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Replicating Business Cycles and Asset Returns with Sentiment and Low Risk Aversion*

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Abstract

This paper develops a real business cycle model with eight fundamental shocks and one “equity sentiment shock” that captures belief-driven fluctuations. I solve for the time series of shock realizations that allow the model to exactly replicate the observed time paths of U.S. macroeconomic variables and asset returns over the past six decades. The representative agent’s perception that movements in equity value are partly driven by sentiment is close to self-fulfilling. The model-identified sentiment shock is strongly correlated with other fundamental shocks and implies “pessimism” relative to fundamental equity value in steady state. Counterfactual scenarios show that the sentiment shock and shocks that appear in the law of motion for capital (representing financial frictions) have large impacts on the levels of macroeconomic variables and the size of the equity risk premium. Other shocks have large impacts on the growth rates of macroeconomic variables. Four of the model-identified shocks help to predict the equity risk premium or the bond term premium in the next quarter. Overall, the results support a narrative in which a large number of correlated shocks have combined to deliver the historical outcomes observed in U.S. data.

Keywords: *Belief-driven business cycles, Sentiment, Animal spirits, Risk aversion, Equity risk premium, Bond term premium.*

JEL Classification: E32, E44, O41.

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1 Introduction

The macroeconomics literature has not reached a consensus in identifying the most important shocks driving U.S. business cycles. Chari, Kehoe, and McGrattan (2007) conclude that productivity and labor supply shocks are the most important. Smet and Wouters (2007) find that shocks to productivity and wage mark-ups account for most of the fluctuations in output over the medium- to long-run. Justiniano, Primiceri, and Tambalotti (2010) conclude that an “investment shock” which appears in the law of motion for capital is the main driver of fluctuations in hours, output, and investment. Christiano, Motto, and Rostagno (2014) conclude that “risk shocks” (defined as the time-varying volatility of firms’ idiosyncratic productivity realizations) are the most important business cycle shocks. Miao, Wang, and Xu (2015) find that a “sentiment shock” (which influences the size of a rational stock price bubble) together with productivity and labor supply shocks are the most important business cycle shocks. Angeletos, Collard, and Dellas (2018) argue that “confidence shocks” (which are orthogonal to fundamental shocks and arise from agents’ non-rational beliefs in the superior accuracy of their own productivity signals) are the main drivers of business cycles. In a follow-up paper, Angeletos, Collard, and Dellas (2020) identify the “main business cycle shock” as a demand shock that does not strictly rely on nominal rigidity, consistent with a confidence- or sentiment-type shock.

In contrast to the studies mentioned above, this paper considers data on both the equity risk premium and the bond term premium while seeking to identify the main drivers of U.S. business cycles and asset returns.¹ As noted by Campbell, Pflueger, and Viceira (2020), an integrated model of macroeconomics and asset pricing imposes valuable discipline on any model that seeks to explain the observed data. I show that a rich combination of correlated shocks, rather than a small number of orthogonal shocks, are needed to fully replicate the patterns of U.S. business cycles and asset returns.

The framework for the analysis is a real business cycle model with eight fundamental shocks and one “equity sentiment shock” that captures belief-driven fluctuations. The eight fundamental shocks influence the representative agent’s risk aversion coefficient, the disutility of labor supply, the productivity of three separate inputs that appear in the law of motion for capital, capital’s share of income, the productivity of hours worked, and the real value of coupon payments from a long-term bond.

Inclusion of the equity sentiment shock is motivated by a large literature that documents links between movements in equity prices and measures of investor or consumer sentiment.²

¹In estimating their model, Christiano, Motto, and Rostagno (2014) consider data on the value of the stock market, credit to nonfinancial firms, the credit spread of bond yields, and the term spread of bond yields. They evaluate their model using data on the cross-sectional dispersion of firm-level stock returns. In estimating their model, Miao, Wang, and Xu (2015) consider data on the value of the stock market and the Chicago Fed’s National Financial Conditions Index (NFCI).

²See, for example, Schmeling (2009), Greenwood and Shleifer (2014), Huang, et al. (2014), Adam, Marcet, and Beutel (2017), and Lansing, LeRoy, and Ma (2020), among others.

Numerous studies find evidence of a significant empirical link between non-fundamental equity price movements and the resulting investment decisions by firms.³ Recently, Bhandari, Borovička, and Ho (2019) and Bianchi, Ludvigson, and Ma (2020) present evidence that survey forecasts of economic activity fluctuate between optimism and pessimism and that these forecast fluctuations appear to be important drivers of the associated macroeconomic variables.

I postulate that the representative agent makes use of a sentiment measure to construct a conditional forecast involving future equity value. Due to the self-referential nature of the model and the near-unity slope of the intertemporal first order condition, the agent’s perception that movements in equity value are partly driven by sentiment is close to self-fulfilling. The agent’s forecast errors for equity value are nearly identical those implied by a hypothetical model-consistent forecast.

Three out of the nine model shocks, including the equity sentiment shock, can be classified as demand shocks. The remaining six shocks, including the labor disutility shock, can be classified as supply shocks. The model builds on the setup in Lansing (2019) to include three additional fundamental shocks. These include a time-varying risk aversion coefficient, a shock that influences the productivity of “investor effort” in the production of new capital, and a shock that influences the real value of bond coupon payments. The additional shocks allow the model to exactly replicate quarterly U.S. data for the real return on equity (including dividends) and the real returns on both 1-period and long-term bonds. In so doing, the model exactly replicates quarterly U.S. data for the equity risk premium and the bond term premium.

As an alternative to estimation, I calibrate the model’s parameters so that the steady state matches the U.S. data in 1972.Q3—a period when U.S. macroeconomic ratios are close to their long-run means. The time series of the nine stochastic shocks are “reverse-engineered” so that the model exactly replicates the observed time paths of eleven macroeconomic variables and asset returns (only nine of which are independent).

The steady-state value of the model-identified sentiment shock is negative, implying that sentiment is “pessimistic” relative to fundamental equity value in steady state. This feature allows the model to replicate the equity risk premium in the data while maintaining a low level of risk aversion. Shifting up the entire sequence of sentiment shocks by a constant amount to achieve a higher steady state value in 1972.Q3 serves to reduce (and eventually eliminate) the model’s mean equity risk premium and the mean bond term premium. I demonstrate analytically how a pessimistic equity market forecast can serve to magnify the equity risk premium.⁴ The model-identified risk aversion coefficient fluctuates between 0.57 and 1.19. Intuitively, the sentiment shock allows the model to replicate the volatile U.S. equity return

³See, for example, Chirinko and Schaller (2001), Goyal and Yamada (2004), Gilchrist, Himmelberg, and Huberman (2005), and Campello and Graham (2013).

⁴Previous studies have shown that incorporating some form of in-sample pessimism about fundamentals or future equity values can magnify the equity risk premium in standard asset pricing models. See, for example, Reitz (1988), Cecchetti, Lam, and Mark (2000), Abel (2002), Cogley and Sargent (2008), Barro (2009), Bidder and Dew-Becker (2016), and Adam and Merkel (2019), among others.

while the mildly time-varying risk aversion coefficient allows the model to replicate the U.S. risk free rate of return.

As a preview of the results, Figure 1 plots the model-identified sentiment shock together with the University of Michigan’s consumer sentiment index. The correlation coefficient between the two series is 0.68. Both series, in turn, are strongly correlated with a stock market valuation ratio defined as the nominal market capitalization of the Standard & Poor’s (S&P) 500 stock index divided by a measure of nominal output. The correlation coefficient between the model-identified sentiment shock and the S&P 500 valuation ratio is 0.70.

The sentiment shock also exhibits a strong positive correlation with the model-identified risk aversion coefficient. More optimistic sentiment together with higher risk aversion leads to a correlated increase in all macroeconomic variables. Equity value increases but bond prices decline (implying an increase in bond yields). Taken together, the combination of these two highly correlated shocks allows the model to capture the features observed during a typical economic boom or recovery.

Given the nine model-identified shock series, I perform counterfactual scenarios that omit one or more shock realizations while leaving the other shock realizations in place. The purpose of the exercise is to identify which shock (or set of shocks) have the largest quantitative impact on a given model variable. Forecast error variance decompositions are problematic here because the model-identified shock innovations are not orthogonal to each other. As an alternative, I compute the mean absolute gaps (measured in percent or percentage points) between the counterfactual paths and the corresponding U.S. data paths. The counterfactual scenarios show that the equity sentiment shock and shocks that appear in the law of motion for capital (representing financial frictions) have the largest impacts on the levels of investment, the capital stock, and equity value. The labor disutility shock has the largest impacts on the levels of output, consumption, and hours worked. To gauge the relative importance of the various shocks for higher frequency movements, I compute the mean absolute gaps using 4-quarter growth rates instead of the levels of the variables. In this case, the factor distribution shock has the largest impact on output growth while the risk aversion shock has the largest impact on consumption growth. The labor disutility shock has the largest impact on hours growth.

Omitting realizations of the financial frictions implied by the capital law of motion shocks serves to significantly magnify the equity risk premium and the bond term premium. In contrast, omitting realizations of the equity sentiment shock (but maintaining pessimism in steady state) serves to shrink these same premia.

Previous studies by Greenwald, Lettau, and Ludvigson (2014) and Lansing (2015) using concentrated capital ownership models (i.e., capital owners versus workers) identify a large role for factor distribution shocks in explaining the equity risk premium in U.S. data. In contrast, omitting realizations of the factor distribution shocks in the representative agent model employed here serves only to mildly shrink the mean equity risk premium relative to

the baseline model. This example shows that conclusions regarding the relative importance of various shocks for macroeconomic or financial variables can be model-specific.

Simple forecasting regressions show that higher values of the sentiment shock, implying more optimism, predict a lower equity risk premium in the next quarter, consistent with the empirical results of Huang, et al. (2014). Higher values of the multiplier shock in the law of motion for capital predict a higher equity risk premium. Higher values of the labor disutility shock predict a smaller bond term premium. A positive investor effort shock predicts both a higher equity risk premium and a higher bond term premium.

In discussing the difficulty of identifying a “main business cycle shock,” Angeletos, Collard, and Dellas (2020) acknowledge (p. 3054) that “In principle, any of the reduced-form objects contained in our anatomy may map into a uninterpretable combination of multiple theoretical shocks...” The results presented here show that multiple theoretical shocks are indeed necessary to fully explain the historical patterns of U.S. business cycles and asset returns.

Layout. The remainder of this paper is organized as follows. Section 2 describes the model and the manner in which I introduce equity sentiment. Section 3 describes the identification of parameter values and shock realizations so that the model exactly replicates quarterly U.S. data from 1960.Q1 to 2019.Q4. Section 4 presents quantitative exercises, including counterfactual scenarios that omit one or more shock realizations. Section 5 concludes. The Appendix provides details of the model solution, the shock identification procedure, and the data sources and methods.

2 Model

The framework for the analysis is a real business cycle model that includes eight fundamental shocks and one “equity sentiment shock” that captures belief-driven fluctuations. The representative agent’s decision problem is to maximize

$$\widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t - \kappa_t C_t) - D \exp(u_t) \frac{(h_{1,t} + h_{2,t})^{1+\gamma}}{1 + \gamma} \right], \quad (1)$$

subject to the budget constraint

$$c_t + i_t = w_t h_{1,t} + r_t k_t, \quad (2)$$

where c_t is consumption, $h_{1,t}$ is hours worked in the production of output, $h_{2,t}$ is hours worked in the production of new capital (called investor effort), i_t is investment, w_t is the real wage per hour, r_t is the real rental rate per unit of capital, and k_t is the stock of physical capital. All quantities are measured in per person terms. The parameter $\beta > 0$ is the agent’s subjective time discount factor. The symbol \widehat{E}_t represents the agent’s subjective expectation, conditional on information available at time t . Under rational expectations, \widehat{E}_t corresponds

to the mathematical expectation operator E_t evaluated using the objective distribution of all shocks, which are assumed known to the rational agent.

To allow for time-varying risk aversion, I assume that the representative agent derives utility from individual consumption c_t measured relative to a reference level that depends on the amount of aggregate consumption per person C_t , which is viewed by the agent as exogenous.⁵ The reference level of consumption is often defined in terms of C_{t-1} or c_{t-1} as opposed to C_t .⁶ But in the continuous-time limit, there is no distinction between the values of C_t or C_{t-1} . Defining the reference level in terms of C_t has the advantage of reducing the number of endogenous state variables and simplifying the equilibrium solution of the model.

The time-varying parameter κ_t determines the agent's coefficient of relative risk aversion η_t according to the relationship

$$\begin{aligned}\eta_t &\equiv -c_t \frac{U_{cc}(c_t, C_t)}{U_c(c_t, C_t)}, \\ &= -c_t \frac{-1/(c_t - \kappa_t C_t)^2}{1/(c_t - \kappa_t C_t)} = \frac{1}{1 - \kappa_t},\end{aligned}\tag{3}$$

where I have imposed the equilibrium condition $c_t = C_t$ in the second line of the expression. The agent's time-varying risk aversion coefficient evolves according to the following stationary law of motion

$$\eta_t = \eta_{t-1}^{\rho_\eta} \bar{\eta}^{1-\rho_\eta} \exp(\varepsilon_{\eta,t}), \quad |\rho_\eta| < 1, \quad \varepsilon_{\eta,t} \sim NID(0, \sigma_{\varepsilon,\eta}^2),\tag{4}$$

which ensures $\eta_t > 0$. The parameter ρ_η governs the persistence of the risk aversion coefficient and $\varepsilon_{\eta,t}$ is a normally and independently distributed (*NID*) innovation with mean zero and variance $\sigma_{\varepsilon,\eta}^2$. The steady state level of risk aversion is given by $\bar{\eta}$. For the quantitative analysis, I will employ $\bar{\eta} = 1$ such that $\bar{\kappa} = 0$.

The agent supplies labor to productive firms in the amount $h_{1,t}$. Following Zhu (1995), the agent also supplies “investor effort” in the amount $h_{2,t}$ that contributes to the production of new physical capital, as described further below. The disutility of total labor supplied $h_{1,t} + h_{2,t}$ is governed by the second term in (1), where $D > 0$, and $\gamma \geq 0$. The Frisch elasticity of labor supply is given by $1/\gamma$. As $\gamma \rightarrow \infty$, the model reduces to one with fixed labor supply. Following Hall (1997), I allow for a “labor disutility shock” u_t that shifts the intratemporal trade-off between consumption and leisure. In support of this idea, Kaplan and Schulhofer-Wohl (2018) find that labor disutility, as measured by “feelings about work” from surveys, has shifted in significant ways since 1950.⁷ The labor disutility shock evolves according to the

⁵Maurer and Meier (2008) find strong empirical evidence for “peer-group effects” on individual consumption decisions using panel data on US household expenditures.

⁶See, for example, Otrok, Ravikumar and Whiteman (2002), Beaubrun-Diant and Tripier (2005), Christiano, Motto, and Rostagno (2014), and Lansing (2015).

⁷More generally, the shock u_t could also be interpreted as a “labor wedge” that captures fluctuations in the effective tax rate on labor income.

following stationary AR(1) process.

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t}, \quad |\rho_u| < 1, \quad \varepsilon_{u,t} \sim NID(0, \sigma_{\varepsilon,u}^2). \quad (5)$$

The representative agent derives income by supplying labor and capital services to identical competitive firms. Firms produce output according to the technology

$$y_t = A k_t^{\alpha_t} [\exp(z_t) h_{1,t}]^{1-\alpha_t}, \quad A > 0, \quad (6)$$

$$z_t = z_{t-1} + \mu + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim NID(0, \sigma_{\varepsilon,z}^2), \quad (7)$$

$$\alpha_t = \alpha_{t-1}^{\rho_\alpha} \bar{\alpha}^{1-\rho_\alpha} \exp(\varepsilon_{\alpha,t}), \quad |\rho_\alpha| < 1, \quad \varepsilon_{\alpha,t} \sim NID(0, \sigma_{\varepsilon,\alpha}^2). \quad (8)$$

In equation (6), z_t represents a “labor productivity shock” that evolves as a random walk with drift. The drift parameter $\mu > 0$ determines the trend growth rate of output per person in the economy. Stochastic variation in the production function exponent α_t represents a “factor distribution shock,” along the lines of Young (2004), Ríos-Rull and Santaeulàlia-Llopis (2010), Lansing (2015), and Lansing and Markiewicz (2018). The logarithm of α_t evolves as a stationary AR(1) process.

Profit maximization by firms yields the factor prices

$$w_t = (1 - \alpha_t) y_t / h_{1,t}, \quad (9)$$

$$r_t = \alpha_t y_t / k_t, \quad (10)$$

which together imply $y_t = w_t h_{1,t} + r_t k_t$. From equation (10), stochastic variation in α_t allows the model to replicate fluctuations in the U.S. capital share of income. Given the time series for α_t , stochastic variation in z_t allows equation (6) to replicate fluctuations in U.S. output.

Resources devoted to investment together with investor effort contribute to the production of new physical capital according to the following law of motion

$$k_{t+1} = B \exp(v_t) k_t^{1-\delta_t-\varphi_t} i_t^{\delta_t} [\exp(z_t) h_{2,t}]^{\varphi_t}, \quad B > 0, \quad (11)$$

$$v_t = \rho_v v_{t-1} + \varepsilon_{v,t}, \quad |\rho_v| < 1, \quad \varepsilon_{v,t} \sim NID(0, \sigma_{\varepsilon,v}^2), \quad (12)$$

$$\delta_t = \delta_{t-1}^{\rho_\delta} \bar{\delta}^{1-\rho_\delta} \exp(\varepsilon_{\delta,t}), \quad |\rho_\delta| < 1, \quad \varepsilon_{\delta,t} \sim NID(0, \sigma_{\varepsilon,\delta}^2), \quad (13)$$

$$\varphi_t = \varphi_{t-1}^{\rho_\varphi} \bar{\varphi}^{1-\rho_\varphi} \exp(\varepsilon_{\varphi,t}), \quad |\rho_\varphi| < 1, \quad \varepsilon_{\varphi,t} \sim NID(0, \sigma_{\varepsilon,\varphi}^2), \quad (14)$$

where the shocks v_t , δ_t , and φ_t can be interpreted as capturing financial frictions that impact the supply of new capital and the price of claims to existing capital. A study by Greenwood, Hercowitz, and Huffman (1988) was the first to demonstrate that shocks of this sort can be an important driving force for business cycle fluctuations.⁸ The log-linear formulation of equation (11) captures the presence of capital adjustment costs.⁹

⁸Other examples along these lines include Ambler and Paquet (1994), Justiniano, Primiceri, and Tambalotti (2010), Waggoner and Zha (2011), and Furlanetto and Seneca (2014).

⁹Lansing (2012) shows that equation (11) with $h_{2,t} = 0$ maps directly to a log-linear approximate version of the law of motion for capital employed by Jermann (1998).

Following Cassou and Lansing (1997) and Lansing and Markiewicz (2018), I allow for a “multiplier shock” v_t that evolves as a stationary AR(1) process. Stochastic variation in v_t allows equation (11) to replicate the time path of U.S. private nonresidential fixed assets. The variable δ_t is an “investment shock” that represents stochastic variation in the elasticity of new capital with respect to new investment. Analogous to equation (6), the productivity of investor effort is influenced by the labor productivity shock z_t . The variable φ_t is an “investor effort shock” that represents stochastic variation in the elasticity of new capital with respect to investor effort. The logarithms of δ_t and φ_t evolve as stationary AR(1) processes.

The first-order conditions with respect to c_t , $h_{1,t}$, $h_{2,t}$, and k_{t+1} are given by

$$\lambda_t = 1/(c_t - \kappa_t C_t) = \eta_t/c_t, \quad (15)$$

$$D \exp(u_t) (h_{1,t} + h_{2,t})^\gamma = \lambda_t w_t, \quad (16)$$

$$D \exp(u_t) (h_{1,t} + h_{2,t})^\gamma = \lambda_t \varphi_t \dot{i}_t / (\delta_t h_{2,t}), \quad (17)$$

$$\lambda_t \dot{i}_t / (\delta_t k_{t+1}) = \beta \widehat{E}_t \lambda_{t+1} [r_{t+1} + (1 - \delta_{t+1} - \varphi_{t+1}) i_{t+1} / (\delta_{t+1} k_{t+1})], \quad (18)$$

where λ_t is the Lagrange multiplier on the budget constraint (2). In equation (15), I have imposed the equilibrium relationships $c_t = C_t$ and $\eta_t = 1/(1 - \kappa_t)$. In deriving equation (18), I start by using the capital law of motion (11) to eliminate \dot{i}_t from the budget constraint (2).

Combining equations (9), (16), and (17) yields the following expression for total hours worked $h_t \equiv h_{1,t} + h_{2,t}$:

$$\underbrace{h_{1,t} + h_{2,t}}_{h_t} = \left\{ \frac{\eta_t}{D \exp(u_t)} \left[(1 - \alpha_t) \frac{y_t}{c_t} + \frac{\varphi_t \dot{i}_t}{\delta_t c_t} \right] \right\}^{1/(1+\gamma)}. \quad (19)$$

Given the time series for the shocks α_t , η_t , δ_t , and φ_t , stochastic variation in u_t allows equation (19) to replicate the time path of U.S. hours worked per person.

Since k_{t+1} is known at time t , equation (18) can be rewritten as follows

$$\underbrace{\dot{i}_t / \delta_t}_{p_{s,t}} = \widehat{E}_t \left\{ M_{t+1} \underbrace{[\alpha_{t+1} y_{t+1} - (1 + \varphi_{t+1} / \delta_{t+1}) i_{t+1}]}_{d_{t+1}} + \underbrace{i_{t+1} / \delta_{t+1}}_{p_{s,t+1}} \right\}, \quad (20)$$

where $M_{t+1} \equiv \beta (\eta_{t+1} / \eta_t) (c_{t+1} / c_t)^{-1}$ is the equilibrium stochastic discount factor. Notice that in steady state, the stochastic discount factor is given by $\bar{M} \equiv \beta \exp(-\mu)$, which does not depend on the steady state value $\bar{\eta}$.

The rewritten first-order condition (20) is in the form of a standard asset pricing equation where $p_{s,t} = \dot{i}_t / \delta_t$ represents the market value of the agent’s equity shares in the firm. The equity shares entitle the agent to a perpetual stream of dividends d_{t+1} starting in period $t + 1$. From equations (16) and (17), we have $\varphi_t \dot{i}_t / \delta_t = w_t h_{2,t}$. Dividends in period t can therefore be written as $d_t = \alpha_t y_t - \dot{i}_t - w_t h_{2,t}$, which shows that the shadow wage bill for investor effort subtracts from the residual cash flow that can be paid out as dividends. Equity shares are assumed to exist in unit net supply.

Stochastic variation in δ_t allows the model to replicate fluctuations in U.S. investment conditional on U.S. equity value. Stochastic variation in φ_t allows the model to replicate fluctuations in U.S. dividends. Stochastic variation in an “equity sentiment shock” s_t (introduced below) allows $p_{s,t}$ in the model to replicate fluctuations in the real market value of the S&P 500 stock index. In so doing, the model’s real equity return, given by $r_{s,t} = (p_{s,t} + d_t)/p_{s,t-1} - 1$, replicates the real return on the S&P 500 stock index.

In addition to equity shares, the representative agent can purchase default free, privately-issued bonds that exist in zero net-supply. One-period discount bonds purchased at the price $p_{b,t}$ yield a single payoff of one consumption unit per bond in period $t + 1$. Long-term bonds (consols) purchased at the ex-coupon price $p_{c,t}$ yield a perpetual stream of stochastically-decaying coupon payments (measured in consumption units) starting in period $t + 1$. The equilibrium prices of the bonds are determined by the following first-order conditions

$$p_{b,t} = E_t M_{t+1}, \quad (21)$$

$$p_{c,t} = E_t M_{t+1} [1 + \bar{\delta}_c \exp(\omega_{t+1}) p_{c,t+1}], \quad (22)$$

where I have imposed $\widehat{E}_t = E_t$ so that the agent’s bond market forecasts are consistent with the actual laws of motion of the objects being forecasted. Consequently, departures from model-consistent expectations are restricted to the equity market and these departures turn out to be very small. The quantity $\delta_{c,t+1} \equiv \bar{\delta}_c \exp(\omega_{t+1})$ is the decay rate of the coupon received in period $t + 1$. The parameter $\bar{\delta}_c \in [0, 1)$ is the steady state decay rate which influences the Macaulay duration of the bond, i.e., the present-value weighted average maturity of the bond’s cash flows.¹⁰ The shock ω_t captures stochastic variation in the real value of the bond coupon payment (for example, due to surprise inflation) and evolves according to the following stationary AR(1) process

$$\omega_t = \rho_\omega \omega_{t-1} + \varepsilon_{\omega,t}, \quad |\rho_\omega| < 1, \quad \varepsilon_{\omega,t} \sim NID(0, \sigma_{\varepsilon,\omega}^2).$$

The risk free rate of return is given by $r_{b,t+1} = 1/E_t M_{t+1} - 1$, which is known at time t . Fluctuations in the risk aversion coefficient η_t influence M_{t+1} and thereby allow the model to replicate the real return on a 3-month U.S. Treasury bill. The risky return on the long-term bond is given by $r_{c,t+1} = (1 + \bar{\delta}_c \exp(\omega_{t+1}) p_{c,t+1})/p_{c,t} - 1$. Given the model-implied time series for M_{t+1} , fluctuations in the coupon decay rate shock ω_t allow the model to replicate the real return on a long-term U.S. Treasury bond.

¹⁰The perpetual stream of coupon payments is given by: $1, \bar{\delta}_c \exp(\omega_{t+1}), \bar{\delta}_c^2 \exp(\omega_{t+1} + \omega_{t+2}), \bar{\delta}_c^3 \exp(\omega_{t+1} + \omega_{t+2} + \omega_{t+3}) \dots$

2.1 Fundamental equity value

Defining the risk adjusted equity value-consumption ratio (a stationary variable) as $x_t \equiv \eta_t p_{s,t}/c_t = \eta_t i_t/(\delta_t c_t)$, the first order condition (20) becomes

$$\begin{aligned} x_t &= \beta \widehat{E}_t \{ \eta_{t+1} \alpha_{t+1} y_{t+1}/c_{t+1} + (1 - \delta_{t+1} - \varphi_{t+1}) x_{t+1} \} \\ &= \beta \widehat{E}_t \{ \underbrace{\eta_{t+1} \alpha_{t+1} + [1 - \delta_{t+1} (1 - \alpha_{t+1}) - \varphi_{t+1}] x_{t+1}}_{\equiv q_{t+1}} \}, \end{aligned} \quad (23)$$

where I have substituted in for M_{t+1} and collected terms dated t on the left side. In the second line, I use the budget constraint (2) at time $t+1$ and the definition of x_{t+1} to make the substitution $y_{t+1}/c_{t+1} = 1 + \delta_{t+1} x_{t+1}/\eta_{t+1}$. At this point, it is convenient to define a nonlinear change of variables such that q_{t+1} represents the composite stationary variable that the agent must forecast.¹¹ The agent's first-order condition (23) becomes $x_t = \beta \widehat{E}_t q_{t+1}$. Now using the definition of q_t to make the substitution $x_t = (q_t - \alpha_t \eta_t) / [1 - \delta_t (1 - \alpha_t) - \varphi_t]$ we obtain the following transformed version of the agent's first order condition

$$q_t = \eta_t \alpha_t + [1 - \delta_t (1 - \alpha_t) - \varphi_t] \beta \widehat{E}_t q_{t+1}. \quad (24)$$

The fundamental equity value is obtained by solving equation (24) under the assumption of rational expectations such that $\widehat{E}_t q_{t+1} = E_t q_{t+1}$. As shown in Appendix A, a log-linear approximate version of the fundamental solution is given by

$$q_t^f = \bar{q}^f \left[\frac{\eta_t}{\bar{\eta}} \right]^{\gamma_\eta} \left[\frac{\alpha_t}{\bar{\alpha}} \right]^{\gamma_\alpha} \left[\frac{\delta_t}{\bar{\delta}} \right]^{\gamma_\delta} \left[\frac{\varphi_t}{\bar{\varphi}} \right]^{\gamma_\varphi}, \quad (25)$$

where $\bar{q}^f \equiv \exp[E \log(q_t^f)]$ and $\gamma_\eta, \gamma_\alpha, \gamma_\delta,$ and γ_φ are solution coefficients that depend on model parameters. Given the value of q_t^f , we can recover the fundamental equity value-consumption ratio as

$$\frac{p_{s,t}^f}{c_t} = \frac{(q_t^f - \alpha_t)/\eta_t}{1 - \delta_t (1 - \alpha_t) - \varphi_t}, \quad (26)$$

which shows that $p_{s,t}^f/c_t$ will only move in response the four fundamental shocks $\eta_t, \alpha_t, \delta_t,$ and φ_t .

2.2 Introducing equity sentiment

Numerous empirical studies starting with Shiller (1981) and LeRoy and Porter (1981) have shown that equity prices appear to exhibit excess volatility when compared to fundamentals, as measured by the discounted stream of ex post realized dividends.¹² To capture the notion

¹¹This nonlinear change of variables technique and the associated solution method is also employed in Lansing (2010, 2016) and Lansing and LeRoy (2014).

¹²Lansing and LeRoy (2014) provide a recent update on this literature.

of sentiment-driven excess volatility, I postulate that the representative agent’s perceived law of motion (PLM) for the composite variable q_t in the first-order condition (24) allows for the possibility of departures from the fundamental value q_t^f . Specifically, the agent’s PLM takes the form

$$q_t = q_t^f \exp(s_t), \quad (27)$$

$$s_t = \bar{s} + \rho_s(s_{t-1} - \bar{s}) + \varepsilon_{s,t}, \quad |\rho_s| < 1, \quad \varepsilon_{s,t} \sim NID(0, \sigma_{\varepsilon,s}^2), \quad (28)$$

where the sentiment shock s_t evolves as a stationary AR(1) process. The PLM predicts that $\log(q_t/q_t^f)$ is increasing in s_t , where q_t is the actual value observed in the data and q_t^f is the value predicted by fundamentals from equation (25).¹³

Given the PLM (27), the agent’s subjective forecast can be computed as follows

$$\begin{aligned} \widehat{E}_t q_{t+1} &= E_t q_{t+1}^f \exp[\bar{s} + \rho_s(s_t - \bar{s}) + \sigma_{\varepsilon,s}^2/2], \\ &= \underbrace{\bar{q}^f \exp(\gamma_\eta^2 \sigma_{\varepsilon,\eta}^2/2 + \gamma_\alpha^2 \sigma_{\varepsilon,\alpha}^2/2 + \gamma_\delta^2 \sigma_{\varepsilon,\delta}^2/2 + \gamma_\varphi^2 \sigma_{\varepsilon,\varphi}^2/2 + \bar{s} + \sigma_{\varepsilon,s}^2/2)}_{\equiv \bar{q}} \\ &\quad \times \left[\frac{\eta_t}{\bar{\eta}} \right]^{\rho_\eta \gamma_\eta} \left[\frac{\alpha_t}{\bar{\alpha}} \right]^{\rho_\alpha \gamma_\alpha} \left[\frac{\delta_t}{\bar{\delta}} \right]^{\rho_\delta \gamma_\delta} \left[\frac{\varphi_t}{\bar{\varphi}} \right]^{\rho_\varphi \gamma_\varphi} \exp[\rho_s(s_t - \bar{s})], \end{aligned} \quad (29)$$

where $E_t q_{t+1}^f$ is the rational “fundamentals-only” forecast that is computed from equation (25). From equation (29), we see that the equity sentiment shock s_t and its associated parameters introduce a wedge between the agent’s subjective forecast and the fundamentals-only forecast that would prevail under rational expectations.¹⁴ The basic structure of equation (29) is consistent with the findings of Frydman and Stillwagon (2018). Using survey data of investors’ expectations about future stock returns, they present evidence that expectations are jointly driven by fundamentals and behavioral factors.¹⁵

Substituting the agent’s subjective forecast (29) into the first order condition (24) yields the actual law of motion (ALM) for q_t . The first order condition is “self-referential,” meaning that the actual value of q_t depends in part on the agent’s subjective forecast $\widehat{E}_t q_{t+1}$. When $\eta_t \simeq 0$, equation (24) resembles a rational bubble condition for which there exists a continuum of self-fulfilling solutions.¹⁶ But even when $\eta_t > 0$, the actual value of q_t can closely approximate

¹³Yu (2013) introduces a persistent sentiment shock that acts as a wedge between the actual versus perceived laws of motion for consumption growth in an endowment economy.

¹⁴The theoretical model presumes that the eight fundamental shocks are uncorrelated with each other. But in the quantitative exercise applied to U.S. data, the model-identified fundamental shocks turn out to be correlated with other fundamental shocks. The fundamentals-only forecast ignores these shock correlations, implying that this forecast is only boundedly rational in the quantitative exercises.

¹⁵Equation (29) can also be mapped into a version of the diagnostic expectations (DE) setup employed by Bordalo, et al. (2021). Under DE, we would have $\widehat{E}_t q_{t+1} = E_t q_{t+1}^f + \theta_t (E_t q_{t+1}^f - \bar{q}^f)$, where θ_t is a time-varying parameter that governs the degree of over- or under-reaction to expected movements in fundamental equity value.

¹⁶Lansing (2010) provides examples of rational bubble solutions in an endowment economy.

the value predicted by the PLM (27) if the slope coefficient applied to $\widehat{E}_t q_{t+1}$ in equation (24) is close to 1.0. This near-unity property of the slope coefficient is satisfied in the quantitative version of the model. Consequently, the agent's perception that movements in equity value are partly driven by sentiment is close to self-fulfilling. As I will show, the agent's subjective forecast errors are close to white noise with near-zero mean, providing no obvious signal that the sentiment-based forecast rule (29) is misspecified.

Given the realized value of q_t , we can recover the equity value-consumption ratio as

$$\frac{p_{s,t}}{c_t} = \frac{(q_t - \alpha_t)/\eta_t}{1 - \delta_t(1 - \alpha_t) - \varphi_t}, \quad (30)$$

where $q_t = q(\eta_t, \alpha_t, \delta_t, \varphi_t, s_t)$. Hence in equilibrium, $p_{s,t}/c_t$ will be partly driven by sentiment because the agent's subjective forecast (29) makes use of the sentiment variable. Using equation (30), we can recover the risk adjusted equity value-consumption ratio as

$$x_t \equiv \frac{\eta_t p_{s,t}}{c_t} = \frac{\eta_t i_t}{\delta_t c_t} = \frac{q_t - \alpha_t}{1 - \delta_t(1 - \alpha_t) - \varphi_t}. \quad (31)$$

Alternatively, since $x_t = \beta \widehat{E}_t q_{t+1}$, we can recover x_t by multiplying the agent's subjective forecast (29) by β .

2.3 Equilibrium macroeconomic variables and asset returns

Given $x_t = x(\eta_t, \alpha_t, \delta_t, \varphi_t, s_t)$ from equation (31), the equilibrium values of the other macroeconomic variables can be computed using the following equations

$$h_t = \left[\frac{(1 - \alpha_t)(\eta_t + \delta_t x_t) + \varphi_t x_t}{D \exp(u_t)} \right]^{1/(1+\gamma)}, \quad (32)$$

$$h_{1,t} = \left[\frac{(1 - \alpha_t)(\eta_t + \delta_t x_t)}{(1 - \alpha_t)(\eta_t + \delta_t x_t) + \varphi_t x_t} \right] h_t, \quad (33)$$

$$h_{2,t} = \left[\frac{\varphi_t x_t}{(1 - \alpha_t)(\eta_t + \delta_t x_t) + \varphi_t x_t} \right] h_t, \quad (34)$$

$$y_t = A k_t^{\alpha_t} [\exp(z_t) h_{1,t}]^{1-\alpha_t}, \quad (35)$$

$$c_t = \left[\frac{\eta_t}{\eta_t + \delta_t x_t} \right] y_t, \quad (36)$$

$$i_t = \left[\frac{\delta_t x_t}{\eta_t + \delta_t x_t} \right] y_t, \quad (37)$$

$$d_t = \alpha_t y_t - (1 + \varphi_t/\delta_t) i_t, \quad (38)$$

where I have made use of equation (19) and the budget relationships $y_t/c_t = 1 + \delta_t x_t/\eta_t$, and $i_t/c_t = \delta_t x_t/\eta_t$.

The equilibrium paths of $p_{s,t}$ and d_t pin down the real equity return $r_{s,t}$. The equilibrium paths of the bond prices $p_{b,t}$ and $p_{c,t}$ are obtained by solving equations (21) and (22). The solutions, which pin down the real bond returns $r_{b,t}$ and $r_{c,t}$, are contained in Appendix B.

3 Parameter values and model-identified shocks

Figure 2 plots the U.S. data versions of ten model variables. The sources and methods used to construct these variables, plus the long-term bond return, are described in Appendix D. Figure 3 plots the U.S. data versions of the ratios c_t/y_t , i_t/y_t , k_t/y_t , and $p_{s,t}/y_t$. From Figure 3, we see that the U.S. macroeconomic ratios are all close to their long-run means in 1972.Q3. As an alternative to estimation, I choose parameters so that the steady-state, trend, or ergodic mean values of the model variables are exactly equal to the values observed in the data in 1972.Q3.¹⁷

The steady state value $\bar{\eta} = 1$ implies $\bar{\kappa} = 0$ such that the representative agent’s utility function exhibits no habit component in steady state. The value of γ is chosen to deliver an aggregate Frisch labor supply elasticity of $1/\gamma = 1$. This value is consistent with the empirical evidence presented by Kneip, Merz, and Storjohann (2020). Using panel data on German men from 2000 to 2013, they estimate an aggregate Frisch elasticity that ranges between 0.85 and 1.06. Given a time endowment normalized to one, the value of the parameter D achieves the steady state target $h_{1,t} + h_{2,t} = 0.3$, implying that the representative agent spends about one-third of available time pursuing market work or investor effort. The values of the parameters A and B achieve the steady state targets of $k_t/y_t = 9.212$ and $k_{t+1}/k_t = \exp(\mu)$, as implied by equations (6) and (11). Table 1 summarizes the model parameter values.

Table 1. Model parameter values

Parameter	Value	Description/Target
$\bar{\eta}$	1	Risk aversion coefficient = 1 in 1972.Q3
γ	1	Frisch labor supply elasticity = $1/\gamma = 1$.
$\bar{\alpha}$	0.361	Capital income share = 0.361 in 1972.Q3.
A	1.001	$k_t/y_t = 9.212$ with $z_t = 0$ in 1972.Q3.
$\bar{\delta}$	0.069	$i_t/y_t = 0.287$ in 1972.Q3.
$\bar{\varphi}$	0.008	$d_t/y_t = 0.040$ in 1972.Q3.
B	1.345	$B(i_t/k_t)^{\delta_t} [\exp(z_t)h_{2,t}/k_t]^{\varphi_t} = \exp(\mu)$ in 1972.Q3.
D	10.489	$h_{1,t} + h_{2,t} = 0.3$ in 1972.Q3.
\bar{s}	-0.211	$p_{s,t}/y_t = 4.171$ in 1972.Q3.
β	1.0003	$r_{b,t} = 0.431\%$ in 1972.Q3.
$\bar{\delta}_c$	0.932	$r_{c,t} = 0.869\%$ in 1972.Q3.

The model-implied values of $p_{s,t}/y_t$ and $r_{b,t}$ in 1972.Q3 depend on numerous model parameters, including \bar{s} , β , and the various shock variances (for details, see Appendices A and B).

¹⁷Meenagh, Minford, and Wickens (2021) provide a critique of Bayesian and maximum likelihood methods of estimating macroeconomic models. They show that both methods can deliver significantly biased estimates of the true model’s parameter values.

Given candidate values for the shock variances, the values of \bar{s} and β are determined iteratively until the model-implied values of $p_{s,t}/y_t$ and $r_{b,t}$ in 1972:Q3 match the corresponding values in U.S. data and the shock variances have converged. The resulting value $\bar{s} = -0.211$ implies that equity sentiment is “pessimistic” relative to fundamental value in steady state. As noted in the introduction, this feature allows the model to replicate the mean equity risk premium in the data without the need for high levels of risk aversion. In section 4.3, I examine the sensitivity of the model’s mean asset returns to alternative values of \bar{s} .¹⁸

Given the parameter values in Table 1, I solve for the sequences of shock realizations that allow the calibrated model to exactly replicate the observed time paths of eleven U.S. macroeconomic variables and asset returns. These include the ten time series plotted in Figure 2 plus the real return on a long-term U.S. Treasury bond. Of these eleven time series, only nine are independent since $y_t = c_t + i_t$ and $r_{s,t} = (p_{s,t} + d_t)/p_{s,t-1} - 1$. The model has nine independent shocks, so each shock series is uniquely identified.¹⁹ The nine model shocks are: s_t (equity sentiment), η_t (risk aversion coefficient), u_t (labor disutility), v_t (capital law multiplier), δ_t (capital law exponent on investment), φ_t (capital law exponent on investor effort), α_t (factor distribution), z_t (labor productivity), and ω_t (bond coupon decay rate). Details of the shock identification procedure are contained in Appendix C.

Table 2 shows the values of the shock parameters implied by the identification exercise. All nine shocks exhibit very strong persistence—a typical result in the business cycle literature.²⁰ The strong persistence allows variables that are presumed stationary in the model (e.g., hours worked per person, capital’s share of income, and the equity value-consumption ratio) to be able to replicate the sustained upward or downward trends observed in U.S. data. Of the nine total shocks, three can be classified as demand shocks while the remaining six can be classified as supply shocks. I classify the labor disutility shock as one that affects labor supply.

Table 2. Model-identified shocks

Shock	Type	Values	
Equity sentiment, s_t	Demand	$\rho_s = 0.9270$	$\sigma_{\varepsilon,s} = 0.0752$
Risk aversion coefficient, η_t	Demand	$\rho_\eta = 0.8510$	$\sigma_{\varepsilon,\eta} = 0.0742$
Labor disutility, u_t	Supply	$\rho_u = 0.8661$	$\sigma_{\varepsilon,u} = 0.0753$
Capital law multiplier, v_t	Supply	$\rho_v = 0.9716$	$\sigma_{\varepsilon,v} = 0.0363$
Capital law exponent on investment, δ_t	Supply	$\rho_\delta = 0.9796$	$\sigma_{\varepsilon,\delta} = 0.0819$
Capital law exponent on investor effort, φ_t	Supply	$\rho_\varphi = 0.9800$	$\sigma_{\varepsilon,\varphi} = 0.1326$
Factor distribution, α_t	Supply	$\rho_\alpha = 0.9806$	$\sigma_{\varepsilon,\alpha} = 0.0128$
Labor productivity, z_t	Supply	$\mu = 0.0052$	$\sigma_{\varepsilon,z} = 0.0222$
Bond coupon decay rate, ω_t	Demand	$\rho_\omega = 0.9886$	$\sigma_{\varepsilon,\omega} = 0.0050$

Figure 4 plots the nine model-identified shock series. By construction, all shocks are equal

¹⁸In a similar framework with only six fundamental shocks, Lansing (2019) sets $\bar{s} = 0$. That version of the model cannot replicate the U.S. equity risk premium or the risk free rate of return.

¹⁹For similar shock identification exercises, but in the context of different models, see Lansing and Markiewicz (2018), Gelain, Lansing, and Natvik (2018), and Buckman, et al. (2020).

²⁰See, for example, Christiano, Motto, and Rostagno (2014, p. 44).

to their steady state or trend values in 1972.Q3.²¹ The equity sentiment shock s_t mimics movements the U.S. equity valuation ratio, as shown earlier in Figure 1. The sentiment shock reaches its maximum value in 1998.Q2, near the peak of the NASDAQ technology stock boom. The two lowest values for the sentiment shock occur during the recession quarters of 1982.Q3 and 2009.Q2, respectively.

The risk aversion coefficient η_t fluctuates between 0.57 and 1.19, with a standard deviation of 0.12. The minimum value occurs in 1982.Q3 and the maximum values occurs in 2001.Q4. Movements in η_t are positively correlated with movements in s_t . This correlation pattern allows the model to match the comovement of U.S. macroeconomic variables over the business cycle and the low volatility of the risk free rate relative to the equity return.

The labor disutility shock u_t exhibits a net downward trend over time, allowing the model to match the net upward trend of total hours worked per person h_t in the data. The net upward trend in h_t occurs despite the net downward trend in labor's share of income resulting from the increase in the factor distribution shock α_t . Movements in u_t are positively correlated with movements in s_t and η_t . This correlation pattern allows the model to match both the amplitude and comovement of macroeconomic variables over the business cycle.

The capital law multiplier shock v_t is positively correlated the other two capital law of motion shocks δ_t and φ_t . Consequently, v_t is almost perfectly negatively correlated with the quantity $1 - \delta_t - \varphi_t$, representing the exponent on k_t in the capital law of motion (11). This correlation pattern allows the model to match the smooth time path of k_t in the data while simultaneously matching the more-volatile time paths of i_t and $p_{s,t}$. Fluctuations in the investor effort shock φ_t allow the model to match the time path of d_t in the data.

The factor distribution shock α_t , representing capital's share of income, fluctuates around its steady state value until experiencing a sustained upward trend starting around 2005. As described in Appendix D, α_t is measured as one minus the ratio of employee compensation to gross value added of the corporate business sector. The labor productivity shock z_t evolves close to trend from around 1970 until the onset of Great Recession in 2008. The shock remains well below trend at the end of the data sample in 2019.Q4. Movements in α_t are negatively correlated with movements in z_t .

Finally, the bond coupon decay rate shock ω_t exhibits a net upward trend over time. This pattern allows the model to match the net increase in the U.S. real bond return that derives mainly from the secular decline in U.S. inflation. Movements in ω_t also allow the model to capture the shifting correlation pattern between returns on bonds versus equities, as documented by Campbell, Pflueger, and Viceira (2020).

Table 3 shows the contemporaneous cross correlations for the various shocks in first-difference form so as to mitigate the influence of the underlying shock trends. There is a strong positive correlation between movements in Δs_t , $\Delta \eta_t$, and Δu_t . There is also a strong

²¹The trend value of z_t is constructed as $\bar{z}_t = \bar{z}_{t-1} + \mu$, where μ is the sample mean of Δz_t and $\bar{z}_t = z_t = 0$ in 1972.Q3.

positive correlation between movements in Δv_t , $\Delta \delta_t$, and $\Delta \varphi_t$. The first group of shocks is negatively correlated with the second group. Movements in $\Delta \alpha_t$ and Δz_t exhibit a strong negative correlation, but these two shocks are mostly weakly correlated with the other shocks. Movements in $\Delta \omega_t$ are weakly correlated with all other shocks.

Table 3. Contemporaneous cross correlations

Shock	Δs_t	$\Delta \eta_t$	Δu_t	Δv_t	$\Delta \delta_t$	$\Delta \varphi_t$	$\Delta \alpha_t$	Δz_t	$\Delta \omega_t$
Δs_t	1.00	0.95	0.93	-0.68	-0.50	-0.53	-0.07	0.20	0.12
$\Delta \eta_t$		1.00	0.97	-0.47	-0.28	-0.37	-0.04	0.18	0.25
Δu_t			1.00	-0.47	-0.32	-0.32	-0.05	0.23	0.25
Δv_t				1.00	0.90	0.74	0.10	-0.10	0.34
$\Delta \delta_t$					1.00	0.44	-0.06	0.06	0.25
$\Delta \varphi_t$						1.00	0.45	-0.38	0.35
$\Delta \alpha_t$							1.00	-0.90	0.01
Δz_t								1.00	0.05
$\Delta \omega_t$									1.00

Note: Correlation coefficients computed for the sample period from 1960.Q2 to 2019.Q4.

4 Quantitative Analysis

4.1 Impulse response functions

Figure 5 plots impulse response functions from the model. The left column panels show the effects of a one standard deviation positive innovation to the equity sentiment shock, implying more optimism. The innovation causes an immediate increase in output, hours worked, and investment. But since the capital stock cannot respond immediately, the initial increase in output is not sufficient to allow both consumption and investment to increase on impact. Consumption drops on impact, but then increases as the capital stock starts rising in response to higher investment. Equity value increases but bond prices decline.

The middle column panels of Figure 5 show the effects of a one standard deviation negative innovation to the risk aversion coefficient, implying less risk aversion. The innovation causes an immediate increase in investment and investor effort, but hours worked in the production of output declines, causing a temporary drop in output and consumption until the capital stock starts rising in response to higher investment. All asset values increase in response to lower risk aversion.

Recall from Table 3 that Δs_t exhibits a strong positive correlation with $\Delta \eta_t$. The right column panels of Figure 5 show the effects of simultaneous positive innovations to both s_t and η_t . More optimistic sentiment together with higher risk aversion delivers a correlated increase in all macroeconomic variables. Equity value increases but bond prices decline (implying an increase in bond yields). Taken together, the combination of these two highly correlated shocks allows the model to capture the features observed during a typical economic boom or recovery.

The combination of more optimistic sentiment and higher risk aversion may seem at odds

during an economic boom. But higher risk aversion induces a shift towards current consumption that is facilitated by an increase in hours worked in the production of output, i.e., the mirror image of the response functions shown in the middle column panels of Figure 5. Rising consumption and hours worked in production are typical features of an economic boom.

Table 3 shows that Δu_t is positively correlated with Δs_t and $\Delta \eta_t$. Although not shown, adding a simultaneous positive innovation to u_t for the exercise in the right column panels of Figure 5 serves to dampen the correlated upward movements in the macroeconomic variables and equity value. The positive correlations among these three shocks allows the model to replicate both the amplitude and comovement of fluctuations in macroeconomic variables over the business cycle.

4.2 Actual versus perceived law of motion

Figure 6 provides insight into the near self-fulfilling nature of the agent's perceived law of motion (27). The top left panel plots the equilibrium quantity $\log(q_t/q_t^f)$ versus the value of the equity sentiment shock s_t . The eight fundamental shocks are all set to their steady state values. The agent's perceived law of motion predicts that $\log(q_t/q_t^f)$ should increase with s_t along the 45-degree line with slope = 1. The actual law of motion implies that $\log(q_t/q_t^f)$ increases along a line with slope $\simeq 0.9$. For any given value of s_t , the value of $\log(q_t/q_t^f)$ predicted by the two lines are nearly the same. For example when $s_t = \bar{s}$, the perceived law of motion predicts $\log(q_t/q_t^f) = -0.211$ whereas the actual law of motion predicts $\log(q_t/q_t^f) = -0.196$. The close approximation of the PLM to the ALM occurs because the slope coefficient applied to the agent's subjective forecast $\widehat{E}_t q_{t+1}$ in the first order condition (24) is always close to 1, as plotted in the top right panel of Figure 6. Consequently, the agent's subjective forecast has a very strong influence on the actual value of q_t .

The bottom left panel of Figure 6 separates the actual law of motion for equity value $p_{s,t}$ into two parts: (1) the fundamental component $p_{s,t}^f$, and (2) the sentiment-driven component given by $p_{s,t} - p_{s,t}^f$.²² The value $\bar{s} = -0.211$ implies pessimism relative to fundamental equity value in steady state. Consequently, the sentiment-driven component of equity value fluctuates almost entirely in negative territory except for two quarters during 1998. But if measured relative to \bar{s} , then equity sentiment can be viewed as becoming more optimistic during the late 1990s (coinciding with the NASDAQ technology stock boom) and becoming more pessimistic during the years 2008 and 2009 (coinciding with the Great Recession). Since 2010, equity sentiment has become more optimistic, contributing to the increase in actual equity value.

The bottom right panel of Figure 6 plots the agent's subjective forecast errors, as measured by $err_t \equiv q_t - \widehat{E}_{t-1} q_t$, where q_t evolves according to the actual law of motion and $\widehat{E}_{t-1} q_t$ is computed using the lagged version of equation (29). Table 4 summarizes the properties of the forecast errors together with those generated by a model-consistent forecast and a

²²The actual law of motion for $p_{s,t}$ is computed by multiplying equation (30) by c_t . The fundamental component is computed by multiplying equation (26) by c_t .

fundamentals-only forecast.²³

Table 4. Properties of forecast errors

Statistic	Subjective Forecast	Model-consistent Forecast	Fundamentals-only Forecast
$Mean(err_t)$	0.05	-0.07	-1.70
$\sqrt{Mean(err_t^2)}$	0.56	0.55	1.88
$Corr(err_t, err_{t-1})$	-0.18	-0.16	0.60
$Corr(err_t, err_{t-2})$	0.07	0.07	0.64
$Corr(err_t, err_{t-3})$	0.08	0.08	0.63

Notes: Forecast errors are computed for the sample period 1960.Q2 to 2019.Q4 using the model together with the identified shock parameters in Table 2. The mean value of q_t in the data is 5.06.

Table 4 shows that the agent’s subjective forecast errors are close to white noise with near-zero mean, giving no obvious signal that the sentiment-based forecast rule (29) is misspecified. The hypothetical model-consistent forecast delivers a slightly lower root mean squared forecast error of 0.55 versus 0.56 for the subjective forecast. Hence, there is little room for the representative agent to improve forecasting performance by employing more sophisticated econometric methods to discover the actual law of motion for q_t .²⁴

The fundamentals-only forecast performs very poorly when attempting to predict the actual value of q_t . This is because the representative agent’s use of the sentiment-based forecast rule serves to shift the moments of q_t . From the perspective of any individual agent, switching to the fundamentals-only forecast would appear to severely reduce forecast accuracy, so there is no incentive to switch.²⁵

4.3 Effect of steady state sentiment

The value $\bar{s} = -0.211$ implies that sentiment is pessimistic relative to fundamental equity value in steady state. As noted earlier, this feature allows the model to replicate the equity risk premium in the data while maintaining a low level of risk aversion. Table 5 shows the sensitivity of the model’s mean asset returns to higher values of sentiment. Specifically, I construct a counterfactual shock scenario that shifts up the entire sequence of model-identified sentiment shocks by a constant amount so as to achieve the steady state value $\bar{s}' > \bar{s}$ in 1972.Q3. The sequences of the eight fundamental shocks are unchanged from the baseline model.

As \bar{s}' increases, the equity return $r_{s,t}$ and the long-term bond return $r_{c,t}$ both decline, while the risk free rate $r_{b,t}$ rises. Increased optimism serves to shrink (and eventually eliminate)

²³The actual law of motion for q_t is obtained by substituting the agent’s subjective forecast (29) into the first order condition (24). The model-consistent forecast is constructed from a log-linearized version of the actual law of motion for q_t . The fundamentals-only forecast is constructed using the fundamental solution (25).

²⁴The forecast error statistics in Table 4 are influenced by the small sample properties of the U.S. data. In long-run model simulations, the model-consistent forecast delivers a mean forecast error of -0.01 , a root mean squared forecast error of 0.57, and a forecast error autocorrelation of 0.00. The corresponding values for the subjective forecast are 0.04, 0.59, and -0.01 , respectively.

²⁵Lansing (2006) explores the concept of “forecast lock-in” using a simple asset pricing model.

both the mean equity risk premium and the mean bond term premium. For example, when sentiment is neutral in steady state such that $\bar{s}' = 0$, the mean equity risk premium relative to $r_{b,t}$ is -0.18% per quarter. In contrast, the baseline model with $\bar{s} = -0.211$ delivers a mean equity risk premium of 1.64% per quarter.

Table 5. Effect of steady state sentiment on mean asset returns

Steady state sentiment	$r_{s,t}$	$r_{b,t}$	$r_{c,t}$	$r_{s,t} - r_{b,t}$	$r_{s,t} - r_{c,t}$	$r_{c,t} - r_{b,t}$
$\bar{s} = -0.211$, Baseline model	2.04	0.41	1.11	1.64	0.93	0.71
$\bar{s}' = -0.1$	1.40	0.76	1.24	0.65	0.16	0.49
$\bar{s}' = 0$	0.89	1.07	1.36	-0.18	-0.47	0.29
$\bar{s}' = 0.1$	0.45	1.40	1.49	-0.95	-1.04	0.09
$\bar{s}' = 0.211$	0.01	1.76	1.63	-1.74	-1.60	-0.12

Notes: Each number is the mean quarterly return (measured in percent) from 1960.Q2 to 2019.Q4 under a given steady state value of the equity sentiment shock in 1972.Q3. The top row shows the mean quarterly returns in U.S. data that are matched in the baseline model with the shock realizations s_t . Other rows use the shock realizations $s'_t = s_t + (\bar{s}' - \bar{s})$, such that $s'_t = \bar{s}'$ in 1972.Q3.

It is straightforward to demonstrate analytically how a pessimistic equity market forecast can magnify the equity risk premium. The equity market first order condition (20) can be written as

$$p_{s,t}/d_t = \widehat{E}_t [M_{t+1} (d_{t+1}/d_t) (1 + p_{s,t+1}/d_{t+1})], \quad (39)$$

where $p_{s,t}/d_t$ is the equity value-dividend ratio and d_{t+1}/d_t is the gross growth rate of dividends. Defining $z_{s,t} \equiv M_t (d_t/d_{t-1}) (1 + p_{s,t}/d_t)$, the first-order condition (39) becomes

$$p_{s,t}/d_t = \widehat{E}_t z_{s,t+1}, \quad (40)$$

which shows that the equity value-dividend ratio is simply the agent's subjective forecast of the composite variable $z_{s,t+1}$. Substituting $p_{s,t}^s/d_t = \widehat{E}_t z_{t+1}^s$ into the definition of $z_{s,t}$ yields the following transformed version of the equity market first order condition

$$z_{s,t} = M_t (d_t/d_{t-1}) (1 + \widehat{E}_t z_{s,t+1}). \quad (41)$$

The gross stock return can now be written as

$$1 + r_{s,t+1} = \left(\frac{1 + p_{s,t+1}/d_{t+1}}{p_{s,t}/d_t} \right) \frac{d_{t+1}}{d_t} = \frac{z_{s,t+1}}{\widehat{E}_t z_{s,t+1}} \frac{1}{M_{t+1}}, \quad (42)$$

where I have eliminated $p_{s,t}/d_t$ using equation (40) and eliminated $p_{s,t+1}/d_{t+1} + 1$ using the definition of $z_{s,t}$ evaluated at time $t + 1$.

Combining equation (42) with the first order condition for 1-period bonds (21) yields the following ratio of the gross equity return to the gross risk free rate:

$$\frac{1 + r_{s,t+1}}{1 + r_{b,t+1}} = \frac{z_{s,t+1}}{\widehat{E}_t z_{s,t+1}} \frac{E_t M_{t+1}}{M_{t+1}}. \quad (43)$$

Taking logs of both sides of equation (43) yields the following compact expression for the realized equity risk premium:

$$\log(1 + r_{s,t+1}) - \log(1 + r_{b,t+1}) = \log(z_{t+1}^s / \widehat{E}_t z_{t+1}^s) - \log(M_{t+1} / E_t M_{t+1}). \quad (44)$$

Pessimism implies that the agent's subjective forecast $\widehat{E}_t z_{s,t+1}$ is consistently below the realization $z_{s,t+1}$, thus serving to increase the magnitude of the first term in equation (44). Moreover, as shown in Appendix B, the steady state value \bar{s} enters negatively into the expressions for both M_{t+1} and $E_t M_{t+1}$. A more negative value of \bar{s} serves to increase $E_t M_{t+1}$ and thereby lower the risk free rate of return via a precautionary savings effect.

4.4 Counterfactual shock scenarios

Along the lines of figures presented by Chari, Kehoe, and McGrattan (2007), Figures 7 and 8 display counterfactual shock scenarios for four model variables: c_t , h_t , k_t , and $p_{s,t}$. In Figure 7, I omit one or more shock realizations (as indicated) while leaving the other shock realizations in place.²⁶ In Figure 8, I add one or more shock realizations when no other shock realizations are present. The purpose of these scenarios is to identify which shock (or set of shocks) has the largest quantitative impact on a given model variable. In Figure 7, a large gap between the counterfactual path and the U.S. data path (solid blue line) implies that the omitted shock plays an important role in allowing the model variable to replicate the path of the corresponding U.S. variable. In Figure 8, a small gap between the counterfactual path and the U.S. data path implies that the added shock plays an important role.

From Figures 7 and 8, we see that the quantitative impact of a given shock can be large for one variable but small for another variable. The fact that some model-identified shocks are highly correlated with other shocks makes it difficult to isolate the quantitative importance of an individual shock, or even a subset of shocks like those that appear in the law of motion for capital. Put another way, all nine shocks work together in allowing the model to exactly replicate the U.S. data.

Forecast error variance decompositions are often used to assess the relative importance of different shock innovations in the context of vector autoregressions (Gorodnichenko and Lee 2019). But in these applications, the shock innovations are first orthogonalized, typically via a Choleski decomposition. Forecast error variance decompositions are problematic here because the model-identified shock innovations are not orthogonal to each other, as evidenced by the results in Table 3.²⁷ The existence of nontrivial covariances among the shock innovations prevents a clear separation of the variance contribution coming from a given innovation. For

²⁶Specifically, I set the given shock realization equal to its steady state or trend value each period. But any shock variance terms that appear in the model's equilibrium solution remain in place.

²⁷The cross correlations among the shock innovations are very similar to the cross correlations among the shock first-differences.

example, the model-consistent forecast error for output growth can be written as

$$\begin{aligned} \log(y_{t+1}/y_t) - E_t \log(y_{t+1}/y_t) = & a_1 \varepsilon_{s,t+1} + a_2 \varepsilon_{\eta,t+1} + a_3 \varepsilon_{u,t+1} + a_4 \varepsilon_{\alpha,t+1} + a_5 \varepsilon_{\delta,t+1} \\ & + a_6 \varepsilon_{\varphi,t+1} + a_7 \varepsilon_{z,t+1}, \end{aligned} \quad (45)$$

where a_i for $i = 1$ to 7 are Taylor-series coefficients.²⁸ Taking the variance of the right-side of equation (45) yields 7 variance terms given by $\sigma_{\varepsilon,s}^2 + \sigma_{\varepsilon,\eta}^2 + \sigma_{\varepsilon,\alpha}^2 + \dots$ and 21 covariance terms given by $2Cov(\varepsilon_{s,t+1}, \varepsilon_{\eta,t+1}) + 2Cov(\varepsilon_{s,t+1}, \varepsilon_{\alpha,t+1}) + 2Cov(\varepsilon_{s,t+1}, \varepsilon_{u,t+1}) + \dots$ Depending on the sign, each covariance term can either magnify or shrink the contribution of a given shock innovation to the total variance of the forecast error.

As an alternative to forecast error variance decompositions, I compute the mean absolute gaps (measured in percent for levels of variables or percentage points for growth rates or asset returns) between the counterfactual paths that omit one or more shock realizations and the corresponding U.S. data paths. The results of the gap computations are shown in Tables 6, 7, and 8, where boldface indicates the largest mean absolute gap for each macroeconomic variable or asset return.

In Table 6, the equity sentiment shock and the capital law of motion shocks have the largest impacts on the variables i_t , k_t , and $p_{s,t}$. The labor disutility shock has the largest impacts on the variables y_t , c_t , and h_t .²⁹ The factor distribution shock has sizable impacts on the variables y_t , c_t , and i_t . The risk aversion shock has relatively mild impacts on the levels of all macroeconomic variables. This is because η_t fluctuates over the narrow range of 0.57 to 1.19 in the baseline model.

Table 6. Mean absolute gaps: Levels of macroeconomic variables

Shock scenario	y_t	c_t	i_t	h_t	k_t	$p_{s,t}$
Baseline model = U.S. data	0	0	0	0	0	0
No equity sentiment shock	8.51	4.52	23.0	3.57	20.9	23.0
No risk aversion shock	2.22	4.58	4.71	4.50	3.83	4.71
No labor disutility shock	10.7	10.7	10.7	11.0	10.4	10.7
No capital law of motion shocks	7.94	6.28	17.8	2.67	20.0	31.5
No factor distribution shock	9.37	8.15	12.9	0.43	9.87	12.9
No labor productivity shock	8.18	8.18	8.18	0	9.79	8.18

Notes: Each number is the mean absolute gap (measured in percent) between the path of the model variable under a given scenario that omits one or more shock realizations and the path of the corresponding U.S. variable. Boldface indicates the largest mean absolute gap for each variable.

The gap measures in Table 6 do not distinguish between high frequency versus low frequency movements in the U.S. macroeconomic variables. The goal of the shock identification exercise is to replicate *all* movements in the U.S. data, not just those associated with business

²⁸Equation (45) does not include a term involving $\varepsilon_{v,t+1}$ because y_{t+1}/y_t depends on k_{t+1} which in turn depends on v_t (but not v_{t+1}) from equation (11).

²⁹In this case, the gaps for numerous macroeconomic variables are exactly the same at 10.7%. This result is due to log utility which delivers simple proportional relationships among some macroeconomic variables.

cycle frequencies. To gauge the relative importance of the various shocks for higher frequency movements, Table 7 shows the mean absolute gaps between the 4-quarter growth rates of the counterfactual paths and the 4-quarter growth rates of the corresponding U.S. data paths.³⁰ The factor distribution shock has the largest impact on the growth rate of y_t . The risk aversion shock has the largest impact on the growth rate of c_t . The equity sentiment shock and the capital law of motion shocks have the largest impacts on the growth rates of i_t , k_t , and $p_{s,t}$. The labor disutility shock has the largest impact on the growth rate of h_t . Overall, the results in Tables 6 and 7 point to the difficulty of identifying the most important shock, or set of shocks, for explaining movements in macroeconomic variables.

Table 7. Mean absolute gaps: 4-quarter growth rates of macroeconomic variables

Shock scenario	y_t	c_t	i_t	h_t	k_t	$p_{s,t}$
Baseline model = U.S. data	0	0	0	0	0	0
No equity sentiment shock	1.54	2.24	6.75	1.67	3.30	6.75
No risk aversion shock	2.06	3.58	2.47	2.86	0.89	2.47
No labor disutility shock	2.50	2.50	2.50	3.93	0.85	2.50
No capital law of motion shocks	1.54	1.60	3.98	0.73	3.68	10.1
No factor distribution shock	2.96	2.36	4.67	0.17	2.06	4.67
No labor productivity shock	2.41	2.41	2.41	0	2.19	2.41

Notes: Each number is the mean absolute gap (measured in percentage points) between the 4-quarter growth rate of the model variable under a given scenario that omits one or more shock realizations and the 4-quarter growth rate of the corresponding U.S. variable. Boldface indicates the largest mean absolute gap for each variable's 4-quarter growth rate.

Table 8 shows the mean absolute gaps for asset returns. Shocks that appear in the capital law of motion (representing financial frictions) and the equity sentiment shock have largest impacts on the size of the equity risk premium relative to both bond returns. The capital law of motion shocks also have a large impact on the bond term premium as does the coupon decay shock (by construction).

³⁰Similar rankings of the relative importance of the various shocks are obtained if the gaps in Table 7 are constructed using detrended versions of the variables (obtained using the Hodrick-Prescott filter with a smoothing parameter of 1600) rather than the 4-quarter growth rates of the variables.

Table 8. Mean absolute gaps: Asset returns

Shock scenario	$r_{s,t}$	$r_{b,t}$	$r_{c,t}$	$r_{s,t} - r_{b,t}$	$r_{s,t} - r_{c,t}$	$r_{c,t} - r_{b,t}$
Baseline model = U.S. data	0	0	0	0	0	0
No equity sentiment shock	4.03	0.44	1.62	4.09	5.58	1.76
No risk aversion shock	2.00	0.88	2.01	1.80	1.07	2.35
No labor disutility shock	1.77	1.01	1.03	2.14	0.77	1.44
No capital law of motion shocks	4.64	2.12	5.78	5.18	5.36	5.94
No factor distribution shock	1.94	0.12	0.52	1.89	2.43	0.56
No labor productivity shock	1.01	0.22	0.01	0.95	1.01	0.22
No bond coupon decay shock	0	0	5.81	0	2.01	5.81

Notes: Each number is the mean absolute gap (measured in percentage points) between the path of the model return under a given scenario that omits one or more shock realizations and the path of the U.S. return. Boldface indicates the largest mean absolute gap for each asset return.

Table 9 shows the mean asset returns for each counterfactual shock scenario. Overall, the results portray a complex picture of the roles played by each of the various shocks in generating the mean asset returns observed in U.S. data. Omitting realizations of the capital law of motion shocks serves to significantly magnify the equity risk premium and the bond term premium. In contrast, omitting realizations of the equity sentiment shock (but maintaining $\bar{s} = -0.211$ in steady state) serves to shrink these same premia. Omitting realizations of the risk aversion shock serves to magnify the equity risk premium but shrink the bond term premium. The same is true for the labor productivity shock, but the magnitudes of the shifts are now smaller. Omitting realizations of the labor disutility shock serves to shrink both the equity risk premium and bond term premium.

Table 9. Counterfactual mean asset returns

Shock scenario	$r_{s,t}$	$r_{b,t}$	$r_{c,t}$	$r_{s,t} - r_{b,t}$	$r_{s,t} - r_{c,t}$	$r_{c,t} - r_{b,t}$
Baseline model = U.S. data	2.04	0.41	1.12	1.64	0.93	0.71
No equity sentiment shock	0.63	0.67	1.35	-0.04	-0.72	0.68
No risk aversion shock	3.00	1.06	1.30	1.93	1.70	0.23
No labor disutility shock	2.06	1.38	1.24	0.69	0.82	-0.13
No capital law of motion shocks	2.15	-1.70	0.73	3.84	1.42	2.42
No factor distribution shock	1.76	0.38	1.06	1.38	0.70	0.67
No labor productivity shock	2.23	0.58	1.11	1.65	1.12	0.53
No bond coupon decay shock	2.04	0.41	-0.84	1.64	2.88	-1.25

Notes: Each number is the mean quarterly return (measured in percent) from 1960.Q2 to 2019.Q4 under a given scenario that omits one or more shock realizations. The top row shows the mean quarterly returns in U.S. data that are matched by construction in the baseline model.

Previous studies by Greenwald, Lettau, and Ludvigson (2014) and Lansing (2015) using concentrated capital ownership models (i.e., capital owners versus workers) identify a large role for factor distribution shocks in explaining the equity risk premium in U.S. data. In contrast, Table 9 shows that omitting realizations of the factor distribution shock serves only to

mildly shrink the equity risk premium (and the bond term premium) relative to the baseline model. As demonstrated numerically by Lansing (2015, p. 83), the equity risk premium in the concentrated capital ownership model is highly sensitive to the presence of factor distribution shocks because these shocks strongly impact the volatility of capital owners’ consumption growth. But in a representative agent framework, the same sequence of factor distribution shocks has much less impact on the volatility of aggregate consumption growth, thereby muting the resulting impact on the equity risk premium. This example shows that conclusions regarding the relative importance of various shocks for macroeconomic or financial variables can be model-specific.

4.5 Cumulative growth impacts of individual shocks

Using data from 1952.Q1 to 2017.Q4, Greenwald, Lettau, and Ludvigson (2020) estimate a concentrated capital ownership model with four types of orthogonal shocks that govern: (1) capital’s share of income, (2) capital owners’ degree of risk aversion, (3) capital owners’ rate of time preference (which influences the risk free rate of return), and (4) the growth rate of real per capita output. They state (p. 3) “*[N]either economic growth, risk premia, nor risk-free interest rates has been the foremost driving force behind the [stock] market’s sharp gains over the last several decades. Instead, the single most important contributor has been a string of factor share shocks that reallocated the rewards of production without affecting the size of those rewards.*”

Table 10 shows the cumulative growth impacts of adding one or more shock realizations when no other shock realizations are present. Each number is the cumulative growth rate (in percent) starting from the common steady state value in 1972.Q3 and ending in 2019.Q4. Boldface indicates the largest cumulative growth rate for each macroeconomic variable.

Table 10. Cumulative growth impacts, 1972.Q3 to 2019.Q4

Shock scenario	y_t	c_t	i_t	h_t	k_t	$p_{s,t}$
Baseline model = U.S. data	179.6	193.1	146.0	13.4	181.1	326.9
Only equity sentiment shock	149.0	154.7	134.8	-1.27	121.3	134.8
Only risk aversion shock	166.0	160.4	180.0	-3.38	178.8	179.9
Only labor disutility shock	214.5	214.5	214.5	18.4	207.8	214.5
Only capital law of motion shocks	150.1	170.8	98.5	1.44	159.8	244.5
Only factor distribution shock	284.6	268.2	325.6	-1.54	327.6	325.6
Only labor productivity shock	107.2	107.2	107.2	0	107.2	107.2

Notes: Each number is the cumulative growth rate (in percent) starting from the model steady state in 1972.Q3 through 2019.Q4. Each scenario adds one or more shock realizations when no other shock realizations are present. Boldface indicates the largest cumulative growth rate for each variable.

Adding realizations of the factor distribution shock α_t delivers the largest cumulative growth rate for all variables except total hours worked per person h_t . Indeed, the factor distribution shock alone delivers a 325.6% cumulative increase in real equity value $p_{s,t}$, which

is very close to the 326.9% cumulative increase observed in U.S. data. The intuition for this result can be seen from Table 3 which shows that $\Delta\alpha_t$ is strongly negatively correlated with Δz_t . Adding a string of mostly positive factor distribution shock innovations from 1972.Q3 onward, while simultaneously omitting a string of mostly negative labor productivity shock innovations, serves to produce large cumulative increases in all model variables except h_t . Despite different modeling choices and different assumptions regarding the orthogonality of shocks, the results in Table 10 are qualitatively consistent with the findings of Greenwald, Lettau, and Ludvigson (2020).

4.6 Predicting excess returns with model-identified shocks

A vast literature, pioneered by Fama and French (1988), examines the “predictability” of excess returns on risky assets. Predictability is typically measured by the size of a slope coefficient and the R-squared statistic in forecasting regressions. In the present context, such regressions provide an alternative way of assessing the relative importance of the various shocks for the equity risk premium or the bond term premium.

Table 11 shows the results of regressing the U.S. equity risk premium, as measured by $r_{t+1}^s - r_{t+1}^b$, on a constant and one model-identified shock at time t . Table 12 shows the corresponding results for the U.S. bond term premium, as measured by $r_{t+1}^c - r_{t+1}^b$. For ease of comparison across regressions, each shock series is first demeaned and normalized by its standard deviation over the sample period 1960.Q2 to 2019.Q3.³¹

Table 11. Predicting the equity risk premium

Shock	Slope coefficient	t -statistic	R^2
s_t	-1.230	-2.425	2.4%
η_t	-0.915	-1.794	1.3%
u_t	-0.717	-1.403	0.8%
v_t	1.093	2.148	1.9%
δ_t	0.506	0.988	0.4%
φ_t	1.446	2.864	3.4%
α_t	0.833	1.631	1.1%
Δz_t	-0.388	-0.756	0.2%
$\omega_{c,t}$	0.831	1.626	1.1%

Notes: Each number is the slope coefficient from the following regression: $r_{t+1}^s - r_{t+1}^b = c_0 + c_1(\text{shock}_t) + \epsilon_{t+1}$. Each model-identified shock series is demeaned and normalized by its standard deviation over the sample period 1960.Q2 to 2019.Q3. Boldface indicates significant at the 5% level.

In Table 11, the equity sentiment shock s_t and the shocks v_t and φ_t that appear in the capital law of motion are statistically significant in predicting the 1-quarter ahead equity risk premium. More optimistic sentiment predicts a smaller equity risk premium in the next

³¹The sample period for the predictability regressions starts in 1960.Q2 because I use Δz_t as a shock regressor instead of z_t which exhibits a unit root in the theoretical model.

quarter. This result is consistent with the findings of Huang, et al. (2014) who show that a version of the investor sentiment index originally constructed by Baker and Wurgler (2007) is a robust negative predictor of 1-month ahead excess stock returns. Lansing, LeRoy, and Ma (2020) show that a variable which interacts the 12-month change in the University of Michigan consumer sentiment index with lagged excess stock returns is a robust negative predictor of 1-month ahead excess stock returns. Higher values of v_t or φ_t predict a larger equity risk premium in the next quarter. This result is intuitive because higher values of these shocks contribute to more capital at time $t + 1$.

As detailed in Appendix C, the identification procedure for the shocks s_t , v_t , and φ_t employs U.S. data on equity value $p_{s,t}$ and equity dividends d_t at time t . Hence, the predictive power of the shocks s_t , v_t , and φ for the equity risk premium is related to the well-documented fact that the equity price-dividend ratio is a robust predictor of excess stock returns (Cochrane 2008). The results in Table 11 help to provide some insight into the microfoundations of the price-dividend ratio’s predictive power.

Table 12 shows that more optimistic sentiment predicts a smaller bond term premium in the next quarter, but this result is not statistically significant. The slope coefficients on the labor disutility shock u_t and the investor effort shock φ_t are both statistically significant, but differ in sign. Higher values of u_t serve to reduce hours worked in production, predicting a smaller bond term premium in the next quarter. Higher values of φ_t serve to increase capital at time $t + 1$, predicting a larger bond term premium in the next quarter. Interestingly, the investor effort shock is the only shock with a statistically significant slope coefficient in both Tables 11 and 12. By this metric, “investor effort” should perhaps be viewed as a key fundamental driver of excess returns on risky assets.

Table 12. Predicting the bond term premium

Shock	Slope coefficient	t -statistic	R^2
s_t	-0.610	-1.703	1.2%
η_t	-0.606	-1.692	1.2%
u_t	-0.981	-2.766	3.1%
v_t	0.583	1.626	1.1%
δ_t	0.235	0.653	0.2%
φ_t	0.823	2.308	2.2%
α_t	0.450	1.251	0.7%
Δz_t	0.149	0.414	0.1%
$\omega_{c,t}$	0.641	1.790	1.3%

Notes: Each number is the slope coefficient from the following regression: $r_{t+1}^c - r_{t+1}^b = c_0 + c_1 (shock_t) + \epsilon_{t+1}$. Each model-identified shock series is demeaned and normalized by its standard deviation over the sample period 1960.Q2 to 2019.Q3. Boldface indicates significant at the 5% level.

4.7 Asset returns from stochastic simulations

As a final quantitative exercise, I compute asset returns from 100,000 stochastic simulations of the baseline model, where each simulation is 239 quarters in length, i.e., the number of quarters in the data from 1960.Q2 to 2019.Q4. For the simulations, the nine stochastic shocks are uncorrelated with each other, as presumed by the theoretical model. The results of the stochastic simulations are presented in Table 13.

Table 13. Distribution of asset returns

Return	U.S. data			Model simulations		
	Mean	Median	[25%, 75%]	Mean	Median	[25%, 75%]
$r_{s,t}$	2.04	2.87	[-2.38, 6.65]	1.77	1.27	[-5.28, 8.28]
$r_{b,t}$	0.41	0.38	[-0.17, 0.90]	0.48	0.42	[-2.40, 3.30]
$r_{c,t}$	1.12	0.54	[-2.61, 4.28]	4.35	1.13	[-13.4, 18.4]

Notes: Quarterly returns in percent. U.S. data sample is 1960.Q2 to 2019.Q4. Model statistics are average values from 100,000 simulations, each 239 quarters in length.

The model’s 25% to 75% distribution bands are quite wide due to the highly persistent nature of the model-identified shocks. This is particularly true for the distribution bands surrounding the simulated long-term bond return $r_{c,t}$. The bond coupon decay shock ω_t has an AR(1) coefficient of $\rho_\omega = 0.9886$, the largest of all nine shocks in Table 2. Moreover, this shock enters directly into the definition of the long-term bond return given by $r_{c,t+1} = [1 + \bar{\delta}_c \exp(\omega_{t+1})p_{c,t+1}]/p_{c,t} - 1$.

Greenwald, Lettau, and Ludvigson (2020, p. 29) state: “[Our] estimates imply that the high returns to holding equity in the post-war period have been driven, in large part, by a highly unusual sample, one characterized by a long string of factors share shocks that redistributed rewards from productive activity toward shareholders.”

The results in Table 13 lend support to the idea that the unique set of shock realizations that account for the historical U.S. data sample have produced higher real equity returns than should be expected going forward, based on the theoretical model. The mean and median equity return computed from the model simulations are 1.77% and 1.27% per quarter, respectively. Both statistics are below the corresponding values of 2.04% and 2.87% observed in the U.S. data since 1960. While this discrepancy may simply reflect statistical sampling variation, it is also partially attributable to the use of a model shock correlation structure that differs considerably from that observed in the historical data sample. Going forward, if the shocks hitting the U.S. economy behave more like those in the theoretical model, then the resulting mean equity return (and the mean equity risk premium) may turn out to be lower than in the past.

5 Conclusion

Pigou (1927, p.73) attributed business cycle fluctuations partly to “psychological causes” which lead people to make “errors of undue optimism or undue pessimism in their business forecasts.” Keynes (1936, p.156) likened the stock market to a “beauty contest” where participants devoted their efforts not to judging the underlying concept of beauty, but instead to “anticipating what average opinion expects the average opinion to be.” More recently, Shiller (2017) argues that investors’ optimistic or pessimistic beliefs about the stock market are similar to fads that can spread through the popular culture like an infectious disease.

This paper seeks to capture the flavor of these ideas by introducing an “equity sentiment shock” in a standard real business cycle model with eight other fundamental shocks. Due to the self-referential nature of the model, the agent’s perception that movements in equity value are partly driven by sentiment is close to self-fulfilling. The agent’s forecast errors for equity value are nearly identical those implied by a hypothetical model-consistent forecast. From the perspective of an individual agent, switching to a fundamentals-only forecast would appear to reduce forecast accuracy, so there is no incentive to switch.

I solve for the time series of shock realizations that allow the model to exactly replicate the observed time paths of U.S. macroeconomic variables and asset returns over the past six decades. The exercise is a transparent way of investigating which shocks are the most important for explaining the movements of a given macroeconomic variable or asset return.

The model-identified sentiment shock is negative in steady state, implying pessimism relative to fundamental equity value. This feature allows the model to replicate the U.S. equity risk premium while maintaining a low level of risk aversion. The sentiment shock exhibits a strong positive correlation with the model-identified risk aversion coefficient. More optimistic sentiment together with higher risk aversion leads to a correlated increase in all macroeconomic variables and equity value, thus capturing the features observed during a typical economic boom or recovery.

Counterfactual scenarios with the model show that the equity sentiment shock and shocks that appear in the law of motion for capital (representing financial frictions) have large impacts on the levels of investment, the capital stock, and equity value. The labor disutility shock has large impacts on the levels of output, consumption, and hours worked. With regard to higher frequency movements, the factor distribution shock has the largest impact on output growth while the risk aversion shock has the largest impact on consumption growth. The labor disutility shock has the largest impact on hours growth.

Omitting realizations of the capital law of motion shocks serves to significantly magnify the equity risk premium and the bond term premium. Omitting realizations of the equity sentiment shock (but maintaining pessimism in steady state) serves to shrink these same premia. Four of the model-identified shocks help to predict the equity risk premium or the bond term premium in the next quarter.

Rather than identifying a “main business cycle shock,” as described by Angeletos, Collard, and Dellas (2020), the quantitative results presented here support a narrative in which a large number of correlated shocks have combined to deliver the historical outcomes observed in U.S. data.

A Appendix: Fundamental equity value

This appendix provides details of the fundamental solution for q_t^f shown in equation (25). First imposing rational expectations and then log linearizing the right-side of the fundamentals-only version of the first order condition (24) yields

$$q_t^f = b_0 \left[\frac{\eta_t}{\bar{\eta}} \right]^{b_1} \left[\frac{\alpha_t}{\bar{\alpha}} \right]^{b_2} \left[\frac{\delta_t}{\bar{\delta}} \right]^{b_3} \left[\frac{\varphi_t}{\bar{\varphi}} \right]^{b_4} E_t \left[\frac{q_{t+1}^f}{\bar{q}^f} \right]^{b_5}, \quad (\text{A.1})$$

where b_i for $i = 0$ to 5 are Taylor-series coefficients and $\bar{q}^f \equiv \exp[E \log(q_t^f)]$. The expressions for the Taylor-series coefficients are

$$b_0 = \bar{\alpha} \bar{\eta} + [1 - \bar{\delta}(1 - \bar{\alpha}) - \bar{\varphi}] \beta \bar{q}^f, \quad (\text{A.2})$$

$$b_1 = \frac{\bar{\alpha} \bar{\eta}}{\bar{\alpha} \bar{\eta} + [1 - \bar{\delta}(1 - \bar{\alpha}) - \bar{\varphi}] \beta \bar{q}^f}, \quad (\text{A.3})$$

$$b_2 = \frac{\bar{\alpha} (\bar{\eta} + \bar{\delta} \beta \bar{q}^f)}{\bar{\alpha} \bar{\eta} + [1 - \bar{\delta}(1 - \bar{\alpha}) - \bar{\varphi}] \beta \bar{q}^f}, \quad (\text{A.4})$$

$$b_3 = \frac{-\bar{\delta}(1 - \bar{\alpha}) \beta \bar{q}^f}{\bar{\alpha} \bar{\eta} + [1 - \bar{\delta}(1 - \bar{\alpha}) - \bar{\varphi}] \beta \bar{q}^f}, \quad (\text{A.5})$$

$$b_4 = \frac{-\bar{\varphi} \beta \bar{q}^f}{\bar{\alpha} \bar{\eta} + [1 - \bar{\delta}(1 - \bar{\alpha}) - \bar{\varphi}] \beta \bar{q}^f}, \quad (\text{A.6})$$

$$b_5 = \frac{[1 - \bar{\delta}(1 - \bar{\alpha}) - \bar{\varphi}] \beta \bar{q}^f}{\bar{\alpha} \bar{\eta} + [1 - \bar{\delta}(1 - \bar{\alpha}) - \bar{\varphi}] \beta \bar{q}^f}, \quad (\text{A.7})$$

A conjecture for the fundamental solution takes the form of equation (25). The conjectured solution is iterated ahead one period and then substituted into the right-side of equation (A.1) together with the laws of motion for η_{t+1} , α_{t+1} , δ_{t+1} and φ_{t+1} from equations (4), (8), (13) and (14), respectively. After evaluating the conditional expectation and then collecting terms, we have

$$\begin{aligned} q_t^f &= b_0 \exp \left[\underbrace{(\gamma_\eta b_5)^2 \sigma_{\varepsilon, \eta}^2 / 2 + (\gamma_\alpha b_5)^2 \sigma_{\varepsilon, \alpha}^2 / 2 + (\gamma_\delta b_5)^2 \sigma_{\varepsilon, \delta}^2 / 2 + (\gamma_\varphi b_5)^2 \sigma_{\varepsilon, \varphi}^2 / 2}_{=\bar{q}^f} \right] \\ &\times \left[\frac{\eta_t}{\bar{\eta}} \right]^{\underbrace{b_1 + \rho_\eta \gamma_\eta b_5}_{=\gamma_\eta}} \times \left[\frac{\alpha_t}{\bar{\alpha}} \right]^{\underbrace{b_2 + \rho_\alpha \gamma_\alpha b_5}_{=\gamma_\alpha}} \times \left[\frac{\delta_t}{\bar{\delta}} \right]^{\underbrace{b_3 + \rho_\delta \gamma_\delta b_5}_{=\gamma_\delta}} \times \left[\frac{\varphi_t}{\bar{\varphi}} \right]^{\underbrace{b_4 + \rho_\varphi \gamma_\varphi b_5}_{=\gamma_\varphi}} \quad (\text{A.8}) \end{aligned}$$

which yields five equations in the five solution coefficients \bar{q}^f , γ_η , γ_α , γ_δ , and γ_φ . For the baseline calibration, the resulting solution coefficients are $\bar{q}^f = 7.184$, $\gamma_\eta = 0.263$, $\gamma_\alpha = 1.094$, $\gamma_\delta = -0.631$, and $\gamma_\varphi = -0.120$.

B Appendix: Equilibrium bond prices

This appendix outlines the solutions for the equilibrium bond prices $p_{b,t}$ and $p_{c,t}$ using equations (21) and (22). The equilibrium stochastic discount factor can be written as follows

$$\begin{aligned} M_{t+1} &= \beta \frac{\eta_{t+1}}{\eta_t} \times \frac{c_t/y_t}{c_{t+1}/y_{t+1}} \times \frac{y_t}{y_{t+1}} \\ &= \beta \left[\frac{\eta_{t+1} + \delta_{t+1} x_{t+1}}{\eta_t + \delta_t x_t} \right] \frac{y_t}{y_{t+1}}, \end{aligned} \quad (\text{B.1})$$

where I have made use of the equilibrium budget relationship $c_t/y_t = \eta_t/(\eta_t + \delta_t x_t)$ from equation (36).

Making use of equation (6), the term y_t/y_{t+1} in equation (B.1) can be written as

$$\frac{y_t}{y_{t+1}} = \frac{\exp(z_t) k_{n,t}^{\alpha_t} h_{1,t}^{1-\alpha_t}}{\exp(z_{t+1}) k_{n,t+1}^{\alpha_{t+1}} h_{1,t+1}^{1-\alpha_{t+1}}}, \quad (\text{B.2})$$

where $k_{n,t} \equiv k_t \exp(-z_t)$ is the normalized capital stock (a stationary variable). Starting from equation (11), the law of motion for the normalized capital stock is given by

$$\begin{aligned} k_{n,t+1} &= \exp(z_t - z_{t+1}) B \exp(v_t) k_{n,t}^{1-\varphi_t} \left[\frac{i_t y_t}{y_t k_t} \right]^{\delta_t} h_{2,t}^{\varphi_t}, \\ &= \exp(z_t - z_{t+1}) B \exp(v_t) k_{n,t}^{1-\varphi_t - \delta_t(1-\alpha_t)} \left[\frac{\delta_t x_t}{\eta_t + \delta_t x_t} A h_{1,t}^{1-\alpha_t} \right]^{\delta_t} h_{2,t}^{\varphi_t}. \end{aligned} \quad (\text{B.3})$$

Equations (33) and (34) can be used to substitute for $h_{1,t}$, $h_{1,t+1}$, and $h_{2,t}$ in equations (B.2) and (B.3). Then, since x_t depends on the equilibrium solution for q_t , equation (31) can be used to make the substitutions $x_t = x(\eta_t, \alpha_t, \delta_t, \varphi_t, s_t)$ and $x_{t+1} = x(\eta_{t+1}, \alpha_{t+1}, \delta_{t+1}, \varphi_{t+1}, s_{t+1})$ in equations (B.1) through (B.3). After these various substitutions, a log-linear approximation of the stochastic discount factor takes the form

$$\begin{aligned} M_{t+1} &\simeq \beta \exp(-\mu) \left[\frac{\eta_t}{\bar{\eta}} \right]^{\text{m}_1} \left[\frac{\alpha_t}{\bar{\alpha}} \right]^{\text{m}_2} \left[\frac{\delta_t}{\bar{\delta}} \right]^{\text{m}_3} \left[\frac{\varphi_t}{\bar{\varphi}} \right]^{\text{m}_4} \left[\frac{k_{n,t}}{\bar{k}_n} \right]^{\text{m}_5} \\ &\quad \times \exp[\text{m}_6 v_t + \text{m}_7 u_t + \text{m}_8 (s_t - \bar{s}) + \text{m}_9 \varepsilon_{\eta,t+1} + \text{m}_{10} \varepsilon_{\alpha,t+1}] \\ &\quad \times \exp[\text{m}_{11} \varepsilon_{\delta,t+1} + \text{m}_{12} \varepsilon_{\varphi,t+1} + \text{m}_{13} \varepsilon_{z,t+1} + \text{m}_{14} \varepsilon_{u,t+1} + \text{m}_{15} \varepsilon_{s,t+1}], \end{aligned} \quad (\text{B.4})$$

where m_1 through m_{15} are Taylor series coefficients and the laws of motions for the shocks have been used to eliminate η_{t+1} , α_{t+1} , δ_{t+1} , φ_{t+1} , z_{t+1} , u_{t+1} , and s_{t+1} . The steady state value of \bar{k}_n is given by $k_t \exp(-\bar{z}_t)$ in 1972.Q3, where \bar{z}_t is the trend value of z_t constructed as $\bar{z}_t = \bar{z}_{t-1} + \mu$ such that μ is the sample mean of Δz_t and $\bar{z}_t = z_t = 0$ in 1972.Q3.

Given equation (B.4), it is straightforward to compute $p_{b,t} = E_t M_{t+1}$ and $r_{b,t} = 1/p_{b,t} - 1$. Since \bar{s} enters negatively in equation (B.4), a more negative value of \bar{s} , implying more

pessimism, serves to increase $E_t M_{t+1}$ and thereby lower the risk free rate of return $r_{b,t}$ via a precautionary savings effect.

The long-term bond pricing equation (22) can be approximated as follows

$$p_{c,t} \simeq E_t M_{t+1} (1 + \bar{\delta}_c \bar{p}_c) \left[\frac{p_{c,t+1}}{\bar{p}_c} \right]^{b_c} \exp(b_c \omega_{t+1}), \quad (\text{B.5})$$

where $b_c = \bar{\delta}_c \bar{p}_c / (1 + \bar{\delta}_c \bar{p}_c)$ is a Taylor series coefficient. A conjectured solution for equation (B.5) takes the form

$$\begin{aligned} p_{c,t} = & \bar{p}_c \left[\frac{\eta_t}{\bar{\eta}} \right]^{n_1} \left[\frac{\alpha_t}{\bar{\alpha}} \right]^{n_2} \left[\frac{\delta_t}{\bar{\delta}} \right]^{n_3} \left[\frac{\varphi_t}{\bar{\varphi}} \right]^{n_4} \left[\frac{k_{n,t}}{\bar{k}_n} \right]^{n_5} \\ & \times \exp [n_6 v_t + n_7 u_t + n_8 (s_t - \bar{s}) + n_9 \omega_t] \end{aligned} \quad (\text{B.6})$$

The conjectured solution (B.6) is iterated ahead one period and then substituted into the long-term bond pricing equation (B.5) together with the expression for M_{t+1} from equation (B.4). Collecting terms and then evaluating the expectation operator yields a set of ten equations in the ten solution coefficients given by \bar{p}_c and n_1 through n_9 . The ergodic mean value \bar{p}_c depends on the coupon decay parameter $\bar{\delta}_c$ and numerous shock variances. I solve for the value of $\bar{\delta}_c$ such that $p_{c,t} = \bar{p}_c = 20$ in 1972.Q3. The target value of \bar{p}_c is arbitrary and has no affect on the model-implied time series for the long-term bond return given by $r_{c,t} = [1 + \bar{\delta}_c \exp(\omega_t) p_{c,t}] / p_{c,t-1} - 1$.

C Appendix: Shock identification procedure

The time series for the factor distribution shock α_t is directly pinned down by U.S. data on capital's share of income. Data for U.S. total hours worked per person h_t are plotted in Figure 2. By equating the right-sides of the two equilibrium conditions (16) and (17), the model-implied time series for $h_{1,t}$ and $h_{2,t}$ are constructed using the following equations

$$h_{1,t} = h_t [1 + (\varphi_t / \delta_t) (i_t / y_t) / (1 - \alpha_t)]^{-1} = h_t \left[\frac{(1 - \alpha_t) y_t}{y_t - d_t - i_t} \right], \quad (\text{C.1})$$

$$h_{2,t} = h_t - h_{1,t} = h_t \left[\frac{\alpha_t y_t - d_t - i_t}{y_t - d_t - i_t} \right], \quad (\text{C.2})$$

where I have made use of $w_t = (1 - \alpha_t) y_t / h_{1,t}$ and $d_t = \alpha_t y_t - (1 + \varphi_t / \delta_t) i_t$. The right-side values of α_t , h_t , y_t , d_t , and i_t in equations (C.1) and (C.2) are given by the U.S. data plotted in Figure 2.

Given the model-implied time series for $h_{1,t}$ and $h_{2,t}$, the times series for the shocks z_t , δ_t ,

φ_t and v_t are uniquely pinned down using the following equations:

$$z_t = \left[\log(y_t) - \log(Ak_t^{\alpha_t} h_{1,t}^{1-\alpha_t}) \right] / (1 - \alpha_t), \quad (\text{C.3})$$

$$\delta_t = i_t / p_{s,t}, \quad (\text{C.4})$$

$$\varphi_t = \delta_t (\alpha_t y_t - d_t - i_t) / i_t \quad (\text{C.5})$$

$$v_t = \log(k_{t+1}/k_t) - \log(B) - \delta_t \log(i_t/k_t) + \varphi_t \log[k_t \exp(-z_t) / h_{2,t}], \quad (\text{C.6})$$

where the right-side values of the macroeconomic variables are given by the U.S. data plotted in Figure 2. If a shock appears on the right side, then it takes on the value identified in a previous equation.³²

Starting from the equilibrium conditions (16), (23), (29), and (21), the time series of realizations for the shocks u_t , s_t , and η_t are determined iteratively by solving the following set of simultaneous equations

$$u_t = \log \{ \eta_t (1 - \alpha_t) / [(c_t / y_t) D h_{1,t} (h_{1,t} + h_{2,t})^\gamma] \}, \quad (\text{C.7})$$

$$s_t = \bar{s} - (1/\rho_s) (\bar{s} + \sigma_{\varepsilon,s}^2/2) + (1/\rho_s) \log \left[\eta_t (p_{s,t}/c_t) / (\beta E_t q_{t+1}^f) \right], \quad (\text{C.8})$$

$$\begin{aligned} \eta_t &= \bar{\eta} \{ p_{b,t} \beta^{-1} \exp(\mu) [\alpha_t / \bar{\alpha}]^{-m_2} [\delta_t / \bar{\delta}]^{-m_3} [\varphi_t / \bar{\varphi}]^{-m_4} [k_{n,t} / \bar{k}_n]^{-m_5} \\ &\quad \times \exp [-m_6 v_t - m_7 u_t - m_8 (s_t - \bar{s}) - m_9 \sigma_{\varepsilon,\eta}^2/2 - m_{10} \sigma_{\varepsilon,\alpha}^2/2 - m_{11} \sigma_{\varepsilon,\delta}^2/2] \\ &\quad \times \exp [-m_{12} \sigma_{\varepsilon,\varphi}^2/2 - m_{13} \sigma_{\varepsilon,z}^2/2 - m_{14} \sigma_{\varepsilon,u}^2/2 - m_{15} \sigma_{\varepsilon,s}^2/2] \}^{1/m_1}, \end{aligned} \quad (\text{C.9})$$

where $p_{b,t}$ is the inverse of the U.S. gross risk free rate of return in quarter t . The fundamentals-only forecast $E_t q_{t+1}^f$ that appears in equation (C.8) is computed using the fundamental solution (25), as shown in equation (29). The fundamentals-only forecast depends on the shocks η_t , α_t , δ_t , and φ_t . Various parameters and shock variances that appear in equations (C.8) and (C.9) are initially undetermined, but influence the computed time series for u_t , s_t , and η_t . These parameters and shock variances include \bar{s} , β , ρ_s , $\sigma_{\varepsilon,s}$, $\sigma_{\varepsilon,\eta}$, and $\sigma_{\varepsilon,u}$. Starting from initial guesses for the various parameters and shock variances, together with initial guesses for the time series of u_t , s_t , and η_t , equations (C.7) through (C.9) are iterated until convergence is achieved. After each iteration, new guesses for the time series of u_t , s_t and η_t are computed as an exponentially-weighted moving average of the current and past values implied by equations (C.7) through (C.9). In practice, convergence to 8 decimal points takes around 70 iterations.

To identify the bond coupon decay rate shock ω_t , I first solve the equilibrium bond price solution (B.6) for $\exp(\omega_t)$, yielding

$$\begin{aligned} \exp(\omega_t) &= \{ p_{c,t} / \bar{p}_c [\eta_t / \bar{\eta}]^{-n_1} [\alpha_t / \bar{\alpha}]^{-n_2} [\delta_t / \bar{\delta}]^{-n_3} [\varphi_t / \bar{\varphi}]^{-n_4} [k_{n,t} / \bar{k}_n]^{-n_5} \\ &\quad \times \exp [-n_6 v_t - n_7 u_t - n_8 (s_t - \bar{s})] \}^{1/n_9}, \end{aligned} \quad (\text{C.10})$$

³²Since the computation of v_t requires data at time $t+1$, I set the end-of-sample shock value to $v_T = \rho_v v_{T-1}$, where $T = 2019.Q4$.

where $p_{c,t} = \bar{p}_c$ in 1972.Q3 such that $\omega_t = 0$. Next, I substitute equation (C.10) into the bond return definition and then solve for $p_{c,t}$, yielding

$$p_{c,t} = \left\{ \bar{p}_c [p_{c,t-1}(1 + r_{c,t})/\bar{\delta}_c - 1/\bar{\delta}_c]^{n_9} [\eta_t/\bar{\eta}]^{n_1} [\alpha_t/\bar{\alpha}]^{n_2} [\delta_t/\bar{\delta}]^{n_3} [\varphi_t/\bar{\varphi}]^{n_4} [k_{n,t}/\bar{k}_n]^{n_5} \right. \\ \left. \times \exp [n_6 v_t + n_7 u_t + n_8 (s_t - \bar{s})] \right\}^{1/(1+n_9)}, \quad (\text{C.11})$$

where $1 + r_{c,t}$ is the U.S. gross real bond return in quarter t . Given the time series for the previously-identified shocks, equation (C.11) is used to construct the equilibrium time series for $p_{c,t}$ for $t > 1972.Q3$, starting with $p_{c,t-1} = \bar{p}_c$. For $t < 1972.Q3$, equation (C.11) is inverted to solve for $p_{c,t-1}$ as a function of $p_{c,t}$ and the previously-identified shocks. Given the equilibrium time series for $p_{c,t}$ from 1960.Q1 to 2019.Q4, equation (C.10) is used to recover the model-implied time series for $\exp(\omega_t)$. The stochastic coupon decay rate is given by $\delta_{c,t} \equiv \bar{\delta}_c \exp(\omega_t)$.

D Appendix: Data sources and methods

I start with data on nominal personal consumption expenditures on nondurable goods plus services (C_t), nominal private nonresidential fixed investment plus nominal personal consumption expenditures on durable goods (I_t), the corresponding price indices for each of the various nominal expenditure categories that sum to C_t and I_t , and U.S. population. All of this data are from the Federal Reserve Bank of St. Louis' FRED database. I define the nominal ratios C_t/Y_t and I_t/Y_t , where $Y_t \equiv C_t + I_t$. The nominal ratios capture shifts in relative prices. I deflate Y_t by an output price index constructed as the weighted-average of the price indices for the various nominal expenditure categories that sum to C_t and I_t , where the weights are the nominal expenditure ratios relative to Y_t . After dividing by U.S. population, the level of real output per person y_t is normalized to 1.0 in 1972.Q3. The real per person series for c_t and i_t are then constructed by applying the nominal ratios C_t/Y_t and I_t/Y_t to the constructed y_t series. In this way, the real per person series for c_t and i_t reflect the same resource allocation ratios as the nominal per person series.

Data for h_t are hours worked of all persons in the nonfarm business sector from FRED, divided by U.S. population and then normalized to equal 0.3 in 1972.Q3.³³

The data for k_t are constructed using the historical-cost net stock of private nonresidential fixed assets plus the historical-cost net stock of consumer durable goods, both in billions of dollars at year end, from the Bureau of Economic Analysis (BEA), NIPA Table 4.3, line 1 and Table 8.3, line 1, respectively. The data are only available at annual frequency, so I first create a quarterly series by log-linear interpolation. The nominal capital stock series is deflated using the output price index described above and then divided by U.S. population. I normalize the real per person series for k_t to deliver a target value of $i_t/k_t = 0.0311$ in 1972.Q3. The target value is arbitrary given that the model parameters B and $\bar{\delta}$ can be adjusted to hit any desired target value. I choose the target value of i_t/k_t to coincide with the steady state value

³³The hours data are from <https://fred.stlouisfed.org/series/HOANBS>.

implied by a model with no capital adjustment costs, such that $i_t/k_t = k_{t+1}/k_t - 1 + \delta'$, where $\delta' = 0.025$ is a typical quarterly depreciation rate for physical capital. For the normalization, I employ the mean quarterly growth rate of the real capital stock per person series which implies $k_{t+1}/k_t = \exp(0.0061)$ in steady state. I calibrate the value of A in the production function (6) to yield $y_t = 1$ in 1972.Q3 when k_t is equal to the normalized capital stock and $z_t = 0$ in 1972.Q3. This procedure delivers a sample mean of $k_t/y_t = 9.159$ from 1960.Q1 to 2019.Q4.

Following Lansing (2015) and Lansing and Markiewicz (2018), capital's share of income is measured as 1 minus the ratio of employee compensation to gross value added of the corporate business sector. Both series are from the BEA, NIPA Table 1.14, lines 1 and 4.

To construct data for $p_{s,t}$, I start with the nominal market capitalization of the S&P 500 stock index from www.siblisresearch.com. The nominal market capitalization is deflated using the output price index described above and then divided by U.S. population to create a series for real equity value per person.

Quarterly data on the nominal end-of-quarter closing value of the S&P 500 stock index, nominal dividends, the nominal risk free rate of return (based on a 3-month Treasury bill), and the nominal return on a long-term Treasury bond (based on a maturity of 20 years) are from Welch and Goyal (2008).³⁴ The gross nominal return on the S&P 500 stock index in quarter t is defined as $(P_t + D_t/4)/P_{t-1}$, where P_t is the end-of-quarter closing value of the index and D_t is cumulative nominal dividends over the past 4 quarters. Gross nominal asset returns are converted to gross real returns by dividing by $1 + \pi_t$ where π_t is the quarterly inflation rate computed using the output price index described above. Given the gross real equity return $1 + r_{s,t}$ and the constructed data for real equity value per person $p_{s,t}$, I compute a consistent series for real dividends per person as $d_t = (1 + r_{s,t})p_{s,t} - p_{s,t-1}$. The gross nominal returns on the 3-month Treasury bill and the long-term Treasury bond are similarly divided by $1 + \pi_t$ to obtain the gross real bond returns $1 + r_{b,t}$ and $1 + r_{c,t}$. The University of Michigan consumer sentiment index plotted in Figure 1 is from www.sca.isr.umich.edu/tables.html.

³⁴Updated data through the end of 2019 are available from Amit Goyal's website: www.hec.unil.ch/agoyal/.

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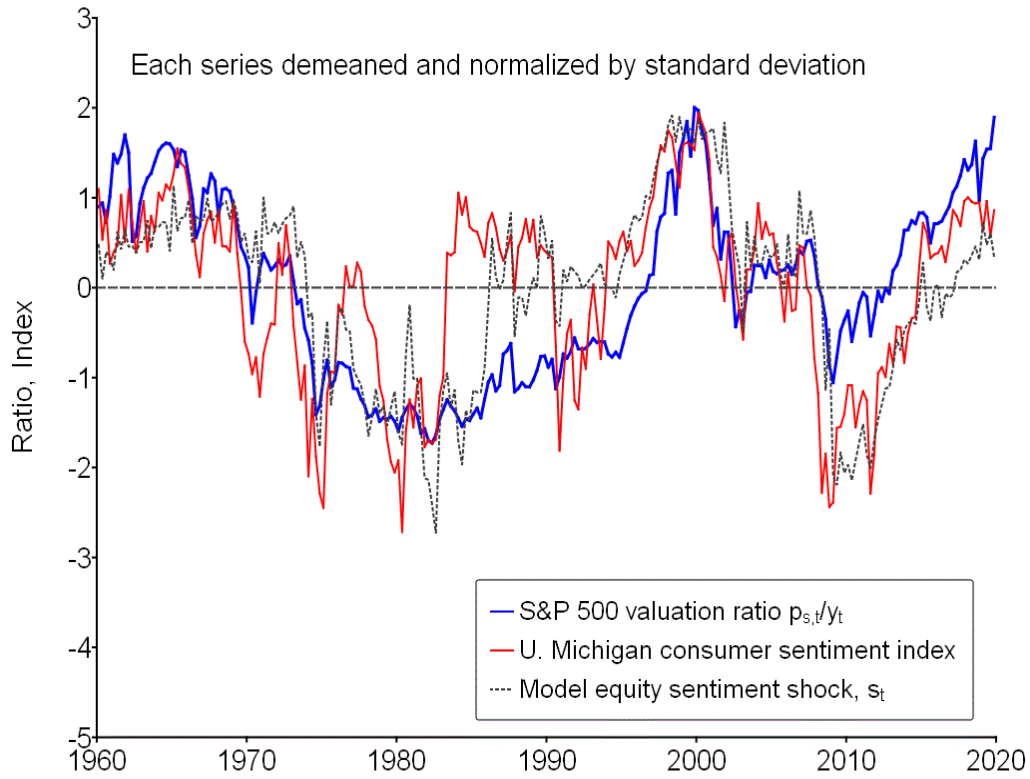
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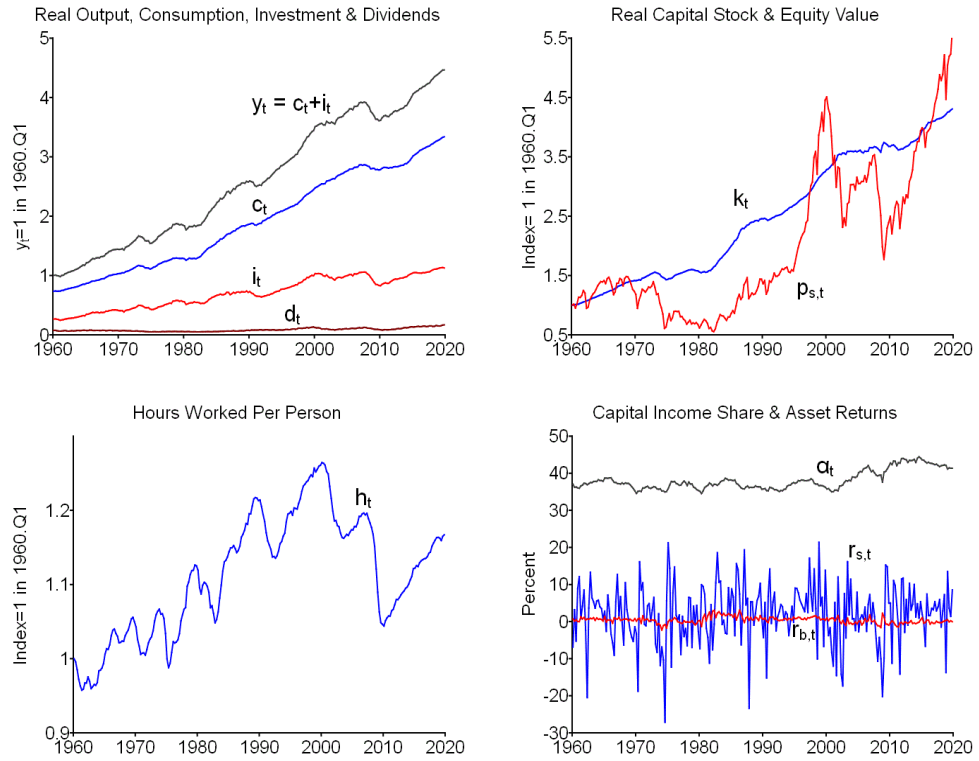
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Figure 1: *Measures of equity value and sentiment*



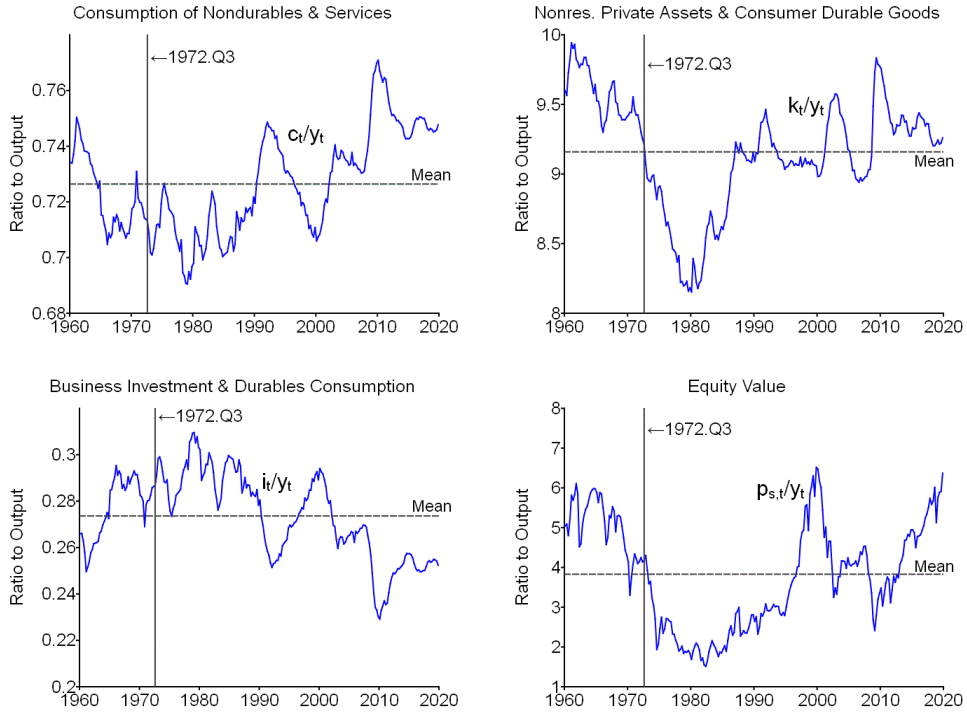
Notes: The model-identified sentiment shock is strongly correlated with the University of Michigan's consumer sentiment index. The correlation coefficient between the two series is 0.68. Both series, in turn, are strongly correlated with a stock market valuation ratio defined as the nominal market capitalization of the S&P 500 stock index divided by a measure of nominal output. The correlation coefficient between the sentiment shock and the S&P 500 valuation ratio is 0.70. Data series are described in Appendix D.

Figure 2: *U.S. macroeconomic variables and asset returns*



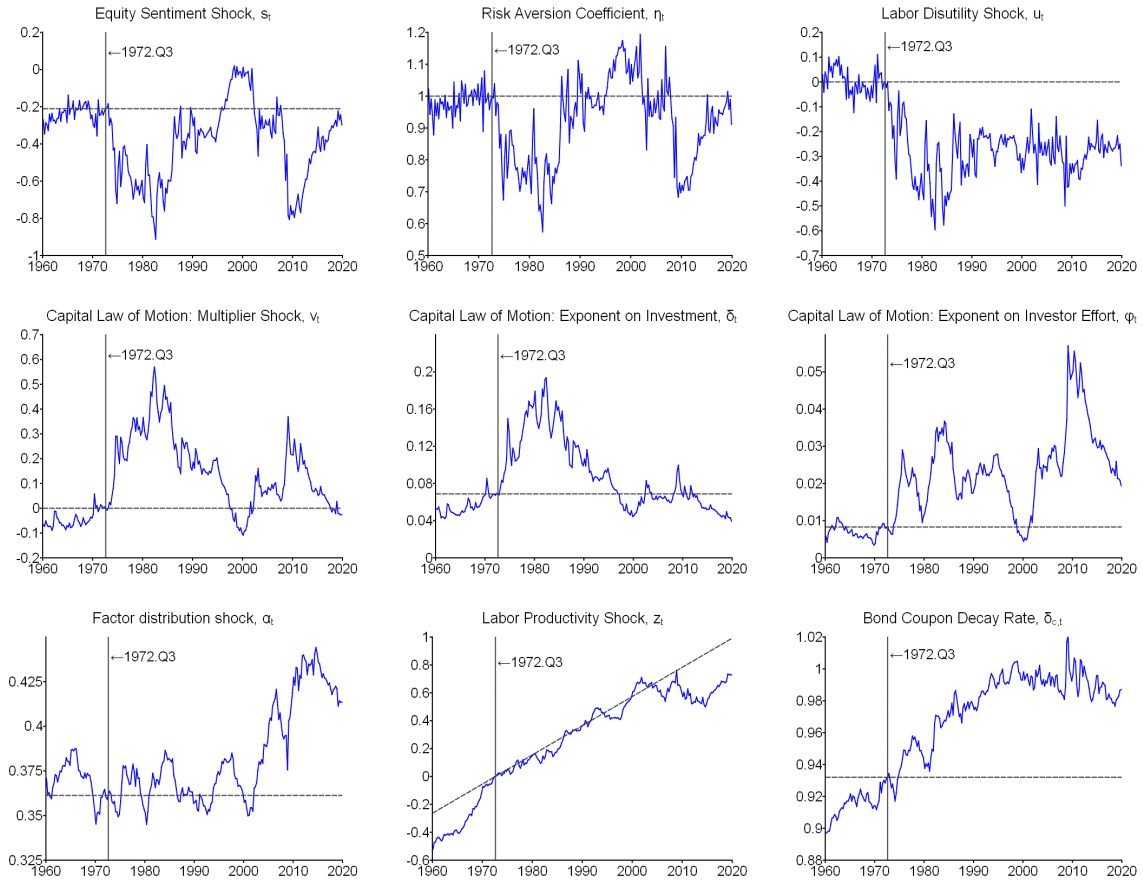
Notes: The baseline simulation exactly replicates the observed U.S. time paths of all ten variables plotted above plus the real return on a long-term U.S. government bond from 1960 to 2019. Data series are described in Appendix D.

Figure 3: *U.S. macroeconomic ratios*



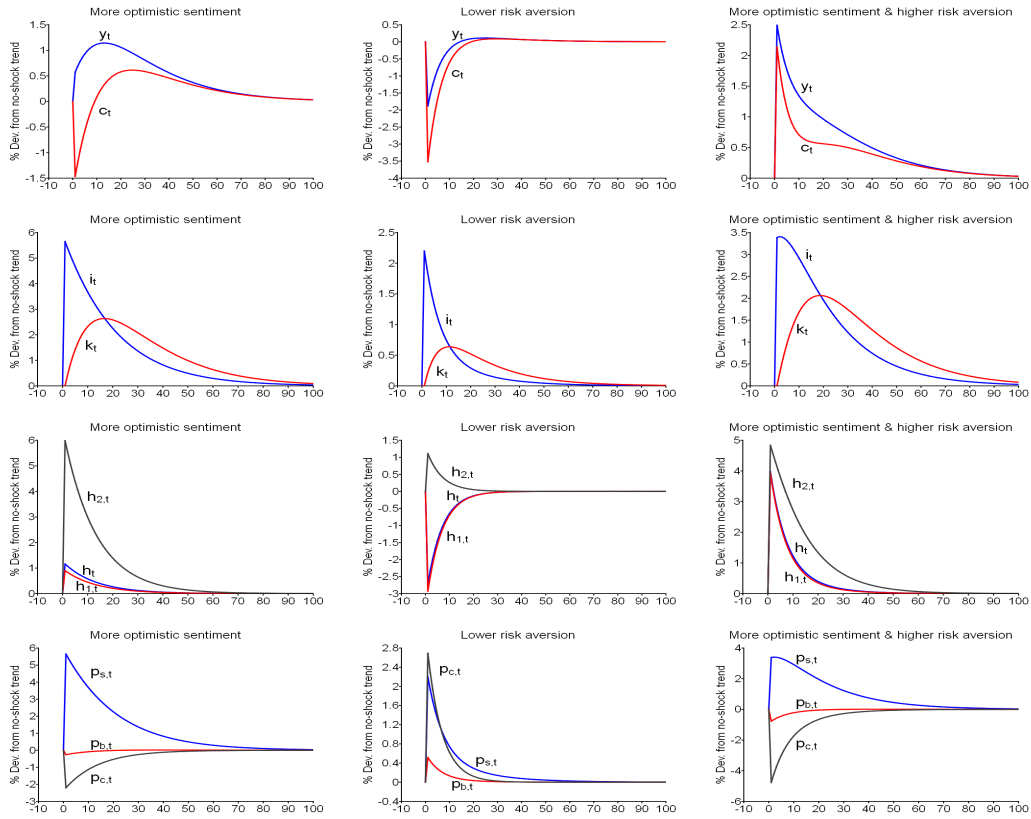
Notes: Parameter values are chosen so that the steady-state, trend, or ergodic mean values of model variables correspond to the values observed in the data in 1972.Q3, a period when the ratios of U.S. macroeconomic variables to output are close to their long-run means.

Figure 4: *Model-identified shocks*



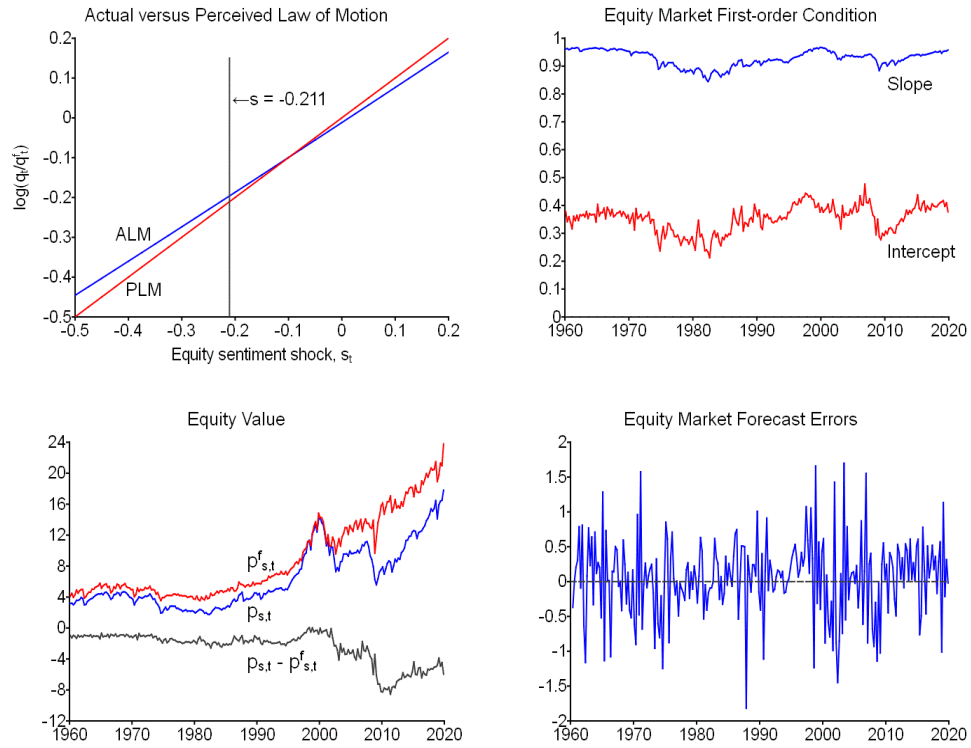
Notes: The figure plots the nine model-identified shock series. By construction, all shocks are equal to their steady state or trend values in 1972.Q3. The equity sentiment shock s_t reaches its maximum value in 1998.Q2 and is positively correlated with the time-varying risk aversion coefficient η_t and the labor disutility shock u_t . The three capital law of motion shocks v_t , δ_t , and φ_t are positively correlated with each other. The first group of shocks is negatively correlated with the second group. Movements in the factor distribution shock α_t are negatively correlated with movements in the labor productivity shock z_t .

Figure 5: *Impulse response functions*



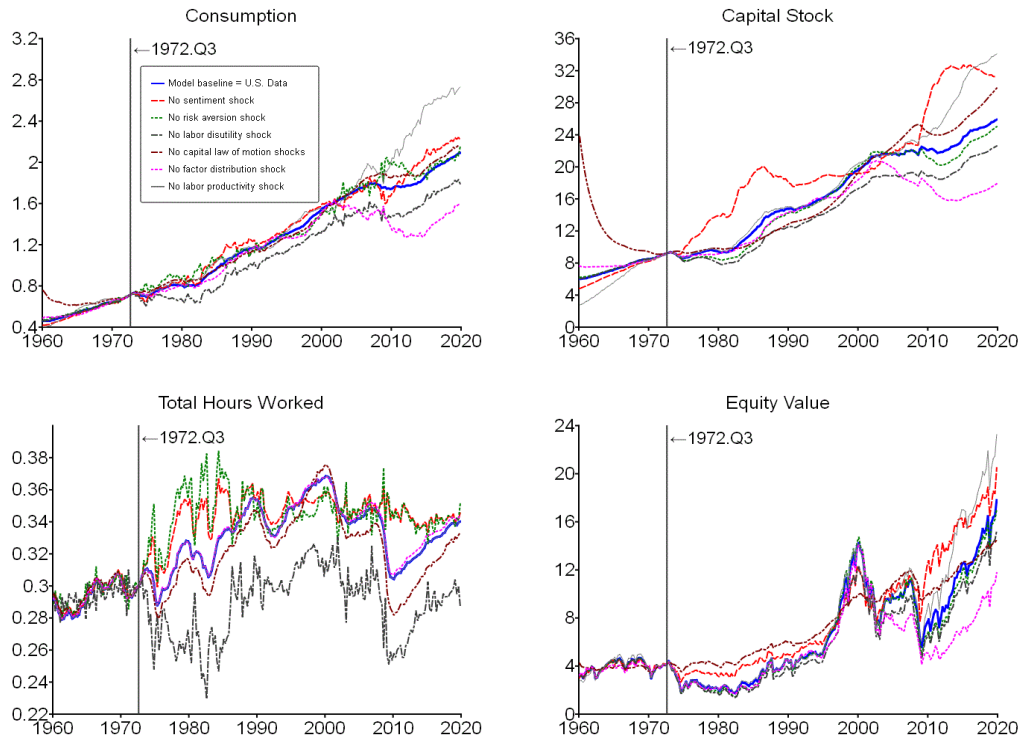
Notes: The left column panels show the effects of a positive innovation to the equity sentiment shock. The middle column panels show the effects of a negative innovation to the risk aversion coefficient. In the data, the sentiment shock exhibits a strong positive correlation with the risk aversion coefficient. The right column panels show the effects of simultaneous positive innovations to both shocks. More optimistic sentiment together with higher risk aversion leads to a correlated increase in all macroeconomic variables. Equity value increases but bond prices decline (implying an increase in bond yields). Taken together, the combination of these two highly correlated shocks allows the model to capture the features observed during a typical economic boom or recovery.

Figure 6: *Equity market variables*



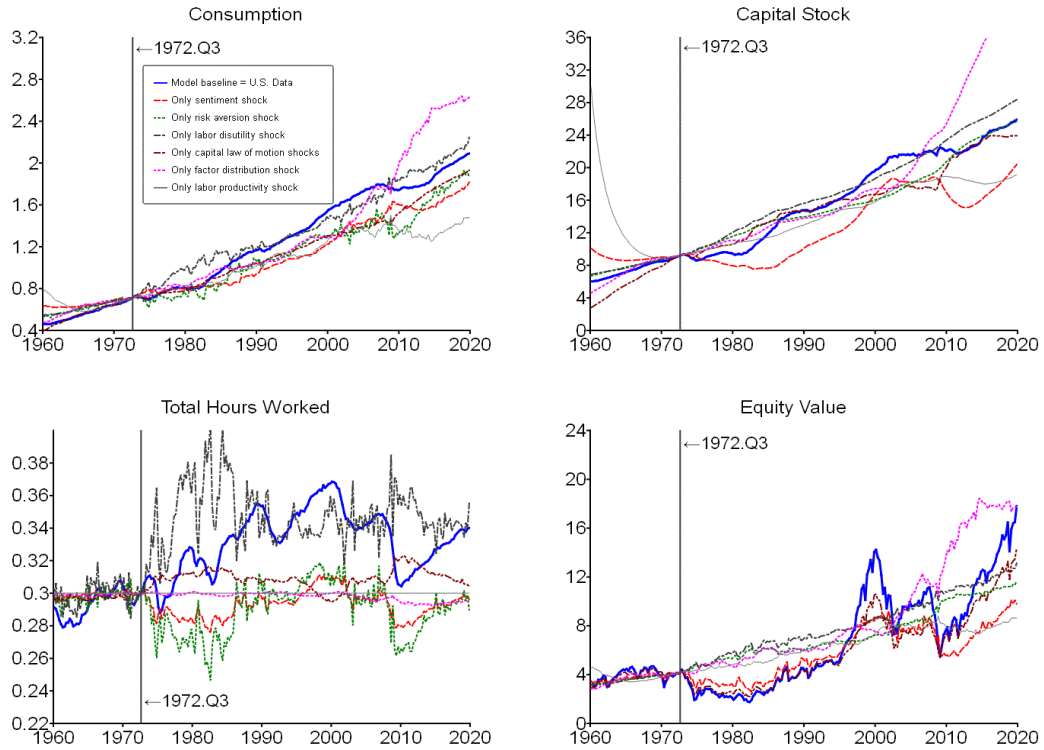
Notes: The representative agent's perceived law of motion (27) predicts values for the quantity $\log(q_t/q_t^f)$ that are very close to those generated by the actual law of motion. This is because the slope of the equity market first order condition (24) is always close to 1. Consequently, the agent's perception that equity value is partly driven by sentiment is close to self-fulfilling. The steady state value of the equity sentiment shock is $\bar{s} = -0.211$, implying pessimism relative to fundamental equity value. Consequently, the sentiment-driven component of equity value, given by $p_{s,t} - p_{s,t}^f$, fluctuates almost entirely in negative territory except for two quarters during 1998. The agent's subjective forecast error $q_t - \hat{E}_{t-1}q_t$ exhibits an autocorrelation coefficient of -0.18 and a mean value of 0.05 .

Figure 7: *Counterfactual shock scenarios*



Notes: The panels show the effects of omitting one or more shock realizations relative to the baseline model that includes all shock realizations. A large gap between the counterfactual path and the U.S. data path (blue line) implies that the omitted shock plays an important role in allowing the model variable to replicate the path of the corresponding U.S. data variable.

Figure 8: *Individual shock scenarios*



Notes: The panels show the effects of adding one or more shock realizations when no other shock realizations are present. A small gap between the shock-induced path and the U.S. data path (blue line) implies that the added shock(s) plays an important role in allowing the model variable to replicate the path of the corresponding U.S. data variable.