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TURBULENT BUSINESS CYCLES

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Abstract. Firm-level evidence suggests that turbulence that reshuffles firms' productivity rankings rises sharply in recessions. An increase in turbulence reallocates labor and capital from high- to low-productivity firms, reducing aggregate TFP and the stock market value of firms. A real business cycle model with heterogeneous firms and financial frictions can generate the observed macroeconomic and reallocation effects of turbulence. In the model, increased turbulence makes high-productivity firms less likely to remain productive, reducing their expected equity values and tightening their borrowing constraints relative to low-productivity firms. This leads to a reallocation that reduces aggregate TFP. Unlike uncertainty, turbulence changes both the conditional mean and the conditional variance of the firm productivity distribution, enabling a turbulence shock to generate a recession with synchronized declines in aggregate activities.

I. Introduction

Recessions are characterized by synchronized declines in aggregate economic activity. They are also characterized by a sharp rise in micro-level turbulence with increased churn in firm productivity rankings. This paper studies the macroeconomic and reallocation effects of

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turbulence shocks over business cycles and propose a theoretical model featuring financial frictions that helps explain the observed effects of turbulence shocks.

We first document several stylized facts about the reallocation and macroeconomic effects of turbulence. For this purpose, we construct an empirical measure of turbulence using data from publicly traded U.S. firms listed in Compustat. Specifically, we first construct a revenue-based measure of firm-level total factor productivity (TFP) following the approach in the literature (Syverson, 2004; Foster et al., 2008; Bloom et al., 2018). We then sort the measured firm-level TFP in each year and estimate the Spearman rank correlations (denoted by ρ_t) between adjacent years following the same approach as that in Bloom et al. (2018). A low Spearman correlation indicates more churning of firm rankings in the productivity distribution: a high-productivity firm this year is less likely to maintain its productivity ranking next year; whereas a low-productivity firm now might become more productive in the future. Turbulence is inversely related to the Spearman correlation of firm-level TFP, and we measure it by $1 - \rho_t$. Relative to Bloom et al. (2018), we go one step further and use an instrumental variable (IV) approach to correct potential attenuation biases due to measurement errors in firm-level TFP.

We document a set of new stylized facts about the macroeconomic and reallocation effects of turbulence over the business cycles. First, turbulence is countercyclical, rising sharply in recessions, in line with the findings of Bloom et al. (2018). Second, an increase in turbulence is associated with reallocation of labor and capital from high- to low-productivity firms. These reallocation effects remain significant after we control for the confounding effects of recessions and uncertainty. Importantly, financing constraints amplify the reallocation effects. Third, reflecting its reallocation effects, turbulence is negatively correlated with aggregate manufacturing TFP and the aggregate stock market value of firms. Finally, at the aggregate level, an increase in turbulence is associated with persistent declines in real GDP, consumption, investment, and employment.[1]

To understand the economic mechanism through which turbulence can drive the observed macroeconomic fluctuations and cross-sectional reallocation, we construct a real business cycle (RBC) model with heterogeneous firms and financial frictions. In the model, firms produce a homogeneous good using capital and labor, subject to idiosyncratic productivity shocks. Firms rely on external financing of working capital, with the borrowing capacity constrained by a fraction of the expected future equity value (Jermann and Quadrini, 2012; Liu

[1]Our measure of turbulence is not a purely exogenous process. In our micro-level regressions, we control for the confounding effects of uncertainty and first-moment shocks. In the regressions using aggregate variables, we orthogonalize the turbulence shock by controlling for the effects of uncertainty and other aggregate shocks.

and Wang, 2014; Lian and Ma, 2021). Firms also face idiosyncratic production distortions, reflecting differential policy interventions or government subsidies at the firm level (Hsieh and Klenow, 2009; Buera and Shin, 2013; Moll, 2014). At each given level of productivity, firms with sufficiently high levels of subsidies choose to operate, facing binding credit constraints while those with low levels of subsidies remain inactive. Given productivity, there is an endogenously determined threshold level of subsidy, at which a firm is indifferent between producing and staying inactive.

To keep the analysis tractable, we consider a simple stochastic process of the idiosyncratic productivity shock that is in line with that for our empirical analysis. Under this stochastic process, a firm can maintain its productivity from the current period to the next period with a time-varying probability ρ_t. With the complementary probability $1 - \rho_t$, the firm's productivity will be an independent and identically distributed (i.i.d.) random variable. A lower value of ρ_t implies more frequent switching in firm productivity rankings between adjacent periods or, equivalently, greater turbulence.

The model predicts that a shock that increases turbulence leads to a recession. With greater turbulence, a high-productivity firm today would be less likely to remain productive in the future and a low-productivity firm today would be more likely to get a better productivity draw in the future. Thus, the expected equity value of a high-productivity firm falls relative to that of a low-productivity firm. Since firms' borrowing capacity depends on the expected equity value, turbulence disproportionately tightens the current-period credit constraints for high-productivity firms and reallocates labor and capital from high- to low-productivity firms. This reallocation reduces aggregate TFP. The endogenous decline in TFP is quantitatively important, enabling the model to generate a recession with synchronized declines in aggregate output, consumption, investment, and labor hours. These model predictions are in line with empirical evidence.

The reallocation effects of turbulence also compress the distributions of labor and capital across firms with different productivity levels, reducing the cross-sectional dispersion of the *levels* of employment, capital, and sales. Since the shock reshuffles expected firm-level productivity, more firms choose to adjust their production, increasing the dispersion of sales *growth* across firms. These model predictions are line with our empirical evidence. Furthermore, the decline in aggregate TFP through the reallocation channel reduces worker wages and capital rents, lowering the productivity threshold for active production. Thus, the share of active low-productivity firms increases. The resulting increases in the left skewness of productivity and of sales in a recession is consistent with empirical evidence (Kehrig, 2015; Salgado et al., 2019). Importantly, in our model, a turbulence shock alone can generate these changes in the firm size distribution that are consistent with those observed in the data during recessions, without relying on other shocks such as a TFP shock.

Financial frictions are crucial for amplifying the macroeconomic effects of turbulence shocks. Since labor and capital are perfectly mobile across firms, competition for input factors from high-productivity firms bids up wages and capital rents. Absent credit constraints and production subsidies, resources would be concentrated in the most productive firms, and the equilibrium allocation would be efficient. Credit constraints and idiosyncratic production distortions restrict the borrowing capacity of high-productivity firms, allowing some low-productivity firms to stay active in production. Such financial frictions lead to steady-state misallocation and they also create room for between-firm reallocation following a turbulence shock. Such reallocation leads to procyclical TFP, enabling the model to generate business cycle comovements.

The presence of financial frictions implies that competitive equilibrium allocations are inefficient. Appropriate policy interventions can potentially mitigate credit constraints and improve allocative efficiency. Since financial frictions are the key transmission channel for turbulence, policy interventions that alleviate credit constraints might mitigate its recessionary effect.

We use our model framework to evaluate the effectiveness of two alternative policy interventions for stabilizing turbulence-driven recessions. The first policy is a borrowing subsidy that reduces the effective costs of hiring capital and labor, therefore reducing the amount of working capital that firms need to finance. The second policy is credit easing, under which the government injects liquidity to enhance the borrowing capacity of active firms. Each policy is transitory and unexpected, and it is triggered by the realization of a turbulence shock, with the same persistence as that of the shock.

Under our calibration, both types of policies are effective for mitigating the recessionary effects of turbulence relative to the *laissez-faire* benchmark economy with no policy interventions. However, the policies operate through different channels and therefore have different implications for reallocation. Borrowing subsidies reduce the effective costs of hiring input factors for all firms, expanding the set of active firms at each level of productivity and boosting aggregate output. However, by enabling a larger fraction of low-productivity firms to stay active, the policy exacerbates misallocation, reducing aggregate TFP relative to the benchmark. The decline in TFP partly offsets the stimulus effects on aggregate output. Credit easing expands the borrowing capacity for active firms. Competition for input factors from high-productivity firms pushes up equilibrium wages and capital rents, forcing some low-productivity firms to stay inactive. This reallocation improves aggregate TFP, contributing to increased output.[2]

II. Related literature

Our work is closely related to the important contribution of Bloom et al. (2018), who study the macroeconomic implications of micro-level uncertainty. They show that, in a real business cycle model with capital and labor adjustment costs, an increase in micro-level uncertainty (i.e., an increase in the standard deviation of the firm-level TFP shocks) reduces net aggregate investment, net hiring, and aggregate output. However, in their model, aggregate consumption rises following an uncertainty shock. To generate a recession with aggregate comovements requires a simultaneous negative shock to the level of aggregate TFP.

Unlike uncertainty, which is a mean-preserving spread of the productivity distribution, turbulence changes not just the conditional variance but also the conditional mean of firm-level productivity. Following an increase in turbulence, firms with high productivity in the current period may not be as productive in the future. Thus, this turbulence-induced changes in conditional expectations of future firm productivity, together with credit constraints, lead to reallocation from high- to low-productivity firms, reducing aggregate TFP. The endogenous decline in TFP in turn leads to a recession with aggregate comovements in our model, without relying on simultaneous shocks to the level of TFP.

Our model highlights the importance of financial frictions for propagating turbulence shocks.[3] In this sense, our work is complementary to the existing studies that emphasize the importance of financial frictions for the transmission of uncertainty shocks (Gilchrist et al., 2014; Christiano et al., 2014; Alfaro et al., 2018; Arellano et al., 2019).[4]

[2]The two types of policy interventions—borrowing subsidies and credit easing—do not necessarily improve welfare relative to the benchmark, because they both incur a deadweight loss. We use these counterfactual policies to highlight the transmission mechanism of turbulence shocks. We do not study optimal policy here because welfare depends on the calibration of the sizes of the deadweight losses (see also Gertler and Karadi (2011)).

[3]The global financial crisis of 2008-2009 has spurred a large literature that incorporates financial frictions into business cycle models, building on the seminal contributions of Bernanke et al. (1999) and Kiyotaki and Moore (1997). Examples include Jermann and Quadrini (2012), Gertler et al. (2012), Liu et al. (2013), Christiano et al. (2014), Gertler and Kiyotaki (2015), and Lian and Ma (2021). For recent surveys of this literature, see Christiano et al. (2018) and Gertler and Gilchrist (2018).

[4]There is a large strand of literature on the macroeconomic effects of uncertainty shocks. Examples include Bloom (2009), Bachmann et al. (2013), Fernández-Villaverde et al. (2015), Jurado et al. (2015), Baker et al. (2016), Leduc and Liu (2016), Basu and Bundick (2017), Bansal et al. (2019), Berger et al. (2020), and many others. For recent surveys of the uncertainty literature, see Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020).

Our work is related to the economic development literature on capital misallocation under financial frictions (Midrigan and Xu, 2014; Moll, 2014; Buera and Shin, 2013; Gopinath et al., 2017; Liu et al., 2021). Indeed, our measure of turbulence is analogous to the persistence of idiosyncratic productivity in the continuous-time model of Moll (2014). In his model, more persistent idiosyncratic productivity shocks create an incentive for firms to save more in order to mitigate the impact of potentially binding credit constraints, resulting in relatively smaller steady-state productivity losses but also slower transitions to the steady-state. Other things being equal, the less persistent the idiosyncratic productivity shocks are, the greater the impact of financial frictions on aggregate productivity (Buera and Shin, 2013). We focus on the business cycle implications of turbulence. Thus, our work complements this development literature.

The countercyclical behavior of turbulence that we find is consistent with other empirical studies based on different data and measurements. For example, Aghion et al. (2021) construct a measure of turbulence based on the rate of new product additions and subtractions (i.e., product churn) using US Census of Manufactures data. They find that product churn rises sharply during recessions. Bernard and Okubo (2016) and Dekle et al. (2021) also report evidence of countercyclical product churn based on Japanese manufacturing data. Similar reallocation effects can arise from labor market churns (Pratap and Quintin, 2011) or supply-chain disruptions (Meier and Pinto, 2024).[5] We add to this empirical literature by documenting the macroeconomic and reallocation effects of turbulence.

To our knowledge, our paper represents a first attempt to study the transmission mechanism of turbulence shocks over the business cycle using firm-level data and a quantitative business cycle model featuring firm heterogeneity and financial frictions.

III. Empirical methodology

This section describes our empirical methods of measuring turbulence and the macroeconomic and reallocation effects of turbulence.

III.1. **Defining turbulence.** We measure turbulence by the Spearman correlations of firm productivity rankings between adjacent years, following the approach of Bloom et al. (2018).

[5]Countercyclical turbulence implies increased cross-firm reallocation in recessions, and this is consistent with some anecdotal observations. For example, following the stock market crash during the global financial crisis, some top firms declared bankruptcy or were bailed out by the government (e.g., Lehman, WaMu, Citigroup, AIG, GM, and Chrysler) while some other firms thrived, particular some startups such as Uber, Venmo, and Airbnb. The COVID-19 recession was also associated with important cross-firm reallocation (Barrero et al., 2021; Davis et al., 2020). To the extent that turbulence reflects supply-chain disruptions (as we show in Appendix C), the evidence of Meier and Pinto (2024) also suggests that turbulence can be an important contributing factor to the brief but sharp COVID-19 recession in the United States.

We construct a measure of firm-specific productivity based on the production function

$$Y_{jt} = z_{jt} F(K_{jt}, N_{jt}), \tag{1}$$

where Y_{jt} denotes value-added output of firm j in period t, K_{jt} and N_{jt} denote capital and labor inputs, respectively, and $F(K, N)$ is the production function. We assume that the idiosyncratic productivity, z_{jt}, follows the stochastic process

$$z_{j,t+1} = \begin{cases} z_{jt} & \text{with prob} \quad \rho_t, \\ \tilde{z}_{j,t+1} & \text{with prob} \quad 1 - \rho_t, \end{cases} \tag{2}$$

where $\tilde{z}_{j,t+1}$ is an i.i.d. random variable across time and across firms, with the cumulative density function $\tilde{G}(z)$.[6]

Under the stochastic process in Eq. (2), a firm's productivity level in period $t+1$ can stay the same as that in period t, in which case the ranking of firm productivity also stays the same. This occurs with the probability ρ_t. With the complementary probability $1 - \rho_t$, the firm's productivity in period $t + 1$ is an i.i.d. random variable $\tilde{z}_{j,t+1}$, which is independent of the period-t productivity, such that firm productivity in $t + 1$ would be uncorrelated with that in t.

The term ρ_t measures the persistence of firm-level TFP. In the extreme case with $\rho_t = 1$ for all t, the productivity level would be permanent: high-productivity firms would remain productive and low-productivity firms would remain unproductive. In the other extreme with $\rho_t = 0$, productivity would be an i.i.d. process, with no persistence. In the more general case with $\rho_t \in (0, 1)$, productivity is persistent, and the persistence is time varying. A decline in ρ_t implies that high-productivity firms in period t would be less likely to remain productive in period $t + 1$ and low-productivity firms in period t would have a chance to become more productive. Thus a decline in ρ_t increases the churn of firm rankings in the productivity distribution. We measure micro-level turbulence by $1 - \rho_t$.

Turbulence is related to but different from the micro-level uncertainty studied by Bloom et al. (2018). An increase in micro-level uncertainty corresponds to a mean-preserving spread of the cross-sectional productivity distribution—an increase in the variance or inter-quartile range (IQR) of productivity. An increase in turbulence also raises the conditional variance of

[6]The countercyclical property of turbulence does not hinge upon this particular productivity process, and it can be obtained with a continuous-state AR(1) process such as that considered by Bloom et al. (2018). The discrete-state TFP process helps simplify the computation and solution of our theoretical model below. To maintain internal consistency, we assume the same productivity process throughout our analysis.

the productivity distribution, as does uncertainty. Thus, turbulence is positively correlated with micro-level uncertainty.[7]

However, unlike uncertainty, turbulence changes not only the conditional variance but also the conditional mean of the firm-level productivity distribution. Through its impact on the conditional mean of firm productivity, a turbulence shock generates between-firm reallocation, which is essential for generating procyclical aggregate productivity and business cycle comovements.

Furthermore, turbulence does not affect the *ex ante* stationary distribution of productivity. That is, a turbulence shock is an *ex ante* distribution-preserving shock, as shown in Proposition 1 below.[8]

Proposition 1. The cross-sectional stationary distribution of idiosyncratic productivity (denoted by $G_t(z)$) is invariant to the realization of ρ_t.

Proof. Under the stochastic process of idiosyncratic productivity specified in Eq. (2), the cumulative density function of productivity is given by

$$\begin{aligned} G_{t+1}(z) &= Pr(z_{t+1} \leq z) \\ &= Pr(z_t \leq z)\rho_t + Pr(\tilde{z} \leq z)(1-\rho_t) \\ &= G_t(z)\rho_t + \tilde{G}(z)(1-\rho_t). \end{aligned} \tag{3}$$

Under the stationarity of the distribution of z, we have $G_t(z) = \tilde{G}(z)$ for all t. Thus, the stationary distribution is independent of the realization of ρ_t. □

III.2. **Measuring turbulence.** If idiosyncratic productivity is perfectly measured, then the Spearman rank correlation of firm productivity between adjacent periods would provide a correct measure of ρ_t and thus of turbulence (i.e., $1-\rho_t$).[9]

III.2.1. *Measurement challenges.* In general, however, firm-level productivity can be measured with errors. It is well-known in the productivity literature that revenue-based measures of firm-level TFP contain not only true productivity but also information about demand conditions (Syverson, 2004). Measurement errors in productivity can pose challenges for measuring turbulence.

[7]For example, the correlation between our turbulence measure and the IQR of firm-level TFP from Compustat data is about 0.55.

[8]The distribution-preserving turbulence that we study here can be viewed a discrete-time counterpart to the persistence of idiosyncratic productivity shocks in the continuous-time models of Moll (2014), which is also orthogonal to the stationary productivity distribution.

[9]We formally show this in Proposition 2 in appendix A.1.

To put the measurement challenges into context, consider the case with i.i.d. measurement errors in observed firm TFP, which is given by

$$\hat{a}_{jt} = \hat{z}_{jt} + \hat{\tau}_{jt}, \tag{4}$$

where $\hat{a}_{jt}$ denotes the observed TFP (in log units) of firm j at time t, $\hat{z}_{jt}$ denotes the true TFP (also in log units) that follows the process in Eq. (2), and $\hat{\tau}_{jt}$ denotes a measurement error that is uncorrelated with $\hat{z}_{jt}$ and is i.i.d. across firms and across time, drawn from the normal distribution $N(0, \sigma_t^2)$. We allow the variance of the measurement error (denoted by σ_t) to be time-varying.

Given the stochastic process of true productivity in Eq. (2), the measured TFP follows the process

$$\begin{aligned} \hat{a}_{j,t+1} &= \begin{cases} \hat{z}_{jt} + \hat{\tau}_{jt+1} & \text{with prob} \quad \rho_t, \\ \tilde{\hat{z}} + \hat{\tau}_{jt+1} & \text{with prob} \quad 1-\rho_t, \end{cases} \\ &= \begin{cases} \hat{a}_{jt} + \underbrace{\hat{\tau}_{jt+1} - \hat{\tau}_{jt}}_{\equiv \hat{e}_{jt+1}} & \text{with prob} \quad \rho_t, \\ \tilde{\hat{z}} + \hat{\tau}_{jt+1} & \text{with prob} \quad 1-\rho_t. \end{cases} \end{aligned} \tag{5}$$

The presence of measurement errors in productivity gives rise to two challenges in estimating the true process of turbulence. The first challenge is heteroskedasticity. With a time-varying volatility of $\hat{\tau}_{jt}$, the variance of the residual term $\hat{e}_{j,t+1}$ in Eq. (5) is also time-varying. Thus, the OLS estimator of the auto-correlation in $\hat{a}_{jt}$ can be biased and inconsistent. We tackle this heteroskedasticity issue by estimating the rank correlations of the observed firm productivity between adjacent periods, because the rank distribution is time-invariant regardless of the functional forms of the underlying distribution of the observed productivity.

The second challenge is the standard endogeneity problem in a dynamic panel model. In the empirical specification (5), the residual term $\hat{e}_{j,t+1}$ is correlated with the independent variable $\hat{a}_{jt}$ because both are functions of the measurement error $\hat{\tau}_{jt}$. In the spirit of Arellano and Bond (1991), we address the endogeneity problem by an instrumental-variable (IV) estimation approach, using the rankings of the lagged productivity $\hat{a}_{j,t-1}$ and $\hat{a}_{j,t-2}$ as instruments for the ranking of $\hat{a}_{jt}$. Since $corr(\hat{a}_{j,t-1}, \hat{e}_{jt+1}) = 0$ and $corr(\hat{a}_{j,t}, \hat{a}_{j,t-1}) > 0$, these IVs satisfy both the exclusion restriction and the relevance condition.

III.2.2. *The data.* To implement the IV approach to measuring turbulence, we use firm-level data from Compustat Fundamentals Annual database. To obtain measures of industry-level employment, payroll, and price indices, we use information from the NBER-CES Manufacturing Industry Database.[10] By combining these two data sources, we obtain an unbalanced panel with 53,285 firm-year observations. This full sample (Sample 1) includes all listed firms in all manufacturing industries covered by NBER-CES in the years from 1958 to 2016.[11] Table A.1 in the Appendix presents the summary statistics of our samples.

Following Bloom et al. (2018), we focus on the subset of firms with 25+ years of observations and use it as our baseline sample (Sample 2) for estimating firm-level TFP. The baseline sample contains about 29304 firm-year observations. Since firms in the baseline sample are older than those in the full sample, they are also larger on average in terms of assets, value added, capital, and employment, although their average growth rates of employment and capital are slower.[12]

III.2.3. *Measured turbulence.* We measure firm-level TFP based on Solow residuals calculated from the constant-returns production function

$$tfp_{ijt} = y_{ijt} - \alpha_{it}k_{ijt} - (1 - \alpha_{it})n_{ijt}, \tag{6}$$

where tfp_{ijt} is a revenue-based measure of TFP (in log units) of firm j in industry i and year t, and y_{ijt}, k_{ijt} and n_{ijt} denote the firm's value added, capital input, and labor input, respectively, all in log units. Following Bloom et al. (2018), we assume that the cost share of capital input α_{it} is common for all firms within an industry i, although it can vary across time.[13]

After obtaining the firm-level TFP, we construct a measure of turbulence using an IV estimation approach. Specifically, we rank firms within each industry (at the 3-digit level) by deciles of their productivity levels. We then estimate the rank correlation of firm TFP

[10]The Compustat database is accessed through Wharton Research Data Service at: https://wrds-web.wharton.upenn.edu/wrds/. The NBER-CES database is accessed through https://www.nber.org/research/data/nber-ces-manufacturing-industry-database.

[11]Our sample does not cover the COVID-19 recession because of the limited availability of industry deflator data from the NBER-CES. We include firms incorporated in the US (Compustat fic='USA') that trade on major stock exchanges (NYSE, AMEX, and NASDAQ, Compustat exchg = 11, 12 or 14), for which the native currency is US dollars (Compustat curcd='USD'). We exclude firm-year observations with obvious errors: missing or nonpositive values in reported revenue, employment, and capital. We remove a firm if it was involved in a major merger or acquisition that affected its asset by more than 10 percent.

[12]In a robustness check, we further narrow down the sample and focus on industries with more than 20 firms in each year. This sample (Sample 3) contains about 19,000 firm-year observations. Firms in Sample 3 have similar characteristics as those in Sample 2.

[13]In our sample, the average value of the cost share of capital (weighted by the value of shipment) is about 0.34. We provide some details of our approach to measuring value added, capital and labor inputs, and the capital share in Appendix A.

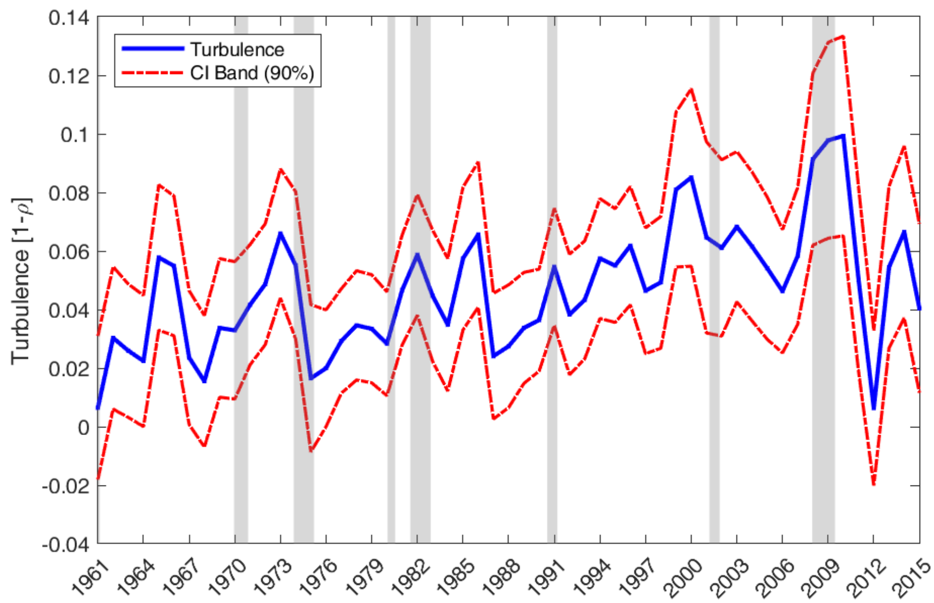

FIGURE 1. Measured turbulence

Note: Turbulence is measured by $1 - \rho_t$ (the blue solid line), where ρ_t is the IV estimator of Spearman correlation of firm TFP rankings between year t and year $t+1$. The dashed lines indicate the 90% confidence bands of the estimates. The gray shaded bars indicate NBER recession dates.

Source: Compustat, NBER-CES, BLS, and authors' calculations.

between year $t+1$ and t, using the rankings in years $t-1$ and $t-2$ as instrument variables for the ranking in year t. The time series of the estimated Spearman correlations corresponds to our measure of ρ_t, and turbulence is measured by $1 - \rho_t$. Our estimated turbulence series based on the IV approach is quite different from the OLS estimation (see Figure A1 in the appendix), reflecting substantial biases stemming from the heteroskedasticity and endogeneity issues discussed earlier.

Figure 1 plots the time series of our IV-based measure of turbulence for the years from 1965 to 2015. The mean, standard deviation, and autocorrelation of the estimated turbulence$(1-\rho_t)$ are 0.047, 0.021, and 0.55, respectively. The figure shows that turbulence is countercyclical, rising sharply in recessions.[14] The measured turbulence is negative correlated wtih

[14]In Appendix A, we show that the baseline estimate of turbulence is robust to alternative samples and alternative approaches to measuring productivity, and it is not driven by a subset of firms ranked on the top or on the bottom of the productivity distribution. Our measured turbulence displays an upward trend up to the global financial crisis. Since an increase in turbulence raises firm-level volatility, the trend increase in

manufacturing TFP and with the stock market value of firms, as shown in Figure A2 in the Appendix.

III.3. **The reallocation effects of turbulence.** Turbulence leads to expected churning of firm productivity rankings, with potential reallocation effects among firms. With an increase in turbulence, a high-productivity firm is less likely to remain productive in future periods, and such expectations could reduce the firm's growth in the current period through, for example, financial frictions as we elaborate below.

To quantify the reallocation effects of turbulence, we estimate the empirical specification

$$x_{jt} = \beta_0 + \beta_1 High\_TFP_{jt} + \beta_2 TURB_t * High\_TFP_{jt} + \phi_{jt} + \mu_j + \eta_t + \epsilon_{jt}, \tag{7}$$

where the dependent variable x_{jt} is the growth rate of firm j's employment, capital, or value-added in year t from $t-1$. The key independent variable is the interaction between turbulence ($TURB_t$) and a dummy indicator of high productivity firms ($High\_TFP_{jt}$).[15] If the coefficient (β_2) of the interaction term is negative, then turbulence would be associated with slower growth of high-productivity firms relative to low-productivity firms. The empirical specification also includes the linear term of the high-productivity indicator, along with a set of controls for time-varying firm characteristics (ϕ_{jt}, including Tobin's Q, ROA, cash ratio and book leverage), in addition to firm fixed effects (μ_j) and year fixed effects (η_t). The term ϵ_{jt} denotes regression errors.

Table 1 shows the estimation results. The baseline estimates of β_2 for employment growth, capital growth, and value-added growth are shown in Columns (1), (3), and (5), respectively. The negative estimated values of β_2 suggest that an increase in turbulence is associated with larger declines in the growth rates of high-productivity firms relative to those of low-productivity firms, and the p-values indicate that those estimates are statistically significant at the 99 percent confidence level. The estimated values of β_2 are also economically meaningful. The point estimates imply that a one-standard-deviation increase in turbulence is associated with a slower employment growth rate for high-productivity firms of about 6.7 percent. The same increase in turbulence also slows the growth rates of capital and value-added of high-productivity firms by about 2.0 percent and 1.2 percent, respectively.[16]

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turbulence is consistent with the empirical evidence that the volatility for publicly traded firms has increased steadily over time, whereas the volatility of privately held firms has declined (Davis et al., 2006).

[15] $High\_TFP_{jt}$ equals one if firm j's TFP level is above the median within its industry and zero otherwise.

[16] In our sample, the measured turbulence has a mean of 0.047 and a standard deviation of 2.1 percent. The average growth rates of employment, capital, and value-added of the high-productivity firms are 1.3%, 4.0%, and 5.6%, respectively. Thus, the impact of a one standard deviation increase in turbulence relative to its mean on the relative growth rates of employment, capital, and value-added are $(-0.878)\times 0.047\times 2.1/1.3 \approx -6.7\%$, $(-0.830)\times 0.047\times 2.1/4.0 \approx -2.0\%$, and $(-0.694)\times 0.047\times 2.1/5.6 \approx -1.2\%$, respectively.

TABLE 1. Impact of turbulence on firms with different levels of productivity

Dep. Var.	Δn_{jt}		Δk_{jt}		Δy_{jt}	
	(1)	(2)	(3)	(4)	(5)	(6)
$High\_TFP_{jt}$	0.001	0.023	0.004	-0.083***	0.073***	0.046***
	(0.006)	(0.016)	(0.008)	(0.021)	(0.007)	(0.017)
$TURB_t * High\_TFP_{jt}$	-0.878***	-0.526***	-0.830***	-1.005***	-0.694***	-0.311**
	(0.089)	(0.103)	(0.127)	(0.142)	(0.108)	(0.123)
$UNC_t * High\_TFP_{jt}$		-0.327***		0.469***		-0.149
		(0.093)		(0.119)		(0.096)
$\Delta GDP_t * High\_TFP_{jt}$		0.892***		0.492***		1.600***
		(0.088)		(0.110)		(0.105)
Constant	-0.019***	-0.018***	-0.035***	-0.035***	-0.017***	-0.015***
	(0.005)	(0.005)	(0.008)	(0.008)	(0.005)	(0.005)
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.042	0.047	0.064	0.065	0.024	0.033
Observations	26,204	25,953	26,204	25,953	26,204	25,953

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital expenditure, and value-added) on the measured turbulence ($TURB_t$) for firms with different levels of TFP. The dummy $High\_TFP_{jt}$ equals one if firm j's TFP is above the median within its industry at period t and zero otherwise. The level of uncertainty (UNC_t) is measured following Bloom et al. (2018) as the dispersion (IQR) of firm-level productivity shock at period $t+1$. ΔGDP_t denotes the growth rate of real output at period t. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The standard errors shown in the parentheses are double clustered by industry and time. The stars denote the p-values: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Since turbulence is countercyclical, the reallocation effects that we have estimated might be confounded by the impacts of recessions or uncertainty. To address this concern, we re-estimate the empirical specification in Eq. (7) by including controls for the direct reallocation effects of business cycles and uncertainty. In particular, we include in our regressions the interactions of the high-productivity indicator with real GDP growth (ΔGDP_t) and with the time series of micro-level uncertainty (UNC_t, measured by the dispersion of firm-level productivity constructed by Bloom et al. (2018)). After controlling for the reallocation effects of recessions and uncertainty, the reallocation effects of turbulence remain large and significant, as shown in Columns (2), (4), and (6) of Table 1. Our estimation further suggests that, unlike uncertainty that could have ambiguous effects on the relative growth rate of high-productivity firms (depending on the measurements of firm growth), turbulence consistently slows the relative growth of high-productivity firms.[17]

The reallocation effects of turbulence are also robust to alternative high-productivity indicators (lagged high-TFP indicators or finer grouping of TFP rankings) and alternative samples (excluding top manufacturing firms or including non-manufacturing industries), as we show in the Appendix (see Appendix A.3). In the Appendix, we also estimate the dynamic reallocation effects of turbulence using the local projections approach of Jordà (2005). The reallocation effects are persistent, lasting more than 5 years after the impact.

Overall, our evidence suggests that turbulence is associated with slower growth of high-productivity firms than that of low-productivity firms, implying reallocation of capital and labor from high- to low-productivity firms.

III.4. **Financing constraints and the reallocation effects of turbulence.** What drives the reallocation effects of turbulence? In a frictionless complete-market environment, turbulence would have no effects on resource allocations across firms. Turbulence increases the risk of future productivity churns but it does not affect the ex ante productivity distribution (Proposition 1). Thus, the propagation of turbulence in the aggregate economy requires some kind of frictions that link the risks of future productivity churns to current production decisions. Financing constraints are one such frictions. If firms need external financing of production, employment, and investment, then increased risks of future productivity churns (i.e., increased turbulence) would have an impact on their borrowing capacities through influencing their stock market values. We now present evidence that financial frictions are important for driving the reallocation effects of turbulence.

III.4.1. *Firm-level evidence.* To examine the extent to which the reallocation effects of turbulence might depend on financial frictions, we estimate the empirical specification

$$\begin{aligned} x_{jt} = \beta_0 + \beta_1 High\_TFP_{jt} + \beta_2 FF_{jt} + \beta_3 TURB_t * High\_TFP_{jt} + \beta_4 TURB_t * FF_{jt} \\ + \beta_5 High\_TFP_{jt} * FF_{jt} + \beta_6 TURB_t * High\_TFP_{jt} * FF_{jt} + \mu_j + \eta_t + \epsilon_{jt}, \end{aligned} \tag{8}$$

where the dependent variable x_{jt} denotes the growth rate of employment, capital, or value-added of firm j in year t from $t-1$, $TURB_t$ denotes measured turbulence, and $High\_TFP_{jt}$ is a dummy variable that equals one if firm j's TFP level is above the median within its industry at period t and zero otherwise. The term FF_{jt} denoted a text-based measure of

[17]Our results here suggest that high-TFP firms are more cyclically sensitive: they respond more than low-TFP firms to turbulence, uncertainty, or recessions. Since firm productivity and firm size are not perfectly correlated, our results do not necessarily contradict the finding of Crouzet and Mehrotra (2020) that large firms (the top 1 percent by size) are less cyclically sensitive than other firms.

firm-specific financial constraints constructed by Hoberg and Maksimovic (2015). A higher value of FF_{jt} indicates higher similarity of firm j to a set of firms known to be at risk of delaying their investments due to difficulty in raising debt as indicated in the Management's Discussion and Analysis (MD&A) section in 10-Ks. In our regressions, we controlling for firm fixed effects (μ_j) and year fixed effects (η_t). The term ϵ_{it} denotes regression errors.[18]

We are interested in how financial frictions could affect the reallocation effects of turbulence across firms with different levels of productivity. Thus, the key independent variable of interest is the triple interaction term $TURB_t * High\_TFP_{jt} * FF_{jt}$. As we have discussed in Section III.3, turbulence slows the relative growth of high-productivity firms, reallocating resources from high- to low-productivity firms. A negative value of the coefficient β_6 of the triple interaction term would imply that tighter financing constraints amplify the reallocation effects of turbulence.

The estimation results shown in Table 2 confirm that (1) turbulence slows the growth rate of high-productivity firms (i.e., the coefficient β_3 for the double interaction term $TURB_t * High\_TFP_{jt}$ is negative) and (2) financing constraints indeed amplify the reallocation effects of turbulence (i.e., the coefficient β_6 for the triple interaction term $TURB_t * High\_TFP_{jt} * FF_{jt}$ is also negative). This firm-level evidence suggests that financial frictions play an important role in driving the reallocation effects of turbulence.

III.4.2. *Industry-level evidence.* We now present industry-level evidence for the importance of financial frictions in driving the reallocation effects of turbulence. We aggregate the firm-level data in Compustat to NAICS 4-digit industry level and estimate the empirical specification

$$\begin{aligned} IQR_{it} = \beta_0 + \beta_1 High\_FF_{it} + \beta_2 TURB_t * High\_FF_{it} + \beta_3 UNC_t * High\_FF_{it} \\ + \beta_4 \Delta GDP_t * High\_FF_{it} + \mu_i + \eta_t + \epsilon_{it}, \end{aligned} \tag{9}$$

where the dependent variable IQR_{it} denotes interquartile range (IQR) of labor (or capital) of firms in industry i in year t, $TURB_t$ denotes measured turbulence, and $High\_FF_{it}$ is a dummy variable that equals one if industry i's financial constraint is above the median level among all NAICS 4-digit industries in year t. We obtain an industry-level measure

[18]For estimating (8), we restrict our sample to the subset of firms for which a measure of financial constraints is available. This restriction reduces our sample size from over 25,000 observations in the baseline sample to 6,629 observations, making it more difficult to obtain precise estimates. Adding to this challenge, including the triple interaction term in the empirical specification (8) increases the number of parameters to be estimated relative to the baseline specification (7). Given our sample limitations, we do not include controls for the effects of recessions and uncertainty (and their interactions with the high-productivity indicator and with the financing constraint indicator), which would have further increased the number of parameters to be estimated, making it even more difficult to obtain precise estimates.

Table 2. Financial frictions and reallocation effects of turbulence: Firm-level evidence

	(1)	(2)	(3)
Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}
$High\_TFP_{jt}$	0.018	0.023	0.155***
	(0.018)	(0.023)	(0.024)
FF_{jt}	-0.576***	-0.595***	-1.631***
	(0.164)	(0.208)	(0.217)
$TURB_t * High\_TFP_{jt}$	-0.560**	-0.709**	-0.965***
	(0.259)	(0.330)	(0.343)
$TURB_t * FF_{jt}$	8.099***	9.110***	22.173***
	(2.465)	(3.133)	(3.263)
$High\_TFP_{jt} * FF_{jt}$	0.389	0.476	1.253***
	(0.246)	(0.312)	(0.325)
$TURB_t * High\_TFP_{jt} * FF_{jt}$	-5.945	-10.255**	-15.939***
	(3.708)	(4.713)	(4.907)
$Constant$	0.057***	0.098***	0.073***
	(0.011)	(0.014)	(0.014)
Firm Fixed Effect	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes
R-squared	0.028	0.027	0.068
Observations	6,629	6,629	6,629

Note: This table shows the estimation results from the empirical specification (8) using firm level data. The 3 columns shows the regressions with the dependent variable being the growth rate of employment, capital expenditure, and value-added, respectively. The independent variables include (1) the dummy variable $High\_TFP_{jt}$ that equals one if firm j's TFP is above the median within its industry at year t and zero otherwise; (2) FF_{jt} that measures firm j's financing constraints in year t; (3) the interaction between turbulence and the high-TFP indicator $TURB_t * High\_TFP_{jt}$; (4) the interaction between turbulence and financing constraints $TURB_t * FF_{jt}$; (5) the interaction between the high-TFP indicator and financial constraints $High\_TFP_{jt} * FF_{jt}$; and (5) the triple interaction $TURB_t * High\_TFP_{jt} * FF_{jt}$. The sample is restricted to the subset of firms with an available measure of financing constraints (FF_{jt}) between 1997 and 2015. The stars denote the p-values: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

of financial constraint by taking the within-industry sales-weighted average of the firm-level KZ indices[19]. In our regressions, we include the interactions of the high-financial constraint indicator with real GDP growth (ΔGDP_t) and with the time series of micro-level uncertainty (UNC_t. we also control for industry fixed effects (μ_i) and year fixed effects (η_t). The term ϵ_{it} denotes regression errors.

Changes in the IQR of employment (or capital) associated with changes in turbulence capture the reallocation effects within an industry. For example, a decline in the IQR of employment in an industry following a rise in turbulence would indicate reallocation of workers from firms with more workers to those with fewer workers. We are interested in how financial frictions could affect the reallocation effects of turbulence. This effect is captured by the coefficient β_2 on the interaction term $TURB_t * High\_FF_{it}$ in Eq. (9). The estimation results are displayed in Table 3.

In the baseline specifications (Columns (1), (3) and (5)), the estimated values of β_2 are negative and statistically significant at the 90 percent confidence level. Thus, an increase in turbulence is associated with greater declines the IQRs of both employment, capital and value-added in industries facing tighter financial constraints. The positive estimates of β_1 indicate that, absent turbulence, an industry with tighter financial constraints has also greater within-industry dispersion of employment and capital.

As we did in estimating equation (7), we show robustness of the results by adding two additional interaction terms between uncertainty or GDP growth rate and the high-financial friction dummy. The average effects of turbulence, uncertainty and recession are absorbed by year-fixed effects. Columns (2), (4), and (6) of Table 3 show that the financial friction amplifies the misallocation effects of turbulence shock after controlling for potential effects of recessions and of uncertainty. Furthermore, unlike turbulence which is associated with significant declines in the IQRs of employment and of capital for firms facing relatively high financial frictions, uncertainty is associated with increases in those variables, although the estimated correlations with uncertainty are statistically insignificant. Overall, our estimation suggests that tighter financial constraints are associated with larger declines in the cross-sectional dispersion of employment, capital and value-added when turbulence rises.

[19]While existing studies have proposed alternative approaches to measure financial constraint at the firm level, to our knowledge there is no consensus on the measure at the industry-level. Since firm-level 10K text cannot be aggregated to the industry level, we instead construct the industry-level index following Kaplan and Zingales (1997) and Lamont et al. (2001), which is a linear function of five categories of accounting variables, namely firm cash flow, long-term debt, dividend-to-asset ratio and Tobin's Q, that best match expert evaluations of 10-K MD&A statements to measure financial constraints. The coefficients associated with flow variables are adjusted to annual frequency, and a higher index value suggests a firm is more constrained.

Table 3. Financial frictions and reallocation effects of turbulence: Industry-level evidence

Dep. Var.	IQR of Employment		IQR of Capital		IQR of Value-added	
	(1)	(2)	(3)	(4)	(5)	(6)
$High\_FF_{it}$	0.185**	-0.138	0.261***	-0.304	0.050	-0.473
	(0.075)	(0.267)	(0.082)	(0.293)	(0.085)	(0.304)
$TURB_t * High\_FF_{it}$	-3.434**	-3.800**	-4.584***	-5.151***	-3.401**	-3.976**
	(1.368)	(1.629)	(1.503)	(1.787)	(1.558)	(1.854)
$UNC_t * High\_FF_{it}$		1.585		2.726		2.557
		(1.598)		(1.753)		(1.818)
$\Delta GDP_t * High\_FF_{it}$		2.378*		4.373***		3.906**
		(1.384)		(1.518)		(1.575)
Constant	2.017***	2.017***	5.737***	5.737***	6.672***	6.672***
	(0.024)	(0.024)	(0.026)	(0.026)	(0.027)	(0.027)
Industry Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Industry Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.649	0.649	0.769	0.770	0.709	0.709
Observations	3,237	3,149	3,237	3,149	3,237	3,149

Note: This table shows the regression of interquartile range of employment, capital and value-added on the measured turbulence ($TURB$) for industries with different levels of financial constraint. The dummy $High\_FF_{it}$ equals one if industry i's financial constraint is above the median. The level of uncertainty (UNC_t) is measured following Bloom et al. (2018) as the dispersion (IQR) of firm-level productivity shock at period $t+1$. ΔGDP_t denotes the growth rate of real output at period t. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years in the sample from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries and year. The stars denote the p-values: $^* p < 0.1$; $^{**} p < 0.05$; $^{***} p < 0.01$.

III.5. **Macroeconomic effects of turbulence.** We now examine the macroeconomic effects of turbulence. For this purpose, we estimate the impulse responses of several key macroeconomic variables to a turbulence shock using the local projections approach of Jordà (2005).[20]

[20]As shown by Plagborg-Møller and Wolf (2021), linear local projections and vector autoregressions (VARs) estimate the same impulse responses when the lag structures are unrestricted, without imposing any parametric assumptions on the data generating process.

Since our empirical measure of turbulence is not exogenous, we include a set of control variables to orthogonalize the effects of turbulence. Specifically, we consider the local projections specification

$$y_{t+h} - y_{t-1} = \beta_0^h + \beta_1^h TURB_t + \beta_2^h UNC_t + \Gamma_{t-1}\Omega^h + \epsilon_{t+h} \quad h = 0, 1, 2, 3, 4. \tag{10}$$

The dependent variable $y_{t+h} - y_{t-1}$ denotes the cumulative changes in the log-level of the variable of interest from year $t-1$ to year $t+h$, where h denotes the projection horizons (number of years). The set of dependent variables that we are interested includes per capita real private consumption expenditure, private fixed investment, private output (i.e., the sum of consumption and investment), hours worked, S&P 500 stock price index, manufacture TFP, and dispersions of sales in both levels and growth rates (measured by the IQR of these variables in our sample). The key independent variable is the annual time series of our measured turbulence ($TURB_t$). The set of control variables includes contemporaneous uncertainty (UNC_t) and lags of a set of macroeconomic variables (Γ_{t-1}), including lagged turbulence ($TURB_{t-1}$), lagged uncertainty (UNC_{t-1}), and the lagged growth rates of all the dependent variables.[21] The term ϵ_{t+h} is the regression residual. The parameter β_1^h measures the impulse responses of the macroeconomic variables to a turbulence shock at horizon h.

Figure 2 plots the estimated impulse responses of the macroeconomic variables to a one-standard-deviation turbulence shock for horizons up to four years.[22] The shock reduces firm value and TFP and leads to a recession with synchronized and persistent declines in aggregate output, consumption, investment, hours worked. A turbulence shock also leads to a decline in the dispersion of sales levels and a rise in the dispersion of sales growth (measured by the IQR of these variables). In comparison, an uncertainty shock leads to a rise in the dispersion of both sales levels and sales growth (see Figure A6 in Appendix A.4).

The macroeconomic effects of turbulence are quantitatively important. For example, a one-standard-deviation increase in turbulence reduces private output by about 0.25 percent on impact, and by around 1.5 percent within three years after the shock. Turbulence is also important for macroeconomic fluctuations in the sense of forecast-error variance decompositions (FEVD). Following the approach of Gorodnichenko and Lee (2020), we estimate that, over the forecasting horizons of up to 4 years, turbulence shocks contribute to between 10

[21] Including these control variables mitigates but does not eliminate concerns of endogeneity of turbulence. Thus, the estimated impulse responses should not be interpreted as causal effects of turbulence.

[22] The estimated standard deviation of turbulence shock is 0.0147.

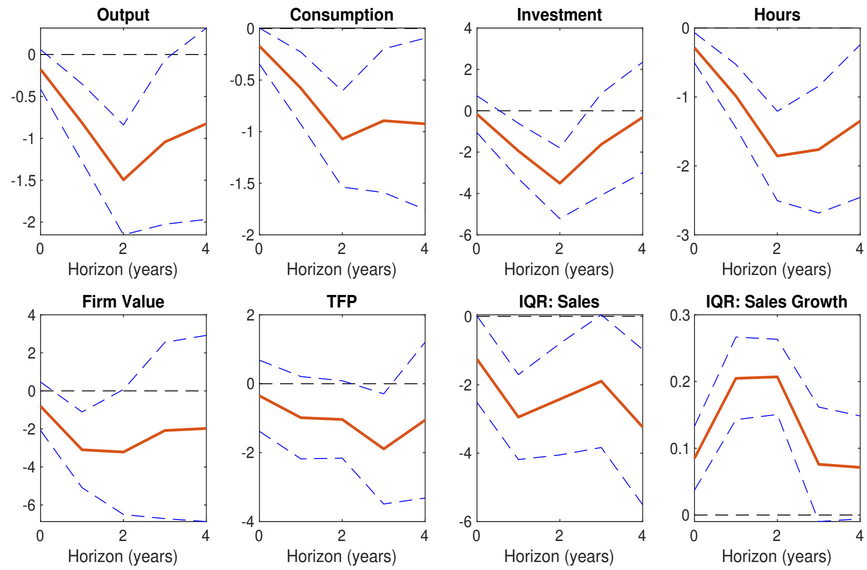

FIGURE 2. Estimated impulse response of macroeconomic variables to a turbulence shock

Note: This figure shows the impulse responses of macroeconomic variables to a one-standard-deviation shock to turbulence estimated from the local projections model (10). The solid lines show the point estimates of the impulse responses (%). The blue dashed lines show the 68% confidence intervals.
Source: BEA, Compustat, NBER-CES, and authors' calculations.

and 15 percent of the forecast-error variances of aggregate output, investment, and consumption. This magnitude of contributions from turbulence is modest, but comparable to those of uncertainty shocks.[23]

IV. A REAL BUSINESS CYCLE MODEL WITH TURBULENCE SHOCKS

We now construct a real business cycle model to examine the economic mechanism through which turbulence can drive macroeconomic fluctuations and cross-sectional reallocation. In light of the empirical evidence presented in Section III, we incorporate into the model two

[23]We discuss the methodology for computing the forecast error variance decomposition in Appendix A.5. We have also calculated historical decompositions of the shock contributions to output fluctuations based on a VAR specification that includes uncertainty, turbulence, and private output growth, with the same Cholesky ordering. We find that shocks to turbulence contributed about 20% of the actual declines in output growth during the Great Recession. Again, this magnitude of contribution is modest, but comparable to that of uncertainty shocks. See Appendix A.6 for details.

key ingredients—firm heterogeneity and financial frictions. We show that these ingredients are both important for the transmission of turbulence shocks.

IV.1. **The model.**

Household. The model economy is populated by a continuum of infinitely lived households with measure one. The representative household has the utility function

$$\mathbf{E}\sum_{t=0}^{\infty}\beta^t\left\{\log(C_t)-\psi\frac{N_t^{1+\gamma}}{1+\gamma}\right\}, \tag{11}$$

where C_t denotes consumption, N_t denotes labor hours, and $\mathbf{E}$ is an expectation operator. The parameter $\beta \in (0,1)$ is a subjective discount factor, $\psi > 0$ measures the relative weight on the disutility of working, and $\gamma \geq 0$ is the inverse Frisch elasticity of labor supply.

All markets are perfectly competitive. The household takes prices as given and maximizes the utility in Eq. (11) subject to the sequence of budget constraints,

$$C_t + K_{t+1} = (R_t + 1 - \delta)K_t + W_t N_t + D_t - T_t, \tag{12}$$

where K_{t+1} denotes the end-of-period capital stock, R_t denotes the capital rental rate, W_t denotes the real wage rate, D_t denotes the dividend income from firms, and T_t denotes a lump-sum tax paid to the government. The capital stock evolves according to the law of motion

$$K_{t+1} = (1-\delta)K_t + [1 - \frac{\Omega_k}{2}(\frac{I_t}{I_{t-1}} - 1)^2]I_t \tag{13}$$

where the parameter $\Omega_k > 0$ measures the size of the investment adjustment costs in the spirit of Christiano et al. (2005).

The household's decision rules are characterized by the following equations, where Λ_t denotes marginal utility.

$$\Lambda_t = \frac{1}{C_t} \tag{14}$$

$$\psi N_t^{\gamma} = \Lambda_t W_t \tag{15}$$

$$1 = q_t\left(1 - \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega_k\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}q_{t+1}\Omega_k\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{16}$$

$$q_t = \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left(R_{t+1} + (1-\delta)q_{t+1}\right), \tag{17}$$

where q_t denotes the shadow value of capital, or Tobin's q.

Firms. There is a continuum of firms, each endowed with a constant-returns technology that produces the final consumption good using capital and labor as inputs. Firms face idiosyncratic productivity shocks drawn at the beginning of each period, before hiring inputs. The production function for an individual firm is given by

$$y_{jt} = A_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha}, \tag{18}$$

where y_{jt} denotes the output produced by firm j in period t, A_t denotes aggregate productivity, and k_{jt} and n_{jt} denote the capital and labor inputs, respectively.

The idiosyncratic productivity shock z_{jt} follows the stochastic process described in Eq. (2), which we rewrite here for convenience of referencing:

$$z_{j,t+1} = \begin{cases} z_{jt} & \text{with prob} \quad \rho_t, \\ \tilde{z}_{j,t+1} & \text{with prob} \quad 1-\rho_t. \end{cases} \tag{19}$$

Here, the term $\tilde{z}_{j,t+1}$ is an i.i.d. random variable with a finite number of states. Specifically, we assume that $\tilde{z}_{j,t+1} = z_j$ with probability π_j, for $j = 1, 2, ..., J$. Without loss of generality, we further assume that $z_1 < z_2 < ... < z_J$. The process features time-invariant cross-sectional distribution of firm productivity such that, regardless of the realization of $\rho_t \in (0, 1)$, there is always a fraction π_j of firms with $z_{jt} = z_j$ in each period. Thus, in a stationary equilibrium, π_j is the measure of firms with productivity z_j.

We measure turbulence by $1 - \rho_t$. If $\rho_t = 1$, then the idiosyncratic productivity z_{jt} would be permanent. If $\rho_t = 0$, on the other hand, then each firm would face i.i.d. shocks to productivity with no persistence. A lower value of ρ_t implies that a high-productivity firm in the current period may not maintain its productivity in the next period, whereas a low-productivity firm in the current period might be able to draw a better productivity in the next period. Thus, a decline in ρ_t reshuffles firms' productivity ranking across time, increasing turbulence.[24]

We assume that the turbulence shock follows the stochastic process

$$\ln(1-\rho_t) = (1-\rho_\rho)\ln(1-\bar{\rho}) + \rho_\rho \ln(1-\rho_{t-1}) + \sigma_\rho \varepsilon_t^{\rho}, \tag{20}$$

where $\bar{\rho}$ denotes the average level of ρ_t and the innovation term ε_t^{ρ} follows a standard normal process. The parameter ρ_ρ and σ_ρ measure the persistence and the volatility of the turbulence shock, respectively.

[24]The turbulence shock in our model is isomorphic to supply-chain disruptions in a simple framework with input-output connections, where the productivity of a final goods producer depends on the quality of its suppliers in a match. If a supply-chain relation is separated, the final good producer needs to find a new supplier with random match quality. We show that an exogenous separation shock to the supply-chain relations can be mapped to the turbulence shock in the baseline model (see Appendix C).

Firms rely on external financing of their working capital. In the beginning of each period, firms need to borrow from a competitive financial intermediary to cover payments for input factors, and these working capital loans are repaid within the period, after firms receive revenues. Following Jermann and Quadrini (2012) and Liu and Wang (2014), we assume that a firm's borrowing capacity is constrained by a fraction θ of its expected equity value in the next period, in line with the empirical evidence of Lian and Ma (2021).

Firms at each level of productivity face idiosyncratic production distortions (denoted by τ_{jt}), reflecting differential policy interventions or government subsidies at the firm level (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Buera and Shin, 2013; Moll, 2014). These production distortions drive a wedge between firms' private and social marginal revenue products. We assume that τ_{jt} is independent of z_{jt} and it is drawn from a continuous i.i.d. distribution $F(\tau_{jt})$.[25] Under credit constraints, the presence of idiosyncratic production distortions allows a fraction of firms at each level of productivity to stay active, enabling turbulence shocks to generate reallocation and endogenous fluctuations in aggregate TFP.

The firms' optimizing problem is characterized by the Bellman equation

$$V_t(z_{jt}, \tau_{jt}) = \max_{k_{jt}, n_{jt}} \tau_{jt} A_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha} - R_t k_{jt} - W_t n_{jt} + \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}), \tag{21}$$

subject to the working capital constraint

$$R_t k_{jt} + W_t n_{jt} \leq \theta \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}) \equiv \theta B_{jt}. \tag{22}$$

Here, the term $V_t(z_{jt}, \tau_{jt})$ denotes the value function of firm j that depends on the firm-level state variables z_{jt} and τ_{jt}. The value function $V_t(z_{jt}, \tau_{jt})$ also depends on aggregate shocks, which are summarized by the time subscript t. The term $M_{t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}$ denotes the stochastic discount factor determined by the marginal utilities of the representative

[25]Our results do not depend on the assumption that τ_{jt} is independent of z_{jt}. In Appendix D, we consider a version of the model where τ_{jt} is correlated with z_{jt} along the lines of Restuccia and Rogerson (2008). We show that such correlations do not affect the main results of our baseline model. Including idiosyncratic distortions also serves a technical purpose in our model with a discrete distribution of idiosyncratic productivity. The continuity of the distribution function $F(\tau_{jt})$ implies a well-defined cutoff point τ_{jt}^* that determines the subset of active firms at each level of productivity z_{jt}. This assumption simplifies the computation of our model significantly. In Appendix E, we present an alternative framework with a continuous-state productivity distribution without idiosyncratic distortions. We show that the macroeconomic effects of changes in turbulence in the steady-state equilibrium are similar to those obtained in our baseline model. However, solving the dynamic equilibrium of that model with aggregate shocks would be more challenging because one would need to discretize the productivity distribution. With discretized productivity, one would also need to assume some smoothing techniques along the lines of Dotsey et al. (1999) to ensure a well-defined threshold level of productivity for active production. The continuous distribution of the idiosyncratic distortions in our baseline model serves such a purpose.

household who owns all firms. The term B_{jt} denotes the expected present value of a firm with current productivity z_{jt}.

Profit maximizing implies the conditional factor demand functions

$$\alpha \frac{\tau_{jt} y_{jt}}{k_{jt}} = (1+\mu_{jt}) R_t, \tag{23}$$

and

$$(1-\alpha) \frac{\tau_{jt} y_{jt}}{n_{jt}} = (1+\mu_{jt}) W_t, \tag{24}$$

where μ_{jt} denotes the Lagrangian multiplier associated with the credit constraint (22). Using the factor demand functions, we can write the firm's flow profit as

$$d_{jt} \equiv \left[\tau_{jt} A_t z_{jt} \left(\frac{\alpha W_t}{(1-\alpha) R_t} \right)^{\alpha} - \frac{W_t}{1-\alpha} \right] n_{jt}. \tag{25}$$

Since production subsidies follow an i.i.d. process, a firm would choose to be active in production if and only if its subsidy τ_{jt} is sufficiently high such that $d_{jt} \geq 0$. It follows that there exists a threshold level of production subsidy τ^*_{jt} such that, if $\tau_{jt} \geq \tau^*_{jt}$, then a firm would be active in production, facing binding credit constraints. Otherwise, the firm would remain inactive. At the threshold level of subsidy, a firm earns zero profit and thus it would be indifferent between producing and staying inactive. The indifference condition determines the threshold level of subsidy

$$\tau^*_{jt} = \frac{R_t^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A_t z_{jt}}. \tag{26}$$

The threshold τ^*_{jt} increases with the factor prices R_t and W_t and decreases with the productivity level z_{jt}. Thus, given the factor prices, the fraction of active firms is larger for firms with higher productivity.

The presence of credit constraints and production distortions creates misallocation of resources. Absent those distortions, all resources would be allocated to the most productive firm (with productivity z_J). However, under those distortions, some low-productivity firms are able to produce because not all high-productivity firms are active. Specifically, at each level of productivity, there is a non-degenerate fraction of firms that are active, with the share of active firms measured by $1 - F(\tau^*_{jt})$ for all $j \in 1, \ldots, J$. Such misallocation opens up a reallocation channel for turbulence shocks, as we show below.

Since active firms face binding credit constraints and inactive firms do not use any input factors, we obtain the conditional demand functions for labor and capital inputs

$$n_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{(1-\alpha)\theta B_{jt}}{W_t}, & if \quad \tau_{jt} \geq \tau^*_{jt} \\ 0, & otherwise. \end{cases} \tag{27}$$

and

$$k_t(z_{jt},\tau_{jt}) = \begin{cases} \frac{\alpha\theta B_{jt}}{R_t}, & if \quad \tau_{jt} \geq \tau_{jt}^* \\ 0, & otherwise. \end{cases} \tag{28}$$

Given the factor demand functions, firm j's value function can be written as

$$V_t(z_{jt},\tau_{jt}) = \max\left\{\frac{\tau_{jt}}{\tau_{jt}^*} - 1, 0\right\}\theta B_{jt} + B_{jt}. \tag{29}$$

Since production subsidies are i.i.d. across time, the average value of a firm with productivity z_{jt} is given by

$$\bar{V}_t(z_{jt}) = \int V_t(z_{jt},\tau)dF(\tau) = \left[1 + \theta\int_{\tau_{jt}^*}(\frac{\tau}{\tau_{jt}^*} - 1)dF(\tau)\right]B_{jt} \equiv \Phi(\tau_{jt}^*)B_{jt}, \tag{30}$$

where the term $\Phi(\tau_{jt}^*) \equiv 1 + \theta\int_{\tau_{jt}^*}(\frac{\tau}{\tau_{jt}^*} - 1)dF(\tau)$ is a decreasing function of the threshold subsidy level τ_{jt}^*.

Given the stochastic process of $z_{j,t+1}$ and the definition of B_{jt} in Eq. (22), we have

$$B_{jt} \equiv \beta\mathbb{E}_t\frac{C_t}{C_{t+1}}\left[\rho_t\bar{V}_{jt+1} + (1-\rho_t)\sum_{i=1}^{J}\pi_i\bar{V}_{it+1}\right]. \tag{31}$$

In a competitive equilibrium, markets for labor, capital, and final consumptions goods all clear. Labor market clearing implies that

$$N_t = \sum_j \pi_j N_{jt} \equiv \sum_j \pi_j \frac{(1-\alpha)\theta B_{jt}}{W_t}\left[1 - F(\tau_{jt}^*)\right]. \tag{32}$$

Capital market clearing implies that

$$K_t = \sum_j \pi_j K_{jt} \equiv \sum_j \pi_j \frac{\alpha\theta B_{jt}}{R_t}\left[1 - F(\tau_{jt}^*)\right]. \tag{33}$$

Goods market clearing implies that

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t, \tag{34}$$

where aggregate output Y_t is given by

$$Y_t \equiv \sum_j \pi_j Y_{jt} = \sum_j \pi_j A_t z_{jt} K_{jt}^{\alpha} N_{jt}^{1-\alpha}. \tag{35}$$

Given aggregate output, aggregate capital and labor inputs, we define aggregate TFP as

$$Z_t \equiv \frac{Y_t}{K_t^{\alpha}N_t^{1-\alpha}} = \frac{\sum_j \pi_j A_t z_{jt} K_{jt}^{\alpha} N_{jt}^{1-\alpha}}{K_t^{\alpha}N_t^{1-\alpha}}. \tag{36}$$

Definition. A competitive equilibrium consists of the sequence of allocations $\{C_t, Y_t, N_t, K_t\}$ and the sequence of prices $\{W_t, R_t\}$ such that (i) taking all prices as given, the allocations solve the household's utility maximizing problem and the firms' profit maximizing problem; and (ii) markets for labor, capital, and goods all clear.

IV.2. **The calibration.** A period in our model corresponds to a year. We set the subject discount factor to $\beta = 0.96$, implying an annualized risk-free interest rate of 4 percent. Based on our estimated firm-level production function parameters using the Compustat and NBER-CES data, we calibrate the cost share of capital to $\alpha = 0.34$. We set the capital depreciation rate to $\delta = 0.10$ to match the average annual investment rate of 10 percent in the U.S. data (Eisfeldt and Rampini, 2006; Clementi and Palazzo, 2016). We assume that labor is indivisible in the sense of Hansen (1985) and Rogerson (1988), implying that $\gamma = 0$. We calibrate the relative utility weight on leisure ψ such that the steady-state labor hours are one-third of the time endowment. We set the parameter θ to 0.35 to match the average ratio of working capital to market equity in the Compustat data.

The presence of the production distortions τ_{jt} can potentially complicate the calibration of the turbulence shock. Our revenue-based measure of firm-level TFP contains not only true productivity shocks z_{jt} but also information about demand conditions summarized in the production distortion term τ_{jt}. Specifically, the model implies that

$$tfp_{jt} = \log(z_{jt}) + \log(\tau_{jt}), \tag{37}$$

where tfp_{jt} is the firm-level TFP (in log units) that we construct based on the production function using data from Compustat and NBER-CES. For tractability, we assume that τ_{jt} is an i.i.d. process with a constant variance σ_τ and that τ_{jt} is uncorrelated with z_{jt}.[26]

If $\sigma_\tau > 0$, then the estimated average value of the Spearman rank correlation of the observed TFP would understate the true value of $\bar{\rho}$.[27] The presence of τ_{jt} in measured TFP would also distort the estimated value of σ_z. Thus, we need to jointly calibrate the values of σ_τ, $\bar{\rho}$, and σ_z.

We implement this calibration by targeting three moments in the model to their counterparts in the firm-level data. Those three moments in the data include (1) the average value of the Spearman rank correlations of plant-level TFP (0.72, estimated by Bloom et al. (2018)),[28] (2) the standard deviation of the firm-level TFP shock (0.247, based on firm-level

[26]In appendix, we derive a version where distortions are correlated with true productivity.

[27]To see this, consider the extreme case with $\bar{\rho} = 1$ (i.e., no changes in firm productivity ranking). Given the noise in observed productivity stemming from τ_{jt}, the estimated Spearman correlation using the observed TFP would be less than one. This is consistent with the upward bias in measuring turbulence based on OLS estimates that we have discussed in Section III.2.

[28]The calculations of Bloom et al. (2018) are based on plant-level data from the Census of Manufactures (CM) and the Annual Survey of Manufactures (ASM), which provide much broader and more granular coverage of U.S. businesses than the firm-level data in the Compustat.

TABLE 4. Calibrated parameters

	Parameter Description	Value	Target
β	Subjective discount factor	0.96	Average real interest rate of 4% per year
α	Capital share	0.34	Average cost share of capital (NBER-CES)
δ	Capital depreciation rate	0.10	Capital depreciation rate of 10% per year
Ω_k	Investment adjustment cost	1.15	Estimated (BEA)
γ	Inverse Frisch elasticity	0	Indivisible labor
ψ	Utility weight on leisure	2.15	Average hours of 1/3 of time endowment
θ	Loan to value ratio	0.35	Working capital to equity ratio (Compustat)
$\bar{\rho}$	Firm-level TFP persistence	0.93	Estimated (Compustat and NBER-CES)
σ_z	Volatility of firm-level TFP shock	0.08	Estimated (Compustat and NBER-CES)
σ_τ	Volatility of production distortion (log)	0.17	Estimated (Compustat and NBER-CES)
μ_τ	Average production distortion (log)	-0.014	Normalized
ρ_ρ	Persistence of turbulence shock	0.605	Estimated to match Bloom et al. (2018)
σ_ρ	Volatility of turbulence shock	0.40	Estimated to match Bloom et al. (2018)

TFP constructed using the Compustat/NBER-CES data), and (3) the average IQR of employment (in log) across firms (2.68, also from the Compustat data). This calibration implies that $\bar{\rho} = 0.93$, $\sigma_z = 0.08$, and $\sigma_\tau = 0.17$.

The presence of τ_{jt} can also affect the calibration of the turbulence shock process (i.e., ρ_ρ and σ_ρ). Given our calibration of σ_τ, $\bar{\rho}$, and σ_z, we use Eq. (37) to simulate the true productivity process z_{jt} and calibrate the two parameters ρ_ρ and σ_ρ to target the persistence and the standard deviation of the turbulence measure based on plant-level TFP (tfp_{jt}) constructed by Bloom et al. (2018). This process leads to our calibration of $\rho_\rho = 0.605$ and $\sigma_\rho = 0.40$.

IV.3. **Impulse responses to a turbulence shock.** To study the macroeconomic and reallocation effects of turbulence shocks, we solve our model based on calibrated parameters. We simulate the model using third-order approximations of the equilibrium conditions around the deterministic steady-state. We then compute impulse responses of several key macroeconomic and distributional variables as deviations of those variables driven by the turbulence shock from their stochastic steady-state levels without the shock.[29]

[29]We follow the approach in Fernández-Villaverde et al. (2011) and Leduc and Liu (2016) to compute the impulse responses. In particular, the model is first simulated for a large number of periods to compute the ergodic mean of each variable. It is then simulated using the ergodic means as a starting point. Finally, impulse responses to a turbulence shock are computed as the differences between the simulated path with

Figure 3 displays the impulse responses to a one-standard-deviation shock to turbulence. The increase in turbulence reduces the probability for a high-productivity firm to remain as productive in the future, and increases the probability for a low-productivity firm to become more productive in the future. Although firms' current productivity has not changed, the changes in the conditional expectations of future productivity reduce the average stock market value of firms, and they reduce the expected stock market value by more for high-productivity firms than for low productivity firms. Thus, high-productivity firms face disproportionately tightened working capital constraints, resulting in reallocation from high- to low-productivity firms and reducing aggregate TFP. The decline in TFP in turn leads to a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, as in the data. The recessionary effects of turbulence are sizable and persistent. For example, a one-standard-deviation turbulence shock leads to a drop in aggregate output of up to 1.5 percent, and output stays persistently below its steady-state level for more than four years after the shock.

The reallocation effects of turbulence also compress the distributions of labor and capital across firms with different productivity levels, since high-productivity firms are shrinking relative to low-productivity firms. Thus, the turublence shock reduces the IQR of both employment and of capital. Accordingly, the IQR of sales levels also declines. Furthermore, the shock reshuffles expected firm-level productivity and thus more firms choose to adjust their production, resulting in increased dispersion of sales growth across firms. These model predictions are line with our empirical evidence (see Figure 2).[30]

Figure 3 further shows that turbulence increases the left-skewness of the cross-sectional distribution of productivity and sales for active firms. The decline in aggregate TFP through reallocation reduces worker wages and capital rents, lowering the productivity threshold for production. Thus, some firms with productivity below the initial (pre-shock) threshold turn from inactive to active in production. As a result, the share of active low-productivity firms increases in a turbulence-induced recession, skewing the cross-sectional distribution of sales

the turbulence shock and the path with no shocks. This solution approach helps capture potential non-linear effects of the shock. Since turbulence shocks in our model have first-moment impact, the impulse responses generated from the third-order approximations are essentially the same as those from first-order approximations (and we have verified this).

[30] For more detailed distributional impacts of a turbulence shock in our model, see Figure A11 in Appendix B.3. One concern related to our productivity process is that the productivity of a firm experiencing a turbulence shock might switch from the very top to the very bottom of the distribution (or vice versa), which would be counterfactual. Under our calibration, however, this is not the case. As we show in Appendix B.4, the switches of a firm's productivity are concentrated in adjacent productivity groups (e.g., between the top quintile and the second quintile), with much smaller probabilities of switching between the top and the bottom of the productivity distribution, both in the model and in the data.

toward those low-productivity firms. This increase in the left skewness of productivity and of sales in a recession is consistent with empirical evidence (Kehrig, 2015; Salgado et al., 2019).

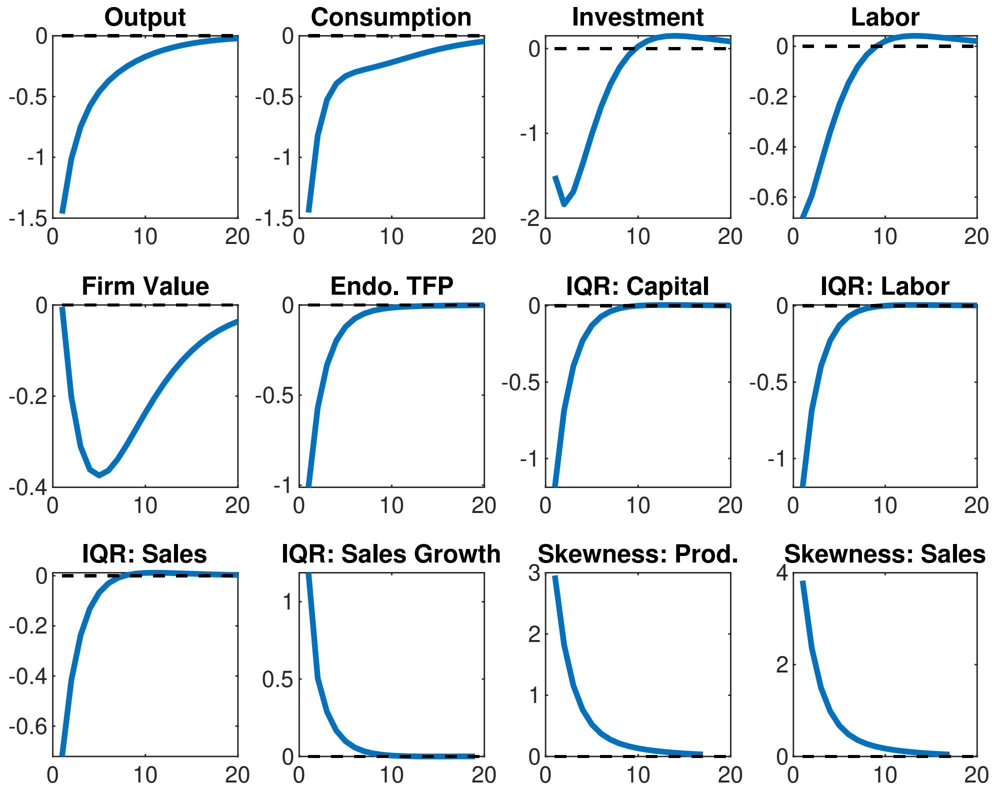

FIGURE 3. Impulse responses to a turbulence shock in the benchmark model

Note: This figure shows the impulse responses to a one-standard-deviation shock to turbulence in the calibrated benchmark model. The interquartile range (IQR) is measured as the difference between the 75th percentile and the 25th percentile of the cross-sectional distribution of the variable of interest. The skewness measures the concentration on the left tail of the distribution. The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

IV.4. **The role of financial frictions.** Our empirical evidence suggests that financial frictions are important for the reallocation effects of turbulence (Section III.4). We now illustrate the importance of financial frictions for propagating turbulence shocks to driven macroeconomic fluctuations. For this purpose, we consider a counterfactual version of our model, in which firms' borrowing capacity does not vary with the expected equity value. Specifically,

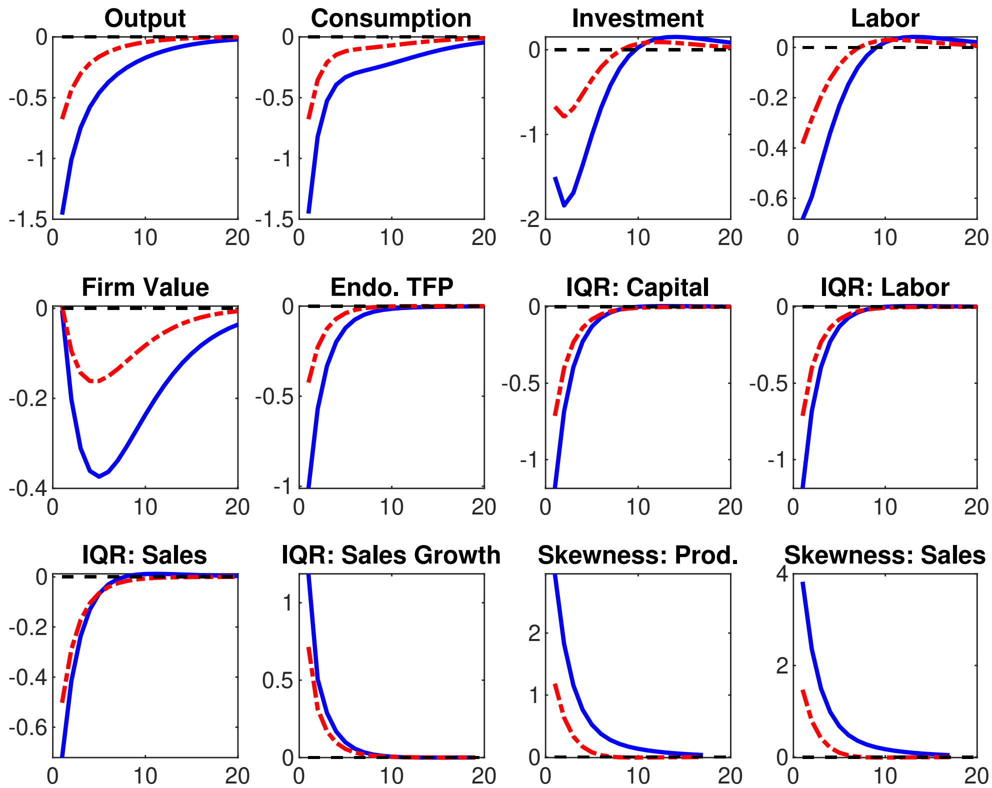

FIGURE 4. Impulse response to a turbulence shock: Benchmark model vs. counterfactual with quasi-fixed borrowing capacity

Note: This figure shows the impulse responses to a one-standard-deviation shock to turbulence in the benchmark model (blue lines) and in the counterfactual with quasi-fixed borrowing capacity (red dash-dotted lines). The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

we replace the working capital constraint with

$$R_t k_{jt} + W_t n_{jt} \leq \theta \beta E_t \frac{C_t}{C_{t+1}} \left[\rho_t \bar{V}_j^{ss} + (1-\rho_t) \sum_{i=1}^{J} \pi_i \bar{V}_i^{ss} \right] \equiv \theta \bar{B}_{jt}, \tag{38}$$

where $\bar{V}_j^{ss}$ denotes the steady-state equity value for firms with productivity z_{jt}. In this counterfactual, a turbulence shock can still influence firms' borrowing capacity by changing the transition probability (ρ_t) of the future productivity distribution, but changes in firms' expected equity value following a turbulence shock would have no effect on the borrowing capacity.

Figure 4 shows the impulse responses in the benchmark model (blue solid lines) and those in the counterfactual under this "quasi-fixed" borrowing capacity (red dash-dotted lines). The figure shows that the recession effects and the reallocation effects of a turbulence shock would be substantially dampened if firms' borrowing capacity could not vary with the expected equity value. This counterfactual illustrates the importance of financial frictions—and in particular, the endogenous variations of the borrowing capacity with expected firm values—for propagating turbulence shocks.

IV.5. **Quantitative importance of turbulence shocks.** Turbulence has quantitatively important recessionary effects, both in the model and in the data. Figure 5 compares the model-implied impulse responses of aggregate output (red solid line) with the empirical estimates of the impulse response (blue solid line). A one-standard-deviation turbulence shock reduces aggregate output by up to 1.5 percent at the peak, both in the data and in the model. The shock has persistent recessionary effects on aggregate output, which stays below the steady state level for at least four years, both in the model and in the data. However the model fails to generate the hump-shaped responses observed in the data, implying that the model's internal propagation mechanism is not sufficiently strong. Overall, these findings suggest that turbulence shocks are important for driving business cycles.[31]

V. Policy interventions

The presence of financial frictions implies that competitive equilibrium allocations are inefficient. Appropriate policy interventions can potentially undo the distortions from financial frictions, stabilizing aggregate output and improving allocative efficiency. To illustrate this point, we consider two alternative policy interventions in response to a recession driven by a turbulence shock. One policy is a borrowing subsidy that reduces firms' borrowing costs. The other is a credit-easing policy that expands firms' borrowing capacity.

Under the borrowing subsidy policy (Policy I), the government subsidizes wages and rents for active firms at an exogenous rate ω_{1t}, such that firms with productivity z_{jt} face the effective credit constraint

$$\tilde{R}_t k_{jt} + \tilde{W}_t n_{jt} \leq \theta B_{jt}, \tag{39}$$

where $\tilde{R}_t = (1-\omega_{1t})R_t$ and $\tilde{W}_t = (1-\omega_{1t})W_t$ denote the after-subsidy capital rental rate and real wage rate, respectively. The subsidies thus reduce the amount of working capital loans that firms need to borrow. The government finances the borrowing subsidies $\Omega_{1t} = \omega_{1t}(W_t N_t + R_t K_t)$ by imposing lump-sum taxes on the representative household. We

[31] We have also compared the theoretical impulse responses of investment, consumption, and labor hours with those in the data (see Appendix B.2). We find that the model-implied responses of these aggregate variables are broadly in line with those estimated from the data, although the calibrated model fails to generate the hump-shaped responses in the data.

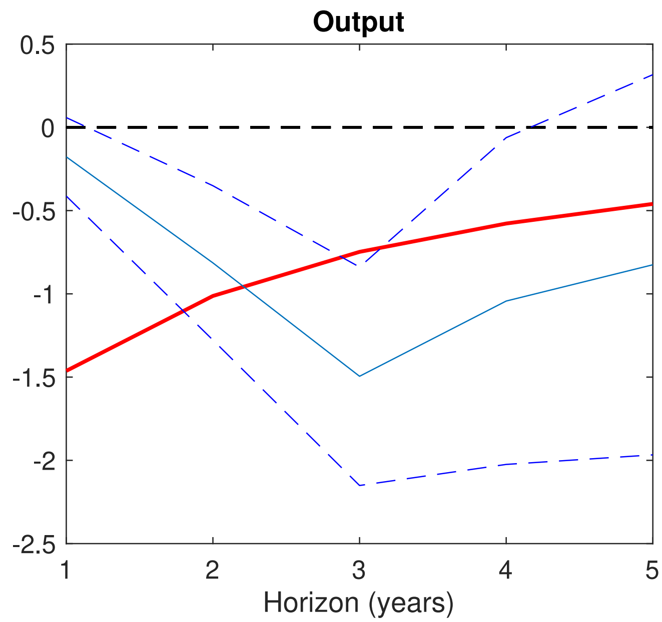

FIGURE 5. Impulse responses of aggregate output to a turbulence shock: Model vs. data

Note: This figure shows the impulse responses of private aggregate output to a one-standard-deviation shock to turbulence in the data (blue solid line) and in the calibrated annual version of the model (red solid line). The dashed lines show the 68% confidence band around the empirical estimates of the impulse responses. The horizontal axis shows the years after the impact of the shock. The vertical axis shows the percent deviations of output in the model from its steady-state level and the percentage changes in output in the data relative to its pre-shock level.

assume that operating this policy incurs a resource cost of $\lambda_1\Omega_{1t}$, where $\lambda_1 \geq 0$ reflects potential deadweight losses associated with the government program.

Under the credit easing policy (Policy II), the government injects liquidity into active firms, such that firms with productivity z_{jt} face the effective credit constraint

$$R_t k_{jt} + W_t n_{jt} \leq \theta(1+\omega_{2t})B_{jt}, \tag{40}$$

where $\omega_{2t}B_{jt}$ is the amount of government transfers to active firms with expected equity value B_{jt}. The total cost of the credit-easing policy is given by $\Omega_{2t} = \omega_{2t}\sum_j \pi_j\theta B_{jt}[1-F(\tau_{jt}^*)]$, which is financed by lump-sum taxes on the household. Similar to the borrowing subsidy policy, we assume that credit easing also incurs a resource cost of $\lambda_2\Omega_{2t}$, where $\lambda_2 \geq 0$ reflects potential deadweight losses in operating the policy.

Following Bloom et al. (2018), we consider transitory and unanticipated policy interventions. A policy would be implemented only if a turbulence shock hits the economy, and the policy intervention has the same persistence as the shock. We evaluate the effectiveness of each of the two alternative policies—borrowing subsidies and credit easing—for mitigating the macroeconomic and reallocation effects of turbulence. For this purpose, we compare the impulse responses of aggregate output and aggregate TFP to a turbulence shock under borrowing subsidies (Policy I) or credit easing (Policy II) to those in the *laissez-faire* benchmark economy without policy intervention (No policy).

In each policy regime, we simulate the model economy for 1000 periods (years) based on third-order approximations of the equilibrium system around the deterministic steady-state. A turbulence shock and a simultaneous policy intervention (if any) are implemented in period 960. The unanticipated policy stimulus has a size of 1 percent of steady-state output, with the same persistence as the turbulence shock. After a policy intervention is implemented, we allow the economy to evolve naturally for the remaining 20 years. We calculate the responses of each endogenous variable to the turbulence shock (with or without a policy intervention) as percent deviations from the stochastic steady-state.

Figure 6 shows the stabilizing effects of the two alternative policies relative to the benchmark economy, conditional on a one-standard-deviation turbulence shock.[32] Compared to the benchmark economy without policy interventions (black bar), Policy I (red bar) is effective for stabilizing the declines in aggregate output (upper panel). By providing borrowing subsidies, the policy stimulates demand for labor and capital, and thus mitigating the declines in equilibrium hours, investment, and output. However, by reducing the effective factor prices, Policy I enables a larger fraction of low-productivity firms to produce, exacerbating misallocation and intensifying the decline in aggregate TFP following a turbulence shock (lower panel).

The credit easing policy is also effective for stabilizing turbulence-driven output declines relative to the benchmark economy (blue bar, upper panel). The policy expands the borrowing capacity for all active firms, enabling a larger share of high-productivity firms to finance working capital and produce. The increase in the share of active high-productivity firms pushes up equilibrium wages and rents, shrinking the set of active low-productivity firms. Thus, Policy II reallocates labor and capital to high-productivity firms, improving aggregate TFP relative to the benchmark (blue bar, lower panel).

These policy experiments suggest that temporary borrowing subsidies or credit easing policies are effective for stabilizing turbulence-driven output fluctuations. However, the reallocation consequences of the two policies are different. While borrowing subsidies exacerbate misallocation, credit easing alleviates it.

[32]The figure here shows the impact effects of a turbulence shock on aggregate output and TFP. The results are qualitatively the same when we consider the cumulative effects (not reported).

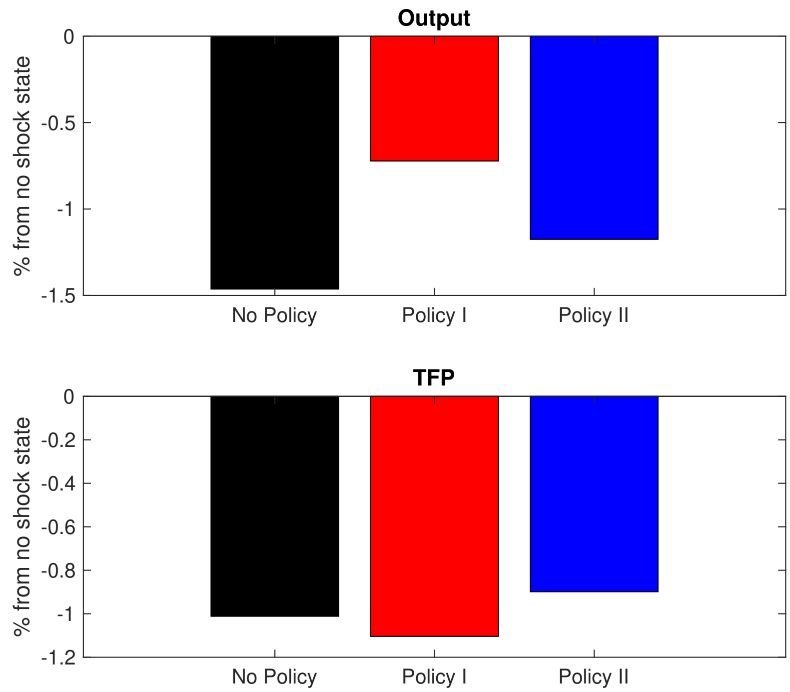

FIGURE 6. Impact effects of a turbulence shock with alternative policy interventions

Note: This figure plots the responses of aggregate output and aggregate TFP in the impact period of a one-standard-deviation turbulence shock. Black bars show the responses in the benchmark economy with no policy interventions. Red bars represent the responses under an unanticipated and temporary borrowing subsidy policy (Policy I). Blue bars represent the responses under an unanticipated and temporary credit easing policy (Policy II).

VI. CONCLUSION

Macroeconomic fluctuations often mask underlying cross currents with important cross-sectional reallocations. We study the implications of turbulence—a form of reallocation shocks—for business cycles. An increase in turbulence changes the conditional distribution of firms' future productivity, leading to reallocations across firms. We document evidence that turbulence is countercyclical, rising sharply in recessions. Turbulence has cross-sectional reallocation effects, the magnitude of which depends on financial frictions. Turbulence is associated with a recession, with synchronized declines in aggregate output, consumption, investment, and labor hours.

Using a real business cycle model augmented with firm heterogeneity and financial frictions, we have highlighted a quantitatively important reallocation channel, through which a turbulence shock drives macroeconomic fluctuations. An increase in turbulence reduces the likelihood for the current high-productivity firms to maintain their productivity rankings in the future, lowering their expected equity values relative to those of the current low-productivity firms. Facing tightened working capital constraints, high-productivity firms pull back hiring of capital and labor relative to low-productivity firms, leading to reallocation from high- to low-productivity firms and reducing aggregate TFP. Such declines in TFP generate a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, as in the data. A one-standard-deviation shock to turbulence leads to a drop in aggregate output of up to 1.5 percent, with the recessionary effects persisting for at least four years, suggesting that turbulence plays an important role in driving business cycles.

Financial frictions are crucial for propagating turbulence shocks in our model. The presence of financial frictions also leads to misallocation. Policy interventions designed to alleviate credit constraints can potentially dampen the impact of turbulence and improve allocative efficiency. However, the particular approach to implementing such policy interventions can produce very different outcomes. For example, borrowing subsidies that reduce the amount of working capital loans that firms need to borrow can effectively boost aggregate output, mitigating the recessionary effects of turbulence. An alternative credit easing policy that expands firms' borrowing capacity can also stimulate aggregate output. However, these two alternative policies have different implications for allocative efficiency. A borrowing subsidy enables more low-productivity firms to stay active, exacerbating misallocation, whereas credit easing allows high-productivity firms to expand production, improving aggregate productivity.

To illustrate the key transmission mechanism of turbulence, we have intentionally kept the model stylized. For example, the model abstracts from firm entries and exits. To the extent that firms rely on external financing and entering (exiting) firms have higher (lower) productivity than incumbent firms, we conjecture that introducing entry and exit decisions could potentially amplify the recessionary effects of turbulence through reallocation. Another direction of generalizing our study is to enrich the model by incorporating other sources of real and nominal frictions. With these additional frictions, the model could better fit time-series data and it could also be used to examine the role of monetary policy in stabilizing macroeconomic fluctuations driven by turbulence. We leave these important subjects for future research.

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Online Appendix: Turbulent Business Cycles

Ding Dong, Zheng Liu, and Pengfei Wang

Appendix A. Additional empirical results

This appendix presents some additional empirical results about the measurement of turbulence, and the macroeconomic and reallocation effects of turbulence.

A.1. **Measuring turbulence under full information.**

Proposition 2. If idiosyncratic TFP is perfectly observable, the Spearman rank correlation of firm productivity between period t and $t+1$ is given by ρ_t.

Proof. Denote by $R_t(z)$ the ranking of productivity z across firms in period t. Under the stochastic process of z_t specified in Eq. (2), we have

$$R_{t+1}(z) = \begin{cases} R_t(z) & \text{with prob} \quad \rho_t, \\ R_t(\tilde{z}) & \text{with prob} \quad 1-\rho_t. \end{cases} \tag{A1}$$

The Spearman rank correlation of firm productivity between t and $t+1$ is thus given by

$$\begin{aligned} r_s &\equiv \frac{Cov(R_{t+1}(z), R_t(z))}{\sqrt{Var(R_{t+1}(z))Var(R_t(z))}} \\ &= \frac{\rho_t Cov(R_t(z), R_t(z)) + (1-\rho_t)Cov(R_t(\tilde{z}), R_t(z))}{Var(R_t(z))} \\ &= \frac{\rho_t Var(R_t(z))}{Var(R_t(z))} = \rho_t, \end{aligned} \tag{A2}$$

Here, the first equality is the definition of the Spearman correlation (denoted by r_s). The second equality follows from Eq. (A1) and the assumption that the stationary productivity distribution $G(z)$ has a finite, discrete number of realizations such that the variance of the ranking of z stays constant over time. The third equality follows from the fact that $R_t(\tilde{z})$ is uncorrelated with $R_t(z)$. $\square$

This relation between ρ_t and the Spearman rank correlation of firm-level productivity provides the theoretical underpinning for our empirical measurement of turbulence.

A.2. **Data and measurements for turbulence.**

TABLE A.1. Summary statistics

	Sample 1			Sample 2		
Variable	Mean	SD	N	Mean	SD	N
Log Asset (1m)	5.5	2.1	53285	6.0	2.1	29304
Log Value-Added (1m)	4.6	2.5	53285	5.3	2.3	29304
Log Capital (1m)	3.6	2.5	53285	4.3	2.4	29304
No. of Workers (1000)	7.3	19.0	53285	11.5	23.9	29304
Log Market Value (1m)	1.7	0.35	47431	1.8	0.36	25788
Value-Added Growth (%)	8.2	33.0	47251	5.3	23.3	27138
Capital Growth (%)	6.6	30.7	47251	4.8	23.9	27138
Employment Growth (%)	5.1	23.1	47251	3.4	19.5	27138
Market-Value Growth(%)	3.4	39.2	41439	3.8	31.3	23496

Note: Sample 1 covers all listed firms in all manufacturing industries (NAICS code 31 to 33). Sample 2 covers firms with 25+ years of observations in all manufacturing industries.

Source: Compustat, NBER-CES, and authors' calculations.

A.2.1. *Summary statistics of data.* For measuring turbulence, we use firm-level data from Compustat Fundamentals Annual database, combined with measures of industry-level employment, payroll, and price indices from the NBER-CES Manufacturing Industry Database. Table A.1 presents the summary statistics of our samples. The full sample (Sample 1) includes all listed firms in all manufacturing industries covered by NBER-CES in the years from 1958 to 2016. Following Bloom et al. (2018), we focus on the subset of firms with 25+ years of observations and use it as our baseline sample (Sample 2).

A.2.2. *OLS versus IV estimates of turbulence.* Our estimated turbulence series based on the IV approach is quite different from the OLS estimation, which can be biased because of the heteroskedasticity and endogeneity issues discussed earlier.

Denote by $\hat{a}_{i,j,t}$ the measured TFP of firm j in industry i and year t and $Rank^{\hat{a}}_{i,j,t}$ the ranking of the firm's TFP within industry i in year t. To obtain the OLS estimator of turbulence, we compute the rolling-window Spearman correlations (ρ_t^{OLS}) between $Rank^{\hat{a}}_{i,j,t}$ and $Rank^{\hat{a}}_{i,j,t+1}$ for each year t. The OLS estimate of turbulence ($Turb_t^{OLS}$) is given by $1-\rho_t^{OLS}$. In particular,

$$Turb_t^{OLS} \equiv 1-\rho_t^{OLS} = 1-Spearman(Rank^{\hat{a}}_{i,j,t}, Rank^{\hat{a}}_{i,j,t+1}) \tag{A3}$$

To obtain the IV-estimate of turbulence, we first compute the IV estimate of the Spearman correlations (ρ_t^{IV}). We estimate ρ_t^{IV} based on rolling-window two-stage least squares, with the year-t TFP ranking $Rank_{i,j,t}^{\hat{a}}$ instrumented by lagged rankings in years $t-1$ and $t-2$. In particular, the first-stage rolling-window regression is given by

$$Rank_{i,j,t}^{\hat{a}} = \beta_{1,t} Rank_{i,j,t-1}^{\hat{a}} + \beta_{2,t} Rank_{i,j,t-2}^{\hat{a}} + \varepsilon_{i,j,t}, \tag{A4}$$

where $\varepsilon_{i,j,t}$ is an error term and $\beta_{1,t}$ and $\beta_{2,t}$ are the time-varying coefficients. In the second stage, we regress the ranking in $t+1$ ($Rank_{i,j,t+1}^{\hat{a}}$) on the ranking in t predicted from the first-stage regressions for each year t. The second-stage regression is given by

$$Rank_{i,j,t+1}^{\hat{a}} = \rho_t \tilde{Rank}_{i,j,t}^{\hat{a}} + u_{i,j,t+1}, \tag{A5}$$

where $\tilde{Rank}_{i,j,t}^{\hat{a}}$ denotes the predicted ranking in year t from the first-stage regression and $u_{i,j,t+1}$ is an error term. The coefficient estimated from the second stage gives ρ_t^{IV} and the IV-based turbulence is given by $Turb_t^{IV} = 1 - \rho_t^{IV}$.

The OLS bias is substantial, as shown in Figure A1. The black line shows the turbulence series estimated using the OLS approach. The blue bars show the turbulence series estimated using the IV approach. The red bars indicate the OLS bias. When we correct for the OLS bias, the average level of the estimated turbulence in our sample is about halved. In other words, OLS estimation would over-state the level of true turbulence. The bias is notably smaller than average during recessions, especially during the 2008-2009 global financial crisis.

A.2.3. *Cyclical properties of turbulence.* Our measure of turbulence is countercyclical and is negatively correlated with manufacturing TFP, as shown in panel A of figure A2. A rise in turbulence (blue line) is typically associated with a decline in TFP relative to trend (red line), and the correlation between the two series is about -0.20.

Turbulence is also negatively correlated with the stock market value of firms, as shown in panel B of figure A2. The market value of assets is calculated based on firms' stock prices at the end of the fiscal year, multiplied by the shares outstanding and deflated by the consumer price index. The correlation between turbulence with the asset value is negative, at -0.19.[33]

A.2.4. *Robustness to alternative measures of TFP.* Our IV-based measure of turbulence is robust to alternative measures of value added, capital input, and labor input for measuring firm-level TFP.

In our benchmark empirical specification, we construct firm-level TFP based on firm-level value added and capital and labor inputs. We measure value added using firm-level sales and the average share of intermediate inputs at the 6 digit industry level, where the intermediate

[33]In Compustat, the firm stock price at the end of the fiscal year is the variable "PRCC_F," the shares outstanding is "CSHO," the firm asset is "AT," and the book equity is "CEQ."

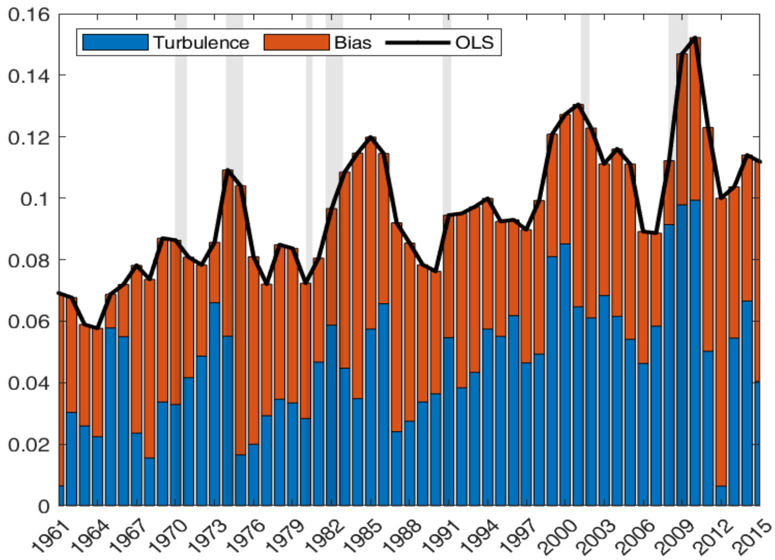

Figure A1. Decomposing churning in firm's TFP ranking

This figure plots the times series of the estimated $1 - \rho_t$ using the OLS (black line) and the IV (blue bar) approaches. The OLS estimates of ρ_t are the rolling-window Spearman correlations of firms' TFP rankings between adjacent years (t and $t+1$). The IV estimates of ρ_t are obtained from two-stage least squares regressions, using the TFP rankings in years $t-1$ and $t-2$ as instruments for the ranking in year t. The red bars show the OLS bias.

input share is the ratio of costs of materials to total value of shipments. We measure capital input using the real book value of a firm and labor input using the number of employees. Figure A3 shows that the turbulence measure is robust to alternative measurements of firm productivity.

The upper panel of the figure compares the benchmark measure of turbulence (blue line) and the alternative measure using a different measure of value added (red line). In particular, we follow the approach in David et al. (2016) and David and Venkateswaran (2019), and construct value added by assuming a constant intermediate input share of 0.5 for all firms. These two alternative measures of turbulence are highly correlated, with a correlation coefficient of 0.86. We have also considered another approach to constructing firm-level value added by subtracting the costs of goods sold from reported sales in Compustat (not shown in the figure). The resulting turbulence measure is also highly correlated with our benchmark measure, with a correlation coefficient of 0.76.

The middle panel of Figure A3 shows that the measure of turbulence is robust to alternative approaches to measuring capital. Here, instead of using the real book value of firms, we

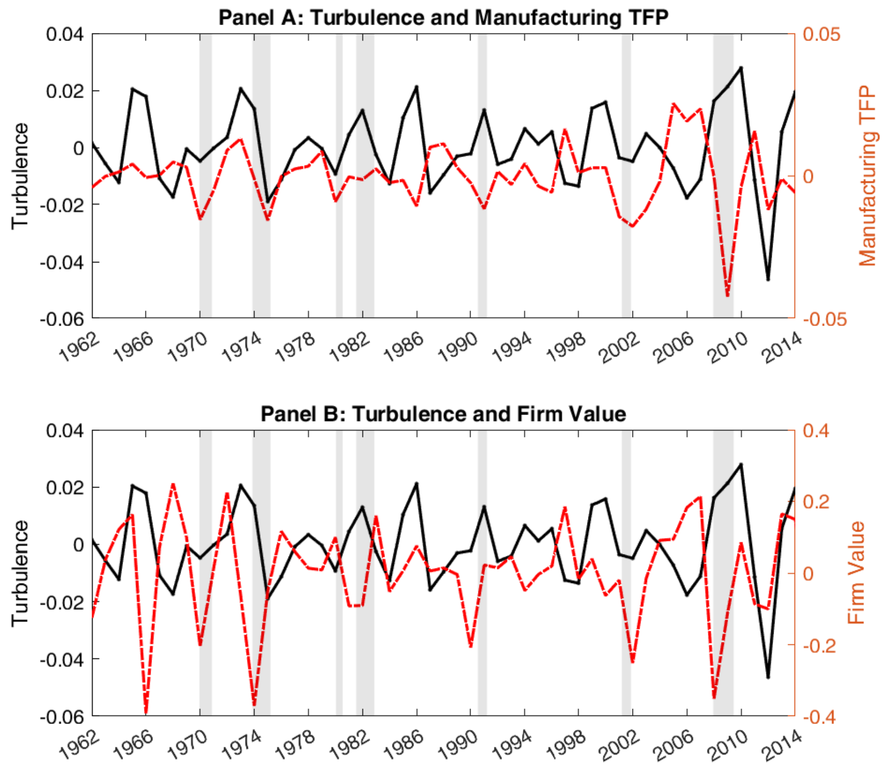

FIGURE A2. Correlation of turbulence with manufacturing TFP and stock market value

Note: The series of manufacturing TFP and firm value are computed as an average of firm-level TFP and stock market value in our benchmark sample. The series are detrended using the HP filter, with a smoothing parameter of 6.25. The gray shaded bars indicate NBER recession dates.

measure capital input using a perpetual inventory method. Specifically, we first fix the initial real value of capital using $PPEGT$ in the first year of our sample. We then construct a measure of net investment using $PPENT_{it} - PPENT_{it-1}$, deflated by industry-specific investment deflators. Finally we iterate forward the law of motion of the capital stock by adding real net investment to the capital stock in the previous period. With this alternative measure of capital, we obtain a turbulence series (red line) that is highly correlated with the benchmark series (blue line), with a correlation coefficient of 0.95.

Finally, we show the measure of turbulence is also robust to alternative approaches to measuring labor input, as shown in the lower panel of Figure A3. Here, instead of measuring labor input by a firm's total payroll, we measure labor input by the number of employees. The

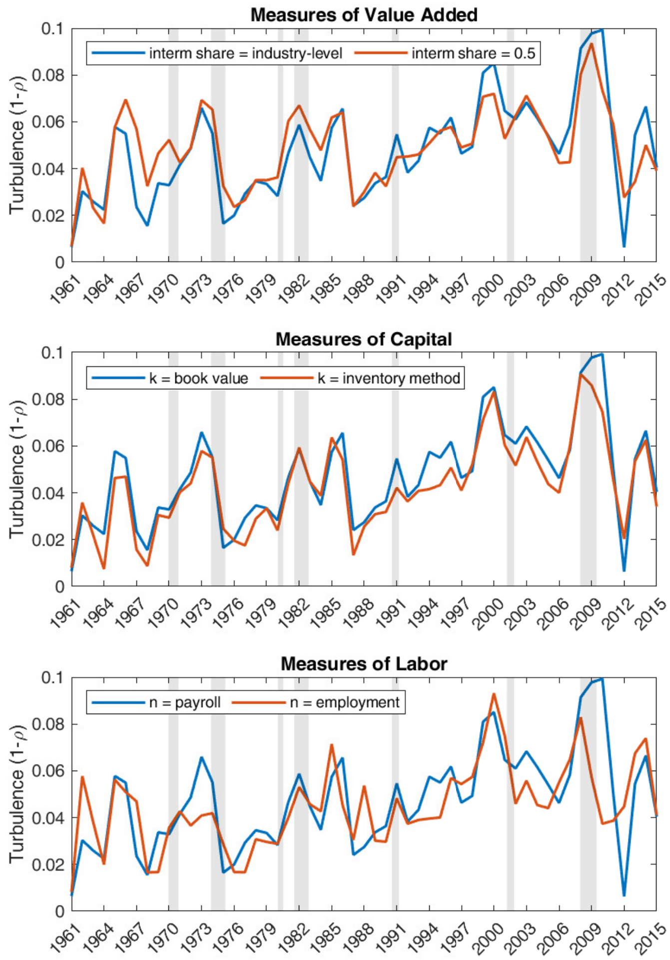

Figure A3. Robustness of Turbulence Measure: Alternative Measures of Firm Productivity

turbulence series under this alternative measure of labor input (red line) is highly correlated with the benchmark series (blue line), with a correlation coefficient of 0.68.

A.2.5. *Robustness to alternative samples.* In our benchmark empirical specification, we measure turbulence based TFP rank correlation of firms within NAICS 3-digit industry, using a sample with firms with observation for at least 25 years. Figure A4 shows that our turbulence measure is robust to alternative samples, alternative definition of industry, and that cyclical patterns of turbulence are not driven by a subset of firms at the top or bottom of productivity, size or age distribution.

In the baseline measure we focus on the sample that contains firms with 25+ years of observations to mitigate effects from entry and exit. The upper left panel of Figure A4

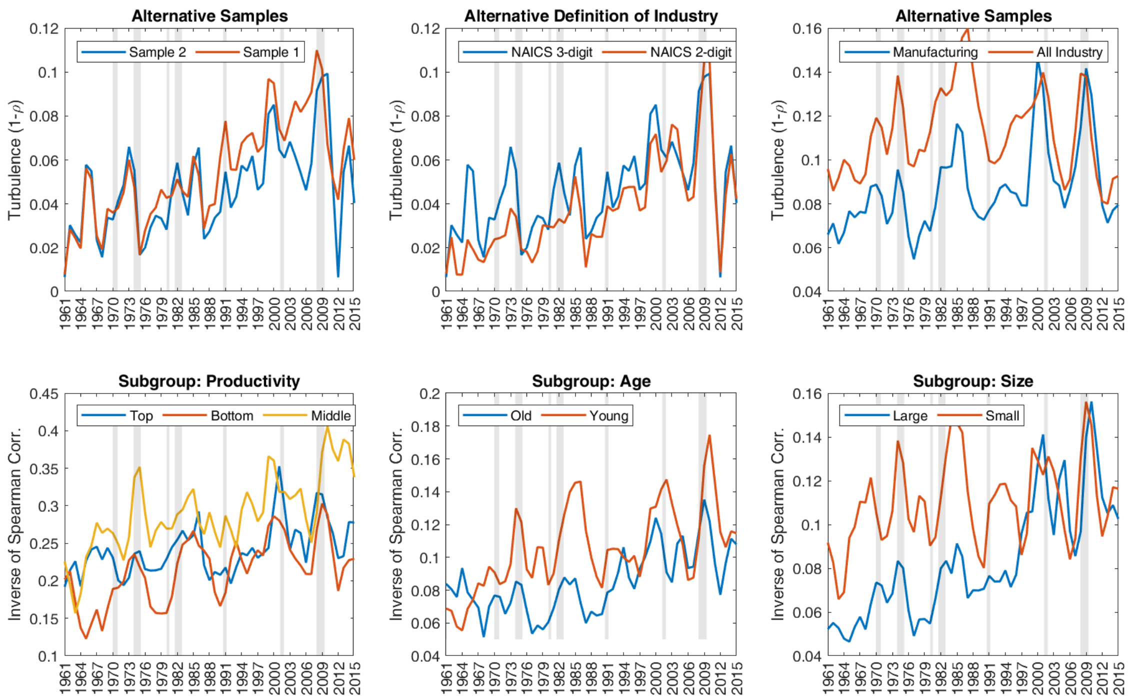

FIGURE A4. Robustness of Turbulence Measure: Alternative Samples and Subgroups

shows that using the sample including firms existing for less than 25 years (sample 1, red line) obtains a measure of turbulence that is highly correlated with the benchmark series, with a correlation coefficient of 0.85. In the baseline measure we define firm productivity ranking based on NAICS 3-digit industry. Ranking firm productivity at NAICS 2-digit level (red line) delivers a similar time series as our baseline measure (blue line) with correlation coefficient of 0.89, as shown in the upper mid panel of Figure A4. Our measure is also robust to ranking firm productivity at NAICS 4-digit industry (not reported here). In our baseline measure, we consider manufacturing sector where industry-level factor prices and shares are available for TFP estimation. As an additional exercise, we construct an MPL-based turbulence measure for all industries beyond manufacturing sector except regulated utility (SIC code 4900-4999), financial industry (SIC code 6000-6999) and public sectors (SIC code 9000-). The series constructed using full industry sample (red line) is highly correlated with MPL-based turbulence measure using manufacturing sector (blue line), with correlation coefficient of 0.58.

In lower panels we plot the inverse of Spearman rank correlation of firm TFP by different subgroups of productivity, age and size. According to the process of idiosyncratic productivity (2), all firms are subject to common risk of churning in productivity ranking regardless of their current productivity ranking. This view is supported by the data. In lower left panel of Figure A4, we plot the conditional Spearman productivity rank correlations of firms at the top and bottom 50% of productivity distribution between year t and t+1. Both the levels and cyclical patterns of two time series track each other well, with correlation coefficient of 0.61.

In lower-mid and lower-right panel we show age and size effects on the measures. Intuitively, TFP rankings of older and larger firms are less turbulent on average, though they exhibit similar cyclical patterns with younger or smaller firms. The upward trend in turbulence of baseline measure is mostly driven by large firms in our sample, echoing findings of Davis et al. (2006). The evidence presented here suggest that cyclical patterns of aggregate turbulence are not driven by a subset of firms at the top or bottom of productivity, age or size distribution.

A.3. **Reallocation effects of turbulence.** The reallocation effects of turbulence are robust to alternative high-TFP indicators, alternative samples, including controls of the potential reallocation effects of uncertainty, recessions, and using industry-level turbulence measures.

A.3.1. *Lagged high-productivity indicators or finer grouping of productivity.* Our baseline regression estimates the effects of turbulence on firms with high versus low levels of TFP, based on current-year firm-level TFP ranking. It is possible that a firm's employment, capital or sales growth might affect its contemporaneous TFP ranking. To mitigate this concern, we construct an alternative indicator of high-productivity firms based on their TFP rankings with a one-year lag. We re-estimate the responses of firm growth to turbulence shocks for firms with different levels of productivity. Table A.2, with the lagged high-TFP indicator, turbulence reduces growth for high-productivity firms relative to low-productivity firms, similar to the results obtained under the benchmark specification.

To examine the robustness of the reallocation effects of turbulence, we also consider an alternative specification in which we sort firm-level TFP into four quartiles and we replace the dummy $High\_TFP_{jt}$ in the baseline specification by the indicator of ranking quartile ($QTile\_TFP_jt$). Consistent with our baseline results, an increase in turbulence is associated with larger declines in employment, capital, sales and market value growth for firms with higher productivity. These reallocation effects are both statistically significant and economically important (Table A.3).

Table A.2. Impact of turbulence on firms with different productivity: Lagged high-TFP indicators

Dep. Var.	Δn_{jt}		Δk_{jt}		Δy_{jt}	
	(1)	(2)	(3)	(4)	(5)	(6)
$High\_TFP_{jt-1}$	0.056***	0.069***	0.050***	-0.035	-0.028**	-0.045**
	(0.010)	(0.013)	(0.013)	(0.027)	(0.012)	(0.020)
$TURB_t * High\_TFP_{jt-1}$	-0.813***	-0.463***	-0.733***	-0.854***	-0.915***	-0.479***
	(0.097)	(0.093)	(0.124)	(0.156)	(0.078)	(0.102)
$UNC_t * High\_TFP_{jt-1}$		-0.290***		0.429***		-0.221**
		(0.070)		(0.125)		(0.102)
$\Delta GDP_t * High\_TFP_{jt-1}$		0.991***		0.642***		1.658***
		(0.158)		(0.089)		(0.204)
$Constant$	-0.045***	-0.044***	-0.058***	-0.057***	0.032***	0.034***
	(0.005)	(0.005)	(0.012)	(0.011)	(0.005)	(0.005)
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Observations	23,358	23,358	23,358	23,358	23,358	23,358
R-squared	0.035	0.041	0.061	0.062	0.034	0.045

Note: This table shows the regression of firm-level employment, capital, and sales growth on the measured turbulence ($Turb$) for firms with different levels of TFP (lagged). The dummy $High\_TFP_{jt-1}$ equals one if firm j's TFP at year $t-1$ is above the median and zero otherwise. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries. The stars denote the p-values: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table A.3. Impact of turbulence on firms with different productivity: Finer Grouping

Dep. Var.	Δn_{jt}		Δk_{jt}		Δy_{jt}	
	(1)	(2)	(3)	(4)	(5)	(6)
$Qtile\_TFP_{j,t}$	-0.014***	-0.008*	-0.012*	-0.045***	0.035***	0.032***
	(0.003)	(0.004)	(0.006)	(0.009)	(0.006)	(0.008)
$TURB_t * Qtile\_TFP_{j,t}$	-0.294***	-0.175***	-0.257***	-0.315***	-0.226***	-0.067*
	(0.025)	(0.026)	(0.042)	(0.053)	(0.025)	(0.032)
$UNC_t * Qtile\_TFP_{j,t}$		-0.105***		0.171***		-0.093***
		(0.020)		(0.033)		(0.021)
$\Delta GDP_t * Qtile\_TFP_{j,t}$		0.324***		0.216***		0.577***
		(0.050)		(0.031)		(0.070)
$Constant$	0.029***	0.031***	0.007	0.007	-0.057***	-0.053***
	(0.009)	(0.009)	(0.022)	(0.022)	(0.012)	(0.012)
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Observations	23,358	23,358	23,358	23,358	23,358	23,358
R-squared	0.049	0.059	0.067	0.071	0.027	0.046

Note: This table shows the regression of firm-level employment, capital, and sales growth on the measured turbulence ($Turb$) for firms with different levels of TFP (by uartiles). The indicator variable $QTile\_TFP_{jt-1}$ denotes the quartile ranking of firm j's TFP at year t within its industry in year t. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries. The stars denote the p-values: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

A.3.2. *Samples excluding large firms.* Our benchmark empirical results suggest that financial frictions are important for amplifying the reallocation effects of turbulence. However, it is plausible that sufficiently large firms are not as financially constrained as small and medium-sized firms. Including large firms in our sample can thus potentially bias the results against finding important reallocation effects of turbulence. This is indeed the case, as we find using a subsample that excludes the top 10% of firms based on their asset sizes. As shown in Table A.4, excluding the large firms from our sample delivers stronger reallocation effect of turbulence.

Table A.4. Impact of turbulence on firms with different productivity: Sample excluding large firms

Dep. Var.	Δn_{jt}		Δk_{jt}		Δy_{jt}	
	(1)	(2)	(3)	(4)	(5)	(6)
$High\_TFP_{jt}$	0.010	0.020	-0.001	-0.088***	0.081***	0.047*
	(0.007)	(0.015)	(0.011)	(0.017)	(0.010)	(0.025)
$TURB_t * High\_TFP_{jt}$	-1.044***	-0.628***	-0.717***	-0.894***	-0.781***	-0.286**
	(0.110)	(0.109)	(0.145)	(0.178)	(0.109)	(0.133)
$UNC_t * High\_TFP_{jt}$		-0.287***		0.457***		-0.149
		(0.097)		(0.085)		(0.123)
$\Delta GDP_t * High\_TFP_{jt}$		0.935***		0.561***		1.663***
		(0.127)		(0.091)		(0.167)
$Constant$	-0.017***	-0.015***	-0.036**	-0.036**	-0.016***	-0.014***
	(0.006)	(0.005)	(0.015)	(0.015)	(0.005)	(0.005)
Firm Characteristics	Yes	Yes	Yes	Yes		
Firm Fixed Effect	Yes	Yes	Yes	Yes		
Year Fixed Effect	Yes	Yes	Yes	Yes		
R-squared	0.042	0.047	0.064	0.065	0.024	0.033
Observations	20,860	20,860	20,860	20,860	20,860	20,860

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital, and sales) on the measured turbulence ($Turb$) for firms with different levels of TFP. The dummy $High\_TFP_{jt}$ equals one if firm j's TFP is above the median within its industry and zero otherwise. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015, excluding firms with asset sizes at the top 10% in each year. The standard errors shown in the parentheses are clustered by industry. The stars denote the p-values: $^{*}\, p < 0.1$; $^{**}\, p < 0.05$; $^{***}\, p < 0.01$.

A.3.3. *Persistence of the reallocation effects.* To examine the persistence of the reallocation effects of turbulence, we estimate the local projections specification

$$\begin{aligned}\Delta^h \log(x_{j,t+h}) &= \beta_0^h + \beta_1^h High\_TFP_{jt} + \beta_2^h TRUB_t * High\_TFP_{jt} + \beta_3^h UNC_t * High\_TFP_{jt} \\ &+ \beta_4^h \Delta GDP_t * High\_TFP_{jt} + \beta_5^h[\log(x_{j,t-1}) - \log(x_{j,t-2})] + \mu_j + \eta_t + \varepsilon_{jt} \quad \text{(A6)}\end{aligned}$$

where $\Delta^h \log(x_{j,t+h}) \equiv \log(x_{j,t+h}) - \log(x_{j,t-1})$ denote the cumulative change in the dependent variable of interest for firm j from the pre-shock period $(t-1)$ to h periods after the shock. The dependent variables that we consider include employment, capital, and sales, all in log units. The coefficients of interest β_2^h are associated with the interaction term between measured turbulence and the dummy variable of high-productivity, which captures the relative impact of turbulence on the cumulative growth of high-productivity firms h years after the turbulence shock, after controlling for the potential confounding effects of uncertainty and recessions.

Figure A5 plots the cumulative responses of employment, capital, and sales of high-productivity firms (relative to low-productivity firms) following a turbulence shock for horizons up to 4 years. The figure suggests that turbulence is associated with persistent reallocation from high- to low-productivity firms.

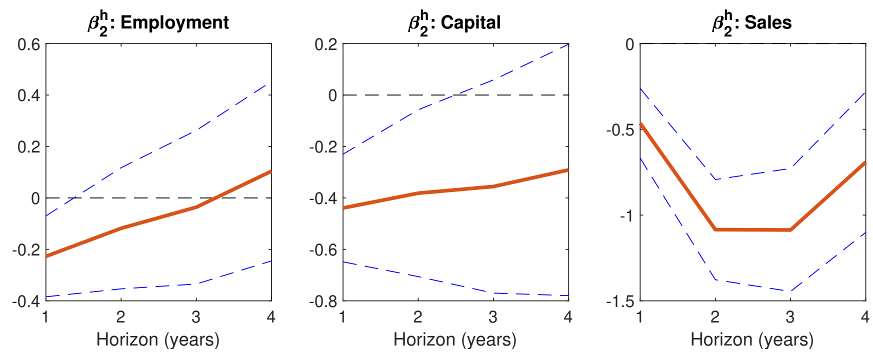

Figure A5. Persistent reallocation effects of turbulence

Note: This figure plots the estimated β_2^h in regression (A6). The variables we consider include employment, capital and sales.

A.3.4. *Sample covering non-manufacturing industries.* Using labor productivity-based measure, which is less demanding on data, helps extend our turbulence series to 2022 and to all industries (except regulated utilities, financial industry and admin sector), although the measure could be more biased for certain industries. Nevertheless, Table A.5 shows that reallocation effects of (labor-productivity-based) turbulence are preserved when we extend our sample to non-manufacturing sectors and till year 2022.

A.4. **Macroeconomic effects of uncertainty.** We argue that turbulence is different from uncertainty because it changes both the conditional mean and conditional variance of firm-level productivity, while uncertainty is a mean-preserving spread. We now examine the macroeconomic effects of an uncertainty shock based on local projections analogous to Eq. (10) in the main text. In particular, we follow the same approach of Bloom et al. (2018) to construct a measure of uncertainty using the Compustat data (since we do not have access to the Census of Manufacturing data).[34] We then estimate the local projections

[34]To measure uncertainty, we first estimate TFP shocks (e_{jt}) as the residual from the first-order autoregression after controlling for firm- and year- fixed effects: $\log(TFP_{jt}) = \rho \log(TFP_{jt-1}) + \mu_j + \lambda_t + e_{jt}$. We define microeconomic uncertainty in period t as the IQR of the TFP shocks (e_{jt}) in period t+1.

TABLE A.5. Reallocation effects of turbulence (labor productivity-based measure): All industries

Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}
	(1)	(2)	(3)
$High\_MPL_{jt}$	0.019*	0.032***	0.114***
	(0.011)	(0.009)	(0.010)
$Turb_t * High\_MPL_{jt}$	-0.035	-0.476***	-0.232***
	(0.091)	(0.071)	(0.081)
Constant	0.050***	0.039***	0.006***
	(0.002)	(0.001)	(0.002)
Firm Fixed Effect	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes
Observations	60,899	60,899	60,899

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital expenditure, and sales) on the measured turbulence ($Turb$) for firms with different levels of labor productivity. The dummy $High\_MPL_{jt}$ equals one if firm j's labor productivity is above the median within its industry and zero otherwise. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2022 in all industries except those in regulated utilities (SIC code 4900-4999), financial industries (SIC code 6000-6999) and administrative sectors (SIC code 9000-). The standard errors shown in the parentheses are clustered by industry. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

specification

$$y_{t+h} - y_{t-1} = \beta_0^h + \beta_1^h TURB_t + \beta_2^h UNC_t + \Gamma_{t-1}\Omega^h + \epsilon_{t+h} \quad h = 0, 1, 2, 3, 4. \tag{A7}$$

The dependent variable $y_{t+h} - y_{t-1}$ denotes the cumulative changes in the log-level of the variable of interest from year $t-1$ to year $t+h$, where h denotes the projection horizons (number of years). The set of dependent variables that we are interested includes real private consumption expenditure, private fixed investment, private output (i.e., the sum of consumption and investment), hours worked, S&P 500 stock price index, manufacture TFP, and dispersions of sales in both levels and growth rates (measured by the IQR of these variables in our sample). The key independent variable is the annual time series of uncertainty (UNC_t). The set of control variables includes contemporaneous turbulence ($TURB_t$) and lags of a set of macroeconomic variables (Γ_{t-1}), including lagged turbulence ($TURB_{t-1}$), lagged uncertainty (UNC_{t-1}), and the lagged growth rates of all the dependent variables. The term ϵ_{t+h} is the regression residual. The parameter β_1^h measures the impulse responses of the macroeconomic variables to a turbulence shock at horizon h.

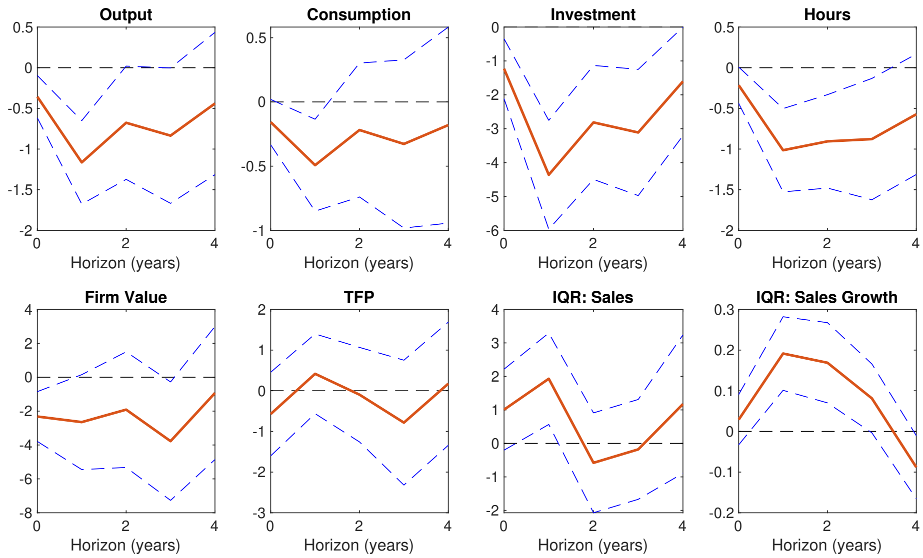

Figure A6. Estimated impulse response of macroeconomic variables to an uncertainty shock

Note: This figure shows the impulse responses of macroeconomic variables to a one-standard-deviation uncertainty shock from the local projections model (A7). The solid lines show the point estimates of the impulse responses. The blue dashed lines show the 68% confidence intervals.
Source: BEA, Compustat, NBER-CES, and authors' calculations.

Figure A6 plots the estimated impulse responses of the macroeconomic variables to a one-standard-deviation uncertainty shock for horizons up to five years.[35] The shock leads to a recession with synchronized and persistent declines in aggregate output, consumption, investment, and hours worked. It also leads to a decline in firm value and a rise in dispersion of sales growth. In contrast to turbulence shocks which reduce dispersion of sales, uncertainty shocks are associated with a rise in the IQR of sales levels. The estimated effects of uncertainty shocks on TFP are insignificant.

A.5. **Forecast error variance decomposition.** To estimate the contributions of a turbulence shock to the fluctuations in macroeconomic variables, we calculate a forecast error variance decomposition (FEVD) based on our estimated local projections, following the approach of Gorodnichenko and Lee (2020).

[35]The standard deviation of estimated uncertainty shocks ($uncshock_t$) from the regression in Eq. (A11) is 0.0177.

To implement the FEVD, we first orthogonalize the turbulence shock by taking the residuals ($turb\_shock_t$) from the regression

$$TURB_t = b_0 + b_1 UNC_t + \Gamma_{t-1} * B + turb\_shock_t, \tag{A8}$$

where $TURB_t$ denotes the time series of our measured turbulence; UNC_t denotes the time-series of micro-level uncertainty constructed following the same approach of Bloom et al. (2018) using Computat data; Γ_{t-1} is the set of control variables in our baseline local projections 10, including lagged uncertainty, lagged turbulence, and the lagged growth rates of all the dependent variables that we study; and B is the vector of coefficients associated with Γ_{t-1}.

Next, we compute the forecast errors for the dependent variable $y_{t+h} - y_{t-1}$ as the residuals from the regression

$$y_{t+h} - y_{t-1} = \beta_0^h + \Gamma_{t-1} * \Omega^h + u_{t+h}, \tag{A9}$$

where Γ_{t-1} is the same vector of control variables, Ω^h denotes the vector of coefficients for the control variables, and the term u_{t+h} denotes regression residuals. The estimated forecast error is then given by $\hat{fe}(y_{t+h}|_{t-1}) = u_{t+h}$.

Third, we measure the contribution of the orthogonalized turbulence shock to the forecast error variance of the dependent variable by the R^2 of the regression

$$\hat{fe}(y_{t+h}|_{t-1}) = \alpha_0^h turb\_shock_{t+h} + \alpha_1^h turb\_shock_{t+h-1} + ... + \alpha_h^h turb\_shock_t + \tilde{v}_{t+h|_{t-1}}. \tag{A10}$$

We repeat the same process to estimate the contribution from uncertainty shocks, with the following exception. In estimating the variant of (A8), we do not include $TURB_t$ in the control. For instance, the uncertainty shocks are estimated from

$$UNC_t = a_0 + \Gamma_{t-1} * A + uncshock_t \tag{A11}$$

In other words, we assume that uncertainty shock can affect contemporaneous turbulence but not vice versa. By doing so, we interpret the forecast error decomposition to the turbulence shocks as a conservative estimate, relative to that to the uncertainty shocks.

Figure A7 presents the estimated contribution to forecast error variance from turbulence shock and uncertainty shocks over the horizon of 0 to 4 years. On average, turbulence shocks contribute to up to 15% of the variance in forecast error of output, investment and consumption, and 10% of the fluctuations in stock value. The magnitudes are comparable, if not larger, than the contribution from uncertainty shocks.

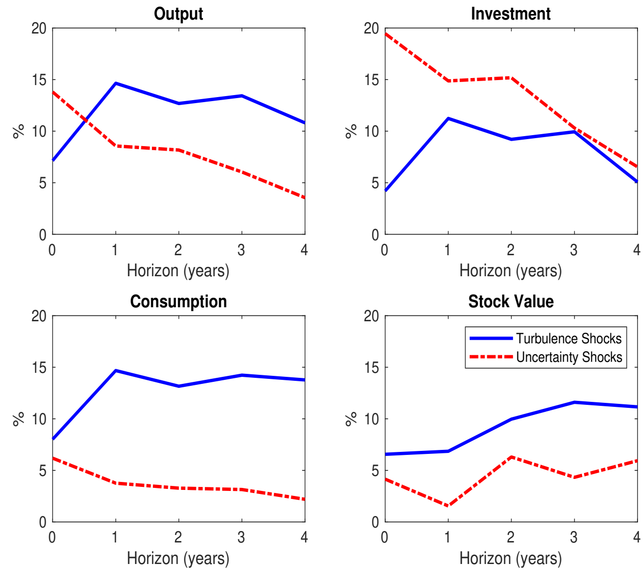

FIGURE A7. Forecast error variance decomposition from local projection

Note: This figure shows the forecast error variance decomposition of output, consumption, investment and stock value to turbulence shocks and uncertainty shocks estimated from the local projections model (10). The blue (red) lines show the contribution of turbulence (uncertainty) shocks in percentage.

A.6. **Historical Shock Decomposition.** To examine the quantitative importance of turbulence shocks over the business cycles, we calculate the historical shock decompositions in a vector-autoregression (VAR) model. We estimate the VAR model

$$AZ_t = BZ_{t-1} + Ce_t, \tag{A12}$$

where $Z_t = [U\hat{N}C_t, TU\hat{R}B_t, \Delta\hat{y}_t]'$ is the vector of variables of interest, including uncertainty $(U\hat{N}C_t)$, turbulence $(TU\hat{R}B_t)$, and private output growth $(\Delta\hat{y}_t)$, all expressed as deviations from the sample means. We impose the Cholesky identification restrictions, such that uncertainty does not respond to contemporaneous changes in turbulence or those in output growth, turbulence responds to contemporaneous changes in uncertainty but not to those in output growth, and output growth responds contemporaneously to changes in both uncertainty and turbulence. Since we have annual data, we focus on one lag in the VAR system. Note that, the VAR specification (A12) is equivalent to the local projections model at the zero horizon $(h = 0)$.

We estimate the VAR model (A12) using our time-series sample. By construction, if all three shocks are turned on, then the VAR model should replicate the actual time series of output growth. The estimated VAR also allows us to calculate the counterfactual path of

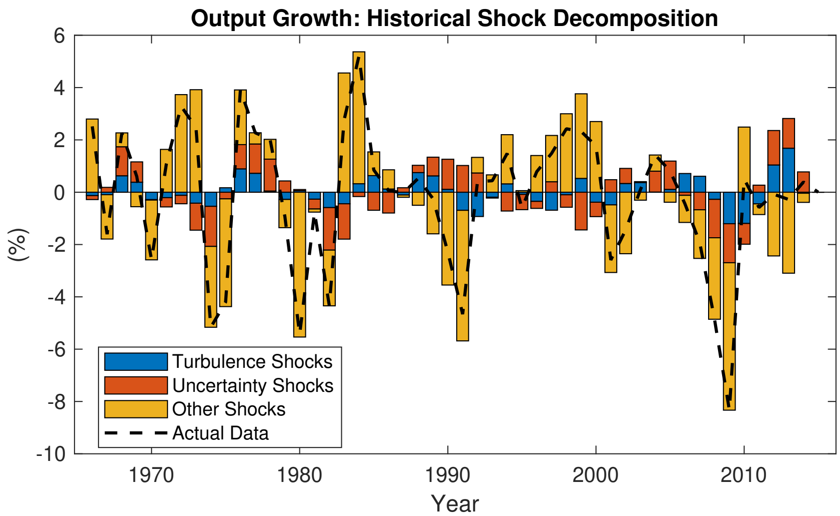

FIGURE A8. Historical shock decomposition from VAR

Note: This figure shows the historical shock decomposition of output growth to turbulence shocks, uncertainty shocks and other shocks estimated from the VAR model (A12). The black dashed line plots the (de-meaned) actual output growth. The blue bars represent the contributions of turbulence shocks, the red bars represent the contributions of uncertainty shocks, and the yellow bars represent the contributions of other shocks.

output growth when only the orthogonalized shock to turbulence is turned on. This counterfactual path is the historical contributions of turbulence shocks to output growth. Similarly, we can compute the historical contributions of uncertainty shocks to output growth. The residuals in output growth that are not explained by these two shocks are the contributions of the other shocks.

Figure A8 plots the time series of de-meaned actual output growth (black dashed line), along with the historical decompositions into the contributions of turbulence shocks (blue bars), of uncertainty shocks (red bars), and of the other shocks (yellow bars). According to our estimates, turbulence shock contributed to around one-fifth of total decline in output growth during the financial crisis periods, a magnitude that is similar to that of uncertainty shocks.

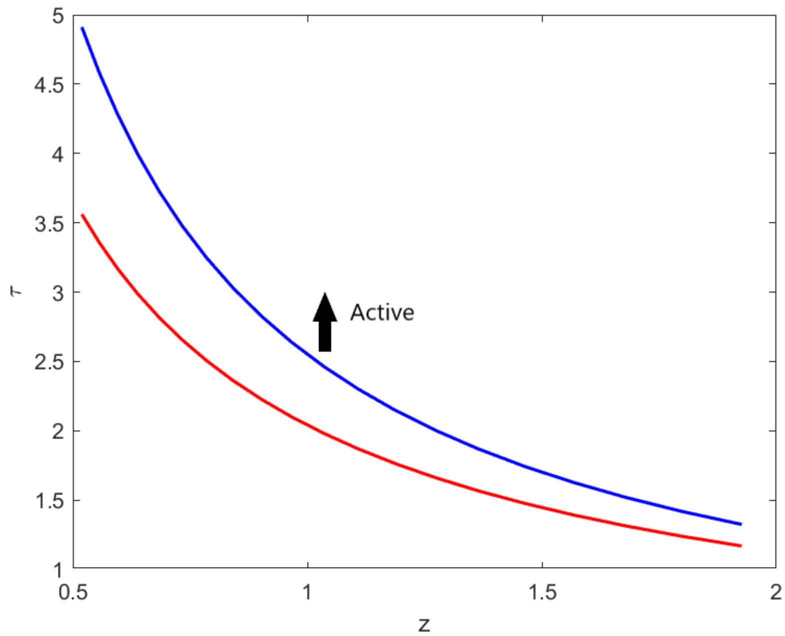

FIGURE A9. Production decisions in the steady-state equilibrium

Note: This figure shows the steady-state relation between the threshold level of subsidy τ^* and firm productivity z (blue line). Firms with (τ, z) lying above the threshold line are active in production and those below the line are inactive. The figure also shows the threshold line (red line) for production decisions in an economy with a higher average level of turbulence (i.e., a lower value of $\bar{\rho}$).

APPENDIX B. ADDITIONAL MODEL RESULTS

We now report some additional results from the baseline theoretical model.

B.1. **Steady-state allocations.** In Section IV.1, we have shown that a firm with productivity z_j chooses to produce (i.e., become active) if and only if its subsidy exceeds the threshold τ_j^*. The threshold level of subsidy is given by Eq. (26), and it is a decreasing function of firm productivity z_j.

Figure A9 illustrates the production decisions for firms with different levels of productivity (z) and subsidies (τ). The downward-sloping curve indicates the threshold function $\tau^*(z)$. At each z, a firm with a subsidy $\tau \geq \tau^*(z)$ chooses to produce. Otherwise, it stays inactive. Thus, the region of active firms are those with (τ, z) lying above the threshold curve.

In an economy with a higher average level of turbulence (i.e., with a lower value of $\bar{\rho}$), a high-productivity firm is less likely to remain productive, reallocating resources to low-productivity firms. Such reallocations reduce aggregate TFP, lowering the factor prices. For any given z, the declines in wages and capital rents reduce the threshold level of subsidy τ^* (see Eq. 26). Therefore, the threshold curve for production decisions shifts downward, with a flatter slope (red line), indicating that the increase in turbulence expands the active regions for low-productivity firms more than it does for high-productivity firms.

The reallocation effects of turbulence illustrated in Figure A9 can be formalized by the following proposition.

Proposition 3. Given the steady-state factor prices R and W, an increase in average turbulence reduces the share of labor hours allocated to high-productivity firms. Specifically, define the relative share of labor hours as $\eta_{ji} \equiv \frac{N_j}{N_i}$, where N_j and N_i denote labor hours allocated to active firms with productivity z_j and z_i, respectively. Without loss of generality, we assume that $z_j > z_i$. Then, we have

$$\frac{\partial \eta_{ji}}{\partial \bar{\rho}} > 0. \tag{A13}$$

Proof. In the steady-state equilibrium, the relative share of labor is given by

$$\eta_{ij} = \frac{\pi_j}{\pi_i} \frac{1 - \beta\bar{\rho}\Phi(\tau_i^*)}{1 - \beta\bar{\rho}\Phi(\tau_j^*)} \frac{\left[1 - F(\tau_j^*)\right]}{\left[1 - F(\tau_i^*)\right]},$$

where we have used aggregated labor demand based on Eq. (27), with the term B_j substituted out using the steady-state version of Eq. (31).

At given values of W and R, the threshold τ_j^* is a function of z_j only. Since $z_j > z_i$, it is easy to show that $\Phi(\tau_j^*) > \Phi(\tau_i^*)$ and that $1 - F(\tau_j^*) > 1 - F(\tau_i^*) > 0$. Thus, we have

$$\frac{\partial \eta_{ji}}{\partial \bar{\rho}} = \frac{\beta[\Phi(\tau_j^*) - \Phi(\tau_i^*)]}{(1 - \beta\bar{\rho}\Phi(\tau_j^*))^2} \frac{\left[1 - F(\tau_j^*)\right]}{\left[1 - F(\tau_i^*)\right]} > 0 \tag{A14}$$

□

When turbulence rises ($\bar{\rho}$ declines), current productivity is less predictive for future productivity, such that productive firms are less likely to stay productive. Thus, an increase in turbulence lowers expected value of productive firms, reducing their borrowing capacity for financing working capital. As a consequence, labor is reallocated to less productive firms.

The analytical results in Proposition 3 are partial equilibrium in nature, because we have assumed that the factor prices W and R are independent of turbulence. However, since the production thresholds for firms with lower productivity are more sensitive to changes in the factor prices, an increase in turbulence that reduces the factor prices would disproportionally expand the active regions for low-productivity firms, reinforcing the misallocation effects of turbulence.

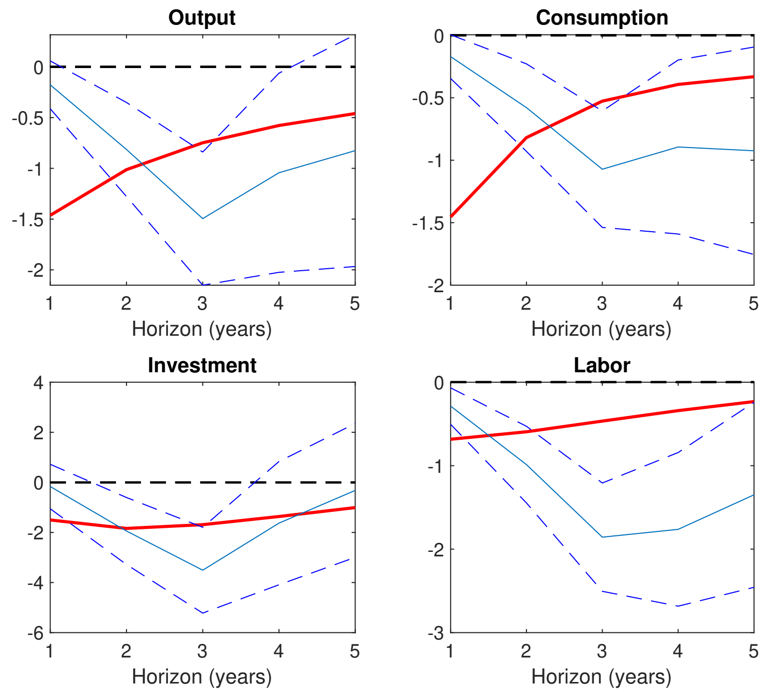

Figure A10. Impulse responses of macroeconomic variables to a turbulence shock: Benchmark model vs. data

Note: This figure shows the impulse responses of aggregate output, consumption, investment, and labor hours to a one-standard-deviation shock to turbulence in the data (the blue solid line) and in the calibrated annual version of the model (the red solid line). The dashed lines show the 68% confidence band around the empirical estimates of the impulse responses. The horizontal axis shows the years after the impact of the shock. The vertical axis shows the percent deviations of output in the model from its steady-state level and the percentage changes in output in the data relative to its pre-shock level.

B.2. **Macroeconomic effects of turbulence: Benchmark model vs. data.** In the text, we show that the benchmark calibrated model generates empirically plausible impulse responses of aggregate output to a turbulence shock. Here, we compare the impulse responses of other macro variables to a turbulence shock from the model against those estimated from the data.

Figure A10 compares the impulse response of output, consumption, investment and hours worked to a one-standard-deviation turbulence shock in the model (blue lines) vs. those in the data (red lines). The responses of output, investment, and labor hours are close to those in the data.

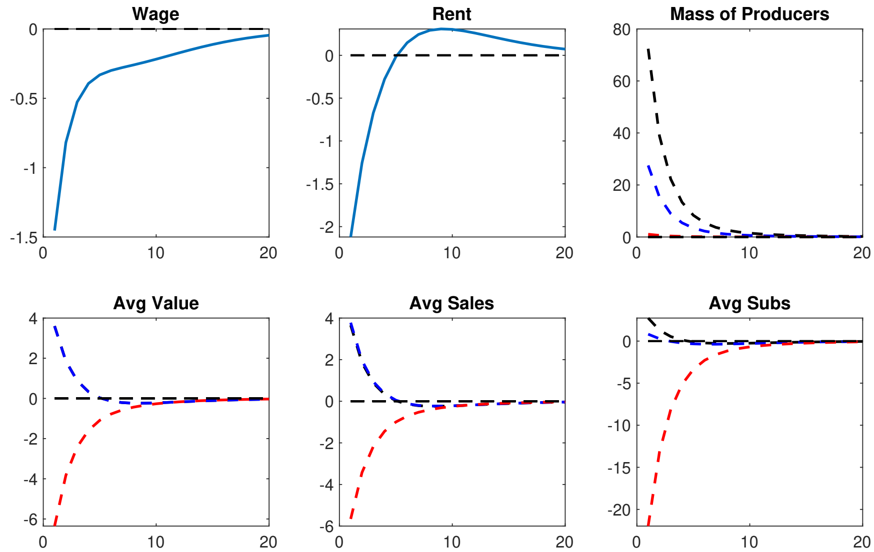

FIGURE A11. Heterogeneity of the effects of a turbulence shock

This figure shows the impulse responses to a turbulence shock in the benchmark model. The black, blue and red dashed lines represent, respectively, the responses of firms at the 25, 50 and 100 percentiles of the productivity distribution. The horizontal axis shows the periods (years) after the impact of the shock. The vertical axis measures percent deviations from the stochastic steady-state in response to a one-standard-deviation shock to turbulence.

B.3. **Heterogeneity of the effects of a turbulence shock.** Figure A11 shows that turbulence has heterogeneous effects on firms at different productivity levels. As discussed in the text, turbulence reduces aggregate TFP and leads to a recession. Thus, aggregate factor demand declines, reducing wages and capital rents. At any given productivity level, the decline in the factor prices lowers the threshold level of subsidy for active production. Thus, the mass of producers at all levels of productivity increases, although the mass of high productivity firms increases less because the shock that reshuffles productivity implies that higher-productivity firms are less likely to remain productive. Since all firms face the same factor prices, firms with lower productivity require higher subsidies to stay active. Therefore, average sales of active low-productivity firms increase relative to those of high-productivity firms, exacerbating misallocation.

B.4. **Transition probabilities of productivity ranking.** One concern related to our productivity process is that the productivity of a firm experiencing a turbulence shock might

Baseline Model					
Rank	1	2	3	4	5
1	0.635	0.243	0.081	0.026	0.015
2	0.242	0.367	0.254	0.110	0.026
3	0.081	0.255	0.332	0.252	0.080
4	0.026	0.109	0.252	0.371	0.242
5	0.015	0.027	0.081	0.241	0.636

TABLE A.6. Transition probabilities of firm productivity rankings: Baseline model

switch from the very top to the very bottom of the distribution (or vice versa), which would be counterfactual. Under our calibration, however, this is not the case.

Table A.6 shows the steady-state transition probabilities for firms in different quintiles of productivity ranking in our baseline model calibrated to match the turbulence process of Bloom et al. (2018) using the Census of Manufacturing data. The table shows that most of the switches are concentrated in adjacent productivity groups (e.g., between the top quintile and the second quintile), with a much smaller probability (less than 2 percent) of switching between the top and the bottom of the productivity distribution.

To examine whether the model-implied transition probabilities are in line with those in the data, we compute the empirical transition probabilities of firms' productivity ranking using the Compustat data (since we do not have access to the Census data). Figure A12 shows the average transition probabilities between productivity groups in the Compustat data (left side of Panel A). Evidently, in the Compustat data, the largest elements of the transition matrix are the diagonal elements, indicating that the productivity ranking is persistent. Most of the switchings are concentrated between firms in adjacent productivity groups (e.g., the first and the second quintiles), with near-zero probabilities of switching between the top and the bottom quintiles.

These transition patterns in the data are similar to those in our baseline model. However, the magnitudes of the transition probabilities are not directly comparable because the empirical transition matrix is calculated using the Compustat data whereas the model is calibrated to the Census data. To make the comparison between the model and the data internally consistent, we recalibrate the model parameters $\bar{\rho}$ and σ_z to match the average transition probabilities in the (1,1) and (2,1) elements of the empirical transition matrix (based on the Compustat data). The rest of the parameters are identical to the baseline model. We solve the steady state model and simulate a panel of 100,000 firms for two periods. We measure firms' TFP from the simulated sample and compute the transition matrix in the same procedure as we do with the data.

Figure A12 shows that the steady-state transition matrix from the simulated model (right side of Panel A) matches the patterns of that in the Compustat data closely. Similar to the data, the model predicts that firms with productivity in the top quintile (group 1) switch to the second quintile (group 2) with a modest probability of about 14 percent, and a much smaller probability to the lower-ranked groups. Conversely, firms in the bottom quintile (group 5) have a very small probability of switching to the top quintiles. Thus, changes in productivity rankings do happen in the model (as in the data), but the switches are concentrated in adjacent productivity groups.

We also report the transition probabilities when the average value of turbulence is one-standard-deviation below (low turbulence, Panel B) or above the steady-state level (high turbulence, Panel C). In both cases, as in the case with the steady state (Panel A), the model predicts that most switches are observed for firms in adjacent productivity groups, with very small probabilities of switching between the top ranked firms and the bottom ranked firms. And those predictions are in line with the data.

Appendix C. A simple microeconomic foundation for turbulence

We present a simple theoretical framework with input-output connections and show that the turbulence shock in our baseline model is isomorphic to supply-chain disruptions.

Consider an economy with a vertical production network consisting of raw material, intermediate good and final good production. Each final goods producer is matched with a unique intermediate good supplier and produces according to the technology

$$y_{ijt} = [a_{ijt} n_{ijt}]^{1-\phi} x_{ijt}^{\phi}$$

where a_{ijt} denotes the productivity level of final good producer i matched with supplier j and n_{ijt} and x_{ijt} denote the labor and intermediate inputs, respectively. The match-specific production efficiency depends on three factors, namely aggregate productivity A_t, i.i.d. productivity shock of final goods producer τ_{it}, as well as the quality of supplier z_{jt}. Specifically, we assume that

$$a_{ijt} = A_t \tau_{it} z_{jt}$$

where $z_{jt} = z_j$ is a permanent productivity shock of supplier j drawn from a finite number of states (with probability π_j), for $j = 1, 2, ..., J$. Without loss of generality, we further assume that $z_1 < z_2 < ... < z_J$. The supplier-customer relations between intermediate and final good producers are subject to separation risks. At the end of each period, existing pairs may

Panel A: Baseline											
Data						Model					
Rank	1	2	3	4	5	Rank	1	2	3	4	5
1	0.827	0.140	0.022	0.007	0.003	1	0.821	0.147	0.012	0.010	0.010
2	0.146	0.617	0.203	0.027	0.006	2	0.147	0.635	0.194	0.015	0.010
3	0.017	0.192	0.566	0.202	0.024	3	0.012	0.194	0.589	0.194	0.012
4	0.009	0.033	0.186	0.609	0.163	4	0.010	0.015	0.194	0.633	0.148
5	0.003	0.006	0.026	0.169	0.839	5	0.010	0.010	0.012	0.148	0.821
Panel B: Low Turbulence											
Data						Model					
Rank	1	2	3	4	5	Rank	1	2	3	4	5
1	0.845	0.135	0.018	0.002	0.000	1	0.844	0.145	0.005	0.003	0.003
2	0.143	0.630	0.190	0.034	0.002	2	0.146	0.649	0.195	0.008	0.003
3	0.015	0.195	0.581	0.190	0.019	3	0.005	0.195	0.601	0.195	0.005
4	0.005	0.026	0.178	0.630	0.162	4	0.003	0.008	0.195	0.648	0.145
5	0.005	0.005	0.020	0.168	0.803	5	0.003	0.003	0.005	0.146	0.844
Panel C: High Turbulence											
Data						Model					
Rank	1	2	3	4	5	Rank	1	2	3	4	5
1	0.784	0.158	0.037	0.009	0.013	1	0.799	0.149	0.019	0.017	0.017
2	0.180	0.572	0.196	0.033	0.019	2	0.149	0.618	0.194	0.022	0.017
3	0.024	0.199	0.519	0.225	0.033	3	0.019	0.194	0.573	0.195	0.019
4	0.010	0.058	0.201	0.540	0.191	4	0.017	0.022	0.195	0.618	0.149
5	0.000	0.008	0.030	0.207	0.756	5	0.017	0.017	0.019	0.149	0.798

FIGURE A12. Transition probabilities of firm productivity rankings: data vs. model (Compustat)

This figure shows the stationary transition probabilities of firm productivity rankings in the data and in the model. The value in cell (i, j) represents the conditional probability of firms' productivity switching from the ith quintile in year t to the jth quintile in year t+1.

separate with the exogenous probability $1 - \rho_t$; and conditional on separation, final goods producers and suppliers form new partnerships for production next period through a random matching process.

Without loss of generality, we assume that raw materials are supplied at the constant relative price q_0 to intermediate good suppliers, who operate with the technology that transforms each unit of raw materials into one unit of intermediate goods.

For given labor input n_{ijt}, the period-t surplus to share between each pair of final- and intermediate- good producers is

$$\tilde{\Pi}_{ijt} = \max_{x_{ijt}} [A_t \tau_{it} z_{jt} n_{ijt}]^{1-\phi} {x_{ijt}}^{\phi} - q_0 x_{ijt} - w_t n_{ijt}$$

The first-order condition w.r.t. x_{ijt} is

$$\phi a_{ijt}^{1-\phi} n_{ijt}^{1-\phi} x_{ijt}^{\phi-1} = q_0$$

or

$$x_{ijt} = \left[\frac{\phi}{q_0}\right]^{\frac{1}{1-\phi}} a_{ijt} n_{ijt}$$

which implies

$$\tilde{\Pi}_{ijt} = (1-\phi)\left[\frac{\phi}{q_0}\right]^{\frac{\phi}{1-\phi}} A_t z_{jt} \tau_{it} n_{ijt} - w_t n_{it}$$

Suppose the final goods producers obtain ω fraction of the total surplus, their problem can be characterized by the following Bellman equation:

$$\begin{aligned}\tilde{V}_t(z_{jt}, \tau_{it}) &= \max_{n_{ijt}} \omega \left((1-\phi)\left[\frac{\phi}{q_0}\right]^{\frac{\phi}{1-\phi}} A_t z_{jt} \tau_{it} n_{ijt} - w_t n_{ijt}\right) + \\ &\quad E_t M_{t+1}\left[\rho_t \tilde{V}_{t+1}(z_{jt}, \tau_{it+1}) + (1-\rho_t)\sum_{j=1}^{J} \pi_j \tilde{V}_{t+1}(z_j, \tau_{it+1})\right],\end{aligned}$$

subject to the constraint

$$w_t n_{ijt} \leq \tilde{\theta} E_t M_{t+1}\left[\rho_t \tilde{V}_{t+1}(z_{jt}, \tau_{it+1}) + (1-\rho_t)\sum_{j=1}^{J} \pi_j \tilde{V}_{t+1}(z_j, \tau_{it+1})\right]$$

For given ϕ and ω, we can normalize $(1-\phi)\left[\frac{\phi}{q_0}\right]^{\frac{\phi}{1-\phi}} = 1$. Define $V_t(z_{jt}, \tau_{it}) \equiv \frac{\tilde{V}_t(z_{jt}, \tau_{it})}{\omega}$, the firm's problem becomes

$$\begin{aligned}V_t(z_{jt}, \tau_{it}) &= \max_{n_{ijt}} (A_t z_{jt} \tau_{it})\, n_{ijt} - w_t n_{ijt} + \\ &\quad E_t M_{t+1}\left[\rho_t V_{t+1}(z_{jt}, \tau_{it+1}) + (1-\rho_t)\sum_{j=1}^{J} \pi_j V_{t+1}(z_j, \tau_{it+1})\right]\end{aligned}$$

subject to the constraint

$$w_t n_{ijt} \leq \tilde{\theta}\omega E_t M_{t+1}\left[\rho_t V_{t+1}(z_{jt}, \tau_{it+1}) + (1-\rho_t)\sum_{j=1}^{J} \pi_j V_{t+1}(z_j, \tau_{it+1})\right]$$

The model is equivalent to our baseline model (21) when $\theta \equiv \tilde{\theta}\omega$.

Appendix D. Correlated Production Distortion

In our baseline model, we assume that production distortion τ_{jt} is i.i.d. and un-correlated with true productivity $\tilde{z}_{jt}$. Now we show the qualitative and quantitative performance of the model remains robust if we introduce correlated distortion. For this purpose, we assume that the term τ_{jt} follows

$$\tau_{jt} = \varepsilon_{jt}\tilde{z}_{jt}^{\eta} \tag{A15}$$

Without loss of generality, the coefficient $\eta \in (-\infty, \infty)$ captures the correlated component in τ_{jt}, and ε_{jt} denotes the orthogonal component. Now we derive the firm's problem under this new specification.

Again, the production function for an individual firm is given by

$$y_{jt} = A_t\tilde{z}_{jt}k_{jt}^{\alpha}n_{jt}^{1-\alpha}, \tag{A16}$$

where y_{jt} denotes the output produced by firm j in period t, A_t denotes aggregate productivity, and k_{jt} and n_{jt} denote the capital and labor inputs, respectively. The idiosyncratic productivity shock $\tilde{z}_{jt}$ follows the stochastic process described in Eq. (2).

Firms at each level of productivity face idiosyncratic production distortions (denoted by τ_{jt}), which is correlated with productivity a la (A15). These production distortions drive a wedge between firms' private and social marginal revenue products. We assume that ε_{jt} is drawn from a continuous i.i.d. distribution $\tilde{F}(\varepsilon_{jt})$.

The firms' optimizing problem is characterized by the Bellman equation

$$V_t(\tilde{z}_{jt}, \varepsilon_{jt}) = \max_{k_{jt}, n_{jt}} \varepsilon_{jt}A_t\tilde{z}_{jt}^{1+\eta}k_{jt}^{\alpha}n_{jt}^{1-\alpha} - R_tk_{jt} - W_tn_{jt} + \mathbb{E}_tM_{t+1}V_{t+1}(\tilde{z}_{jt+1}, \varepsilon_{jt+1}), \tag{A17}$$

subject to the working capital constraint

$$R_tk_{jt} + W_tn_{jt} \leq \theta\mathbb{E}_tM_{t+1}V_{t+1}(\tilde{z}_{jt+1}, \varepsilon_{jt+1}) \equiv \theta B_{jt}. \tag{A18}$$

Here, the term $V_t(\tilde{z}_{jt}, \varepsilon_{jt})$ denotes the value function of firm j that depends on the firm-level state variables $\tilde{z}_{jt}$ and ε_{jt}. The value function $V_t(\tilde{z}_{jt}, \varepsilon_{jt})$ also depends on aggregate shocks, which are summarized by the time subscript t. The term $M_{t+1} = \beta\frac{\Lambda_{t+1}}{\Lambda_t}$ denotes the stochastic discount factor determined by the marginal utilities of the representative household who owns all firms. The term B_{jt} denotes the expected present value of a firm with current productivity $\tilde{z}_{jt}$.

Profit maximizing implies the conditional factor demand functions

$$\alpha\frac{\tau_{jt}y_{jt}}{k_{jt}} = (1 + \mu_{jt})R_t, \tag{A19}$$

and

$$(1-\alpha)\frac{\tau_{jt}y_{jt}}{n_{jt}} = (1 + \mu_{jt})W_t, \tag{A20}$$

where μ_{jt} denotes the Lagrangian multiplier associated with the credit constraint (22). Using the factor demand functions, we can write the firm's flow profit as

$$d_{jt} \equiv \left[\varepsilon_{jt} A_t \tilde{z}_{jt}^{1+\eta} \left(\frac{\alpha W_t}{(1-\alpha) R_t}\right)^{\alpha} - \frac{W_t}{1-\alpha}\right] n_{jt}. \tag{A21}$$

Since production subsidies follow an i.i.d. process, a firm would choose to be active in production if and only if its subsidy ε_{jt} is sufficiently high such that $d_{jt} \geq 0$. It follows that there exists a threshold level of production subsidy ε_{jt}^* such that, if $\varepsilon_{jt} \geq \varepsilon_{jt}^*$, then a firm would be active in production, facing binding credit constraints. Otherwise, the firm would remain inactive. At the threshold level of subsidy, a firm earns zero profit and thus it would be indifferent between producing and staying inactive. The indifference condition determines the threshold level of subsidy

$$\varepsilon_{jt}^* = \frac{R_t^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha} A_t \tilde{z}_{jt}^{1+\eta}}. \tag{A22}$$

The threshold ε_{jt}^* increases with the factor prices R_t and W_t and decreases with the productivity level $\tilde{z}_{jt}$. Thus, given the factor prices, the fraction of active firms is larger for firms with higher productivity.

As in the baseline model, the presence of credit constraints and production distortions creates misallocation of resources. Specifically, at each level of productivity, there is a non-degenerate fraction of firms that are active, with the share of active firms measured by $1 - \tilde{F}(\varepsilon_{jt}^*)$ for all $j \in 1, \ldots, J$. Such misallocation opens up a reallocation channel for turbulence shocks, as we show below.

Since active firms face binding credit constraints and inactive firms do not use any input factors, we obtain the conditional demand functions for labor and capital inputs

$$n_t(\tilde{z}_{jt}, \varepsilon_{jt}) = \begin{cases} \frac{(1-\alpha)\theta B_{jt}}{W_t}, & if \quad \varepsilon_{jt} \geq \varepsilon_{jt}^* \\ 0, & otherwise. \end{cases} \tag{A23}$$

and

$$k_t(\tilde{z}_{jt}, \varepsilon_{jt}) = \begin{cases} \frac{\alpha\theta B_{jt}}{R_t}, & if \quad \varepsilon_{jt} \geq \varepsilon_{jt}^* \\ 0, & otherwise. \end{cases} \tag{A24}$$

Given the factor demand functions, firm j's value function can be written as

$$V_t(\tilde{z}_{jt}, \varepsilon_{jt}) = \max\left\{\frac{\varepsilon_{jt}}{\varepsilon_{jt}^*} - 1, 0\right\} \theta B_{jt} + B_{jt}. \tag{A25}$$

Since production subsidies are i.i.d. across time, the average value of a firm with productivity $\tilde{z}_{jt}$ is given by

$$\bar{V}_t(\tilde{z}_{jt}) = \int V_t(\tilde{z}_{jt}, \varepsilon) d\tilde{F}(\varepsilon) = \left[1 + \theta \int_{\varepsilon_{jt}^*} (\frac{\varepsilon}{\varepsilon_{jt}^*} - 1) d\tilde{F}(\varepsilon)\right] B_{jt} \equiv \Phi(\varepsilon_{jt}^*) B_{jt}, \tag{A26}$$

where the term $\Phi(\varepsilon^*_{jt}) \equiv 1 + \theta \int_{\varepsilon^*_{jt}} (\frac{\varepsilon}{\varepsilon^*_{jt}} - 1) d\tilde{F}(\varepsilon)$ is a decreasing function of the threshold subsidy level ε^*_{jt}.

Given the stochastic process of $\tilde{z}_{j,t+1}$ and the definition of B_{jt}, we have

$$B_{jt} \equiv \beta \mathbb{E}_t \frac{C_t}{C_{t+1}} \left[\rho_t \bar{V}_{jt+1} + (1-\rho_t) \sum_{i=1}^{J} \pi_i \bar{V}_{it+1} \right]. \tag{A27}$$

The model is equivalent to our baseline model by defining $\tilde{F}(\varepsilon) \equiv F(\tau)$ and $\tilde{z}_{jt} \equiv z_{jt}^{1/(1+\eta)}$. As long as $\eta > -1$, the qualitative predictions of our model are preserved. The quantitative effects of turbulence is reinforced if $\eta > 0$.

Appendix E. A model with a continuous-state productivity process and without firm-level distortions

In our baseline model, we assume that firms are subject to persistent idiosyncratic productivity shocks $z_{j,t}$ with a discrete distribution, and i.i.d. production distortion shocks $\tau_{j,t}$. In this section, we present an alternative model with a single, continuous idiosyncratic state variable, which yields a threshold based on productivity. The main insights from the baseline model are preserved in this model.

E.1. **The model environment.** There is a continuum of firms, each endowed with a constant-returns technology that produces the final consumption good using capital and labor as inputs. The production function for an individual firm is given by

$$y_{jt} = z_{jt} [k_{jt}^{\alpha} n_{jt}^{1-\alpha}]^{\frac{\nu-1}{\nu}}, \tag{A28}$$

where y_{jt} denotes the output produced by firm j in period t, and k_{jt} and n_{jt} denote the capital and labor inputs, respectively. The parameter $\nu \in (1, +\infty)$ captures the decreasing return to scale.

Firms face idiosyncratic productivity shocks drawn at the beginning of each period, before hiring inputs. The idiosyncratic productivity shock z_{jt} (in log) follows the stochastic process similar to that described in Eq. (2),

$$\hat{z}_{j,t+1} = \rho_t \hat{z}_{j,t} + \sqrt{1-\rho_t^2} \varepsilon_{j,t+1}, \quad \varepsilon_{j,t+1} \sim N(0, \sigma_z^2) \tag{A29}$$

where $\hat{z}_{j,t} \equiv \log(z_{j,t})$ and $\varepsilon_{j,t}$ is i.i.d. random variable drawn from a time-invariant distribution with C.D.F $G(\varepsilon)$. The process is isomorphic to the TFP process in Moll (2014) in continuous time.

Similar to the baseline model, we can measure turbulence by $1 - \rho_t$. If $\rho_t = 1$, then the idiosyncratic productivity z_{jt} would be permanent. If $\rho_t = 0$, on the other hand, then each firm would face i.i.d. shocks to productivity with no persistence. A lower value of ρ_t implies that a high-productivity firm in the current period may not maintain its productivity in the next period, whereas a low-productivity firm in the current period might be able to draw a better productivity in the next period. Thus, a decline in ρ_t reshuffles firms' productivity ranking across time, increasing turbulence. As in our baseline model with a discrete-state productivity process, changes in turbulence (measured by $1-\rho_t$) affect both the conditional mean and conditional variance of firm-level productivity.[36]

Moreover, given the process of (A29), we can prove the following proposition:

Proposition 4. If the initial distribution of $\hat{z}_0$ is the same as that of ε, the distribution of $\hat{z}_t$ for all t is also the same as the distribution of ε, regardless of the value of ρ_t. In other word, we show that if $\hat{z}_0 \sim G_\varepsilon$, then $\hat{z}_t \sim G_\varepsilon$ for all t.

Proof. Assume that $\hat{z}_t \sim G_\varepsilon$ for some $t \geq 0$ such that we can write:

$$\hat{z}_t = \varepsilon_t \tag{A32}$$

where ε_t is a random variable with the same distribution as ε. We have:

$$\hat{z}_{t+1} = \rho_t \varepsilon_t + \sqrt{1-\rho_t^2}\varepsilon_{t+1} \tag{A33}$$

Both ε_t and ε_{t+1} are i.i.d. random variables with distribution G_ε. Since ε_t and ε_{t+1} are independent and identically distributed, their linear combination:

$$\rho_t \varepsilon_t + \sqrt{1-\rho_t^2}\varepsilon_{t+1} \tag{A34}$$

will also have the same distribution G_ε. □

The firms' optimizing problem is characterized by the Bellman equation

$$V_t(z_{jt}) = \max_{k_{jt}, n_{jt}} z_{jt}[k_{jt}^{\alpha} n_{jt}^{1-\alpha}]^{\frac{\nu-1}{\nu}} - R_t k_{jt} - W_t n_{jt} + \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1} \mid z_{j,t}), \tag{A35}$$

subject to the working capital constraint

$$R_t k_{jt} + W_t n_{jt} \leq \theta \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1} \mid z_{j,t}) \equiv \theta B_t(z_{jt}) \tag{A36}$$

Here, the term $V_t(z_{jt})$ denotes the value function of firm j that depends on the firm-level state variable z_{jt}. The value function $V_t(z_{jt})$ also depends on aggregate shocks, which are summarized by the time subscript t. The term $M_{t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}$ denotes the stochastic discount factor determined by the marginal utilities of the representative household who owns all firms. The term B_{jt} denotes the expected present value of a firm with current productivity z_{jt}.

[36] The conditional mean and variance of $z_{j,t+1}$ are

$$E_t[\hat{z}_{j,t+1}] = E_t[\rho_t \hat{z}_{j,t}] + \sqrt{1-\rho_t^2} E_t[\varepsilon_{j,t+1}] = \rho_t \hat{z}_{j,t} \tag{A30}$$

and

$$Var_t[\hat{z}_{j,t+1}] = Var_t[\sqrt{1-\rho_t^2}\varepsilon_{j,t+1}] = (1-\rho_t^2)\sigma_z^2 \tag{A31}$$

respectively.

Profit maximizing implies

$$k_{jt} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t}n_{jt}, \tag{A37}$$

It follows that there exists a threshold level of production subsidy z_t^* such that, if $z_{jt} \geq z_t^*$, then a firm would be facing binding credit constraints. Otherwise, the firm would choose an interior optimal size. At the threshold level of productivity, the optimal size of production coincides with level implied from binding constraint, and the indifference condition determines the threshold level of productivity

$$\left(\frac{\nu-1}{\nu}\right) z_t^* = [\theta B_t(z_t^*)]^{\frac{1}{\nu}} \left[(\frac{W_t}{1-\alpha})^{1-\alpha} (\frac{R_t}{\alpha})^{\alpha} \right]^{\frac{\nu-1}{\nu}} \tag{A38}$$

The threshold z_t^* increases with the factor prices R_t and W_t and decreases with the aggregate productivity level A_t. We obtain the conditional demand functions for labor and capital inputs

$$n_t(z_{jt}) = \begin{cases} \frac{(1-\alpha)\theta B_t(z_{jt})}{W_t}, & if \quad z_{jt} \geq z_t^* \\ \frac{1-\alpha}{W_t}[(\frac{\nu-1}{\nu})z_{jt}]^{\nu} \left[(\frac{1-\alpha}{W_t})^{1-\alpha} (\frac{\alpha}{R_t})^{\alpha} \right]^{\nu-1}, & otherwise. \end{cases} \tag{A39}$$

and

$$k_t(z_{jt}) = \begin{cases} \frac{\alpha\theta B_t(z_{jt})}{R_t}, & if \quad z_{jt} \geq z_t^* \\ \frac{\alpha}{R_t}[(\frac{\nu-1}{\nu})z_{jt}]^{\nu} \left[(\frac{1-\alpha}{W_t})^{1-\alpha} (\frac{\alpha}{R_t})^{\alpha} \right]^{\nu-1}, & otherwise. \end{cases} \tag{A40}$$

Idiosyncratic output functions are

$$y_t(z_{jt}) = \begin{cases} z_{jt} \left[(\frac{1-\alpha}{W_t})^{1-\alpha} (\frac{\alpha}{R_t})^{\alpha} \theta B_{jt} \right]^{\frac{\nu-1}{\nu}}, & if \quad z_{jt} \geq z_t^* \\ (\frac{\nu-1}{\nu})^{\nu-1} (z_{jt})^{\nu} \left[(\frac{1-\alpha}{W_t})^{1-\alpha} (\frac{\alpha}{R_t})^{\alpha} \right]^{\nu-1}, & otherwise. \end{cases} \tag{A41}$$

Given the factor demand functions, firm j's value function can be written as (for ease of notation, we denote $z \equiv z_{j,t}$ and $z' \equiv z_{j,t+1}$)

$$V_t(z) = \begin{cases} z \left[(\frac{1-\alpha}{W_t})^{1-\alpha} (\frac{\alpha}{R_t})^{\alpha} \theta B_t(z) \right]^{\frac{\nu-1}{\nu}} - \theta B_t(z), & if \quad z \geq z_t^* \\ \frac{1}{\nu-1} \left(\frac{\nu-1}{\nu}\right)^{\nu} (z)^{\nu} \left[(\frac{1-\alpha}{W_t})^{1-\alpha} (\frac{\alpha}{R_t})^{\alpha} \right]^{\nu-1}, & otherwise. \end{cases} \tag{A42}$$

where $B_t(z)$ can be represented by

$$B_t(z) \equiv \mathbb{E}_t M_{t+1} V_{t+1}(z' \mid z) = \beta \mathbb{E}_t \frac{C_t}{C_{t+1}} \left[\int_{\varepsilon} V_{t+1}(z') dG(\varepsilon) \right], \tag{A43}$$

given the stochastic process of $\log(z') \equiv \rho_t \log(z) + \sqrt{1-\rho_t^2}\varepsilon$, and the definition of $B_t(z)$ in Eq. (A36).

In a competitive equilibrium, markets for labor, capital, and final consumptions goods all clear. Labor market clearing implies that

$$N_t = \int^{z_t^*} \frac{1-\alpha}{W_t}[(\frac{\nu-1}{\nu})z]^{\nu}\left[(\frac{1-\alpha}{W_t})^{1-\alpha}(\frac{\alpha}{R_t})^{\alpha}\right]^{\nu-1} dG(z) + \int_{z_t^*} \frac{(1-\alpha)\theta B_t(z)}{W_t} dG(z). \quad \text{(A44)}$$

Capital market clearing implies that

$$K_t = \int^{z_t^*} \frac{\alpha}{R_t}[(\frac{\nu-1}{\nu})z]^{\nu}\left[(\frac{1-\alpha}{W_t})^{1-\alpha}(\frac{\alpha}{R_t})^{\alpha}\right]^{\nu-1} dG(z) + \int_{z_t^*} \frac{\alpha\theta B_t(z)}{R_t} dG(z) \quad \text{(A45)}$$

Goods market clearing implies that

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t, \quad \text{(A46)}$$

where aggregate output Y_t is given by

$$Y_t = \int^{z_t^*} (\frac{\nu-1}{\nu})^{\nu-1}(z)^{\nu}\left[(\frac{1-\alpha}{W_t})^{1-\alpha}(\frac{\alpha}{R_t})^{\alpha}\right]^{\nu-1} dG(z) + \int_{z_t^*} z\left[(\frac{1-\alpha}{W_t})^{1-\alpha}(\frac{\alpha}{R_t})^{\alpha}\theta B_{jt}\right]^{\frac{\nu-1}{\nu}} dG(z). \quad \text{(A47)}$$

We define aggregate TFP as

$$TFP = \frac{Y_t}{[K_t^{\alpha}N_t^{1-\alpha}]^{\frac{\nu-1}{\nu}}} \quad \text{(A48)}$$

The household problem remains identical to our baseline model.

Definition. A competitive equilibrium consists of the sequence of allocations $\{C_t, Y_t, N_t, K_t\}$ and the sequence of prices $\{W_t, R_t\}$ such that (i) taking all prices as given, the allocations solve the household's utility maximizing problem and the firms' profit maximizing problem; and (ii) markets for labor, capital, and goods all clear.

E.2. **Turbulence and Steady State Allocation.** We now provide some analytical characterization of the steady-state equilibrium and show how the steady-state allocations vary with the average level of turbulence $(1-\rho)$. To do so we first discretize the value for $z_{j,t}$ using the Tauchen method, and then solve the stationary value function $(V(z))$ for each state of $z_{j,t}$ through value function iteration.

The main insights from our baseline model are preserved in this alternatively setting with single idiosyncratic state variable. Figure A13 plots the steady-state relation between the turbulence and aggregate output, consumption, investment, wage rate, TFP and interquartile range of labor. In an economy with a higher average level of turbulence (i.e., with a lower value of ρ), a high-productivity firm is less likely to remain productive, reallocating resources to low-productivity firms. Such reallocations reduce aggregate TFP, lowering the factor prices, generating a synchronized decline in output, consumption and investment.

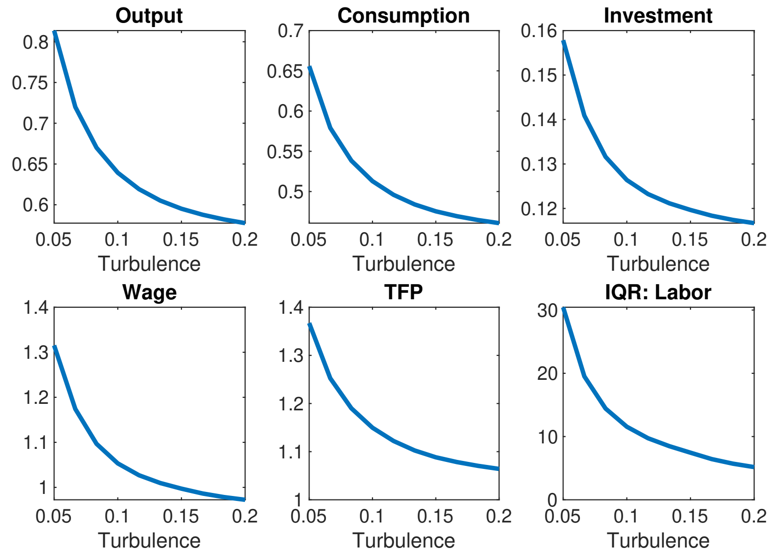

FIGURE A13. Turbulence and the steady-state equilibrium

Note: This figure shows the steady-state relation between the turbulence and aggregate output, consumption, investment, wage rate, TFP and interquartile range of labor. We set the parameter governing decreasing return to scale $\nu = 10$. The rest of the parameters are identical to our baseline model.