors, and we allow for less than full depreciation of capital within the period.

The empirical specification in Section 3 leads us to distinguish between persistent and transitory sector-level changes that can arise from either aggregate or idiosyncratic forces. We consider preferences and technologies that are unit elastic in which case the economy evolves along a balanced growth path in the long run. Capital accumulation, however, allows for variations in output growth off that balanced growth path. Given linkages across sectors, structural change in one sector affects not only its own value added growth but also that of all other sectors. In particular, capital induces network effects that amplify the effects of sector-specific changes on GDP growth and that we summarize in terms of sectoral multipliers.

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<sup>15</sup>See Oliner et al. (2007) for the role of technological improvements in the IT sector as a driver of TFP growth in the Durable Goods.

<sup>16</sup>The decomposition of the trend components into common and sector-specific components is sensitive to constraints placed on the factor loadings,  $\lambda_{\tau,j}^x$ . We impose no constraints, besides the scale normalization that the average value is positive. Petrella (2020) imposes the additional constraint that  $\lambda_{\tau,j}^z \geq 0$  for all sectors and finds a relatively larger role for common technology shocks.

## 4.1 Economic Environment

Consider an economy with  $n$  distinct sectors of production indexed by  $j$  (or  $i$ ). A representative household derives utility from these  $n$  goods according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \prod_{j=1}^n \left( \frac{c_{j,t}}{\theta_j} \right)^{\theta_j}, \quad \sum_{j=1}^n \theta_j = 1, \quad \theta_j \geq 0,$$

where  $\theta_j$  is the household's expenditure share on final good  $j$ .<sup>17</sup>

Each sector produces a quantity,  $y_{j,t}$ , of good  $j$  at date  $t$ , using a value added aggregate,  $v_{j,t}$ , and a materials aggregate,  $m_{j,t}$ , using the technology,

$$y_{j,t} = \left( \frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left( \frac{m_{j,t}}{1 - \gamma_j} \right)^{(1-\gamma_j)}, \quad \gamma_j \in [0, 1]. \quad (3)$$

The quantity of materials aggregate,  $m_{j,t}$ , used in sector  $j$  is produced with the technology,

$$m_{j,t} = \prod_{i=1}^n \left( \frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}}, \quad \sum_{i=1}^n \phi_{ij} = 1, \quad \phi_{ij} \geq 0, \quad (4)$$

where  $m_{ij,t}$  denotes materials purchased from sector  $i$  by sector  $j$ . The notion that every sector potentially uses materials from every other sector introduces a first source of interconnectedness in the economy. An input-output (IO) matrix is an  $n \times n$  matrix  $\Phi$  with typical element  $\phi_{ij}$ . The columns of  $\Phi$  add up to the degree of returns to scale in materials for each sector, in this case unity. The row sums of  $\Phi$  summarize the importance of each sector as a supplier of materials to all other sectors. Thus, the rows and columns of  $\Phi$  reflect “sell to” and “buy from” shares, respectively, for each sector.

The value added aggregate,  $v_{j,t}$ , used in sector  $j$  is produced using capital,  $k_{j,t}$ , and labor,  $\ell_{j,t}$ , according to

$$v_{j,t} = z_{j,t} \left( \frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j} \left( \frac{\ell_{j,t}}{1 - \alpha_j} \right)^{1-\alpha_j}, \quad \alpha_j \in [0, 1]. \quad (5)$$

Capital accumulation in each sector follows

$$k_{j,t+1} = x_{j,t} + (1 - \delta_j)k_{j,t},$$

where  $x_{j,t}$  represents investment in new capital in sector  $j$ , and  $\delta_j \in (0, 1)$  is the depreciation rate specific to that sector. Investment in each sector  $j$  is produced using the quantity,  $x_{ij,t}$ , of sector  $i$  goods by way of

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<sup>17</sup>Here, the representative household is assumed to have full information with respect to transitory and permanent changes, as well as between idiosyncratic and common changes, as described in Section 3. In Section 3.11 of the [Technical Appendix](#), we also consider an imperfect information case in which the representative household cannot distinguish between permanent and transitory components of exogenous changes to the environment. In this alternative scenario, the household faces an additional filtering problem in which it must infer estimates of these components in deciding how much to consume and save in the face of exogenous disturbances.

the technology,

$$x_{j,t} = \prod_{i=1}^n \left( \frac{x_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}}, \quad \sum_{i=1}^n \omega_{ij} = 1, \quad \omega_{ij} \geq 0. \quad (6)$$

Thus, there exists a second source of interconnectedness in this economy in that new capital goods in every sector are potentially produced using the output of other sectors. This additional source of linkages in the economy has often been absent from structural multi-sector studies though it is shown to be a key propagation mechanism over the business cycle in recent work by [vom Lehn and Winberry \(2019\)](#). Similarly to the IO matrix, a Capital Flow matrix is an  $n \times n$  matrix  $\Omega$  with typical element  $\omega_{ij}$ . The columns of  $\Omega$  add up to the degree of returns to scale in investment for each sector or 1 in this case. The row sums of  $\Omega$  indicate the importance of each sector as a supplier of new capital to all other sectors.

The resource constraint in each sector  $j$  is given by

$$c_{j,t} + \sum_{i=1}^n m_{ji,t} + \sum_{i=1}^n x_{ji,t} = y_{j,t}. \quad (7)$$

Structural change in a sector is captured by the composite variable,  $A_{j,t}$ , which reflects the joint behavior of TFP and labor growth. In particular, under the maintained assumptions, sectoral value added may be expressed alternatively as

$$v_{j,t} = A_{j,t} \left( \frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j},$$

where

$$\Delta \ln A_{j,t} = \Delta \ln z_{j,t} + (1 - \alpha_j) \Delta \ln \ell_{j,t}. \quad (8)$$

In this paper, we condition on the observed joint behavior of  $\{\Delta \ln z_{j,t}, \Delta \ln \ell_{j,t}\}$  in each sector  $j$  and interpret these changes as describing the process of structural change in different sectors. Sectoral changes in labor input can of course reflect some reallocation of that input across sectors. Therefore, in counterfactual exercises that focus on individual sectors in [Section 5](#), we consider two polar cases comprising no labor mobility and complete mobility across sectors. In the first case, changes in labor input are treated as entirely sector-specific. In the polar opposite case, changes in labor input in a sector are entirely reallocated to other sectors in proportion to their employment shares.<sup>18</sup>

Each component of  $\{\Delta \ln z_{j,t}, \Delta \ln \ell_{j,t}\}$  is modeled after the empirical specification in [equation \(2\)](#) with

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<sup>18</sup>Observe in [Figures 2 and 3](#) that trend movements in TFP and labor growth often appear unrelated for a given sector. For example, trend TFP growth in Construction has seen a large decline since the 1950s while trend labor growth in that sector has been close to flat and dominated by its common trend or demographics (see [Figure 5](#)). Similarly, trend TFP growth in the Durable Goods sector in 2016 is close to where it started in 1950 ([Figure 3](#)), with several notable up-and-down swings along the way, while trend labor growth in that sector has steadily declined during the same period. As noted earlier, this lack of correlation is interesting but not one that we attempt to resolve in this paper.



one small change. Specifically, we let

$$\tau_{c,t}^x = (1 - \rho)g_c^x + \rho\tau_{c,t-1}^x + \eta_{c,t}^x, \quad (9)$$

and

$$\tau_{j,t}^x = (1 - \rho)g_j^x + \rho\tau_{j,t-1}^x + \eta_{j,t}^x, \quad (10)$$

where  $x = z$  or  $\ell$ , and  $\eta_{c,t}^x$  and  $\eta_{j,t}^x \forall j$  are mean-zero random variables. We assume that  $\rho < 1$  in which case the economy is characterized by a balanced growth path in the long run that we describe below. For values of  $\rho$  arbitrarily close to 1, however, the processes for  $\tau_{c,t}^x$  and  $\tau_{j,t}^x$  become those described in Section 3. Put another way, as in the empirical section, we allow the growth rates of sectoral TFP and labor to have both transitory and persistent - though not quite permanent - components off the balanced growth path. In the quantitative application, we consider values of  $\rho$  close to 1 and study transition paths implied by the observed secular behavior of  $\Delta \ln z_{j,t}$  and  $\Delta \ln \ell_{j,t}$ .

## 4.2 Model Parameters

Our choice of model parameters follows mostly Foerster et al. (2011) and is governed by the BEA Input-Output (IO) and Capital Flow accounts, and Fixed Asset Tables.

In our benchmark economy, the consumption bundle shares,  $\{\theta_j\}$ , value-added shares in gross output  $\{\gamma_j\}$ , capital shares in value added,  $\{\alpha_j\}$ , and material bundle shares,  $\{\phi_{ij}\}$ , are obtained from the 2015 BEA Make and Use Tables. The Make Table tracks the value of production of commodities by sector, while the Use Table measures the value of commodities used by each sector. We combine the Make and Use Tables to yield, for each sector, a table whose rows show the value of a sector's production going to other sectors (materials) and households (consumption), and whose columns show payments to other sectors (materials) as well as labor and capital. Thus, a column sum represents total payments from a given sector to all other sectors, while a row sum gives the importance of a sector as a supplier to other sectors. We then calculate material bundle shares,  $\{\phi_{ij}\}$ , as the fraction of all material payments from sector  $j$  that goes to sector  $i$ . Similarly, value-added shares in gross output,  $\{\gamma_j\}$ , are calculated as payments to capital and labor as a fraction of total expenditures by sector  $j$ , while capital shares in value added,  $\{\alpha_j\}$ , are payments to capital as a fraction of total payments to labor and capital. The consumption bundle parameters,  $\{\theta_j\}$ , are likewise payments for consumption to sector  $j$  as a fraction of total consumption expenditures.

Recall that GDP growth in Figure 1 was computed using different definitions of value added shares. While this did not lead to meaningful differences in aggregation, to the degree that these shares are ultimately determined by the nature of input-output relationships, and that these relationships have changed over time, the model might nevertheless yield material differences in the implied sectoral multipliers we derive below. Thus we also consider a version of the model informed by the 1960 and 1997 Make and Use Tables.

The parameters that determine the production of investment goods,  $\{\omega_{ij}\}$ , are chosen similarly in accordance with the BEA Capital Flow table from 1997, the most recent year in which this flow table is available.

The Capital Flow table shows the flow of new investment in equipment, software, and structures towards sectors that purchase or lease it. By matching commodity codes to sectors, we obtain a table that has entries showing the value of investment purchased by each sector from every other sector. A column sum represents total payments from a given sector for investment goods to all other sectors, while a row sum shows the importance of a sector as a supplier of investment goods to other sectors. Hence, the investment bundle shares,  $\{\omega_{ij}\}$ , are estimated as the fraction of payments for investment goods from sector  $j$  to sector  $i$ , expressed as a fraction of total investment expenditures made by sector  $j$ .

The capital depreciation rates are chosen to be consistent with capital accumulation as described in the BEA's Fixed Asset Tables. We construct Divisia aggregates for our 16-sector aggregation from detailed real net capital stocks and investment, and calculate capital depreciation rates such that net-stocks and investment are consistent with the capital accumulation equation in each sector. Because the implied depreciation rates vary over time, we fix each sector's depreciation rate at its post-2000 average as a benchmark.

For ease of presentation, we use the following notation throughout the paper: we denote the vector of household expenditure shares by  $\Theta = (\theta_1, \dots, \theta_n)$ , the matrix summarizing value added shares in gross output by sector,  $\Gamma_d = \text{diag}\{\gamma_j\}$ , the matrix of input-output linkages by  $\Phi = \{\phi_{ij}\}$ , the capital flow matrix by  $\Omega = \{\omega_{ij}\}$ , the matrix summarizing capital shares in value added by sector,  $\alpha_d = \text{diag}\{\alpha_j\}$ , and the matrix summarizing sector-specific depreciation rates by  $\delta_d = \text{diag}\{\delta_j\}$ .

### 4.3 Some Benchmark Results in an Economy Without Growth

A special case of the economic environment presented above is one where  $\alpha_j = 0 \forall j$ , and  $\{\ln z_{j,t}, \ln \ell_{j,t}\}$  are modeled as stationary processes in levels rather than growth rates, in which case  $\ln A_{j,t}$  is also stationary in levels around a constant mean in each sector. This special case reduces to the economy studied in [Long and Plosser \(1983\)](#) - though in that paper, materials,  $m_{j,t}$ , are used with a one-period lag. Denoting aggregate value added or GDP by  $V_t$ , we have that in this economy,

$$\frac{\partial \ln V_t}{\partial \ln A_{j,t}} = s_j^v, \quad (11)$$

where  $s_j^v$  is sector  $j$ 's value added share in GDP, and where these shares may be summarized in a vector,  $s^v = (s_1^v, \dots, s_n^v)$ , given by  $s^v = \Theta(I - (I - \Gamma_d)\Phi')^{-1}\Gamma_d$ . Consistent with [Hulten \(1978\)](#) or more recently [Gabaix \(2011\)](#), a sector's value added share entirely summarizes the effects of structural change in that sector on the level of GDP. Accordingly, [Acemoglu et al. \(2012\)](#) refer to the object  $\Theta(I - (I - \Gamma_d)\Phi')^{-1}\Gamma_d$  as *the influence vector*. We state Hulten's insight in equation (11) in terms of value added shares rather than Domar weights (i.e. shares of gross output in GDP). This is because, consistent with our empirical measures, TFP scales value added in equation (5) rather than gross output in equation (3). An alternative exercise relying instead on gross output TFP would immediately yield Domar weights as the partial derivative in equation (11).

When at least some sectors use capital in production, so that  $\alpha_j > 0$  for some  $j$ , the economy becomes

dynamic and, absent shocks, converges to a steady state in levels in the long-run. Getting rid of the  $t$  subscripts to denote variables in that steady state, and letting  $A = (A_1, \dots, A_n)$  represent the long-run vector of composite exogenous sectoral states, a version of equation (11) holds in the limit as  $\beta \rightarrow 1$ ,

$$\frac{\partial \ln V}{\partial \ln A_j} = \eta s_j^v, \quad (12)$$

where  $\eta$  is an adjustment factor approximately equal to the inverse of the mean labor share across sectors. In particular, when sectors use capital with the same intensity,  $\alpha_j = \alpha \forall j$ , then  $\eta = \frac{1}{1-\alpha}$ . The value added shares in this case (rather than Domar weights),  $s^v$ , are given by  $\Theta[\Gamma_d^{-1}(I - (I - \Gamma_d)\Phi') - \alpha_d\Omega']^{-1}/\Theta[\Gamma_d^{-1}(I - (I - \Gamma_d)\Phi') - \alpha_d\Omega']^{-1}\mathbf{1}$ , where  $\mathbf{1}$  is a unit vector of size  $n$ . When  $\beta < 1$ , the influence vector also depends on sectoral depreciation rates,  $\delta_j$ , and equation (12) holds as an approximation that depends on  $\frac{\beta}{1-\beta(1-\delta_j)} \times \delta_j$  which, for standard calibrations of  $\beta$ , is close to 1.<sup>19</sup> When preferences and technology are Cobb-Douglas, neither expressions (11) nor (12) has sectoral states,  $A_j$ , affecting value added shares,  $s_j^v$ .

#### 4.4 Balanced Growth

The effects of structural changes are less straightforward during transitions to the steady state in an economy with capital. Furthermore, because the data described in Section 3 reveals important persistent components in sectoral growth rates, we frame the effects of sectoral structural change on the aggregate economy in terms of growth rate elasticities,  $\partial \Delta \ln V_t / \partial \Delta \ln A_{j,t}$ .

In an economy with steady state growth, sectoral value added shares in GDP will depend on the entire distribution of growth rates characterizing structural changes,  $\Delta \ln A_j$ ,  $j = 1, \dots, n$ , in addition to reflecting the sectoral network implicit in the IO and capital flow matrices. As described above, these shares are then used to calculate overall GDP growth,  $\Delta \ln V$ , by way of a Divisia index. More importantly, because of production linkages, structural change in a sector,  $\Delta \ln A_j$ , potentially helps determine value added growth in every other sector along the balanced growth path. This mechanism, therefore, amplifies the effects of sector-specific structural change on GDP growth in a way that can be summarized in terms of a multiplier for each sector. In some sectors, these multipliers can scale the impact of structural changes on GDP growth by multiple times their share in the economy.

Consider a non-stochastic steady state path where all disturbances - persistent and transitory as well as idiosyncratic and common - are set to zero. The non-stochastic steady state is defined by a balanced growth path determined by constant growth in each sector,

$$\Delta \ln A_{j,t} = \bar{g}_j = \bar{g}_j^z + (1 - \alpha_j) \bar{g}_j^\ell, \quad (13)$$

where

$$\bar{g}_j^z = \lambda_{j,\tau}^z g_c^z + g_j^z \text{ and } \bar{g}_j^\ell = \lambda_{j,\tau}^\ell g_c^\ell + g_j^\ell. \quad (14)$$

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<sup>19</sup>See Appendix B.

In other words, sectoral structural growth in the steady state,  $\bar{g}_j$ , reflects steady state sectoral TFP growth,  $\bar{g}_j^z$ , and sectoral labor growth,  $\bar{g}_j^\ell$ . The long-run growth rates of TFP and labor in each sector in turn reflect aggregate (common) components,  $(\lambda_{j,\tau}^z g_c^z, \lambda_{j,\tau}^\ell g_c^\ell)$ , and idiosyncratic components,  $(g_j^z, g_j^\ell)$ , respectively.

#### 4.4.1 Sectoral Growth in Value Added

Let  $\Delta \ln \mu_i^v = (\Delta \ln \mu_{1,t}^v, \dots, \Delta \ln \mu_{n,t}^v)'$  denote the vector value added growth by sector. Then, along the balanced growth path,  $\Delta \ln \mu^v$  is constant and given by

$$\Delta \ln \mu^v = \left[ I + \alpha_d \Omega' \underbrace{(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d) \Phi')^{-1}}_{\Xi'} \Gamma_d \right] \bar{g}_a, \quad (15)$$

where  $\bar{g}_a = (\bar{g}_1, \dots, \bar{g}_n)'$  is the vector of sectoral structural growth rates.<sup>20</sup> Equation (15) describes how structural growth in a given sector,  $\bar{g}_j$ , affects value added growth in all other sectors,  $\Delta \ln \mu_i^v$ . This relationship involves the direct effects of sectors' structural growth on their own value added growth,  $I \bar{g}_a$ , and the indirect effects that sectors have on other sectors through the economy's sectoral network of materials and investment,  $\alpha_d \Omega' \Xi' \bar{g}_a$ . Specifically,

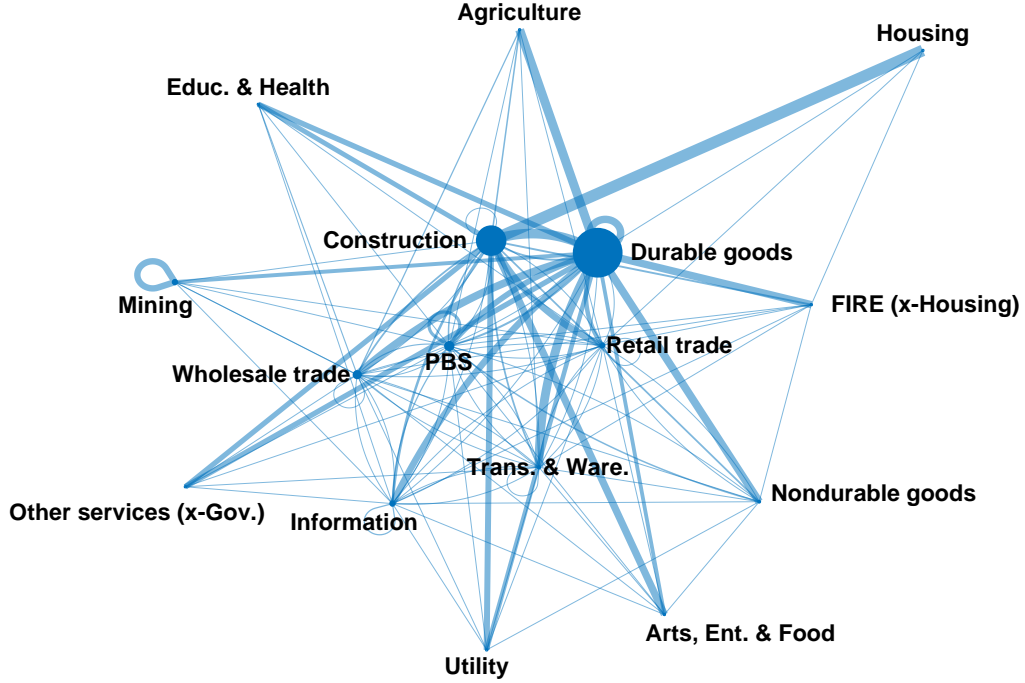
$$\frac{\partial \Delta \ln \mu_j}{\partial \bar{g}_j} = 1 + \alpha_j \sum_{k=1}^n \omega_{kj} \xi_{jk} \quad \text{and} \quad \frac{\partial \Delta \ln \mu_i}{\partial \bar{g}_j} = \alpha_i \sum_{k=1}^n \omega_{ki} \xi_{jk}, \quad (16)$$

where  $(\xi_{j1}, \dots, \xi_{jn})$  is a column of  $\Xi'$  equal to the Leontief inverse,  $(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d) \Phi')^{-1}$ , diagonally weighted by the matrix of value added shares in gross output,  $\Gamma_d$ , in equation (15). Thus, along the balanced growth path, sectoral linkages make it possible for a structural change in a given sector  $j$ ,  $\partial \bar{g}_j$ , to affect value added growth in every other sector,  $i$ , so long as that sector uses capital in production,  $\alpha_i > 0$ . Otherwise, value added growth in a sector with  $\alpha_j = 0$  is entirely determined by its own structural growth rate,  $\frac{\partial \Delta \ln \mu_j}{\partial \bar{g}_j} = 1$ . In this sense, the presence of capital accumulation plays a central role for the sectoral growth implications of production linkages.

To gain intuition into how structural growth in different sectors help determine value added growth in other sectors, observe that the effect of a change in sector  $j$ 's structural growth rate on sector  $i$  is given by the  $(i, j)$  element of  $\alpha_d \Omega' \Xi'$ . Each of these  $(i, j)$  elements in turn contains all of the elements of the vector  $(\xi_{j1}, \dots, \xi_{jn})$  in the  $j^{\text{th}}$  column of the weighted Leontief inverse,  $\Xi'$ , as described in equation (16). The  $j^{\text{th}}$  column of the weighted Leontief inverse in turn will reflect the  $j^{\text{th}}$  column of the transposed capital flow matrix,  $\Omega'$  (i.e. the  $j^{\text{th}}$  row of  $\Omega$  or the degree to which sector  $j$  produces new capital for other sectors), as well as the  $j^{\text{th}}$  column of the transposed IO matrix,  $\Phi'$ , (i.e. the  $j^{\text{th}}$  row of  $\Phi$  or the degree to which sector  $j$  produces materials for other sectors). To see this, observe that the Leontief inverse,  $(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d) \Phi')^{-1}$ , can be alternatively expressed as the limit of  $(\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi') +$

<sup>20</sup>See Appendix C. Observe also that preference parameters are absent from equation (15) in that balanced growth relationships are ultimately statements about technologies and resource constraints only.

Figure 7: Investment Network



Notes: This figure shows the investment network as a graph, where capital flows are represented by edges between nodes representing sectors. See the Capital Flow Table in Section 4 of the [Technical Appendix](#).

$(\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi')^2 + (\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi')^3 + \dots + (\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi')^n$ . Each column of  $\Omega'$  and  $\Phi'$  is weighted by that column's corresponding sector's share of value added and materials in gross output respectively,  $\Gamma_d \Omega'$  and  $(I - \Gamma_d) \Phi'$ . Ultimately, sectors that play a central role in producing capital or materials for other sectors will be associated with a column of the weighted Leontief inverse,  $\Xi'$ , whose elements are relatively large. The individual elements of the Neumann series,  $(\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi')^k$ ,  $k = 1, \dots, n$ , describe feedback effects in which a change in the structural growth rate of some sector,  $j$ , impacts the price and quantities of  $j$ 's goods purchased by another sector,  $i$ , which in turn impacts the price and quantities of  $i$ 's goods purchased by other sectors including  $j$ . This process then feeds back into the prices and quantities of goods that sector  $j$  sells to sector  $i$  in the next round, and so on.

The production of investment goods in the U.S. turns out to be concentrated in relatively few sectors, with many sectors not producing any capital for other sectors while Construction and Durable Goods produce close to 80 percent of capital in almost every sector. Put another way, as shown in Figure 7, we can think of the Construction and Durable goods sectors as investment hubs in the production network.<sup>21</sup>

<sup>21</sup>Table A6 in Section 4.1 of the [Technical Appendix](#) shows the Capital Flow matrix of the U.S. economy,  $\Omega$ , for the 16 sectors

Construction comprises residential and non-residential structures, including infrastructure such as power plants or pipelines used to transport crude oil for example, but also the maintenance and repair of highways, bridges, and other surface roads. The bulk of capital produced by the Durable Goods sector resides in Motor Vehicles, Machinery, and Computer and Electronic Products. Other sectors recorded as producing capital goods for the U.S. economy include Wholesale Trade, Retail Trade, and Professional and Business Services. In the Professional and Business Services sector, the notion of capital produced for other sectors is overwhelmingly composed of Computer System Designs and Related Services. As a practical matter, the distinction between materials and investment goods is not always straightforward. The BEA distinguishes between materials and capital goods by estimating the service life of different commodities and, consistent with a time period in this paper, commodities expected to be used in production within the year are defined as materials.<sup>22</sup> From the Capital Flow table, we expect that columns of the Leontief inverse,  $\Xi'$ , associated with Construction and Durable Goods will have relatively large elements.

Compared to the Capital Flow table, the production of materials is considerably less concentrated in that all sectors produce materials for all other sectors, though to varying degrees.<sup>23</sup> From the Make-Use table, Professional and Business Services, Finance and Insurance, and to a degree Nondurable Goods, all play an important role in providing intermediate inputs to the U.S. economy. While Professional and Business Services figures prominently in the IO table, this sector is not nearly as dominant as Durable Goods or Construction are in the capital flow table. In contrast to the sectors that play a key role in the U.S. production network, output produced in sectors such as Agriculture, Forestry, Fishing, and Hunting, Entertainment and Food Services, or Housing, is mostly consumed as a final good. Therefore, columns of the Leontief inverse associated with these sectors will have elements that tend to be small.

Putting the information from the Capital Flow and Make-Use tables together, weighted by the diagonal matrix  $\Gamma_d$ , gives us the generalized Leontief inverse matrix  $\Xi'$ .<sup>24</sup> As expected, elements of the Construction and Durable Goods columns of the Leontief inverse are uniformly large, while those of the Agriculture, Forestry, Fishing, and Hunting or Entertainment and Food Services columns are comparatively unimportant. Because columns of the Leontief inverse determine the extent to which structural change in one sector affects value added growth in every other sector, these columns also ultimately make up a network multiplier for each sector that summarizes the importance of structural change in that sector for aggregate GDP growth.

Before turning our attention to sectoral multipliers, we make one last observation regarding the role of

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considered in this paper.

<sup>22</sup>As noted in [Foerster et al. \(2011\)](#), there are a couple of notable measurement issues related to the construction of the Capital Flow table. First, the table accounts for the purchases of new capital goods but not used assets. Thus, for example, a firm's purchase of a used truck in a sector from another sector will not be recorded as investment in the capital flow table even though the truck's remaining service life may be well in excess of a year. Second, [McGrattan and Schmitz \(1999\)](#) note that a non-trivial portion of maintenance and repair takes place using within sector resources, yet many of the diagonal elements of the Capital Flow table are very small or zero. Results presented here are robust up to an adjustment that assumes that an additional 25 percent of capital expenditures takes place within sectors.

<sup>23</sup>Table A7 in Section 4.2 of the [Technical Appendix](#) shows the IO matrix or Make-Use table for the U.S. economy,  $\Phi$ . The values shown in that table are from the 2015 BEA Make and Use Tables but the next section shows results using values from different Make and Use Tables over the decades.

<sup>24</sup>See Table A8 in Section 4.3 of the [Technical Appendix](#) which describes the generalized Leontief Inverse.

sectoral linkages along the balanced growth path. In particular, consider the special case in which each sector produces its own capital,  $\Omega = I$ , as in Horvath (1998) or Dupor (1999), and its own materials,  $\Phi = I$ . In other words, production requires capital and materials but both are sector specific. In that case, there are effectively no sectoral linkages and we have that

$$\Delta \ln \mu_j^v = \frac{1}{1 - \alpha_j} \bar{g}_j^z + \bar{g}_j^\ell, \quad (17)$$

which is the equation that obtains in the one-sector growth model but sector by sector.

#### 4.4.2 Aggregate Balanced Growth Implications in a Canonical Model with Linkages

Given the vector of value added growth,  $\Delta \ln \mu^v$ , the Divisia aggregate index of GDP growth is  $\Delta \ln V = s^v \Delta \ln \mu^v$  or

$$\Delta \ln V = \sum_{j=1}^n s_j^v \left[ \bar{g}_j + \sum_{i=1}^n \alpha_j \omega_{ij} \sum_{k=1}^n \xi_{ki} \bar{g}_k \right], \quad (18)$$

so that, holding shares constant,

$$\frac{\partial \Delta \ln V}{\partial \bar{g}_j} = s_j^v + s_j^v \alpha_j \sum_{k=1}^n \omega_{kj} \xi_{jk} + \sum_{i \neq j} s_i^v \alpha_i \sum_{k=1}^n \omega_{ki} \xi_{jk}, \quad (19)$$

where the second and third terms in this last expression capture the weighted network sectoral effects discussed above in the context of the (diagonally weighted) Leontief inverse,  $\Xi'$ .<sup>25</sup>

When all sectors use little or no capital in production,  $\alpha_j = 0 \forall j$ , the effects of structural change in a given sector  $j$ ,  $\partial \bar{g}_j$ , on GDP growth will be well approximated by its share in the economy,  $s_j^v$ . This case recovers a version of Hulten's theorem (1978) but in growth rates. More generally, equation (19) suggests the presence of a network multiplier effect that varies by sector and that depends not only on the importance of sectors as suppliers of capital and materials to other sectors,  $\omega_{ki}$  and  $\xi_{jk}$ , but also on the extent to which the latter sectors use capital in their own production,  $\alpha_i$ . The effects of sectoral change,  $\partial \bar{g}_j$ , on GDP growth may be summarized as a direct effect,  $s^v I$ , and an additional indirect effect resulting from sectoral linkages,  $s^v \alpha_d \Omega' \Xi'$ . Hence, we define the combined direct and indirect effects of structural change on GDP growth in terms of network multipliers,  $s^v (I + \alpha_d \Omega' \Xi')$ .

Table 4 shows the direct and combined effects of structural change in the different sectors we consider on GDP growth given the input-output and capital flow matrices. The importance of Construction and Durable Goods as suppliers of investment goods results in their having not only a large value added share

<sup>25</sup>In Section 3.5 of the [Technical Appendix](#), we show that along the balanced growth path, sectoral value added shares in GDP,  $s^v$ , are functions of the model's underlying parameters including, in the general case, the distribution of sectoral structural growth rates,  $\bar{g}_a$ . Furthermore, changes in sectoral shares induced by a change in structural growth in a sector  $k$ ,  $\frac{\partial s_j^v}{\partial \bar{g}_k}$  tend to be small and somewhat inconsequential for overall growth, consistent with Figure 1, and the notion that since shares must sum to 1,  $\sum_j \frac{\partial s_j^v}{\partial \bar{g}_k} = 0$ .



Table 4: Sectoral Network Multipliers

Sector	$s^v$	$s^v \alpha_d \Omega' \Xi'$	$s^v (I + \alpha_d \Omega' \Xi')$
Agriculture	0.03	0.01	0.03
Mining	0.02	0.03	0.05
Utilities	0.02	0.01	0.03
Construction	0.05	0.12	0.17
Durable goods	0.13	0.28	0.42
Nondurable goods	0.09	0.03	0.13
Wholesale trade	0.07	0.08	0.15
Retail trade	0.08	0.02	0.11
Trans. & Ware.	0.04	0.03	0.07
Information	0.05	0.03	0.08
FIRE (x-Housing)	0.10	0.03	0.14
PBS	0.09	0.16	0.25
Educ. & Health	0.06	0.00	0.06
Arts, Ent., & Food svc.	0.04	0.01	0.04
Oth. serv. (x-Gov)	0.03	0.01	0.04
Housing	0.09	0.00	0.09

Notes: This table decomposes each sector's total multiplier (column 3) into a direct effect (column 1) and an indirect effect (column 2). The sums do not necessarily add up because of rounding.

in GDP, 5 and 13 percent respectively, but also in their having large spillover effects on other sectors. In particular, the network multipliers for the Construction and Durable Goods sectors come out to more than 3 times their share, 0.17 and 0.42 respectively. Considering that trend TFP growth in Construction fell by nearly 5 percentage points between 1950 and 2016 in Figure 3, this gives us, all else equal, a roughly 0.7 percentage point effect on trend GDP growth from TFP changes in Construction alone. In practice, the aggregate growth effects from combined changes in TFP and labor in Construction are likely smaller since labor growth has been moderately positive in that sector throughout the post-war. Nevertheless, the effect from Construction is large not only because of its central role as a producer of capital, including commercial and residential structures for all sectors, but also because capital depreciates only partially in any given year. It is apparent from Table 4 that the effects of structural change on GDP growth are always at least as large as sectoral shares. The sectoral network multipliers roughly double the share of Professional and Business Services, from 0.09 to 0.25, and Wholesale Trade from 0.07 to 0.15. In other sectors, such as Agriculture, Forestry, Fishing and Hunting, Entertainment and Food Services, or Housing, the network multipliers are small or negligible as suggested by the corresponding columns of the weighted Leontief inverse. Because the same network relationships embodied in the Capital Flow table,  $\Omega$ , and Make-Use table,  $\Phi$ , determine the importance that sectors have in the economy both as a share of value added and through their spillover

Table 5: Sectoral Network Multipliers Under Alternative Calibrations

Sector	Benchmark	Mean Shares, First 15 Years	Mean Shares, Last 15 Years	1997 IO Table	1960 IO Table
Agriculture	0.03	0.06	0.02	0.03	0.04
Mining	0.05	0.06	0.05	0.04	0.04
Utilities	0.03	0.03	0.02	0.03	0.03
Construction	0.17	0.17	0.17	0.15	0.17
Durable goods	0.42	0.48	0.35	0.39	0.42
Nondurable goods	0.13	0.16	0.10	0.13	0.14
Wholesale trade	0.15	0.15	0.14	0.14	0.13
Retail trade	0.11	0.12	0.09	0.11	0.11
Trans. & Ware.	0.07	0.08	0.06	0.06	0.07
Information	0.08	0.07	0.09	0.08	0.07
FIRE (x-Housing)	0.14	0.12	0.17	0.14	–
PBS	0.24	0.20	0.28	0.20	0.16
Educ. & Health	0.06	0.03	0.09	0.06	0.06
Arts, Ent., & Food svc.	0.04	0.04	0.05	0.04	0.04
Oth. serv. (x-Gov)	0.04	0.04	0.03	0.04	0.04
Housing	0.09	0.08	0.11	0.09	–
Addendum: FIRE + Housing	0.24	0.20	0.28	0.24	0.23

Notes: This table shows the sectoral multiplier (see Table 4) for the baseline calibration (column 1), for alternative value-added share weights (columns 2 and 3) and for alternative IO tables (columns 4 and 5).

effects, sectors with relatively larger shares in GDP will also tend to be associated with large network multipliers.

A key implication of Table 4 is that the effects of sectoral change on GDP growth arise in part through a composition effect. Therefore, secular changes in GDP growth can take place without observable changes in aggregate TFP growth. For example, consider purely idiosyncratic changes in TFP growth,  $\partial \bar{g}_j^z$ , that leave aggregate TFP growth unchanged,  $\sum_{j=1}^n s_j^y \partial \bar{g}_j^z = 0$ . Despite aggregate TFP growth not changing, these idiosyncratic changes may nevertheless have an effect on GDP growth since the sum of sectoral multipliers is larger than 1.

#### 4.4.3 Robustness of the Sectoral Multipliers

The key take away from Table 4 is that the influence of sectors on aggregate growth generally exceed their value added share in GDP, especially in Construction and Durable Goods whose multipliers amount to more than three times their respective share in the economy. As the table makes clear, this observation of course

depends on what shares are being used and how the sectors interact through input-output and capital good linkages. Table 5, therefore, explores the degree to which sectoral multipliers are sensitive to the definition of shares or input-output table. (Data limitations require us to use the 1997 capital flow table throughout.)

The first column of Table 5 reproduces our benchmark sectoral multipliers shown in the last column of Table 4; recall that these results are based on constant shares computed as averages over the full sample (1950-2016) and the 2015 Make and Use Tables. The second column of Table 5 shows the sectoral multipliers obtained using constant mean shares calculated only over the first 15 years of the sample (1950-1964). The third column shows these multipliers computed instead using constant mean shares from the last 15 years of the sample (2002-2016). The fourth and fifth columns of Table 5 shows the sectoral multipliers implied by the Make and Use Tables from 1997 and 1960 respectively.

While there are differences across the columns of Table 5, the general lesson remains the same. The sum of the multipliers always exceeds 1 and varies from 1.7 to 1.9 across columns. Construction and Durable Goods consistently have an outsized influence on aggregate growth regardless of the calculation in Table 5 given their role as input suppliers to the rest of the economy. Moreover, the ranking of sectoral multipliers by sector is also generally consistent across columns. The Make and Use table from 1960 does not allow us to separate FIRE and Housing so that the last row of Table 5 gives a multiplier for the combined sectors of about 0.24 on average across columns.

The various sectoral multiplier calculations we have just carried out depend on the balanced growth equations (15) and (18), which hold only in the steady state and ignore important endogenous dynamics driven by capital accumulation. In other words, these equations all miss dynamic terms that capture deviations from steady state growth. Thus, we now turn our attention to the model solution and associated Markov decision and policy rules.

## 5 Quantitative Findings

Having described the balanced growth path implications of sectoral structural change, we now interpret the secular trend in aggregate GDP growth estimated in Section 2 as part of a slow-moving transition towards a balanced growth path. Therefore, to assess how sectoral trends in different sectors play out in other sectors over time, as well as in terms of overall GDP growth, we first work out the transition dynamics of the model.

Given the economy's balanced growth path, we normalize the model's variables by appropriate factors (these are also the factors used in Appendix C) so as to make the model's optimality conditions and resource constraints stationary in the detrended variables. The scaling factors used in detrending differ with the variables under consideration but always involve the entire distribution of growth rates summarized in  $\bar{g}_a$ . Detrended variables are constant along the balanced growth path and while consumption, investment, materials, and gross output all grow at the same rate within each sector, these variables grow at different rates across sectors. Prices in different sectors, as well as their implied price indices, also all grow at different rates along the balanced growth path but in such a way as to generate constant shares along that path. After

linearizing the model's dynamic equations around the steady state in detrended variables, Markov decision and policy rules may be readily obtained using standard linear rational expectations solution toolkits, including in this case [King and Watson \(2002\)](#).<sup>26</sup>

## 5.1 Historical Decompositions

In exploring the aggregate implications of structural change in individual sectors, the previous section highlighted the importance of accounting for production linkages and their network effects. In other words, in any counterfactual exercise, individual sectors cannot be modeled in isolation. In this section, we wish to recover the dynamic implications of the model for sectoral value added growth,  $\Delta \ln v_{j,t}$ , and aggregate GDP growth,  $\Delta \ln V_t = \sum_{j=1}^n s_j^v \Delta \ln v_{j,t}$ , when driven by the different sectoral processes estimated in Section 3.

In Appendix D, we show that during transitions to the balanced growth path, the vector of value added growth rates,  $\Delta \ln v_t$ , evolves according to

$$\Delta \ln v_t = (I + \alpha_d \Omega' \Xi') \bar{g}_a + \alpha_d \Delta \hat{k}_t + \hat{A}_t + \alpha_d \Omega' \Xi' \hat{A}_{t-1}, \quad (20)$$

where the vector  $\hat{k}_t = (\hat{k}_{1,t}, \dots, \hat{k}_{n,t})'$  denotes the log deviations of detrended capital from its appropriate level and  $\hat{A}_t = (\hat{A}_{1,t}, \dots, \hat{A}_{n,t})'$  with  $\hat{A}_{j,t} = \Delta \ln A_{j,t} - \bar{g}_j$ . The vector  $\hat{k}_t$  follows from the model's equilibrium Markov decision rules and is thus a function of previous endogenous states,  $\hat{k}_{t-1}$ , and the various shocks affecting the economy (i.e. persistent and transitory, as well as idiosyncratic and common). The first term on the right-hand side of (20) captures the sectoral steady state growth paths described in equation (15). The additional terms represent the dynamics of value added growth induced by the driving processes,  $\hat{A}_t$ , and by endogenous adjustments to capital in different sectors,  $\Delta \hat{k}_t$ . In the absence of shocks,  $\Delta \hat{k}_t$  and  $\hat{A}_t$  are zero in the long run and  $\Delta \ln v_t = (I + \alpha_d \Omega' \Xi') \bar{g}_a$  as in equation (15).

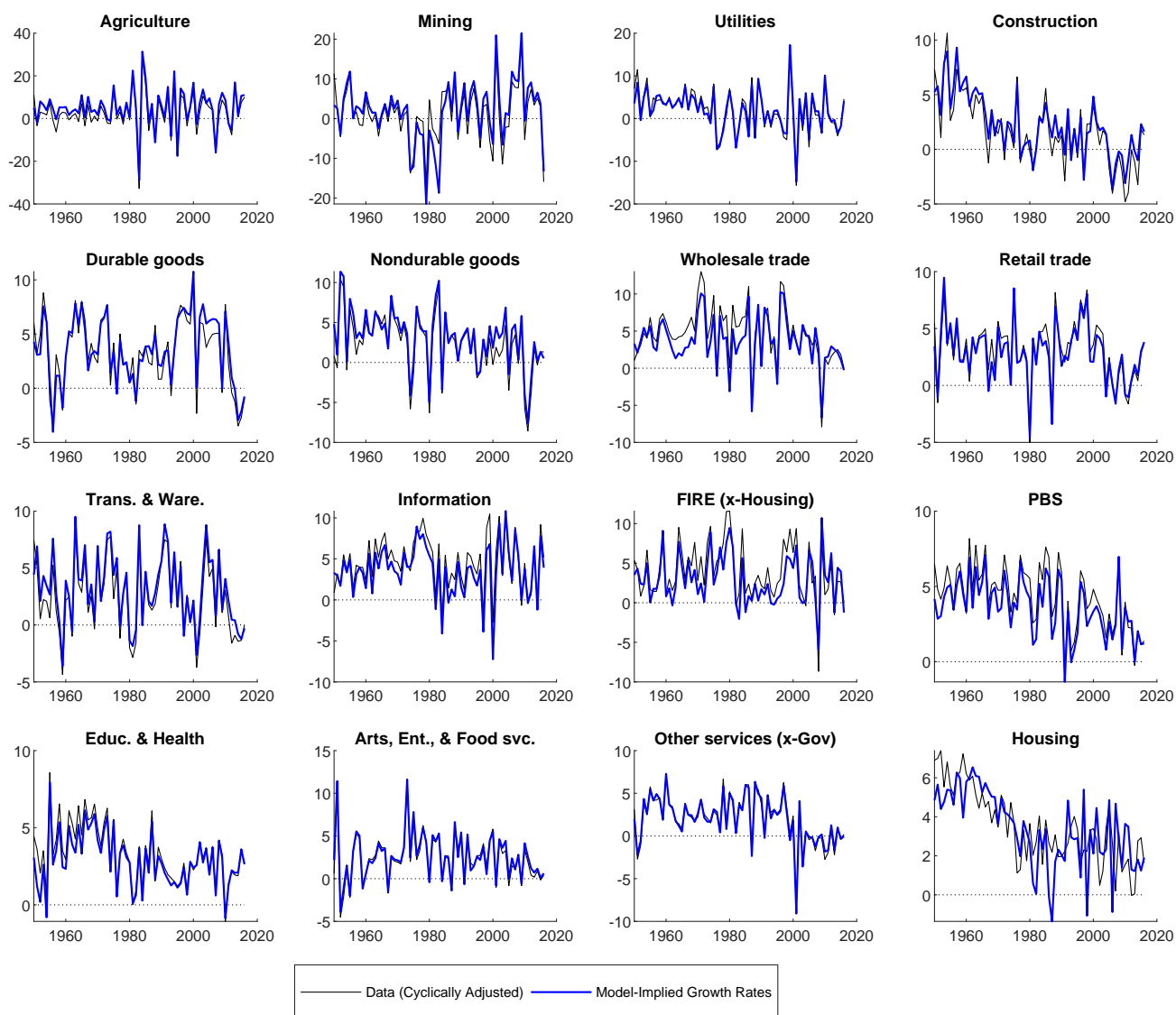
Equation (20) implies both contemporaneous and lagged effects of structural changes,  $\hat{A}_t$ , on sectoral value added growth,  $\Delta \ln v_t$ , towards balanced growth. The contemporaneous effect of a structural change in sector  $j$ ,  $\partial \Delta \ln A_{j,t}$ , on its own value added,  $\Delta \ln v_{j,t}$ , is one-for-one whereas this effect is zero on value added growth in other sectors. It follows that in the period in which a sector experiences a structural change, the effect of that change on GDP growth is entirely captured by that sector's share in GDP,  $\frac{\partial \Delta \ln V_t}{\partial \Delta \ln A_{j,t}} = s_j^v$ . Thus, a version of Hulten's theorem (1978) holds in growth rates on impact.

In subsequent periods, the change in GDP growth stemming from sectoral structural changes will reflect the network effects of production linkages,  $s^v \alpha_d \Omega' \Xi' \Delta \ln A_t$ . Along transitions, structural changes will also affect GDP growth indirectly in-so-far as they are reflected by endogenous changes in the capital stock of different sectors,  $s^v \alpha_d \Delta \hat{k}_t$ , through optimal investment decisions.

Figure 8 illustrates the behavior of value added growth in each sector,  $\Delta \ln v_{j,t}$ , in the data and in the model. As the figure makes clear, the model performs remarkably well in capturing sectoral value

<sup>26</sup>For a detailed description of this sequence of steps and all their subsequent implications for this section, see Sections 3.5 - 3.10 of the [Technical Appendix](#).

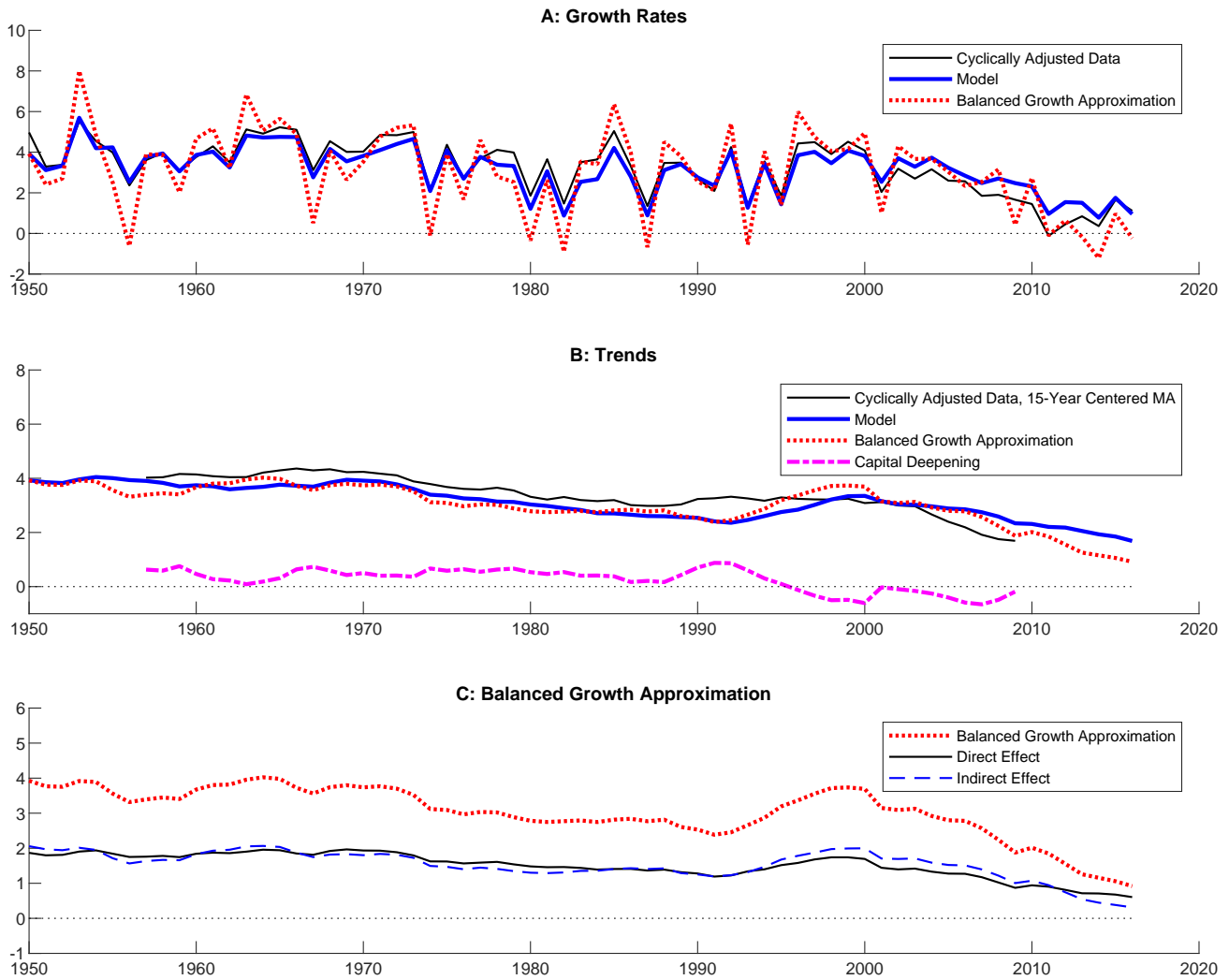
Figure 8: Sectoral Value-Added Growth Rates – Data and Model  
(Percentage points at an annual rate)



Notes: The figure shows the growth rate in value added for each sector. The model-implied value added is computed from equation (20) with values of  $\hat{A}_{j,t}$  from the DFM model.

added growth simultaneously across virtually every sector. The fit is particularly close in some key sectors highlighted above, such as Construction and Durable Goods, as well as most other sectors. The model misses somewhat on the dynamics of Wholesale Trade in the early part of the sample. In Figure 8, a downward trend is clearly noticeable in Construction more than in any other sector. This feature of the

Figure 9: GDP Growth - Data and Model



Notes: Panel (A) shows the cyclically adjusted GDP growth rate (thin black line), the model-implied GDP growth rate (solid blue line), and balanced growth approximation (dashed red line) using the estimated permanent and transitory components of TFP and labor growth. Panel (B) shows 15-year centered moving averages of the cyclically-adjusted data (thin black line) and the model-implied values of growth rates using equation (20) and the  $\tau$ -components of  $\hat{A}_{j,t}$ . The dashed lines are balanced growth approximations using equation (18) but with the disturbances to TFP and labor growth in place of  $\bar{g}_j$ . Capital deepening (magenta dash-dot line) is the difference between measured trend GDP growth and model-implied trend GDP growth, using the balanced growth approximation. Panel (C) decomposes the model-implied balanced growth approximation (red dotted line) into the direct contributions from input growth (thin black line) and their indirect effects via endogenous capital growth (dashed blue line).

data is significant in that the Construction sector was also associated with a large multiplier in the previous section.

Figure 9A illustrates the behavior of GDP growth in the model and in the data. The dotted red line depicts the growth rate of GDP as approximated by the balanced growth formula in equation (18) but driven by the full set of disturbances to TFP and labor growth in place of  $\bar{g}_j$  in that equation. In other words, the red line shows the model’s predicted GDP growth rate implied by the dynamics embodied in the exogenous driving processes but absent any internal (endogenous) dynamics from the model. The figure shows that this predicted growth rate is able to move ‘locally’ with the data. However, it is also considerably more volatile than actual GDP growth. In contrast, the blue line depicts the behavior of GDP growth in the full model as implied by the law of motion for sectoral value added growth in equation (20) together with the Markov decision rules for  $\hat{k}_t$ . This line is considerably smoother and closely matches the actual behavior of GDP growth (in black). The model underpredicts GDP growth slightly in the late 1990s and overstates the level of GDP growth somewhat in the 2000s but it matches its overall rate of decline almost exactly during the latter period.

Figure 9B illustrates the implications of the dynamic multi-sector model for trend GDP growth. Specifically, it illustrates the model’s solution when driven only by the disturbances associated with the persistent components of labor and TFP growth,  $\Delta\tau_{c,t}^x$  and  $\Delta\tau_{j,t}^x$ ,  $x = \ell, z$ . The dotted red line represents the trend growth rate implied by the balanced growth approximation (18) when driven only by these shocks. The blue line in Figure 9B is the model-implied trend growth rate of GDP taking into account the model’s internal dynamics. The black line depicts a 15–year centered moving average of actual (cyclically adjusted) GDP growth. Absent the transitory shocks, both the balanced growth approximation (the red dotted line) and the full model solution (the blue solid line) are now close. The full model indicates that trend GDP growth rate has steadily declined by about 2.2 percent over the post-war period.

## 5.2 Capital’s Contribution to Long-Run GDP Growth

As discussed above, the presence of investment hubs amplifies the aggregate effects of sector-specific changes in the trend growth rates of inputs. Thus, we now discuss the overall contribution of capital accumulation to long-run GDP growth (the black solid line in Figure 9B) given our estimates of trend growth in TFP and labor.

In the expression for GDP growth along the balanced growth path, equation (18), the direct contribution of observed input growth is  $\sum_{j=1}^n s_j^y \bar{g}_j$  while the indirect contribution of those inputs via capital growth is  $\sum_{i=1}^n \alpha_j \omega_{ij} \sum_{k=1}^n \xi_{ki} \bar{g}_k$ . As noted, the dotted red line in Figure 9B depicts an approximation of equation (18) where the constant growth rates,  $\bar{g}_k$ , are replaced by their time-varying counterparts,  $\bar{\tau}_{k,t}$ . In Figure 9C, we then decompose this model-implied trend into the direct effects of input growth and the indirect effect of those inputs through capital growth.

Figure 9C indicates that the direct and indirect contributions of input growth to the model-implied trend growth rate of GDP are roughly equal over most of the sample. Put another way, induced trend capital growth (dashed blue line) accounts for about 1/2 of GDP growth on the (time-varying) balanced growth path. In the one-sector growth model interpretation of aggregate GDP, the contribution of induced capital

growth to GDP growth on the balanced growth path is the aggregate capital income share,  $\alpha$ , or around 0.4 in our dataset.<sup>27</sup> The larger contribution of capital growth to trend GDP growth in this context stems from production linkages and represents another way of gauging the aggregate effects of sectoral multipliers.

Figure 9B also shows that for most of the sample, the model-implied trend in GDP growth understates observed long-run GDP growth as measured its 15-year centered moving average. Given our estimates of trend growth in sectoral TFP and labor, the excess of observed long-run GDP growth (the black solid line) over its model-implied trend counterpart (the red dotted line) reflects mainly an excess of actual long-run capital growth over its model-implied trend, that is capital deepening. This difference is plotted as the solid magenta line in Figure 9B. Thus, capital deepening contributed around 0.5 percentage points to long-run growth for most of the post-war period. The early 1990's saw a surge in capital deepening which reversed course shortly thereafter and collapsed in the late 1990's, well before the Great Recession. This pattern of capital accumulation is consistent with average investment rates studied by [Alexander and Eberly \(2018\)](#). Remarkably, the contributions of capital deepening to long-run growth have been negative for most of these past two decades.

### 5.3 Implications of Changing Sectoral Trends for GDP Growth

The multi-sector balanced growth model of Section 4 provides an accounting framework for decomposing the changes in the trend growth rate of GDP shown in Figure 9B into sources associated with common and sector-specific shocks. More precisely, using equation (20) and the definition of  $A_t$ , the model-based trend in GDP growth rates can be written as its balanced growth value, say  $\mu_{\Delta V}$ , and a distributed-lag of past trend shocks:

$$\Delta \ln V_t = \mu_{\Delta V} + \left[ \beta_{c,z}(L)\Delta\tau_{c,t}^z + \beta_{c,\ell}(L)\Delta\tau_{c,t}^\ell \right] + \sum_{j=1}^n \left[ \beta_{j,z}(L)\Delta\tau_{j,t}^z + \beta_{j,\ell}(L)\Delta\tau_{j,t}^\ell \right], \quad (21)$$

where the distributed lag coefficients,  $\beta(L)$ , are functions of the model parameters and the factor loadings that link the common shocks to the sectors. Figure 10 plots the historical contribution of each sector to  $\Delta \ln V_t$ , that is  $\zeta_{j,t} = \beta_{j,z}(L)\Delta\tau_{j,t}^z + \beta_{j,\ell}(L)\Delta\tau_{j,t}^\ell$ ; Figure 14 plots the contribution of the common shocks,  $\zeta_{c,t} = \beta_{c,z}(L)\Delta\tau_{c,t}^z + \beta_{c,\ell}(L)\Delta\tau_{c,t}^\ell$  (in each figure, pre-sample values of  $\Delta\tau$  are set to zero, so initial values of  $\zeta$  are equal to zero.)

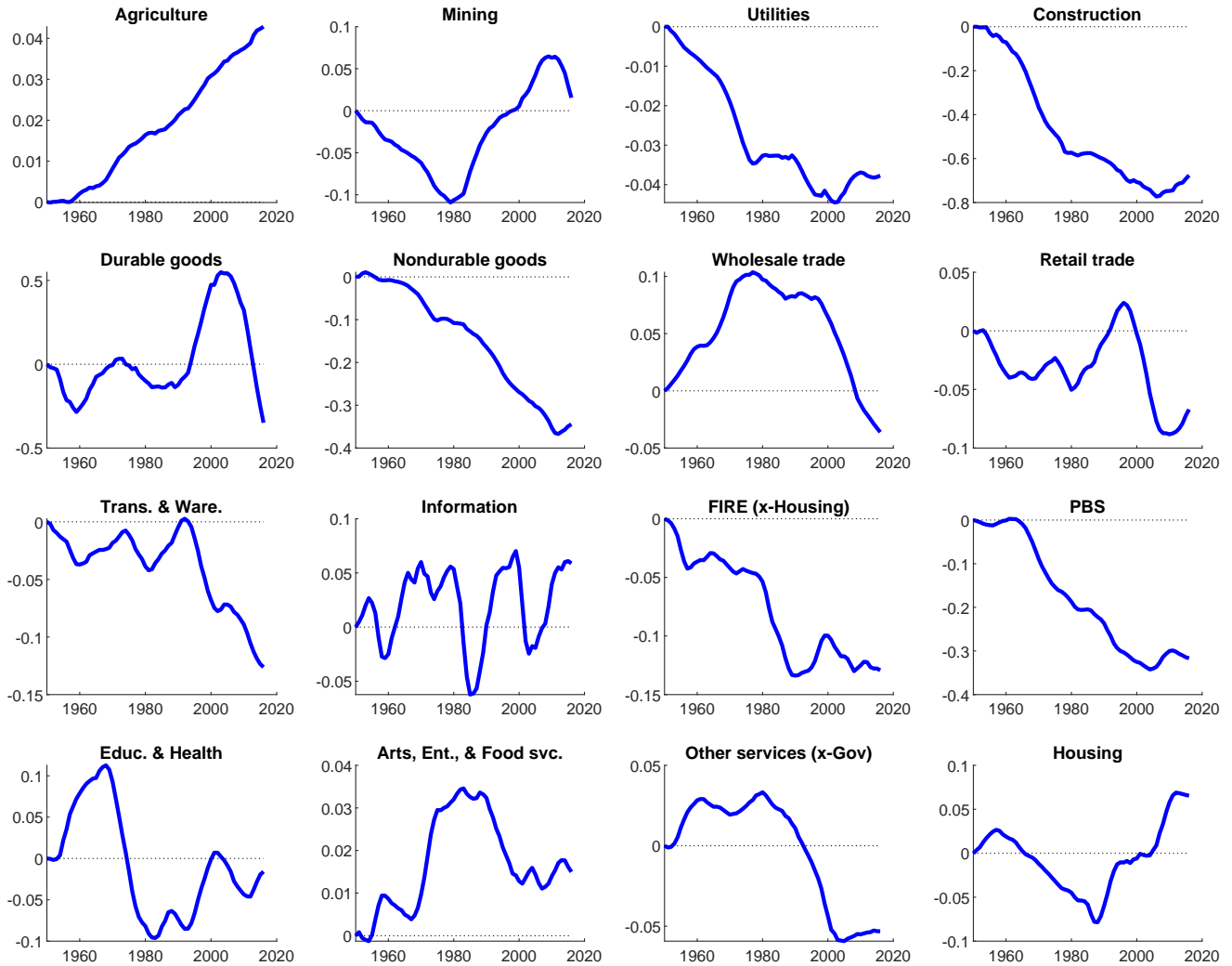
The sectoral results shown in Figure 10 suggest that, over the entire post-war period, the Construction sector is the largest single source of decline in trend GDP growth at nearly 0.7 percentage points. That is, 30 percent of the trend decline in GDP growth in the last 60 years is associated with this sector alone. Other sectors that contributed materially to the fall in trend GDP growth in the last 6 decades include Professional and Business Services and Nondurable Goods with both sectors contributing more than 0.3 percentage point loss. The Durable Goods sector contributed large decade-long up-and-down swings in

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<sup>27</sup>See Section 3.7 in the [Technical Appendix](#).



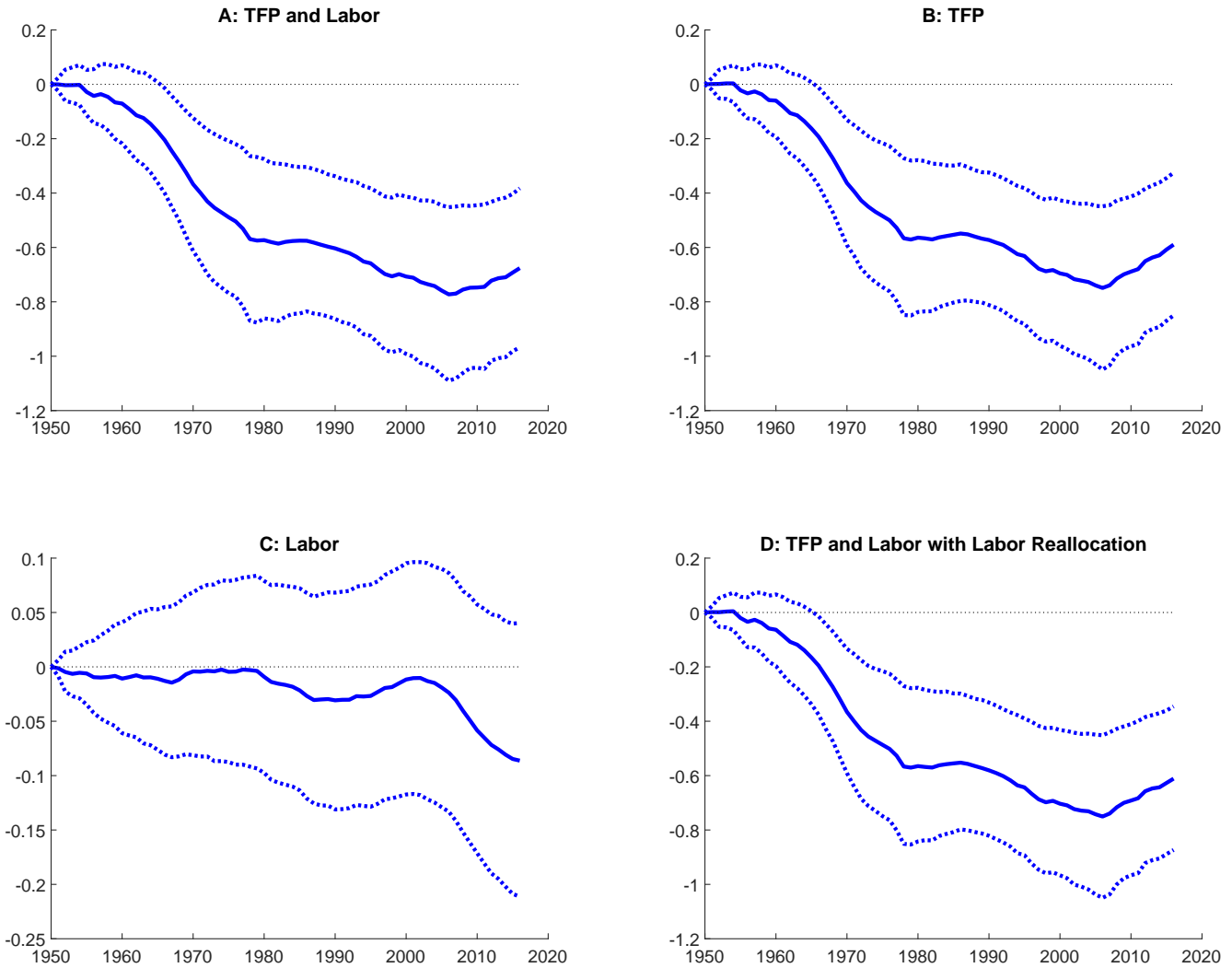
Figure 10: Model-Based Historical Decomposition of Trend GDP Growth Rates:  
Contributions from Sector-Specific Trends



Notes: These figures show the effect of the sector-specific trend shocks ( $\Delta\tau_{j,t}^z$  and  $\Delta\tau_{j,t}^\ell$ ) on the growth rate of GDP using the model-based dynamic multipliers. The trend growth rates are normalized to 0 in 1950.

trend GDP growth between 1950 and the end of the sample. The most-recent upswing in Durable Goods, contributing by itself almost 0.5 percentage points to trend GDP growth between the late 1980s and 2000, partly reflects technical progress in the production of semiconductors that was much publicized during that period. The subsequent decrease in Durable Goods accounts for nearly half (0.75 percentage points) of the

Figure 11: Model-Based Historical Decomposition of Trend Growth Rates:  
Contributions from Construction



Notes: Panels (A)-(C) show the effect of the sector-specific trend shocks ( $\Delta\tau_{j,t}^z$  and  $\Delta\tau_{j,t}^\ell$ ) on the growth rate of GDP using the model-based dynamic multipliers. Panel (D) shows these effects after reallocating  $\Delta\tau_{j,t}^\ell$  to the other sectors. Blue dots indicate 68 percent credible intervals conditional on balanced growth model parameters. The trend growth rates are normalized to 0 in 1950.

1.5 percentage point post-1999 decline in the growth rate of GDP seen in Figure 9B. Finally, some sectors, such as Information and Wholesale Trade, have somewhat offset the longer run decrease in GDP growth.

Figure 11 looks more closely at the aggregate effects of structural change in the construction sector. Panel A shows the composite effect of construction-specific changes in trend values of labor and TFP that was

shown previously in Figure 10. Panels B and C show the separate effects from labor ( $\beta_{j,\ell}(L)\Delta\tau_{j,t}^\ell$ ) and TFP ( $\beta_{j,z}(L)\Delta\tau_{j,t}^z$ ). Evidently, decreases in the rate of growth of TFP in the construction sector explain most of its deleterious effect on trend GDP growth. Of course, one interpretation of these historical decompositions is as counterfactual paths of GDP growth setting all other shocks to zero. In this interpretation, Panel A of Figure 11 shows the path of the trend growth rate of GDP under the counterfactual absence of changes in common and sector-specific shocks other than construction ( $\Delta\tau_{c,t}^z = \Delta\tau_{c,t}^\ell = \tau_{j,t}^z = \Delta\tau_{j,t}^\ell = 0$  for  $j \neq$  Construction). This interpretation of Figure 11 raises the question of how to account for the reduction in labor associated with  $\Delta\tau_{j,t}^\ell$  in the construction sector. Did it disappear, as implicitly assumed in Panels A and C, or did it move to other sectors? Panel D shows the counterfactual historical path of trend growth rates in GDP after allocating all of the sectoral losses of labor in Construction to the other sectors in proportion to their labor shares, so there is no change in the aggregate supply of labor. Because the behavior of labor growth is not the dominant force driving secular changes in the Construction sector, panels A and D show similar historical paths.

Figures 12 and 13 repeat this exercise for two other important sectors: Nondurable Goods and Professional and Business Services. Taken together, Figures 12 and 13 underscore the importance of sector-specific structural change for trend GDP growth. Remarkably, the Construction, Nondurable Goods, and Professional and Business Services sectors account for around 60 percent of the 2.2 percentage point decrease in trend GDP growth since 1950.

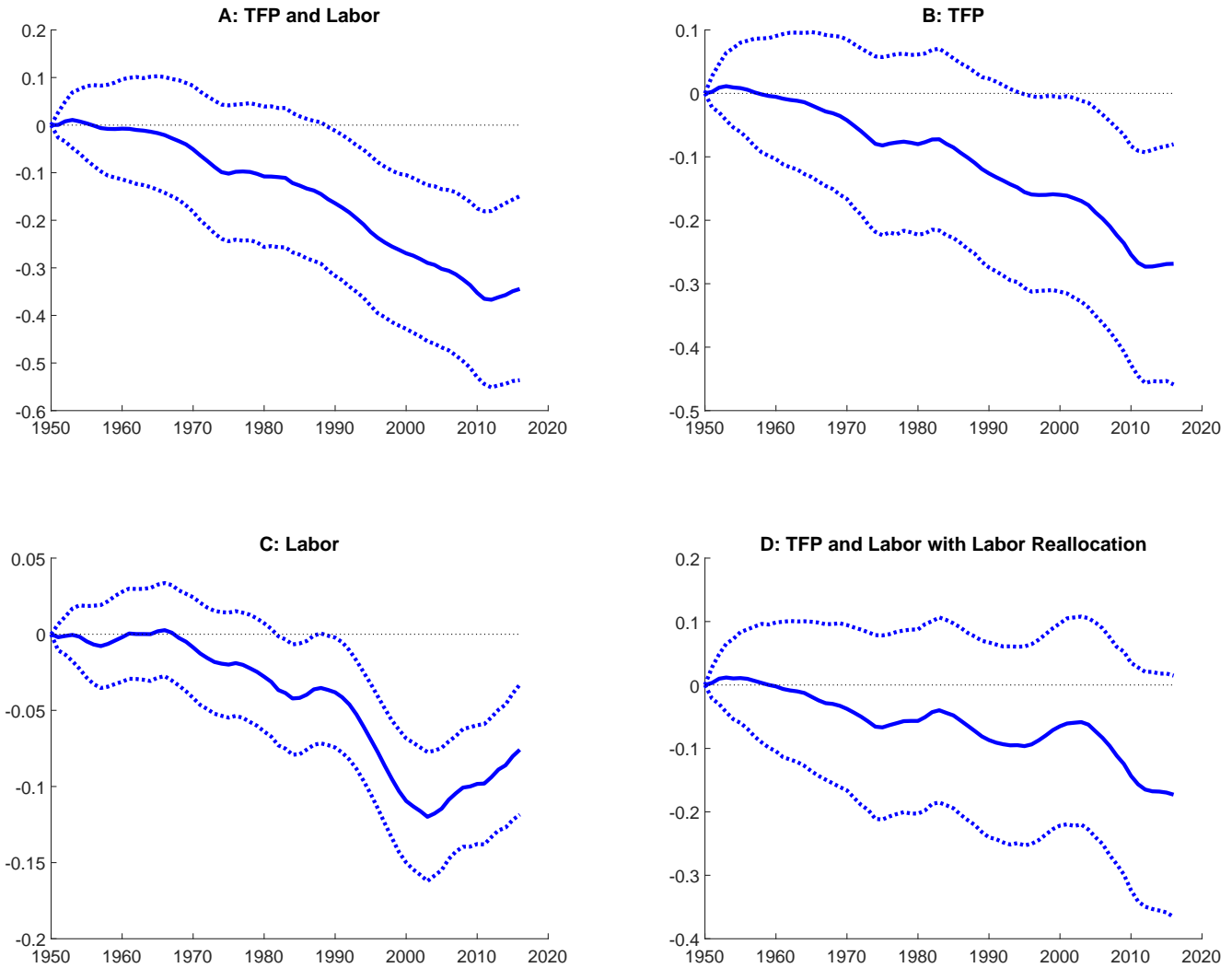
In contrast, Figure 14 illustrates the implications of the common or aggregate trends in labor and TFP growth estimated in Section 4. Figure 14 suggests that these common trends have not made a material contribution to the aggregate trend decline in GDP growth since 1950. Thus, sector-specific rather than aggregate factors shaped the major part of the secular behavior of GDP growth over the period 1950 – 2016.

## 5.4 Implications for Future Trend Growth

Given the trends in labor growth and TFP growth estimated in this paper, and their implications for the aggregate U.S. economy through sectoral multipliers, a natural question is: what do those trends imply going forward? As indicated by equation (20), disturbances to trend labor growth and TFP growth are endogenously propagated over time through investment decisions and thus have persistent effects. The random walk dynamics of the trend components of labor growth and TFP growth imply that forecasts of these components are equal to their value in 2016, the last year of the data.

Figure 15 shows predicted trend growth rates for GDP from the dynamic multi-sector model as past disturbances play out through the model’s internal dynamics. The figure indicates that absent the realization of positive and persistent disturbances to TFP and labor growth, trend GDP growth will continue to fall by around 0.60 percentage points over the next 10 years. In this scenario, both trend labor and TFP growth remain constant, but the internal dynamics of capital accumulation imply continued slowing of trend GDP growth. The 68 percent credible bands indicate there is substantial uncertainty about the exact level of trend GDP going forward but, absent positive shocks, the model predicts a slowdown ahead nonetheless.

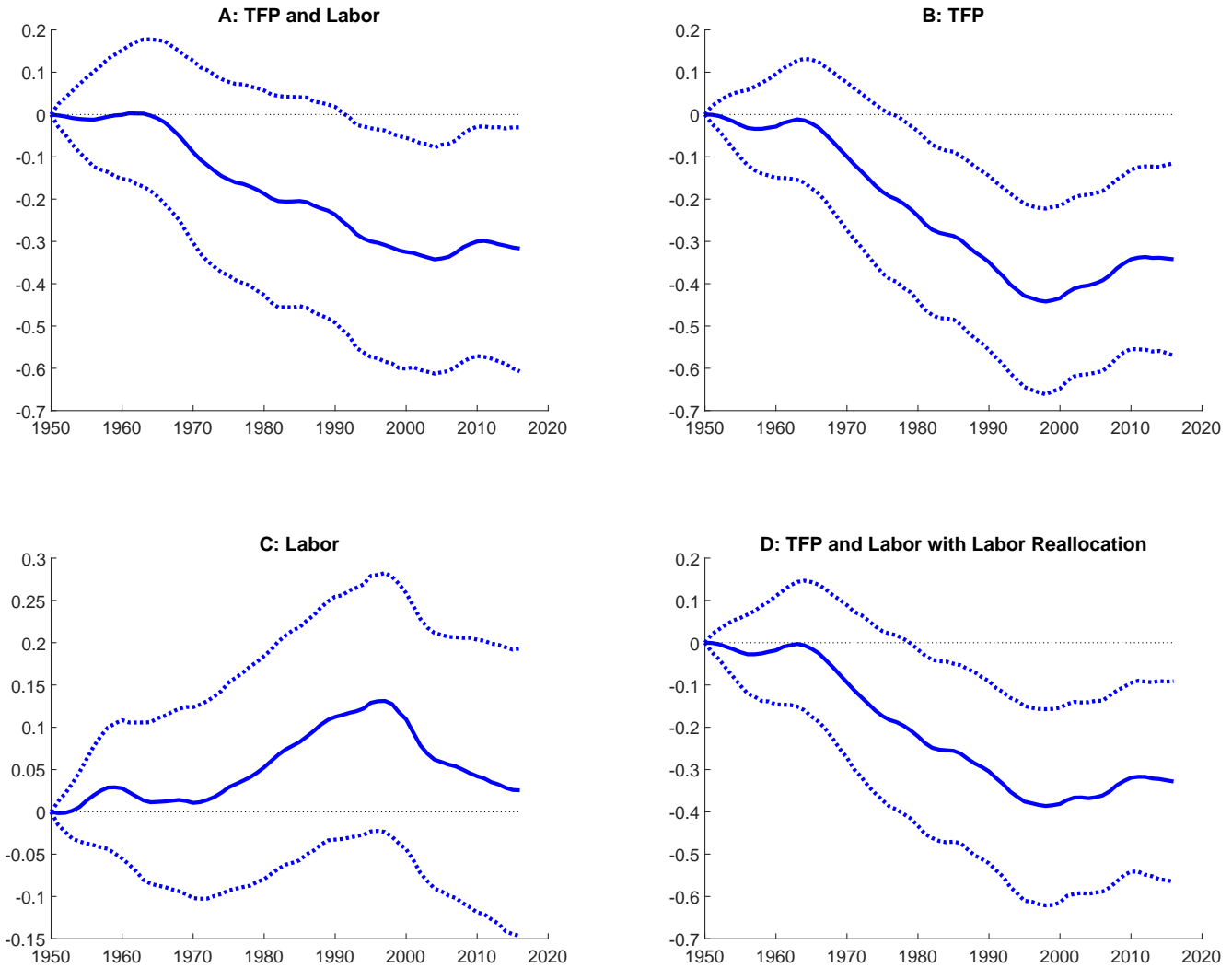
Figure 12: Model-Based Historical Decomposition of Trend Growth Rates:  
Contributions from Nondurable Goods



Notes: See notes to Figure 11.

Table 6 shows the contributions of the common and sector-specific trend components in generating the decline in trend GDP growth of around 0.65 percentage points going forward. While there is a role for common trends, they play a smaller role than the sector-specific trends, particularly in TFP growth. The decomposition into each individual sector shows that a decline in Durable Goods is the key driver of lower trend GDP growth in the future. The fall in large part reflects the propagation of the collapse associated with the Durable Goods sector that began in 2000 (see Figure 10). This effect is mitigated somewhat by

Figure 13: Model-Based Historical Decomposition of Trend Growth Rates:  
Contributions from Professional and Business Services

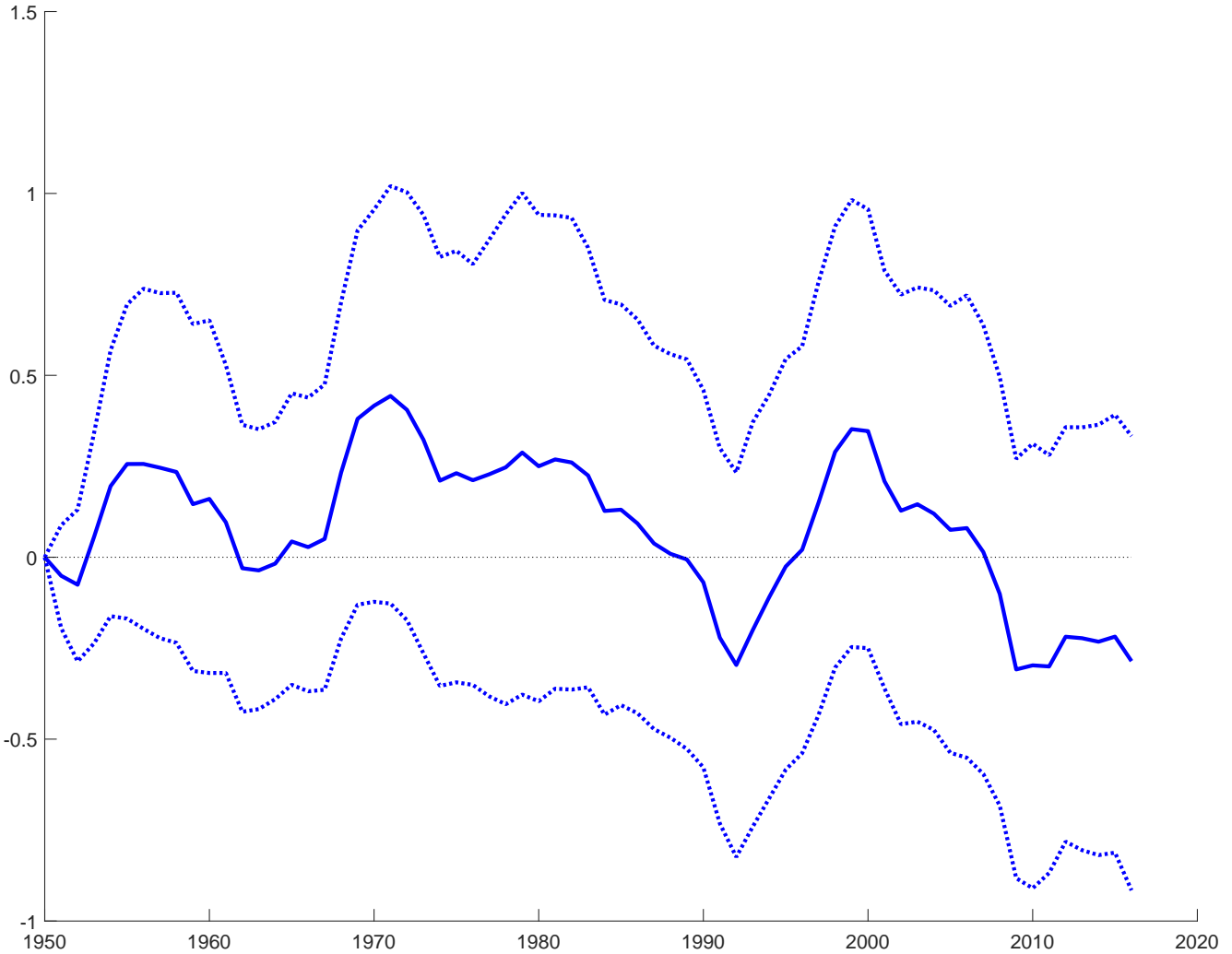


Notes: See notes to Figure 11.

a turn-around in Construction. However, the marked slowdown in Durable Goods affects other sectors through production linkages, and through capital accumulation implies a pervasive impact that is sustained 10 years into the future.<sup>28</sup>

<sup>28</sup>Of course, our data stop in 2016 and given the recent pandemic, any persistent effects of the crisis on growth would only lower our estimates of trend growth going forward.

Figure 14: Model-Based Historical Decomposition of Trend Growth Rates:  
Contributions from Common Trends

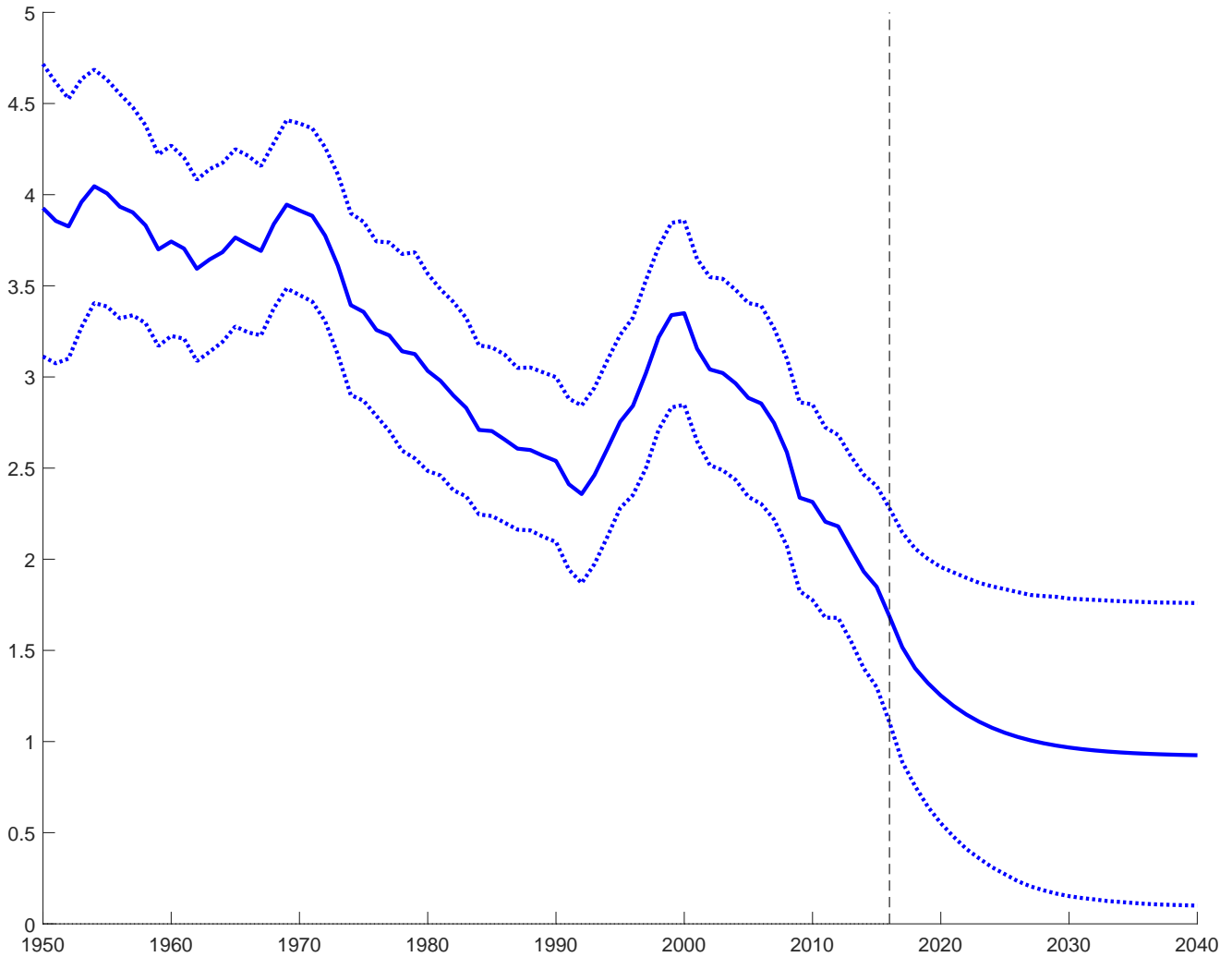


Notes: This figure show the effect of the common trend shocks ( $\Delta\tau_{c,t}^z$  and  $\Delta\tau_{c,t}^\ell$ ) on the growth rate of GDP using the model-based dynamic multipliers. Blue dots indicate 68 percent credible intervals conditional on balanced growth model parameters. The trend growth rates are normalized to 0 in 1950.

## 6 Concluding Remarks

In this paper, we estimate trends in TFP and labor growth across major US production sectors and explore the role they have played in shaping the secular behavior of GDP growth. We find that trends in TFP and labor growth have generally decreased across a majority of sectors since 1950. More than 3/4 of the

Figure 15: In-Sample and Predicted Out-of-Sample Trend Growth Rate of GDP



Notes: Blue dots indicate 68 percent credible intervals conditional on balanced growth model parameters.

secular decline in aggregate TFP growth results from the combination of sector-specific rather than aggregate disturbances. Similarly, trend labor growth has also been dominated by sector-specific factors, especially after 1980 and the latter part of the post-war period.

We embed these findings into a dynamic multi-sector framework in which materials and capital used by different sectors are produced by other sectors. The presence of capital, in particular, allows changes in TFP or labor growth in a given sector to affect value added growth in every other sector. This feature leads to quantitatively important sectoral multiplier effects on GDP growth that reflect the importance of

Table 6: Model-based Forecasts of Changes in Trend GDP Growth

	2016 – 2021			2021 – 2026		
	VA	Labor	TFP	VA	Labor	TFP
Aggregate	-0.49	-0.15	-0.34	-0.16	-0.03	-0.13
Common	-0.16	-0.05	-0.11	-0.03	0.00	-0.04
Sector specific (total)	-0.33	-0.10	-0.23	-0.13	-0.03	-0.10
Sector specific ( <i>by sector</i> )						
Agriculture	-0.00	-0.00	0.00	-0.00	-0.00	-0.00
Mining	-0.02	-0.01	-0.01	-0.01	-0.00	-0.00
Utilities	-0.00	-0.00	0.00	-0.00	-0.00	0.00
Construction	0.02	-0.01	0.02	0.01	-0.00	0.01
Durable goods	-0.31	-0.08	-0.22	-0.12	-0.03	-0.09
Nondurable goods	0.00	0.00	0.00	0.00	-0.00	0.00
Wholesale trade	-0.01	-0.00	-0.00	-0.00	-0.00	-0.00
Retail trade	0.00	0.00	0.00	-0.00	0.00	-0.00
Trans. & Ware.	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Information	-0.00	-0.00	0.00	0.00	-0.00	0.00
FIRE (x-Housing)	-0.00	0.00	-0.00	-0.00	0.00	-0.00
PBS	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Educ. & Health	-0.00	0.01	-0.01	-0.00	0.00	-0.01
Arts, Ent., & Food svc.	-0.00	-0.00	0.00	0.00	-0.00	0.00
Oth. serv. (x-Gov)	0.00	0.00	0.00	0.00	0.00	0.00
Housing	-0.01	0.00	-0.01	-0.00	0.00	-0.00

different sectors as suppliers of capital or materials to other sectors. The strength of these linkages result in GDP growth multipliers that for some sectors can be as large 3 times their value added share.

Ultimately, sector-specific rather than aggregate factors in TFP and labor growth explain the major part of low frequency variations in U.S. GDP growth. Changing sectoral trends in the last 6 decades, translated through the economy's production network, have on net lowered trend GDP growth by around 2.2 percentage points. The Construction sector, more than any other sector, stands out for its contribution to the trend decline in GDP growth over the post-war period, accounting for 30 percent of this decline. Moreover, the process of capital accumulation means that these structural changes have endogenously persistent effects. Thus, absent the realization of predominantly positive and persistent disturbances to TFP and labor growth,



we estimate that trend GDP growth will continue to fall over the next 10 years.

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## Appendix A: Estimation of the Multivariate Unobserved Components Model

The DFM has the form:  $\Delta \ln x_{j,t} = \lambda_{j,\tau}^x \tau_{c,t}^x + \lambda_{j,\varepsilon}^x \varepsilon_{c,t}^x + \tau_{j,t}^x + \varepsilon_{j,t}^x$ , where  $x = z$  or  $\ell$  and  $j = 1, \dots, n$ . The  $\tau$  components evolve as random walks and the  $\varepsilon$  components as white noise. More specifically, we assume that the  $2(n+1) \times 1$  vector  $\zeta_t = \left( \Delta \tau_{c,t}^x, \varepsilon_{c,t}^x, \{\Delta \tau_{j,t}^x, \varepsilon_{j,t}^x\}_{j=1}^n \right)$  is *i.i.d.* with  $\zeta \sim N(0, \Sigma_\zeta)$ , where  $\Sigma_\zeta$  is a diagonal matrix. The factor loadings and scale of the common factors are not separately identified; thus we normalize  $var(\Delta \tau_{c,t}^x) = var(\varepsilon_{c,t}^x) = 1$ ,  $\sum_{j=1}^n \lambda_{j,\tau}^x \geq 0$ , and  $\sum_{j=1}^n \lambda_{j,\varepsilon}^x \geq 0$  to identify the scale and sign of the factors.

We estimate the model using Bayesian methods. We use independent priors for all the parameters. The priors for the factor loadings are  $\lambda_{j,\tau}^x \sim N(0, 1)$  and  $\lambda_{j,\varepsilon}^x \sim N(0, 16)$ . The priors for  $var(\Delta \tau_{j,t}^x)$  and  $var(\varepsilon_{j,t}^x)$  are inverse Gamma priors with shape and scale parameters that correspond to  $T_{prior}$  observations with  $s_{prior}^2 = \omega_{prior}^2 / T_{prior}$ . We use  $T_{prior} = 2$  and  $s_{prior}^2 = 0.25^2$  for  $var(\Delta \tau_{j,t}^x)$  and  $T_{prior} = 2$  and  $s_{prior}^2 = 3.00^2$  for  $var(\varepsilon_{j,t}^x)$ .

The posterior is approximated using Gibbs-MCMC methods. Let  $X$  denote the data,  $\zeta$  denote the values for  $\zeta_t$  for  $t = 1, \dots, T$ , and  $\theta$  denote the parameter values  $(\lambda_{j,\tau}^x, \lambda_{j,\varepsilon}^x, var(\Delta \tau_{j,t}^x), var(\varepsilon_{j,t}^x))$  for  $j = 1, \dots, n$ . The first Gibbs step draws  $\zeta | (X, \theta)$  using standard formulae related to the Kalman smoother. The second Gibbs step draws  $\theta$  from the posterior of  $\theta | (X, \zeta)$  that is also known in closed form from familiar linear regression formulae. The estimates reported in the paper rely on one million MCMC draws after 5k initial draws.

Section 2 of the [Technical Appendix](#) contains additional details.

## Appendix B: The Influence Vector in a Model with Capital

The competitive equilibrium of the economy described in the main text is defined as a set of gross output prices,  $p_{j,t}^y$ , value added prices,  $p_{j,t}^v$ , materials prices,  $p_{j,t}^m$ , and investment goods prices,  $p_{j,t}^x$ , for all sectors  $j$  and dates  $t$ , and allocations,  $(c_{j,t}, C_t, x_{ij,t}, m_{ij,t}, x_{j,t}, m_{j,t}, v_{j,t}, y_{j,t}, k_{j,t+1}, \ell_{j,t})$ , such that

- i) the representative household maximizes its utility,
- ii) the representative firm in each sector maximizes profits
- iii) all markets clear.

Absent frictions, equilibrium allocations may be found by solving a planner's problem whose corresponding decentralization and definitions of prices are straightforward.

Suppose that sectoral structural changes embodied in  $\{z_{j,t}, \ell_{j,t}\}$  are stationary in levels, with non-stochastic steady state denoted by  $\{z_j, \ell_j\}$ . Then, in the absence of shocks, the economy converges to a steady state in the long-run indexed by  $A = (A_1, \dots, A_n)$ , where  $\ln A_j = \ln z_j + (1 - \alpha_j) \ln \ell_j$ ,  $j = 1, \dots, n$ .

In the steady state, first-order conditions imply a set of relationships between the prices of materials, investment, and value added,

$$\Theta \ln p^y = 0,$$

$$\begin{aligned}\ln p^v &= \Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] \ln p^y, \\ \ln p^x &= \Omega' \ln p^y,\end{aligned}$$

where  $p^v = (p_1^v, \dots, p_n^v)'$ ,  $p^x = (p_1^x, \dots, p_n^x)'$ , and  $p^y = (p_1^y, \dots, p_n^y)'$ .

From the definition of value added, we have that

$$\ln v_j = \ln A_j + \alpha_j \ln \left( \frac{k_j}{\alpha_j} \right),$$

and from the Euler equation governing the optimal choice of capital in each sector,

$$\frac{k_j}{\alpha_j} = \left( \frac{p_j^v v_j}{p_j^x} \right) \left( \frac{\beta}{1 - \beta(1 - \delta_j)} \right).$$

Combining these expressions gives

$$\ln v_j = \ln A_j + \alpha_j \ln (p_j^v v_j) - \alpha_j \ln p_j^x + \alpha_j \ln \left( \frac{\beta}{1 - \beta(1 - \delta_j)} \right),$$

or in matrix form,

$$(I - \alpha_d) \ln (p^v \cdot v) = \ln A + \ln p^v - \alpha_d \ln p^x + \alpha_d \ln \Delta_d,$$

where  $(p^v \cdot v)$  represents the vector of nominal value added,  $\{p_j^v v_j\}$ ,  $\alpha_d = \text{diag}(\alpha_j)$ , and  $\Delta_d = \text{diag} \left( \frac{\beta}{1 - \beta(1 - \delta_j)} \right)$ . Substituting for investment and value added prices on the right-hand-side of this last expression, we obtain

$$\ln p^y = (\Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] - \alpha_d \Omega')^{-1} [(I - \alpha_d) \ln (p^v \cdot v) - \ln A - \alpha_d \ln \Delta_d]. \quad (\text{B.1})$$

From the resource constraints in each sector  $j$ , and the optimal allocation of materials and investment in the economy, it follows that

$$\frac{p_j^v v_j}{\gamma_j} = \theta_j C + \sum_{i=1}^n \phi_{ji} (1 - \gamma_j) \frac{p_i^v v_i}{\gamma_i} + \sum_{i=1}^n \omega_{ji} \frac{\beta \delta_j}{1 - \beta(1 - \delta_j)} \alpha_i p_i^v v_i,$$

or in matrix form,

$$\Gamma_d^{-1} (p^v \cdot v) = \Theta' C + \Phi (I - \Gamma_d) \Gamma_d^{-1} (p^v \cdot v) + \Omega \Delta_d \delta_d \alpha_d (p^v \cdot v), \quad (\text{B.2})$$

so that

$$\frac{(p^v \cdot v)}{C} = ([I - \Phi(I - \Gamma_d)] \Gamma_d^{-1} - \Omega \Delta_d \delta_d \alpha_d)^{-1} \Theta'.$$

Aggregate GDP (in units of the final consumption bundle) is then given by

$$V = \mathbf{1}'(p^v \cdot v) = \mathbf{1}'\psi C,$$

where  $\psi = ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d)^{-1}\Theta'$ .

Substituting the expression for value added in (B.2) into the equation for gross output prices, (B.1), and using the fact that the ideal price index for the final consumption bundle implies  $\Theta \ln p^y = 0$ , we obtain

$$\ln C = \frac{\Theta (\Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] - \alpha_d\Omega')^{-1} [\ln A + \alpha_d \ln \Delta_d - (I - \alpha_d) \ln \psi]}{\Theta (\Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] - \alpha_d\Omega')^{-1} (I - \alpha_d)\mathbf{1}},$$

where note that  $\Theta (\Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] - \alpha_d\Omega')^{-1} (I - \alpha_d)\mathbf{1} = 1$ .

It follows that

$$\frac{\partial \ln V}{\partial \ln A_j} = \frac{\partial \ln C}{\partial \ln A_j} = \left\{ ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\alpha_d)^{-1} \Theta' \right\}_j.$$

At the same time, the vector of value added shares,  $s^v$ , is given by

$$\frac{(p^v \cdot v)}{V} = \frac{([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d)^{-1} \Theta'}{\mathbf{1}' ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d)^{-1} \Theta'}.$$

Therefore, in the limit case where  $\beta \rightarrow 1$ ,  $\Delta_d\delta_d \rightarrow I$ , and

$$\frac{\partial \ln V}{\partial \ln A_j} = \eta s_j^v,$$

where  $\eta = \mathbf{1}' ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\alpha_d)^{-1} \Theta'$  is approximately the inverse of the mean labor share across sectors. To see this, observe that  $\eta$  can also be expressed as  $\eta = \frac{\mathbf{1}' ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\alpha_d)^{-1} \Theta'}{\mathbf{1}' (I - \alpha_d) ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\alpha_d)^{-1} \Theta'}$ . Thus, when  $\alpha_j = \alpha \forall j$ ,  $\eta = \frac{1}{1-\alpha}$ . See section 3 of the [Technical Appendix](#) for a full description of the problem and solution.

## Appendix C: Balanced Growth with Sectoral Linkages in Materials and Investment

We begin by normalizing some of the model's key variables with respect to sector-specific factors,  $\mu_{j,t}$ , to be determined below, so as to yield a system of equations that is stationary in the normalized variables. If all growth rates are constant, the resource constraint in any individual sector implies that all variables in that constraint must grow at the same rate along a balanced growth path. Thus, define  $\tilde{y}_{j,t} = y_{j,t}/\mu_{j,t}$ ,

$\tilde{c}_{j,t} = c_{j,t}/\mu_{j,t}$ ,  $\tilde{m}_{ji,t} = m_{ji,t}/\mu_{j,t}$ , and  $\tilde{x}_{ji,t} = x_{ji,t}/\mu_{j,t}$ . Then, the economy's resource constraint becomes

$$\tilde{c}_{j,t} + \sum_{i=1}^n \tilde{m}_{ji,t} + \sum_{i=1}^n \tilde{x}_{ji,t} = \tilde{y}_{j,t}.$$

Similarly, the production of investment goods may be re-written as

$$\tilde{x}_{j,t} = \prod_{j=1}^n \left( \frac{\tilde{x}_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}},$$

where  $\tilde{x}_{j,t} = x_{j,t}/\prod_{j=1}^n \mu_{i,t}^{\omega_{ij}}$ , while the capital accumulation equation becomes

$$\tilde{k}_{j,t+1} = \tilde{x}_{j,t} + (1 - \delta_j) \tilde{k}_{j,t} \prod_{j=1}^n \left( \frac{\mu_{i,t-1}}{\mu_{i,t}} \right)^{\omega_{ij}},$$

where  $\tilde{k}_{j,t+1} = k_{j,t+1}/\prod_{j=1}^n \mu_{i,t}^{\omega_{ij}}$ .

The expression for value added may be written as

$$v_{j,t} = A_{j,t} \left( \frac{\tilde{k}_{j,t} \prod_{j=1}^n \mu_{i,t-1}^{\omega_{ij}}}{\alpha_j} \right)^{\alpha_j},$$

so that, defining  $\tilde{v}_{j,t} = v_{j,t}/A_{j,t} \left( \prod_{j=1}^n \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}$ , it becomes

$$\tilde{v}_{j,t} = \left( \frac{\tilde{k}_{j,t}}{\alpha_j} \right)^{\alpha_j}.$$

The composite bundle of materials used in sector  $j$  may be expressed as

$$\tilde{m}_{j,t} = \prod_{j=1}^n \left( \frac{\tilde{m}_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}},$$

with  $\tilde{m}_{j,t} = m_{j,t}/\prod_{j=1}^n \mu_{i,t}^{\phi_{ij}}$ . Therefore, gross output in terms of detrended variables is given by

$$\tilde{y}_{j,t} \mu_{j,t} = \left( \frac{\tilde{v}_{j,t} A_{j,t} \prod_{j=1}^n \mu_{i,t-1}^{\alpha_j \omega_{ij}}}{\gamma_j} \right)^{\gamma_j} \left( \frac{\tilde{m}_{j,t} \prod_{j=1}^n \mu_{i,t}^{\phi_{ij}}}{1 - \gamma_j} \right)^{1 - \gamma_j},$$

or alternatively,

$$\tilde{y}_{j,t} = \left( \frac{\tilde{v}_{j,t}}{\gamma_j} \right)^{\gamma_j} \left( \frac{\tilde{m}_{j,t}}{1 - \gamma_j} \right)^{1 - \gamma_j} \left[ \frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \prod_{j=1}^n \mu_{i,t-1}^{\gamma_j \alpha_j \omega_{ij}} \mu_{i,t}^{(1 - \gamma_j) \phi_{ij}} \right].$$



We can now use the expression in square brackets to solve for the normalizing factors,  $\mu_{j,t}$ , as a function of the model's underlying parameters.

First, re-write the term in square brackets as

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \left( \prod_{j=1}^n \frac{\mu_{i,t-1}^{\gamma_j \alpha_j \omega_{ij}}}{\mu_{i,t}^{\gamma_j \alpha_j \omega_{ij}}} \right) \left( \prod_{j=1}^n \mu_{i,t}^{(1-\gamma_j)\phi_{ij}} \mu_{i,t}^{\gamma_j \alpha_j \omega_{ij}} \right),$$

where this last expression involves the growth rate of  $\mu_{i,t}$ . Then, along a balanced growth path where all variables grow at constant rates, the term  $\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \prod_{j=1}^n \mu_{i,t}^{(1-\gamma_j)\phi_{ij}} \mu_{i,t}^{\gamma_j \alpha_j \omega_{ij}}$  must be constant. Thus, we choose  $\mu_{j,t}$  such that

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \prod_{j=1}^n \mu_{i,t}^{\gamma_j \alpha_j \omega_{ij} + (1-\gamma_j)\phi_{ij}} = 1.$$

This choice is without loss of generality since any other constant in place of 1 will be differenced out along the balanced growth path.

Taking logs of both sides of the above expression, we have

$$\gamma_j \ln A_{j,t} - \ln \mu_{j,t} + \sum_{i=1}^n (\gamma_j \alpha_j \omega_{ij} + (1 - \gamma_j) \phi_{ij}) \ln \mu_{i,t} = 0,$$

or in vector form,

$$\Gamma_d \ln A_t - \ln \mu_t + \Gamma_d \alpha_d \Omega' \ln \mu_t + (I - \Gamma_d) \Phi' \ln \mu_t = 0,$$

which gives us

$$\ln \mu_t = \Xi' \ln A_t \tag{C.1}$$

where

$$\Xi' = (I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d) \Phi')^{-1} \Gamma_d$$

is the Leontief inverse,  $(I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d) \Phi')^{-1}$ , diagonally weighted by value added shares in gross output,  $\Gamma_d$ .

Going back to equation (13) in the main text, and writing the vector of productivity growth rates as  $\Delta \ln A_t = \bar{g}_a$ , it follows that

$$\Delta \ln \mu_t = \Xi' \bar{g}_a = \Xi' (\bar{g}_z + (I - \alpha_d) \bar{g}_\ell).$$

Thus, let  $\mu_{j,t}^v$  denote the normalizing factor for value added in sector  $j$  defined above,

$$\mu_{j,t}^v = A_{j,t} \left( \prod_{j=1}^n \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}.$$

Then, using equation (C.1), we have that

$$\ln \mu_t^v = \ln A_t + \alpha_d \Omega' \Xi' \ln A_{t-1},$$

or, along a steady state growth path,

$$\Delta \ln \mu_t^v = \left[ I + \alpha_d \Omega' \underbrace{(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d) \Phi')^{-1} \Gamma_d}_{\Xi'} \right] \bar{g}_a. \quad (\text{C.2})$$

See section 3 of the [Technical Appendix](#) for a full description of the problem and solution.

## Appendix D: The Dynamics of Value Added Growth

We wish to recover the implications of the model for sectoral value added growth,  $\Delta \ln v_{j,t}$ , and aggregate GDP growth,  $\Delta \ln V_t$ .

Observe that

$$\Delta \ln \tilde{v}_{j,t} = \alpha_j \Delta \ln \tilde{k}_{j,t},$$

and that  $\Delta \ln \tilde{k}_{j,t} = \Delta \hat{k}_{j,t}$ , where  $\Delta \hat{k}_{j,t}$  follows directly from the Markov decision rules. Then, by definition of  $\tilde{v}_{j,t}$ ,

$$\begin{aligned} \Delta \ln v_{j,t} &= \Delta \ln \tilde{v}_{j,t} + \Delta \ln A_{j,t} + \sum_{i=1}^n \alpha_j \omega_{ij} \Delta \ln \mu_{i,t-1} \\ &= \alpha_j \Delta \hat{k}_{j,t} + \hat{A}_{j,t} + \bar{g}_j + \sum_{i=1}^n \alpha_j \omega_{ij} \Delta \ln \mu_{i,t-1}, \end{aligned}$$

or, in vector form,

$$\Delta \ln v_t = \alpha_d \Delta \hat{k}_t + \hat{A}_t + \bar{g}_a + \alpha_d \Omega' \Delta \ln \mu_{t-1}.$$

The sectoral detrending factors,  $\mu_{j,t}$ , solve

$$\Delta \ln \mu_t = \Xi' \Delta \ln A_t,$$

where

$$\Xi' = (I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d) \Phi')^{-1} \Gamma_d.$$

Then,

$$\Delta \ln v_t = \alpha_d \Delta \hat{k}_t + \hat{A}_t + \bar{g}_a + \alpha_d \Omega' \Xi' (\hat{A}_{t-1} + \bar{g}_a)$$

or

$$\Delta \ln v_t = (I + \alpha_d \Omega' \Xi') \bar{g}_a + \alpha_d \Delta \hat{k}_t + \hat{A}_t + \alpha_d \Omega' \Xi' \hat{A}_{t-1},$$

with  $\Delta \ln V_t = s^v \Delta \ln v_t$ . When all shocks are set to zero, so that  $\Delta \hat{k}_t = \hat{A}_t = \hat{A}_{t-1} = 0$ , we recover the sectoral balanced growth paths for sectoral value added,

$$\Delta \ln v_t = (I + \alpha_d \Omega' \Xi') \bar{g}_a.$$