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# Anchored Inflation Expectations and the Flatter Phillips Curve*

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**Abstract**

Conventional versions of the Phillips curve cannot account for inflation dynamics during and after the U.S. Great Recession, leading many to conclude that the Phillips curve relationship has weakened or even disappeared. We show that if agents solve a signal extraction problem to disentangle temporary versus permanent shocks to inflation, then agents' inflation expectations should have become more "anchored" over the Great Moderation period. An estimated New Keynesian Phillips curve that accounts for the increased anchoring of expected inflation exhibits a stable slope coefficient over the period 1960 to 2019. Out-of-sample forecasts show that this model can account for the "missing disinflation" during the U.S. Great Recession and the "missing inflation" during the subsequent recovery. We use a simple three-equation New Keynesian model to show that an increase in the Taylor rule coefficient on inflation (or the output gap) serves to endogenously anchor agents' subjective inflation expectations and thereby "flatten" the reduced-form Phillips curve.

Keywords: *Inflation expectations, Phillips curve, Inflation puzzles, Kalman filter*
JEL Classification: E31, E37

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# 1 Introduction

Starting with the seminal contributions of Phelps (1967) and Friedman (1968), the expectations-augmented Phillips curve, which links inflation to expected inflation and some measure of economic activity, has become a key element in monetary economic models. But over the past decade, inflation in the U.S. appears to have deviated from the behavior predicted by the Phillips curve. First, the absence of a persistent decline in inflation during the Great Recession (the "missing disinflation,"Coibion and Gorodnichenko, 2015a), and, subsequently, the absence of re-inflation during the recovery (the "missing inflation," Constâncio, 2015), has led some to argue that the Phillips curve relationship has weakened or even disappeared (Hall 2011, Powell 2019). Indeed, Hall and Sargent (2018) recently issued the following negative declaration: "*[T]he Phillips curve has little value as a component of a model of inflation. It is not a description of the actual behavior of inflation, and it is incapable of dealing with the important question of what happens when macroeconomic policy undergoes major reform.*"

Figure 1 shows the evolution of key macroeconomic variables from 2006 to 2019. During the Great Recession from 2007.q4 to 2009.q2, the output gap declined sharply by around 6 percentage points. From a historical perspective, a recession of this magnitude should have delivered substantial disinflationary pressures. But in the wake of the Great Recession, core CPI inflation declined by less than 2 percentage points. Moreover, core CPI inflation sharply increased from 2010 and 2012 while the CBO output gap remained in negative territory. The absence of a large disinflation has become known as "the missing disinflation puzzle." Figure 1 shows that long-run inflation expectations, as measured by 10-year ahead CPI inflation forecasts from the Survey of Professional Forecasters (SPF) and the Livingston Survey, remained nearly constant during the Great Recession. More recently, however, long-run inflation expectations have gradually declined and are currently hovering about 25 basis points (bp) below their pre-recession levels. Similarly, core CPI inflation has remained about 50 bp below its pre-recession level. The Fed's preferred inflation measure, namely the headline PCE inflation rate, has remained mostly below the Fed's 2% target since 2012. The absence of re-inflation during the recovery from the Great Recession has become known as the "missing inflation puzzle."

These inflation puzzles have led some to conclude that the historically observed statistical relationship between economic slack and inflation, i.e., the Phillips curve, has changed. Figure 2 provides reduced-form evidence that the Phillips curve has become "flatter" over time. The figure plots the CBO output gap against the 4-quarter change in the 4-quarter core

CPI inflation rate, both before and after the millennium. Indeed, inflation appears to have become less sensitive to economic slack over the past 20 years compared to the previous 40 year period. Numerous empirical studies attribute this finding, at least partially, to a decline in the structural slope parameter of the Phillips curve (Ball and Mazumder 2011, IMF 2013, Blanchard, Cerutti and Summers 2015).[1]

In an influential speech, Bernanke (2007) points out that the "anchoring" of agents' inflation expectations would make reduced-form versions of the Phillips curve appear flatter. Intuitively, if agents set prices and wages with reference to their long-run inflation expectations, and if these expectations become insensitive to changes in economic activity, then inflation itself will become less sensitive to economic activity. Most economists would agree that inflation expectations are well-anchored, and that this anchoring served to mute disinflationary pressures during the Great Recession (Williams 2009, Stock and Watson, 2010, Blanchard, et al. 2015). Nevertheless, the term "anchored" remains somewhat loosely-defined in the literature. Few papers have attempted to formalize the idea in the context of a theoretical equilibrium model or in the empirical estimation of the Phillips curve. Consequently, it remains unclear to *what extent* expectations are anchored, *why* expectations have become more anchored over time and, importantly, *how* this anchoring affects the quantitative relationship between inflation and economic slack.

In this paper, we postulate that agents have an imperfect understanding of the inflation process but nevertheless behave in a boundedly-rational manner. Along the lines of Stock and Watson (2007, 2010), agents construct their inflation forecasts using a univariate time-series model for inflation which allows for both temporary and permanent shocks.[2] This approach has several appealing properties. First, it allows for a straightforward interpretation of the concept of "anchored" expectations. In our model, the representative agent's subjective inflation forecast, $\widetilde{E}_t\pi_{t+1}$, is determined by the Kalman filter:

$$\widetilde{E}_t\pi_{t+1} = \widetilde{E}_{t-1}\pi_t + \lambda_\pi(\pi_t - \widetilde{E}_{t-1}\pi_t),$$

where $\lambda_\pi \in (0, 1]$ is the Kalman gain that governs the sensitivity of inflation expectations to short-run inflation surprises. A low value of $\lambda_\pi$ implies that agents do not revise their inflation forecasts much in response to inflation surprises, implying that expectations are

[1] A Phillips curve slope parameter which depends positively on the level (or volatility) of inflation can be motivated by theories of costly price adjustment (Ball, Mankiw, and Romer 1988). Lansing (2019a) presents some empirical evidence in support of this idea.

[2] But in contrast to Stock and Watson (2007, 2010), agents in our model abstract from stochastic volatility in the two types of shocks.

well-anchored. This idea seems to capture the definition provided by Bernanke (2007): *"I use the term "anchored" to mean relatively insensitive to incoming data. So, for example, if the public experiences a spell of inflation higher than their long-run expectation, but their long-run expectation of inflation changes little as a result, then inflation expectations are well anchored."* Second, this type of forecast rule is strongly supported by survey data on actual expectations, including inflation expectations, as measured by the Survey of Professional Forecasters (Coibion and Gorodnichenko 2015b). Third, as pointed out by Stock and Watson (2010), deriving inflation forecasts from a univariate time series model may be viewed as "near rational" given that numerous studies show that inflation is well-described by univariate statistical processes (Stock and Watson 2007, Lansing 2009, Kozicki and Tinsley 2012, Clark and Doh 2014, Cecchetti, et al. 2017, Dotsey, et. al. 2018).

Our approach has several important implications. First, in line with Bernanke's hypothesis, we find that inflation expectations have become increasingly anchored since the early-1980s, as reflected by a lower estimated value of $\lambda_\pi$. Second, holding the structural slope parameter of the Phillips curve constant, we show that a lower value of $\lambda_\pi$ implies that inflation becomes less sensitive to variations in economic slack.[3] Thus, the anchoring of inflation expectations can help explain the observed flattening of the reduced-form Phillips curve. Indeed, by allowing for time variation in $\lambda_\pi$, an estimated version of a New Keynesian Phillips curve (NKPC) exhibits a stable and highly statistically significant slope parameter over the period 1960 to 2019. To our knowledge, our paper is the first to document a stable and significant NKPC relationship in U.S. data over the past 60 years.

We use our estimated NKPC to generate out-of-sample forecasts for the Great Recession period. The exercise shows that the estimated NKPC accounts very well for the behavior of inflation and long run inflation expectations over the period 2007 to 2019. We find that inflation expectations were well-anchored (but not perfectly anchored) prior to the outbreak of the Great Recession. Well-anchored inflation expectations imply a muted response of inflation to a highly-negative output gap. However, a long-lasting negative gap episode gradually induces a moderate but persistent decline in long-run inflation expectations. As a result, inflation persistently undershoots the Fed's target inflation rate. According to our model of the NKPC, there is no missing disinflation puzzle in the wake of the Great Recession and no missing inflation puzzle during the subsequent recovery.

Why have U.S. inflation expectations become more firmly anchored over the Great Moderation period? Variations in the degree of anchoring are difficult to explain in the context of

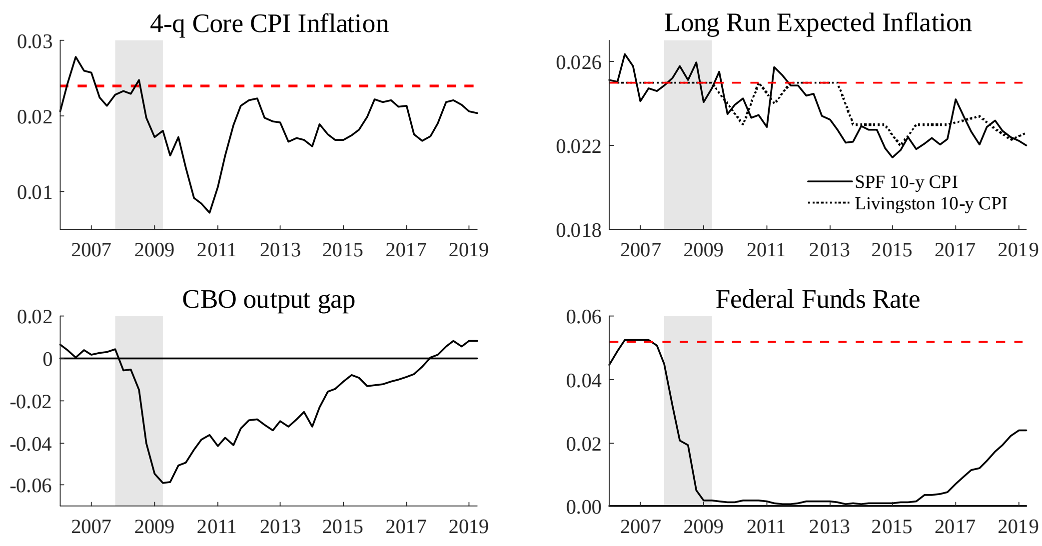
[3] A similar point is made in Stock and Watson (2010).

models where rational agents have full knowledge of the economy and the objectives of the central bank. In this case, agents' long-run inflation expectations do not vary over time in response to new information. However, as Bernanke (2007) points out: *"[L]ong-run inflation expectations do vary over time. That is, they are not perfectly anchored in real economies; moreover, the extent to which they are anchored can change, depending on economic developments and (most important) the current and past conduct of monetary policy."* An implication of the Lucas (1976) critique is that the reduced-form Phillips curve may not be stable over time if the behavior of the monetary authority changes. Put another way: a shift in the monetary policy rule can alter the coefficients of a forecast based on the distributed lags of past inflation. We illustrate this idea using a simple three-equation New Keynesian model augmented by our proposed specification for inflation expectations.[4] Specifically, we show that the optimal Kalman gain in the agent's subjective inflation forecast is uniquely pinned down by the structural parameters of the model. A stronger monetary policy response to inflation (or the output gap) serves to reduce the agent's chosen value of $\lambda_\pi$, making expected inflation more anchored. This result is consistent with a popular view among economists that a more "hawkish" monetary policy accounts for the improved anchoring U.S. inflation expectations starting with the Volcker disinflation of the early 1980s. A stronger monetary policy response to inflation also enables our model to account for the observed declines in macroeconomic volatility and inflation persistence, as well as a reduced sensitivity of inflation to cost-push type shocks (Williams 2006, Mishkin 2007, Bernanke 2010).

The apparent flattening of the Phillips curve has become an important issue for U.S. monetary policy. Former Fed Chair Janet Yellen (2019) remarks: *"If the Phillips curve is very flat, and if inflation expectations are insensitive to fluctuations in actual inflation, the Federal Reserve may be able to run the economy hot, yielding significant benefits to workers, while imposing only minimal costs on society in terms of the higher inflation."* But as noted by Fed Vice Chair Richard Clarida (2019), if inflation does rise above target, a flatter Phillips curve also raises the cost (in terms of lost output or higher unemployment) of bringing inflation back down. Our empirical results indicate that the underlying structural relationship between inflation and economic slack remains alive and well. According to our model, the apparent flattening of the reduced-form Phillips curve over time can be driven by an increase in the monetary policy response to inflation (or the output gap) which, in turn, serves to anchor agents' inflation expectations.

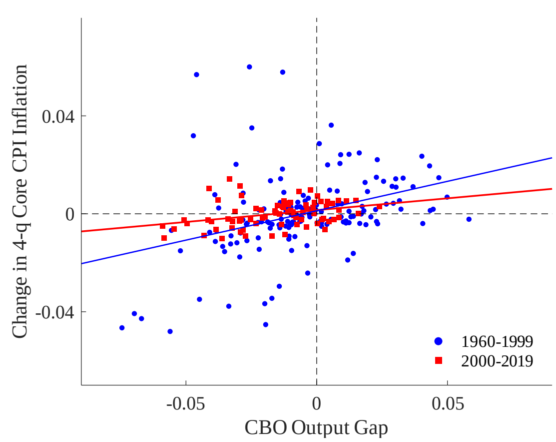
[4] We assume an analogous specification for output gap expectations.

Figure 1: *Key Macroeconomic Variables 2006.q1 to 2019.q2*




Notes: Gray bars indicate the Great Recession from 2007.q4 to 2009.q2. Dashed red lines indicate pre-recession levels as measured by the average level of each variable over the four quarters prior to the recession, i.e., from 2006.q4 to 2007.q3.
Sources: Congressional Budget Office, U.S. Bureau of Labor Statistics, Board of Governors of the Federal Reserve System, Federal Reserve Bank of Philadelphia

Figure 2: *The "Flattening" of the Phillips Curve*




Note: The figure plots fitted lines of the form: $\pi_{4,t} - \pi_{4,t-4} = c_0 + c_1 y_t$, where $\pi_{4,t}$ is the 4-quarter core CPI inflation rate and $y_t$ is the CBO output gap.

## 1.1 Related Literature

The inflation puzzles have spurred a large literature exploring potential explanations. Daly, et al. 2012 and Daly and Hobijn 2014 argue that downward nominal wage rigidity can prevent wages (and prices) from declining substantially during a recession. In contrast, Coibion and Gorodnichenko (2015a) find no evidence of missing disinflation in wages during the Great Recession. Instead, these authors argue that the missing disinflation is explained by a sharp rise in household inflation expectations between 2009 and 2011 which, in turn, can be traced to higher oil prices. Del Negro, et al. (2015) find that the presence of financial frictions can help explain the missing disinflation by reducing the estimated slope of the NKPC. Christiano, et al. (2015) show that a decline in productivity growth combined with an increase in the cost of working capital can help account for the small observed drop in inflation during the Great Recession.

According to Lindé and Trabandt (2019), the introduction of additional strategic complementarities in price and wage setting can resolve both inflation puzzles in a nonlinear solution of the New Keynesian model. The nonlinear solution delivers flatter price and wage Phillips curves when the output gap is negative.

Accounts of the missing inflation puzzle often invoke the role played by the zero lower bound (ZLB) on nominal interest rates. Hills, Nakata, and Schmidt (2016) show that the risk of encountering the ZLB in the future can shift agents' expectations such that the central bank undershoots its inflation target in the present. Bianchi, et al. (2019) make a similar point. Lansing (2019b) considers a New Keynesian model with an occasionally binding ZLB and two local rational expectations equilibria, labeled the "targeted" and "deflation" solutions respectively. The model can produce persistent inflation undershooting if agents' forecast rules assign a nontrivial weight to the deflation equilibrium.

While the above-mentioned theories offer some important insights about inflation dynamics, our paper presents a simple model that can account for key features of U.S. inflation data over the past 40 years. As emphasized by Miskin (2007): *"[W]hat is particularly attractive about highlighting a better anchoring of inflation expectations as probably the primary factor driving the changes in inflation dynamics is that this one explanation covers so many of the stylized facts—an application of Occam's razor."*

Our theoretical framework is related to Lansing (2009) and Carvalho, et al. (2019) who develop partial equilibrium models in which the concept of central bank credibility or anchored inflation expectations is linked to agents' signal extraction problem for unobserved trend inflation. To our knowledge, our paper is the first to formalize the link between the degree of anchoring in agents' inflation expectations and the coefficients of the monetary policy rule in a general equilibrium model.

The empirical approach in our paper is closely related to Stock and Watson (2010) and Stock (2011). These authors derive a measure of expected inflation from the unobserved components-stochastic volatility (UC-SV) model of Stock and Watson (2007). They find support for a stable slope coefficient in reduced-form Phillips curve regressions using the so-called "unemployment recession gap," defined as the unemployment rate minus the minimum value of the unemployment rate over the current and past 11 quarters. For standard gap measures, however, stability of the slope coefficient is generally rejected. In contrast to these authors, we do not treat expected inflation as pre-determined in our regressions. Instead, we estimate the Kalman gain $\lambda_\pi$ and the NKPC slope parameter simultaneously using standard gap measures.

In recent work, Ball and Mazumder (2019a,b) argue that inflation puzzles in the U.S. and the euro area can be partially resolved by replacing standard measures of core inflation with the weighted-median of industry inflation rates—a measure that is less volatile than conventional core measures. In contrast, our specification of agents' inflation expectations resolves both inflation puzzles when we use core CPI inflation.

Our paper is also related to the literature on time-varying trend inflation (Ascari and Sbordone 2014). Cogley and Sbordone (2008) derive a version of the NKPC that explicitly allows for variations in trend inflation. In their model, a time-varying vector autoregression is used to both construct a measure of trend inflation and to represent agents' subjective beliefs. Under these assumptions, the inclusion of ad-hoc inflation lags in the NKPC (the so-called "hybrid" NKPC) is no longer necessary to match inflation persistence in U.S. data.[5] In our model, agents' subjective beliefs about the time series process for inflation introduces endogenous persistence that can match the data without the need for ad-hoc inflation lags.

The remaining sections of the paper proceed as follows. Section 2 briefly illustrates the "puzzling" inflation dynamics implied by versions of the NKPC with either rational or backward-looking expectations. Section 3 presents our model of inflation expectations that formalizes the idea of anchoring in terms of the Kalman gain parameter. Section 4 investigates the empirical implications of our model for the estimated slope coefficient in the NKPC and for out-of-sample forecasts both during and after the Great Recession. Section 5 uses a simple New Keynesian equilibrium model to examine the theoretical links between the mon-

[5] These inflation lags can be motivated by price indexation (Christiano, et al. 2005) or rule-of-thumb behavior (Gali and Gertler 1999).

etary policy rule coefficients and the degree of anchoring in agents' expectations. Section 6 concludes.

# 2 Inflation Puzzles in Standard Models

The starting point for our analysis is the standard NKPC:

$$\pi_t = \beta \widetilde{E}_t \pi_{t+1} + \kappa y_t + u_t, \qquad \beta \in [0,1),\;\; \kappa > 0,\;\; u_t \sim N\left(0, \sigma_u^2\right), \tag{1}$$

where $\pi_t$ is the quarterly inflation rate (log difference of the price level), $y_t$ is the output gap (the log deviation of real output from potential output), $u_t$ is an *iid* cost-push shock, and $\beta$ is the representative agent's subjective time discount factor. The symbol $\widetilde{E}_t$ represents the agent's subjective expectations operator. Under rational expectations (RE), $\widetilde{E}_t$ becomes the mathematical expectation operator $E_t$. Equation (1) can be derived from the sticky price model of Calvo (1983) or the menu cost model of Rotemberg (1982).

For illustration purposes in this section, we complete the model using a simple autoregressive process for the output gap:[6]

$$y_t = \rho y_{t-1} + \upsilon_t, \qquad \rho \in [0,1), \quad \upsilon_t \sim N\left(0, \sigma_\upsilon^2\right). \tag{2}$$

The $h$-quarter ahead rational inflation forecast is given by

$$E_t \pi_{t+h} = \frac{\rho^h \kappa}{1 - \rho\beta}\, y_t, \tag{3}$$

which implies that expected inflation at any horizon $h$ is perfectly correlated with the current output gap $y_t$.

In contrast with equation (3), a backward-looking (BL) Phillips curve would typically assume that the agent's inflation forecast is constructed as a distributed lag of past inflation rates, with the lag coefficients summing to unity. For example, Ball and Mazumder (2011) develop a model where expected inflation is given by

$$\widetilde{E}_t \pi_{t+1} = \left(\pi_{t-1} + \pi_{t-2} + \pi_{t-3} + \pi_{t-4}\right)/4. \tag{4}$$

For the following exercise, we substitute either the rational forecast (3) or the backward-looking forecast (4) into the NKPC to construct a projected inflation path from 2007 to

[6] None of our conclusions depend on this assumption. Inflation puzzles also arise in fully specified New Keynesian models with a zero lower bound on the nominal interest rate (see, for example, Aruoba and Schorfheide 2016).

2019, conditional on the observed path for the CBO output gap. We set the discount factor to $\beta = 0.995$, corresponding to an annual real interest rate of 2 percent. We estimate the autoregressive parameter of $\rho = 0.95$ using data for the CBO output gap from 1984.q1 to 2007.q3. To be conservative, we set $\kappa = 0.04$ which is somewhat lower than values typically employed in the literature.[7]

Using the CBO output gap plotted in Figure 1 as a driving variable, we construct paths for inflation and expected inflation for the period 2007.q4 to 2019.q2 using each of the two forecast rules (3) or (4). In the case of the rational forecast, we add a constant term to the right side of equation (1) to bring the level of $\pi_t$ up to the value observed in the data prior to the start of the exercise.[8]

Figure 3 shows that both forecast rules imply pronounced deflation episodes during and after the Great Recession. The RE model exhibits a missing inflation puzzle after 2009 in the sense that inflation increases steadily in the model, but not in the data. By the year 2018, the RE model predicts an inflation rate that exceeds its pre-recession level. The right panel shows that 10-year ahead expected inflation in the RE model looks nothing like the 10-year ahead expected inflation rate from the SPF, which remains tightly anchored to its pre-recession level. The BL model predicts declining inflation and expected inflation from 2007.q3 onward. The predictions of both models can be improved by employing an extremely low (near-zero) value of the slope parameter $\kappa$. Nevertheless, both models still fail to qualitatively match the patterns observed in the data.

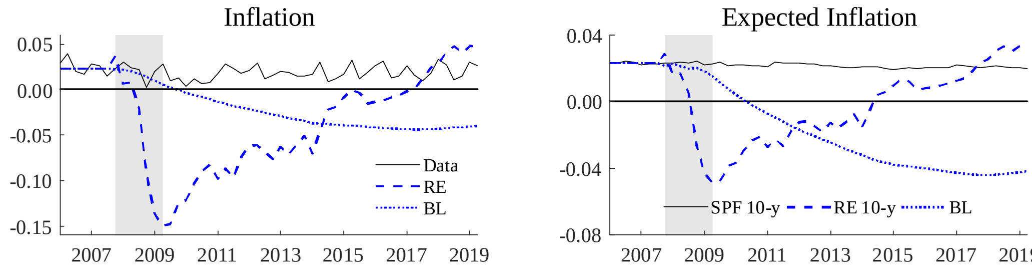
# 3 A Model of Inflation Expectations

The NKPC (1) is derived under the assumption of a constant (zero) steady state inflation rate. In reality, however, trend inflation is positive and time-varying. Ascari (2004) and Sahuc (2006) demonstrate that the functional form of the NKPC changes under positive (and potentially time-varying) trend inflation. However, Cogley and Sbordone (2008) show that a standard NKPC expression can be derived under subjective expectations and time-varying trend inflation in terms of the inflation gap (the deviation of inflation from trend inflation)

[7] For example Aruoba and Schorfheide (2016) estimate $\kappa \simeq 0.10$ on U.S. data using Bayesian methods. In Gali (2008), $\kappa \simeq 0.13$ (chapter 3).

[8] The constant term corresponds to an annualized inflation rate of 2.3%. This value also roughly coincides with pre-recession forecasts of long-run headline CPI inflation from the SPF and Livingston surveys, but adjusted downward by 20 bp to account for the persistent gap between headline and core CPI inflation prior to the recession.

Figure 3: *Inflation Puzzles in Standard Versions of the Phillips Curve*




Notes: Model-implied inflation and expected inflation paths are expressed as annualized quarterly rates. In the right panel, "RE 10-y" is the rational expectation of the average quarterly inflation rate over the next 40 quarters from equation (3), whereas "BL" is the 1-quarter ahead backward-looking forecast from equation (4). "SPF 10-y" is 10-year ahead headline CPI inflation forecasts from the SPF, but adjusted downward by 20 bp to account for the persistent gap between headline and core CPI inflation prior to the recession.

if non-resetting firms index their price to past prices and/or to current trend inflation.[9] The derivation of equation (1) also makes use of the Law of Iterated Expectations, which may not be satisfied under subjective expectations. However, Adam and Padula (2011) show that if agents are unable to predict revisions to their own or other agents' forecasts, then subjective expectations will satisfy the Law of Iterated Expectations, thereby recovering a Phillips curve that resembles equation (1). Coibion and Gorodnichenko (2017) show that SPF inflation forecasts do in fact appear to satisfy the Law of Iterated Expectations. As we will show, our model of subjective expectations closely tracks long-run inflation expectations from the SPF.

If agents have imperfect knowledge about the underlying inflation process and/or policymakers' objective function, then they must make statistical inferences about the evolution of the unobservable inflation trend. Numerous studies show that inflation or expected inflation appears to be well-described by simple univariate time series models. According to Stock and Watson (2010), *"it is exceedingly difficult to improve systematically upon simple univariate forecasting models [of inflation]."* Along the lines of Stock and Watson (2007, 2010), we assume that agents employ the following time series model for inflation:

$$\begin{bmatrix} \pi_t \\ \overline{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \overline{\pi}_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \zeta_t \\ \eta_t \end{bmatrix}, \qquad \begin{array}{c} \zeta_t \sim N\left(0, \sigma_\zeta^2\right), \\ \eta_t \sim N\left(0, \sigma_\eta^2\right), \\ Cov\left(\zeta_t, \eta_t\right) = 0, \end{array} \tag{5}$$

where $\overline{\pi}_t$ is the unobservable inflation trend, $\zeta_t$ is a transitory shock that pushes $\pi_t$ away from

[9]Indexation is frequently assumed in models with positive trend inflation (Ireland 2007, Schorfheide 2005, Smets and Wouters 2003).

trend, and $\eta_t$ is permanent shock (uncorrelated with $\zeta_t$) that shifts the trend over time. But in contrast to Stock and Watson (2007, 2010), we assume that agents abstract from stochastic volatility in the two types of shocks. Given the time series model (5), the optimal subjective forecast $\widetilde{E}_t \pi_{t+1}$ is the Kalman filter estimate of the unobserved inflation trend $\overline{\pi}_t$, as given by

$$\begin{aligned} \widetilde{E}_t \pi_{t+1} = \widetilde{E}_t \overline{\pi}_t &= \lambda_\pi \pi_t + (1-\lambda_\pi)\, \widetilde{E}_{t-1} \pi_t, \\ &= \lambda_\pi \left[ \pi_t + (1-\lambda_\pi)\, \pi_{t-1} + (1-\lambda_\pi)^2\, \pi_{t-2} + ... \right], \end{aligned} \tag{6}$$

where $\lambda_\pi \in (0, 1]$ is the Kalman gain parameter which depends on the relative variances of the two types of shocks (Muth 1960). Equation (6) implies that the agent's subjective inflation forecast at time $t$ is an exponentially-weighted moving average of current and past inflation rates.

A large body of empirical evidence suggests that moving-average type forecast rules, such as (6), provide a good description of actual expectations as measured by surveys. This evidence includes household expectations about future house price growth (Piazzesi and Schneider 2009, Case, et al. 2012, Jurgilas and Lansing 2013), investor's expectations about future stock returns (Vissing-Jorgensen 2003, Greenwood and Shleifer 2014, Barberis, et al. 2015, Adam, et al. 2017), economists' long-run productivity growth forecasts (Edge, et al. 2011), inflation forecasts of households and professionals (Mankiw, et al. 2003, Lansing 2009, Kozicki and Tinsley 2012, Coibion and Gorodnichenko 2015b, 2018), and forecasts of other key macroeconomic variables (Coibion and Gorodnichenko 2012, Bordalo, et al. 2018). In a prominent paper, Coibion and Gorodnichenko (2015b) show that ex-post mean inflation forecast errors from the SPF can be predicted using ex-ante mean forecast revisions, consistent with a forecast rule of the form (6).

The optimal value of $\lambda_\pi$ minimizes the one-step ahead mean squared forecast error, as given by $E(\pi_{t+1} - \widetilde{E}_t \pi_{t+1})^2$. It can be shown that the unique steady state solution for the optimal Kalman gain is given by

$$\lambda_\pi = \frac{-\,\phi_\pi + \sqrt{\phi_\pi^2 + 4\phi_\pi}}{2}, \tag{7}$$

where $\phi_\pi \equiv \sigma_\eta^2 / \sigma_\zeta^2$ is the perceived signal-to-noise ratio for inflation.[10] As can be seen from equation (6), $\lambda_\pi$ governs the sensitivity of forecast revisions to inflation surprises. The value of $\lambda_\pi$ can therefore be viewed as measuring the degree to which inflation expectations remain anchored, with a lower value implying more anchoring. As $\phi_\pi \to \infty$, we have $\lambda_\pi \to 1$.

[10] For details of the derivation, see Nerlove (1967, pp. 141-143). His results are expressed as a formula for $1 - \lambda$.

Intuitively, a high signal-to-noise ratio implies that inflation is driven mostly by the permanent shock $\eta_t$. Consequently, agents are quick to revise their estimate of trend inflation in response to incoming data, implying that expectations are poorly anchored. In contrast, a low signal-to-noise ratio implies that inflation is driven mostly by the transitory shock $\zeta_t$. As $\phi_\pi \to 0$, we have $\lambda_\pi \to 0$. In this special case, agents do not revise their forecasts at all in response to recent forecast errors, implying that expectations are perfectly-anchored at the long-run mean inflation rate. Along these lines, Lansing (2009) notes that the perceived signal-to-noise ratio can be viewed as an inverse measure of the central bank's credibility for maintaining a stable inflation target.

We now consider whether an agent's perceived-optimal value of $\lambda_\pi$ computed from U.S. inflation data has changed over time. Table 1 shows the values of $\lambda_\pi$ that minimize the 1-quarter ahead mean squared forecast errors for quarterly core CPI inflation rates across three subsamples. Specifically, we compute the value of $\lambda_\pi$ that solves:

$$\min_{\lambda_\pi} \sum_{k=0}^{h} \frac{1}{h} (\pi_{t-k} - \widetilde{E}_{t-k-1} \pi_{t-k})^2 \tag{8}$$

where $\pi_t$ is the observed quarterly inflation rate, $h$ is the number of forecast errors in the sub-sample, and $\widetilde{E}_{t-k-1}\pi_{t-k}$ is constructed using equation (6). Table 1 shows that the perceived-optimal value of $\lambda_\pi$ has declined substantially from around 0.5 in the Great Inflation Era to near-zero in the Great Recession Era. This pattern is driven by a decline in the inflation signal-to-noise ratio. Put another way, unexpected changes in core CPI inflation are much less persistent today than in earlier decades. Consequently, inflation expectations, as governed by (6) should have become more anchored over the past 30 to 40 years. Consistent with these results, Coibion and Gorodnichenko (2015b) find that their version of $\lambda_\pi$ has decreased since the early 1980s. Similarly, Stock and Watson (2007) find that the estimated gain parameter in a time-varying unobserved components model of inflation has declined. Other papers that find empirical evidence of more-anchored inflation expectations over the Great Moderation Era include Williams (2006), Lansing (2009), IMF (2013), Blanchard, et al. (2015), and Carvalho, et al. (2019).

Table 1: Ex-Post Optimal Kalman Gain

| | Great Inflation Era 1960.q1 to 1983.q4 | Great Moderation Era 1984.q1 to 2007.q3 | Great Recession Era 2007.q4 to 2019.q2 |
|---|---|---|---|
| $\lambda_\pi$ | 0.489 | 0.216 | 0.041 |

## 4 The Flatter Phillips Curve

We now examine how a trend toward more-anchored inflation expectations, i.e., a lower value of $\lambda_\pi$, influences the estimated value of $\kappa$, which represents the structural slope parameter of the NKPC. Substituting the agent's subjective inflation forecast (6) into the NKPC (1) and then solving for $\pi_t$ yields

$$\pi_t = \frac{\beta\left(1-\lambda_\pi\right)}{1-\beta\lambda_\pi}\widetilde{E}_{t-1}\pi_t + \frac{1}{1-\beta\lambda_\pi}\left(\kappa y_t + u_t\right). \tag{9}$$

Equation (9) implies the following partial derivative of inflation with respect to the output gap:

$$\frac{\partial \pi_t}{\partial y_t} = \frac{\kappa}{1-\beta\lambda_\pi}. \tag{10}$$

The above derivative can be viewed measuring the slope of a reduced-form Phillips curve that is constructed by regressing $\pi_t$ on a constant and $y_t$. Lower values of $\lambda_\pi$, implying more-anchored inflation expectations, serve to "flatten" the reduced-form Phillips curve even if the true NKPC slope parameter $\kappa$ remains unchanged.[11]

Has the value of $\kappa$ changed over time? To answer this question, we use equation (9) to simultaneously estimate $\lambda_\pi$ and $\kappa$. Specifically, we run the following regression

$$\pi_t = \frac{\beta(1-\widehat{\lambda}_\pi)}{1-\beta\widehat{\lambda}_\pi}\widetilde{E}_{t-1}\pi_t \;+\; \frac{\widehat{\kappa}}{1-\beta\widehat{\lambda}_\pi}y_t \;+\; \varepsilon_t, \tag{11}$$

where $\varepsilon_t$ is an error term and $\widetilde{E}_{t-1}\pi_t$ evolves according to the following law of motion:

$$\widetilde{E}_{t-1}\pi_t = \widehat{\lambda}_\pi \pi_{t-1} \;+\; (1-\widehat{\lambda}_\pi)\widetilde{E}_{t-2}\pi_{t-1}. \tag{12}$$

For the first period to be forecasted, $t = t_0$, we assume that the value of $\widetilde{E}_{t_0-2}\pi_{t_0-1}$ in equation (12) is exogenously given by the average of the eight most recent inflation rates, i.e., $0.125\sum_{k=2}^{9}\pi_{t_0-k}$. Our estimation procedure implies that the agent's forecast of inflation at any time $t$ makes use of all current and past inflation rates observed within a given sample period. We estimate the system (11) and (12) using nonlinear least squares under the assumption that $\beta = 0.995$.[12] We use quarterly data for core CPI inflation and the CBO output gap

[11] Lower values of $\lambda_\pi$ also imply that $\pi_t$ becomes less sensitive to the cost-push shock $u_t$, e.g., a shock to food or energy prices. This prediction of our model is consistent with U.S. data (Bernanke 2007, Mishkin 2007, Hooker 2012).

[12] In the New Keynesian literature, the NKPC is typically estimated with instrumental variables using lagged variables as instruments which should be orthogonal to the current period cost-push shock. However, as discussed by Mavroeidis, et al. (2014), if cost-push shocks exhibit autocorrelation, then shocks will still be correlated with the lagged variables and the exclusion restriction will not be satisfied. Our use of core inflation helps to control for the direct impacts of cost-push shocks on inflation.

over the period 1960.q1 to 2019.q2. As in Table 1, we split the data into three subsamples. The estimation results are shown in Table 2.

Table 2: NKPC Parameter Estimates

| | Great Inflation Era 1960.q1 to 1983.q4 | Great Moderation Era 1984.q1 to 2007.q3 | Great Recession Era 2007.q4 to 2019.q2 |
|---|---|---|---|
| $\widehat{\kappa}$ | 0.041** | 0.028*** | 0.041*** |
| | (0.019) | (0.010) | (0.011) |
| $\widehat{\lambda}_{\pi}$ | 0.410*** | 0.188*** | 0.008 |
| | (0.126) | (0.045) | (0.007) |
| Adj. $R^2$ | 0.66 | 0.54 | 0.07 |
| Obs. | 96 | 95 | 47 |

Notes: The asterisks *** and ** denote significance at the 1 percent and 5 percent levels, respectively. The estimation uses quarterly inflation rates (not annualized). Newey-West standard errors (3 lags) are shown in parentheses.

Table 2 shows that the estimated value of the NKPC slope parameter $\widehat{\kappa}$ remains relatively stable and highly statistically significant across all three subsamples. In contrast, the estimated value of the Kalman gain parameter $\widehat{\lambda}_{\pi}$ declines over time, going from a value around 0.4 during the Great Inflation Era to around 0.2 during the Great Moderation Era. During the Great Recession Era, the estimated value of $\lambda_{\pi}$ is not statistically different from zero, implying that inflation expectations have been well-anchored during and after the Great Recession. If we fix the value of $\lambda_{\pi}$ in our regressions to say $\lambda_{\pi} = 0.25$, then we obtain the usual result that $\widehat{\kappa}$ declines over time and eventually becomes statistically insignificant. In Table 2A of the appendix, we demonstrate that our results are robust to changes in both the inflation measure (use of core PCE inflation instead of core CPI inflation) and the measure of economic slack (use of the unemployment gap instead of the output gap).[13]

The top panel of Figure 4 plots values of $\widehat{\lambda}_{\pi}$ from a series of 10-year rolling regressions.[14] The estimated value of the Kalman gain increases during the 1960s and 1970s, but then starts to decline in the early 1980s. This downward trend continues until the late 1990s. But since then, $\widehat{\lambda}_{\pi}$ remains roughly constant at a low positive value. These results indicate that the expectations anchoring process was completed roughly 20 years ago. Mishkin (2007) and

[13] As shown in Table 2A, we do observe a decline in the magnitude of $\widehat{\kappa}$ for the unemployment gap when going from the Great Inflation Era to the Great Moderation Era. But then $\widehat{\kappa}$ remains roughly constant after this initial decline.

[14] The initial condition $\widetilde{E}_{t_0-2}\overline{\pi}_{t_0-1}$ for each rolling regression is the model-implied trend estimate for that quarter from the preceeding rolling regression.

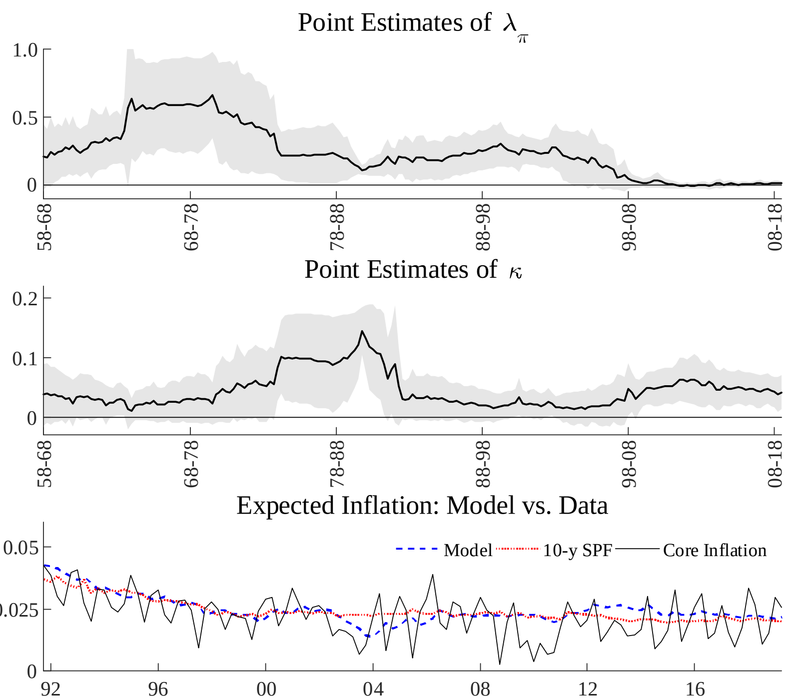
Bernanke (2007) employ narrative approaches and reach a similar conclusion regarding the timing of the expectations anchoring process. The middle panel of Figure 4 plots values of $\widehat{\kappa}$ from the same series of 10-year rolling regressions. With exception of the 1980s, the point estimates are relatively stable.

The agent's time series model for inflation (5) implies that the subjective forecast for inflation is the same for all future horizons. The lower panel of Figure 4 compares expected inflation from the model to long-run expected inflation from the Survey of Professional Forecasters. Specifically, we compute expected inflation implied by the forecast rule (6) using the trailing 10-year rolling regression values of $\widehat{\lambda}_{\pi}$, as plotted in the top panel of Figure 4. The model-implied forecasts track the 10-year ahead CPI inflation forecast from the SPF relatively well. The correlation coefficient between the model-implied forecasts and the SPF forecasts is 0.90.[15] Notably, the model-implied inflation forecast does not decline between 2008 and 2012, despite a simultaneous decline in core CPI inflation. This pattern is a direct result of well-anchored inflation expectations. In the mid-2000s, the model-implied forecast for core CPI inflation is persistently below the SPF forecasts of headline CPI inflation. This result can be attributed to the large and persistent gap between headline and core CPI inflation rates during this period.

Figure 5 shows how different values of $\lambda_{\pi}$ influence the model-implied paths of inflation and expected inflation in response to an exogenous path for the output gap $y_t$. Specifically, we employ the CBO output gap from 2007.q4 to 2019q.2 as a driving variable in equation (9). We set $u_t = 0$ for all $t$ and choose an initial value for $\widetilde{E}_{t-1}\pi_t$ that corresponds to an annual inflation rate of 2.3%. We employ three different values of $\lambda_{\pi}$, corresponding to the three subsample point estimates shown earlier in Table 2. To make the simulations comparable, we use our baseline value $\kappa = 0.04$ in all three cases, which roughly corresponds to the point estimates for the Great Inflation and Great Recession subsamples in Table 2. Clearly, inflation dynamics depend crucially on the value of $\lambda_{\pi}$. When expectations are poorly-anchored ($\lambda_{\pi} = 0.410$), inflation declines by roughly 10 percentage points over the simulation, not unlike what happened during the Volcker disinflation. But when expectations are well-anchored ($\lambda_{\pi} = 0.008$), the post-2007 output gap series has practically no impact on the model-implied expected inflation series.

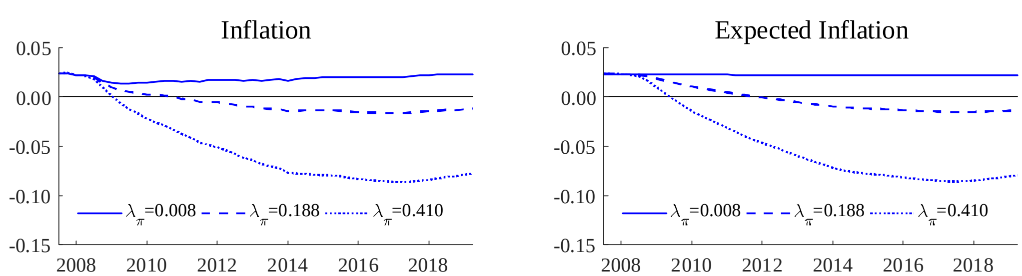
[15] Similarly, the correlation coefficient between the model-implied forecasts and 10-year ahead CPI inflation forecast from the Livingston survey is 0.92.

Figure 4: *Results from 10-year Rolling Regressions*




Notes: Gray areas represent 95 percent confidence intervals on point estimates from 10-year rolling regressions. Model-implied inflation expectations are expressed as annualized quarterly rates. Long-run expectations in the data is the 10-year ahead forecast of headline CPI inflation from the SPF, but adjusted downward by 20 bp to account for the persistent gap between headline and core CPI inflation prior to the recession.

Figure 5: *Inflation Response to the Great Recession with Different Degrees of Anchoring*




Note: Model-implied inflation and expected inflation are expressed as annualized quarterly rates.

## 4.1 Out-of-Sample Forecasts: Resolving the Inflation Puzzles

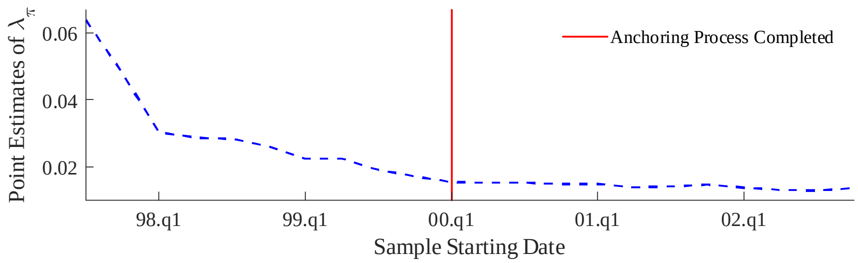
In this section, we show that the estimated version of our NKPC given by equations (11) and (12) can account for the "puzzling" behavior of inflation since 2007. We estimate equations (11) and (12) using data from 2000.q1 to 2007.q3 and then use the estimated equations to compute a projected path for inflation from to 2007.q4 to 2019.q2, again using the CBO output gap as a driving variable.[16] The 2000.Q1 starting date for the estimation corresponds to the date when the expectations anchoring process appears to have been completed—as shown earlier in the top panel of Figure 4. To further illustrate this idea, Figure 6 plots regression values of $\widehat{\lambda}_{\pi}$ for various sample starting dates, while keeping the ending date fixed at 2019.q2.[17] The figure confirms that the downward trend in $\widehat{\lambda}_{\pi}$ appears to have ended around 2000.q1. The point estimates for $\kappa$ and $\lambda_{\pi}$ using data from 2000.q1 to 2007.q3 are $\widehat{\kappa} = 0.058$ (significant at the 1 percent level) and $\widehat{\lambda}_{\pi} = 0.017$ (not statistically different from zero). These results are fairly similar to the estimates obtained for the Great Recession Era starting in 2007.q4, as reported earlier in Table 2.

The out-of-sample inflation forecasts from this exercise are plotted in Figure 7. The model-implied paths for inflation and long-run expected inflation track well with the corresponding series in the data. As noted earlier in the context of equation (10), a low value of $\lambda_{\pi}$, implying well-anchored inflation expectations, reduces the sensitivity of inflation to movements in the output gap. According to our model, there is no missing disinflation in the wake of the Great Recession. However, because inflation expectations are not perfectly anchored, i.e. $\lambda_{\pi} > 0$, the model-implied path for long-run expected inflation will gradually decline when inflation remains persistently low, as it does in the data. While the decline in long-run expected inflation is modest (around 40 bp in the model and 25 bp in the SPF), it is highly persistent. The low level of expected inflation in the model serves to keep actual inflation low, even after the output gap has fully recovered. Thus, according to the model, there is no missing inflation in the years since the Great Recession ended.

[16] As in Figure 4, the initial condition $\widetilde{E}_{t_0-2}\overline{\pi}_{t_0-1}$ is given by the trend estimate for that quarter from the preceeding rolling regression (ending in 2007.q2), which corresponds to an annualized inflation rate close to 2.3%

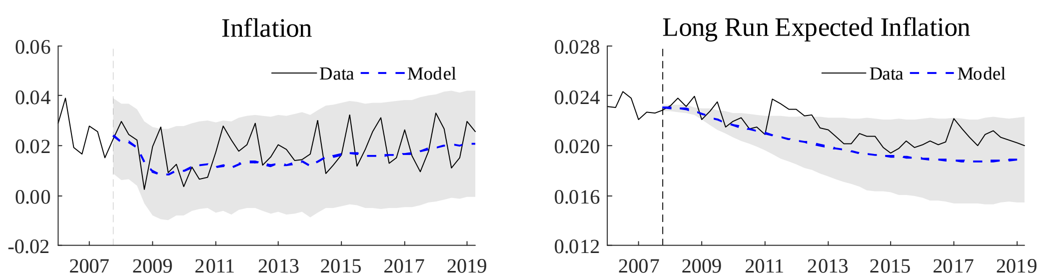
[17] Using 2019.q2 as the ending date instead of 2007.q3 increases the statistical power of these regressions without changing the qualitative conclusions.

Figure 6: *Point Estimates of the Kalman Gain for Subsamples Ending in 2019.q2*




Notes: The figure shows regression values of $\widehat{\lambda}_\pi$ for various sample starting dates, while keeping the ending date fixed at 2019.q2. The inflation expectations anchoring process appears to have been completed around 2000.q1.

Figure 7: *Out-of-Sample Forecasts: 2007.q4 to 2019.q2*




Notes: Gray areas indicate 95 percent confidence bands. Model-implied inflation and expected inflation are expressed as annualized quarterly rates. Long-run expectations in the data is the 10-year ahead forecast of headline CPI inflation from the SPF, but adjusted downward by 20 bp to account for the persistent gap between headline and core CPI inflation prior to the recession.

# 5 Policy and Anchored Expectations in Equilibrium

What accounts for the anchoring of U.S. inflation expectations, as reflected by the decline is the estimated value of $\lambda_\pi$? Many economists believe that the start of the anchoring phenomenon can be traced to a shift in monetary policy under Fed Chairman Paul Volcker in the early-1980s. Indeed, at the peak of the Great Inflation, Volcker himself (1979, pp. 888-889) emphasized the importance of inflation expectations: *"Inflation feeds in part on itself, so part of the job of returning to a more stable and more productive economy must be to break the grip of inflationary expectations."* In this section, we show that a more "hawkish" monetary policy can indeed serve to anchor the agent's subjective inflation expectations in a simple New Keynesian model.

## 5.1 New Keynesian equilibrium model

We employ a three-equation New Keynesian model along the lines of Clarida, et al. (2000) consisting of the NKPC (1), an IS curve, and a monetary policy rule. The IS curve (which is derived from the agent's consumption Euler equation) is given by:

$$y_t = \widetilde{E}_t y_{t+1} - \alpha(i_t - \widetilde{E}_t \pi_{t+1}) + v_t, \qquad \alpha > 0, \;\; v_t \sim N\left(0, \sigma_v^2\right), \tag{13}$$

where $i_t$ is the deviation of the nominal policy interest rate from its steady state value, $\alpha$ is the inverse of the coefficient of relative risk aversion, and $v_t$ is an *iid* demand shock that is uncorrelated with the cost-push shock.

Monetary policy is governed by a Taylor-type rule (Taylor 1993):

$$i_t = \mu_\pi \pi_t + \mu_y y_t, \tag{14}$$

where $\mu_\pi$ and $\mu_y$ determine the response of the policy interest rate to inflation and the output gap. We assume that the Taylor principle is satisfied such that $\mu_\pi > 1$.

The model contains two subjective expectations, namely $\widetilde{E}_t \pi_{t+1}$ and $\widetilde{E}_t y_{t+1}$. As before, $\widetilde{E}_t \pi_{t+1}$ is given by equation (6) which is the perceived-optimal forecast rule for $\pi_{t+1}$ conditional on the agent's time series model for inflation (5). We postulate that the agent employs an analogous time series model for the output gap which is given by:

$$\begin{bmatrix} y_t \\ \overline{y}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \overline{y}_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \chi_t \\ \varphi_t \end{bmatrix}, \qquad \begin{array}{c} \chi_t \sim N\left(0, \sigma_\chi^2\right), \\ \varphi_t \sim N\left(0, \sigma_\varphi^2\right), \\ Cov\left(\chi_t, \varphi_t\right) = 0, \end{array} \tag{15}$$

where $\overline{y}_t$ is the perceived long-run output gap, $\chi_t$ is a transitory shock and $\varphi_t$ is permanent shock (uncorrelated with $\chi_t$). A technical point worth noting is that while the CBO output gap appears to be stationary, it is highly persistent. For example, the CBO output gap remained in negative territory for nearly a decade from 2008.q1 through 2017.q3. The autoregressive coefficient in quarterly data from 1984.q1 to 2019.q2. is 0.95. The agent's use of a time series model that exhibits a unit root can be viewed as a local approximation that is convenient for forecasting purposes.

Conditional on the agent's time series model for the output gap (15), the perceived-optimal forecast rule for $y_{t+1}$ is given by

$$\begin{aligned} \widetilde{E}_t y_{t+1} = \widetilde{E}_t \overline{y}_t &= \lambda_y y_t + (1-\lambda_y)\, \widetilde{E}_{t-1} y_t \ , \\ &= \lambda_y \left[ y_t + (1-\lambda_y)\ y_{t-1} + (1-\lambda_y)^2\, y_{t-2} + ... \right] , \end{aligned} \tag{16}$$

where $\lambda_y \in (0,1]$ is the optimal Kalman gain. Analogous to equation (7), the solution for $\lambda_y$ is

$$\lambda_y = \frac{-\,\phi_y + \sqrt{\phi_y^2 + 4\phi_y}}{2}, \tag{17}$$

where $\phi_y \equiv \sigma_\varphi^2 / \sigma_\chi^2$ is the perceived signal-to-noise ratio for the output gap. Our model specification is consistent with the findings of Coibion and Gorodnichenko (2015b) who identify different degrees of information rigidity across macroeconomic variables, implying the use of different Kalman gains when professionals forecast these variables.

## 5.2 Consistent Expectations Equilibrium

Rational expectations are sometimes called "model consistent expectations." A more precise term would be "true-model consistent expectations," because the maintained assumption is that agents know the true model of the economy. In reality, agents do not know the true model of the economy, but they can observe economic data. In this section, we solve for a "consistent expectations equilibrium" in which the parameters of the representative agent's subjective forecast rules are consistent with: (1) the perceived laws of motion for $\pi_t$ and $y_t$, and (2) the observed moments of $\Delta\pi_t$ and $\Delta y_t$ in the model-generated data.[18]

**Proposition 1.** *If the representative agent's perceived law of motion for inflation is given by equation (5), then the perceived optimal value of the Kalman gain parameter $\lambda_\pi$ is uniquely pinned down by the autocorrelation of observed inflation changes, $Corr\,(\Delta\pi_t, \Delta\pi_{t-1})$.*

[18] This type of boundedly-rational equilibrium concept was developed by Hommes and Sorger (1998). A closely-related concept is the "restricted perceptions equilibrium" described by Evans and Honkopohja (2001, Chapter 13).

*Proof*: From (5), we have $\Delta\pi_t = \eta_t + \zeta_t - \zeta_{t-1}$. Since $\eta_t$ and $\zeta_t$ are perceived to be independent, we have $Cov\,(\Delta\pi_t, \Delta\pi_{t-1}) = -\sigma_\zeta^2$ and $Var\,(\Delta\pi_t) = \sigma_\eta^2 + 2\sigma_\zeta^2$. Combining these two expressions and solving for the signal-to-noise ratio yields

$$\phi_\pi \ = \ \frac{-1}{Corr\,(\Delta\pi_t, \Delta\pi_{t-1})} \ - \ 2, \tag{18}$$

where $\phi_\pi \ \equiv \ \sigma_\eta^2 / \sigma_\zeta^2$ and $Corr\,(\Delta\pi_t, \Delta\pi_{t-1}) \ = \ Cov\,(\Delta\pi_t, \Delta\pi_{t-1}) \, / Var\,(\Delta\pi_t)$. The above expression shows that $Corr\,(\Delta\pi_t, \Delta\pi_{t-1})$ uniquely pins down $\phi_\pi$. The value of $\phi_\pi$, in turn, uniquely pins down $\lambda_\pi$ from equation (7). From the agent's perspective, the shocks $\zeta_t$ and $\eta_t$ are not directly observable, but the signal-to-noise ratio can be inferred from observed inflation data. ■

Proposition 1 shows that the observed data statistic $Corr\,(\Delta\pi_t, \Delta\pi_{t-1})$ is used to pin down the value of $\lambda_\pi$ which, in turn, governs the weights assigned to past rates of inflation in the agent's subjective forecast rule (6). This result is reminiscent of the "accelerationist controversy" identified by Sargent (1971, p. 35) who argued persuasively that any forecast weighting scheme involving past rates of inflation should *"be compatible with the observed evolution of the rate of inflation."*

Analogous to equation (18), the perceived signal-to-noise ratio for the output gap $\phi_y$ is uniquely pinned down by the observed data statistic $Corr\,(\Delta y_t, \Delta y_{t-1})$. The value of $\phi_y$, in turn, uniquely pins down $\lambda_y$ from equation (17).

Given the values of $\phi_\pi$, $\phi_y$, $\lambda_\pi$, and $\lambda_y$ together with the agent's perceived-optimal forecast rules (6) and (16), the actual law of motion (ALM) for the economy is governed by the three model equations (1), (13), and (14). The ALM can written in the following matrix form:

$$\mathbf{Z}_t = \mathbf{A}\mathbf{Z}_{t-1} + \mathbf{B}\mathbf{U}_t, \tag{19}$$

where $\mathbf{Z}_t \equiv \begin{bmatrix} \pi_t & y_t & i_t & \widetilde{E}_t\pi_{t+1} & \widetilde{E}_t y_{t+1} \end{bmatrix}'$ and $\mathbf{U}_t \equiv \begin{bmatrix} u_t & v_t \end{bmatrix}'$. The variance-covariance matrix $\mathbf{V}$ of the left-side variables in equation (19) can be computed using the formula:

$$vec\,(\mathbf{V}) = [\mathbf{I} - \mathbf{A} \otimes \mathbf{A}]^{-1} \; vec(\mathbf{B}\mathbf{\Omega}\mathbf{B}'), \tag{20}$$

where $\mathbf{\Omega}$ is the variance-covariance matrix of the two fundamental shocks $u_t$ and $v_t$. Given the moments of $\pi_t$ and $y_t$ from equation (20), we can derive analytical expressions for $Corr\,(\Delta\pi_t, \Delta\pi_{t-1})$ and $Corr\,(\Delta y_t, \Delta y_{t-1})$ in terms of $\phi_\pi$, $\phi_y$, $\lambda_\pi$, and $\lambda_y$.

**Definition 1.** *A consistent expectations equilibrium is defined as the actual law of motion (19), and the associated Kalman gain parameters* $\left(\lambda_\pi^*, \lambda_y^*\right)$, *such that* $\left(\lambda_\pi^*, \lambda_y^*\right)$ *represents the* *fixed points of the following multidimensional nonlinear maps:*

$$\lambda_\pi^* = \frac{-\phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right) + \sqrt{\phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right)^2 + 4\phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right)}}{2},$$

$$\text{where} \qquad \phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right) = \frac{-1}{Corr\left(\Delta\pi_t, \Delta\pi_{t-1}\right)} - 2, \tag{21}$$

$$\lambda_y^* = \frac{-\phi_y\left(\lambda_\pi^*, \lambda_y^*\right) + \sqrt{\phi_y\left(\lambda_\pi^*, \lambda_y^*\right)^2 + 4\phi_y\left(\lambda_\pi^*, \lambda_y^*\right)}}{2},$$

$$\text{where} \qquad \phi_y\left(\lambda_\pi^*, \lambda_y^*\right) = \frac{-1}{Corr\left(\Delta y_t, \Delta y_{t-1}\right)} - 2, \tag{22}$$

*and where the statistics $Corr\left(\Delta\pi_t, \Delta\pi_{t-1}\right)$ and $Corr\left(\Delta y_t, \Delta y_{t-1}\right)$ are computed from the actual law of motion* (19).

To obtain a graphical representation of the equilibrium, it is useful to express the nonlinear maps (21) and (22) in terms of the following functions:

$$\mathrm{f}_\pi\left(\lambda_\pi^*, \lambda_y^*\right) \equiv \lambda_\pi^* - \frac{-\phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right) + \sqrt{\phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right)^2 + 4\phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right)}}{2}, \tag{23}$$

$$\mathrm{f}_y\left(\lambda_\pi^*, \lambda_y^*\right) \equiv \lambda_y^* - \frac{-\phi_y\left(\lambda_\pi^*, \lambda_y^*\right) + \sqrt{\phi_y\left(\lambda_\pi^*, \lambda_y^*\right)^2 + 4\phi_y\left(\lambda_\pi^*, \lambda_y^*\right)}}{2}. \tag{24}$$

A consistent expectations equilibrium must therefore satisfy the following two conditions:

$$\mathrm{f}_\pi\left(\lambda_\pi^*, \lambda_y^*\right) = 0, \tag{25}$$

$$\mathrm{f}_y\left(\lambda_\pi^*, \lambda_y^*\right) = 0. \tag{26}$$

If only one pair $(\lambda_\pi^*,\ \lambda_y^*)$ satisfies both (25) and (26) with $\phi_\pi$ and $\phi_y$ as defined in (21) and (22), then the equilibrium is unique.

### 5.2.1 Numerical Solution for the Equilibrium

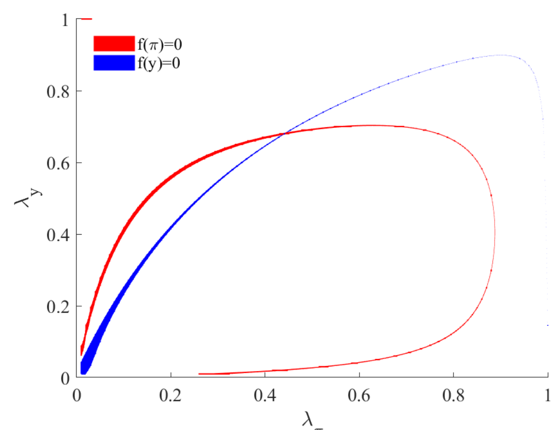
The complexity of the equilibrium conditions (25) and (26) necessitates a numerical solution for the equilibrium. We consider a textbook calibration of the New Keynesian model using the parameter values shown in Table 3. As in previous sections, we set $\beta = 0.995$ and $\kappa = 0.04$. We employ a coefficient of relative risk aversion $(1/\alpha)$ equal to 2, a typical value. The coefficients in the Taylor-type rule are $\mu_\pi = 1.5$ and $\mu_y = 0.5/4$ (Taylor 1999, Gali 2008). The standard deviations of the shocks, $\sigma_u = \sigma_v = 0.2$, are chosen to roughly match the observed volatility of core CPI inflation and the CBO output gap in the Great Moderation Era.

Figure 8 plots the equilibrium conditions (25) and (26) in $(\lambda_\pi, \lambda_y)$ space. As shown, the model has a unique fixed point equilibrium at $\left(\lambda_\pi^*, \lambda_y^*\right) = (0.436, 0.677)$. At the fixed point, we have $Corr\left(\triangle\pi_t, \triangle\pi_{t-1}\right) = -0.43$ and $Corr\left(\triangle y_t, \triangle y_{t-1}\right) = -0.29$, which in turn imply $\phi_\pi^* = 0.338$ and $\phi_y^* = 1.420$.[19]

Table 3: Baseline Parameter Values

| Parameter | Value | Description |
|---|---|---|
| $\beta$ | 0.995 | Subjective time discount factor. |
| $\kappa$ | 0.04 | Slope coefficient in NKPC. |
| $1/\alpha$ | 2 | Coefficient of relative risk aversion. |
| $\mu_\pi$ | 1.5 | Policy rule response to inflation. |
| $\mu_y$ | 0.5/4 | Policy rule response to output gap. |
| $\sigma_u$ | 0.20 | Std. dev. of cost push shock in percent. |
| $\sigma_v$ | 0.20 | Std. dev. of aggregate demand shock in percent. |

Figure 8: *Uniqueness of the Consistent Expectations Equilibrium*




Note: The figure plots the consistent equilibrium conditions (25) and (26) in $(\lambda_\pi, \lambda_y)$ space. The model has a unique fixed point equilibrium at $\left(\lambda_\pi^*, \lambda_y^*\right) = (0.436, 0.677)$.

[19] Although not plotted here, we have verified that the model's consistent expectations equilibrium is convergent under a real time learning algorithm in which the agent's estimates of the population statistics $Corr\left(\triangle\pi_t, \triangle\pi_{t-1}\right)$ and $Corr\left(\triangle y_t, \triangle y_{t-1}\right)$ are computed using past data generated by the model itself. Details are available upon request.

## 5.3 Monetary Policy Regime Change

A large literature has identified shifts in the conduct of U.S. monetary policy starting with the Volcker disinflation of the early 1980s (Clarida, et al. 2000, Orphanides 2004). Other research has documented a substantial decrease in U.S. macroeconomic volatility starting roughly around the same time (McConnell and Perez-Quirós 2000). In this section, we examine the implications of a shift towards a more hawkish monetary policy in the context of our New Keynesian consistent expectations model. First, we compute the fixed point values of $\lambda_\pi^*$ and $\lambda_y^*$ for different combinations of the policy rule coefficients $\mu_\pi$ and $\mu_y$. All other parameters take on the values shown in Table 3. The results of the exercise are plotted in Figure 9.

The left panel of Figure 9 shows that, all else equal, a stronger monetary response to inflation (higher $\mu_\pi$ coefficient) reduces the equilibrium value of $\lambda_\pi^*$, thereby serving to endogenously anchor the agent's inflation expectations. Intuitively, the anchoring occurs because higher values of $\mu_\pi$ serve to push the statistic $Corr\left(\triangle\pi_t, \triangle\pi_{t-1}\right)$ further into negative territory, implying a faster reversion of inflation to steady state in response to a shock. The right panel of Figure 9 shows that, all else equal, a stronger monetary response to the output gap (higher $\mu_y$ coefficient) also reduces the equilibrium value of $\lambda_\pi^*$. In this case, the stronger response to the output gap works via the NKPC slope parameter $\kappa$ to once again bring about a faster reversion of inflation to steady state.

We now consider whether plausible shifts in the policy rule coefficients and shock volatilities can simultaneously explain both the anchoring of U.S. inflation expectations and the decline in U.S. macroeconomic volatility observed over the past several decades. Specifically, we solve for the values of $\mu_\pi$, $\mu_y$, $\sigma_u$ and $\sigma_v$ that enable our model to approximately match the standard deviations of inflation, the output gap and the federal funds rate, as well as the statistic $Corr\left(\triangle\pi_t, \triangle\pi_{t-1}\right)$, in two different subsamples of U.S. data. Again, all other parameters take on the values shown in Table 3. The results of this exercise are shown in Table 4. The + superscripts indicate the moments in the data that we aim to match.

For the model to reproduce the high macroeconomic volatility of the Great Inflation Era, the policy rule coefficients must be very low. The model-implied values of $\mu_\pi = 1.11$ and $\mu_y = 0$ would place the rational expectations version of the model near the threshold of equilibrium indeterminacy. Whether or not the U.S. economy was in a state of equilibrium indeterminacy in the pre-Volcker era remains unsettled (Clarida, et al. 2000, Orphanides 2004).

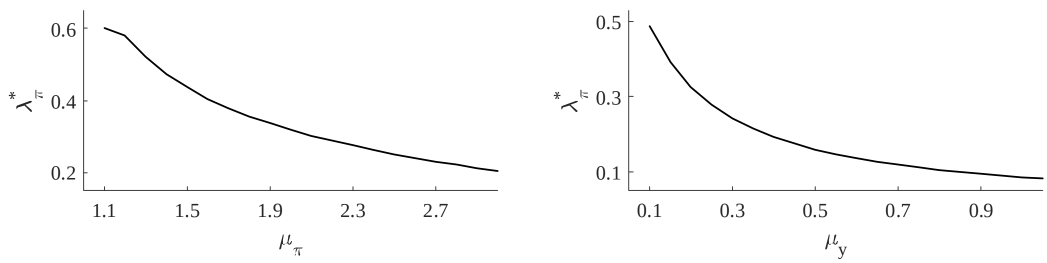
Table 4 shows that a transition to a more hawkish monetary policy regime serves to endogenously anchor the agent's subjective inflation expectations as the model-implied value of $\lambda_\pi^*$ declines from 0.46 to 0.34. Table 4 also shows that a plausible increase in the Taylor coefficients from $(\mu_\pi,\ \mu_y) = (1.11,\ 0.00)$ to $(\mu_\pi,\ \mu_y) = (2.27,\ 0.15)$ helps the model to replicate the substantial decline in macroeconomic volatility observed in the data. The decline in macroeconomic volatility can be partially attributed to the anchoring process. Well-anchored inflation expectations imply that inflation is less responsive to shocks. The model-implied value of $\lambda_y^*$ declines from 0.79 to 0.67 such that the output gap is also less responsive to shocks. Thus, by responding more strongly to inflation and the output gap, monetary policy helps to anchor expectations and as a result, the policy interest rate becomes *less* volatile during the Great Moderation Era.

A notable feature of our model is that it can account for most of the persistence of $\pi_t$ and $y_t$ observed in the U.S. data without relying on other devices (e.g., consumption habits, ad-hoc inflation lags in the NKPC, or highly persistent shocks) that are typically required in RE models (Smets and Wouters 2003). Importantly, Table 4 shows that the shift to a more hawkish monetary policy enables our model to reproduce another stylized fact of the Great Moderation, namely, a decline in inflation persistence. Specifically, the model-implied value of $Corr\,(\pi_t, \pi_{t-1})$ goes from 0.67 in the Great Inflation Era to 0.40 in the Great Moderation Era.

According to the model, the standard deviation of the cost-push shock $\sigma_u$ decreases during the Great Moderation—in line with most accounts of the period. According to Blinder (1982), oil and food price shocks, coupled with pent-up inflation from the release of the Nixon wage-price controls in 1974, can account for most of the increase in inflation during the 1970s. Furthermore, he argues that the absence of these same factors can account for most of the decline in inflation during the early 1980s.

The model-implied value of $\sigma_v$ (demand shock standard deviation) goes up during the Great Moderation. Typically, however, "good luck" explanations of the Great Moderation identify declines in the volatilities of both supply- and demand-side shocks during this period (Sims and Zha 2006). Hence, our model implies that the observed decline in the volatility of the output gap is primarily due to improved monetary policy, as opposed to good luck.

Figure 9: *Effect of Policy Rule Coefficients on Equilibrium Kalman Gain for Inflation*




Note: The figure shows that, all else equal, a stronger monetary response to inflation (higher $\mu_\pi$ coefficient) or a stronger monetary response to the output gap (higher $\mu_y$ coefficient) act to reduce the equilibrium value of $\lambda^*_\pi$, thereby serving to endogenously anchor the agent's inflation expectations.

Table 4: Monetary Policy Regime Change

| | Great Inflation Era 1960.q1 to 1983.q4 | | Great Moderation Era 1984.q1 to 2007.q3 | |
|---|---|---|---|---|
| Statistic | Model | Data | Model | Data |
| $\mu_\pi$ | 1.11 | – | 2.27 | – |
| $\mu_y$ | 0.00 | – | 0.15 | – |
| $\sigma_u$ | 0.37 | – | 0.20 | – |
| $\sigma_v$ | 0.12 | – | 0.25 | – |
| $\lambda^*_\pi$ | 0.46 | 0.41 | 0.34 | 0.19 |
| $\lambda^*_y$ | 0.79 | – | 0.67 | – |
| $Std.\ Dev.\,(4\pi_t)^+$ | 3.50 | 3.50 | 1.24 | 1.24 |
| $Corr\,(\pi_t, \pi_{t-1})$ | 0.67 | 0.75 | 0.40 | 0.60 |
| $Corr\,(\triangle\pi_t, \triangle\pi_{t-1})^+$ | $-0.42$ | $-0.42$ | $-0.46$ | $-0.46$ |
| $Std.\ Dev.\,(y_t)^+$ | 2.84 | 2.84 | 1.46 | 1.46 |
| $Corr\,(y_t, y_{t-1})$ | 0.93 | 0.93 | 0.90 | 0.95 |
| $Corr\,(\triangle y_t, \triangle y_{t-1})$ | $-0.18$ | 0.22 | $-0.30$ | 0.20 |
| $Std.\ Dev.\,(4i_t)^+$ | 3.87 | 3.80 | 2.37 | 2.37 |
| $Corr\,(i_t, i_{t-1})$ | 0.67 | 0.94 | 0.32 | 0.98 |

Notes: Parameter values are reported in Table 3. Standard deviations are expressed in percent. Data values are for core CPI inflation, the CBO output gap and the federal funds rate. The data values for $\lambda^*_\pi$ correspond to the estimates in Table 2. The + superscripts indicate the moments in the data that we aim to match. Model statistics are computed analytically using equation (20).

# 6 Conclusion

Standard versions of the New Keynesian Phillips curve cannot account for the "missing disinflation" during the U.S. Great Recession and the "missing inflation" during the subsequent recovery, leading many to believe that the Phillips curve relationship has weakened or even disappeared. In this paper, we formalize the idea of anchored inflation expectations in the context of a model where agents forecast inflation using a simple univariate time series process—along the lines of Stock and Watson (2007, 2010). We show that the anchoring of inflation expectations implied by our model can help explain the observed flattening of the reduced-form Phillips curve even while the true underlying slope parameter of the NKPC remains stable and statistically significant. In an out-of-sample forecast from 2007 to 2019, we show that an estimated version of the NKPC can account for the dynamics of inflation and long-run expected inflation in U.S. data, thereby resolving the inflation puzzles. Finally, we use a simple New Keynesian equilibrium model to show that a stronger monetary policy response to either inflation or the output gap serves to endogenously anchor agents' subjective inflation expectations, leading to lower macroeconomic volatility and reduced inflation persistence.

## References


Adam, K., A. Marcet, and J. Beutel (2017) "Stock Price Booms and Expected Capital Gains," *American Economic Review* 107, 2352-2408.

Adam, K. and M. Padula (2011) "Inflation Dynamics and Subjective Expectations in the United States," *Economic Inquiry* 49, 13–25.

Ascari, G. (2004): "Staggered Prices and Trend Inflation: Some Nuisances," *Review of Economic Dynamics* 7, 642-667.

Ascari, G. and A.M. Sbordone (2014) "The Macroeconomic of Trend Inflation," *Journal of Economic Literature* 52(3), 679-739.

Aruoba, B. and F. Schorfheide (2016) "Inflation During and After the Zero Lower Bound." In: *Inflation Dynamics and Monetary Policy,* Proceedings of the Jackson Hole Economic Policy Symposium, Federal Reserve Bank of Kansas City, pp. 359-436.

Ball, L. and S. Mazumder (2011) "Inflation Dynamics and the Great Recession," *Brookings Papers on Economic Activity* (Spring), 337-405.

Ball, L. and S. Mazumder (2019a) "The Nonpuzzling Behavior of Median Inflation," NBER Working Paper 25512.

Ball, L. and S. Mazumder (2019b) "A Phillips Curve for the Euro Area," NBER Working Paper 26450.

Ball, L., N.G. Mankiw, and D. Romer (1988) "The New Keynesian Economics and the Output-Inflation Trade-Off,". *Brookings Papers on Economic Activity*, 1-82.

Barberis, N., R. Greenwood, L. Jin, and A. Shleifer (2015) "X-CAPM: An Extrapolative Capital Assset Pricing Model," *Journal of Financial Economics* 115, 1-24.

Bernanke, B. (2007) "Inflation Expectations and Inflation Forecasting," Speech at the Monetary Economics Workshop of the NBER Summer Institute, Cambridge, Massachusetts (July 10).

Bernanke, B. (2010) "The Economic Outlook and Monetary Policy," Speech at the Federal Reserve Bank of Kansas City Economic Symposium, Jackson Hole, Wyoming (August 27).

Bianchi, F., L. Melosi, and M. Rottner (2019) "Hitting the Elusive Inflation Target," Federal Reserve Bank of Chicago, Working Paper No. 2019-07.

Blanchard, O., E. Cerutti, and L. Summers (2015) "Inflation and Activity: Two Explorations and Their Monetary Policy Implications," In: *Inflation and Unemployment in Europe: Conference proceedings. ECB Forum on Central Banking.* European Central Bank.

Blinder, A.S. (1982) "The Anatomy of Double-Digit Inflation in the 1970s." In: R. Hall, ed., *Inflation: Causes and Effects.* Chicago: University of Chicago Press, pp. 261-282.

Bordalo, P., N. Gennaioli, Y. Ma, and A. Shleifer. (2018) "Overreaction in Macroeconomic Expectations," NBER Working Paper 24932.

Calvo, G.A. (1983) "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12, 383-398.

Case, K.E., R. J. Shiller, and A.K. Thompson (2012) "What Have They Been Thinking? Homebuyer Behavior in Hot and Cold Markets," *Brookings Papers on Economic Activity* (Fall), 265-298.

Carvalho, C., S. Eusepi, E. Moench, and B. Preston (2019) "Anchored Inflation Expectations," Working Paper.

Cecchetti, S.G., M.E. Feroli, P. Hooper, A.K. Kashyap, and K.L. Schoenholtz (2017) "Deflating Inflation Expectations: the Implications of Inflation's Simple Dynamics," Working Paper, U.S. Monetary Policy Forum.

Christiano, L.J., M. Eichenbaum, and C.L. Evans (2005) "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113, 1-45.

Christiano, L.J., Eichenbaum, M.S., and Trabandt, M. (2015) "Understanding the Great Recession," *American Economic Journal: Macroeconomics* 7, 110-167.

Clarida, R.H. (2019) "The Federal Reserve's Review of Its Monetary Policy Strategy, Tools, and Communication Practices,", Speech at "A Hot Economy: Sustainability and Trade-Offs," Federal Reserve Bank of San Francisco (September 26).

Clarida, R.H., J. Gali, and M.J. Gertler (2000) "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics* 115, 147-180.

Clark, T.E. and T. Doh (2014) "Evaluating Alternative Models of Trend Inflation," *International Journal of Forecasting* 30, 426-448.

Cogley, T. and A.M. Sbordone (2008) "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips curve," *American Economic Review* 98, 2101-2126.

Coibion, O. and Y. Gorodnichenko (2011) "Monetary Policy, Trend Inflation and the Great Moderation: An Alternative Interpretation," *American Economic Review* 101, 341–370.

Coibion, O. and Y. Gorodnichenko. (2012) "What Can Survey Forecasts Tell Us about Information Rigidities?" *Journal of Political Economy* 120, 116-159.

Coibion, O. and Y. Gorodnichenko (2015a) "Is the Phillips curve Alive and Well After All? Inflation Expectations and the Missing Disinflation," *American Economic Journal: Macroeconomics* 7, 197-232.

Coibion, O. and Y. Gorodnichenko (2015b) "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review* 105, 2644-2678.

Coibion, O., Y. Gorodnichenko and R. Kamdar (2018) "The Formation of Expectations, Inflation and the Phillips curve," *Journal of Economic Literature* 56, 1447-1491.

Constâncio, V. (2016) "Understanding Inflation Dynamics and Monetary Policy, " In: *Inflation Dynamics and Monetary Policy,* Proceedings of the Jackson Hole Economic Policy Symposium, Federal Reserve Bank of Kansas City.

Daly, M., B. Hobijn, and B. Lucking (2012) “Why Has Wage Growth Stayed Strong?” *FRBSF Economic Letter* 2012-10 (April 2).

Daly, M. and B. Hobijn (2014) “Downward Nominal Wage Rigidities Bend the Phillips curve, ” *Journal of Money, Credit and Banking* 46(S2), 51-93.

Del Negro, M., M.P. Giannoni, and F. Schorfheide (2015) “Inflation in the Great Recession and New Keynesian Models,”.*American Economic Journal: Macroeconomics* 7, 168-196.

Dotsey, M., S. Fujita, and T. Stark (2018) “Do Phillips Curves Conditionally Help to Forecast Inflation?” *International Journal of Central Banking* 14, 43-92.

Edge, R.M., T. Laubach, and J.C. Williams (2007) “Learning and Shifts in Long-run Productivity Growth,” *Journal of Monetary Economics* 54, 2421-2438.

Evans, G.W. and S. Honkapohja (2001) *Learning and Expectations in Economics.* Princeton University Press, Princeton.

Friedman, M. (1968) “The Role of Monetary Policy,” *American Economic Review* 58, 1-15.

Gali, J. (2008) “Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework,” Princeton University Press.

Gali, J. and M. Gertler (1999) “Inflation Dynamics: A Structural Econometric Analysis,” *Journal of Monetary Economics* 44, 195-222.

Greenwood, R. and A. Shleifer (2014) “Expectations of Returns and Expected Returns,” *Review of Financial Studies* 27, 714-746.

Gürkaynak, R., B. Sack, and E. Swanson (2005) “The Sensitivity of Long-term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models,” *American Economic Review* 95, 425-436.

Hall, R.E. (2011) “The Long Slump,” *American Economic Review* 101, 431-469.

Hall, R.E. and T.J. Sargent (2018) “Short-Run and Long-Run Effects of Milton Friedman’s Presidential Address,” *Journal of Economic Perspectives* 32(1), Winter, 121–134.

Hills, T.S., T. Nakata, and S. Schmidt (2016) “The Risky Steady State and the Interest Rate Lower Bound, Federal Reserve Board, Finance and Economics Discussion Series Paper 2016-009.

Hommes, C. and G. Sorger (1998) “Consistent Expectations Equilibria,” *Macroeconomic Dynamics 2*, 287-321.

Hommes, C. and M. Zhu (2013) “Behavioral Learning Equilibria,” *Journal of Economic Theory* 150, 778-814.

Hooker, M. (2002) “Are Oil Shocks Inflationary? Asymmetric and Nonlinear Specifications versus Changes in Regime,” *Journal of Money, Credit, and Banking* 34, 540-561.

International Monetary Fund (2013) “The Dog That Didn’t Bark: Has Inflation Been Muzzled or Was It Just Sleeping?” *World Economic Outlook, Chapter 3.*

Ireland, P.N. (2007) “Changes in the Federal Reserve’s Inflation Target: Causes and Consequences,” *Journal of Money, Credit, and Banking* 39, 1851-1882.

Jurgilas, M. and K.J. Lansing (2013) "Housing Bubbles and Expected Returns to Home Ownership: Lessons and Policy Implications". In: M. Balling and J. Berg (eds.), *Property Prices and Real Estate Financing in a Turbulent World*. Brusssells/Vienna: Société Universitaire Européenne de Recherches Financières (SUERF), 2013/4, pp. 101-128.

Kozicki, S. and P.A. Tinsley (2012) "Effective Use of Survey Information in Estimating the Evolution of Expected Inflation," *Journal of Money, Credit and Banking* 44, 145-169.

Lansing, K.J. (2009) "Time-Varying U.S. Inflation Dynamics and the New Keynesian Phillips Curve," *Review of Economic Dynamics* 12, 304-326.

Lansing, K.J. (2019a) "Improving the Phillips Curve wtih an Interaction Variable," *FRBSF Economic Letter* 2019-13 (May 6).

Lansing, K.J. (2019b) "Endogenous Forecast Switching Near the Zero Lower Bound," *Journal of Monetary Economics*, forthcoming.

Lindé, J. and M. Trabandt (2019) "Resolving the Missing Deflation Puzzle," CEPR Discussion Paper 13690.

Lucas, R.E. (1976) "Econometric Policy Evaluation: A Critique," *Carnegie Rochester Conference Series on Public Policy* 1, 19-46.

Mankiw, N.G., R. Reis, and J. Wolfers (2003) "Disagreement about Inflation Expectations." In: M. Gertler and K. Rogoff (Eds.), *NBER Macroeconomics Annual 2003.* Cambridge MA: MIT Press, pp. 209-248.

Mavroeidis, S., M. Plagborg-Møller, and J.H. Stock (2014) "Empirical Evidence on Inflation Expectations in the New Keynesian Philips curve," *Journal of Economic Literature* 52(1), 124-188.

McConnell, M. M. and G. Perez-Quirós (2000) "Output Fluctuations in the United States: What Has Changed since the Early 1980s?" *American Economic Review* 90, 1464-1476.

Mishkin, F.S. (2007) "Inflation Dynamics", NBER Working Paper 13147.

Muth, J.F. (1960) "Optimal Properties of Exponentially Weighted Forecasts," *Journal of the American Statistical Association* 55, 299-306.

Nerlove, M. (1967) "Distributed Lags and Unobserved Components in Economic Time Series." In: W. Fellner (Ed.), *Ten Economic Studies in the Tradition of Irving Fischer.* New York: John Wiley & Sons, pp. 127-169.

Orphanides, A. (2004) "Monetary Policy Rules, Macroeconomic Stability and Inflation: A View from the Trenches," *Journal of Money, Credit, and Banking* 36, 151-175.

Phelps, E.S. (1967) "Phillips Curves, Expectations of Inflation, and Optimal Unemployment over Time," *Economica* 34, 254-281.

Phillips, A.W. (1958) "The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957," *Economica* 25, 283-299.

Powell, J. (2019) Testimony before the Senate Banking Committee, Washington D.C. (July 11).

Rotemberg, J.J. (1982) "Sticky Prices in the United States," *Journal of Political Economy* 90, 1187–1211.

Sahuc, J.G. (2006) "Partial Indexation, Trend Inflation, and the Hybrid Phillips Curve," *Economic Letters* 90, 42-50.

Sargent, T. (1971) "A Note on the 'Accelerationist' Controversy," *Journal of Money, Credit, and Banking* 3, 721-725.

Schorfheide, F. (2005) "Learning and Monetary Policy Shifts," *Review of Economic Dynamics* 8, 392–419.

Sims, C.A. and T. Zha (2006) "Were There Regime Switches in U.S. Monetary Policy?" *American Economic Review* 96, 54-81.

Smets, F. and R. Wouters (2003) "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association* 1, 1123-1175.

Stock, J.H. (2011) Comment on "Inflation Dynamics and the Great Recession," *Brookings Papers on Economic Activity* (Spring), 387-402.

Stock, J.H. and M.W. Watson (2007) "Why Has U.S. Inflation Become Harder to Forecast?" *Journal of Money, Credit and Banking* 39, 3-34.

Stock, J.H. and M.W. Watson (2010) "Modeling Inflation after the Crisis." In: *Macroeconomic Challenges: The Decade Ahead,* Proceedings of the Jackson Hole Economic Policy Symposium, Federal Reserve Bank of Kansas City, pp. 173–220.

Taylor, J.B. (1993) "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214.

Trehan, B. (2015) "Survey Measures of Expected Inflation and the Inflation Process, " *Journal of Money, Credit and Banking* 47, 207-222.

Vissing-Jørgensen, A. (2003) "Perspectives on Behavioral Finance: Does 'Irrationality' Disappear With Wealth? Evidence from Expectations and Actions." *NBER Macroeconomics Annual* 18, 139-194.

Volcker, P.A. (1979) "Statement" Before the Joint Economic Committee, *Federal Reserve Bulletin*, 65 (November), 888-890.

Williams, J.C. (2006) "Inflation Persistence In An Era of Well-Anchored Inflation Expectations," *FRBSF Economic Letter* 2006-27 (October 13).

Williams, J.C. (2009) "The Risk of Deflation," *FRBSF Economic Letter* 2009-12 (March 27).

Yellen, J. (2019) Remarks at "What's (Not) Up With Inflation?" Conference Hosted by the Hutchins Center on Fiscal and Monetary Policy at the Brookings Institution, Washington, D.C. (October 3).

# A Appendix: Robustness of NKPC Estimates

Table 2A shows the results of estimating equation (11) using two alternative specifications: (1) $\pi_t$ is measured by core PCE inflation instead of core CPI inflation, and (2) economic slack is measured by the unemployment gap instead of the CBO output gap. The point estimate of the NKPC slope parameter $\widehat{\kappa}$ increases slightly over time when we use core PCE inflation. In contrast, the absolute value of $\widehat{\kappa}$ tends to decline over time when we use the unemployment gap. Taken together with our baseline results in Table 2, the evidence supports the view that $\widehat{\kappa}$ has remained relatively stable and significant over time.

Table 2A: NKPC Parameter Estimates, Robustness

| | Great Inflation Era 1961.q3 to 1983.q4 | Great Moderation Era 1984.q1 to 2007.q3 | Great Recession Era 2007.q4 to 2019.q2 |
|---|---|---|---|
| | | A. Core PCE | |
| $\widehat{\kappa}$ | 0.008 | 0.013** | 0.032*** |
| | (0.007) | (0.007) | (0.012) |
| $\widehat{\lambda}_\pi$ | 0.712*** | 0.330*** | 0.028** |
| | (0.183) | (0.072) | (0.015) |
| Adj. $R^2$ | 0.84 | 0.71 | 0.04 |
| Obs. | 90 | 95 | 47 |
| | Great Inflation Era 1960.q1 to 1983.q4 | Great Moderation Era 1984.q1 to 2007.q3 | Great Recession Era 2007.q4 to 2019.q2 |
| | | B. Unemployment Gap | |
| $\widehat{\kappa}$ | −0.102*** | −0.064*** | −0.048*** |
| | (0.029) | (0.016) | (0.014) |
| $\widehat{\lambda}_\pi$ | 0.335*** | 0.170*** | 0.012* |
| | (0.104) | (0.041) | (0.008) |
| Adj. $R^2$ | 0.68 | 0.57 | 0.08 |
| Obs. | 96 | 95 | 47 |

Notes: The asterisks ***/ **/* denote significance at the 1/5/10 percent levels, respectively. Newey-West standard errors (3 lags) shown in parentheses. Due to limited data availability, the regression for the Great Inflation Era using Core PCE inflation starts in 1961.q3.