Turbulent Business Cycles

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The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.
Abstract. Recessions are associated with sharp increases in turbulence that reshuffles firms’ productivity rankings. To study the business cycle implications of turbulence shocks, we use Compustat data to construct a measure of turbulence based on the (inverse of) Spearman correlations of firms’ productivity rankings between adjacent years. We document evidence that turbulence rises in recessions, reallocating labor and capital from high- to low-productivity firms and reducing aggregate TFP and the stock market value of firms. A real business cycle model with heterogeneous firms and financial frictions can generate the observed macroeconomic and reallocation effects of turbulence. In the model, increased turbulence makes high-productivity firms less likely to remain productive, reducing their expected equity values and tightening their borrowing constraints relative to low-productivity firms. Thus, labor and capital are reallocated to low-productivity firms, reducing aggregate TFP and generating a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, in line with empirical evidence.

I. Introduction

Recessions are characterized by declines in aggregate economic activity. They are also characterized by a sharp rise in micro-level turbulence with increased churning of firms’ productivity rankings. This paper studies the macroeconomic and reallocation effects of turbulence shocks over the business cycles.

We develop an empirical measure of turbulence using data from publicly traded U.S. firms listed in Compustat. We first construct a measure of firm-level total factor productivity (TFP) following the approach in the literature (Syverson, 2004; Foster et al., 2008; Bloom et al., 2018). We then sort the firm-level TFP in each year and estimate the Spearman rank correlations (denoted by $\rho_t$) between adjacent years. A low Spearman correlation indicates
more churning of firm rankings in the productivity distribution: a high-productivity firm this
year are less likely to maintain its productivity ranking next year; whereas a low-productivity
firm now might become more productive in the future. Turbulence is inversely related to the
Spearman correlation of firm-level TFP; and in particular, it is measured by $1 - \rho_t$.

Our measure of turbulence is countercyclical, rising sharply in recessions. We present firm-
level evidence that an increase in turbulence reallocates labor and capital from high- to low-
productivity firms. Partly reflecting its reallocation effects, turbulence is negative correlated
with aggregate manufacturing TFP and the stock market value of firms. Turbulence is also
associated with synchronized and persistent declines in real GDP, consumption, investment,
and employment.

To understand the economic mechanism through which turbulence can drive macroeco-
nomic fluctuations and cross-sectional reallocation, we construct a real business model with
heterogeneous firms and financial frictions. In the model, firms produce a homogeneous good
using capital and labor, subject to idiosyncratic productivity shocks. Firms rely on external
financing of working capital, with the borrowing capacity constrained by a fraction of the the-
expected future equity value (Jermann and Quadrini, 2012; Liu and Wang, 2014; Lian and
Ma, 2021). Firms also face idiosyncratic production distortions, reflecting differential policy
interventions or government subsidies at the firm level (Hsieh and Klenow, 2009; Buera and
Shin, 2013; Moll, 2014). At each given level of productivity, firms with sufficiently high levels
of subsidies choose to operate, facing binding credit constraints. Those with low levels of
subsidies remain inactive. There is an endogenous threshold level of subsidy, at which a firm
is indifferent between producing and staying inactive.

Under the stochastic process of the idiosyncratic productivity shocks, a firm can maintain
its productivity from the current period to the next period with a time-varying probability $\rho_t$.
With the complementary probability $1 - \rho_t$, the firm’s productivity will be randomly drawn
from a stationary, i.i.d. distribution. As we have alluded to, a lower value of $\rho_t$ implies more
churning of firms’ productivity rankings between adjacent periods, or equivalently, greater
turbulence.

The model predicts that a shock that increases turbulence leads to a recession. With
greater turbulence, a high-productivity firm today would be less likely to remain produc-
tive in the future and a low-productivity firm today would be more likely to get a better
productivity draw in the future. Thus, the expected equity value of a high-productivity
firm falls relative that of a low-productivity firm. Since firms’ borrowing capacity depends
on the expected equity value, turbulence tightens the current-period credit constraints for
high-productivity firms disproportionately, and reallocating labor and capital from high- to
low-productivity firms. This reallocation reduces aggregate TFP. The endogenous decline
in TFP is quantitively important, enabling the model to generate a recession with synchronized declines in aggregate output, consumption, investment, and labor hours. These model predictions are in line with empirical evidence.

Financial frictions are crucial for amplifying the macroeconomic effects of turbulence shocks. Since labor and capital are perfectly mobile across firms, competition for input factors from high-productivity firms bids up wages and capital rents. Absent credit constraints, resources would be concentrated in the most productive firms and the equilibrium allocation would be efficient. The heterogeneity stemming from credit constraints and production distortions allows a fraction of firms at each level of productivity to stay active in production. Such financial frictions lead to steady-state misallocation; and they also create room for turbulence shocks to generate reallocation effects across firms with different productivity levels.

The presence of financial frictions implies that competitive equilibrium allocations are inefficient. Appropriate policy interventions can potentially mitigate credit constraints and improve allocative efficiency. Since financial frictions are the key transmission channel for turbulence, policy interventions that alleviate credit constraints might mitigate its recessionary effect.

We use our model framework to evaluate the effectiveness of two alternative policy interventions for stabilizing turbulence-driven recessions. The first policy is a borrowing subsidy that reduces the effective costs of hiring capital and labor, and therefore reducing the amount of working capital loans that firms need to borrow. The second policy is credit easing, under which the government injects liquidity directly to active firms to enhance their borrowing capacity. Each policy is transitory and unexpected, and it is triggered by the realization of a turbulence shock, with the same persistence as that of the shock.

Compared to the laissez-faire benchmark economy with no policy interventions, both types of policies are effective for stabilizing output fluctuations. However, borrowing subsidies exacerbate misallocation, reducing aggregate TFP relative to the benchmark. In contrast, the credit easing policy does not confront a tradeoff between output stability and allocative efficiency: it is effective for both stabilizing aggregate output and improving TFP relative to the benchmark.

Borrowing subsidies reduce firms’ effective costs of hiring input factors, stimulating demand for labor and capital. Thus, it expands the set of active firms at each level of productivity, boosting aggregate output. However, by enabling a larger fraction of low-productivity firms to stay active, the policy exacerbates misallocation, reducing aggregate TFP relative to the benchmark. The decline in TFP partly offsets the stimulus effects on aggregate output.
Credit easing expands the borrowing capacity for active firm. Competition for input factors from high-productivity firms pushes up equilibrium wages and capital rents, forcing some low-productivity firms to stay inactive. This reallocation improves aggregate TFP, contributing to increased output. Thus, the credit easing policy helps mitigate the declines in both aggregate output and TFP relative to the benchmark.\footnote{The two types of policy interventions—borrowing subsidies and credit easing—do not necessarily improve welfare relative to the benchmark, because they both incur a deadweight loss. We use these counterfactual policies to highlight the transmission mechanism of turbulence shocks. We do not study optimal policy here because welfare depends on the calibration of the sizes of the deadweight losses (Gertler and Karadi, 2011).}

II. Related literature

Our work is closely related to the important contribution of Bloom et al. (2018), who study the macroeconomic implications of micro-level uncertainty. They show that, in a real business cycle model with capital and labor adjustment costs, an increase in micro-level uncertainty (i.e., an increase in the standard deviation of the firm-level TFP shocks) reduces net aggregate investment, net hiring, and aggregate output. However, in their model, aggregate consumption rises following an uncertainty shock. Generating a recession with aggregate comovements requires a simultaneous negative shock to the level of aggregate TFP.\footnote{There is large literature on the macroeconomic effects of uncertainty shocks. Examples include Bloom (2009), Christiano et al. (2014), Fernández-Villaverde et al. (2015), Jurado et al. (2015), Baker et al. (2016), Leduc and Liu (2016), Basu and Bundick (2017), Bansal et al. (2019), and many others. For recent surveys of the uncertainty literature, see Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020).}

A turbulence shock in our model can generate a recession with synchronized declines in macro aggregates. Turbulence increases the conditional variance of the firms’ TFP distribution, similar to a micro-level uncertainty shock. However, unlike uncertainty, turbulence in our model operates through a reallocation channel stemming from financial frictions.\footnote{The global financial crisis of 2008-2009 has spurred a large literature that incorporates financial frictions into business cycle models, building on the seminal contributions of Bernanke et al. (1999) and Kiyotaki and Moore (1997). Examples include Jermann and Quadrini (2012), Gertler et al. (2012), Liu et al. (2013), Christiano et al. (2014), Gertler and Kiyotaki (2015), and Lian and Ma (2021). For recent surveys of this literature, see Christiano et al. (2018) and Gertler and Gilchrist (2018).} An increase in turbulence changes the conditional expectations of future productivity distribution, such that firms with high productivity in the current period may not be as productive in the future. These changes in expected productivity distribution reduce the expected equity value and tighten the borrowing constraints for high-productivity firms more than for
low-productivity firms, leading to reallocations that reduce aggregate TFP. The decline in TFP in turn leads to a recession with aggregate comovements.

Arellano et al. (2019) highlight the importance of financial frictions for the transmission of uncertainty shocks. In their model, hiring inputs is risky because firms need to finance the input costs before they receive revenues, and firms face idiosyncratic productivity shocks between paying for the inputs and receiving the revenue. An increase in the volatility of idiosyncratic shocks raises default risks, and firms respond by pulling back hiring and reducing production. Since firms are ex ante identical, they make identical hiring decisions. Thus, an increase in uncertainty (i.e., firm-level volatility) does not lead to reallocation of capital and labor inputs. In our model, however, reallocation is the central mechanism for propagating turbulence shocks.

Our work is related to the economic development literature on capital misallocation under financial frictions (Midrigan and Xu, 2014; Moll, 2014; Buera and Shin, 2013; Liu et al., 2021). Indeed, our measure of turbulence is similar to the persistence of idiosyncratic productivity in the continuous-time model of Moll (2014) and Dou et al. (2021). More persistent productivity shocks imply relatively smaller steady-state productivity losses but also slower transitions to the steady state (Moll, 2014). Other things being equal, the less persistent the idiosyncratic productivity shocks are, the greater the impact of financial frictions on aggregate productivity (Buera and Shin, 2013). We focus on the business cycle implications of turbulence. Thus, our work complements this development literature.

To our knowledge, our paper represents a first attempt to study the transmission mechanism of turbulence shocks over the business cycles using firm-level data and a quantitative business cycle model featuring firm heterogeneity and financial frictions.

III. Measuring micro-level turbulence and its macroeconomic effects

This section describes our empirical methods for measuring micro-level turbulence and documents some stylized facts about the macroeconomic effects of turbulence.

III.1. Empirical methodology. Consider the production function for firm $j$ in period $t$

$$Y_{jt} = z_{jt} F(K_{jt}, N_{jt}),$$

where $Y_{jt}$ denotes value-added output, $K_{jt}$ and $N_{jt}$ denote capital and labor inputs, respectively, and $F(K, N)$ is the production function. The term $z_{jt}$ denotes firm $j$’s TFP that follows the stochastic process

$$z_{jt+1} = \begin{cases} z_{jt} & \text{with prob } \rho_t, \\ \bar{z} & \text{with prob } 1 - \rho_t, \end{cases}$$

where $\bar{z}$ is a constant.
where $\tilde{z}$ is an independently identically distributed (i.i.d.) random variable with the cumulative density function $\tilde{G}(z)$.

The term $\rho_t$ measures the persistence of firm-level TFP. In the extreme case with $\rho_t = 1$ for all $t$, a firm’s productivity level would be permanent: high-productivity firms would remain productive and low-productivity firms would remain unproductive. In the other extreme with $\rho_t = 0$, firm productivity would be an i.i.d. process, with no persistence. In the more general case with $\rho_t \in (0, 1)$, firm productivity is persistent, and the persistence is time varying. A decline in $\rho_t$ implies that high-productivity firms in period $t$ would be less likely to remain productive in period $t + 1$ and low-productivity firms in period $t$ would have a chance to draw a high productivity in period $t + 1$. Thus a decline in $\rho_t$ increases the churn of firm rankings in the productivity distribution. We measure micro-level turbulence by $1 - \rho_t$.

Turbulence is related to, but different from the micro-level uncertainty studied by Bloom et al. (2018). An increase in micro-level uncertainty corresponds to a mean-preserving spread of the cross-sectional productivity distribution—an increase in the variance or inter-quartile range (IQR) of productivity. An increase in turbulence also raises the conditional variance of the productivity distribution, as does uncertainty. Thus, turbulence is positively correlated with micro-level uncertainty.\(^4\) However, unlike uncertainty, turbulence does not affect the ex ante stationary distribution of productivity—it is an ex ante distribution-preserving shock, as we show in Proposition 1 below.\(^5\)

**Proposition 1.** The cross-sectional stationary distribution of idiosyncratic productivity (denoted by $G_t(z)$) is invariant to the realization of $\rho_t$.

**Proof.** Under the stochastic process of idiosyncratic productivity specified in Eq. (2), the cumulative density function of productivity is given by

\[
G_{t+1}(z) = Pr(z_{t+1} \leq z) = Pr(z_t \leq z) \rho_t + Pr(\tilde{z} \leq z)(1 - \rho_t) = G_t(z) \rho_t + \tilde{G}(z)(1 - \rho_t)
\]

(3)

Under the stationarity of the distribution of $z$, we have $G_t(z) = \tilde{G}(z)$ for all $t$. Thus, the stationary distribution is independent of the realization of $\rho_t$. \(\square\)

\(^4\)For example, the correlation between our turbulence measure and the IQR of firm-level TFP from Compustat data is about 0.55.

\(^5\)The distribution-preserving turbulence that we study here can be viewed a discrete-time counterpart to the persistence of idiosyncratic productivity shocks in the continuous-time models of Moll (2014) and Dou et al. (2021), which is also orthogonal to the stationary productivity distribution.
Table 1. Summary statistics of our samples with publicly traded manufacturing firms

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sample 1</th>
<th>Sample 2</th>
<th>Sample 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>N</td>
</tr>
<tr>
<td>Log Asset (1m)</td>
<td>5.6</td>
<td>2.0</td>
<td>51817</td>
</tr>
<tr>
<td>Log Value-Added (1m)</td>
<td>4.7</td>
<td>2.4</td>
<td>51817</td>
</tr>
<tr>
<td>Log Capital (1m)</td>
<td>3.7</td>
<td>2.5</td>
<td>51817</td>
</tr>
<tr>
<td>No. of Workers (1000)</td>
<td>4.2</td>
<td>2.0</td>
<td>51817</td>
</tr>
</tbody>
</table>

Note: Sample 1 covers all listed firms in all manufacturing industries (NAICS code 31 to 33). Sample 2 covers the firms with 25+ years of observations in all manufacturing industries. Sample 3 covers the manufacturing firms with 25+ years of observations in those industries with at least 20 firms in each year.

Source: Compustat, NBER-CES, and authors’ calculations.

III.1.1. Data. We use firm-level data from Compustat Fundamentals Annual database. To obtain measures of industry-level employment, payroll, and price indices, we use information from the NBER-CES Manufacturing Industry Database.6 By combining these two data sources, we obtain an unbalanced panel with 51,817 firm-year observations. This full sample (“Sample 1”) includes all listed firms in all manufacturing industries covered by NBER-CES in the years from 1958 to 2016.7

Following Bloom et al. (2018), we focus on the subset of firms with 25+ years of observations in our sample. We use this pseudo-balanced panel as our baseline sample (“Sample 2”) for estimating firm-level TFP. The baseline sample contains 28,476 firm-year observations. In a robustness check, we further narrow down the sample and focus on industries with more than 20 firms in each year. This sample (“Sample 3”) contains 22,320 firm-year observations. Table 1 presents the summary statistics of our samples.

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7We include firms incorporated in the US (Compustat fic = ‘USA’) that trade on major stock exchanges (NYSE, AMEX, and NASDAQ, Compustat exchg = 11, 12 or 14), for which the native currency is US dollars (Compustat curcd = ‘USD’). We exclude firm-year observations with obvious errors: missing or non-positive values in reported revenue, employment, and capital. We remove a firm if it was involved in a major merger or acquisition that affected its asset by more than 10%. 
III.1.2. Measuring turbulence. We construct a measure of micro-level turbulence based on firm-level TFP.

Following the approach in the literature (Syverson, 2004; Foster et al., 2008; Bloom et al., 2018), we measure firm-level TFP based on the Solow residual

\[ tfp_{ijt} = y_{ijt} - \alpha_{it}k_{ijt} - (1 - \alpha_{it})n_{ijt} \] (4)

where \( tfp_{ijt} \) denotes the TFP (in log units) of firm \( j \) in industry \( i \) and year \( t \), and \( y_{ijt}, k_{ijt} \) and \( n_{ijt} \) denote the firm’s value added, capital input, and labor input, respectively, all in log units. Following Bloom et al. (2018), we assume that the cost share of capital input \( \alpha_{it} \) is common for all firms within an industry \( i \), but varies across time.\(^8\)

After obtaining firm-level TFP, we construct a measure of turbulence following the approach of Bloom et al. (2018). Specifically, we rank firms by deciles of productivity levels. We then compute the Spearman rank correlations of firm TFP between year \( t \) and year \( t + 1 \). The time series of the Spearman correlations corresponds to our measure of \( \rho_t \). Turbulence is measured by \( 1 - \rho_t \). Intuitively, a decline in \( \rho_t \) implies that a high-productivity firm in year \( t \) would be less likely to remain as productive in \( t + 1 \), whereas a low-productivity firm in year \( t \) might get a high productivity draw in year \( t + 1 \). A reduction in \( \rho_t \) reshuffles productivity rankings across firms between \( t \) and \( t + 1 \), increasing turbulence.\(^9\)

III.1.3. Cyclical properties of turbulence. Figure 1 plots the time series of firm-level turbulence from 1960 to 2015. The mean, standard deviation and autocorrelation of the estimated \( \rho_t \) are 0.90, 0.026 and 0.66, respectively. The figure shows that turbulence is countercyclical, rising sharply in recessions.\(^10\)

Our measure of turbulence is negatively correlated with manufacturing TFP, as shown in panel A of figure 2. A rise in turbulence (the blue line) is typically associated with a decline

\(^8\)In our sample, the average value of the cost share of capital (weighted by the value of shipment) is about 0.34. In the Appendix, we provide details of our approach to measuring value added, capital and labor inputs, and the capital share.

\(^9\)Bloom et al. (2018) also estimate the Spearman correlation of plant rankings in the TFP distribution across adjacent years using Census Manufacturing (CM) data (see Figure A1 in their online appendix at https://nbloom.people.stanford.edu/sites/g/files/sbiybj4746/f/rubc_appendix_0.pdf). Despite the difference in the samples, our measure of \( \rho_t \) is highly correlated with theirs, with a correlation coefficient of 0.49.

\(^10\)In the Appendix, we show that the baseline estimate of \( \rho_t \) is robust to alternative samples and alternative approaches to measuring TFP. Our measured turbulence displays an upward trend. Since an increase in turbulence raises firm-level volatility, the trend increase in turbulence is consistent with the empirical evidence that the volatility for publicly traded firms has increased steadily over time, whereas the volatility of privately held firms has declined (Davis et al., 2006).
Figure 1. Measured micro-level turbulence

*Note:* Turbulence is measured by $1 - \rho_t$, where $\rho_t$ is the Spearman correlation of firm TFP rankings between year $t$ and year $t+1$. The grey shaded bars indicate NBER recession dates.

*Source:* Compustat, NBER-CES, BLS, and authors’ calculations.

In TFP relative to trend (the red line), and the correlation between the two series is about -0.28.

Turbulence is also negatively correlated with the stock market value of firms, as shown in panel B of figure 2. We consider two alternative measures of the firm value. One is the market value of assets (“market value”, the red dashed line) calculated based on firms’ stock prices at the end of the fiscal year, multiplied by the shares outstanding and deflated by the consumer price index. The other is the market value of equity (“equity value”, the red solid line) calculated based on market equity plus assets, net of book equity following Eisfeldt and Rampini (2006). The correlations between turbulence with the market value and the equity value are both negative, at -0.35 and -0.43, respectively.\(^{11}\)

\(^{11}\)In Compustat, the firm stock price at the end of the fiscal year is the variable “PRCC.F”, the shares outstanding is “CSHO”, the firm asset is “AT”, and the book equity is “CEQ”. 
Figure 2. Correlation between Turbulence and TFP & Firm Values

Note: The series of manufacturing TFP is computed as an average of firm-level TFP in our benchmark sample, weighted by sales. The TFP index is detrended by the HP filter with a smoothing parameter of 100. The market equity and the market value are both detrended by the HP filter with a smoothing parameter of 100. The grey shaded bars indicate NBER recession dates.

III.1.4. Reallocation effects of turbulence. To examine the reallocation effects of turbulence, we sort firms into different TFP bins and estimate the empirical specification

$$x_{jt} = \beta_0 + \beta_1 Turb_t + \beta_2 Turb_t \ast High_{TFP_{jt}} + \mu_j + \epsilon_{jt}$$

(5)

where the dependent variable $x_{jt}$ denotes employment growth (or capital growth) of firm $j$ in year $t$ relative to the previous year, $Turb_t$ denotes measured turbulence, and $High_{TFP_{jt}}$ is a dummy variable that equals one if firm $j$'s TFP level is above the median and zero
otherwise. To mitigate potential biases associated with firm entries and exits, we focus on a pseudo balanced panel with firms appearing at least 25 years in our sample from 1958 to 2016 (Sample 2), following Bloom et al. (2018). In the empirical specification (5), \( \mu_j \) denotes firm fixed effects and \( \epsilon_{jt} \) denotes regression errors.

The parameter \( \beta_0 \) is a constant intercept. The parameter \( \beta_1 \) measures the relation between turbulence and employment (or capital) growth for low-productivity firms. The sum of \( \beta_1 \) and \( \beta_2 \) measures the relation between turbulence and employment (or capital) growth for high-productivity firms. The estimation results are displayed in Table 2 (Columns (1) and (3)).

In the baseline regressions for both employment growth and capital growth, the estimated values of \( \beta_1 \) and \( \beta_2 \) are both negative and statistically significant at the 99-percent confidence level. Thus, an increase in turbulence is associated with declines in employment and capital growth for all firms, and the declines are more pronounced for high-productivity firms. The point estimates imply that a one standard-deviation increase in turbulence reduces annual employment growth by about 3.1 percent and capital growth by a little under 2.9 percent for high-productivity firms. The same increase in turbulence reduces employment growth for low-productivity firms by about 2.3 percent and capital growth by about 2.2 percent.\(^{12}\) Thus, turbulence has economically important impacts on firm-level activity, with high-productivity firms suffering more from turbulence than low-productivity firms. These findings suggest that turbulence reallocates capital and labor from high- to low-productivity firms, contributing to reducing aggregate TFP.

To examine the robustness of the reallocation effects of turbulence, we also consider an alternative specification in which we sort firm-level TFP into four quartiles and we replace the dummy \( High_{TFP} \) in the baseline specification by the dummy indicators of the top three quartiles of the productivity distribution (denoted by \( z_{2,j,t} \), \( z_{3,j,t} \), and \( z_{4,j,t} \)). Implicitly, we treat the firms in the first quartile \( (z_{1,j,t}) \) of the productivity distribution as the reference group. The estimation results displayed in Columns (2) and (4) of the table confirm the reallocation effects of turbulence. An increase in turbulence is associated with declines in employment growth and capital growth for all firms, and the negative effects are more pronounced for firms with higher productivity. These reallocation effects are again statistically significant and economically important.

\(^{12}\)The standard deviation of our measured turbulence is 2.6 percent. The impact of a one standard deviation increase in turbulence on employment growth for high-productivity firms is \( 2.6 \times (-0.877 - 0.325) = -3.125 \) percent. The impact on capital growth is \( 2.6 \times (-0.842 - 0.260) = -2.865 \) percent. For low-productivity firms, the impacts are given by \( 2.6 \times (-0.877) = -2.28 \) percent on employment growth and \( 2.6 \times (-0.842) = -2.189 \) percent on capital growth.
### Table 2. Impact of turbulence on firms with different productivity

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>Employment growth</th>
<th>Capital growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$Turb_t$</td>
<td>-0.877***</td>
<td>-0.753***</td>
</tr>
<tr>
<td></td>
<td>(0.65)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>$Turb_t \times High_{TFP_{jt}}$</td>
<td>-0.325***</td>
<td>-0.260***</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>$Turb_t \times z_{2,j,t}$</td>
<td>-0.154***</td>
<td>-0.156***</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>$Turb_t \times z_{3,j,t}$</td>
<td>-0.343***</td>
<td>-0.300***</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>$Turb_t \times z_{4,j,t}$</td>
<td>-0.617***</td>
<td>-0.521***</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.070)</td>
</tr>
<tr>
<td>constant</td>
<td>0.191***</td>
<td>0.182***</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.045)</td>
</tr>
<tr>
<td></td>
<td>0.266***</td>
<td>0.257***</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.058)</td>
</tr>
</tbody>
</table>

Firm Fixed Effect | Yes | Yes | Yes | Yes
Observations | 23495 | 23493 | 23495 | 23493

Note: This table shows the regression of firm-level employment growth (or capital growth) on the measured turbulence ($Turb$) for firms with different levels of TFP. In the baseline specification (Columns (1) and (3)), the dummy $High_{TFP_{jt}}$ equals one if firm $j$’s TFP is above the median and zero otherwise. In the alternative specification (Columns (2) and (4)), we sort the Compustat firms into four quartiles based on their TFP levels. The dummy variables $z_{2,j,t}$, $z_{3,j,t}$, and $z_{4,j,t}$ indicate whether a firm $j$’s TFP is in the second, third, or fourth quartile in year $t$. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

### III.1.5. Macroeconomic effects of turbulence.

We now examine the macroeconomic effects of turbulence. For this purpose, we estimate the impulse responses of several key macroeconomic variables to a turbulence shock using the local projections approach of Jordà (2005). We consider the empirical specification

$$x_{t+h} - x_{t-1} = \beta_0^h + \beta_1^h Turb_t + \epsilon_{t+h} \quad h = 0, 1, 2, 3, 4. \quad (6)$$
The term $x_{t+h} - x_{t-1}$ denotes the cumulative changes in the log-level of the variable of interest, including per capita real consumption, investment, private output (i.e., the sum of consumption and investment), hours worked, and the interquartile ranges of capital and employment from year $t - 1$ to year $t + h$, where $h$ denotes the projection horizons (number of years). The independent variable $\text{turb}_t \equiv \log(1 - \rho_t)$ denotes the log-level of turbulence in year $t$. Our measured annual series of turbulence has a first-order autocorrelation of 0.606 and a standard deviation of 0.2633, implying a standard deviation of the innovation of 0.2094. The term $\epsilon_{t+h}$ is the regression residual. The parameter $\beta_1^h$ measures the impulse
responses of the macroeconomic variables to a turbulence shocks at the horizon \( h \) (measured in years).\textsuperscript{13}

Figure 3 plots the estimated impulse responses of the macroeconomic variables to a one standard deviation turbulence shock (i.e., an increase in turbulence of 0.2094) for horizons up to five years. The shock leads to a recession with synchronized and persistent declines in aggregate output, consumption, investment, and hours worked. It also leads to reallocations across firms that reduce the interquartile ranges of capital and employment persistently. These macroeconomic effects of turbulence are quantitatively important. For example, a one standard deviation increase in turbulence reduces per capita output by a little over 0.5% on impact.

IV. A REAL BUSINESS CYCLE MODEL WITH TURBULENCE SHOCKS

To understand the economic mechanism through which turbulence can drive macroeconomic fluctuations and cross-sectional reallocation, we construct a real business model with heterogeneous firms and financial frictions.

IV.1. The model. The model economy is populated by a continuum of infinitely lived households with measure one. The representative household has the utility function

\begin{equation}
E \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right\},
\end{equation}

where \( C_t \) denotes consumption, \( N_t \) denotes labor hours, and \( E \) is an expectation operator. The parameter \( \beta \in (0, 1) \) is a subjective discount factor, \( \psi > 0 \) measures the relative weight on the dis-utility of working, and \( \gamma \geq 0 \) is the inverse Frisch elasticity of labor supply.

All markets are perfectly competitive. The household takes all prices as given and maximizes the utility in Eq. (7) subject to the sequence of budget constraints

\begin{equation}
C_t + K_{t+1} = (R_t + 1 - \delta)K_t + W_t N_t + D_t - T_t,
\end{equation}

where \( K_{t+1} \) denotes the end-of-period capital stock, \( R_t \) denotes the capital rental rate, \( W_t \) denotes the real wage rate, \( D_t \) denotes the dividend income from firms, and \( T_t \) denotes a lump-sum tax paid to the government.

There is a continuum of firms, each endowed with a constant-returns technology that produces the final consumption good using capital and labor as inputs. Firms face idiosyncratic productivity shocks drawn at the beginning of each period, before hiring inputs. The

\textsuperscript{13}The aggregate time series data are reported by the U.S. Bureau of Economic Analysis (BEA) and are available through FRED of the St. Louis Fed.
The production function for an individual firm is given by
\[ y_{jt} = A_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha}, \] (9)
where \( y_{jt} \) denotes the output produced by firm \( j \) in period \( t \), and \( k_{jt} \) and \( n_{jt} \) denote the capital and labor inputs, respectively.

The term \( A_t \) denotes an aggregate productivity shock that follows the AR(1) process
\[ \ln(A_t) = \rho_A \ln(A_{t-1}) + \sigma_A \varepsilon_A^t, \] (10)
where the innovation term \( \varepsilon_A^t \) follows a standard normal process. The parameter \( \rho_A \) and \( \sigma_A \) measure the persistence and the volatility, respectively, of the aggregate productivity shock.

The idiosyncratic productivity shock \( z_{jt} \) follows the stochastic process described in Eq. (2), which we rewrite here for convenience of referencing:
\[ z_{j,t+1} = \begin{cases} z_{jt} & \text{with prob } \rho_t, \\ \tilde{z} & \text{with prob } 1 - \rho_t. \end{cases} \] (11)

Here, the term \( \tilde{z} \) is an i.i.d. random variable with a finite number of states. Specifically, we assume that \( \tilde{z} = z_j \) with probability \( \pi_j \), for \( j = 1, 2, ..., J \). Without loss of generality, we further assume that \( z_1 < z_2 < ... < z_J \). The process features time-invariant cross-sectional distribution of firm productivity such that, regardless of realization on \( \rho_t \in (0, 1) \), there are always \( \pi_j \) fraction of firms with \( z_{jt} = z_j \) in each period. Thus, in a stationary equilibrium, \( \pi_j \) is the measure of firms with productivity \( z_j \).

We measure turbulence by \( 1 - \rho_t \). If \( \rho_t = 1 \), then the idiosyncratic productivity \( z_{jt} \) would be permanent. If \( \rho_t = 0 \), on the other hand, then each firm would face i.i.d. shocks to productivity with no persistence. A lower value of \( \rho_t \) implies that a high-productivity firm in this period may not maintain its productivity in the next period, whereas a low-productivity firm in this period might be able to draw a better productivity next period. Thus, a decline in \( \rho_t \) reshuffles firms’ productivity ranking across time, increasing turbulence.\(^{14}\)

We assume that the turbulence shock follows the stochastic process
\[ \ln(1 - \rho_t) = (1 - \rho_p) \ln(1 - \bar{\rho}) + \rho_p \ln(1 - \rho_{t-1}) + \sigma_p \varepsilon_p^t, \] (12)
where \( \bar{\rho} \) denotes the average level of \( \rho_t \) and the innovation term \( \varepsilon_p^t \) follows a standard normal process. The parameter \( \rho_p \) and \( \sigma_p \) measure the persistence and the volatility of turbulence shock, respectively.

\(^{14}\)As we have shown in Proposition 1, turbulence does not alter the ex ante stationary distribution of the idiosyncratic productivity. Thus, it is conceptually different from other types of micro-level uncertainty such as that studied by Bloom et al. (2018).
Firms rely on external financing of their working capital. In the beginning of each period, firms need to borrow from a competitive financial intermediary to cover payments for input factors, and these working capital loans are repaid within the period, after firms receive revenues. Following Jermann and Quadrini (2012) and Liu and Wang (2014), we assume that a firm’s borrowing capacity is constrained by a fraction $\theta$ of its expected equity value in the next period, in line with the empirical evidence of Lian and Ma (2021).

Firms at each level of productivity also face idiosyncratic production distortions (denoted by $\tau_{jt}$), reflecting differential policy interventions or government subsidies at the firm level (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Buera and Shin, 2013; Moll, 2014). These production distortions drive a wedge between firms’ private and social marginal revenue products. We assume that $\tau_{jt}$ is drawn from a continuous i.i.d. distribution $F(\tau_{jt})$. Under credit constraints, the presence of idiosyncratic production distortions allows a fraction of firms at each level of productivity to stay active, enabling turbulence shocks to generate reallocation and endogenous fluctuations in aggregate TFP.\(^{15}\)

The firms’ optimizing problem is characterized by the Bellman equation

$$V_t(z_{jt}, \tau_{jt}) = \max_{k_{jt}, n_{jt}} \tau_{jt} A_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha} - R_t k_{jt} - W_t n_{jt} + \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}), \quad (13)$$

subject to the working capital constraint

$$R_t k_{jt} + W_t n_{jt} \leq \theta \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}) \equiv \theta B_{jt}. \quad (14)$$

Here, the term $V_t(z_{jt}, \tau_{jt})$ denotes the value function of firm $j$ that depends on the firm-level state variables $z_{jt}$ and $\tau_{jt}$. The value function $V_t(z_{jt}, \tau_{jt})$ also depends on aggregate shocks, which are summarized by the time subscript $t$. The term $M_{t+1} = \beta \frac{C_t}{C_{t+1}}$ denotes the stochastic discount factor determined by the marginal utilities of the representative household who owns all firms. The term $B_{jt}$ denotes the expected present value of a firm with current productivity $z_{jt}$.

Profit maximizing implies the conditional factor demand functions

$$\frac{\tau_{jt} y_{jt}}{k_{jt}} = (1 + \mu_{jt}) R_t, \quad (15)$$

and

$$\frac{(1 - \alpha) \tau_{jt} y_{jt}}{n_{jt}} = (1 + \mu_{jt}) W_t, \quad (16)$$

\(^{15}\)Including idiosyncratic distortions also serves a technical purpose in our model with a discrete distribution of idiosyncratic productivity. The continuity of the distribution function $F(\tau_{jt})$ implies a well-defined cutoff point $\tau_{jt}^*$ for each level of productivity $z_{jt}$.\)
where $\mu_{jt}$ denotes the Lagrangian multiplier associated with the credit constraint (14). Using the factor demand functions, we can write the firm’s flow profit as

$$d_{jt} \equiv \left[ \tau_{jt} A_t z_{jt} \left( \frac{\alpha W_t}{(1-\alpha)R_t} \right)^\alpha - \frac{W_t}{1-\alpha} \right] n_{jt}. \quad (17)$$

A firm would choose to produce if and only if $d_{jt} \geq 0$. It follows that there exists a threshold level of production subsidy $\tau_{jt}^*$ such that, if $\tau_{jt} \geq \tau_{jt}^*$, then a firm would be active in production, facing binding credit constraints. Otherwise, the firm would remain inactive. At the threshold level of subsidy, a firm earns zero profit and thus it would be indifferent between producing and staying inactive. The indifference condition determines the threshold level of subsidy

$$\tau_{jt}^* = \frac{R_t^\alpha W_t^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha} A_t z_{jt}}. \quad (18)$$

The threshold $\tau_{jt}^*$ increases with the factor prices $R_t$ and $W_t$. The threshold $\tau_{jt}^*$ decreases with the productivity level $z_{jt}$, implying that the fraction of active firms is higher for firms with higher productivity.

The presence of credit constraints and production distortions creates misallocation of resources. Absent those distortions, all resources would be allocated to the most productive firm (with productivity $z_J$). However, under those distortions, some low-productivity firms are able to produce because not all high-productivity firms are active. Specifically, at each level of productivity, there is a non-degenerate fraction of firms that are active, with the share of active firms measured by $1 - F(\tau_{jt}^*)$ for all $j \in 1, \ldots, J$. Such misallocation opens up a reallocation channel for turbulence shocks, as we show below.

Since active firms face binding credit constraints and inactive firms do not use any input factors, the conditional demand functions for labor and capital inputs are given by

$$n_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{(1-\alpha)\theta B_{jt}}{W_t}, & \text{if } \tau_{jt} \geq \tau_{jt}^* \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

and

$$k_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{\alpha \theta B_{jt}}{R_t}, & \text{if } \tau_{jt} \geq \tau_{jt}^* \\ 0, & \text{otherwise} \end{cases} \quad (20)$$

Given the factor demand functions, firm $j$’s value function can be written as

$$V_t(z_{jt}, \tau_{jt}) = \max \left\{ \frac{\tau_{jt}}{\tau_{jt}^*} - 1, 0 \right\} \theta B_{jt} + B_{jt} \quad (21)$$

Since production subsidies are i.i.d. across time, the average value of a firm with productivity $z_{jt}$ is given by

$$\bar{V}_t(z_{jt}) = \int V_t(z_{jt}, \tau) dF(\tau) = \left[ 1 + \theta \int_{\tau_{jt}^*}^\infty \left\{ \frac{\tau}{\tau_{jt}^*} - 1 \right\} dF(\tau) \right] B_{jt} \equiv \Phi(\tau_{jt}^*) B_{jt}, \quad (22)$$
where the term $\Phi(\tau^*_j) \equiv 1 + \theta \int_{\tau^*_j}^{\tau_{\tau^*_j}} (\tau_{\tau^*_j} - 1) dF(\tau)$ is a decreasing function of the threshold subsidy level $\tau^*_j$.

Given the stochastic process of $z_{j,t+1}$ and the definition of $B_{jt}$ in Eq. (14), we have

$$B_{jt} \equiv \beta E_t \frac{C_t}{C_{t+1}} \left[ \rho_t \bar{V}_{jt+1} + (1 - \rho_t) \sum_{i=1}^{J} \pi_i \bar{V}_{it+1} \right]$$

(23)

In a competitive equilibrium, markets for labor, capital, and final consumptions goods all clear. Labor market clearing implies that

$$N_t = \sum_j \pi_j n_{jt} \equiv \sum_j \pi_j \frac{(1 - \alpha) \theta B_{jt}}{W_t} \left[ 1 - F(\tau^*_j) \right].$$

(24)

Capital market clearing implies that

$$K_t = \sum_j \pi_j k_{jt} \equiv \sum_j \pi_j \frac{\alpha \theta B_{jt}}{R_t} \left[ 1 - F(\tau^*_j) \right].$$

(25)

Goods market clearing implies that

$$Y_t = C_t + K_{t+1} - (1 - \delta) K_t,$$

(26)

where aggregate output $Y_t$ is given by

$$Y_t \equiv \sum_j \pi_j y_{jt} = \sum_j \pi_j \bar{A}_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha}.$$

(27)

Given aggregate output, aggregate capital and labor inputs, we define aggregate TFP as

$$Z_t \equiv \frac{Y_t}{K_t^{\alpha} N_t^{1-\alpha}} = \frac{\sum_j \pi_j \bar{A}_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha}}{K_t^{\alpha} N_t^{1-\alpha}}.$$

(28)

Definition. A competitive equilibrium consists of the sequence of allocations $\{C_t, Y_t, N_t, K_t\}$ and the sequence of prices $\{W_t, R_t\}$ such that (i) taking all prices as given, the allocations solve the household’s utility maximizing problem and the firms’ profit maximizing problem; and (ii) markets for labor, capital, and goods all clear.

IV.2. Steady-state allocations. We now provide some analytical characterization of the steady state equilibrium and show how the steady-state allocations vary with the average level of turbulence $(1 - \bar{\rho})$.

In Section IV.1, we have shown that a firm with productivity $z_j$ chooses to produce (i.e., become active) if and only if its subsidy exceeds the threshold $\tau^*_j$. The threshold level of subsidy is given by Eq. (18), and it is a decreasing function of firm productivity $z_j$.

Figure 4 illustrates the production decisions for firms with different levels of productivity $(z)$ and subsidies $(\tau)$. The downward-sloping curve indicates the threshold function $\tau^*(z)$. At each $z$, a firm with a subsidy $\tau \geq \tau^*(z)$ chooses to produce. Otherwise, it stays inactive. Thus, the region of active firms are those with $(\tau, z)$ lying above the threshold curve.
Figure 4. Production decisions in the steady state equilibrium

Note: This figure shows the steady-state relation between the threshold level of subsidy $\tau^*$ and firm productivity $z$ (the blue line). Firms with $(\tau, z)$ lying above the threshold line are active in production and those below the line are inactive. The figure also shows the threshold line (the red line) for production decisions in an economy with a higher average level of turbulence (i.e., a lower value of $\bar{\rho}$).

In an economy with a higher average level of turbulence (i.e., with a lower value of $\bar{\rho}$), a high-productivity firm is less likely to remain productive, reallocating resources to low-productivity firms. Such reallocations reduce aggregate TFP, lowering the factor prices. For any given $z$, the declines in wages and capital rents reduce the threshold level of subsidy $\tau^*$ (see Eq. (18)). Therefore, the threshold curve for production decisions shifts downward, with a flatter slope (see the red line in the figure), indicating that the increase in turbulence expands the active regions for low-productivity firms more than it does for high-productivity firms.

The reallocation effects of turbulence illustrated in Figure 4 can be formalized by the following proposition.

Proposition 2. Given the steady-state factor prices $R$ and $W$, an increase in average turbulence reduces the share of labor hours allocated to high-productivity firms. Specifically, define the relative share of labor hours as $\eta_{ji} \equiv \frac{N_j}{N_i}$, where $N_j$ and $N_i$ denote labor hours allocated to active firms with productivity $z_j$ and $z_i$, respectively. Without loss of generality,
we assume that $z_j > z_i$. Then, we have
\[
\frac{\partial \eta_{ji}}{\partial \bar{\rho}} > 0.
\]
(29)

Proof. In the steady state equilibrium, the relative share of labor is given by
\[
\eta_{ij} = \frac{\pi_j}{\pi_i} \frac{1 - \beta \bar{\rho} \Phi(\tau_{j}^{*}) \left[1 - F(\tau_{j}^{*})\right]}{1 - \beta \bar{\rho} \Phi(\tau_{i}^{*}) \left[1 - F(\tau_{i}^{*})\right]},
\]
where we have used aggregated labor demand based on Eq. (19), with the term $B_j$ substituted out using the steady-state version of Eq. (23).

For given $W$ and $R$, the threshold $\tau_{j}^{*}$ is a function of $z_j$ only. Since $z_j > z_i$, it is easy to show that $\Phi(\tau_{j}^{*}) > \Phi(\tau_{i}^{*})$ and that $1 - F(\tau_{j}^{*}) > 1 - F(\tau_{i}^{*}) > 0$. Thus, we have
\[
\frac{\partial \eta_{ji}}{\partial \bar{\rho}} = \frac{\beta [\Phi(\tau_{j}^{*}) - \Phi(\tau_{i}^{*})] \left[1 - F(\tau_{j}^{*})\right]}{(1 - \beta \bar{\rho} \Phi(\tau_{j}^{*}))^2 \left[1 - F(\tau_{i}^{*})\right]} > 0
\]
(30)

When turbulence rises ($\bar{\rho}$ declines), current productivity is less predictive for future productivity, such that productive firms are less likely to stay productive. Thus, an increase in turbulence lowers expected value of productive firms, reducing their borrowing capacity for financing working capital. As a consequence, labor is reallocated to less productive firm.

The analytical results in Proposition 2 are partial equilibrium in nature, because we have assumed that the factor prices $W$ and $R$ are independent of turbulence. However, since the production thresholds for low-productivity firms are more sensitive to changes in the factor prices, an increase in turbulence that reduces the factor prices would disproportionally expand the active regions for low-productivity firms, reinforcing the misallocation effects of turbulence. We quantify the general equilibrium effects of a turbulence shock in the next section.

IV.3. The calibration. We solve the equilibrium dynamics numerically based on calibrated parameters. Table 3 displays the calibrated parameter values.

A period in our model corresponds to a quarter of a year. We set the subject discount factor to $\beta = 0.99$, implying an annualized risk-free interest rate of 4%. We calibrate the cost share of capital to $\alpha = 0.34$ using the NEBR-CES data following the approach of Bloom et al. (2018). We set the capital depreciation rate to $\delta = 0.025$ to match the average annual investment rate of 10% in the U.S. data (Eisfeldt and Rampini, 2006; Clementi and Palazzo, 2016). We set the inverse Frisch elasticity of labor supply to $\gamma = 5$. We calibrate the relative utility weight on leisure $\psi$ such that the steady-state labor hours is one-third of the time endowment. We set the parameter $\theta$ to 0.35 to match the average ratio of working capital to market equity in the Compustat data.
We calibrate the turbulence shock process $1 - \rho_t$ based on the Spearman rank correlations of firm-level TFP using annual data from Compustat and NBER-CES (see Section III). The mean value of $\rho_t$ is 0.90 at the annual frequency. Thus, we set the quarterly persistence of firm-level TFP to $\bar{\rho} = 0.9^{0.25} = 0.974$. The annual series of turbulence (in log levels) has a first-order autocorrelation of 0.606 and a standard deviation of 0.2633. Translating to quarterly values, we obtain an autocorrelation of $\rho_{\sigma} = 0.8823$ and a standard deviation of the innovation of $\sigma_{\rho} = 0.124$.\(^{16}\)

We assume that the idiosyncratic production distortion $\tau_{jt}$ follows a log-normal distribution with a positive support and a time-varying volatility. Specifically, we assume that the standard deviation of log($\tau_{jt}$) follows the stationary stochastic process

$$\ln(\sigma_{\tau,t}) = (1 - \rho_{\sigma}) \ln(\sigma_{\tau}) + \rho_{\sigma} \ln(\sigma_{\tau,t-1}) + \sigma_{\sigma} \varepsilon_t^\sigma, \quad (31)$$

where the innovation term $\varepsilon_t^\sigma$ follows a standard normal process. The parameter $\rho_{\sigma}$ and $\sigma_{\sigma}$ measure the persistence and the standard deviation of the volatility shock, respectively. The uncertainty measure $\ln(\sigma_{\tau,t})$ here parallels the micro-level uncertainty in Bloom et al. (2018).

We calibrate the average volatility to $\sigma_{\tau} = 0.60$ to match the observed average IQR of employment across firms from 1960 to 2015 (the average IQR is 17). We normalize the mean value of $\log(\tau_{jt})$ to $\mu_{\tau} = -0.5\sigma_{\tau}^2$ such that the unconditional mean of $\tau_{jt}$ is one (i.e., $E(\tau_{jt}) = 1$).

Given the calibrated parameters process of $\tau_{jt}$, we calibrate the volatility of the idiosyncratic productivity shock $\tilde{z}_{jt}$ based on our measured firm-level TFP. Our revenue-based measure of firm-level TFP includes both exogenous technology shocks $z_{jt}$ and production distortions $\tau_{jt}$. Specifically, the model implies that

$$tfp_{jt} = \log(z_{jt}) + \log(\tau_{jt}), \quad (32)$$

where $tfp_{jt}$ is the firm-level TFP (in log units) that we have constructed based on the production function. The standard deviation of our measured $tfp_{jt}$ is $\sigma_{tp} = 0.607$. Given our assumption that $\log(z_{jt})$ and $\log(\tau_{jt})$ are independent processes, the unconditional standard deviation of $\log(z_{jt})$ (which is also the standard deviation of $\log(\tilde{z}_{jt})$) at the quarterly frequency is given by $\sigma_z = \sqrt{\sigma_{tfp}^2 - \sigma_{\tau}^2} = 0.05$.

We calibrate the parameters in the aggregate shocks following the literature. We set the quarterly persistence of aggregate TFP shocks to $\rho_A = 0.95$, following the real business cycle literature (Cooley and Prescott, 1995). We set the persistence of the micro-level uncertainty

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\(^{16}\) The quarterly autocorrelation is thus given by $\rho_{\rho} = 0.606^{0.25} = 0.8823$. The standard deviation of the innovation to the turbulence shock at the annual frequency is $0.2633 \sqrt{(1 - 0.606^{2.5})} = 0.2094$. Converting this to the quarterly frequency, the standard deviation of the shock innovation is $\sigma_{\rho} = \frac{0.2094}{\sqrt{1 + \rho_{\rho}^2 + \rho_{\sigma}^2 + \rho_{\rho}^4}} = 0.124$.
Table 3. Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.99</td>
<td>Average real interest rate of 4% per year</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>0.34</td>
<td>Average cost share of capital (NBER-CES)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
<td>Capital depreciation rate of 10% per year</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Inverse Frisch elasticity</td>
<td>5</td>
<td>Frisch elasticity of labor supply of 0.2</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Utility weight on leisure</td>
<td>1396</td>
<td>Average hours worked of 1/3</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Loan to value ratio</td>
<td>0.35</td>
<td>Working capital to equity ratio (Compustat)</td>
</tr>
<tr>
<td>$\bar{\rho}$</td>
<td>Firm-level TFP persistence</td>
<td>0.974</td>
<td>Estimated (Compustat and NBER-CES)</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Persistence of turbulence shock</td>
<td>0.8823</td>
<td>Estimated (Compustat and NBER-CES)</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>Volatility of turbulence shock</td>
<td>0.124</td>
<td>Estimated (Compustat and NBER-CES)</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>Average production distortion (log)</td>
<td>-0.18</td>
<td>Estimated (Compustat and NBER-CES)</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>Volatility of production distortion (log)</td>
<td>0.60</td>
<td>Estimated (Compustat and NBER-CES)</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>Persistence of productivity shock</td>
<td>0.95</td>
<td>Cooley and Prescott (1995)</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>Persistence of uncertainty shock</td>
<td>0.75</td>
<td>Bloom et al. (2018)</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Volatility of aggregate TFP shock</td>
<td>0.01</td>
<td>Normalized</td>
</tr>
<tr>
<td>$\sigma_\sigma$</td>
<td>Volatility of uncertainty shock</td>
<td>0.01</td>
<td>Normalized</td>
</tr>
</tbody>
</table>

To $\rho_\sigma = 0.75$, consistent with Leduc and Liu (2016) and Bloom et al. (2018). We normalize the standard deviations of these aggregate shocks to one percent (0.01).

IV.4. Impulse responses of turbulence shock. To examine the macroeconomic effects of turbulence shocks, we solve our model based on calibrated parameters. We simulate the model using third-order approximations of the equilibrium conditions around the deterministic steady state. We then compute impulse responses of several key macroeconomic and distributional variables as deviations of those variables driven by the turbulence shock from their stochastic steady state levels without the shock.\footnote{We follow the approach in Fernández-Villaverde et al. (2011) and Leduc and Liu (2016) to compute the impulse responses. In particular, the model is first simulated for a large number of periods to compute the ergodic mean of each variable. It is then simulated using the ergodic means as a starting point. Finally, impulse responses to a turbulence shock are computed as the differences between the simulated path with the turbulence shock and the path with no shocks. This solution approach helps capture potential non-linear effects of the shock. Since turbulence shocks in our model have first-moment impact, the impulse responses generated from the third-order approximations are essentially the same as those from first-order approximations (and we have verified this).}

Figure 5 displays the impulse responses to a one standard deviation shock to turbulence. An increase in turbulence reduces the chance for a current high-productivity firm to remain as
productive in the future, and it also increases the chance for a current low-productivity firm to get a higher productivity draw in the future. Thus, the expected equity values of high-productivity firms declines relative to those of low-productivity firms, disproportionately tightening the borrowing capacity of high-productivity firms in the current period. Facing tightened credit constraints, high-productivity firms pull back hirings, reallocating labor and capital to low-productivity firms. Since high-productivity firms use more capital and labor in the steady state than low-productivity firms, the increase in turbulence reduces the interquartile ranges of both labor and capital across firms (as shown in the figure), indicating reallocation of those input factors.

The reallocation effects of the turbulence shock reduce aggregate productivity, leading to a decline in aggregate TFP and the expected equity value of firms, as shown in the figure. Since the turbulence shock reduces aggregate TFP, it leads to a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, as in the data.

The recessionary effects of turbulence are sizable and persistent. For example, a one standard deviation turbulence shock leads to a drop in aggregate output of about 0.4% on impact, and output stays below its steady-state level for more than five years after the shock.
Figure 5. Impulse response to a turbulence shock in the benchmark model

Note: This figure shows the impulse responses to a one standard deviation shock to turbulence in the calibrated benchmark model. The horizontal axis shows the periods (quarters) since the impact of the shock. The vertical axis shows the percentage deviations of each variable from its stochastic steady state level.

IV.5. The role of financial frictions. Financial frictions are important for the propagation of turbulence shocks. To illustrate this point, we consider a counterfactual version of our model, in which firms’ borrowing capacity does not vary with the expected equity value. Specifically, we replace the working capital constraint by

\[ R_t k_{jt} + W_t n_{jt} \leq \theta \beta E_t \frac{C_t}{C_{t+1}} \left[ \rho_t \bar{V}^{ss} + (1 - \rho_t) \sum_{i=1}^{J} \pi_i \bar{V}^{ss}_i \right] \equiv \theta \bar{B}_{jt}, \]  

where \( \bar{V}^{ss}_j \) denotes the steady-state equity value for firms with productivity \( z_{jt} \). In this counterfactual, a turbulence shock can still influence firms’ borrowing capacity by changing the transition probability (\( \rho_t \)) of the future productivity distribution, but changes in firms’ expected equity value following a turbulence shock would have no effect on the borrowing capacity.

Figure 6 shows the impulse responses in the benchmark model (the blue solid lines) and those in the counterfactual under this ‘quasi-fixed’ borrowing capacity (the red dash-dotted lines). The figure shows that the recession effects and the reallocation effects of a turbulence
shock would be substantially dampened if firms’ borrowing capacity could not vary with the expected equity value. This counterfactual illustrates the importance of financial frictions—and in particular, the endogenous variations of the borrowing capacity with expected firm values—for propagating turbulence shocks.

IV.6. Impulse responses to a micro-level uncertainty shock. Turbulence is related to but different from micro-level uncertainty studied in the literature (Bloom et al., 2018). Uncertainty increases the dispersion without changing the mean of the idiosyncratic firm-level shocks. Turbulence reshuffles firms’ ranking in future productivity distributions, which also raises the conditional variance of the idiosyncratic shocks, but it does not change the ex ante unconditional distribution of productivity (see Proposition 1). More importantly, our model suggests that turbulence works through a reallocation channel stemming from financial frictions.
To illustrate the relation between turbulence and uncertainty, we report in Figure 7 the impulse responses of the key macroeconomic and distributional variables following an increase in uncertainty, measured by $\sigma_{\tau,t}$, the time-varying volatility of the production distortions.

Similar to a turbulence shock, an increase in uncertainty leads to declines in aggregate output, consumption, and investment. It also reduces the interquartile ranges of capital and labor, indicating reallocation from high- to low-productivity firms, and therefore reducing aggregate TFP. The decline in aggregate TFP further contributes to the declines in output, consumption, and investment.

A key difference is that uncertainty boosts labor hours whereas turbulence reduces them. At each level of productivity ($z_j$), only those firms with sufficiently high subsidies would find it profitable to produce (i.e., those firms with subsides $\tau_j \geq \tau^*_j$). A mean-reserving spread in the distribution of $\tau$ (i.e., an increase in uncertainty) expands the right tail of the subsidy...
distribution, raising the average subsidies for active firms.\textsuperscript{18} Thus, active firms choose to increase production, boosting labor demand and the real wage rate. The increase in wages, combined with the increase in the marginal utility of consumption, stimulates labor supply, raising equilibrium labor hours.\textsuperscript{19}

For the same reason, uncertainty also raises demand for capital. Since the capital stock is predetermined, the capital rental rate rises persistently, creating an incentive for households to increase investment. However, uncertainty also exacerbates misallocation because the increased average subsidies for active firms benefit low-productivity firms more than high-productivity firms.\textsuperscript{20} This exacerbated misallocation reduces aggregate TFP and household income. Under our calibration, the negative income effect dominates the intertemporal substitution effect, reducing equilibrium investment.\textsuperscript{21}

Despite the increase in equilibrium labor hours, aggregate output declines following the uncertainty shock because the decline in aggregate TFP more than offsets the increase in labor hours.\textsuperscript{22}

\textbf{IV.7. Impulse responses to an aggregate productivity shock.} The macroeconomic and reallocation effects of turbulence are also different from those associated with other aggregate shocks such as a TFP shock.

Figure 8 displays the impulse responses to a negative TFP shock. Similar to a turbulence shock, a negative TFP shock generates a recession with synchronized declines in aggregate output, consumption, investment, and labor hours. The shock also reduces average firm value.

\textsuperscript{18}Uncertainty also expands the left tail of the distribution, but this does not affect production decisions because it affects only inactive firms.

\textsuperscript{19}We have examined alternative calibrations of the Frisch elasticity of labor supply and found that, with a larger Frisch elasticity, the expansionary effect of uncertainty on labor hours becomes stronger.

\textsuperscript{20}See Appendix A.3.

\textsuperscript{21}The responses of investment to uncertainty depend on calibrated parameters, and in particular, on the size of the elasticity of intertemporal substitution (EIS). Under a sufficiently large EIS, we find that the intertemporal substitution effect can dominate the income effect, such that investment increases following an uncertainty shock (not reported in the paper).

\textsuperscript{22}It is well-known that real business cycle models have difficulties in generating macroeconomic comovements following an uncertainty shocks (Gilchrist and Williams, 2000). For example, the model in Bloom et al. (2018) implies a counterfactual increase in aggregate consumption following a micro-level uncertainty shock. They show that, to generate comovements in their model requires a simultaneous negative first-moment shock to TFP. Our model with financial frictions features a reallocation channel, through which uncertainty reduces aggregate TFP, and is thus more promising for generating aggregate comovements. However, under our calibration, uncertainty still boosts labor hours. New Keynesian models with sticky prices can generate comovements following an uncertainty shock (Basu and Bundick, 2017; Leduc and Liu, 2016).
However, the TFP shock has different reallocation effects than turbulence. A decline in aggregate TFP raises the threshold of active production for firms at each level of productivity, shrinking the set of active firms. Since labor and capital are perfectly mobile across firms, all firms face the same wages and capital rents. A decline in aggregate TFP would therefore force more low-productivity firms into the inactive regions than high-productivity firms. This “cleansing effect” reallocates resources to more productive firms, raising the endogenous component of aggregate TFP, mitigating the recession.

![Figure 8. Impulse response to a negative aggregate TFP shock in the benchmark model](image)

*Figure 8.* Impulse response to a negative aggregate TFP shock in the benchmark model

*Note:* This figure shows the impulse responses to a one standard deviation negative shock to aggregate TFP. The horizontal axis shows the periods (quarters) since the impact of the shock. The vertical axis shows the percentage deviations of each variable from its stochastic steady state level.

### IV.8. Quantitative importance of turbulence shocks.

Turbulence has quantitatively important recessionary effects, both in the model and in the data. In our quarterly model, a one standard deviation shock to turbulence leads to a drop in aggregate output of 0.4% and the recessionary effects persist for more than 5 years. Indeed, the magnitude of the recessionary effects of turbulence in the model is comparable to our empirical estimates discussed in Section III.1.5 .
Figure 9. Impulse response to turbulence shock: Model vs. data

Note: This figure shows the impulse responses of private aggregate output to a one standard deviation shock to turbulence in the data (the black solid line) and in the calibrated annual version of the model (the blue solid line). The dashed lines show the 68% confidence band around the empirical estimates of the impulse responses. The horizontal axis shows the years after the impact of the shock. The vertical axis shows the percentage deviations of output in the model from its steady state level and the percentage changes in output in the data relative to its pre-shock level.

Since our empirical measure of turbulence uses annual data, we calibrate an annual version of the model to facilitate comparisons of the impulse responses following a turbulence shock. In the annual model, we set the subjective discount factor to $\beta = 0.96$ and the capital depreciation rate to $\delta = 0.10$. We calibrate the turbulence shock process based on our empirical measure of turbulence. In particular, the annual series of our turbulence measure (in log units) has a mean of $\log(0.1)$, a standard deviation of 0.2633, and a first-order autocorrelation of 0.606. The rest of the structural parameters are identical to those in the benchmark model.

Figure 9 compares the model-implied impulse responses of aggregate output (the blue solid line) with the empirical estimates of the impulse response (the black solid line). A one standard deviation turbulence shock reduces aggregate output by a bit over 0.5% on impact, both in the data and in the model. The shock has persistent recessionary effects on aggregate output both in the model and in the data, although the theoretical impulse
responses miss the hump estimated from the data. Despite the simplicity of the model, it is remarkable that the path of the theoretical impulse responses lies within the 68% confidence band around the empirical estimates (the dashed lines) for at least the first five years after the impact of the shock. These findings suggest that turbulence plays an important role in driving business cycles.\footnote{We have also compared the theoretical impulse responses of investment, consumption, and labor hours with those in the data (see Appendix A.4). We find that, similar to the case of output, the model-implied responses of investment and consumption are both in line with those estimated from the data, although the calibrated model fails to generate the turbulence-driven large declines in labor hours observed in the data.}

V. Policy interventions

Appropriate policy interventions can potentially undo the financial frictions, stabilizing aggregate output, and improving allocative efficiency. To illustrate this point, we consider two alternative policy interventions in response to a recession driven by a turbulence shock. One policy is a borrowing subsidy that reduces firms’ borrowing costs. The other is a credit-easing policy that expands firms’ borrowing capacity.

Under the borrowing subsidy policy (Policy I), the government subsidizes wages and rents for active firms at an exogenous rate $\omega_{1t}$, such that firms with productivity $z_{jt}$ face the effective credit constraint

$$\tilde{R}_t k_{jt} + \tilde{W}_t n_{jt} \leq \theta B_{jt},$$

(34)

where $\tilde{R}_t = (1 - \omega_{1t})R_t$ and $\tilde{W}_t = (1 - \omega_{1t})W_t$ denote the after-subsidy capital rental rate and real wage rate, respectively. The subsidies thus reduce the amount of working capital loans that firms need to borrow. The government finances the borrowing subsidies $\Omega_{1t} = \omega_{1t}(W_t N_t + R_t K_t)$ by imposing lump-sum taxes on the representative household. We assume that operating this policy incurs a resource cost of $\lambda_1 \Omega_{1t}$, where $\lambda_1 \geq 0$ reflects potential deadweight losses associated with the government program.

Under the credit easing policy (Policy II), the government injects liquidity to active firms, such that firms with productivity $z_{jt}$ face the effective credit constraint

$$R_t k_{jt} + W_t n_{jt} \leq \theta(1 + \omega_{2t})B_{jt},$$

(35)

where $\omega_{2t} B_{jt}$ is the amount of government transfers to active firms with expected equity value $B_{jt}$. The total cost of the credit-easing policy is given by $\Omega_{2t} = \omega_{2t} \sum_j \pi_j \theta B_{jt}[1 - F(\tau_{j1})]$, which is financed by lump-sum taxes on the household. Similar to the borrowing subsidy policy, we assume that credit easing also incurs a resource cost of $\lambda_2 \Omega_{2t}$, where $\lambda_2 \geq 0$ reflects potential deadweight losses in operating the policy.

Following Bloom et al. (2018), we consider transitory and unanticipated policy interventions. A policy would be implemented only if a turbulence shock hits the economy, and the
policy intervention has the same persistence as the shock. We evaluate the effectiveness of each of the two alternative policies—borrowing subsidies and credit easing—for mitigating the macroeconomic and reallocation effects of turbulence. For this purpose, we compare the impulse responses of aggregate output and aggregate TFP to a turbulence shock under borrowing subsidies (“Policy I”) or credit easing (“Policy II”) to those in the laissez-faire benchmark economy without policy intervention (“No policy”).

In each policy regime, we simulate the model economy for 1000 periods (quarters) based on third-order approximations of the equilibrium system around the deterministic steady state. A turbulence shock and a simultaneous policy intervention (if any) are implemented in period 960. The unanticipated policy stimulus has a size of 3% of steady state output, with the same persistence as the turbulence shock. After a policy intervention is implemented, we allow the economy to evolve naturally for the remaining 40 quarters. We calculate the responses of each endogenous variable to the turbulence shock (with or without a policy intervention) as percentage deviations from the stochastic steady state.

Figure 10 shows the stabilizing effects of the two alternative policies relative to the benchmark economy, conditional on a one-standard-deviation turbulence shock.\(^\text{24}\) Compared to the benchmark economy without policy interventions (the black bar), Policy I (the red bar) is effective for stabilizing the declines in aggregate output (see the upper panel of the figure). By providing borrowing subsidies, the policy stimulates demand for labor and capital, and thus mitigating the declines in equilibrium hours, investment, and output. However, by reducing the effective factor prices, Policy I enables a larger fraction of low-productivity firms to produce, exacerbating misallocation and intensifying the decline in aggregate TFP following a turbulence shock (see the lower panel).

The credit easing policy is also effective for stabilizing turbulence-driven output declines (see the blue bar vs. the black bar in the upper panel of the figure). The policy expands the borrowing capacity for all active firms, enabling a larger share of high-productivity firms to finance working capital and produce. The increase in the share of active high-productivity firms pushes up equilibrium wages and rents, shrinking the set of active low-productivity firms. Thus, Policy II reallocates labor and capital to high-productivity firms, improving aggregate TFP relative to the benchmark (see the blue bar vs. the black bar in the lower panel).

\(^{24}\)The figure here shows the impact effects of turbulence on aggregate output and TFP. The results are qualitatively the same when we consider the cumulative effects (not reported).
Figure 10. Impact effects of a turbulence shock with alternative policy interventions

Note: This figure plots the responses of aggregate output and aggregate TFP in the impact period of a one-standard deviation turbulence shock. Black bars show the responses in the benchmark economy with no policy interventions. Red bars represent the responses under an unanticipated and temporary borrowing subsidy policy (‘Policy I’). Blue bars represent the responses under an unanticipated and temporary credit easing policy (‘Policy II’).

These policy experiments suggest that temporary borrowing subsidies or credit easing policies are effective for stabilizing turbulence-driven output fluctuations. However, the reallocation consequences of the two policies are different. While borrowing subsidies exacerbate misallocation, credit easing alleviates it.

VI. Conclusion

Macroeconomic fluctuations often mask underlying cross currents with important cross-sectional reallocations. We have studies the implications of turbulence—a form of reallocation shocks—for business cycles. An increase in turbulence changes the conditional distribution of firms’ future productivity, leading to reallocations across firms. We measure
turbulence based on firm-level TFP data and document evidence that turbulence is countercyclical, rising sharply in recessions. It is negatively correlated with average firm equity values and aggregate TFP. An increase in turbulence is associated with synchronized declines in aggregate output, consumption, investment, and labor hours.

Using a real business cycle model augmented with firm heterogeneity and financial frictions, we have highlighted a quantitatively important reallocation channel, through which a turbulence shock drives macroeconomic fluctuations. An increase in turbulence reduces the likelihood for the current high-productivity firms to maintain their productivity rankings in the future, lowering their expected equity values relative to those of the current low-productivity firms. Facing tightened working capital constraints, high-productivity firms pull back hiring of capital and labor relatively to low-productivity firms, leading to reallocation from high- to low-productivity firms and reducing aggregate TFP. The declines in the endogenous TFP generate a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, as in the data. A one standard deviation shock to turbulence leads to a drop in aggregate output of about 0.5%, with the recessionary effects persisting for more than five years, suggesting that turbulence plays an important role in driving business cycles.

Financial frictions are crucial for propagating turbulence shocks in our model. The presence of financial frictions also leads to misallocation. Policy interventions designed to alleviate credit constraints can potentially dampen the impact of turbulence and improve allocative efficiency. However, the particular approach to implementing such policy interventions can produce very different outcomes. For example, borrowing subsidies that reduce the amount of working capital loans that firms need to borrow can effectively boost aggregate output, mitigating the recessionary effects of turbulence. An alternative credit easing policy that expands firms’ borrowing capacity can also stimulate aggregate output. However, these two alternative policies have different implications for allocative efficiency. A borrowing subsidy enables more low-productivity firms to stay active, exacerbating misallocation, whereas credit easing allows high-productivity firms to expand production, improving aggregate productivity.

To illustrate the key transmission mechanism of turbulence, we have intentionally kept the model stylized. To match the time-series data, however, the model needs to be enriched by incorporating other real and nominal rigidities, such as habit formation, investment adjustment costs, and nominal rigidities. Introducing nominal rigidities would also allow for examining the role of monetary policy in stabilizing macroeconomic fluctuations driven by turbulence. We leave these important subjects for future research.
References


A. Robustness of the turbulence measure. Our measure of turbulence is robust to alternative measures of value added, capital input, and labor input. It is also robust to the data samples used.

In our benchmark empirical specification, we construct firm-level TFP based on firm-level value added and capital and labor inputs. We measure value added using firm-level sales and the average share of intermediate inputs at the 6 digit industry level, where the intermediate input share is the ratio of costs of materials to total value of shipments. We measure capital input using the real book value of a firm and labor input using the number of employees. We also focus on the sample that contains firms with 25+ years of observations (Sample 2). Figure A1 shows that the turbulence measure is robust to alternative measurements of value added, capital and labor inputs, and alternative samples.

The upper left panel of the figure compares the benchmark measure of turbulence (the blue line) and the alternative measure using a different approach to measure value added (the red line). In particular, we follow the approach in David et al. (2016) and David and Venkateswaran (2019), and construct value added by assuming a constant intermediate input share of 0.5 for all firms. These two alternative measures of turbulence are highly correlated, with a correlation coefficient of 0.8522. We have also considered another approach to constructing firm-level value added by subtracting the costs of goods sold from reported sales in Compustat (not shown in the figure). The resulting turbulence measure is also highly correlated with our benchmark measure, with a correlation coefficient of 0.7583.

The upper right panel of Figure A1 shows that the measure of turbulence is robust to alternative approaches to measuring capital. Here, instead of using the real book value of firms, we measure capital input by using a perpetual inventory approach. Specifically, we first initialize the value of capital as real value of first entry of $PPEGT$. We then construct a measure of net investment using $PPENT_t - PPENT_{t-1}$, deflated by industry-specific investment deflators. Finally we iterate forward capital accumulation by adding real net investment to capital stock in previous period. With this alternative measure of capital, we obtain a turbulence series (the red line) that is highly correlated with the benchmark series (the blue line), with a correlation coefficient of 0.9179.

The measure of turbulence is also robust to alternative approaches to measuring labor input, as shown in the lower left panel of Figure A1. Here, instead of measuring labor input
by the number of employees, we measure labor input by a firm’s total payroll. The turbulence series under this alternative measure of labor input (the red line) is highly correlated with the benchmark series (the blue line), with a correlation coefficient of 0.7804.

Finally, the lower right panel of the figure shows that the turbulence measure is robust to alternative data samples.

A.2. Robustness of the reallocation effects of turbulence. We now show that the reallocation effects of turbulence are robust to our empirical specifications.

In the baseline specification, we examine the reallocation effects of turbulence by estimating the responses of firm-level employment growth and capital growth to a turbulence shock interacted with an indicator of high-productivity firms. We construct the indicator of high productivity based on the firms’ current-year TFP ranking. It is possible that the current-year TFP ranking might be endogenous to employment growth or capital growth. To address this issue, we construct an alternative indicator of high-productivity firms based on
Table A.1. Impact of turbulence on firms with different productivity

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>Employment growth</th>
<th>Capital growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$Turb_t$</td>
<td>-1.029***</td>
<td>-1.000***</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>$Turb_t \times High_{TFP_{jt-1}}$</td>
<td>-0.100***</td>
<td>-0.216***</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>$Turb_t \times z_{2,j,t-1}$</td>
<td>-0.027</td>
<td>-0.133***</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.059)</td>
</tr>
<tr>
<td>$Turb_t \times z_{3,j,t-1}$</td>
<td>-0.080</td>
<td>-0.235***</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>$Turb_t \times z_{4,j,t-1}$</td>
<td>-0.222***</td>
<td>-0.447***</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>constant</td>
<td>0.207***</td>
<td>0.205***</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>Firm Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>21944</td>
<td>21922</td>
</tr>
</tbody>
</table>

Note: This table shows the regression of firm-level employment growth (or capital growth) on the measured turbulence ($Turb$) for firms with different levels of TFP. In the baseline specification (Columns (1) and (3)), the dummy $High_{TFP_{jt-1}}$ equals one if firm $j$’s TFP is above the median in period $t-1$ and zero otherwise. In the alternative specification (Columns (2) and (4)), we sort the Compustat firms into four quartiles based on their TFP levels. The dummy variables $z_{2,j,t-1}$, $z_{3,j,t-1}$, and $z_{4,j,t-1}$ indicate whether a firm $j$’s TFP is in the second, third, or fourth quartile in year $t-1$. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

their TFP rankings with a one-year lag. We reestimate the responses of employment (or capital) growth to turbulence shocks for firms with different levels of productivity. The results are shown in Table A.1. We find that, with this alternative TFP rankings, turbulence reduces employment and capital growth for high-productivity firms relative to low-productivity firms, as we found in the benchmark specification.
This figure shows the impulse responses of to an uncertainty shock in the benchmark model. The black, blue and red dashed lines represent the responses of firms at the 25%, 50% and 100% percentile of the productivity distribution. The horizontal axis shows the periods (quarters) after the impact of the shock. The vertical axis measures percentage deviations from the stochastic steady state in response to a one standard deviation shock to uncertainty.

A.3. **Heterogeneous impacts of a micro-level uncertainty shock.** A shock to micro-level uncertainty measured by the standard deviation $\sigma_{\tau t}$ of the idiosyncratic production distortions has heterogeneous impacts on firms with different productivity levels. As shown in Figure A2, a mean-preserving spread in production distortions (i.e., an increase in uncertainty) raises the average subsidies for active firms, raising aggregate demand for labor and capital, and boosting wages and rents. By increasing the average subsidies for low-productivity firms relative to high-productivity firms, uncertainty boosting the relative share of low-productivity firms in production, exacerbating misallocation.

A.4. **Macroeconomic effects of turbulence: Benchmark model vs. data.** In the text, we show that an annual version of the benchmark calibrated model generates empirically plausible impulse responses of aggregate output to a turbulence shock. Here, we compare the impulse responses of other macro variables to a turbulence shock from the model against those estimated from the data.
Figure A3. Impulse responses of macroeconomic variables to turbulence shock: Benchmark model vs. data

Note: This figure shows the impulse responses of aggregate output, consumption, investment, and labor hours to a one standard deviation shock to turbulence in the data (the black solid line) and in the calibrated annual version of the model (the blue solid line). The dashed lines show the 68% confidence band around the empirical estimates of the impulse responses. The horizontal axis shows the years after the impact of the shock. The vertical axis shows the percentage deviations of output in the model from its steady state level and the percentage changes in output in the data relative to its pre-shock level.

Figure A3 compares the impulse response of output, consumption, investment and hours worked to one standard deviation turbulence shock in the model (blue lines) vs. the data (black lines). The responses of output, investment, and consumption are close to those in the data in the sense that they lie within the 68% confidence bands of the empirical impulse responses (at least in the first two years). However, under the calibrated Frisch elasticity of labor supply (0.2), the model fails to generate the observed large contraction in labor hours following a turbulence shock.