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Aggregate Implications of Changing Sectoral Trends*

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Abstract

We find disparate trend variations in TFP and labor growth across major U.S. production sectors and study their implications for the post-war secular decline in GDP growth. Capital accumulation and the network structure of U.S. production amplify the effects of sector-specific changes in the trend growth rates of TFP and labor on trend GDP growth. We summarize this amplification effect in terms of sectoral multipliers that, for some sectors, can exceed 3 times their value added shares in the economy. We estimate that sector-specific factors have historically accounted for approximately 3/4 of long-run changes in GDP growth, leaving common or aggregate factors to explain only 1/4 of those changes. Trend GDP growth fell by nearly 3 percentage points over the post-war period with the Construction sector alone contributing roughly 1 percentage point of that decline between 1950 and 1980. Idiosyncratic changes to trend growth in the Durable Goods sector then contributed an almost 2 percentage point decline in trend GDP growth between 2000 and the end of our sample in 2018. Remarkably, no sector has contributed any steady significant increase to the trend growth rate of GDP in the past 70 years.

Keywords: trend growth, sectoral linkages, investment network
JEL Codes: C32, E23, O41

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1 Introduction

Following the so-called Great Recession of 2008-2009, U.S. GDP recovered only very gradually which resulted in a conspicuously low average growth rate in the ensuing decade. Fernald, Hall, Stock, and Watson (2017) found this weak recovery stemmed mainly from slow growth in total factor productivity (TFP) and a fall in labor input, and note that these adverse forces preceded the Great Recession. Antolin-Diaz, Drechsel, and Petrella (2017) likewise document a slowdown in output growth that predates the Great Recession.[1] This paper studies what has in fact been a steady decline in trend GDP growth over the entire post-war period, $1950 - 2018$. We explore the implications of TFP and labor inputs in accounting for this secular decline but we do so at a disaggregated sectoral level. We document disparate trend variations in TFP and labor growth across sectors and estimate the extent to which these trends are driven by idiosyncratic rather than common factors. We then study the implications of our empirical findings for trend growth within a multi-sector framework with production linkages that mimic those of the US economy including, crucially, in the production of investment goods.

We find that common trend factors play a relatively small role in explaining sectoral trends in labor and TFP growth. For example, in Durable Goods, only 3 percent of the overall trend variation in labor and TFP growth is explained by their respective common trend factors. These findings, therefore, highlight the quantitative importance of idiosyncratic forces not only for business cycle fluctuations (see Gabaix (2011), Foerster, Sarte, and Watson (2011), and Atalay (2017)), but also for variations in trends. There are, however, exceptions in that in some service sectors, the trend variation in labor is explained to a greater degree by the common trend factor. Common trends explain a higher fraction of aggregate trend variation in labor and TFP growth because aggregation reduces the importance of sector-specific trends. We estimate that approximately 1/3 of the variation in the trend growth rate of aggregate TFP is common across sectors while roughly 2/3 is common for labor. One cannot, however, directly infer from these findings the role that common and sectoral growth trends in labor and TFP play in the overall trend growth rate of GDP. The reason is that capital accumulation and production linkages can considerably amplify the influence of particular sectors on the aggregate economy.

To explore the historical implications of changing sectoral trends for the long-run evolution of GDP growth, we derive balanced growth accounting equations in a dynamic multi-sector framework where sectors use not only materials but also investment goods produced in other sectors. We then use these new growth accounting equations to assess the aggregate effects of observed sectoral changes in the trend growth rates of labor and TFP. Our analysis, therefore, extends the work of Greenwood, Hercowitz, and Krusell (1997) to an environment with multiple investment sectors and multiple intermediate goods sectors that are also

[1] Cette, Fernald, and Mojon (2016) suggest that a slowdown in productivity growth that began prior to the Great Recession reflects in part the fading gains from the Information Technology (IT) revolution. This view is consistent with the long lags associated with the productivity effects of IT adoption found by Basu and Fernald (2001), and the collapse of the dot-com boom in the early 2000s. Decker, Haltiwanger, Jarmin, and Miranda (2016) point to a decline in business dynamism that began in the 1980s as an additional force underlying slowing economic activity.

interconnected in production.[2] At the same time, its focus on estimating common and idiosyncratic sources of sectoral trends, and what their aggregate implications are for the long-run, differentiates it from the literature building on Greenwood et al. (1997). Specifically, Fisher (2006), Justiniano, Primiceri, and Tambalotti (2010, 2011) and Basu, Fernald, Fisher, and Kimball (2013) are primarily concerned with the business cycle implications of sectoral shocks, and in particular, investment-specific shocks. More recently, vom Lehn and Winberry (2019) show that the input-output network of investment goods is critical in accounting for shifts in the cyclicality and relative volatilites of aggregate time series since the 1980s.

We show that capital accumulation and the network structure of U.S. production can markedly amplify the effects of sector-specific changes in the trend growth rates of TFP and labor on trend GDP growth. We summarize this amplification mechanism in terms of sectoral multipliers that reflect the knock-on effects induced by production linkages. The size of a sector's multiplier depends on its importance as a supplier of investment goods, and to a lesser degree materials, to other sectors. Given observed U.S. production linkages, we find that the influence of individual sectors on GDP growth may be as large as 3 times their share in the economy.

Combining our empirical findings with the amplification effects of sectoral multipliers, we find that sector-specific trends have accounted for roughly 3/4 of the trend variation in GDP growth over the post-war period, leaving aggregate or common factors to explain only 1/4 of those changes. These findings arise in part because the knock-on or indirect effects of sector-specific changes in TFP and labor are large in some sectors, especially those producing capital goods. Thus, U.S. trend GDP growth fell by nearly 3 percentage points between 1950 and 2018 with the Construction sector alone contributing roughly 1 percentage point of that decline between 1950 and 1980. The Durable Goods sector, after contributing to an economic expansion in the 1990's, then contributed another 2 percentage point decline in trend GDP growth between 2000 and 2018. Remarkably, no sector has contributed any steady significant increase to the trend growth rate of GDP over the post-war period.

Our paper also falls within the literature on equilibrium multi-sector models first developed by Long and Plosser (1983) and later Horvath (1998, 2000) and Dupor (1999). Since then, a large body of work has explored important features of those models for generating aggregate fluctuations from idiosyncratic shocks. We maintain the original assumptions of competitive input and product markets as well as constant-returns-to-scale technologies. Even absent non-log-linearities in production, and beyond the role of idiosyncratic shocks in explaining aggregate cyclical variations, the analysis reveals that sector-specific changes also dominate trend variations in U.S. GDP growth.[3]

[2]Ngai and Pissarides (2007) provide a seminal study of balanced growth in a multi-sector environment. They consider both multiple intermediates and multiple capital-producing sectors but not at the same time. Importantly, their analysis abstracts from pairwise linkages in both intermediates and capital-producing sectors that play a key role in this paper. Ngai and Samaniego (2009) extend the model in Greenwood et al. (1997) to three sectors which allows for an input-output network in intermediate goods in carrying out growth accounting. Duarte and Restuccia (2020) include input-output linkages across sectors in a multi-sector environment abstracting from capital and study the implications of cross-country productivity differences in non-traditional service sectors.

[3]See Gabaix (2011), Foerster, Sarte, and Watson (2011) and Atalay (2017) for assessments of the importance of idiosyncratic

This paper is organized as follows. Section 2 gives an overview of the behavior of trend GDP growth over the past 70 years. Section 3 provides an empirical description of the trend growth rates of TFP and labor growth by industry and estimates the contributions of sector-specific and common factors to these trends. Section 4 develops the implications of these changes at the sector level in the context of a dynamic multi-sector model with production linkages in materials and investment. This model serves as the balanced growth accounting framework that we use to determine the aggregate implications of changes in the sectoral trend growth rates of labor and TFP. Section 5 presents our quantitative findings. Section 6 concludes and discusses possible directions for future research. An online Technical Appendix contains a detailed description of the data, statistical methods, economic model, discussions of departures from our benchmark assumptions, and includes additional figures and tables referenced in the text.

2 The Long-Run Decline in U.S. GDP Growth

Figure 1 shows the behavior of U.S. GDP growth over the post-WWII period. Here, annual GDP growth is measured as the share-weighted value added growth from 16 sectors comprising the private U.S. economy; details are provided in the next section.

Panel A shows aggregate private-sector growth rates computed by chain-weighting the sectors, and by using three alternative sets of fixed sectoral shares computed as averages over the entire sample $(1950-2018)$, over the first fifteen years of the sample $(1950-1964)$ and over the final fifteen years $(2004-2018)$. Panel A shows large variation in GDP growth rates – the standard deviation is 2.5 percent over the period $1950-2018$ – but much of this variation is relatively short-lived and is associated with business cycles and other relatively transitory phenomena. Moreover, to the extent that sectoral shares have changed slowly over time, these share shifts have little effect in Panel A. In other words, changes in aggregate growth largely stem from changes within sectors rather than between them. Our interest, however, is in longer-run variation.

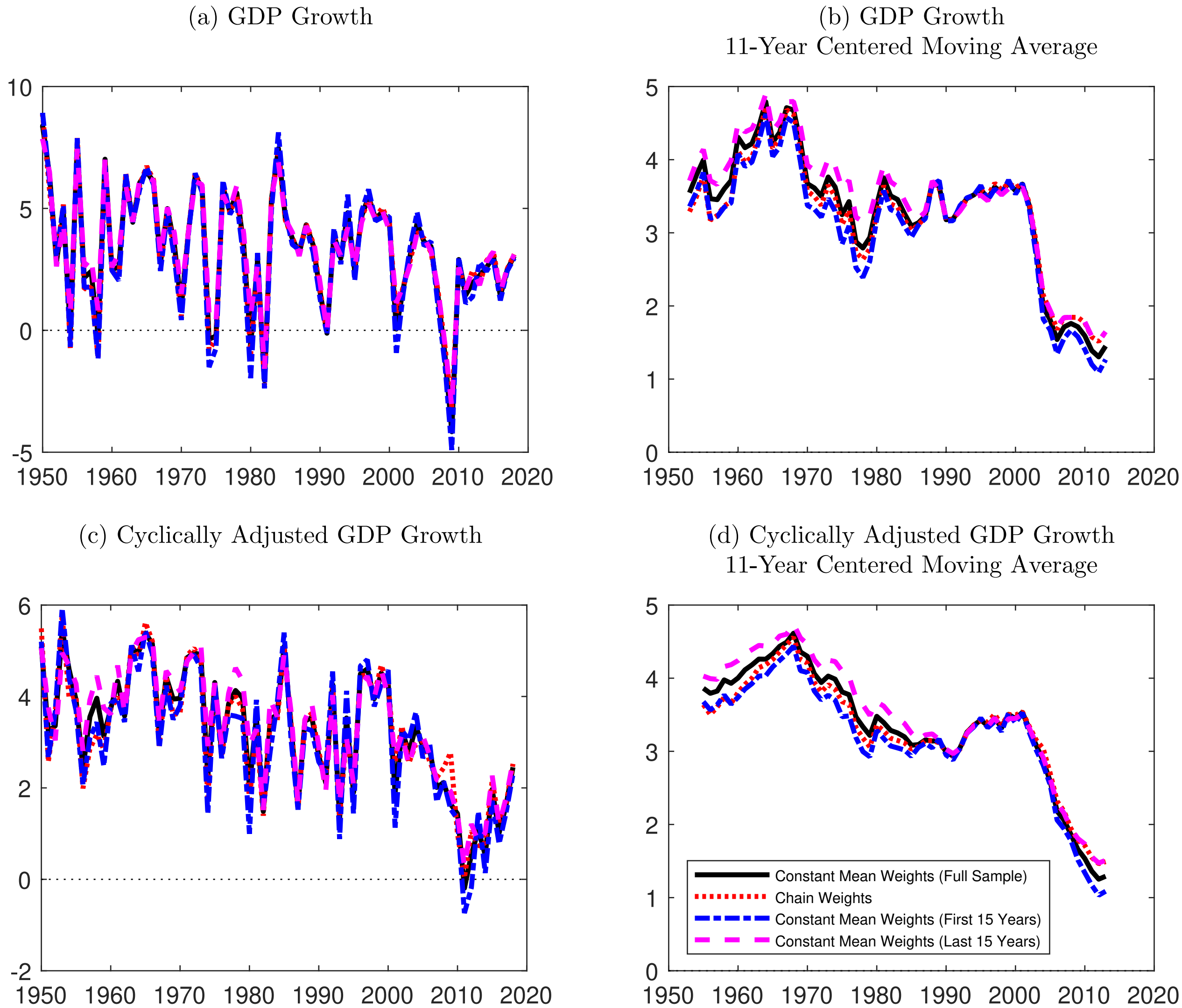
Panel B, therefore, plots centered $11-$year moving averages of the annual growth rates. Here too there is variability. In the 1950s and early 1960s average annual growth exceeded 4 percent. This fell to 3 percent in the 1970s, rebounded to nearly 4 percent in the 1990s, but plummeted to less than 2 percent in the 2000s (See Table 1). At these lower frequencies, the effects of slowly shifting shares over the sample become more visible.

Panels C and D refine these calculations by eliminating the cyclical variation using an Okun's law regression in GDP growth rates as in Fernald et al. (2017).[4] Thus, panel C plots the residuals from a regression of GDP growth rates onto a short distributed lead and lag of changes in the unemployment

shocks in driving business cycle fluctuations. We build on Acemoglu et al. (2012), Baqaee and Farhi (2019) and Miranda-Pinto (2019) by studying an explicitly dynamic framework along with an empirical model that parses out common and idiosyncratic components of sectoral trend input growth, as well as their implications for the observed behavior of the trend growth rates of sectoral value added and GDP since 1950.

[4]Compared to other measures of cyclical slack or resource utilization, Fernald et al. (2017) point out that the civilian unemployment rate has two key advantages. First, it has been measured using essentially the same survey instrument since 1948. Second, changes in the unemployment rate have nearly a mean of zero over long periods.

Figure 1: U.S. GDP Growth Rates 1950-2018
(percentage points at an annual rate)



Notes: Growth rates are share-weighted value added growth rates from 16 sectors making up the private U.S. economy. Cyclical adjustment uses a regression on leads and lags of the first-difference in the unemployment rate.

rate $(\Delta u_{t+1}, \Delta u_t, \Delta u_{t-1})$. This cyclical adjustment eliminates much of the cyclical variability evident in panel A. In addition, the 11$-$year moving average in Panel D now produces a more focused picture of the trend variation in the growth rate of private GDP. Again, time-varying share weights have discernible but relatively small effect on the aggregate growth rate or its 11-year moving average.

The numbers reported in Table 1 frame the key question of this paper: why did the average growth rate of GDP fall from 4 percent per year in the 1950s to just over 3 percent in the 1980s and 1990s, and then further decline precipitously in the 2000s? As the different columns of the table make clear, this question

Table 1: Average GDP Growth Rates

	Const Mean Weights Full Sample		Time-Varying Weights		Const Mean Weights First 15 Years		Const Mean Weights Last 15 Years	
Dates	Growth rates	Cyc-adj rates	Growth rates	Cyc-adj rates	Growth rates	Cyc-adj rates	Growth rates	Cyc-adj rates
1950-2018	3.3	3.2	3.3	3.2	3.2	3.1	3.4	3.4
1950-1966	4.5	4.2	4.3	4.0	4.3	4.0	4.6	4.3
1967-1983	3.1	3.6	3.0	3.5	2.8	3.4	3.4	3.9
1984-2000	3.9	3.4	3.9	3.4	4.0	3.4	3.8	3.4
2001-2018	1.9	1.8	2.0	2.0	1.7	1.7	2.0	2.0

Notes: The values shown are averages of the series plotted in Figure 1 over the periods shown.

arises regardless of the shares used in constructing GDP. We look to inputs – specifically TFP and labor at the sectoral level – for the answer. That is, interpreting long-run variations of the data as a time-varying balanced growth path, changes in trend GDP growth are in part determined by changes in the trend growth rates of those two inputs. However, as the analysis in Section 4 makes clear, not all sectoral inputs are created equal. Sectors not only differ in their size, that is, their value-added share in GDP, but also in the share of materials or capital that they provide to other sectors. Put another way, input variation across sectors is also a particularly important driver of low frequency movements in aggregate GDP growth.

Before investigating these input-output interactions, we begin by describing the sectoral data, how these data are measured, how we construct trend growth rates, and how sectoral value-added as well as labor and TFP inputs have evolved over the post-WWII period. In much of our analysis we construct aggregates using the constant weights computed using full-sample averages. As Figure 1 and Table 1 suggest, results using these constant shares are robust to alternative weighting schemes.

3 An Empirical Description of Trend Growth in TFP and Labor

As a first step, we estimate an empirical model of TFP and labor growth for different sectors of the U.S. economy. Our paper applies as a benchmark the insights of Hulten (1978) on the interpretation of aggregate total factor productivity (TFP) changes as a weighted average of sector-specific value-added TFP changes. In particular, under constant-returns-to-scale and perfect competition in product and input markets, the sectors' weights are the ratios of their valued added to GDP.[5]

We calculate standard TFP growth rates at the sectoral level following Jorgenson et al. (2017) among

[5] In the absence of constant-returns-to-scale or perfect competition, Basu and Fernald (1997, 2001) and Baqaee and Farhi (2018) show that aggregate TFP changes also incorporate reallocation effects. These effects reflect the movement of inputs between low and high returns to scale sectors stemming from changes in relative sectoral TFP.

others, construct trend growth rates using a 'low-pass' filter, and estimate a statistical model to decompose these trend growth rates into common and sector-specific components.

3.1 Data

Sectoral TFP growth rates are calculated using KLEMS data from of the Bureau of Economic Analysis and the Bureau of Labor Statistics Integrated Industry-Level Production Accounts (ILPA).[6] These data are attractive for our purposes because they provide a unified approach to the construction of gross output, the primary inputs capital and labor, as well as intermediate inputs ('materials') for a large number of industries. The KLEMS data are based on U.S. National Income and Product Accounts (NIPA) and consistently integrate industry data with Input-Output tables and Fixed Asset tables.

The KLEMS dataset contains quantity and price indices for inputs and outputs across 61 private industries. The growth rate of any one industry's aggregate is defined as a Divisia index given by the value-share weighted average of its disaggregated component growth rates. Labor input is differentiated by gender, age, education, and labor status. Labor input growth is then defined as a weighted average of growth in annual hours worked across all labor types using labor compensation shares of each type as weights. Similarly, intermediate input growth reflects a weighted average of the growth rate of all intermediate inputs averaged using payments to those inputs as weights. Finally, capital input growth reflects a weighted average of growth rates across 53 capital types using payments to each type of capital as weights. Capital payments are based on implicit rental rates consistent with a user-cost-of-capital approach. Total payments to capital are the residuals after deducting payments to labor and intermediate inputs from the value of production. Put another way, there are no economic profits.

An industry's value added TFP growth rate is defined in terms of its Solow residual, specifically output growth less the revenue-share weighted average of input growth rates. This calculation is consistent with the canonical theoretical framework we adopt in Section 4 where all markets operate under perfect competition and production is constant-returns-to-scale. For earlier versions of Jorgenson's KLEMS data up to 1990, Basu and Fernald (1997, 2001) compute total payments to capital as the sum of rental rates implied by the user-cost-of-capital and find small industry profits on average that amount at most to three percent of gross output. In the presence of close to zero profits, elasticities to scale and markups are equivalent. More recently, an active debate has emerged on the extent to which the competitive environment has changed in the U.S. over the last two decades. On the one hand, Barkai (2017), also applying the user-cost-of-capital framework but using post 1990 data, finds substantial profit shares over that period. On the other hand, Karabarbounis and Neiman (2018) argue that the user-cost-of-capital framework, to the extent that it implies high profit shares starting in the 1990s, also implies unreasonably high profit shares in the 1950s.[7] In this

[6] The ILPA KLEMS data extend earlier work by Jorgenson and his collaborators, e.g., Jorgenson et al. (2017).

[7] In addition, De Loecker and Eeckhout (2017), estimating industry production functions from corporate balance sheets, present evidence of rising markups and returns to scale since the 1980s. However, Traina (2018) argues that the evidence on rising markups from corporate balance sheets depends crucially on the measurement of variable costs and weights in aggregation. Similarly, Rossi-Hansberg et al. (2020) show that while sales concentration has unambiguously risen at the national level since

paper, we maintain the assumptions of competitive markets and constant-returns-to-scale as a benchmark from which to study the aggregate implications of sectoral changes in labor and TFP inputs.

Our calculations rely on the official 2020 version of the ILPA KLEMS dataset which covers the period 1987-2018, and the experimental ILPA KLEMS dataset for the period $1947 - 2016$.[8] To simplify the presentation and analysis, we carry out the empirical work using private industries at the two-digit level. In particular, we aggregate the 61 private industries included in the two KLEMS datasets into 16 two-digit private industries following the procedure in Hulten (1978).[9] Another advantage of the aggregation into two-digit industries is that any differences between the two KLEMS datasets are attenuated and we feel comfortable splicing the two datasets in 1987.[10] That is, we use the growth rates calculated using the experimental ILPA data before 1987 and using the official ILPA data after that date.

Table 2 lists the 16 sectors we consider. For each sector, the table shows average cyclically adjusted growth rates of value added, labor, and value added TFP over $1950 - 2018$, and it also shows their average shares in aggregate value added and labor input. The aggregate growth rates in the bottom row are the value-weighted averages of the sectoral growth rates with average value added and labor shares used as fixed weights.

Clearly sectors grow at different rates and this disparity is hidden in studies that only consider aggregates. Average real value added growth rates range from 1.4 percent in Mining to 4.9 percent in Information, bracketing the aggregate value added growth rate of 3.3 percent. With the exception of the Durable Goods sector, most sectors with growth rates that exceed the aggregate growth rate provide services. Similarly, labor input growth rates range from -1.3 percent in Agriculture to 3.5 percent in Professional and Business Services (PBS), bracketing the average aggregate growth rate of 1.6 percent. Again, most sectors with labor input growth rates that exceed the aggregate growth rate provide services. Finally, TFP growth rates range from -0.4 percent in Utilities to 3.1 percent in Agriculture, bracketing the average aggregate TFP growth rate of 0.8 percent. Sectoral TFP growth rates are less aligned with either value added or labor input growth rates. There are four sectors with TFP declines, namely Utilities, Construction, FIRE (x-Housing), and Education and Health, as well as a number of sectors with stagnant TFP levels. Negative TFP growth rates are a counter-intuitive but known feature of disaggregated industry data. These are in part attributed to measurement issues with respect to output though land and other regulations are also clearly a factor in

the 1980s, concentration has steadily declined at the Core-Based Statistical Area, county, and ZIP code levels over the same period. While these facts can seem conflicting, the authors present evidence that large firms have become bigger through the opening of more establishments or stores in new local markets, but this process has lowered concentration in those markets.

[8]The official ILPA dataset for 1987-2018 is downloaded from `https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems` and the experimental ILPA dataset for 1947-2016 is downloaded from `https://www.bls.gov/mfp/special_requests/tables_detail.xlsx`. See Fleck et al. (2014) and Corby et al. (2020) for a detailed description of the official ILPA data, and Eldridge et al. (2020) for the experimental ILPA data.

[9]A detailed description is provided in Section 7 of the online-only Technical Appendix to this paper, Foerster et al. (2021).

[10]While the two ILPAs are related, they are not exactly identical for the time period in which they overlap. Since both datasets are constructed to be consistent with the BEA's input-output tables, they mostly agree on industry details and both cover the same 61 private industries. Nevertheless, there remain differences but these are reflected mostly in the levels of the variables rather than their growth rates.

Table 2: 16 Sector Decomposition of the U.S. Private Economy
(1950-2018)

	Sectors	Average growth rate Cyclically adjusted data (Percentage points at an annual rate)			Average share (Percentage points)	
		Value Added	Labor	TFP	Value Added	Labor
1	Agriculture	2.41	-1.29	3.12	2.69	3.23
2	Mining	1.38	0.37	0.39	2.11	1.55
3	Utilities	2.09	1.00	-0.42	2.37	1.04
4	Construction	1.69	1.76	-0.23	4.99	7.62
5	Durable Goods	3.65	0.54	2.10	13.32	15.5
6	Nondurable Goods	2.27	0.14	0.83	9.20	8.80
7	Wholesale Trade	4.61	1.67	1.81	7.15	6.63
8	Retail Trade	3.13	1.19	1.07	8.18	9.61
9	Trans. & Ware.	2.58	0.91	1.27	4.16	5.03
10	Information	4.93	1.35	1.04	4.97	3.74
11	FIRE (x-Housing)	3.88	2.77	-0.03	9.97	7.53
12	PBS	4.45	3.51	0.36	8.79	11.25
13	Educ. & Health	3.43	3.34	-0.29	6.22	9.35
14	Arts, Ent., & Food Svc.	2.48	1.79	0.36	3.74	4.56
15	Other Services (x-Gov)	1.99	0.52	1.04	2.94	4.37
16	Housing	3.45	0.86	0.24	9.20	0.20
	Aggregate	3.32	1.55	0.82	100	100

Notes: The values shown are average annual growth rates for the 16 sectors. The row labelled "Aggregate" is the constant share-weighted average of the 16 sectors.

sectors such as Construction.[11]

To a first approximation, the contributions of the different sectors to aggregate outcomes are given by the nominal value added and labor input shares in the last two columns of Table 2. In those columns, two notable contributors to value added and TFP are Durable Goods and FIRE excluding Housing. The two largest contributors to labor payments are Durable Goods and Professional and Business Services. Over time, the shares of goods-producing sectors has declined while the shares of services-producing sectors has increased. However, despite these changes, aggregating sectoral outputs and inputs using constant mean

[11]See for example Herkenhoff, Ohanian, and Prescott (2018).

shares, as opposed to time-varying shares, has little effect on the measurement of aggregate outputs and inputs (Figure 1).

3.2 Empirical Framework

The empirical framework used to characterize the long-run properties of the data proceeds in 3 steps. First, we carry out a cyclical adjustment of sectoral TFP and labor raw growth rates to eliminate some of their cyclical variability. Second, we make use of methods discussed in Müller and Watson (2020) to extract smooth trends capturing the long-run evolution of the data. Finally, we carry out a factor analysis that explores the relative importance of common and sector-specific factors in driving these smooth trend components.

3.2.1 Cyclical Adjustment

Let $\Delta\widetilde{x}_{i,t}$ denote the growth rate (100 $\times$ the first difference of the logarithm) of annual measurements of labor or TFP in sector i at date t. These sectoral growth rates are volatile and, in many sectors, much of the variability is associated with the business cycle. Our interest is in trend (i.e., low-frequency) variation, which is more easily measured after cyclically adjusting the raw growth rates. Thus, as with the cyclically adjusted measure of GDP shown in Figure 1, we follow Fernald et al. (2017) and cyclically adjust these growth rates using the change in the unemployment rate, Δu_t, as a measure of cyclical resource utilization. That is, we estimate

$$\Delta\widetilde{x}_{i,t} = \mu_i + \beta_i(L)\Delta u_t + e_{i,t},$$

where $\beta_i(L) = \beta_{i,1}L + \beta_{i,0} + \beta_{i,-1}L^{-1}$ and the leads and lags of Δu_t capture much of the business-cycle variability in the data. Throughout the remainder of the paper, we use $\Delta x_{i,t} = \Delta\widetilde{x}_{i,t} - \hat{\beta}_i(L)\Delta u_t$, where $\widehat{\beta}_i(L)$ denotes the OLS estimator, and where $x_{i,t}$ represents the implied cyclically adjusted value of sectoral TFP (denoted $z_{i,t}$) or labor input (denoted $\ell_{i,t}$) growth rates.

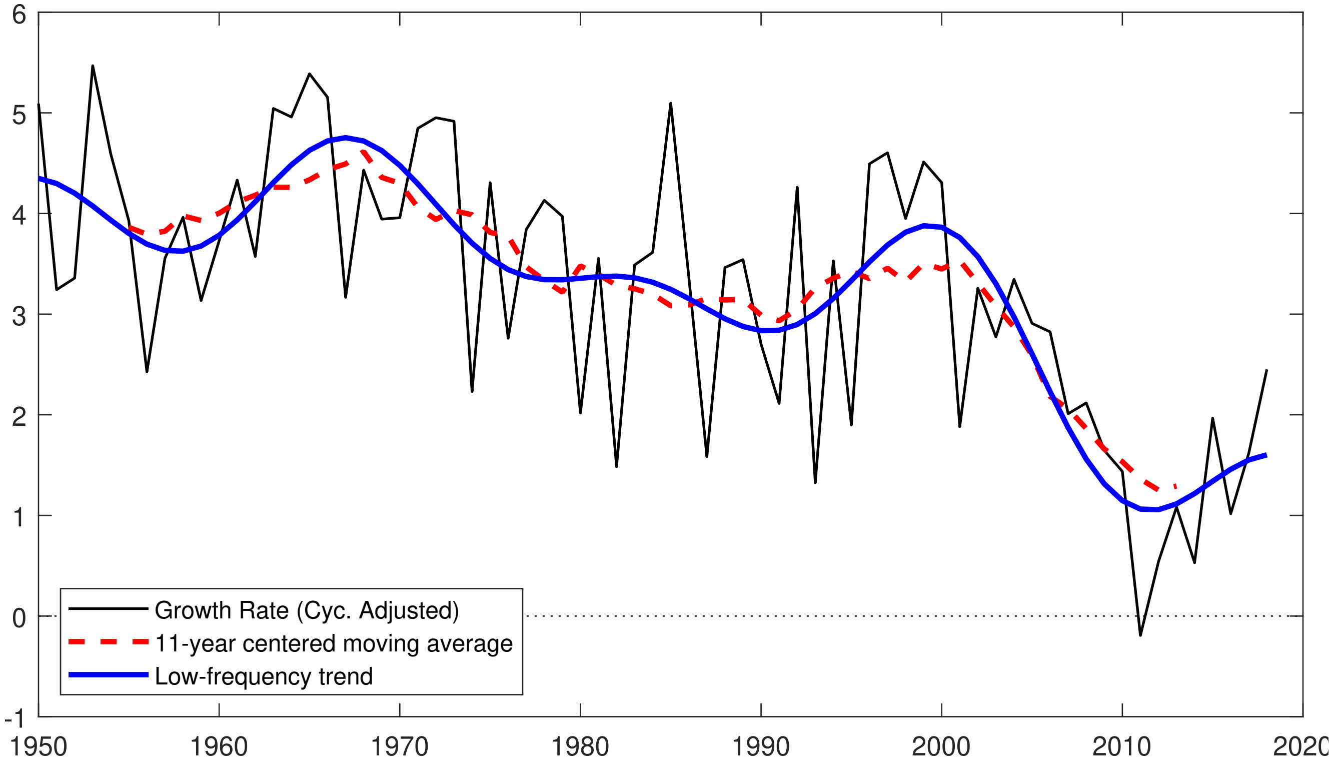
3.2.2 Extracting Low-Frequency Trends

We begin by extracting low-frequency trends from the data using a framework presented in Müller and Watson (2008). That framework is useful because on the one hand, it yields smooth trends that capture the long-run evolution of the growth rate of GDP and the associated growth rates of sectoral labor and TFP, and on the other hand, it simultaneously provides a convenient framework for statistical analysis. We give an overview below of the approach here. Müller and Watson (2020) provides a detailed *Handbook* discussion of statistical analysis using this framework.[12]

To extract low frequency trends in the growth rates of GDP, TFP and labor input, generically denoted by Δx_t, we regress these series onto a constant and a set of low-frequency periodic functions. In particular,

[12] The framework and methods are closely related to well-known spectral analysis methods using low-frequency Fourier transforms of the data. See Müller and Watson (2020) for a detailed discussion and references.

Figure 2: Trend Rate of Growth of GDP
(percentage points at an annual rate)



Notes: The low-frequency trend captures variability for periodicities longer than 17 years.

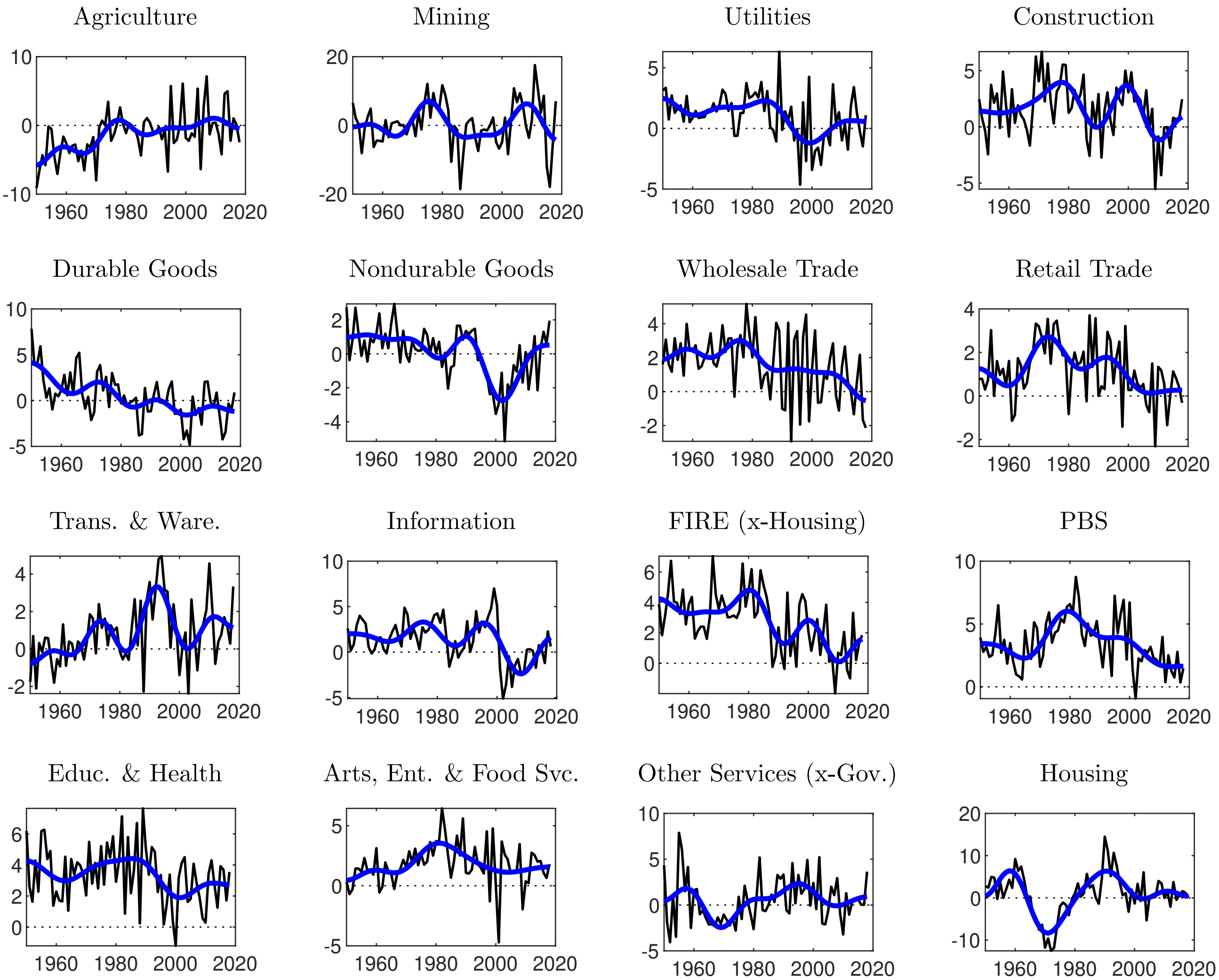
let $\Psi_j(s) = \sqrt{2}\cos(js\pi)$ denote a cosine function on $s \in [0, 1]$ with period $2/j$. The fitted values from the OLS regression of Δx_t onto a constant and $\Psi_j((t-1/2)/T)$ for $j = 1, ..., q$ and $t = 1, ..., T$ capture the low-frequency variability in the sample corresponding to periodicities longer than $2T/q$. Moreover, let $\mathbf{\Psi}(s)$ denote the vector of regressors $[\Psi_1(s), ..., \Psi_q(s)]'$ with periods 2 through $2/q$, $\mathbf{\Psi}_T$ the $T \times q$ matrix with t^{th} row $\mathbf{\Psi}((t-1/2)/T)'$ and $\mathbf{\Psi}_T^0 = [\mathbf{1}_T, \mathbf{\Psi}_T]$ where $\mathbf{1}_T$ is a $T \times 1$ vector of ones. The specific form used for the cosine weights implies that the columns of $\mathbf{\Psi}_T^0$ are orthogonal with $T^{-1}\mathbf{\Psi}_T^{0\prime}\mathbf{\Psi}_T^0 = \mathbf{I}_{q+1}$. Thus, the OLS coefficients from the regression of Δx_t onto $\mathbf{\Psi}_T^0$, that is $\left(\mathbf{\Psi}_T^{0\prime}\mathbf{\Psi}_T^0\right)^{-1}\mathbf{\Psi}_T^{0\prime}\Delta x_{1:T}$, amount to $q+1$ weighted averages of the data, $T^{-1}\mathbf{\Psi}_T^{0\prime}\Delta x_{1:T}$, which we partition as $(\overline{x}, \mathbf{X})$ where $\overline{x}$ is the sample mean of Δx_t. In our application, $T = 69$ so that with $q = 8$, the regression captures long-run variation with periodicities longer than 17.25 ($= 2 \times 69/8$) years. These are the low-frequency growth rate trends analyzed in this paper.[13]

Figure 2 plots the growth rates of (cyclically-adjusted) GDP, its centered 11-year moving average, and its trend computed as the fitted values from the low-frequency regression we have just described.[14] The

[13] Calculations presented in Müller and Watson (2008) show that these low-frequency projections approximate a low-pass filter for periods longer than $2T/q$. That said, there is some leakage from higher frequencies and this makes the cyclical adjustment discussed above useful.

[14] An 11-year moving average is a crude low-pass filter with more than half of its spectral gain associated with periods longer

Figure 3: Labor Growth Rates and Trends by Sector
(percentage points at an annual rate)



Notes: Each panel shows the cylically adjusted growth rate of labor for each sector in black, along with its low-frequency trend in blue.

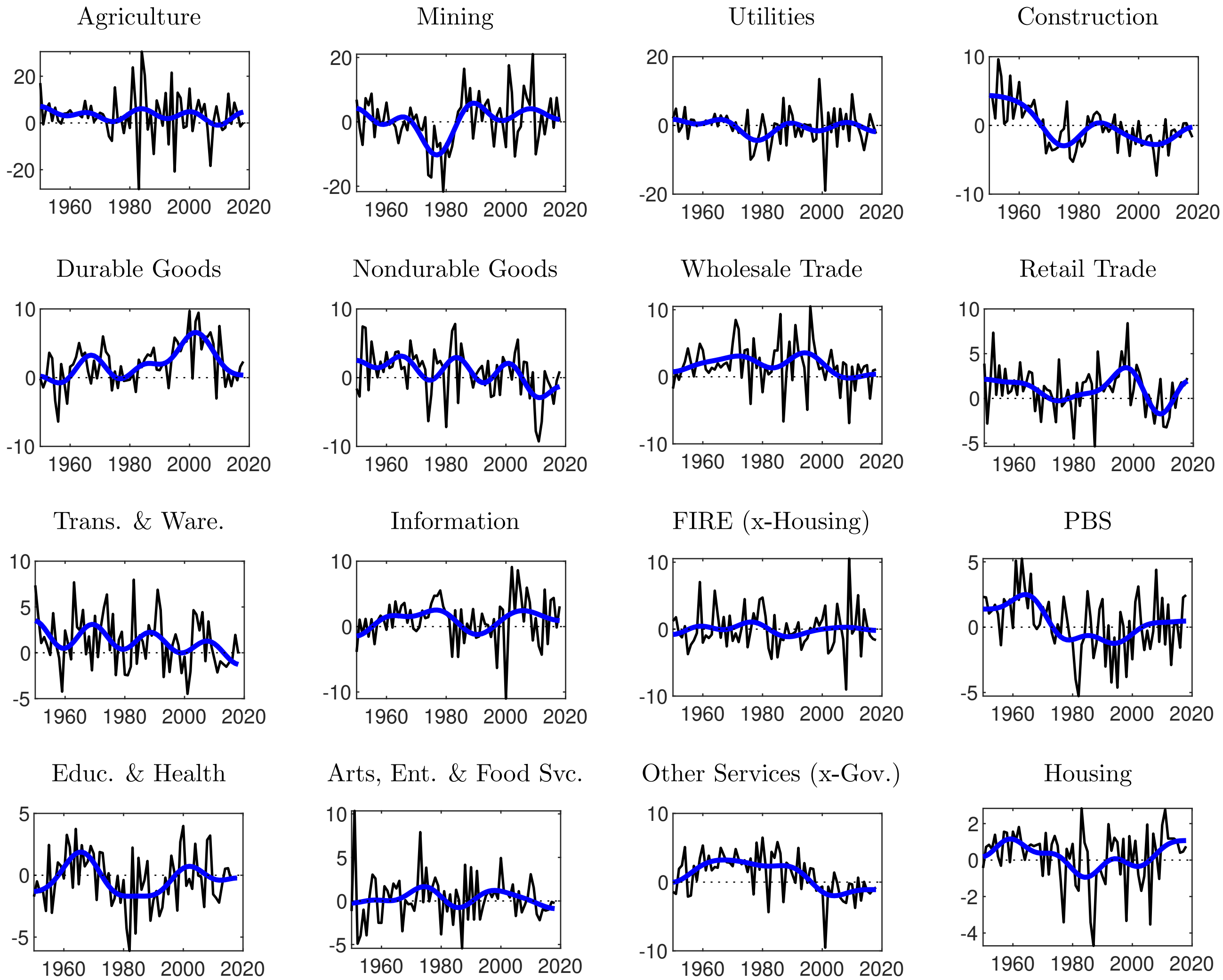
low-frequency trend smooths out the higher-frequency variation in the 11-year moving average. While the aggregate importance of sectoral shocks is known for business cycles – generally cycles with periods ranging from 2 to 8 years – our interest here is on the role of sectoral shocks for the aggregate trend variations shown in Figure 2.[15] Thus, we will focus on cycles longer than 17 years as captured by the Ψ-weighted averages of the data.

Figures 3 and 4 plot the cyclically adjusted growth rates of labor and TFP for each of the 16 sectors

than 17 years.

[15]See for example Gabaix (2011), Foerster et al. (2011) or Atalay (2017) for empirical analyses of the role of sectoral shocks at quarterly and business cycle frequencies.

Figure 4: TFP Growth Rates and Trends by Sector
(percentage points at annual rate)



Notes: See notes for Figure 3.

along with their low frequency trends. The disparity in experiences across different sectors stands out. In particular, the trends show large variations across sectors and through time. For example, labor input was contracting at nearly 4 percent per year in Agriculture in the 1950s but stabilized near the end of the sample. In contrast, labor input in the Durables and Nondurable Goods sectors was increasing in the 1950s but has been contracting since the mid-1980s. At the same time, the trend growth rate of labor in several service sectors exhibit large ups and downs over the sample. Similar disparities are apparent in the sectoral growth rates of TFP. Trend TFP growth in Construction, for example, was around 5 percent in the 1950s, declined over the next couple of decades, and flattened out thereafter. In contrast, TFP trend growth in Durable Goods increased somewhat steadily from the 1950s to 2000 but has since collapsed by more than 5 percentage points. In Sections 4 and 5, we quantify the aggregate implications of these sectoral variations in labor and TFP inputs.

3.2.3 Statistical Properties of Low Frequency Trends

By construction, the low frequency trends are highly serially correlated, and this needs to be accounted for in the statistical analysis. As it turns out, this is relatively straightforward given the framework described above. We highlight a few key features of this framework and refer the reader to Müller and Watson (2020) and references therein for more detail.

To fix notation, let g_t denote the trend growth rate constructed from our data on TFP or labor input, Δx_t. That is, g_t is the fitted value from the OLS regression of Δx_t onto a constant and the q periodic functions $\Psi_j((t-1/2)/T)$, and is the low-frequency trend plotted in Figures 2-4. We saw earlier that because the regressors are mutually orthogonal, the OLS regression coefficients are $(\overline{x}, \mathbf{X})$ where $\overline{x}$ is the sample mean of Δx_t and $\mathbf{X}$ is the $q \times 1$ vector of OLS regression coefficients from the regression of Δx_t onto $\Psi_j((t-1/2)/T)$ for $j = 1, ..., q$. The elements of $\mathbf{X}$ are called the *cosine transforms* of Δx. Importantly, because the regressors are deterministic, the stochastic process for g_t is completely characterized by the probability distribution of the $(q+1)$ random variables $(\overline{x}, \mathbf{X})$, and variation in g_t over the sample is determined by the $q \times 1$ vector $\mathbf{X}$.

As noted above, the cosine transforms are weighted averages of the sample values of Δx_t, $T^{-1}\mathbf{\Psi}'_T \Delta x_{1:T}$, and a central limit result in Müller and Watson (2020) provides sufficient conditions under which $\mathbf{X}$ is normally distributed when T is large. Importantly, these conditions allow for a wide range of persistent processes so that Δx_t may be, for example, stationary and $I(0)$, $I(1)$ (that is, have a unit root), or generated by other highly persistent processes. In these cases, $\mathbf{X} \overset{a}{\sim} N(0, \Omega)$ where the covariance matrix Ω is determined by the low frequency second moments of the Δx process, that is the low frequency (pseudo-) spectrum.

In the empirical analysis below, we use a 'local-level-model' parameterization of the low frequency spectrum that linearly combines a flat spectrum (from an $I(0)$ component) and a steeply decreasing spectrum (from an $I(1)$ component). In this model, Δx_t behaves like the sum of independent $I(0)$ and $I(1)$ processes over the long run. The resulting covariance matrix, Ω, depends on two parameters, (σ^2, γ), where σ is an overall scale parameter and γ governs the relative importance of the $I(0)$ and $I(1)$ components; larger values of σ produce a more variable low frequency trend and larger values of γ produce a more persistent trend. For this model, the covariance matrix Ω has the form:

$$\Omega = \sigma^2 D(\gamma), \tag{1}$$

where D is diagonal.[16]

[16] See Section 1 of the Technical Appendix or Müller and Watson (2020) for an explicit formula of the diagonal elements of $D(\gamma)$.

A key implication of these results is that the original sample of T observations on Δx_t contains only q pieces of independent information on the long-run properties of Δx. In our context, the $T = 69$ annual observations contain only $q = 8$ observations describing the long-run variation for periods longer than 17 years. This makes precise the intuition that a statistical analysis of long-run growth is inherently a 'small sample' problem. Conveniently, however, this small sample problem involves variables that are averages of the T observations – the elements of $\mathbf{X}$ – and that are, therefore, (approximately) normally distributed and readily analyzed using standard statistical methods.

3.2.4 Decomposition of Trend Growth Rates into Common and Sector-Specific Factors

Examination of the trends plotted in Figures 3 and 4 suggests that some of the trend variation may be common across sectors while some are sector-specific. In addition, in some sectors, trend variation in labor appears to be correlated with trend variation in TFP (and interestingly this correlation generally appears to be negative). We now outline an empirical model that captures these features.

Let $\Delta \ln \ell_{i,t}$ denote the rate of growth of labor input in sector i in period t, and let $\Delta \ln z_{i,t}$ denote the rate of growth of TFP. Consider the factor model

$$\begin{bmatrix} \Delta \ln \ell_{i,t} \\ \Delta \ln z_{i,t} \end{bmatrix} = \begin{bmatrix} \lambda_i^\ell & 0 \\ 0 & \lambda_i^z \end{bmatrix} \begin{bmatrix} f_t^\ell \\ f_t^z \end{bmatrix} + \begin{bmatrix} u_{i,t}^\ell \\ u_{i,t}^z \end{bmatrix}, \tag{2}$$

where $f_t = (f_t^\ell \ f_t^z)'$ are unobserved common factors, $\lambda_i = (\lambda_i^\ell \ \lambda_i^z)'$ are factor loadings, and $u_{i,t} = (u_{i,t}^\ell \ u_{it}^z)'$ are sector-specific disturbances (sometimes referred to as *uniquenesses*). Denote the trend growth rates in $(\Delta \ln \ell_{i,t}, \Delta \ln z_{i,t}, f_t^\ell, f_t^z, u_{i,t}^\ell, u_{i,t}^z)$ by respectively $(g_{i,t}^\ell, g_{i,t}^z, g_{f,t}^\ell, g_{f,t}^z, g_{u,i,t}^\ell, g_{u,i,t}^z)$ and let $(\mathbf{X}_i^\ell, \mathbf{X}_i^z, \mathbf{F}^\ell, \mathbf{F}^z, \mathbf{U}_i^\ell, \mathbf{U}_i^z)$ represent the associated cosine transforms. In other words, $\mathbf{X}_i^\ell$ is the $q \times 1$ vector of OLS coefficients associated with $\Psi_j((t-1/2)/T)$, $j = 1, ..., q$, in the regression of $\Delta \ln \ell_{i,t}$ on a constant and these periodic functions, and similarly for $\mathbf{X}_i^z$ and so on. Pre-multiplying each element in equation (2) by $T^{-1}\psi_t'$, where $T^{-1}\psi_t'$ is the t^{th} row of $\mathbf{\Psi}_T$, and summing, yields a factor decomposition of the trends and cosine transforms of the form (abstracting from the constant),

$$\begin{bmatrix} \mathbf{X}_i^\ell \\ \mathbf{X}_i^z \end{bmatrix} = \begin{bmatrix} \lambda_i^\ell I_q & 0 \\ 0 & \lambda_i^z I_q \end{bmatrix} \begin{bmatrix} \mathbf{F}^\ell \\ \mathbf{F}^z \end{bmatrix} + \begin{bmatrix} \mathbf{U}_i^\ell \\ \mathbf{U}_i^z \end{bmatrix}, \tag{3}$$

which characterizes the low-frequency variation in the data. We estimate a version of (3) and use it to describe the common components, $(g_{f,t}^\ell, g_{f,t}^z)$, and sector-specific components, $(g_{u,i,t}^\ell, g_{u,i,t}^z)$, of the trend growth rates in sectoral labor input and TFP.[17]

To estimate equation (3) requires that we parameterize the covariance matrices for $(\mathbf{F}, \mathbf{U})$. We use the local-level parameterization of Ω in (1) to characterize the covariance matrix of each of the components

[17] See Müller et al. (2020) for a related application studying long run growth and long horizons forecasts for per-capita GDP values of a panel of 113 countries.

in $(\mathbf{F}^{\ell}, \mathbf{F}^{z}, \{\mathbf{U}_i^{\ell}, \mathbf{U}_i^{z}\}_{i=1}^{16})$, where each component has its own value of (σ, γ); thus for example $\mathrm{Var}(\mathbf{F}^{\ell}) = \sigma_{F,\ell}^2 D(\gamma_F^{\ell})$, and similarly for $\mathbf{F}^z$ and each of the $\mathbf{U}_i^{\ell}$ and $\mathbf{U}_i^{z}$ components . We assume that $\mathbf{F}$ and $\mathbf{U}$ are uncorrelated, as are $\mathbf{U}_i$ and $\mathbf{U}_j$ for $i \neq j$. We allow $\mathbf{F}^{\ell}$ and $\mathbf{F}^{z}$ to be correlated by introducing a covariance parameter $\sigma_{F,\ell z}$ and letting $\mathrm{Cov}(\mathbf{F}^{\ell}, \mathbf{F}^{z}) = \sigma_{F,\ell z} D(\gamma_F^{\ell})^{1/2} D(\gamma_F^{z})^{1/2}$. We use an analogous parameterization for the covariance between each of the sectoral values of $\mathbf{U}_i^{\ell}$ and $\mathbf{U}_i^{z}$. Thus, each pair $(\mathbf{F}^{\ell}, \mathbf{F}^{z})$ or $(\mathbf{U}_i^{\ell}, \mathbf{U}_i^{z})$ is characterized by a parameter pair, $\gamma = (\gamma^{\ell}, \gamma^{z})$, that governs persistence and a 2×2 covariance matrix, say Σ, that includes $(\sigma_{\ell}^2, \sigma_z^2, \sigma_{\ell z})$.

The model is estimated using Bayes methods. While large-sample Bayes and Frequentist methods often coincide, the analysis of long-run trends is predicated on a small sample: in our application, the variation in each trend is characterized by only $q = 8$ observations. Hence, large-sample frequentist results are irrelevant for our 'small-sample' empirical problem, and Bayes analysis will in general depend on the specifics of the chosen priors.[18] Thus, we now turn to the priors that we use.

The empirical model is characterized by three sets of parameters: a set of 2×2 covariance matrices, $(\Sigma_{FF}, \{\Sigma_{UU,i}\}_{i=1}^{16})$, that govern the variability and covariability of (ℓ, z) pairs of $\mathbf{F}$ and $\mathbf{U}$, the low frequency persistence parameters, $((\gamma_F^{\ell}, \gamma_F^{z}), \{(\gamma_{U,i}^{\ell}, \gamma_{U,i}^{z}\}_{i=1}^{16})$, and the factor loadings $\{\lambda_i^{\ell}, \lambda_i^{z}\}_{i=1}^{16}$.

We use relatively uninformative priors for the Σ matrices and γ. Specifically, the prior assumes that each set of parameters is independently distributed. We use a standard conjugate prior for each of the Σ matrices which is the inverse-Wishart with $\nu = 0.01$ degrees of freedom and scale $\nu \mathbf{I}_2$. There isn't a conjugate prior for γ. We use a prior with $\ln(\lambda) \sim U(0, \ln(500))$. This puts relatively more weight on small values of γ, i.e., small weight on the $I(1)$ component of the local-level model (consistent with a body of evidence beginning in Stock and Watson (1998)) but allows for low-frequency behavior dominated by $I(1)$ dynamics and thus allows for a wide range of persistence patterns.

The prior for the factor loadings is more informative. Let $\lambda^{\ell} = (\lambda_1^{\ell}, ..., \lambda_{16}^{\ell})'$ and note that the scale of λ^{ℓ} and $\mathbf{F}^{\ell}$ are not separately identified. Thus, we normalize $s_{\ell}' \lambda^{\ell} = 1$, where s_{ℓ} denotes the vector of average sectoral labor shares shown in Table 2. This imposes a normalization where the growth of aggregate labor, say $\Delta \ln \ell_t = \sum_{i=1} s_{\ell,i} \Delta \ln \ell_{i,t}$, satisfies $\Delta \ln \ell_t = f_t^{\ell} + \sum_i s_{\ell,i} u_{i,t}^{\ell}$. That is, a one unit change in f^{ℓ} corresponds to a unit change in the long-run growth rate of aggregate labor.

The prior for λ^{ℓ} is $\lambda^{\ell} \sim N(\mathbf{1}, \mathbf{P}_{\ell})$ where $\mathbf{1}$ is a vector of 1s and $\mathbf{P}_{\ell} = \eta^2(\mathbf{I}_{16} - s_{\ell}(s_{\ell}' s_{\ell})^{-1} s_{\ell}')$ which enforces the constraint that $s_{\ell}' \lambda^{\ell} = 1$. The parameter η governs how aggressively the estimates of λ_i^{ℓ} are shrunk toward their mean of unity. Our benchmark model uses $\eta = 1$, so the prior puts approximately 2/3 of its weight on values of λ_i^{ℓ} between 0 and 2. Smaller values of η tighten the constraint, making negative factor loadings less likely, while larger values of η loosen it. To gauge the robustness of our conclusions to the choice of η, we will also show results with $\eta = 1/2$ and $\eta = 2$. We use an analogous prior for λ^z.

[18] Frequentist methods for small-sample problems such as these are discussed, for example, in Müller and Watson (2008, 2016, 2018). As practical matter, these methods apply only to univariate and bivariate settings. Our application here involves 32 time series.

3.3 Estimated Sectoral and Aggregate Trend Growth Rates in Labor and TFP

Section 1 of the Technical Appendix contains details of the estimation method and empirical results for the low-frequency factor model. For our purposes, the key results are summarized in a table and three figures.

Table 3 reports the posterior medians for λ along with 68 percent credible intervals.[19] Also reported is the fraction of the trend variability in each sector explained by the common trend factors, $(g^{\ell}_{f,t}, g^{z}_{f,t})$, which is denoted by R^2_{ℓ} and R^2_z in the table. Finally, the table also reports the correlation between the sector-specific labor and TFP trends, $(g^{\ell}_{u,i,t}, g^{z}_{u,i,t})$, in each sector and the correlation between the common trends $(g^{\ell}_{f,t}, g^{z}_{f,t})$.

Looking first at the median values of the factor loadings, Agriculture, Finance, Insurance and Real Estate (FIRE (excluding housing)), and Professional and Business Services (PBS) have the largest factor loadings for labor, and Transportation and Warehousing, and Durable and Nondurable Goods have the smallest. Utilities, Durable Goods and Construction have the largest loadings for TFP, while FIRE (x-housing) and Arts, Entertainment and Food Services have the smallest. The 68 percent credible intervals are relatively wide and give a quantitative sense of how information about the long-run is limited in our sample: the average width is 1.3 for λ^{ℓ} and 1.9 for λ^{z}. That said, for the majority of sectors, the posterior puts relatively little weight on negative values of the factor loadings. Section 5 summarizes the paper's key results for alternative models with tighter ($\eta = 0.5$) and looser ($\eta = 2.0$) priors for the factor loadings that give rise to correspondingly tighter or looser posteriors. Detailed results for these alternative models are available in Section 1 of the Technical Appendix.

The sectoral R^2 values are typically low indicating that common trend factors play a relatively muted role in explaining overall sectoral trends. For example, in Durable Goods, only 3 percent of the overall trend variation in labor and TFP growth is explained by their respective common trend factors. Notable exceptions for R^2_{ℓ} arise in several service sectors, for example in FIRE (x-housing) where 76 percent of the trend variation in labor is explained by the common trend factor. Interestingly, the posterior suggests that the sector-specific trends in labor and TFP are generally negatively correlated, rather dramatically so for Professional and Business Services. This negative correlation may reflect economic forces (such as input substitution) or potentially correlated measurement error in the measures of labor input and TFP.

The final row of the table shows the results for aggregate values of labor and TFP. By construction, the share-weighted factor loadings sum to unity. The common trends, $(g^{\ell}_{f,t}, g^{z}_{f,t})$, are also negatively correlated. The R^2 values are higher for the aggregates because aggregation reduces the importance of the sector-specific trends. The point estimates suggest that roughly 2/3 of the variation in the trend growth rate of labor is common across sectors while roughly 1/3 is common for TFP. However, one cannot directly infer from these findings the role that common growth trends in labor and TFP play in the overall trend growth rate of GDP. The reason is production linkages. In particular, the effective weight that each sector has in the aggregate

[19] Throughout the paper, we report 68 equal-tail percent credible intervals. Section 1 of the Technical Appendix also reports selected 90 percent credible intervals, which in some cases are markedly wider. We remind the reader that these long-run empirical results use only $q = 8$ independent observations on labor input and TFP for each of the 16 sectors.

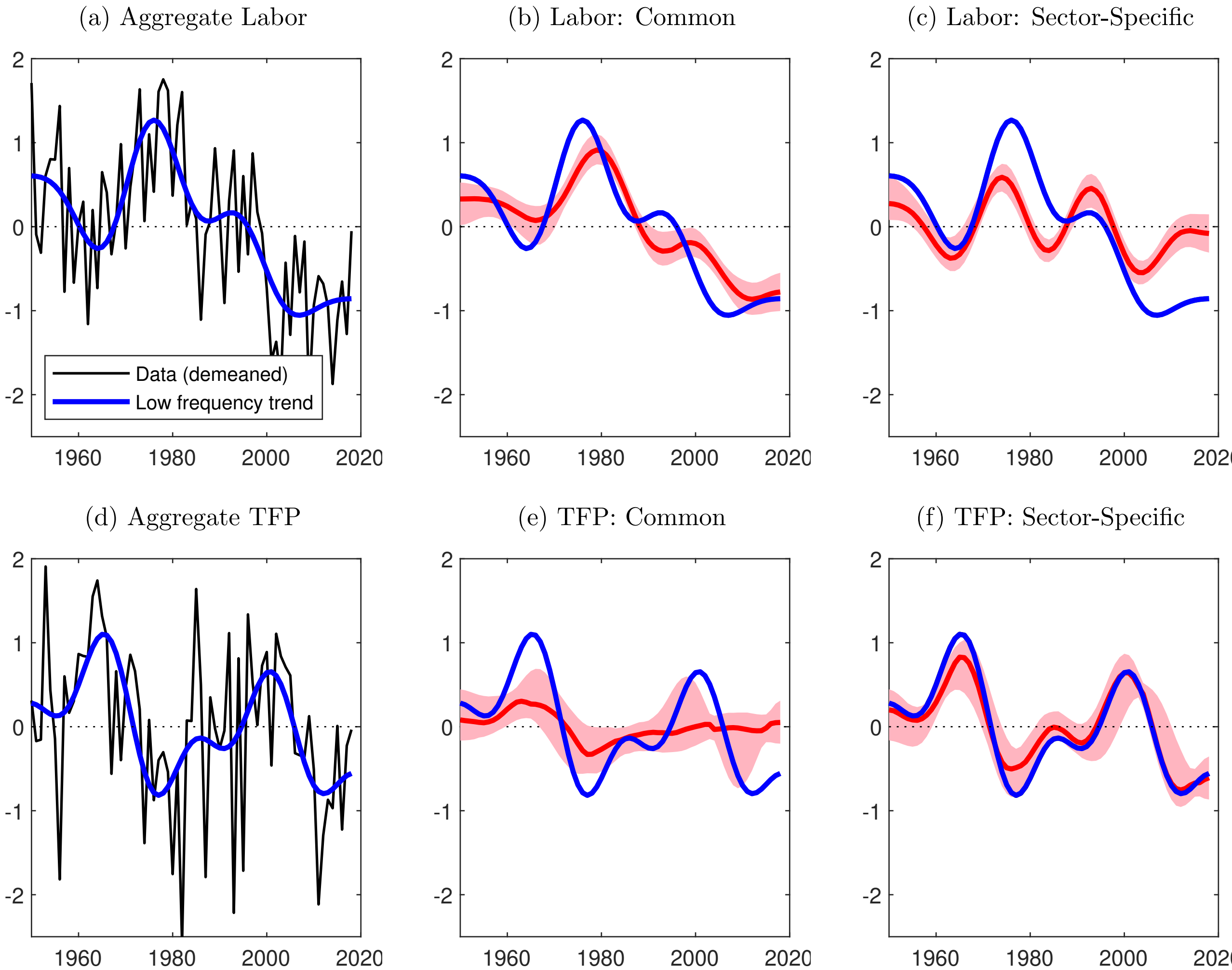
Table 3: Changes in Trend Value of Labor and TFP Growth Rates

	Sector	λ^ℓ	λ^z	R^2_ℓ	R^2_z	$corr(\ell, z)$
1	Agriculture	2.01	0.59	0.21	0.02	-0.32
		(1.24, 2.71)	(-0.59, 1.64)	(0.06, 0.44)	(0.00, 0.13)	(-0.52, -0.15)
2	Mining	0.73	1.10	0.01	0.01	-0.35
		(-0.17, 1.64)	(0.10, 2.09)	(0.00, 0.07)	(0.00, 0.04)	(-0.63, -0.06)
3	Utilities	1.13	1.36	0.24	0.05	0.22
		(0.41, 1.82)	(0.36, 2.35)	(0.04, 0.58)	(0.00, 0.29)	(-0.06, 0.58)
4	Construction	1.55	1.26	0.33	0.02	-0.25
		(0.95, 2.08)	(0.21, 2.66)	(0.10, 0.61)	(0.00, 0.19)	(-0.55, -0.04)
5	Durable Goods	0.40	1.31	0.03	0.03	-0.35
		(-0.23, 1.03)	(0.44, 2.17)	(0.00, 0.18)	(0.00, 0.15)	(-0.63, -0.05)
6	Nondurable Goods	0.59	1.22	0.06	0.04	-0.36
		(-0.20, 1.38)	(0.36, 2.13)	(0.01, 0.29)	(0.00, 0.23)	(-0.65, -0.06)
7	Wholesale Trade	1.09	0.88	0.53	0.04	0.20
		(0.62, 1.49)	(0.06, 1.74)	(0.17, 0.81)	(0.00, 0.20)	(-0.06, 0.53)
8	Retail Trade	0.80	1.14	0.26	0.05	0.06
		(0.26, 1.29)	(0.17, 2.82)	(0.04, 0.60)	(0.00, 0.85)	(-0.25, 0.62)
9	Trans. & Ware.	-0.04	0.88	0.05	0.06	0.06
		(-0.75, 0.72)	(-0.02, 1.79)	(0.00, 0.23)	(0.00, 0.28)	(-0.25, 0.36)
10	Information	1.34	0.77	0.22	0.03	-0.25
		(0.69, 2.01)	(-0.18, 1.81)	(0.04, 0.51)	(0.00, 0.19)	(-0.56, -0.00)
11	FIRE (x-Housing)	1.92	0.35	0.76	0.08	0.01
		(1.34, 2.48)	(-0.42, 1.34)	(0.35, 0.92)	(0.01, 0.40)	(-0.41, 0.40)
12	PBS	1.87	0.90	0.64	0.06	-0.92
		(1.48, 2.29)	(-0.01, 1.80)	(0.31, 0.87)	(0.00, 0.39)	(-0.98, -0.67)
13	Educ. & Health	0.59	1.36	0.16	0.10	-0.63
		(-0.06, 1.05)	(0.24, 2.49)	(0.01, 0.56)	(0.01, 0.54)	(-0.88, -0.26)
14	Arts, Ent., & Food Svc.	1.19	0.37	0.37	0.05	-0.18
		(0.69, 1.75)	(-0.39, 1.31)	(0.11, 0.67)	(0.00, 0.26)	(-0.51, 0.02)
15	Other Services (x-Gov)	0.68	0.74	0.06	0.02	-0.07
		(-0.10, 1.48)	(-0.10, 1.63)	(0.01, 0.23)	(0.00, 0.10)	(-0.35, 0.17)
16	Housing	0.82	0.75	0.01	0.10	0.07
		(-0.13, 1.74)	(0.08, 1.50)	(0.00, 0.04)	(0.01, 0.44)	(-0.21, 0.40)
	Aggregate	1.0	1.0	0.67	0.30	-0.29
				(0.48, 0.82)	(0.10, 0.58)	(-0.72, -0.13)

Notes: The estimates are posterior medians with 68 percent credible intervals shown parentheses. The entries under $\text{cor}(\ell, z)$ are the correlations between $g^\ell_{u,i,t}$ and $g^z_{u,i,t}$ for the rows corresponding to sectors, and correlations between $g^\ell_{f,t}$ and $g^z_{f,t}$ for the row labeled *Aggregate*.

economy can differ considerably from its value added share in GDP. Thus, as we show below, idiosyncratic trends in sectors such as Construction or Durable Goods, with extensive linkages to other sectors as input providers, will have outsize influence on the aggregate trend.

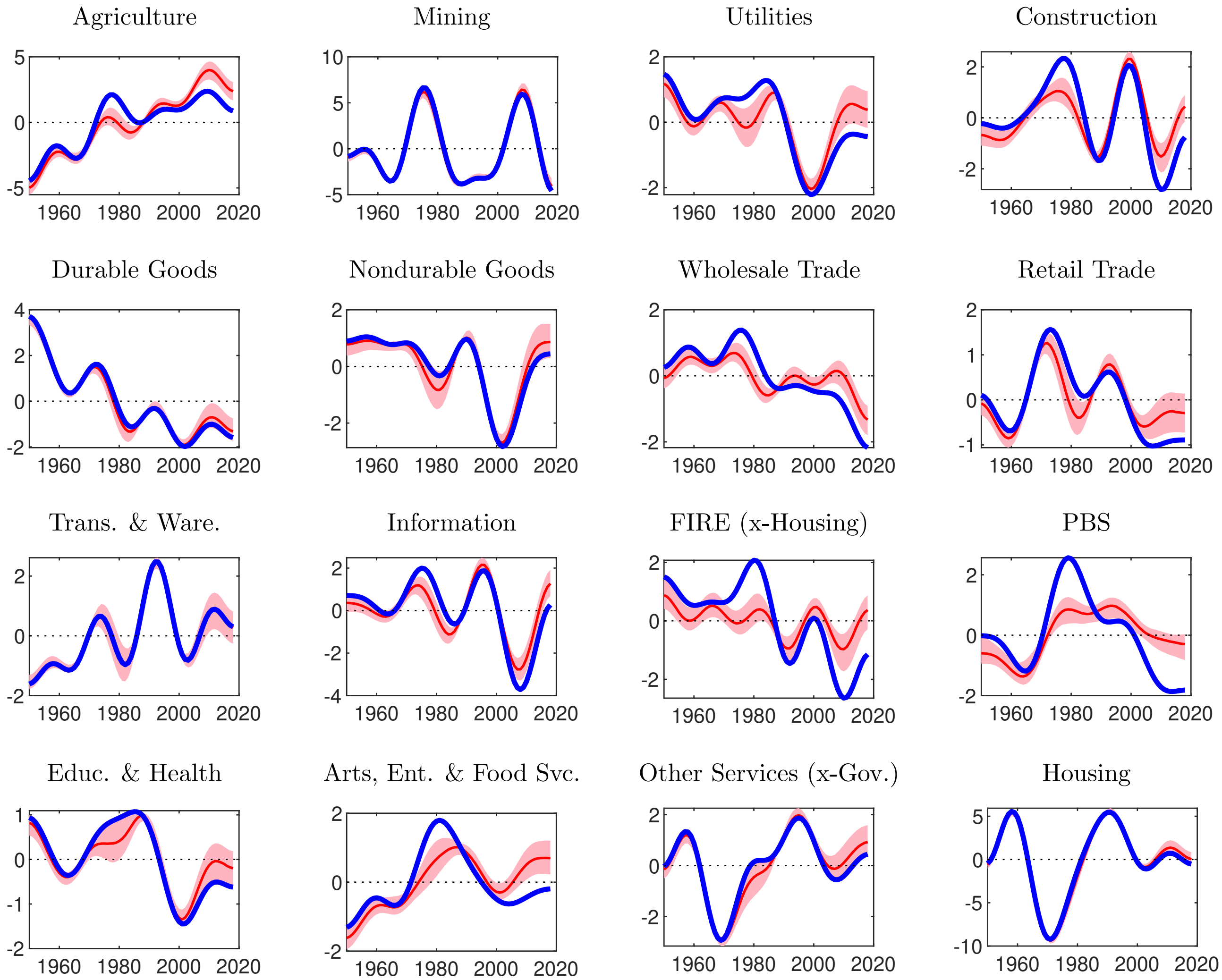
Figure 5: Aggregate Trend Growth Rates in Labor and TFP: Common and Sector-Specific Components (percentage points at annual rate)



Notes: Panels (a) and (d) show the growth rates (deviated from their sample mean) and the low-frequency trend. The other panels show the low-frequency trend and its decomposition into common and sector-specific components. The red lines denote the posterior median and the shaded areas are (pointwise) equal-tail 68% credible intervals.

Figure 5 shows a historical decomposition of the trends in aggregate labor and TFP growth rates arising from the common factors, (g_f^ℓ, g_f^z), and sector specific components, $\{g_{u,i}^\ell, g_{u,i}^z\}_{i=1}^{16}$. Panels (a) and (d) show the (demeaned) values of the aggregate growth rates with the associated low frequency trend. The other panels decompose the trend into its common (panels (b) and (e)) and sector-specific components (panels (c) and (f)). This decomposition relies on standard signal extraction formulas to compute the posterior distribution of $(\mathbf{F}, \mathbf{U})$ given $\mathbf{X}$ and the resulting trends, and the figure includes 68% (pointwise) credible intervals for common and sector-specific trends that incorporate uncertainty about the model's parameter values. Figure 5, panel (b), suggests that much of the increase in the trend growth rate of aggregate labor in

Figure 6: Labor Trends and Sector-Specific Components
(percentage points at annual rate)

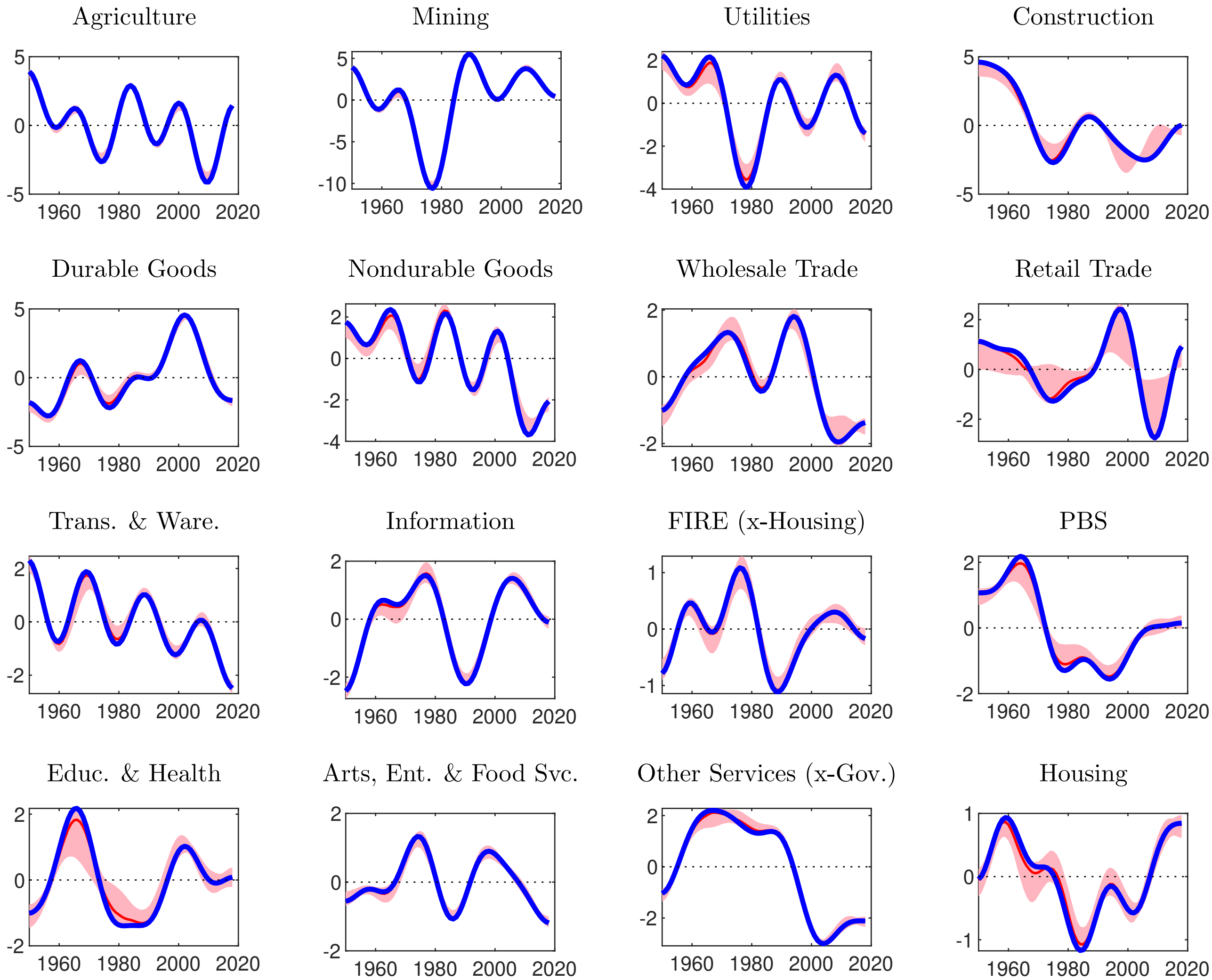


Notes: Each panel shows the low-frequency trend for sectoral growth rate (in blue) and its sector-specific component (in red). The red lines denote the posterior median and the shaded areas are (pointwise) equal-tail 68% credible intervals.

the 1960s and 70s, and subsequent decline in the 80s and 90s (both typically associated with demographics), are captured by the model's common factor in labor. Sector-specific labor factors, for the most part, played a supporting role. In contrast, while the model's aggregate common factor played a role in the decline of trend TFP growth the 1970s, the low frequency variation in the series since then has been associated almost exclusively with sector-specific sources.

Figures 6 and 7 present the trend growth rates for each of the sectors (shown previoulsy in Figure 5) along with the estimated sector-specific ($g^{\ell}_{u,i,t}, g^{z}_{u,i,t}$) components. Consistent with the R^2 values shown in Table 3, much of the variation in the trend growth rates of sectoral TFP and labor is associated with sector-

Figure 7: TFP Trends and Sector-Specific Components
(percentage points at annual rate)



Notes: See notes to Figure 6.

specific factors, and this is particularly true for TFP. Notable in Figures 6 and 7 is the negative correlation in the low frequency components of labor and TFP sectoral growth. Addressing this somewhat surprising finding is beyond the scope of this paper but we nevertheless underscore it as an interesting observation.

4 Sectoral Trends and the Aggregate Economy

Given the evolution of sectoral trend growth rates for labor and TFP over the past 70 years, this section explores their historical implications for the aggregate economy and GDP growth. The key consideration here is that production sectors are interconnected because each sector uses capital goods and materials produced in other sectors. Therefore, we consider a multi-sector growth model that features these interactions and allows for less than full sector-specific capital depreciation within the period. Consistent with our TFP calculations in Section 3, the model features competitive product and input markets.

The empirical specification in Section 3 distinguishes between idiosyncratic and common sources of changes in sectoral trend growth rates. While the importance of idiosyncratic disturbances is well documented for aggregate cyclical fluctuations, here we study the relative historical importance of sector-specific trends for the trend growth rate of GDP. We consider a structural framework with preferences and technologies that are unit elastic so that the economy evolves along a balanced growth path in the long run. Given linkages across sectors, changes in the growth rate of labor or TFP in one sector affect not only its own value added but also that of all other sectors. Specifically, we show that capital induces network effects that amplify the repercussions of sector-specific sources of growth on the aggregate economy and that we summarize in terms of sectoral multipliers. The magnitude of the multiplier associated with a given sector depends importantly on the extent to which it serves as supplier of capital goods and, to a lesser degree, materials to other sectors.

This section begins by outlining the general n-sector model that we use in our quantitative analysis. After introducing the general model, we present several special cases using $n = 2$ sectors to highlight the important mechanisms at work, and then return to the general n-sector model.

4.1 Economic Environment

Consider an economy with n distinct sectors of production indexed by j (or i). A representative household derives utility from these n goods according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \prod_{j=1}^{n} \left(\frac{c_{j,t}}{\theta_j} \right)^{\theta_j}, \ \sum_{j=1}^{n} \theta_j = 1, \ \theta_j \geq 0,$$

where θ_j is the household's expenditure share on final good j.

Each sector produces a quantity, $y_{j,t}$, of good j at date t, using a value added aggregate, $v_{j,t}$, and a materials aggregate, $m_{j,t}$, using the technology,

$$y_{j,t} = \left(\frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{m_{j,t}}{1-\gamma_j} \right)^{(1-\gamma_j)}, \ \gamma_j \in [0, 1].$$

The quantity of materials aggregate, $m_{j,t}$, used in sector j is produced with the technology,

$$m_{j,t} = \prod_{i=1}^{n} \left(\frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}}, \ \sum_{i=1}^{n} \phi_{ij} = 1, \ \phi_{ij} \geq 0,$$

where $m_{ij,t}$ denotes materials purchased from sector i by sector j. The notion that every sector potentially uses materials from every other sector introduces a first source of interconnectedness in the economy. An input-output (IO) matrix is an $n \times n$ matrix Φ with typical element ϕ_{ij}. The columns of Φ add up to the degree of returns to scale in materials for each sector, in this case unity. The row sums of Φ summarize the importance of each sector as a supplier of materials to all other sectors. Thus, the rows and columns of Φ reflect "sell to" and "buy from" shares, respectively, for each sector.

The value added aggregate, $v_{j,t}$, from sector j is produced using capital, $k_{j,t}$, and labor, $\ell_{j,t}$, according to

$$v_{j,t} = z_{j,t} \left(\frac{k_{j,t}}{\alpha_j}\right)^{\alpha_j} \left(\frac{\ell_{j,t}}{1-\alpha_j}\right)^{1-\alpha_j}, \ \alpha_j \in [0,1].$$

Capital accumulation in each sector follows

$$k_{j,t+1} = x_{j,t} + (1-\delta_j)k_{j,t},$$

where $x_{j,t}$ represents investment in new capital in sector j, and $\delta_j \in (0,1)$ is the depreciation rate specific to that sector. Investment in each sector j is produced using the quantity, $x_{ij,t}$, of sector i goods by way of the technology,

$$x_{j,t} = \prod_{i=1}^{n} \left(\frac{x_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}}, \ \sum_{i=1}^{n} \omega_{ij} = 1, \ \omega_{ij} \geq 0.$$

Thus, there exists a second source of interconnectedness in this economy in that new capital goods in every sector are potentially produced using the output of other sectors. This additional source of linkages in the economy has often been absent from structural multi-sector studies (for example, Acemoglu et al., 2012, 2017; Baqaee and Farhi, 2019) though it is shown to be a key propagation mechanism over the business cycle in recent work by vom Lehn and Winberry (2019). Similarly to the IO matrix, a Capital Flow matrix is an $n \times n$ matrix Ω with typical element ω_{ij}. The columns of Ω add up to the degree of returns to scale in investment for each sector which is unity in this model. The row sums of Ω indicate the importance of each sector as a supplier of new capital to all other sectors.

The resource constraint in each sector j is given by

$$c_{j,t} + \sum_{i=1}^{n} m_{ji,t} + \sum_{i=1}^{n} x_{ji,t} = y_{j,t}.$$

Sectoral change is defined by changes in the composite variable, $A_{j,t}$, that reflect the joint behavior of both TFP and labor growth. In particular, under the maintained assumptions, sectoral value added may be alternatively expressed as

$$v_{j,t} = A_{j,t} \left(\frac{k_{j,t}}{\alpha_j}\right)^{\alpha_j},$$

where

$$\Delta \ln A_{j,t} = \Delta \ln z_{j,t} + (1-\alpha_j)\Delta \ln \ell_{j,t}. \tag{4}$$

In this paper, we condition on the observed joint behavior of TFP and labor growth rates, $\{\Delta \ln z_{j,t}, \Delta \ln \ell_{j,t}\}$, in each sector j and derive their implications for aggregate value added or GDP growth. The model allows us to flesh out and quantify the way in which capital and the network features of production amplify the aggregate implications of sectoral growth. In particular, we show that sectors that act mainly as producers of capital for other sectors, and to a lesser degree materials, have an outsize influence on GDP growth. We provide general growth accounting expressions that quantify this effect for every sector given its specific production linkages to all other sectors.

While we condition on observed labor growth rates, the growth accounting expressions we derive are largely unchanged in a model where the allocation of labor is endogenous. In particular, a conventional treatment of labor supply produces a growth expression that is isomorphic to that presented below. In that expression, the way in which capital accumulation and the network features of production determine the influence of different sectors on aggregate growth is unchanged, as are the effects of long-run changes in TFP growth on GDP growth. The key difference is that with endogenous labor supply, the common and idiosyncratic components of labor input now carry a structural interpretation. Specifically, the common component is associated with broad demographics such as population growth and how these demographics affect labor input in each sector. The idiosyncratic component reflects sector-specific factors such as those which determine the disutility cost of working in different sectors, including a sector-specific Frisch elasticity, or sector-specific labor quality adjustments.[20]

For ease of presentation, we use the following notation throughout the paper: we denote the vector of household expenditure shares by $\Theta = (\theta_1, ..., \theta_n)$, the matrix summarizing value added shares in gross output in different sectors by $\Gamma_d = diag\{\gamma_j\}$, the matrix of input-output linkages by $\Phi = \{\phi_{ij}\}$, the capital flow matrix by $\Omega = \{\omega_{ij}\}$, the matrix summarizing capital shares in value added in different sectors by $\alpha_d = diag\{\alpha_j\}$, and the matrix summarizing sector-specific depreciation rates by $\delta_d = diag\{\delta_j\}$.

4.2 Balanced Growth and Sectoral Multipliers

We consider a balanced growth path where the growth rates of TFP and labor in sector j are given by g_j^z and g_j^ℓ respectively. From equation (4), it follows that along that path,

$$\Delta \ln A_{j,t} = g_j^a = g_j^z + (1 - \alpha_j)\, g_j^\ell.$$

Furthermore, as highlighted in our empirical section, we let

$$g_j^z = \lambda_j^z g_f^z + g_{u,j}^z \text{ and } g_j^\ell = \lambda_j^\ell g_f^\ell + g_{u,j}^\ell.$$

[20] See section 4 of the Technical Appendix. The interpretation or identification of sources of labor growth will necessarily depend on the particular model of endogenous labor supply under consideration. Because our focus is on growth accounting (rather than counterfactuals), we take the observations on labor growth as given whatever their underlying forces. Ngai and Pissarides (2007) explore an alternative framework where the reallocation of labor among consumption goods sectors is an outcome of unbalanced growth among those goods while, at the same time, preserving balanced growth at the aggregate level. Absent from their work, however, are the network considerations and the role of capital in determining network multipliers that are central to this paper.

In other words, composite sources of sectoral growth in the steady state, g_j^a, reflect steady state sectoral TFP growth, g_j^z, and sectoral labor growth, g_j^ℓ. The growth rates of these inputs in turn reflect both common (aggregate) factors, $(\lambda_j^z g_f^z, \lambda_j^\ell g_f^\ell)$, and unique idiosyncratic components, $(g_{u,j}^z, g_{u,j}^\ell)$.

We now show that because of production linkages, sources of change in an individual sector, g_j^a, help determine value added growth in every other sector along the balanced growth path. These linkages, therefore, amplify the effects of sector-specific change on GDP growth, particularly for sectors that produce capital for other sectors. One can summarize this amplification effect by way of a multiplier for each sector. As we will see, these multipliers scale the influence of some sectors on GDP growth by up to multiple times their share in the economy.

Let $g_t^v = (g_{1,t}^v, ...g_{n,t}^v)'$ denote the vector of value added growth by sector at any date t. Then, along the balanced growth path, g_t^v is constant and given by

$$g^v = \left[I + \alpha_d \Omega' \underbrace{\left(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d)\Phi' \right)^{-1} \Gamma_d}_{\Xi'} \right] g^a, \tag{5}$$

where $g^a = (g_1^a, ..., g_n^a)'$.[21] Equation (5) describes how the sources of growth in a given sector, g_j^a, affects value added growth in all other sectors, g_i^v. This relationship involves the direct effects of sectors' TFP and labor growth on their own value added growth, Ig^a, and the indirect effects that sectors have on other sectors through the economy's sectoral network of investment and materials, $\alpha_d \Omega' \Xi' g^a$. Specifically,

$$\frac{\partial g_j^v}{\partial g_j^a} = 1 + \alpha_j \sum_{k=1}^{n} \omega_{kj} \xi_{jk} \text{ and } \frac{\partial g_i^v}{\partial g_j^a} = \alpha_i \sum_{k=1}^{n} \omega_{ki} \xi_{jk}, \tag{6}$$

where $(\xi_{j1}, ..., \xi_{jn})$ is the jth column of Ξ' which denotes the generalized Leontief inverse, $(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d)\Phi')^{-1}\Gamma_d$, in equation (5). Thus, along the balanced growth path, sectoral linkages make it possible for sources of change by way of TFP or labor in a given sector j, ∂g_j^a, to affect value added growth in every other sector, i, so long as that sector uses capital in production, $\alpha_i > 0$. Otherwise, value added growth in a sector with $\alpha_j = 0$ is entirely determined by its own input growth rates, $\frac{\partial g_j^v}{\partial g_j^a} = 1$. In this sense, the presence of capital accumulation plays a central role for the sectoral growth implications of production linkages. In the following subsection we illustrate the mechanisms at play in a series of simplified models with $n = 2$ sectors.

Given the vector of value added growth rates, g^v, the Divisia aggregate index of GDP growth is $g^V = s^{v\prime} g^v$ where $s^v = (s_1^v, ..., s_n^v)$ is a vector of sectoral value added shares in GDP. Alternatively,

$$g^V = \sum_{j=1}^{n} s_j^v \left[g_j^a + \sum_{i=1}^{n} \alpha_j \omega_{ij} \sum_{k=1}^{n} \xi_{ki} g_k^a \right], \tag{7}$$

[21]See Appendix A for the derivation. Observe also that preference parameters are absent from equation (5) in that balanced growth relationships are ultimately statements about technologies and resource constraints.

so that, holding shares constant,

$$\frac{\partial g^V}{\partial g_j^a} = s_j^v + s_j^v \alpha_j \sum_{k=1}^{n} \omega_{kj} \xi_{jk} + \sum_{i \neq j} s_i^v \alpha_i \sum_{k=1}^{n} \omega_{ki} \xi_{jk}, \tag{8}$$

where the second and third terms in this last expression reflect network sectoral effects by way of the generalized Leontief inverse, Ξ'.[22] Equation (8) defines the *sectoral multiplier* for sector j.

When no sector uses capital in production ($\alpha_j = 0\ \forall j$), the sources of input growth a given sector j, g_j^a, affect GDP growth through that sector's share in the economy only, $\frac{\partial g^V}{\partial g_j^a} = s_j^v$. This case recovers a version of Hulten's theorem (1978) - discussed below - but in growth rates. More generally, equation (8) suggests the presence of a network multiplier effect that varies by sector and that depends not only on the importance of sectoral interactions, ξ_{jk}, but also on the extent to which sectors use capital produced by other sectors in their own production, ω_{ki} and α_i. In equation (8), sources of growth in sector j influence every other sector k through the network of production linkages summarized by the generalized Leontief inverse, ξ_{jk}. Induced changes in sector k in turn potentially affect investment in every other sector i, ω_{ki}, (including back to j). The net effect on GDP growth is the sum of all these interactions. Conveniently, the effects of sectoral changes, ∂g^a, on GDP growth may be thought of as a direct effect, $s^v I$, and an additional indirect effect resulting from sectoral linkages, $s^{v\prime} \alpha_d \Omega' \Xi'$. Hence, we define the combined direct and indirect effects of structural change on GDP growth in terms of sectoral multipliers, $s^{v\prime}(I + \alpha_d \Omega' \Xi')$.

4.3 Examples and Relationship to Greenwood, Hercowitz, Krusell (1997)

To gain intuition, this section discusses expressions (5) and (7) above in the context of special cases exemplified in previous work. In particular, we provide examples of sectoral multipliers in Greenwood, Hercowitz, and Krusell (1997) (henceforth GHK (1997)) and variations thereof. The Technical Appendix also discusses the link with Ngai and Pissarides (2007). These examples help underscore the role of capital-producing sectors for the strength of sectoral multipliers. In these examples, goods and factor markets are perfectly competitive and factors of production are freely mobile across sectors. However, as we also make clear, the way in which sectoral sources of growth are amplified at the aggregate level is invariant to the assumption of factor mobility. Details are included in Section 3 of the Technical Appendix.

To establish the link between our economic environment and that of GHK (1997), observe first that the one-sector environment featuring an aggregate production function in GHK (1997) also has an interpretation as a two-sector economy.[23] Under that interpretation, one sector produces consumption goods (sector 1) and the other investment goods (sector 2), and each sector's production function has the same capital

[22]In general, sectoral value added shares in GDP, s^v, will also be functions of the model's underlying parameters including the vector of sources of sectoral growth, g^a. However, changes in sectoral shares induced by an exogenous change in a sector k, $\frac{\partial s_j^v}{\partial g_k^a}$, will be mostly inconsequential for overall growth, consistent with Figure 1 and the notion that since shares must sum to 1, $\sum_j \frac{\partial s_j^v}{\partial g_k^a} = 0$.

[23]See Greenwood, Hercowitz, and Krusell (1997), Section V. A.

elasticity, α. For simplicity, we focus on the discussion in section III of GHK (1997) which abstracts from the distinction between equipment and structures. Thus, consider a two-sector economy with production given by

$$c_t = y_{1,t} = z_{1,t} k_{1,t}^{\alpha} \ell_{1,t}^{1-\alpha},$$

$$x_t = y_{2,t} = z_{2,t} k_{2,t}^{\alpha} \ell_{2,t}^{1-\alpha},$$

$$k_{t+1} = x_t + (1-\delta) k_t,$$

where factors are freely mobile, $k_t = k_{1,t} + k_{2,t}$ and $\ell_t = \ell_{1,t} + \ell_{2,t}$, and the constant scale factors in production (which simplify the algebra in the full model) have been dropped. Under the maintained assumptions, this two-sector environment reduces to the one-sector framework with aggregate production described in GHK (1997). Put another way, there exists a one-sector interpretation of the two-sector economy with associated resource constraint,

$$c_t + q_t x_t = z_{1,t} k_t^{\alpha} \ell_t^{1-\alpha},$$

where $q_t = \frac{z_{1,t}}{z_{2,t}}$ is the relative price of investment goods and where aggregate output (in units of consumption goods), $y_t = c_t + q_t x_t$, is a function of total factor endowment only, $z_{1,t} k_t^{\alpha} \ell_t^{1-\alpha}$. To the extent that technical progress in the investment sector, $z_{2,t}$, is generally more pronounced than in the consumption sector, $z_{1,t}$, the relative price of investment goods will decline over time as emphasized by GHK (1997).

We now derive the balanced growth path (BGP) in GHK (1997) and discuss its implications for sectoral multipliers. That is, we highlight how capital accumulation amplifies sectoral drivers of growth. This also means that capital producing sectors will tend to have an outsize effect on the aggregate economy relative to sectors that produce mainly consumption goods.

Along the BGP, all variables grow at constant but potentially different rates. From the market clearing conditions and the form of production technologies, it follows that sectoral output growth rates, g_i^v, are given by,

$$g_i^v = \left(g_i^z + (1-\alpha) g^{\ell} \right) + \alpha g^k = g_i^a + \alpha g^k, \quad i = 1, 2. \tag{9}$$

Equation (9) makes clear that any amplification of sectoral sources of growth, g_i^a, can only take place through capital accumulation. In this case, it follows from the capital accumulation equation that along the BGP, capital grows at the same rate as investment which, in the capital goods producing sector, is also that of output. Thus, we have that

$$g_2^v = g^k = \frac{1}{1-\alpha} g_2^a \quad \text{and} \quad g_1^v = g_1^a + \frac{\alpha}{1-\alpha} g_2^a. \tag{10}$$

Note that the assumption of factor mobility across sectors has only minor implications for the characterization of the BGP. First, even with sector-specific investment, the resource constraint for investment implies that investment and capital grow at the same rate in each sector. Second, with sector-specific labor, the expression for output growth remains as in equation (9) with the only difference being that sector-specific labor growth rates, g_i^ℓ, now replace the aggregate labor growth rate, g^ℓ, so that $g_i^a = g_i^z + (1-\alpha)g_i^\ell$.

Aggregate GDP growth is defined as the Divisia index of sectoral value-added growth rates weighted by their respective value added shares. Because GHK (1997) do not consider intermediate goods, there is no distinction between gross output and value added in equation (9). Thus, from equation (10), aggregate GDP growth is

$$g^V = s_1^v \left(g_1^a + \frac{\alpha}{1-\alpha} g_2^a \right) + s_2^v \frac{1}{1-\alpha} g_2^a, \tag{11}$$

or alternatively,

$$g^V = s_1^v g_1^a + s_2^v g_2^a + \frac{\alpha}{1-\alpha} g_2^a, \tag{12}$$

where s_i^v is sector i's value-added share in GDP.

In this economy, sector 2 is the sole producer of capital for both sectors 1 and 2 and has both a direct and indirect effect on the aggregate economy. The indirect effect stems from the fact that capital accumulation amplifies the role of sectoral sources of growth. In equation (11), sector 2 contributes $\frac{\alpha}{1-\alpha} g_2^a > 0$ to value added growth in sector 1 and scales its contributions from TFP and labor to its own value added growth by $\frac{1}{1-\alpha} > 1$. Thus, in equation (12), the direct aggregate effect of an expansion in sector 2 by way of TFP or labor growth is its share, s_2^v, while its indirect aggregate effect is $\frac{\alpha}{1-\alpha} > 0$. It follows that sector 2's sectoral multiplier, $\partial g^V / \partial g_2^a$, is $s_2^v + \frac{\alpha}{1-\alpha}$. In contrast, because sector 1 produces goods that are only fit for consumption, it only has a direct effect on the aggregate economy. Its sectoral multiplier, $\partial g^V / \partial g_1^a$, is then simply its share in GDP, s_1^v.

A straightforward application of the general framework laid out in the previous section produces the same balanced growth path and sectoral multipliers for sectors 1 and 2 that we have just discussed. In particular, the GHK (1997) economy is a special case with $n = 2$ and, since sector 2 is the only sector producing investment goods, $\omega_{2j} = 1, j = 1, 2$ (and $\omega_{1j} = 0, j = 1, 2$). In addition, each good is produced without intermediate inputs, $\gamma_j = 1, j = 1, 2$, and the sectors use the same production functions, $\alpha_j = \alpha, j = 1, 2$, except for the scale factors, $z_{j,t}, j = 1, 2$. With these restrictions, the parameters of the model are summarized by $\Gamma_d = I$, $\alpha_d = \alpha I$ and

$$\Omega' = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix},$$

where this last matrix reflects the production structure whereby all capital in the economy is produced by sector 2. Then, the generalized Leontief inverse is

$$\Xi' = (I - \alpha\Omega')^{-1} = \begin{pmatrix} 1 & \frac{\alpha}{1-\alpha} \\ 0 & \frac{1}{1-\alpha} \end{pmatrix}.$$

Observe that the first and second rows of the generalized Leontief inverse contain the coefficients that determine the contributions of sources of growth in sectors 1 and 2 to value added growth in sectors 1 and 2 in equation (10). The model thus delivers the same balance growth path for GDP with associated sectoral multipliers given by the elements of $s^{v\prime}\left[I + \alpha_d \Omega' \Xi'\right]$,

$$\frac{\partial g^V}{\partial g_1^a} = s_1^v, \quad \text{and} \quad \frac{\partial g^V}{\partial g_2^a} = s_2^v + \frac{\alpha}{1-\alpha}.$$

As discussed above, as the sole producer of capital goods, sector 2 has both a direct effect, s_2^v, and an indirect effect, $\frac{\alpha}{1-\alpha}$, on GDP growth. In contrast, sector 1 only has a direct effect on GDP growth, s_1^v.

Actual production linkages are generally more involved than those we have just discussed. Importantly, even in the context of two sectors and no materials, the simple fact that factor income shares differ across sectors is enough to prohibit an aggregate production function and thus a one-sector interpretation of the economic environment.[24] In this case, the amplification of sources of sectoral growth on GDP growth now depends on a value-added-share weighted average of capital elasticities. In particular, long-run GDP growth is now given by

$$g^V = s_1^v g_1^a + s_2^v g_2^a + \frac{(s_1^v \alpha_1 + s_2^v \alpha_2)}{1-\alpha_2} g_2^a.$$

Thus, the sectoral multipliers for sectors 1 and 2 are now respectively,

$$\frac{\partial g^V}{\partial g_1^a} = s_1^v \quad \text{and} \quad \frac{\partial g^V}{\partial g_2^a} = s_2^v + \frac{(s_1^v \alpha_1 + s_2^v \alpha_2)}{1-\alpha_2}.$$

With different factor shares, sector 2 continues to have an additional indirect effect on GDP growth, $\frac{(s_1^v \alpha_1 + s_2^v \alpha_2)}{1-\alpha_2}$, that depends for the most part on its own capital elasticity, α_2. As $\alpha_2 \to 0$, this indirect effect tends to $s_1^v \alpha_1 < 1$. Hence, even when sector 2 uses mostly labor in production, it nevertheless has an effect on aggregate growth over and above its direct effect (i.e., its value added share, s_2^v) since it remains a supplier of capital goods to sector 1. In this case, however, this indirect effect is entirely determined by parameters of sector 1, specifically its importance as measured by its value added share in GDP, s_1^v, scaled by the intensity with which it uses capital to produce consumption goods, α_1.[25]

Actual production linkages are more involved still in that they also reflect a network of materials between sectors. Thus, we now introduce intermediate goods into the GHK (1997) environment. With intermediate inputs, additional sectoral contributions to value-added growth continue to arise through the capital growth rate. However, when the consumption sector (sector 1) also produces materials for the investment goods sector (sector 2), the growth rate of capital depends on conditions in both sectors 1 and 2. This means that in contrast to the previous two examples, both sectors 1 and 2 will have indirect effects on long-run GDP growth over and above their share in the economy.

We illustrate these points via a simple network of intermediate goods. Here, sector 1 produces not only consumption goods but also materials, $m_{1,t}$, used by sector 2. Similarly, sector 2 still produces capital

[24] See Greenwood, Hercowitz, and Krusell (1997), Section V. A.

[25] Observe also that as $\alpha_2 \to 1$, the indirect effect becomes ill-defined since the derivation of the BGP assumes exogenous forces, g_i^z and g_i^ℓ, whereas in the limit where the capital elasticity is one, the model becomes an AK-type endogenous growth model.

goods for both sectors but also materials, $m_{2,t}$, used by sector 1. Since sector 1 now produces consumption goods and intermediate goods, we refer to sector 1 as the non-durables sector. Thus, in terms of our general notation, we have that $\gamma_i \neq 1$ and $\omega_{2,i} = 1$ for $i = 1, 2$. Moreover, the relevant resource constraints in sectors 1 and 2 are now

$$c_t + m_{1,t} = y_{1.t} = \left[z_{1,t} k_{1,t}^{\alpha_1} \ell_{1,t}^{1-\alpha_1}\right]^{\gamma_1} m_{2,t}^{1-\gamma_1},$$

and

$$x_t + m_{2,t} = y_{2,t} = \left[z_{2,t} k_{2,t}^{\alpha_2} \ell_{2,t}^{1-\alpha_2}\right]^{\gamma_2} m_{1,t}^{1-\gamma_2},$$

while the rest of the production side of the economy is as in the previous examples.

As before, with perfect factor mobility it follows that $g^x = g^k = g_1^k = g_2^k$ and $g^\ell = g_1^\ell = g_2^\ell$, while from the goods market clearing conditions, we have that $g_1^y = g^c = g_1^m$ and $g_2^y = g^x = g_2^m$. The form of production, which now uses intermediate inputs, implies that gross output growth rates are

$$g_1^y = \gamma_1 \left[g_1^z + \alpha_1 g_2^y + (1-\alpha_1) g^\ell\right] + (1-\gamma_1) g_2^y,$$

and

$$g_2^y = \gamma_2 \left[g_2^z + \alpha_2 g_2^y + (1-\alpha_2) g^\ell\right] + (1-\gamma_2) g_1^y.$$

Therefore, solving for the growth rate of new capital goods, we obtain

$$g_2^y = \frac{(1-\gamma_2)\gamma_1 g_1^a + \gamma_2 g_2^a}{\Delta} = g^k, \tag{13}$$

where $\Delta = 1 - \gamma_2 \alpha_2 - (1-\gamma_2)\left[\gamma_1 \alpha_1 + (1-\gamma_1)\right]$.

With intermediate inputs, sectoral value added growth differs from gross output growth. In particular, value added growth is still determined as in the two previous examples without intermediate inputs, that is equation (9), $g_i^v = g_i^z + \alpha_i g^k + (1-\alpha_i) g^\ell = g_i^a + \alpha_i g^k$.

Long-run GDP growth, therefore, is given by

$$g^V = s_1^v g_1^a + s_2^v g_2^a + (s_1^v \alpha_1 + s_2^v \alpha_2) g^k, \tag{14}$$

where g^k follows from equation (13). Two important observations emerge relative to the previous examples. First, because the non-durable goods sector now produces intermediate inputs for the investment sector, the growth rate of (new) capital goods in equation (13) reflects sources of growth in both sectors, g_1^a and g_2^a. Hence, unlike in the previous examples, both sectors 1 and 2 in equation (14) will have an additional indirect effect on long-run GDP growth, $(s_1^v \alpha_1 + s_2^v \alpha_2)\frac{\partial g^k}{\partial g_1^a}$ and $(s_1^v \alpha_1 + s_2^v \alpha_2)\frac{\partial g^k}{\partial g_2^a}$ respectively, over and above their shares in the economy, s_1^v and s_2^v. Second, from equation (14), the indirect effect from sector 2 on GDP growth will dominate that from sector 1 if and only if its contributions to overall capital growth, $\frac{\partial g^k}{\partial g_2^a}$, are larger than the corresponding contributions from sector 1, $\frac{\partial g^k}{\partial g_1^a}$. Going back to equation (13), this condition holds if and only if

$$\gamma_2 > (1-\gamma_2)\gamma_1.$$

Put differently, this condition implies that the effect from a one percent change in TFP growth in sector 2 on overall capital growth, g^k, is larger than the corresponding effect from sector 1 transmitted through intermediate inputs. It will fail to hold, for example, in economies where the value added share in gross output of the capital sector, γ_2, is small. In that case, the main input into the production of capital goods are intermediate inputs from the non-durables sector. Therefore, it is that sector's conditions that matter most.

In the general framework we lay out, this economic environment is conveniently summarized by

$$\Omega' = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \ \alpha_d = \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix}, \ \Gamma_d = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}, \text{ and } \Phi' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Sectoral multipliers are then immediately given by the elements of $s^{v\prime}(I + \alpha_d \Omega' \Xi')$ or

$$\frac{\partial g^V}{\partial g_1^a} = s_1^v + \frac{(s_1^v \alpha_1 \gamma_1 (1-\gamma_2) + s_2^v \alpha_2 \gamma_1 (1-\gamma_2))}{\Delta},$$

and

$$\frac{\partial g^V}{\partial g_2^a} = s_2^v + \frac{s_1^v \alpha_1 \gamma_2 + s_2^v \alpha_2 \gamma_2}{\Delta},$$

which reproduces the intuition given above.

More generally, the key lesson from these examples is that network production linkages and capital accumulation are the key components underlying the existence of sectoral multipliers along the balanced growth path. Furthermore, the implied amplification of idiosyncratic sources of growth on GDP growth can arise in any sector, including those producing only non-durable goods so long as these goods contribute intermediate inputs, however indirectly, to sectors producing capital goods.

Finally, we relate our work to that of Hulten (1978) and, more recently, Baqaee and Farhi (2019). In its simplest form, Hulten (1978) states that the effects of a shock to productivity in a given sector on GDP is that sector's ratio of gross output to GDP, namely its Domar weight. This result hinges in part on interpreting TFP as scaling gross output. When TFP is instead interpreted as scaling value added as we do here, a sector's influence on GDP becomes its value added share in GDP.[26]

Baqaee and Farhi (2019) explore the role of non-linearities in generating aggregate effects from sectoral shocks over and above the linear approximations highlighted by Hulten (1978). Our work focuses instead

[26]These results are evidently related. When sectoral TFP, z_j, is measured as scaling value added, $\widetilde{z}_{j,t} = z_{j,t}^{\gamma_j}$ becomes the relevant scalar for sectoral gross output, where γ_j is j's value added share in gross output, $\frac{p_j^v v_j}{p_j^y y_j}$. Hulten's (1978) theorem then states that $\frac{\partial \ln V_t}{\partial \ln \widetilde{z}_{j,t}} = \mathscr{D}_j$, where $\mathscr{D}_j$ is sector j's Domar weight or ratio of gross output to GDP, $\frac{p_j^y y_j}{V}$. It immediately follows from the definition of $\widetilde{z}_j$ that $\frac{\partial \ln V_t}{\partial \ln z_{j,t}} = \gamma_j \mathscr{D}_j$, where $\gamma_j \mathscr{D}_j$ is then simply sector j's value added share in GDP, s_j^v.

on other key assumptions, unexplored in Hulten (1978) and related work, that nevertheless are central for understanding the aggregate implications of sectoral trends. One is the role that capital plays as part of a production network in amplifying the effects of sectoral changes on long-run GDP growth. Here, the long-run dynamics of capital accumulation are central to that role. Another is that Hulten (1978) and subsequent work focus mostly on level effects, $\partial \ln V / \partial \ln A$. Instead, motivated by Figure 1, we are interested in the effects of sectoral (trend) growth rates on GDP growth over the long-run, $\partial \Delta \ln V / \partial \Delta \ln A$.

Importantly, Hulten (1978)'s basic insight remains nested by a static version of our economic environment without capital and where the focus is on levels rather than growth rates. Section 5 of the Technical Appendix shows how this 'levels' insight changes in the steady state of a dynamic economy with capital. In particular, the effect of a productivity shock in a sector on the level of GDP is given by its value added share (or Domar weight) scaled by the inverse of the average labor income share.

4.4 The General Case

Using the insights from the special cases we have described, we separate value added growth, g^v, and gross output growth, g^y, implicit in equation (5), as follows,

$$g^v = g^a + \alpha_d \Omega' g^y \text{ and } g^y = \Xi' g^a, \text{ with } \Xi' = \left[I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d) \Phi' \right]^{-1} \Gamma_d. \tag{15}$$

The first expression, $g^v = g^a + \alpha_d \Omega' g^y$, states that value added growth in each sector is the sum of direct effects from sources of growth, g^a, and indirect effects arising from capital accumulation induced by gross output growth, $\alpha_d \Omega' g^y$. Thus, a sector will benefit from indirect effects only to the extent that it uses capital goods. The second expression, $g^y = \Xi' g^a$, then states that gross output growth in each sector is a multiple of the sources of growth in the different sectors, g^a, determined by the generalized Leontief inverse, Ξ'.

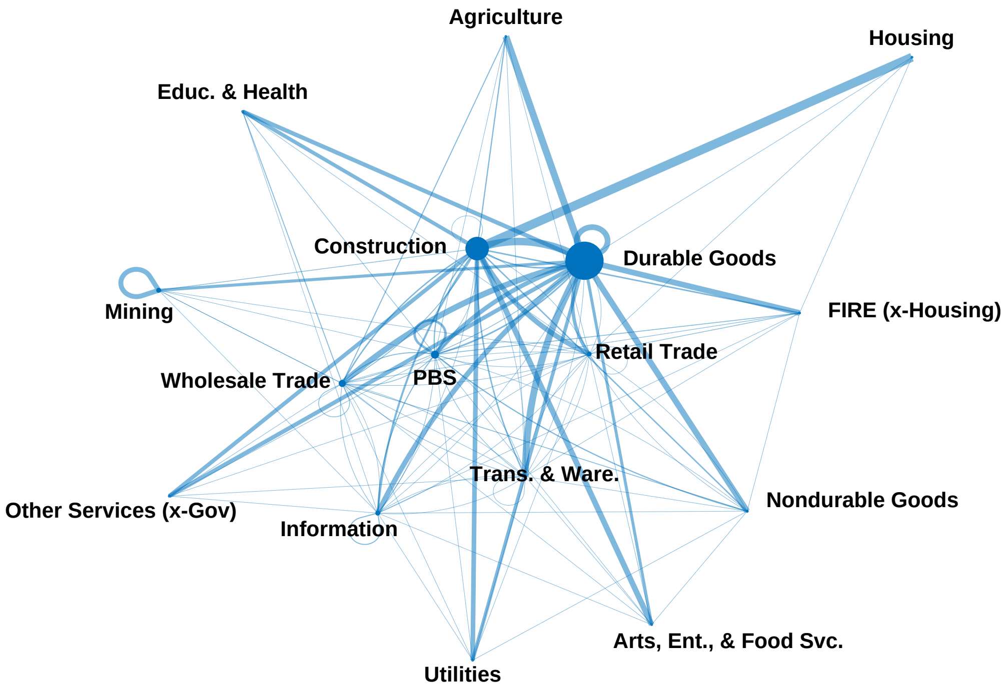
We can write the generalized Leontief inverse as a geometric expansion,

$$\Xi' = \Gamma_d + \sum_{k=1}^{\infty} \left[\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi' \right]^k \Gamma_d,$$

which allows the following interpretation of the network effects discussed above. Consider an initial growth vector, g^a, which has an initial impact on gross output, $\Gamma_d g^a$. This in turn leads to a second round of gross output growth induced by capital growth, through Ω', and intermediate input growth, through Φ'. This then creates a third round of gross output growth, and so on. The generalized Leontief inverse is the cumulative sum of this sequence of multiplier effects.

Note that in the geometric expansion of Ξ', the j^{th} column of the transposed capital flow matrix, Ω' (i.e., the j^{th} row of Ω) reflects the degree to which sector j produces new capital for other sectors. Similarly, the j^{th} column of the transposed IO matrix, Φ', (i.e., the j^{th} row of Φ) reflects the degree to which sector j produces materials for other sectors). Each row of Ω' and Φ' is weighted by the contributions of capital and intermediate inputs to that row's corresponding sector, that is its share of capital and materials in gross

Figure 8: Investment Network



Notes: This figure shows the investment network as a graph, where capital flows are represented by edges between nodes representing sectors. A sector with a larger node indicates that other sectors spend a larger share of their capital expenditures on average from that sector. A wider edge between two nodes reflects larger bi-directional capital flows relative to all other capital flows. See the Capital Flow Table in Section 8 of the Technical Appendix.

output, $\alpha_d \Gamma_d$ and $(I - \Gamma_d)$ respectively. The generalized Leontief inverse then inherits these effects such that its j^{th} column reflects the degree to which the j^{th} sector contributes to the production of capital goods directly and indirectly through the production of intermediate goods.

5 Quantitative Findings

This section puts together the empirical findings from section 3 and model insights from section 4. It shows that sector-specific trends have likely played a dominant role in driving the trend rate of growth in GDP over the postwar period. We estimate that this trend rate of growth has fallen by almost 3 percentage points between 1950 and today.

5.1 Model Parameters

Our choice of model parameters follows mostly Foerster et al. (2011) and is governed by the BEA Input-Output (IO) and Capital Flow accounts.

In our benchmark economy, the consumption bundle shares, $\{\theta_j\}$, value-added shares in gross output $\{\gamma_j\}$, capital shares in value added, $\{\alpha_j\}$, and material bundle shares, $\{\phi_{ij}\}$, are obtained from the 2015 BEA Make and Use Tables. The Make Table tracks the value of production of commodities by sector, while the Use Table measures the value of commodities used by each sector. We combine the Make and Use Tables to yield, for each sector, a table whose rows show the value of a sector's production going to other sectors (materials) and households (consumption), and whose columns show payments to other sectors (materials) as well as labor and capital. Thus, a column sum represents total payments from a given sector to all other sectors, while a row sum gives the importance of a sector as a supplier to other sectors. We then calculate material bundle shares, $\{\phi_{ij}\}$, as the fraction of all material payments from sector j that goes to sector i. Similarly, value-added shares in gross output, $\{\gamma_j\}$, are calculated as payments to capital and labor as a fraction of total expenditures by sector j, while capital shares in value added, $\{\alpha_j\}$, are payments to capital as a fraction of total payments to labor and capital. The consumption bundle parameters, $\{\theta_j\}$, are likewise payments for consumption to sector j as a fraction of total consumption expenditures.

The parameters that determine the production of investment goods, $\{\omega_{ij}\}$, are chosen similarly in accordance with the BEA Capital Flow table from 1997, the most recent year in which this flow table is available. The Capital Flow table shows the flow of new investment in equipment, software, and structures towards sectors that purchase or lease it. By matching commodity codes to sectors, we obtain a table that has entries showing the value of investment purchased by each sector from every other sector. A column sum represents total payments from a given sector for investment goods to all other sectors, while a row sum shows the importance of a sector as a supplier of investment goods to other sectors. Hence, the investment bundle shares, $\{\omega_{ij}\}$, are estimated as the fraction of payments for investment goods from sector j to sector i, expressed as a fraction of total investment expenditures made by sector j.

Conditional on these parameters, equation (5) gives sectoral value added growth along the balanced growth path. In constructing aggregate GDP growth from these sectoral value added growth rates, we rely on the same actual constant mean value added shares from the KLEMS data that were used in our empirical analysis. Recall also that in Figure 1, we explored using different definitions of value added shares in calculating GDP growth. While this did not lead to meaningful differences in aggregate growth, to the extent that these shares are changing over time as do input-output relationships, the model might nevertheless yield material differences in the implied sectoral multipliers described above. Thus, we also consider versions of the model informed by the mean value added shares for the first and last 15 years, and the 1960 and 1997 Make and Use Tables.

5.2 Production Linkages in the U.S. Economy

The production of investment goods in the U.S. turns out to be concentrated in relatively few sectors, with many sectors not producing any capital for other sectors while Construction and Durable Goods produce close to 80 percent of the capital in almost every sector. Put another way, as shown in Figure 8, we can think of the Construction and Durable Goods sectors as investment hubs in the production network.[27] Construction comprises residential and non-residential structures, including infrastructure such as power plants or pipelines for example, but also the maintenance and repair of highways, bridges, and other surface roads. The bulk of capital produced by the Durable Goods sector resides in motor vehicles, machinery, and computer and electronic products. Other sectors recorded as producing capital goods for the U.S. economy include Wholesale Trade, Retail Trade, and Professional and Business Services. In the Professional and Business Services sector, the notion of capital produced for other sectors is overwhelmingly composed of computer system designs and related services. As a practical matter, the distinction between materials and investment goods is not always straightforward. The BEA distinguishes between materials and capital goods by estimating the service life of different commodities and, consistent with a time period in this paper, commodities expected to be used in production within the year are defined as materials.[28] From the Capital Flow table, we expect that columns of the Leontief inverse, Ξ', associated with Construction and Durable Goods will have relatively large elements.

Compared to the Capital Flow table, the production of materials is considerably less concentrated in that all sectors produce materials for all other sectors though to varying degrees.[29] From the Make-Use table, Professional and Business Services, Finance and Insurance, and to a degree Nondurable Goods, all play an important role as suppliers of intermediate inputs including to the capital goods sectors. While Professional and Business Services figures prominently in in the IO table, this sector is not nearly as dominant as Durable Goods or Construction are in the Capital flow table.

In contrast to the sectors that play a key role in the U.S. production network, output produced in sectors such as Agriculture, Forestry, Fishing, and Hunting, Entertainment and Food Services, or Housing, is mostly consumed as a final good. Therefore, columns of the Leontief inverse associated with these sectors will have elements that tend to be comparatively small.[30]

[27] Table A5 in Section 8 of the Technical Appendix shows the Capital Flow matrix of the U.S. economy, Ω, for the 16 sectors considered in this paper.

[28] As noted in Foerster et al. (2011), there are a couple of notable measurement issues related to the construction of the Capital Flow table. First, the table accounts for the purchases of new capital goods but not used assets. Thus, for example, a firm's purchase of a used truck in a sector from another sector will not be recorded as investment in the capital flow table even though the truck's remaining service life may be well in excess of a year. Second, McGrattan and Schmitz (1999) note that a non-trivial portion of maintenance and repair takes place using within sector resources, yet many of the diagonal elements of the Capital Flow table are very small or zero. Results presented here are robust up to an adjustment that assumes that an additional 25 percent of capital expenditures takes place within sectors.

[29] Table A6 in Section 8 of the Technical Appendix shows the IO matrix or Make-Use table for the U.S. economy, Φ. The values shown in that table are from the 2015 BEA Make and Use Tables but the next section shows results using values from different Make and Use Tables over the decades.

[30] See Table A7 in Section 8 of the Technical Appendix which describes the generalized Leontief Inverse.

Table 4: Sectoral Multipliers

Sector	s^v	$s^{v\prime}\alpha_d\Omega'\Xi'$	$s^{v\prime}\left(I+\alpha_d\Omega'\Xi'\right)$
Agriculture	0.03	0.01	0.03
Mining	0.02	0.03	0.05
Utilities	0.02	0.01	0.03
Construction	0.05	0.12	0.17
Durable Goods	0.13	0.28	0.42
Nondurable Goods	0.09	0.03	0.13
Wholesale Trade	0.07	0.08	0.15
Retail Trade	0.08	0.02	0.11
Trans. & Ware.	0.04	0.03	0.07
Information	0.05	0.03	0.08
FIRE (x-Housing)	0.10	0.03	0.14
PBS	0.09	0.16	0.25
Educ. & Health	0.06	0.00	0.06
Arts, Ent., & Food Svc.	0.04	0.01	0.04
Other Services (x-Gov)	0.03	0.01	0.04
Housing	0.09	0.00	0.09

Notes: This table decomposes each sector's total multiplier (column 3) into a direct effect (column 1) and an indirect effect (column 2). The sums do not necessarily add up because of rounding.

5.3 Sectoral Multipliers

Table 4 shows what the direct and combined effects of sources of growth in different sectors are on GDP growth given the input-output and capital flow matrices. The importance of Construction and Durable Goods as suppliers of investment goods means not only that their value added share in GDP is large, 5 and 13 percent respectively, but also that they have large spillover effects on other sectors. In particular, the network multipliers for the Construction and Durable Goods sectors are more than 3 times their share, 0.17 and 0.42 respectively. Considering that trend TFP growth in Construction fell by almost 5 percentage points between 1950 and 2018 in Figure 4, this gives us, all else equal, a roughly 0.85 percentage point contribution to the decline in trend GDP growth from TFP changes in Construction alone. Similarly, the over 6 percent collapse in the trend growth rate of TFP in Durable Goods since 2000 would have on its own contributed roughly a 2.5 percent decline in trend GDP growth over the same period. The effects from Construction and Durable Goods are particularly pronounced because of their central role as producers of capital goods for all sectors, ranging from commercial and residential structures to motor vehicles and electronics. It is apparent from Table 4 that the effects of sectoral change on GDP growth are always at least as large as sectors' value added shares in GDP. Sectoral network multipliers roughly double the share of Professional and Business Services, from 0.09 to 0.25, and Wholesale Trade, from 0.07 to 0.15. In other sectors, such as Agriculture,

Table 5: Sectoral Network Multipliers Under Alternative Calibrations

Sector	Benchmark	Mean Shares, First 15 Years	Mean Shares, Last 15 Years	1997 IO Table	1960 IO Table
Agriculture	0.03	0.06	0.02	0.03	0.04
Mining	0.05	0.06	0.05	0.04	0.04
Utilities	0.03	0.03	0.02	0.03	0.03
Construction	0.17	0.17	0.17	0.15	0.17
Durable Goods	0.42	0.48	0.35	0.39	0.42
Nondurable Goods	0.13	0.16	0.10	0.13	0.14
Wholesale Trade	0.15	0.15	0.14	0.14	0.13
Retail Trade	0.11	0.12	0.09	0.11	0.11
Trans. & Ware.	0.07	0.08	0.06	0.06	0.07
Information	0.08	0.07	0.09	0.08	0.07
FIRE (x-Housing)	0.14	0.12	0.17	0.14	–
PBS	0.24	0.20	0.28	0.20	0.16
Educ. & Health	0.06	0.03	0.09	0.06	0.06
Arts, Ent., & Food Svc.	0.04	0.04	0.05	0.04	0.04
Other Services (x-Gov)	0.04	0.04	0.03	0.04	0.04
Housing	0.09	0.08	0.11	0.09	–
Addendum: FIRE + Housing	0.24	0.20	0.28	0.24	0.23

Notes: This table shows the sectoral multiplier (see Table 4) for the baseline calibration (column 1), for alternative value-added share weights (columns 2 and 3) and for alternative IO tables (columns 4 and 5).

Forestry, Fishing and Hunting, or Housing, the network multipliers are small with little or negligible indirect effects since these sectors produce mainly final consumption goods. Because the same network relationships embodied in the Capital Flow table, Ω, and Make-Use table, Φ, determine the importance that sectors have in the economy both as a share of value added and through their spillover effects, sectors with relatively larger shares in GDP will also tend to be associated with large network multipliers.

A key implication of Table 4 is that the effects of sectoral change on GDP growth arise in part through a composition effect. Therefore, secular changes in GDP growth can take place without observable changes in aggregate TFP growth. For example, consider purely idiosyncratic changes in TFP growth, $\partial g_{u,j}^{u,z}$, that leave aggregate TFP growth unchanged, $\sum_{j=1}^{n} s_j^v \partial g_{u,j}^z = 0$. In other words, the direct effect of sectoral TFP growth in this case is zero. Despite aggregate TFP growth not changing, these idiosyncratic changes may nevertheless have an (indirect) effect on GDP growth since the sum of sectoral multipliers is larger than 1.

5.4 Robustness of the Sectoral Multipliers

The key take away from Table 4 is that the influence of sectors on aggregate growth generally exceed their value added share in GDP, especially in Construction and Durable Goods whose multipliers amount to more than three times their respective share in the economy. As the table makes clear, this observation of course depends on what shares are being used and how the sectors interact through input-output and capital linkages. Table 5, therefore, explores how sectoral multipliers change with the definition of shares or input-output table. (Data limitations require us to use the 1997 capital flow table throughout.)

The first column of Table 5 reproduces our benchmark sectoral multipliers shown in the last column of Table 4; recall that these results are based on constant value added shares computed as averages over the full sample (1950-2018) and the 2015 Make and Use Tables. The second column of Table 5 shows the sectoral multipliers obtained using constant mean shares calculated only over the first 15 years of the sample (1950-1964). The third column shows these multipliers computed instead using constant mean shares from the last 15 years of the sample (2002-2016). The fourth and fifth columns of Table 5 shows the sectoral multipliers implied by the Make and Use Tables from 1997 and 1960 respectively.

While there are differences across the columns of Table 5, the general lesson remains the same. The sum of the multipliers always exceeds 1 and varies from 1.7 to 1.9 across columns. Construction and Durable Goods consistently have an outsize influence on aggregate growth regardless of the calculation in Table 5 given their central as input suppliers. Moreover, the ranking of sectoral multipliers by sector is also generally consistent across columns. The Make and Use table from 1960 does not allow us to separate FIRE and Housing so that the last row of Table 5 gives a multiplier for the combined sectors, about 0.24 on average across columns.

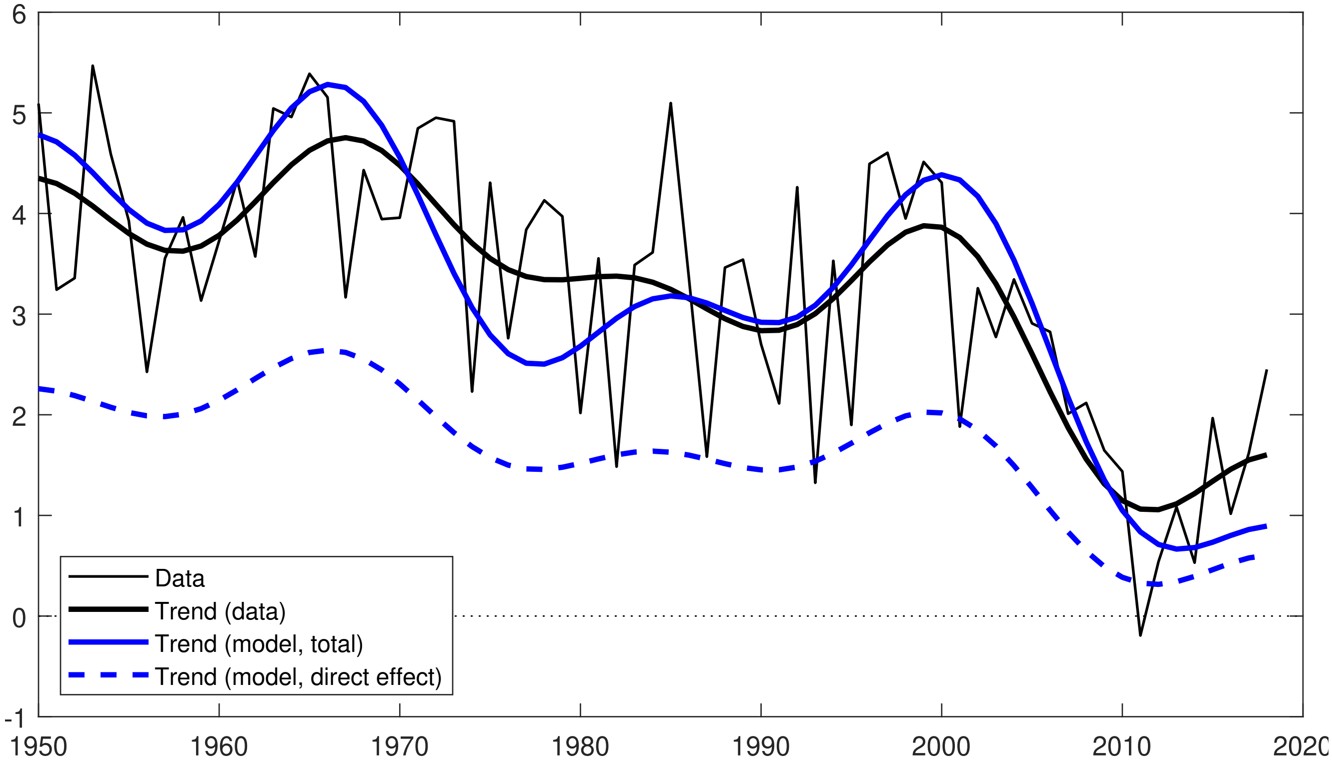
5.5 Historical Decomposition of the Trend Growth Rate of GDP

The various sectoral multiplier calculations we have just carried out depend on the balanced growth equations (5) and (7). These equations hold only in steady state and ignore endogenous transitional dynamics that are potentially important in explaining variations over the business cycle. However, because our empirical focus is on variations in growth rates with periodicities longer than 17 years, we abstract from these transitional dynamics and apply the formulas (5) and (7) directly to the trend growth rates of TFP and labor extracted in section 3, $g^z_{i,t}$ and $g^\ell_{i,t}$, as an approximation.[31] In addition, we then explore how our estimates of common, $(\lambda^z_i g^z_{i,t}, \lambda^\ell_i g^\ell_{i,t})$, and sector-specific, $(g^z_{u,i,t}, g^\ell_{u,i,t})$, trend input growth have historically contributed to the trend growth rates of sectoral value added and GDP. Thus, we compute the trend growth rates of sectoral value added as,

$$g^v_t = \left[I + \alpha_d \Omega' \Xi'\right] \left(\lambda^z g^z_{f,t} + g^z_{u,t} + (I - \alpha_d)\left(\lambda^\ell g^\ell_{f,t} + g^\ell_{u,t}\right)\right),$$

[31] A previous working paper version where linearized transition dynamics are calculated shows this approximation to be generally close at these low frequencies.

Figure 9: Trend Growth Rate in GDP: Data and Model
(percentage points at annual rate)



Notes: The figure shows the cyclically adjusted GDP growth rate (thin black line) along with its low-frequency trend (thick black line). Also shown are the model-implied trend using the low-frequency trends of labor and TFP growth (solid blue line), and the trend implied by only the direct effects of labor and TFP based solely on value-added shares (dashed blue line).

where $g_t^v = (g_{1,t}^v, ..., g_{n,t}^v)$, which then gives trend GDP growth as

$$g_t^V = s^{v\prime} g_t^v.$$

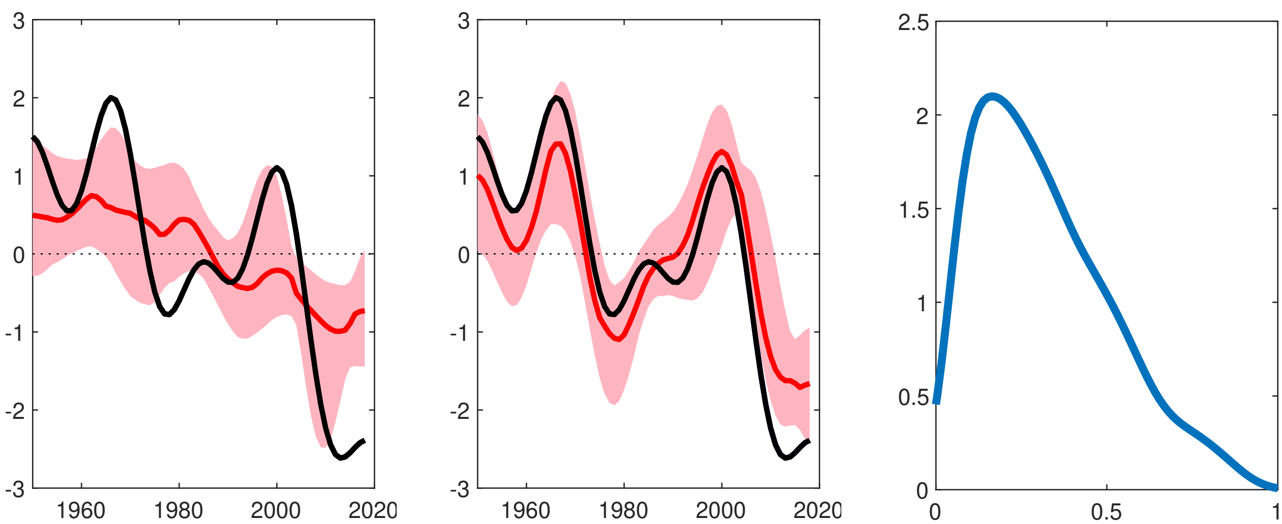
Figure 9 depicts the annual growth rate of GDP and its trend in black (previously shown in Figure 2) together with the corresponding trend growth rate computed from the balanced growth multipliers (in solid blue) and its contribution from the direct effect using sectors' value added shares only (in dashed blue), $s^{v\prime} I \left(g_t^z + (I - \alpha_d) g_t^\ell \right)$. In all, trend GDP growth fell by nearly 3 percent over the postwar period. Importantly, the sizable gap between the model trend with direct effects only and the complete model trend implies that the indirect effects stemming from network production linkages constitute a significant component of trend GDP growth. There is a notable discrepancy between model and data in the 1970s, when the balanced growth multipliers suggest a larger decline in trend GDP growth rates than in the data. In that period, periodicities longer that 17 years may not be adequate to capture the required adjustment to capital implied by the model.

Figure 10: Decomposition of the Trend Growth Rate in GDP
(percentage points at annual rate)

(a) Common Factor

(b) Sector-Specific

(c) Posterior Distribution for Fraction of Variance Attributed to Common Factor (R_f^2)



Notes: Panels (a) and (b) show the (demeaned) model-implied trend GDP growth (black line), and its decomposition into changes due to the common factor and sector-specific factors (red lines), respectively. The red lines denote the posterior median and the shaded areas are (pointwise) equal-tail 68% credible intervals. Panel (c) shows the posterior distribution for the fraction of the variance in trend GDP growth attributed to the common factor.

Figure 10 decomposes the trend growth rate of GDP implied by the model into its components derived from common factors and sector-specific factors.[32] The model indicates that sector-specific or unique factors in trend labor and TFP growth have historically accounted for roughly 3/4 of the long-run changes in GDP growth. Conversely, only about 1/4 of the variation in trend GDP growth since 1950 has come from common sources of input growth. This is despite common factors explaining roughly 2/3 of the variation in the trend growth rate of aggregate labor noted in Section 3. To understand this finding, recall that some sectors that have large sectoral multipliers such as Durable Goods or Construction, Table 3, also have large variations in trend input growth that are almost entirely driven by idiosyncratic factors, Figures 6 and 7.

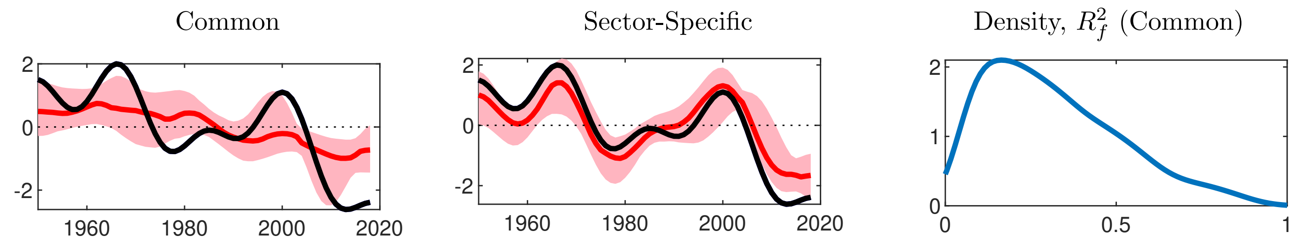
Panel (c) plots the posterior density for R_f^2, the fraction of the variance in trend GDP growth explained by common sources. The median of the posterior for R_f^2 is 0.26, the mode is less than 0.20, and 70 percent of the posterior mass is associated with values of R_f^2 that are less than 0.40.[33] Thus, these results suggest

[32]The overall trend, $s^{v\prime}\left[I+\alpha_d\Omega'\Xi'\right]\left(g_t^z+(I-\alpha_d)g_t^\ell\right)$, is in black and the posterior median estimates of common components, $s^{v\prime}\left[I+\alpha_d\Omega'\Xi'\right]\left(\lambda^z g_{f,t}^z+(I-\alpha_d)\lambda^\ell g_{f,t}^\ell\right)$, and sector-specific components, $s^{v\prime}\left[I+\alpha_d\Omega'\Xi'\right]\left(g_{u,t}^z+(I-\alpha_d)g_{u,t}^\ell\right)$, along with their 68 percent credible intervals are in red.

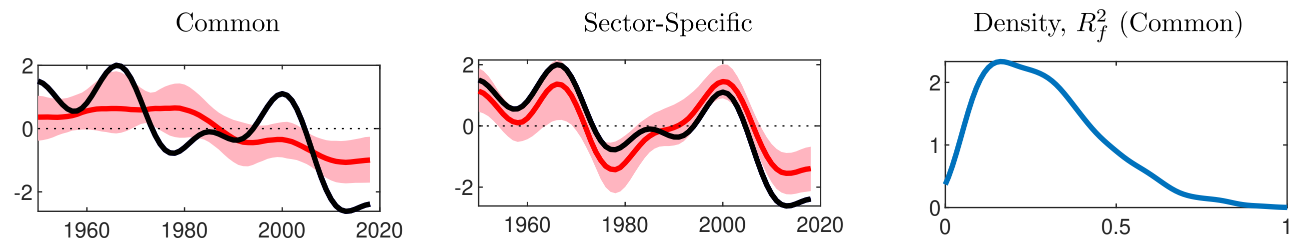
[33]Readers may be interested in the implied prior for R_F^2 induced by the priors for the model parameters. The small value of $\nu=0.01$ generates an improper prior for Σ, making the prior for R_F^2 difficult to characterize. However, a larger value of $\nu=2$ generates a proper prior, and the resulting prior for R_F^2 has the majority of its mass concentrated around the extremes of $R_F^2=0$ and $R_F^2=1$, each associated with the possibility of an extremely large value of $\sigma_{u,i}$ or σ_F. The posterior for R_F^2

Figure 11: Robustness to Changes in Statistical Model

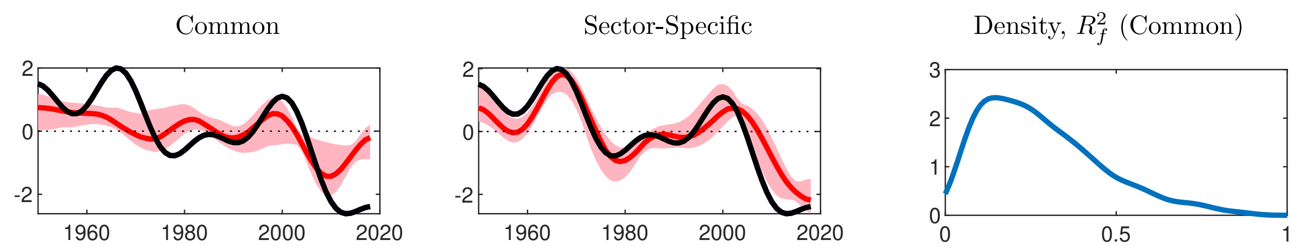
(a) Model: $q = 8$, $\eta = 1.0$, $R^2_f = 0.28\ (0.12, 0.52)$



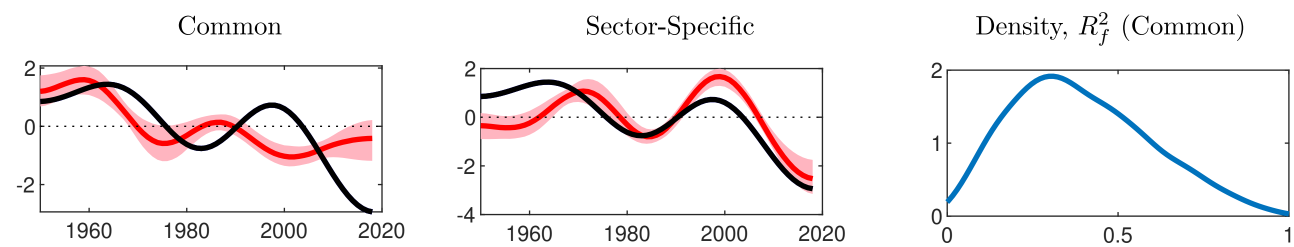
(b) Model: $q = 8$, $\eta = 0.5$, $R^2_f = 0.26\ (0.12, 0.46)$



(c) Model: $q = 8$, $\eta = 2.0$, $R^2_f = 0.25\ (0.11, 0.45)$



(d) Model: $q = 6$, $\eta = 1.0$, $R^2_f = 0.37\ (0.19, 0.60)$



Notes: See notes for Figure 10. Each vertical panel of the Figure shows results for a different specification of the prior distribution, η, or long-run periodicities, q.

that most of the long-run evolution of GDP growth has historically stemmed from sector-specific factors.

The results reported thus far use the benchmark priors. Recall from Section 3 that these priors were relatively uninformative except for the factor loadings. In particular, the prior for λ^ℓ was $\lambda^\ell \sim N(\mathbf{1}, \mathbf{P}_\ell)$,

shown in Figure 10 differs markedly from this prior, and thus reflects information in the sample data.

where $\mathbf{P}_\ell = \eta^2(\mathbf{I}_{16} - s_\ell(s_\ell' s_\ell)^{-1}s_\ell')$ and an analogous prior was used for λ^z. These priors enforced the normalization that $s_\ell'\lambda^\ell = s_v'\lambda^z = 1$, so that unit changes in f_t^ℓ and f_t^z lead to unit changes in the long-run growth rate of aggregate labor and TFP. The parameter η then governed how aggressively the estimates of λ_i^ℓ or λ_i^z are shrunk toward their mean of unity. The benchmark results use $\eta = 1$. Smaller values of η shrink the estimates closer to 1 while larger values of η allows them to deviate from 1 more than the baseline model. Thus, we now explore the robustness of our findings to alternative priors, $\eta = 1/2$ and $\eta = 2$. In addition, we also run the model using $q = 6$ which captures long-run variations with periodicities longer than $2 \times 69/6 = 23$ years.

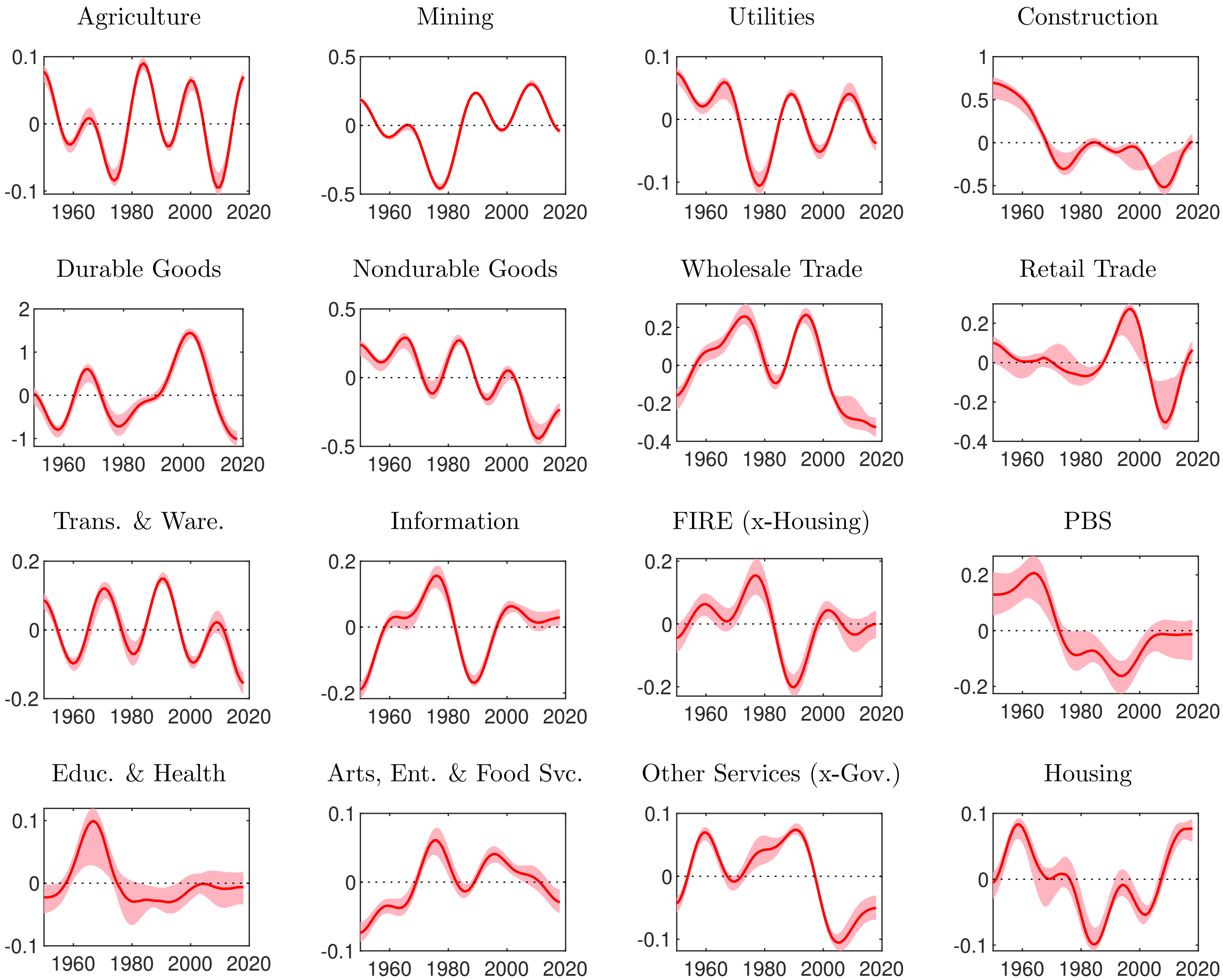
Figure 11 summarizes the findings from these robustness exercises by reproducing Figure 10 for each of these alternative models It is clear from the figure that across all cases, contributions from common sources of trend input growth to the long-run evolution of GDP growth remain limited. Median estimates of R_f^2 range from 0.28 to 0.37 with posterior distributions that are clearly not uniform across values of R_f^2 and that place the bulk of their mass between 0 and 0.5. We thus conclude that the result that sector-specific forces are the primary driver of trend GDP are robust to these changes in the prior for the factor loadings and to increasing the periodicity used to define the long-run trends.

Given that sector-specific (rather than common) trends have played a dominant role in driving trend GDP growth over the postwar period, Figure 12 gives the historical trend contributions to aggregate GDP growth from the sector-specific components for each sector. Two sectors clearly stand out, Construction and Durable Goods. Recall that U.S. trend GDP growth fell by approximately 3 percentage points between 1950 and 2018. Comparing the beginning and the end of the sample, Figure 12 indicates that Durable Goods alone contributed around 1 percentage point of that decline and Construction 0.75 percentage points. However, there are also important differences in the timing and variation of those sectoral contributions. Construction contributed roughly a 1 percentage point decline in trend GDP growth between 1950 and 1980 and was essentially flat thereafter. In contrast, Durable Goods played a key role in raising trend GDP growth in the 1980's and 1990's before contributing an almost 2 percentage point decline in trend GDP growth between 2000 and the end of the sample in 2018. Non-Durable goods also notably contributed to the post-war decline in trend GDP growth at roughly 0.5 percentage points over the entire sample period, though offset somewhat by Mining after 1980. Strikingly, many other sectors show relatively flat contributions to aggregate trend growth over 1950 to 2018, between -0.1 and 0.1 percentage points. Perhaps even more surprising, no sector has contributed any steady significant increase to the trend growth rate of GDP over that period.

6 Concluding Remarks

In this paper, we study how trends in TFP and labor growth across major U.S production sectors have helped shape the secular behavior of GDP growth. We find that aggregate trends in TFP and labor growth have generally decreased across a majority of sectors since 1950. Common trends in sectoral TFP growth

Figure 12: Sector-Specific Contributions to the Trend Growth Rate of GDP
(percentage points at annual rate)



Notes: Each panel shows the implications of sector-specific trends for the trend growth rate of GDP using the model-based multipliers. The solid lines denote the posterior median and the shaded areas are (pointwise) equal-tail 68% credible intervals.

contributed around 1/3 of the secular decline in aggregate TFP growth. Common sectoral trends in sectoral labor growth contributed about 2/3 of the secular decline in aggregate labor growth.[34]

We embed these findings into a dynamic multi-sector framework in which materials and capital used by different sectors are produced by other sectors. These production linkages along with capital accumulation mean that changes in the growth rate of labor or TFP in one sector affect not only its own value added but

[34]One caveat is that those findings emerge in the context of a closed economy. Cavallo and Landry (2010) point out that the fraction of investment coming from abroad has increased since 1967. See also Basu et al. (2013). Thus, some of our conclusions could be sensitive to this observation.

also that of all other sectors. In particular, capital induces network effects that amplify the repercussions of sector-specific sources of growth on the aggregate economy and that we summarize in terms of sectoral multipliers. Quantitatively, these multipliers scale up the influence of some sectors by multiple times their value added share in the economy.

Ultimately, we find that sector-specific factors in TFP and labor growth historically explain 3/4 of low frequency variations in U.S. GDP growth, leaving common or aggregate factors to explain only 1/4 of these variations. Changing sectoral trends in the last 7 decades, translated through the economy's production network, have on net lowered trend GDP growth by close to 3 percentage points. The Construction and Durable Goods sectors, more than any other sector, stand out for their contribution to the trend decline in GDP growth over the post-war period. Strikingly, no sector has contributed any steady or significant increase to the trend growth rate of GDP in the last 70 years.

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Appendix A: Balanced Growth with Production Linkages

Consider the case where $\Delta \ln z_{j,t}$ and $\Delta \ln \ell_{j,t}$ are growing at constant rates, g_j^z and g_j^ℓ respectively, given by $\lambda_j^z g_f^z + g_{u,j}^z$ and $\lambda_j^\ell g_f^\ell + g_{u,j}^\ell$. Then,

$$\Delta \ln A_{j,t} \equiv g_j^a = \lambda_j^z g_f^z + g_{u,j}^z + (1-\alpha_j)\left(\lambda_j^\ell g_f^\ell + g_{u,j}^\ell\right), \tag{A.1}$$

and we denote by $\widetilde{A}_{j,t}$ the gross growth rate of $A_{j,t}$,

$$\widetilde{A}_{j,t} = \frac{A_{j,t}}{A_{j,t-1}} = e^{g_j^a} \approx 1 + g_j^a.$$

The balanced growth path of the economy is one in which, given the constant exogenous growth rates of TFP, $g_j^z = \lambda_j^z g_f^z + g_{u,j}^z$, and labor input, $g_j^\ell = \lambda_j^\ell g_f^\ell + g_{u,j}^\ell$, all other variables grow at constant rates and all shares are constant. Thus, to derive the aggregate balanced growth path, we need to normalize the model's variables in such a way that these normalized variables (generally denoted by a '$\sim$' over the variable) are constant along that path. Because different sectors will generally grow at different rates along the balanced growth path, the factors used to normalize variables will be sector-specific. Generically, we denote these normalizing factors by $\mu_{j,t}$ (or functions thereof). Solving for those factors below will yield a system of equations that is stationary in the normalized variables along the economy's steady state growth path as well as the growth rates of all variables along that path.

Making the model Stationary

If all growth rates are constant, the resource constraint in any individual sector implies that all the variables in that constraint must grow at the same rate. Thus, define $\widetilde{y}_{j,t} = y_{j,t}/\mu_{j,t}$, $\widetilde{c}_{j,t} = c_{j,t}/\mu_{j,t}$, $\widetilde{m}_{ji,t} = m_{ji,t}/\mu_{j,t}$, and $\widetilde{x}_{ji,t} = x_{ji,t}/\mu_{j,t}$. The goal in this subsection is to solve for the normalizing factors, $\mu_{j,t}$, as a function of the model's underlying parameters only (and in particular the constant growth rates of TFP and labor input).

The economy's resource constraint becomes

$$\widetilde{c}_{j,t} + \sum_{i=1}^{n} \widetilde{m}_{ji,t} + \sum_{i=1}^{n} \widetilde{x}_{ji,t} = \widetilde{y}_{j,t}.$$

Given the above definitions, the production of investment goods may be re-written as

$$\widetilde{x}_{j,t} = \prod_{i=1}^{n} \left(\frac{\widetilde{x}_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}},$$

where $\widetilde{x}_{j,t} = x_{j,t} / \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}}$. Under this normalization, the capital accumulation equation is

$$k_{j,t+1} = \widetilde{x}_{j,t} \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}} + (1 - \delta_j) k_{j,t},$$

and so becomes

$$\widetilde{k}_{j,t+1} = \widetilde{x}_{j,t} + (1 - \delta_j) \widetilde{k}_{j,t} \prod_{i=1}^{n} \left(\frac{\mu_{i,t-1}}{\mu_{i,t}} \right)^{\omega_{ij}},$$

where $\widetilde{k}_{j,t+1} = k_{j,t+1} / \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}}$.

The expression for value added may be written as

$$v_{j,t} = A_{j,t} \left(\frac{\widetilde{k}_{j,t} \prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}}}{\alpha_j} \right)^{\alpha_j},$$

so that defining

$$\widetilde{v}_{j,t} = \frac{v_{j,t}}{A_{j,t} \left(\prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}}, \tag{A.2}$$

where $A_{j,t} \left(\prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}$ is the scaling factor that makes normalized value added, $\widetilde{v}_{j,t}$, constant along the balanced growth path, we have

$$\widetilde{v}_{j,t} = \left(\frac{\widetilde{k}_{j,t}}{\alpha_j} \right)^{\alpha_j}.$$

The composite bundle of materials used in sector j may be expressed as

$$\widetilde{m}_{j,t} = \prod_{i=1}^{n} \left(\frac{\widetilde{m}_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}},$$

with $\widetilde{m}_{j,t} = m_{j,t} / \prod_{i=1}^{n} \mu_{i,t}^{\phi_{ij}}$.

Given these normalizations, gross output may be written as

$$\widetilde{y}_{j,t}\mu_{j,t} = \left(\frac{\widetilde{v}_{j,t} A_{j,t} \prod_{i=1}^{n} \mu_{i,t-1}^{\alpha_j \omega_{ij}}}{\gamma_j} \right)^{\gamma_j} \left(\frac{\widetilde{m}_{j,t} \prod_{i=1}^{n} \mu_{i,t}^{\phi_{ij}}}{1-\gamma_j} \right)^{1-\gamma_j},$$

which, collecting terms, gives

$$\widetilde{y}_{j,t} = \left(\frac{\widetilde{v}_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{\widetilde{m}_{j,t}}{1-\gamma_j} \right)^{1-\gamma_j} \left[\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \prod_{i=1}^{n} \mu_{i,t-1}^{\gamma_j \alpha_j \omega_{ij}} \mu_{i,t}^{(1-\gamma_j)\phi_{ij}} \right].$$

We can now use the expression in square brackets to solve for the normalizing factors, $\mu_{j,t}$, as a function of the model's underlying parameters.

First, re-write the term in square brackets as

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \left(\prod_{i=1}^{n} \frac{\mu_{i,t-1}^{\gamma_j \alpha_j \omega_{ij}}}{\mu_{i,t}^{\gamma_j \alpha_j \omega_{ij}}} \right) \left(\prod_{i=1}^{n} \mu_{i,t}^{(1-\gamma_j)\phi_{ij}} \mu_{i,t}^{\gamma_j \alpha_j \omega_{ij}} \right),$$

where this last expression involves the growth rate of $\mu_{i,t}$. Then, without loss of generality with respect to growth rates, we choose $\mu_{j,t}$ in every sector such that, on the steady state growth path,[35]

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \prod_{i=1}^{n} \mu_{i,t}^{\gamma_j \alpha_j \omega_{ij} + (1-\gamma_j)\phi_{ij}} = 1.$$

Sectoral Value Added Growth

Taking logs of both sides of the above expression, we have

$$\gamma_j \ln A_{j,t} - \ln \mu_{j,t} + \sum_{i=1}^{n} (\gamma_j \alpha_j \omega_{ij} + (1-\gamma_j)\phi_{ij}) \ln \mu_{i,t} = 0,$$

or in vector form,

$$\Gamma_d \ln A_t - \ln \mu_t + \Gamma_d \alpha_d \Omega' \ln \mu_t + (I - \Gamma_d)\Phi' \ln \mu_t = 0,$$

which gives us

$$\ln \mu_t = \Xi' \ln A_t, \tag{A.3}$$

where

$$\Xi' = \left(I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d)\Phi' \right)^{-1} \Gamma_d,$$

[35] This is without loss of generality since in the derivations of growth rates below, any constant κ may be used instead of 1.

with $\Xi = \{\xi_{ij}\}$ is the generalized Leontief inverse.

Going back to equation (A.1), and writing the vector of productivity growth rates as $\Delta \ln A_t = g^a$ where $g^a = (g_1^a, ..., g_n^a)$, it follows that

$$\Delta \ln \mu_t = \Xi' g^a = \Xi' \left(\lambda^z g_f^z + g_u^z + (I - \alpha_d) \left(\lambda^\ell g_f^\ell + g_u^\ell \right) \right),$$

where g_f^z and g_f^ℓ are common or aggregate factors in the growth rates of TFP and labor respectively, λ^z and λ^ℓ are loading vectors as defined in the main text, and $g_u^z = (g_{u,1}^z, ..., g_{u,n}^z)$ and $g_u^\ell = (g_{u,1}^\ell, ..., g_{u,n}^\ell)$ are vectors of (unique) idiosyncratic TFP and labor input growth rates.

Recall from equation (A.2) above that the normalizing factor for value added in sector j is $A_{j,t} \left(\prod_{i=1}^n \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}$. Thus, define this factor by $\mu_{j,t}^v$,

$$\mu_{j,t}^v = A_{j,t} \left(\prod_{j=1}^n \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}.$$

In particular, since $\mu_{j,t}^v$ is the normalizing factor that makes value added in sector j constant along the balanced growth path, it follows that $\mu_{j,t}^v$ grows at the same rate as j's value added along that path, denoted g_j^v. Then, using equation (A.3), we have that

$$\ln \mu_t^v = \ln A_t + \alpha_d \Omega' \Xi' \ln A_{t-1},$$

or

$$g^v = \left[I + \alpha_d \Omega' \Xi' \right] g^a,$$

where $g^v = (g_1^v, ..., g_n^v)$ is a vector that summarizes value added growth in every sector. Alternatively,

$$g^v = \left[I + \alpha_d \Omega' \Xi' \right] \left(\lambda^z g_f^z + g_u^z + (I - \alpha_d) \left(\lambda^\ell g_f^\ell + g_u^\ell \right) \right).$$