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Abstract

We study the conditions under which fiscal foresight – forward-looking agents anticipating future policy changes – distorts economic behavior through undesired intertemporal tradeoffs. Somewhat surprisingly, fiscal foresight is far from sufficient for policy and incentives to perversely affect firm behavior. Three necessary conditions are identified for distorting behavior: storable output, diminishing returns, and a non-competitive output market. These conditions suggest that the estimated impacts of fiscal policies may be sensitive to underlying economic characteristics and that policies targeted to specific firms or industries with unique characteristics may not be generalizable.

Keywords: Fiscal foresight, intertemporal tradeoffs, real distortions, fiscal policy

JEL codes: E62 (fiscal policy), H20 (taxation, subsidies, and revenue: general)

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Fiscal Foresight And Real Distortions To Firm Behavior: Anticipatory Dips And Compensating Rebounds

I. Introduction

The vast majority of fiscal policies are anticipated before they go into effect. Anticipation arises for two reasons: a lag between when the policy is formally adopted and when it is implemented (implementation lag) and a lag between when the policy is discussed, deliberated, and amended and formally adopted (preview lag). Fiscal foresight occurs when forward-looking agents anticipate a future policy change. The quantitative importance of fiscal foresight is a key policy question and has been the subject of much previous empirical research and debate that has not yielded a consensus.[1]

This paper examines the theoretical underpinnings of fiscal foresight in the context of business incentives. The policy stimulates economic activity when implemented according to standard channels. Here we are interested in the effects of fiscal foresight that arise before implementation and distort firm behavior through perverse intertemporal tradeoffs. Somewhat surprisingly, fiscal foresight and anticipation of the implementation date do not necessarily impact real variables such as factor demands, output, and sales. Moreover, when there is an impact, it usually does not result in distortions during the implementation period. Necessary conditions for fiscal foresight to distort firm behavior through intertemporal tradeoffs are derived: storable output, diminishing returns, and a non-competitive output market.[2] If any of these conditions are absent, fiscal foresight will not impact real activity. These conditions suggest that the estimated impacts of fiscal policies may be sensitive to underlying economic characteristics and that policies targeted to specific firms or industries with unique characteristics may not be generalizable.

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[1] Several empirical studies of fiscal multipliers distinguish between anticipated and unanticipated changes in aggregate fiscal policy, and the results are mixed. Auerbach and Gale (2009), Caggiano, Castelnuovo, Colombo, and Nodari (2015), Chirinko and Wilson (2021), Kriwoluzky (2012), Leduc and Wilson (2013), Leeper, Richter, and Walker (2012), Mertens and Ravn (2012), Ramey (2011), and Yang (2006), among others, report a significant role for anticipated shocks in terms of their co-movements with macroeconomic activity, pointing to the importance of fiscal foresight. By contrast, the VAR analyses of fiscal shocks by Blanchard and Perotti (2002) and Perotti (2012) and the single-equation narrative analysis by Romer and Romer (2010) do not support the quantitative importance of fiscal foresight.

[2] A non-competitive output market includes any market structure in which the firm does not take prices as given, such as monopoly, Bertrand, Cournot, and Stackelberg competition.

Our theoretical framework applies to any setting in which there is an anticipated change in a tax policy that differentially affects firm demand for a factor of production. We will call this factor “labor”, consider the tax policy a “credit” (i.e., a reduction in an existing tax or an introduction of a subsidy or a negative tax) on labor, and refer to this policy as a delayed tax credit (DTC).

The dynamic optimization problem is specified in Section II, and the forward-looking firm chooses labor, sales, and inventories to maximize discounted profits. The labor choice determines output, the sales choice determines the output price, and inventories are subject to an isoperimetric constraint so that its beginning and ending values are equal. The associated first-order conditions and steady-state values are derived in Section III. The adoption of a DTC is then analyzed in three perfect foresight models of increasing generality in terms of the responses of these real variables away from the steady-state.

Section IV evaluates a simple DTC in a model where there are no costs to inventory imbalances. This section derives the essential results of the paper -- the three necessary conditions for fiscal foresight to distort economic activity. Perverse behaviors are manifested in an anticipatory dip (AD) in labor demand before the tax credit policy is implemented and a compensating rebound (CR) afterwards.[3] An inventory technology provides the means to shift production across periods. Decreasing returns and a non-competitive output market provide the motivation to smooth real activity across periods. All three elements are needed to provide the firm the means and motivation to shift employment, output, and sales intertemporally and generate ADs and CRs.

The next two sections generalize the model to account for two common features in real-world business tax credits. Credits are frequently complicated by a rolling base. For example, job creation tax credits often are based on employment above a rolling average of recent employment (or simply last year’s employment) and R&D tax credits often are a function of the current R&D to sales ratio relative to its recent rolling average. Section V adds the rolling base feature to the optimization problem and analyzes its interesting dynamics. The firm has a target

[3] Note that the distorting behavior is symmetric to the case of a tax increase instead of a tax cut or credit. That is, a delayed tax increase on labor yields an anticipatory spike in labor demand prior to implementation followed by a compensating decline after implementation.

inventory/sales ratio, and Section VI imposes a cost when the actual inventory/sales ratio differs from the target. The key qualitative results remain robust to these extensions, though the quantitative effects change. Section VII discusses our results relative to prior theoretical models, and Section VIII offers a brief summary.

II. Optimization Problem

Cash flow in period t is composed of four elements. First, revenues (REV_t) accrue to the firm from sales (S_t) in a market where the firm may have market power ($P_t = P[S_t], P'[S_t] \leq 0$). The demand curve is linear with slope ($-\beta / 2$) and a constant term equal to ($1+\beta$). The linearity assumption is made for convenience; the parametric restriction as a simple device for assuring that, in the steady-state (SS), the firm faces an elastic demand curve for any positive value of β ,

$$REV\left[S_t\right] \equiv P_t * S_t = \alpha * S_t - (\beta / 2) * S_t^2 , \tag{1a}$$

$$\alpha \equiv 1 + \beta , \tag{1b}$$

$$\left| \frac{dS_t}{dP_t} \frac{P_t}{S_t} \right|_{SS} = (1 + 2 / \beta) > 1 \qquad 0 < \beta < \infty , \tag{1c}$$

where we assume in equation (1c) that the steady-state value of S_t equals one (an assumption verified in Section III).

Second, labor is the only factor of production, and production cost ($COST_t$) is the product of an exogenous wage (w) and labor input (L_t),

$$COST\left[L_t\right] \equiv w * L_t \ . \tag{2}$$

Third, the firm smooths production intertemporally by adjusting the end-of-the-period inventory stock (I_t). The firm has an exogenous target inventory-to-sales ratio (ζ). Deviations from this target result in the following quadratic cost,

$$f[I_{t-1}, S_t] \equiv (\mu / 2) * \left(I_{t-1} - \zeta * S_t\right)^2 \qquad \mu \geq 0 . \tag{3}$$

Such a cost is standard in the inventory literature (cf., Ramey and West, 1999, equation 3.1) and represents inventory holding and stock-out costs. If $\zeta = 0$, $f[.]$ is linear, and $(\mu / 2)$ equals the cost of borrowed funds, then equation (3) would represent the carrying cost of inventory.

Fourth, the firm receives a tax credit equal to the product of the legislated tax credit rate (τ_t), the wage rate, and the level of credit-qualifying employment. Because the previous period is not a fixed interval at a point in time but rather a window that moves forward in time with employment, this type of credit is known as a "rolling base" credit. The rolling base feature of these credits has important implications on the incentives from and the costs of tax credit programs and are frequently adopted. These implications are examined in Section V below. Here we assume a rolling base ($BASE_t$) and the tax credit received by the firm is defined as follows,[4]

$$g[L_t, L_{t-1} : \tau_t] \equiv \tau_t * w * (L_t - BASE_t), \tag{4a}$$

$$BASE_t \equiv L_{t-1}. \tag{4b}$$

The tax credit rate is noted explicitly in equation (4a) as a conditioning variable given its central role in the subsequent analysis. An implicit assumption in the theoretical model is that the firm has positive taxable profits and/or that the credit is fully refundable (i.e., the firm does not need taxable profits to receive the credit).

In maximizing cash flow qua profits over the planning period, the firm faces production function, inventory accumulation, and isoperimetric constraints. The production function depends only on labor,[5]

$$Q_t = L_t^{(1/\gamma)}, \tag{5}$$

where $\gamma > 1$ so that returns to labor are decreasing. The latter property will be required for satisfying the second-order conditions and the uniqueness of the steady-state. The

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[4] An alternative to a rolling base is a fixed base defined, for example, as an average of employment in a fixed interval, $BASE_t \equiv (L_s + L_{s-1}) / 2$, $s < t$ and s fixed.

[5] This formulation of the production function is consistent with a constant returns-to-scale production function with labor and a fixed factor as arguments, where the latter is normalized to one and fixed during the length of the period over which we evaluate the impact of the DTC.

end-of-period inventory stock is accumulated according to the following recursive equation,

$$I_t = Q_t - S_t + I_{t-1}. \tag{6}$$

Equation (6) will be appended to the optimization problem with a time-varying shadow price, θ_t. The final constraint concerns the inventory stock at the end of the planning period. The firm begins the planning period with an inventory stock, I_0. If left unconstrained, the firm will end the planning period at time T with the inventory stock completely depleted, and some of its profit will be illusory. To avoid this extreme inventory drawdown that would distort profits and employment decisions, we require that $I_T = I_0$, which, after repeated substitution with equation (6), is equivalent to the following isoperimetric constraint,

$$I_T - I_0 = 0 = \sum_{t=1}^{T} \left(Q_t - S_t \right). \tag{7}$$

Equation (7) is a weaker constraint than the special case of assuming the firm starts with zero inventory because, in this special case, the firm might not have the possibility of allowing sales to exceed output in the early periods. This constraint will be appended to the optimization problem with a time-invariant shadow price, ϕ. This constant shadow price of output plays a critical role in fiscal foresight distorting the intertemporal allocation of labor, output, and sales for a firm facing a delayed tax credit and hence in the emergence of an AD and CR.

Combining the four relations defining cash flow (CF_t), discounting CF_t by a constant discount factor (R^t depending on a constant discount rate ρ), assuming that cash flows accrue at the end of the period, substituting L_t for Q_t with equation (5), and appending the two constraints, we write the dynamic optimization problem as follows,

$$\Pi_0 = \underset{\{L_t, S_t, I_t\}}{\text{Max}} \sum_{t=1}^{T} R^t \left\{ CF\left[L_t, S_t, I_{t-1}, L_{t-1} : \tau_t\right] + \theta_t \left(I_t - L_t^{(1/\gamma)} + S_t - I_{t-1} \right) + \phi \sum_{t=1}^{T} \left(L_t^{(1/\gamma)} - S_t \right) \right\}, \tag{8a}$$

$$R^t \equiv (1+\rho)^{-t} \qquad \rho > 0, \tag{8b}$$

$$CF\left[L_t, S_t, I_{t-1}, L_{t-1} : \tau_t\right] \equiv \left(REV\left[S_t\right] - COST\left[L_t\right] - f[I_{t-1}, S_t] + g[L_t, L_{t-1} : \tau_t] \right). \tag{8c}$$

III. First Order Conditions And The Steady-State

The firm maximizes discounted cash flows by appropriate choices of labor, sales, and the inventory stock. Given the inventory accumulation constraint, the latter variable is predetermined by the choices of labor and sales, and it could be eliminated from equation (8) with equation (6). We include I_t explicitly in equation (8) to facilitate the interpretation of the first-order conditions and to define θ_t. We begin with the perturbation of equation (8) with respect to I_t,

$$I_t: \quad -R*\mu\left(I_t-\zeta*S_{t+1}\right)+\theta_t-R*\theta_{t+1}=0, \tag{9a}$$

$$\rightarrow$$

$$\theta_t=\sum_{s=0}^{T} R^{t-s-1}\ \mu\left(I_{t+s}-\zeta*S_{t+s+1}\right). \tag{9b}$$

Equation (9a) is a first-order difference equation in θ_t.[6] It can be solved by recursive substitution for θ_{t+s} and by imposing the terminal condition that θ_T equals zero (discussed below). This solution is presented in equation (9b) and defines θ_t as the shadow price of adding a unit of inventory in period t and keeping that unit in inventory until period T. If in period t, the inventory stock exceeds its target level $\left(I_{t+s}-\zeta*S_{t+s+1}>0\right)$, an addition to inventory aggravates the imbalance, is costly to the firm, and $\theta_t>0$.[7] This imbalance is monetized by μ and is discounted by R. If in period t, the inventory stock is below its target level, then the additional unit is beneficial to the firm, the incremental cost is negative, and $\theta_t<0$. In the steady-state, the inventory stock equals its target level, the inventory imbalance is zero, and $\theta_t=0$.

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[6] If the cash flow term defining the inventory costs (equation (3)) had also included an inventory carrying cost ($c*I_{t-1}$), this additional cost term would have merely redefined θ_t.

[7] We assume that the inventory imbalance is reduced monotonically to zero. Given the quadratic specification, inventory imbalances of either sign are penalized, and it would be unnecessarily costly for the firm to overshoot the steady-state value.

The key decisions made by the firm concern labor and sales. The first-order condition for labor is as follows,

$$L_t:\ w*\left(1-\tau_t+\tau_{t+1}*R\right)+\theta_t*\left(L_t^{((1-\gamma)/\gamma)}/\gamma\right)=\phi*\left(L_t^{((1-\gamma)/\gamma)}/\gamma\right), \tag{10a}$$

$$\rightarrow$$

$$MPL\left[L_t\right]=w_t^{EFF-N}\big/\left(\phi-\theta_t\right)\equiv w_t^{EFF-R} \qquad \begin{aligned} w_t^{EFF-N} &\equiv w*\left(1-\tau_t+\tau_{t+1}*R\right) \\ w_t^{EFF-R} &\equiv w_t^{EFF-N}\big/\left(\phi-\theta_t\right) \\ MPL\left[L_t\right] &\equiv L_t^{((1-\gamma)/\gamma)}/\gamma \quad , \end{aligned} \tag{10b}$$

$$\rightarrow$$

$$L_t=\left(\gamma*w_t^{EFF-R}\right)^{(\gamma/(1-\gamma))}, \qquad (\gamma/(1-\gamma))<0 \tag{10c}$$

$$Q_t=\left(\gamma*w_t^{EFF-R}\right)^{(1/(1-\gamma))}. \qquad (1/(1-\gamma))<0 \tag{10d}$$

The two terms on the left side of equation (10a) define the total cost from hiring an incremental worker (and hence producing incremental output). The first term reflects labor costs represented by the effective nominal wage rate, w_t^{EFF-N}, which is equal to the sum of the cost of hiring labor (w), less the tax credit ($-w*\tau_t$) received in period t and, owing to the rolling base feature of the tax credit, plus the tax credit that will not be received in period t+1 ($w*\tau_{t+1}*R$) (discussed in more detail in Section V). Taken together, these latter two terms form the effective tax credit rate that differs markedly from the legislated tax credit rate, τ. The second term is the cost of adding to an inventory imbalance. If θ_t is positive due to a positive inventory imbalance, incremental output from a new hire increases the imbalance and is costly to the firm. These two incremental costs equal the benefit from an additional hire, represented by the term on the right side of equation (10a). This term is the constant shadow price of output, ϕ, multiplied by the marginal product of labor.

These relations are rearranged into a more concise expression in equation (10b), which shows that labor is optimally chosen so that its marginal product is set equal to the effective real wage rate (w_t^{EFF-R}). This factor price equals the effective nominal wage rate (w_t^{EFF-N}) "deflated" by the true price of output, which is its shadow price (ϕ) net of any cost due to an inventory imbalance (θ_t). Equation (10c) is a rearrangement of equation (10b) and relates L_t to the production function parameter, shadow prices, and the effective real wage rate. (In the derivations below, it proves more convenient to return to the notation with the effective nominal wage rate.) Equation (10d) is the corresponding expression for Q_t. Note that the object $(\phi - \theta_t)$ appearing in the numerators is always positive given the wage rate term in equation (10a).

The second key choice by the firm concerns sales determined by the following first-order condition,

$$S_t: \quad (\alpha - \beta * S_t) + \mu * \zeta (I_{t-1} - \zeta * S_t) + \theta_t = \phi, \tag{11a}$$

$\rightarrow$

$$S_t = \frac{\alpha + \mu * \zeta * I_{t-1} + \theta_t - \phi}{\beta + \mu * \zeta^2} \ . \tag{11b}$$

Equation (11a) is a perturbation of equations (8) that impacts cash flow in three ways. The first term in equation (11a) is the marginal revenue, which decreases in the level of sales because of the non-competitive output market. The second term reflects the cash flow from a change in the target and depends on the sign of the inventory imbalance. An increase in sales (and hence an increase in the target level of inventory) reduces a positive imbalance and adds to cash flow. The impact is negative when the inventory imbalance is negative. This effect disappears if the target level is zero ($\zeta = 0$). The third term is the shadow price of inventory imbalances. The shadow price's impact on an incremental sale is opposite to its impact on labor because Q_t (dependent on L_t) and S_t have opposite but numerically identical effects on the inventory stock. These three terms define the total cash flow from an incremental sale and, under profit-maximization, equal the constant shadow price of output, ϕ. If the second and third terms are zero, then ϕ equals the price of output ($\alpha - \beta * S_t$). Equation (11b) is a rearrangement of equation (11a) that relates S_t to demand curve parameters, the predetermined lagged inventory stock, and shadow prices.

Lastly, perturbations of the shadow prices yield the per-period inventory accumulation constraint and the planning-period isoperimetric constraint, respectively,

$$\theta_t: \quad I_t = L_t^{(1/\gamma)} - S_t + I_{t-1}, \tag{12}$$

$$\phi: \quad \sum_{t=1}^{T}\left(L_t^{(1/\gamma)} - S_t\right) = 0, \tag{13}$$

These first-order conditions form the basis of our analysis of the steady-state and the response to a DTC. We analyze a steady-state defined by four characteristics:

1) the inventory stock equals its target value ($I_{SS} = \zeta * S_{SS}$), (14a)
2) tax credits are absent (hence $w_{SS}^{EFF-N} = w$), (14b)
3) sales equals output ($S_{SS} = Q_{SS}$), (14c)
4) the wage rate is normalized ($w = (1/\gamma) < 1$). (14d)

The first characteristic implies that $\theta_{SS} = 0$ and, combined with the second characteristic, that labor, output, and sales given by equations (10c), (10d) and (11b), respectively, can be written as follows,

$$L_{SS} = \left(\frac{\phi_{SS}}{\gamma * w}\right)^{(\gamma/(\gamma-1))}, \tag{15a}$$

$$Q_{SS} = \left(\frac{\phi_{SS}}{\gamma * w}\right)^{(1/(\gamma-1))} = \left(L_{SS}\right)^{1/\gamma}, \tag{15b}$$

$$S_{SS} = \frac{\alpha - \phi_{SS}}{\beta}. \tag{15c}$$

The third and fourth characteristics of the steady-state, along with the constraint in equation (1b), imply the following solution for the shadow price of output,

$$Q_{SS} - S_{SS} = 0,$$
$$\rightarrow$$
$$h[\phi_{SS}] \equiv (\phi_{SS})^{(1/(\gamma-1))} - \frac{\alpha - \phi_{SS}}{\beta} = 0, \qquad (16)$$
$$h[\phi_{SS} = 1] = 0.$$

This solution is unique for non-negative values of ϕ_{SS}. From equation (16), we know that $h[\phi_{SS} = 0] = -((1+\beta)/\beta) < 0$. For $\forall \phi_{SS} > 0$, $\gamma > 1$, and $\beta > 0$, h[.] is positively sloped,

$$h'[\phi_{SS}] = (\beta/(\gamma-1))(\phi_{SS})^{((2-\gamma)/(\gamma-1))} > 0. \qquad (17)$$

Thus, $h[\phi_{SS}]$ crosses the horizontal axis only once.

This solution also satisfies the second-order conditions for a maximum. The matrix of second derivatives for the two choice variables, L and S (computed from equations (10a) and (11a) with the steady-state restrictions), is as follows,[8]

$$\begin{bmatrix} ((1-\gamma)/\gamma^2) * L^{((1-2\gamma)/\gamma)} & 0 \\ 0 & -\beta \end{bmatrix}, \qquad (18)$$

which is negative definite for $\gamma > 1$ and $\beta > 0$.

With this value for the shadow price of output, $S_{SS} = Q_{SS} = L_{SS} = 1$. The usefulness of these results is that the optimal choices of sales, output, and labor in the initial steady state are each 1. Therefore, the effects of introducing a DTC can be easily computed as a deviation from unity.

[8] The first-order condition for I_t vanishes in the steady-state.

IV. Responses To A Delayed Tax Credit (DTC): No Rolling Base; No Inventory Costs

We now examine the firm's responses to the introduction of a DTC. In order to highlight three different channels of influence, we will examine in this section a special case of the model in which the DTC does not have a rolling base and deviations from target inventory are costless. Section V introduces the rolling base. Section VI contains a general model with the rolling base and costly inventory deviations. The model presented in this section captures our essential results for two intertemporal tradeoffs – anticipatory dips and compensating rebounds – that distort economic decisions.

We divide the timeline for a firm facing a DTC into two intervals,

BEFORE: The months between the adoption date and the implementation date; that is, the implementation period. The beginning of the BEFORE interval is defined by the adoption date for convenience and, without loss in generality, could be extended backwards to include the preview lag discussed at the beginning of this paper.

AFTER: The months on and after the implementation date.

The adoption and implementation dates define these two DTC intervals that may exhibit different real responses. When the implementation date occurs after the adoption date, forward-looking firms anticipate the forthcoming decline in the effective wage. With this fiscal foresight, they may have an incentive to initially decrease employment and output during the implementation period and then compensate for this decrease by raising employment and output sharply at the implementation date. There may be related effects on sales. We refer to this potential negative effect on employment as an Anticipatory Dip (AD), and the subsequent offsetting positive effect as a Compensating Rebound (CR). Each is potentially driven by fiscal foresight. However, a DTC policy may not translate into impactful incentives and real actions. It is the purpose of the remaining part of this paper to uncover the necessary conditions under which an AD and a CR will occur as a result of a DTC.

We assume that the firm begins in the steady-state with no tax credit. At the beginning of the planning period, policymakers adopt a permanent tax credit with an implementation date in the future. This situation describes a DTC and leads to some very interesting dynamic behavior for employment, output, and sales. There are two restrictive assumptions adopted in this section and relaxed in the subsequent two sections -- the rolling base is eliminated so that $BASE_t$ equals a constant in equation (4b) and costs associated with deviations from the target inventory/sales ratio are absent ($\mu = 0$). The first-order conditions for labor and sales for this restricted model are as follows,

$$L_t = \left(\frac{\phi}{\gamma * w_t^{EFF-N}} \right)^{(\gamma/(\gamma-1))} \qquad w_t^{EFF-N} \equiv w \qquad t \in \{BEFORE\}, \tag{19a}$$

$$L_t = \left(\frac{\phi}{\gamma * w_t^{EFF-N}} \right)^{(\gamma/(\gamma-1))} \qquad w_t^{EFF-N} \equiv w * (1 - \tau_t) \qquad t \in \{AFTER\}, \tag{19b}$$

$$S_t = \frac{\alpha - \phi}{\beta} \qquad t \in \{BEFORE, AFTER\}, \tag{19c}$$

$$Q_t = \left(\frac{\phi}{\gamma * w_t^{EFF-N}} \right)^{(1/(\gamma-1))} = (L_t)^{(1/\gamma)} \qquad t \in \{BEFORE, AFTER\}. \tag{19d}$$

The introduction of the DTC can be understood by considering several discrete steps. First, it lowers w_t^{EFF-N} in the AFTER interval. Thus, L_t rises at the time of the implementation date and stays permanently higher. These initial hiring and production plans lead to an imbalance with S_t, which, for the moment, remains fixed. An appropriate change in the shadow price of output restores the balance over the planning period. In equations (19), the decline in ϕ (below its initial steady state value of 1) has three effects:

- raising S_t uniformly in both intervals,
- lowering L_t and Q_t in the BEFORE interval,
- also lowering L_t and Q_t in the AFTER interval; however, this decrease is more than offset by the stimulus from the lower effective nominal wage rate if the elasticity of ϕ_{SS} with respect to w_{SS}^{EFF-N} ($\varepsilon[\phi_{SS}, w_{SS}^{EFF-N}]$) is less than one .

To evaluate this elasticity, we rewrite the steady-state relation $h[\phi_{SS}]$ (equation 16) in terms of w_{SS}^{EFF-N} (which does not generally appear in $h[\phi_{SS}]$ because of the normalization, $w = (1/\gamma) < 1$),

$$h\left[\phi_{SS}\right] = \beta\left(\phi_{SS}[w_{SS}^{EFF-N}] / (\gamma * w_{SS}^{EFF-N})\right)^{(1/(\gamma-1))} - \alpha + \phi_{SS}[w_{SS}^{EFF-N}] \equiv \chi\left[w_{SS}^{EFF-N}\right]. \tag{20a}$$

In any steady-state, $Q_{SS} = S_{SS}$ and hence $\chi'\left[w_{SS}^{EFF-N}\right] = 0$ through an adjustment in ϕ to the change in w_{SS}^{EFF-N}. Differentiating $\chi\left[w_{SS}^{EFF-N}\right]$ with respect to w_{SS}^{EFF-N}, setting the derivative equal to zero, and evaluating this derivative at the original steady-state, we obtain

$$\varepsilon\left[\phi_{SS}, w_{SS}^{EFF-N}\right] = \left(\beta / (\beta + \gamma - 1)\right) < 1, \tag{20b}$$

provided $\beta > 0$ and $\gamma > 1$.

The above analysis generates an Anticipatory Dip, AD, due to fiscal foresight.[9] Even though the effective nominal wage rate in the BEFORE interval does not change, the effective real wage rate rises and, consequently, employment in that interval falls relative to its prior steady-state value. This change represents a shift in production from high-cost to low-cost periods as the firm, foreseeing the future changes in the effective real wage rates, adopts an intertemporal production plan that minimizes average production costs and satisfies endogenous sales and fixed inventory constraints.

The AD directly leads to the Compensating Rebound, CR. The level of sales following from the DTC is determined by a lower steady-state value of the shadow price of output, and thus sales rise in both intervals. When an AD occurs in the BEFORE interval, output (and employment) during the AFTER interval must be larger in order to compensate for the lost output and to meet the inventory constraint. This extra employment is the CR.

An inventory technology, decreasing returns to labor, and a non-competitive output market are necessary elements for the AD and CR to emerge. If an inventory technology is absent, then Q_t must equal S_t in each period, and the firm no longer has a separate sales

[9] Our Anticipatory Dip differs from an Ashenfelter Dip (Ashenfelter, 1978; Heckman and Smith, 1999). While both Dips involve transitory declines of employment and earnings, respectively, prior to the implementation of a policy, the channels differ. The Ashenfelter Dip is a selection phenomenon driven by low opportunity costs; an Anticipatory Dip is driven by intertemporal tradeoffs.

decision.[10] In this case, the inability to change inventory across periods prevents the firm from taking advantage of the differential production costs due to the delayed implementation of the tax credit program. There are no interrelations among anticipated future tax incentives and current employment and output decisions. While employment and output in the AFTER period will rise as a result of the tax credit, the dynamic optimization problem becomes a sequence of static problems and there are no impacts in the BEFORE interval.

Decreasing returns to labor are also necessary for the AD and CR. Per equations (19), a tax credit has two opposing effects on L_t and Q_t in the AFTER interval -- a direct stimulus from the tax credit and an indirect counter-stimulus from the lower shadow price of output. As shown in equation (20b), decreasing returns ($\gamma > 1$) is necessary for a net positive stimulus to emerge in the AFTER interval. If the production technology is linear and returns are constant, then the tax credit does not impact real behavior in the AFTER interval, and there is no subsequent effect in the BEFORE interval. Moreover, decreasing returns is required to satisfy the second-order conditions (equation (18)).

A non-competitive output market is the third necessary condition. If the firm faces a perfectly elastic demand curve, then production in either the BEFORE or AFTER interval could be sold without the penalty from declining marginal revenues. In this case, the dynamic elements in the optimization problem disappear, and the dynamic optimization problem again becomes a sequence of static problems. Employment, output, and sales will rise, but only in the AFTER interval. Section IV showed that movements in ϕ were critical for the increases in the AFTER interval to affect real activity in the BEFORE interval. When the demand curve is perfectly elastic, this channel is frozen because ϕ is fixed by the exogenous price of output. Real variables cannot change in the BEFORE interval, and an AD does not exist.

In sum, an inventory technology provides the means to shift production across periods. Decreasing returns and a non-competitive output market provide the motivation to smooth real activity across periods. All three elements are needed to provide the firm the means and motivation to shift employment, output, and sales intertemporally and generate ADs and CRs.

[10] If an inventory technology is not available to the firm, the inventory accumulation constraint $\left(I_t = Q_t - S_t + I_{t-1}\right)$ would be removed from the optimization problem (equation (8a)) and S_t would be replaced by Q_t for all t.

V. Responses To A Delayed Tax Credit (DTC): No Inventory Costs

We now relax one of the two restrictions in the model in Section IV and analyze the effects of the rolling base on the response to the DTC. The qualitative effects on employment and output are identical to those documented in Section IV, though the channels of influence differ. With a rolling base, the effective nominal wage rate (equation (10b)) is impacted differently in the BEFORE interval, the AFTER interval (in equations (21), the AFTER interval excludes the terminal period, T), and the terminal period,

$$w_t^{EFF-N} \equiv w*\left(1-0+\tau_{t+1}*R\right) \qquad t \in \{BEFORE\}, \qquad (21a)$$

$$w_t^{EFF-N} \equiv w*\left(1-\tau_t+\tau_{t+1}*R\right) \qquad t \in \{AFTER\}, \qquad (21b)$$

$$w_t^{EFF-N} \equiv w*\left(1-\tau_t(\rho/(1+\rho))\right) \qquad t \in \{AFTER\} \text{ if } \tau_{t+1}=\tau_t. \qquad (21b')$$

$$w_t^{EFF-N} \equiv w*\left(1-\tau_t+0\right) \qquad t = T. \qquad (21c)$$

Somewhat paradoxically, the introduction of the DTC raises the effective wage rate in the BEFORE interval. With a delayed tax credit, the firm is not eligible to receive the tax credit in the BEFORE interval, and hence obtains no benefits. However, any hiring in the BEFORE interval raises the employment base above which subsequent employment must rise in order to qualify for the credit. Hence, employment in the BEFORE interval lowers the value of the credit in future periods. Owing to fiscal foresight, the firm internalizes this cost when choosing employment in the BEFORE interval. This negative effect on profitability is measured in equation (21a) by the product of the wage rate, the tax credit, and a discount factor.

As in Section IV, in the AFTER interval, the DTC lowers the effective nominal wage rate. However, the quantitative impact is dramatically altered by the rolling base feature of the tax credit. The $\rho/(1+\rho)$ term in equation (21b', which assumes that $\tau_{t+1}=\tau_t$)[11] reflects that, with a rolling base, eligible incremental employment receives a tax credit today but at the expense of eliminating the tax credit on incremental employment tomorrow. This latter cost is discounted,

[11] If this assumption is relaxed, equation (21b') can be written in the following more general form: $w_t^{EFF-N} \equiv w*\left(1-\tau_t[(\rho+g_{t+1}^{\tau})/(1+\rho)]\right)$, where $g_{t+1}^{\tau} \equiv (\tau_{t+1}/\tau_t)-1.0$.

and the stimulus from the tax credit falls as the discount rate declines. Since the discount rate is generally a small number, the rolling base feature drives a large wedge between the legislated and effective tax credits during the AFTER interval. Assuming an expected long-run nominal return on equity of 10% and an expected long-run inflation rate of 3%, ρ is 7%, and $(\rho / (1+\rho)) \approx 0.065$. Hence, during the AFTER interval, the effective tax credit rate is only 6.5% of the legislated credit rate, a reduction by approximately a factor of 15.

The dampening of the tax credit occurring in periods 0 to T-1 does not extend to the terminal period. At the end of the planning horizon, the dampening effect is absent because current employment no longer affects the rolling base. Equation (21c) shows that, in period T, the effective nominal wage rate is lowered by the statutory credit rate.

While the rolling base affects the pattern of incentives over the planning horizon, it does not affect their present value. Consider a situation where the statutory rate of the tax credit of $\$\tau$ is offered only in the first period for an incremental hire. Since cash flows accrue at the end of the period, the present value is $\$\tau / (1+\rho)$. This figure should be contrasted with the cumulative incentive of $\$\tau\ (\rho / (1+\rho))$ for T-1 periods and then $\$\tau$ in period T. The present value of this stream of incentives associated with the rolling base (PV_{RB}) is computed using an annuity formula as follows,

$$
\begin{aligned}
PV_{RB} &= \sum_{t=1}^{T-1} (1+\rho)^{-t}\,\$\tau\,(\rho/(1+\rho)) \;+\; (1+\rho)^{-T}\,\$\tau \\
&= \$\tau\,(\rho/(1+\rho)) \sum_{t=1}^{T-1} (1+\rho)^{-t} \;+\; (1+\rho)^{-T}\,\$\tau \\
&= \$\tau\,(\rho/(1+\rho))\,(1/\rho)\left[1-(1+\rho)^{-(T-1)}\right] \;+\; (1+\rho)^{-T}\,\$\tau \\
&= \$\tau/(1+\rho)
\end{aligned}
\qquad (22)
$$

Equation (22) establishes the equivalence between the present value of tax incentives subject to a rolling base and the present value of the statutory tax incentive taken in the first period at the statutory rate.[12] In other words, though a rolling base greatly reduces the present value of a tax

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[12] This result holds even if the firm's planning horizon is infinitely long ($T \to \infty$) or the discount rate is zero ($\rho = 0$).

credit relative to the present value of the same credit with a fixed base ($PV_{FB} = \$\tau\,(1/\rho)\,[1-(1+\rho)^{-(T-1)}] > PV_{RB}$), it does not reduce PV_{RB} to zero. Rather, the present value of a rolling base credit is reduced to the present value of a one-time credit. Even with a discount rate of zero, PV_{RB} remains positive. To our knowledge, these results have not been recognized previously in the literature, which has instead focused on the dampening effect of the rolling base and its sensitivity to the discount rate (Eisner, Albert, and Sullivan, 1984; Altshuler, 1988; Hall, 1993, forthcoming; Bloom, Schankerman, and Van Reenen, 2013).

VI. Responses To A Delayed Tax Credit (DTC): The General Model

We are now in a position to analyze the general model in which the tax credit is delayed, a rolling-base determines the amount of the tax credit and, unique to this section, inventory imbalances are costly. The latter effect is introduced by allowing $\mu > 0$. The first-order conditions for the general model are modified by including terms containing the cost of inventory imbalances (μ interacted with the inventory/sales target, ζ) and the shadow price of adding to inventory imbalances (θ_t) and the AFTER interval again excludes the terminal period, T,

$$L_t = \left(\frac{\phi - \theta_t}{\gamma * w_t^{EFF-N}} \right)^{(\gamma/(\gamma-1))} \qquad w_t^{EFF-N} \equiv w * \left(1 + \tau_{t+1} * R\right) \qquad t \in \{BEFORE\}\ , \tag{23a}$$

$$L_t = \left(\frac{\phi - \theta_t}{\gamma * w_t^{EFF-N}} \right)^{(\gamma/(\gamma-1))} \qquad w_t^{EFF-N} \equiv w * \left(1 - \tau_t + \tau_{t+1} * R\right) \qquad t \in \{AFTER\}, \tag{23b}$$

$$L_t = \left(\frac{\phi - \theta_t}{\gamma * w_t^{EFF-N}} \right)^{(\gamma/(\gamma-1))} \qquad w_t^{EFF-N} \equiv w * \left(1 - \tau_t\right) \qquad t = T, \tag{23b'}$$

$$S_t = \frac{\alpha + \mu * \zeta * I_{t-1} + \theta_t - \phi_{SS}}{\beta + \mu * \zeta^2} \qquad t \in \{BEFORE, AFTER\}, \tag{23c}$$

$$Q_t = \left(\frac{\phi - \theta_t}{\delta * w_t^{EFF-N}} \right)^{(1/(\gamma-1))} = \left(L_t\right)^{(1/\gamma)} \qquad t \in \{BEFORE, AFTER\}. \tag{23d}$$

The introduction of costly inventory imbalances modifies the quantitative but not the qualitative effects of the DTC analyzed above. As discussed in prior sections, we start with S_t and ϕ held at their initial steady-state levels and then, initially, allow employment to increase in the AFTER interval and decrease in the BEFORE interval. The BEFORE response in employment results in an inventory drawdown, $\theta_t < 0$ (per equation (9b)), and incremental employment in all periods becomes more valuable by reducing the inventory imbalance. Consequently, an unambiguous implication of the general model is that inventory costs lead to a smaller fall in employment in the BEFORE interval.

The change in employment in the AFTER interval is subject to two contrasting effects. Since the inventory drawdown in the BEFORE period is lower, the need to replenish inventory and employment is lower than under the scenario in Section V. However, there is an added incentive to hire labor and produce output to eliminate the costly inventory imbalance; this channel spurs employment. Relative to the prior scenario, the net effect of the introduction of inventory costs on employment is ambiguous in the AFTER interval.

Inventory and sales respond differently in subsequent periods relative to the scenario in Section V. In the face of a negative inventory imbalance, an incremental sale aggravates the imbalance, is less valuable in this more general model, and $\theta_t < 0$. The inventory imbalance is largest in the BEFORE interval and diminishes over time. This decrease results in an increase in θ_t that stimulates sales over time. Rather than being constant over the planning period, sales in the general model rises over time. As in all models considered here, ϕ adjusts so that the inventory imbalance is eliminated by the end of the planning period at which time $\theta_T = 0$.

VII. Discussion

The literature examining the empirical importance of fiscal foresight (cf. fn. 1) has far outpaced the number of theoretical studies giving explicit attention to decision margins. Five models are reviewed briefly. Each contains forward-looking agents constrained by one of two adjustment frictions. In all five models, anticipated tax policies result in real activity during the implementation period, though in the same direction as the eventually realized incentive (with the exception of one special case) and hence no distorting ADs or CRs exist.

The partial equilibrium models of Abel (1982) and Auerbach (1989) are closest to the one developed in the current study. Their models also focus on one factor of production but, in their cases, it is capital, an assumption that resonates with their reliance on convex adjustment costs as the key friction. While these modeling assumptions are appropriate in some circumstances, there are other situations where non-convex adjustment costs are the more relevant friction and labor taxation is the policy under consideration.[13] Convex adjustment costs force firms to smooth production across intervals and, in sharp contrast to the AD, increase capital accumulation prior to the implementation date. Auerbach's model utilizes the Jorgensonian user cost and a partial adjustment framework and, if adjustment costs are absent, no distorting behavior emerges (Auerbach, p. 950). In Abel's q model, anticipated tax policy also increases capital accumulation during the implementation period. Moreover, on the implementation date, there is an additional increase in capital accumulation. His analysis "…makes it clear that an intertemporal substitution explanation must be more sophisticated than simply buying capital when it is cheap…" (Abel, p. 367).

The general equilibrium models Judd (1985), Yang (2005), and Leeper, Walker, and Yang (2013) introduce smoothing through consumption behavior in response to future tax changes. Judd solves his model analytically using Laplace transforms. It is similar to the models in Abel, Auerbach, and the current paper owing to its theoretical structure and its focus on business taxes. Yang (2005), and Leeper, Walker, and Yang (2013) focus on household taxes, solve a second-order linear difference equation, and use simulations of VARMA models to

[13] For example, the value of business economic development incentives offered by states is titled toward labor. In 2015, the percentages across labor, property, and tangible and intangible capital are 50%, 27%, and 23%, respectively (Bartik, 2017, Table 29).

quantify the effects of fiscal foresight.[14] As with the Auerbach and Abel papers, these three papers also find that capital accumulation rises today in anticipation of a tax decrease tomorrow. General equilibrium models can be useful for analyzing aggregate fiscal policies likely to be salient to a large number of economic actors but are less relevant for evaluating more targeted business incentives. For example, state or local corporate tax incentives such as JCTCs seem unlikely to be salient to the vast majority of households and hence unlikely to influence households' consumption behavior due to expectations of future tax policy.

The partial equilibrium optimizing model developed in this paper emphasizes inventory and pricing decisions, does not rely on adjustment costs, and illustrates that fiscal foresight results in intertemporal distortions.

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[14] The simulation models in these two papers depend on a specific timing assumption -- that current investment has no effect on current output; relaxing this assumption may have significant effects on the model dynamics.

VII. Summary

The path from fiscal foresight to real distortions in a choice theoretic model has been surprisingly long. In our partial equilibrium model, we found that there are three necessary conditions to be satisfied before an anticipated tax can lead to perverse firm behavior such as anticipatory dips (AD) and compensating rebounds (CR). An inventory technology provides the means to shift production across periods. Decreasing returns and a non-competitive output market provide the motivation to smooth real activity before and after the implementation date. All three elements are needed to provide firms the means and motivation to shift employment, output, and sales intertemporally and generate ADs and CRs.

These results indicate that the effects of fiscal foresight will be limited to sectors with non-competitive pricing and storable output. These conditions suggest that the estimated impacts of fiscal policies may be sensitive to underlying economic characteristics and that policies targeted to specific firms or industries with unique characteristics may not be generalizable.

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