

# ONLINE APPENDIX: ROBOTS OR WORKERS? A MACRO ANALYSIS OF AUTOMATION AND LABOR MARKETS

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ABSTRACT. This appendix provides some derivation details and further results in Leduc and Liu (2019).

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## Supplemental Appendices: For Online Publication

### APPENDIX A. DERIVATIONS OF HOUSEHOLD'S OPTIMIZING CONDITIONS

Denote by  $V_t(B_{t-1}, N_{t-1})$  the value function for the representative household. The household's optimizing problem can be written in the recursive form

$$V_t(B_{t-1}, N_{t-1}) \equiv \max \ln C_t - \chi N_t + \beta \mathbb{E}_t \theta_{t+1} V_{t+1}(B_t, N_t), \quad (\text{A.1})$$

subject to the budget constraint

$$C_t + \frac{B_t}{r_t} = B_{t-1} + w_t N_t + \phi(1 - N_t) + d_t - T_t, \quad (\text{A.2})$$

and the law of motion for employment

$$N_t = (1 - \delta_t) N_{t-1} + q_t^u u_t, \quad (\text{A.3})$$

where the measure of job seekers is given by

$$u_t = 1 - (1 - \delta_t) N_{t-1}. \quad (\text{A.4})$$

The household chooses  $C_t$ ,  $B_t$ , and  $N_t$ , taking prices and the average job finding rate as given.

Denote by  $\Lambda_t$  the Lagrangian multiplier for the budget constraint (A.2). The first-order condition with respect to consumption implies that

$$\Lambda_t = \frac{1}{C_t}. \quad (\text{A.5})$$

The optimizing decision for  $B_t$  implies that

$$\frac{\Lambda_t}{r_t} = \beta \mathbb{E}_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial B_t}. \quad (\text{A.6})$$

Using the envelope condition with respect to  $B_{t-1}$ , we obtain the intertemporal Euler equation

$$1 = \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} r_t. \quad (\text{A.7})$$

Denote by  $\mu_{nt}$  the Lagrangian multiplier associated with the employment law of motion (A.3). The first-order condition with respect to  $N_t$  implies that

$$\mu_{nt} = \Lambda_t \left( w_t - \phi - \frac{\chi}{\Lambda_t} \right) + \beta \mathbb{E}_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial N_t}. \quad (\text{A.8})$$

After substituting out  $u_t$  in Eq. (A.3) using Eq. (A.4), we obtain the envelope condition with respect to  $N_{t-1}$

$$\frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_{t-1}} = \mu_{nt} (1 - \delta_t) (1 - q_t^u). \quad (\text{A.9})$$

Define the employment surplus as  $S_t^H \equiv \frac{\mu n_t}{\Lambda_t}$ . The first-order condition (A.8), together with the envelope condition (A.9), implies the Bellman equation

$$S_t^H = w_t - \phi - \frac{\chi}{\Lambda_t} + \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \delta_{t+1}) (1 - q_{t+1}^u) S_{t+1}^H. \quad (\text{A.10})$$

#### APPENDIX B. SUMMARY OF EQUILIBRIUM CONDITIONS: BENCHMARK MODEL

A search equilibrium is a system of 18 equations for 18 variables summarized in the vector

$$[C_t, r_t, Y_t, m_t, u_t, v_t, q_t^u, q_t^v, q_t^a, N_t, U_t, \eta_t, J_t^e, J_t^v, J_t^a, A_t, x_t^*, w_t].$$

We write the equations in the same order as in the dynare code.

(1) Household's bond Euler equation:

$$1 = \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} r_t, \quad (\text{B.1})$$

(2) Matching function

$$m_t = \mu u_t^\alpha v_t^{1-\alpha}, \quad (\text{B.2})$$

(3) Job finding rate

$$q_t^u = \frac{m_t}{u_t}, \quad (\text{B.3})$$

(4) Vacancy filling rate

$$q_t^v = \frac{m_t}{v_t}, \quad (\text{B.4})$$

(5) Employment dynamics

$$N_t = (1 - \delta_t) N_{t-1} + m_t, \quad (\text{B.5})$$

(6) Number of searching workers

$$u_t = 1 - (1 - \delta_t) N_{t-1}, \quad (\text{B.6})$$

(7) Unemployment

$$U_t = 1 - N_t, \quad (\text{B.7})$$

(8) Vacancy dynamics

$$v_t = (1 - q_{t-1}^v) (1 - q_t^a) v_{t-1} + \delta_t N_{t-1} + \eta_t, \quad (\text{B.8})$$

(9) Automation dynamics

$$A_t = (1 - \rho^o) A_{t-1} + q_t^a (1 - q_{t-1}^v) v_{t-1}, \quad (\text{B.9})$$

(10) Employment value

$$J_t^e = Z_t - w_t + \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} [\delta_{t+1} J_{t+1}^v + (1 - \delta_{t+1}) J_{t+1}^e], \quad (\text{B.10})$$

(11) Vacancy value

$$J_t^v = -\kappa + q_t^v J_t^e + (1 - q_t^v) \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} [(1 - q_{t+1}^a) J_{t+1}^v + q_{t+1}^a J_{t+1}^a]. \quad (\text{B.11})$$

(12) Automation value

$$J_t^a = Z_t \zeta_t - \kappa_a + (1 - \rho^o) \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} J_{t+1}^a, \quad (\text{B.12})$$

(13) Automation threshold

$$x_t^* = J_t^a - J_t^v, \quad (\text{B.13})$$

(14) Robot adoption

$$q_t^a = \left( \frac{x_t^*}{\bar{x}} \right)^{\eta_a}, \quad (\text{B.14})$$

(15) Vacancy creation

$$\eta_t = \left( \frac{J_t^v}{\bar{e}} \right)^{\eta_e}, \quad (\text{B.15})$$

(16) Aggregate output

$$Y_t = Z_t N_t + Z_t \zeta_t A_t. \quad (\text{B.16})$$

(17) Resource constraint

$$C_t + \kappa v_t + \kappa_a A_t + \frac{\eta_a}{1 + \eta_a} q_t^a x_t^* (1 - q_{t-1}^v) v_{t-1} + \frac{\eta_e}{1 + \eta_e} \eta_t J_t^v = Y_t, \quad (\text{B.17})$$

(18) Nash bargaining wage

$$\frac{b}{1-b} (J_t^e - J_t^v) = w_t - \phi - \chi C_t + \mathbb{E}_t \frac{\beta \theta_{t+1} C_t}{C_{t+1}} (1 - q_{t+1}^u) (1 - \delta_{t+1}) \frac{b}{1-b} (J_{t+1}^e - J_{t+1}^v), \quad (\text{B.18})$$

### APPENDIX C. ADDITIONAL IMPULSE RESPONSES

We report some additional impulse responses in this section.

**Impulse responses to a job separation shock in the benchmark model.** A job separation shock raises both unemployment and vacancies and mechanically boosts hiring through the matching function, as shown in Figure C.1. This finding is consistent with Shimer (2005), who argues that the counterfactual implication of the job separation shock for the correlation between unemployment and vacancies renders the shock unimportant for explaining observed labor market dynamics. The shock reduces the automation probability. Labor productivity increases slightly, since the decline in employment outpaces the decline in aggregate output. The shock also leads to small declines in real wages and the labor income share.

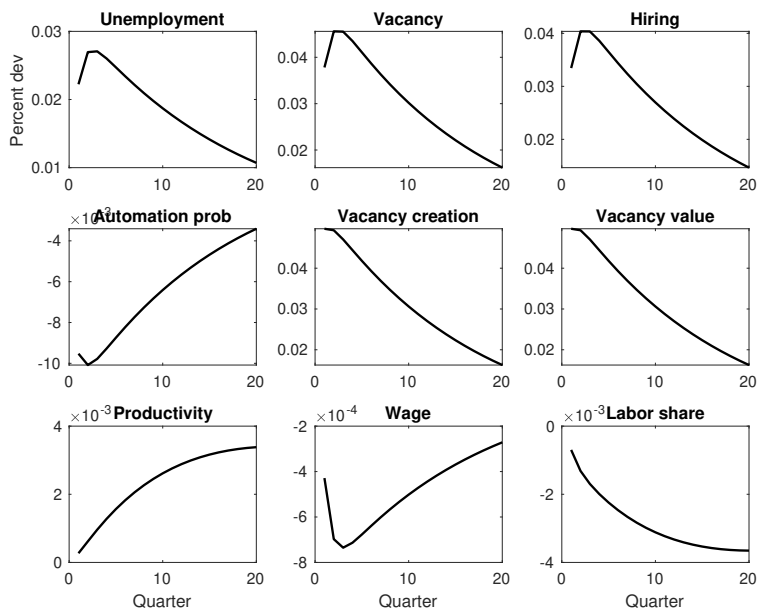


FIGURE C.1. Impulse responses to a job separation shock in the benchmark model.

**Impulse responses to a neutral technology shock: benchmark vs. no-automation counterfactuals.** We compare the impulse responses to a positive neutral technology shock in the benchmark model with those in two counterfactuals: (1) reducing worker bargaining weight and (2) raising the worker’s value of non-market activity (i.e., the unemployment insurance benefits). In both counterfactuals, we turn off the automation channel by keeping the automation-related variables to their steady-state levels and shutting off the automation-specific shock.

Figure C.2 compares the impulse responses to a positive neutral technology shock in the benchmark model (the black solid lines) to the no-automation counterfactual (the blue dashed lines) and the low worker bargaining weight case (the red dashed lines).

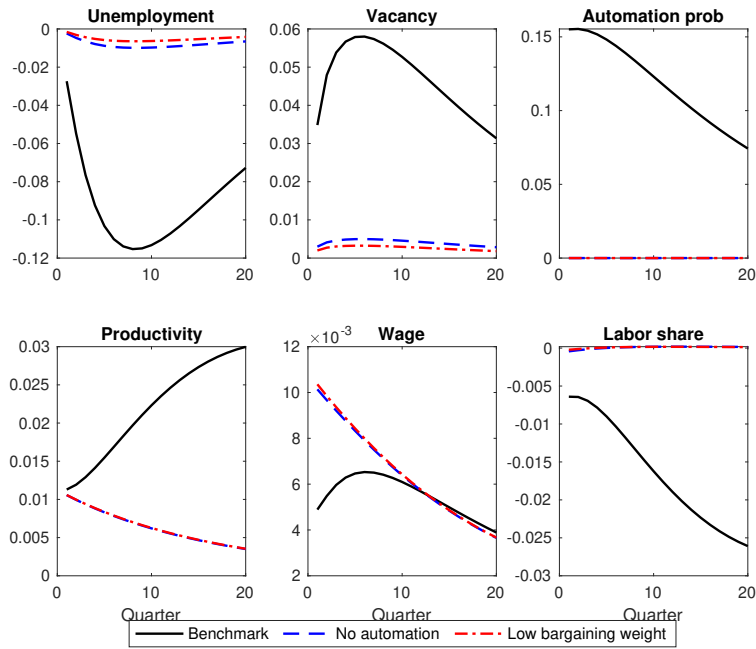


FIGURE C.2. Impulse responses to a positive neutral technology shock in the benchmark model (black solid lines), the counterfactual with no automation (blue dashed lines), and the counterfactual with no automation and low worker bargaining power (red dot-dash lines).

Figure C.3 compares the impulse responses to a positive neutral technology shock in the benchmark model (the black solid lines) to the no-automation counterfactual (the blue dashed lines) and the high UI benefit counterfactual (the red dashed lines).

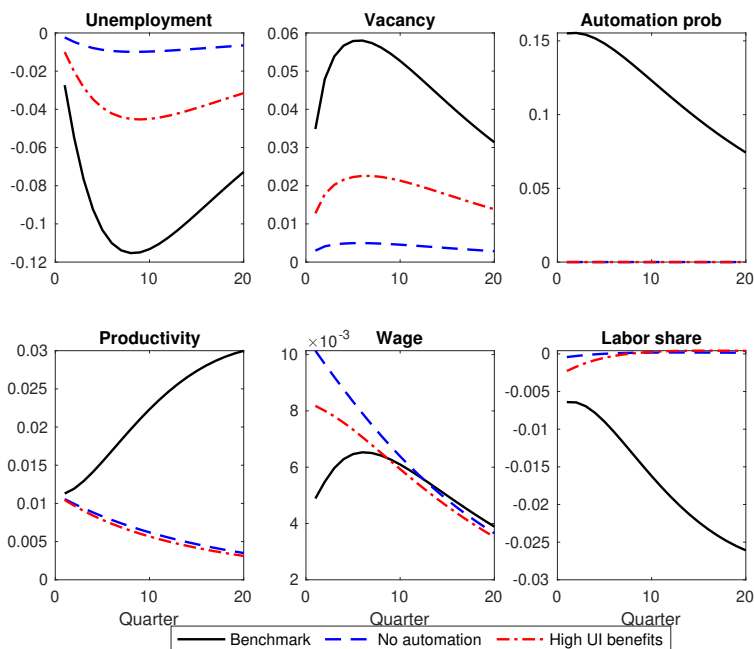


FIGURE C.3. Impulse responses to a positive neutral technology shock in the benchmark model (black solid lines), the counterfactual with no automation (blue dashed lines), and the counterfactual with no automation and high unemployment insurance (UI) benefits (red dot-dash lines).

**Impulse responses to a neutral technology shock: benchmark vs. low search frictions.** Figure C.4 shows the impulse responses of the macro variables following a positive neutral technology shock, and compares the impulse responses from the benchmark model (the black solid lines) with those from the counterfactual with low search frictions (the blue dashed lines). Although both models have the automation channel operating, the benchmark model produces much stronger amplification effects of the shock on unemployment and vacancies than does the counterfactual with low search frictions.

In the benchmark model, automation displaces workers directly, but it also creates jobs by raising the present value of a job vacancy and therefore boosting the incentive for vacancy creation. In an economy with a spot labor market without search frictions, an employment relation would cease to be a long-term relation, and the option of automation in the future would not directly affect current hiring decisions. In that case, robots would replace workers, and increased automation in response to a positive technology shock would raise unemployment and reduce vacancies. The counterfactual model with lower search frictions lies between our benchmark model and the spot labor market, with mitigated job-creating

effect of automation, and thus more muted responses in unemployment and vacancies than those in the benchmark economy.<sup>1</sup>

With low search frictions, the present value of a vacancy responds less to the technology shock (because the model becomes closer to a spot labor market). Since the neutral technology shock directly raises the productivity of both workers and robots, the value of automation rises on impact. Thus, the automation threshold (i.e.,  $x_t^* = J_t^a - J_t^v$ ) and the automation probability rises more sharply than in the benchmark model, as shown in Figure C.4, leading to stronger increases in labor productivity. Although the real wage rate also increase more than that implied by the benchmark model, the productivity effects dominate, leading to a more pronounced and persistent decline in the labor share.

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<sup>1</sup>We have also considered the case with a higher average job separation rate of  $\bar{\delta} = 0.8$  (not reported in the paper). In that case, we find that a positive neutral technology shock raises unemployment and lowers vacancies, because the forward-looking job-creating effect becomes further mitigated.



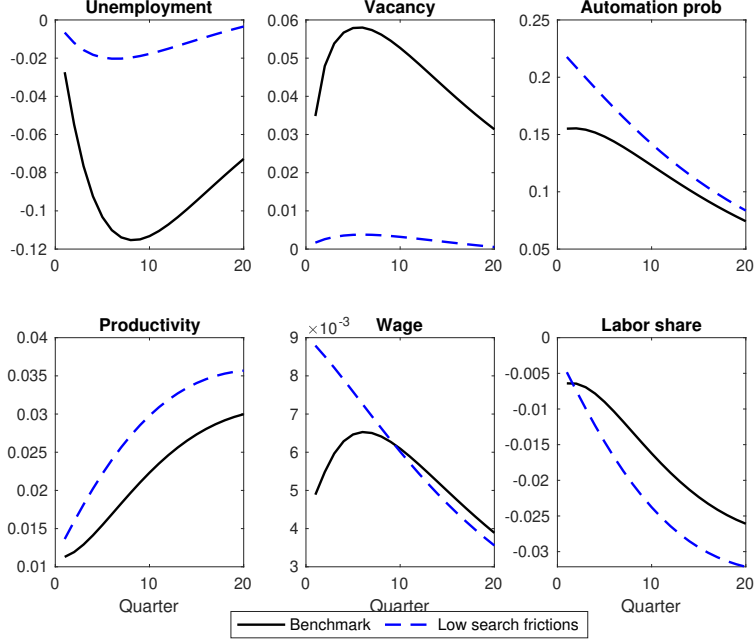


FIGURE C.4. Impulse responses to a positive neutral technology shock in the benchmark model (black solid lines) and the counterfactual with low search frictions (blue dashed lines).

#### APPENDIX D. SUMMARY OF EQUILIBRIUM CONDITIONS: MODEL WITH AUTOMATED JOBS

A search equilibrium is a system of 18 equations for 18 variables summarized in the vector

$$[C_t, r_t, Y_t, m_t, u_t, v_t, q_t^u, q_t^v, q_t^a, N_t, U_t, \eta_t, J_t^e, J_t^v, J_t^a, A_t, x_t^*, w_t].$$

(1) Household's bond Euler equation:

$$1 = E_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} r_t, \quad (\text{D.1})$$

(2) Matching function

$$m_t = \mu u_t^\alpha v_t^{1-\alpha}, \quad (\text{D.2})$$

(3) Job finding rate

$$q_t^u = \frac{m_t}{u_t}, \quad (\text{D.3})$$

(4) Vacancy filling rate

$$q_t^v = \frac{m_t}{v_t}, \quad (\text{D.4})$$

(5) Employment dynamics

$$N_t = (1 - \delta_t)(1 - q_t^a)N_{t-1} + m_t, \quad (\text{D.5})$$

(6) Number of searching workers

$$u_t = 1 - (1 - \delta_t)(1 - q_t^a)N_{t-1}, \quad (\text{D.6})$$

(7) Unemployment

$$U_t = 1 - N_t, \quad (\text{D.7})$$

(8) Vacancy dynamics

$$v_t = (1 - q_{t-1}^v)v_{t-1} + \delta_t N_{t-1} + \eta_t, \quad (\text{D.8})$$

(9) Automation dynamics

$$A_t = (1 - \rho^o)A_{t-1} + q_t^a(1 - \delta_t)N_{t-1}, \quad (\text{D.9})$$

(10) Employment value

$$J_t^e = Z_t - w_t + \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} \left\{ \delta_{t+1} J_{t+1}^v + (1 - \delta_{t+1}) [q_{t+1}^a J_{t+1}^a + (1 - q_{t+1}^a) J_{t+1}^e] \right\}, \quad (\text{D.10})$$

(11) Vacancy value

$$J_t^v = -\kappa + q_t^v J_t^e + (1 - q_t^v) \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} J_{t+1}^v, \quad (\text{D.11})$$

(12) Automation value

$$J_t^a = Z_t \zeta_t - \kappa_a + (1 - \rho^o) \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} J_{t+1}^a, \quad (\text{D.12})$$

(13) Automation threshold

$$x_t^* = J_t^a - J_t^e, \quad (\text{D.13})$$

(14) Robot adoption

$$q_t^a = \left( \frac{x_t^*}{\bar{x}} \right)^{\eta_a}, \quad (\text{D.14})$$

(15) Vacancy creation

$$\eta_t = \left( \frac{J_t^v}{\bar{e}} \right)^{\eta_e}, \quad (\text{D.15})$$

(16) Aggregate output

$$Y_t = Z_t N_t + Z_t \zeta_t A_t. \quad (\text{D.16})$$

(17) Resource constraint

$$C_t + \kappa v_t + \kappa_a A_t + \frac{\eta_a}{1 + \eta_a} q_t^a x_t^* (1 - \delta_t) N_{t-1} + \frac{\eta_e}{1 + \eta_e} \eta_t J_t^v = Y_t, \quad (\text{D.17})$$

(18) Nash bargaining wage

$$\frac{b}{1-b} (J_t^e - J_t^v) = w_t - \phi - \chi C_t + \mathbb{E}_t \frac{\beta \theta_{t+1} C_t}{C_{t+1}} (1 - q_{t+1}^u) (1 - \delta_{t+1}) \frac{b}{1-b} (J_{t+1}^e - J_{t+1}^v), \quad (\text{D.18})$$

APPENDIX E. SUMMARY OF EQUILIBRIUM CONDITIONS: MODEL WITH HETEROGENEOUS  
 WORKER SKILLS

A search equilibrium in the model with heterogeneous workers is a system of 19 equations for 19 variables summarized in the vector

$$[C_t, r_t, Y_t, m_t, u_t, v_t, q_t^u, q_t^v, q_t^a, N_t, U_t, \eta_t, J_t^e, J_t^v, J_t^a, A_t, x_t^*, w_{nt}, w_{st}].$$

We write the equations in the same order as in the dynare code.

(1) Household's bond Euler equation:

$$1 = \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} r_t, \quad (\text{E.1})$$

(2) Matching function

$$m_t = \mu u_t^\alpha v_t^{1-\alpha}, \quad (\text{E.2})$$

(3) Job finding rate

$$q_t^u = \frac{m_t}{u_t}, \quad (\text{E.3})$$

(4) Vacancy filling rate

$$q_t^v = \frac{m_t}{v_t}, \quad (\text{E.4})$$

(5) Employment dynamics

$$N_t = (1 - \delta_t) N_{t-1} + m_t, \quad (\text{E.5})$$

(6) Number of searching workers

$$u_t = 1 - (1 - \delta_t) N_{t-1}, \quad (\text{E.6})$$

(7) Unemployment

$$U_t = 1 - N_t, \quad (\text{E.7})$$

(8) Vacancy dynamics

$$v_t = (1 - q_{t-1}^v)(1 - q_t^a)v_{t-1} + \delta_t N_{t-1} + \eta_t, \quad (\text{E.8})$$

(9) Automation dynamics

$$A_t = (1 - \rho^o) A_{t-1} + q_t^a (1 - q_{t-1}^v) v_{t-1}, \quad (\text{E.9})$$

(10) Employment value

$$J_t^e = Z_t - w_{nt} + \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} [\delta_{t+1} J_{t+1}^v + (1 - \delta_{t+1}) J_{t+1}^e], \quad (\text{E.10})$$

(11) Vacancy value

$$J_t^v = -\kappa + q_t^v J_t^e + (1 - q_t^v) \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} [(1 - q_{t+1}^a) J_{t+1}^v + q_{t+1}^a J_{t+1}^a]. \quad (\text{E.11})$$

(12) Automation value

$$J_t^a = \alpha_a Z_t \zeta_t^{\alpha_a} \left( \frac{\bar{s}}{A_t} \right)^{1-\alpha_a} - \kappa_a + (1 - \rho^o) \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} J_{t+1}^a, \quad (\text{E.12})$$

(13) Automation threshold

$$x_t^* = J_t^a - J_t^v, \quad (\text{E.13})$$

(14) Robot adoption

$$q_t^a = \left( \frac{x_t^*}{\bar{x}} \right)^{\eta_a}, \quad (\text{E.14})$$

(15) Vacancy creation

$$\eta_t = \left( \frac{J_t^v}{\bar{e}} \right)^{\eta_e}, \quad (\text{E.15})$$

(16) Aggregate output

$$Y_t = Z_t N_t + Z_t (\zeta_t A_t)^{\alpha_a} \bar{s}^{1-\alpha_a}, \quad (\text{E.16})$$

(17) Resource constraint

$$C_t + \kappa v_t + \kappa_a A_t + \frac{\eta_a}{1 + \eta_a} q_t^a x_t^* (1 - q_{t-1}^v) v_{t-1} + \frac{\eta_e}{1 + \eta_e} \eta_t J_t^v = Y_t, \quad (\text{E.17})$$

(18) Nash bargaining wage

$$\frac{b}{1-b} (J_t^e - J_t^v) = w_{nt} - \phi - \chi C_t + \mathbb{E}_t \frac{\beta \theta_{t+1} C_t}{C_{t+1}} (1 - q_{t+1}^u) (1 - \delta_{t+1}) \frac{b}{1-b} (J_{t+1}^e - J_{t+1}^v), \quad (\text{E.18})$$

(19) Skilled wage

$$w_{st} = (1 - \alpha_a) Z_t \left( \frac{\zeta_t A_t}{\bar{s}} \right)^{\alpha_a}, \quad (\text{E.19})$$

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