FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

Supply or Demand? Policy Makers' Confusion in the Presence of Hysteresis

Antonio Fatás
INSEAD

Sanjay R. Singh
Federal Reserve Bank of San Francisco
University of California, Davis

June 2023

Working Paper 2023-21

https://www.frbsf.org/economic-research/publications/working-papers/2023/21/

Suggested citation:

Fatás, Antonio, Sanjay R. Singh. 2023 "T Supply or Demand? Policy Makers' Confusion in the Presence of Hysteresis," Federal Reserve Bank of San Francisco Working Paper 2023-21. https://doi.org/10.24148/wp2023-21

Supply or Demand? Policy Makers' Confusion in the Presence of Hysteresis *

Antonio Fatás† Sanjay R. Singh§

Abstract: *Policy makers need to separate between temporary demand-driven shocks and permanent shocks in order to design optimal aggregate demand policies. In this paper we study the case of a central bank that ignores the presence of hysteresis when identifying shocks. By assuming that all low-frequency output fluctuations are driven by permanent technology shocks, monetary policy is not aggressive enough in response to demand shocks. In addition, we show that errors in assessing the state of the economy can be self-perpetuating if seen through the lens of the mistaken views of the policy maker. We show that a central bank that mistakes a demand shock for a supply shock, will produce permanent effects on output through their suboptimal policies. Ex-post, the central bank will see an economy that resembles what they had forecast when designing their policies. The shock is indeed persistent and this persistence validates their assumption that the shock was a supply-driven one. The interaction between forecasts, policies and hysteresis creates the dynamics of self-perpetuating errors that is the focus of this paper.*

1 Introduction

The concept of potential output is central to the study of business cycles as well as in the design of optimal fiscal and monetary policies.[1] Because potential output is not observable, its measurement requires assumptions about the specific model generating the stochastic movements in output. Most common methods to measure potential output rely on statistical filters that separate high frequency from low frequency movements in output. In practice, different filters will put different weights to changes in output that are driven by either the supply-side or the demand-side dynamics with implications for optimal policy (Nelson and Plosser, 1982; Barnett

*We thank the editor Evi Pappa and two anonymous refereees, as well as Pablo Cuba-Borda, Keshav Dogra, John G. Fernald, Omar Rachedi, Michaela Schmöller, Alan M. Taylor, and Mauricio Ulate for comments and useful discussions. The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Federal Reserve System. All errors are ours.

†INSEAD (antonio.fatas@insead.edu).

§Federal Reserve Bank of San Francisco and Department of Economics, UC Davis (sjrsingh@ucdavis.edu).

[1] The output gap is included in monetary policy reaction functions, such as in the Taylor rule (Woodford, 2001). In the case of fiscal policy, cyclically adjusted measures of government balances, an indicator of the policy stance, are based on a measure of the output gap (Fatás and Mihov, 2012) and debt sustainability analysis requires an understanding of potential output (Giorno et al., 1995; Gaspar, 2020).

et al., 2009). As time passes, the data will validate or reject the model and corrections will be made. Using a wrong measure of potential output can lead to suboptimal stabilization policies and excessive GDP volatility.

However, this statistical approach to measuring potential output is not appropriate in the presence of hysteresis. In models with hysteresis, potential output is dependent on history (Cerra et al., 2022). Even if one were to identify the size and the type of shock that hit the economy, a long-term forecast of GDP also depends on the future behavior of agents, including the policymaker. The amount of knowledge required to define and estimate potential output is much larger than without hysteresis. Errors in assessing these dynamics can lead to significant costs because there can be persistent effects of transitory shocks.

There is a second complication that can make these policy errors persist over time and which constitutes the focus of our paper. There is a potential vicious cycle in the way policymakers construct their forecasts, design their policies and learn from the consequences on economic activity. This vicious cycle might lead to lack of learning about the true model because policy makers ignore the consequences of their actions. This lack of learning comes from the fact that there are two possible economic models that generate identical scenarios for observables.

To illustrate these dynamics, we present in this paper an extreme form of errors in measuring potential output. A central bank designs optimal monetary policy under the assumption that the persistence in GDP is a result of shocks to total factor productivity (TFP).[2] But this assumption is wrong. In the correct model, the persistence is the outcome of hysteresis. Demand-driven fluctuations lead, through hysteresis, to permanent changes in output. We show that in this environment, the central banker is not aggressive enough and her actions generate dynamics of GDP that are consistent with her mistaken beliefs. Output is as persistent as the central banker had assumed but the persistence is entirely due to the mistaken policy that she had implemented. Not only the mistakes are very costly but they might never get corrected because the actual behavior of GDP resembles the policymaker's forecasts from a mistaken model. Policy makers who believe that persistence is caused by supply shocks will continue believing in their assumptions and will make the same mistake again in the future. Suboptimal policy regimes self

[2]This is not just a hypothetical scenario. There is evidence that fluctuations in GDP lead too often to revisions of estimates of potential output, suggesting that policy makers overestimate the permanent nature of temporary fluctuations (Fatás, 2019; Coibion et al., 2018).

perpetuate.

This paper is organized as follows. Section 2 presents a literature review with an emphasis on implications of measuring potential output. Section 3 introduces introduces a standard new-Keynesian model as a framework to talk about optimal policies. Section 4 shows how the mistaken views of central bankers about the true model can generate suboptimal policies. Section 5 discusses extensions to the baseline framework and Section 6 concludes.

2 Treatment of Potential Output

2.1 In Traditional Models

The notion of potential output remains central to modern macroeconomic models where we can find several possible definitions of it. In models without inefficiencies, output is always at its efficient level. In the presence of transitory shocks, output deviates temporarily from its deterministic trend but it converges to that trend in the long-run. There can also be shocks that permanently shift the trend or potential output. In this environment, trend output always remains well defined and can be measured as the long-term output forecast.[3]

In models with inefficiencies, the steady state output is not the welfare maximizing level of output. During the transition dynamics, the output will not be at its efficient level. For example, in a new Keynesian model, the inefficiencies arise from both price rigidities and imperfect competition. In these models it is common to focus on the inefficiencies related to cyclical dynamics and define potential output as the level that would prevail if all prices were set flexibly in the current period and all future periods taking as given the evolution of the state variable. This is also referred to as the *natural* output in the literature (Blanchard, 2018).[4] We refer to the distance between the actual and the natural output as the output gap.[5] In the long run, the effects of

[3]See, for example, Nelson and Plosser (1982). Beveridge and Nelson (1981) produce a trend decomposition using this approach in a standard RBC model.

[4]This level is different from the efficient level of output because of the presence of steady-state monopolistic competition distortions. We could also use as a reference the level of output that would prevail if all inefficiencies or price rigidities had never been in place, what Neiss and Nelson (2003) define as the unconditional equilibrium level of output.

[5]Under the assumption that those other inefficiencies (such as imperfect competition) are constant over the business cycle, the distance between the natural and efficient level of output will be constant (Blanchard and Galí, 2007). In this case, whether we measure the output gap relative to natural or efficient, its evolution will be identical even if there is a difference in levels. From a policy point of view, standard New Keynesian models have a well-known prescription for stabilization policies: eliminate the output gap. (Blanchard and Galí, 2007). This takes care of the inefficiencies

transitory shocks die out and we return to the steady state where output converges to the natural output. In that sense, in these models, the long-term level of natural output is also associated to the concept of trend output (Vetlov et al., 2011).

However, the potential output is not observable so estimating the potential output in real time requires a full understanding of the economic model driving fluctuations and the particular shocks that are hitting the economy at any point in time. One approach is to estimate the potential output by relying on a DSGE model to identify the structural shocks (Andrés et al., 2005; Edge et al., 2008). This approach requires an enormous amount of knowledge of the structure of the model and the parameter values as well as the ability to estimate in real time the current set of shocks. Most estimates of potential output are produced using a minimum set of assumptions about the true economic model (Basu and Fernald, 2009). The main framework is one where growth is exogenous and fluctuations are caused by both supply and demand shocks and we rely on reduced-form models or statistical filters to extract a measure of the output trend (Chalaux and Guillemette, 2019; De Resende, 2014; Shackleton, 2018).[6].

Errors in estimating potential output lead to suboptimal monetary and fiscal policies but these errors only affect the volatility of GDP as prices ultimately adjusted and GDP returns to trend. (Orphanides (2003), Orphanides et al. (2000), Ehrmann and Smets (2003), or Gorodnichenko and Shapiro (2007)). In addition, there is an element of learning, even if mistakes are made. The economic outcomes help policy makers identify their errors and improve their knowledge about the model that drives cycles. As an example, if there is a negative supply shock that is interpreted as demand deficiency, this is seen by policy makers as an opening of the output gap and signal to them that they need to engage in expansionary policies. But, as they do, those expansionary

that are responsible for the cyclical component of GDP. Other inefficiencies might persist, such as those derived from imperfect competition, but those should be addressed by other policies. In some models, price rigidities might not be the only friction that is relevant to the business cycle. If more than one of these frictions is in place, policy makers may trade-off output gap stabilization against other cyclical frictions (Blanchard and Galí, 2010; Queralto, 2022).

[6] In fact, in some contexts, a certain statistical filter might provide a good approximation to a model-based measure of potential output. For example, under certain assumptions about the type of shocks that are driving fluctuations, natural output can be a smooth series that can be well approximated by a standard econometric filter (Basistha and Nelson (2007)). However, in the presence of markup shocks, Justiniano and Primiceri (2008) show that while the unconditional efficient output is similar to the HP filtered output of the CBO estimate, the natural level of output tends to be highly volatile. The statistical filters based methodologies have come under criticism because their estimates of potential output are too sensitive to changes in GDP. For example, IMF or European Commission estimates of potential output are revised by 0.6-0.8% for every 1% surprise in GDP (Fatás, 2019). These estimates tend to be too optimistic during good years and too pessimistic during bad years (Mc Morrow et al., 2017; Coibion et al., 2018; Kuusi, 2017)

policies generate price pressures and inflation that indicates that the original assessment of the output gap was mistaken. In the long run, as GDP always returns to trend, policy makers will be able to correctly identify the magnitude of the permanent component of the shock.

2.2 Models with hysteresis

In models with hysteresis, even transitory shocks can affect GDP permanently (Cerra et al., 2022).[7] In these models, the balanced-growth path is a function of the history of the economy. Fluctuations in potential output arise from a wide range of shocks. This long term forecast also depends on all current and future policies as well as the reaction of the private sector.

This difficulty in defining potential output in these models represents a challenge to the traditional methodologies used to estimate it. The knowledge required to estimate potential output is much larger than before. With traditional models we simply relied on our ability to identify supply and demand shocks but now, with hysteresis, we also need an understanding of how the dynamics of any shock can affect the potential output, which is now endogenous. In practice, this means that statistical methods that rely on the behavior of output at different frequencies are ill-suited to measuring potential output. These dynamics of output are consistent with a variety of models that are observationally equivalent.

As it was the case before, errors in measuring potential output lead to suboptimal economic policies but now, in the presence of hysteresis, the costs of these errors is much larger because their effects are permanent. In addition, getting the estimates of potential output wrong might not generate the type of learning that we saw in models without hysteresis. As a result, errors in assessing the state of the economy can be self-perpetuating if seen through the lens of the mistaken views of a policy maker. In the next sections we present an illustrative example of how these dynamics play out and where a central banker that is mistaken about the true economic model can be responsible for generating hysteresis. And this hysteresis ends up validating the mistaken logic of the policy maker that persistence of GDP can be used to infer the existence and

[7] See Cerra and Saxena (2008); Jordà et al. (2021) for evidence on hysteresis or Jordà et al. (2013) for evidence on deep and protracted recovery/non-recovery from financial crises relative to normal recessions for advanced economies. Fatás and Mihov (2013) show the slow recovery post the Great-Recession for the US. For additional references, see, for example, Acharya et al. (2022); Annicchiarico and Pelloni (2021); Anzoategui et al. (2019); Benigno and Fornaro (2018); Bianchi et al. (2019); Fornaro and Wolf (2023); Garga and Singh (2021); Guerron-Quintana and Jinnai (2019); Moran and Queraltó (2018); Queralto (2020); Schmöller and Spitzer (2021); Vinci and Licandro (2021).

magnitude of supply shocks.

3 Monetary policy in a standard new-Keynesian model

Our starting point is an environment where growth is exogenous and business cycles are driven by a combination of permanent technology shocks and temporary demand shocks. While the main message of our paper is relevant to both monetary and fiscal policy, we will focus here on a central bank that designs a monetary policy rule that is optimal in this context.

We follow Giannoni (2014) and consider a stylized new Keynesian framework, variables expressed as log-linear deviations from respective steady state values. x_t is output gap measured in terms of deviations from stationary level of natural output, π_t is log-deviation of inflation from the target rate, i_t is the log-deviation of level of nominal interest rate from the steady state, and r_t^e is the log-deviation of natural real interest rate from the steady state. Equation (1) is derived from inter-temporal consumption Euler equation, combined with economy's resource constraint.[8] Equation (2) describes the new Keynesian Phillips curve.

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^e \right) \tag{1}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{2}$$

The natural (and efficient) rate of interest is defined as[9]

$$r_t^e \equiv \sigma E_t \left[\left(y_{t+1}^e - y_t^e \right) + z_{t+1} \right] + b_t \tag{3}$$

where z_{t+1} is log-deviation of TFP growth rate in period $t+1$ relative to the steady state TFP growth rate, y_t^e is the stationary level of output (in log-deviations) that would prevail in the absence of nominal rigidities,[10] and b_t is a temporary demand disturbance in period t. The coefficient σ denotes the inter-temporal elasticity of substitution, κ is the slope of the Phillips

[8]There is no government spending or storage. Output is equal to consumption.

[9]We assume the presence of lumpsum taxes to finance a production subsidy that offsets monopoly distortions in the intermediate goods sector.

[10]Actual output in the absence of nominal rigidities would grow at the rate of TFP growth in the steady state.

curve, and β is the discount factor. Finally, $x_t \equiv y_t - y_t^e$ denotes the output gap.

We assume that the natural level of (stationary output) is constant, $y_t^e = 0$, and exogenous shocks to TFP growth rate z_{t+1}, and demand disturbances b_t are the only source of fluctuations in this economy. We model the following process for the evolution of TFP growth, and demand disturbance respectively:

$$z_t = \rho z_{t-1} + \epsilon_t \tag{4}$$

$$b_t = \rho b_{t-1} + \epsilon_t^b \tag{5}$$

The central bank follows an interest rate rule. In Taylor-type interest rate rules typically interest rates depend on both the output gap and inflation. In our specification we write a rule that only includes the output gap and ignores inflation. We do this because we want to stress the influence of potential output estimates on policy rather than issues related to the measurement of inflation. Our main argument is about policy makers that confuse the persistence of GDP with the effects of permanent supply shocks, instead of hysteresis.

If our focus was on fiscal policy, the output gap is the key variable that enters decisions on the cyclicality of budget balances or the sustainability of debt. Estimates of the output gap by governments, the US CBO or international organization do not make use of inflation (Arnold, 2009).

For monetary policy, inflation is a key input in the central bankers' decision making given that their objectives tend to be established as an inflation target. However, output gap estimates by central banks are also typically calculated independent of inflation. Furthermore, in our current setting, a central bank can achieve optimality by only reacting to one of the two variables, so our rule happens to be optimal. The reason is that under the assumption that there are no shocks to the Phillips curve, optimal policy can be achieved just by central bank reacting to either the output gap or the inflation rate. In Section 5.2, we extend our results to environments where inflation is also included in the monetary policy rule by introducing price-markup shocks.

The central bank interest rate rule can then be written as:

$$i_t = \psi_x x_t \tag{6}$$

where the weight ψ_x is chosen optimally to minimize a quadratic welfare loss criterion given by:

$$E[L] = (1-\beta)E\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2\right] \tag{7}$$

λ_x, and λ_i are welfare weights in the objective function.[11]

Proposition 1. $\psi_x = \frac{\lambda_x(1-\beta\rho)^2+\kappa^2}{\lambda_i(\sigma(1-\rho)(1-\beta\rho)-\rho\kappa)(1-\beta\rho)} > 0$ *solves the optimal policy problem of a central bank that minimizes the loss function in Equation (7).*

In Appendix A, we provide the derivation for the coefficient ψ_x under optimal policy following Giannoni (2014).[12] The model can be solved using a method of undermined coefficients (Galí, 2015).[13]

The results so far are standard in the literature and we have established a simple Taylor-type rule that the central bank needs to follow in order to minimize the expected loss. The central bank reacts to the output gap in order to minimize volatility in inflation, interest rate and the output gap.

4 Endogenous growth model and policy makers' confusion

Having established a baseline for how central banks should behave in a standard New Keynesian environment with exogenous growth, we now put policy makers in an environment that is very different but we assume that their beliefs are that the first setting is still the relevant one. The second environment has growth being endogenous and where we only have demand shocks. Now, the endogeneity of growth generates hysteresis effects and demand shocks have permanent effects on output. The reason for only including demand shocks in this second environment is just to keep the model as simple as possible.

[11] Giannoni (2007, 2014) assumes this welfare loss function with penalty for interest rate variability based on Friedman (1969) transactions costs (Woodford, 2003a). In Section 5.3, we discuss this loss function further.

[12] Proof of all other propositions can be also be found in Appendices at the end of the paper.

[13] While this optimal rule may not always guarantee equilibrium determinacy (Blanchard and Kahn, 1980), we solve the model using an equilibrium selection device commonly used in the literature (Eggertsson and Woodford, 2003; Werning, 2011; Cochrane, 2017). Under this device, the economy returns back to the same steady state after shocks have dissipated.

If central banks were aware of the true model, they would need to react to demand shocks in order to close the output gap. But under our assumption that the central bank is mistaken about the true underlying model and believes that the economy is driven by the environment described in the previous section, we will see inaction to fluctuations caused by wrongly identified shocks. The central bank believes that growth is exogenous and the fluctuations in the long-run trend of the economy are outside of their influence, because they are driven by supply shocks. Their inference about the existence of demand and supply shocks comes from their observation of the dynamics of output. The more persistent output is, the more weight central bankers will put on supply shocks. In many ways this can be interpreted as if central banks makes use of a standard filter applied to GDP to separate permanent and transitory shocks.[14]

Our goal is to show that the evolution of GDP under these two environments can be observationally equivalent, which will make the central bank be reassured that their beliefs about the model are correct. The persistence of GDP observed under the true model, when growth is endogenous, is the outcome of hysteresis and will mimic the one observed in the exogenous growth model with supply shocks. What is interesting is that the persistence of GDP in the endogenous growth environment is partially caused by the mistakes of the central bank. And this persistence, an outcome of mistakes of central banks, is used to reinforce the wrong assumptions that led to those mistakes in the first place.

4.1 The endogenous growth model with mis-measured output gap

The model is characterized by a similar set of equations as the previous one except for the fact that growth is now endogenous. Productivity growth is assumed to react to the output gap via a hysteresis parameter (η). This reduced form relationship is intended to capture the effects of the cycle on productivity growth in a stylized manner. Micro-foundations for such a relationship, based on a learning-by-doing mechanism, can be found for example in Stadler (1990) or more recently in Queralto (2022), among other papers in the endogenous growth business cycle

[14] Orphanides and Norden (2002) argue that including other variables such as inflation to estimate potential output does not deliver a more reliable estimate than univariate filters. However, Coibion et al. (2018) results suggest that in the context of their theoretical framework, with growth being exogenous, the use of other variables such as inflation could potentially improve the identification of shocks.

literature.

$$x_t = E_t x_{t+1} + E_t z_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - \tilde{r}_t^e \right) \tag{8}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{9}$$

$$z_{t+1} = \eta x_t \tag{10}$$

Only temporary demand shocks affect the natural interest rate in this economy.[15] The natural rate process is given by:

$$\tilde{r}_t^e = \rho_e \tilde{r}_{t-1}^e + \epsilon_t^e$$

As for the policy rule, we use the same rule as before where the central bank is reacting to the output gap. The difference is that the central bank does not react to the true output gap. Instead, and because they are assuming the wrong economic model, they infer the output gap by observing the persistence of GDP, a signal of the importance of supply versus demand shocks. If we represent by x_t^{cb} the output gap as estimated by the central bank, we can write the policy rule as

$$i_t = \psi_x x_t^{cb}$$

Since this interest rate rule is effectively a peg, an ismorphism with an endogenous interest rate rule can be shown by modeling monetary policy shocks. The error in measuring the output gap can be considered a shock to the policy function.[16]

We will also consider an optimal rule under endogenous growth. In Appendix D, we provide the derivation for the optimal coefficient to the Taylor rule $i_t = \psi_x^{en} x_t$ in the endogenous growth environment.[17]

[15]Notice, an implication of modeling equation 10 is that the forward guidance puzzle is amplified in this model relative to the case when $\eta = 0$. We largely work with current, unanticipated iid shocks to show our main results.

[16]In fact, we will build the observational equivalence proof shortly using the following rule, under endogenous growth: $i_t = \psi_x x_t + \epsilon_t^i$, where x_t is now the actual gap and we can think of ϵ_t^i as the interest error that is introduce by the central bank because of their confusion about the economic model.

[17]Note that there can be local indeterminacy for some parameterizations. We select an equilibrium using the minimum state variable criterion, as before, assuming the economy returns to the original stationary steady state when there are no shocks to the natural rate.

4.2 Output persistence equivalence and central bank confusion

We now show, in Proposition 2, that there exists a sequence of shocks in the exogenous growth and endogenous growth environment such that the output path is identical under the two settings.

Proposition 2. *Assume all shocks are iid (i.e. $\rho = \rho_e = 0$). There exists a sequence of shocks $\{\epsilon_t^b, \epsilon_t\}$ in the exogenous growth environment, and shocks $\{\epsilon_t^e\}$ in the endogenous growth environment such that the output path is identical.*

The central bank, under the belief that the true economic model is that of exogenous growth with supply and demand shocks will observe a path for output that is consistent with their beliefs. They will follow their policy rule and implement an interest rate that is optimal given their mistaken perception of the output gap. In reality, this interest rate is suboptimal, does not properly minimize the output gap, and this output gap via hysteresis generates that exact output path that the mistaken central bank was expecting.

The true output gap and inflation in the endogenous growth model are actually lower than the counterfactual equilibrium when the central bank had correctly measured the output gap, or conducted optimal policy consistent with endogenous growth model. Moreover, in the endogenous growth environment, the potential output is endogenous to the policy regime in place.[18] An implication of Proposition 2 is that the central bank policy regime is responsible for the decline in potential output following a negative demand shock $\epsilon_1^e < 0$. We analytically prove this result in Proposition 3:

Proposition 3. *Let $\epsilon_1^e < 0$. If the central bank targeted the correct output gap in the endogenous growth environment, the output gap, the potential output and the level of the long-run output are higher than the scenario in Proposition 2.*

We define *output hysteresis* as the gap between the potential output and its initial deterministic trend level. In this environment, a mistaken central banker causes excess output hysteresis because of an incorrect model being used to make inference on output gap. Furthermore, if the

[18] Potential output or the natural output (interchangeably) is defined as the level of output that would prevail if all prices were set flexibly in the current period and all future periods taking as given the evolution of the state variables. See (Garga and Singh, 2021, Sec 2.1) for a more elaborate discussion on the natural and the potential output in models with hysteresis.

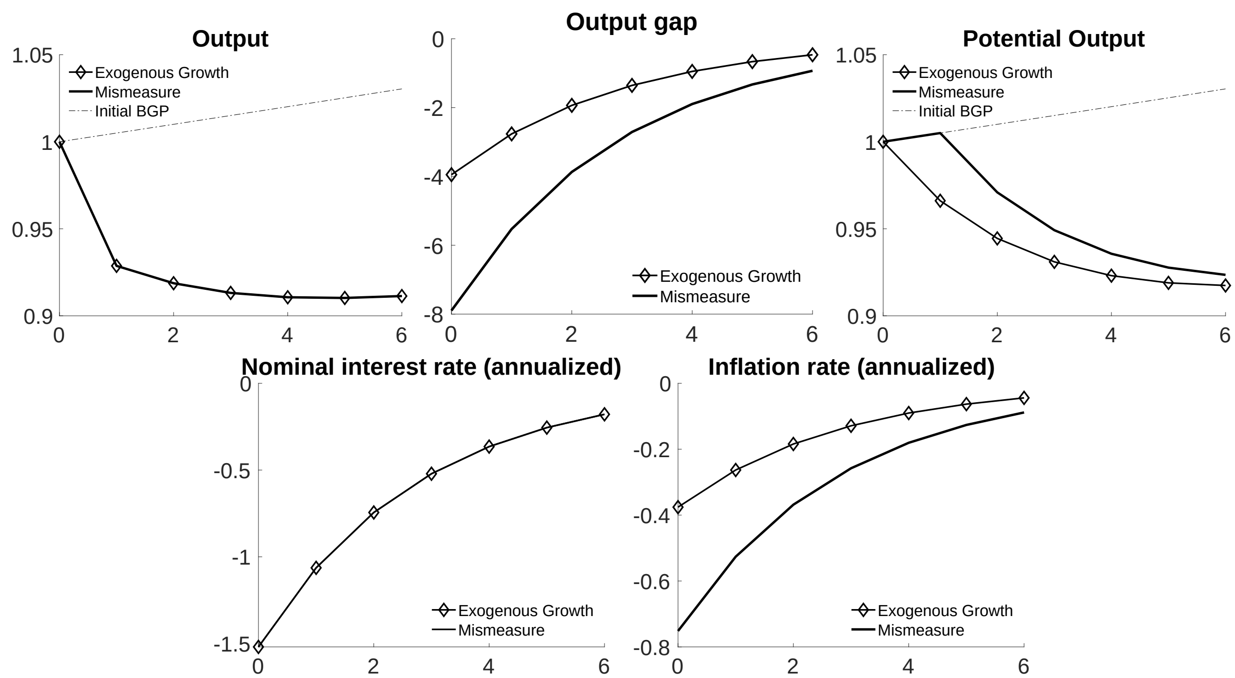
central bank used an optimal policy rule, with coefficient on output gap optimized for the endogenous growth environment, they could achieve even more gains in averting output hysteresis and achieve higher welfare. Proposition 4 formally states this result:

Proposition 4. *Let $\epsilon_1^e < 0$. If the central bank followed an optimal Taylor rule in the endogenous growth environment, the output gap, the potential output and the level of the long-run output are higher than the scenario in Proposition 3.*

4.3 Graphical Illustration

To illustrate our results graphically, we use the calibration of parameters as in Woodford (2003a) and Giannoni (2014): $\beta = 0.99$, $\sigma = 0.1571$, $\kappa = 0.0238$, $\lambda_i = 0.236$, and $\lambda_x = 0.048$. Optimal Taylor rule coefficient under exogenous growth is given by: $\psi_x = 0.096$. We set $\eta = 0.2$ for illustration.

Figure 1: Model response of output, the output gap, the potential output, the nominal interest rate, and the inflation rate to a decline in r-star in the endogenous growth model



Notes: “Exogenous Growth" refers to simulation of the economy under the baseline new Keynesian model, with exogenous growth, presented in Section 3 in response to TFP growth shock and stationary demand shocks. “Mismeasure" refers to simulation of the economy to a natural rate shock in the endogenous growth economy presented in Section 4.1. Under the “Mismeasure" scenario, policy maker incorrectly believes output gap is same as the one generated by the exogeneous growth model.

Figure 1 plots the baseline results. We shock the r-star in the endogenous model (the true model) and we compare the responses of output, the output gap, the potential output, the interest rate and the inflation rate in this scenario to our scenario where central banks believe in the exogenous growth model. The solid line represents the endogenous growth model responses, and the solid line with diamonds represents the responses in the exogenous growth model. By construction we have the same path for output. This identical path supports the confusion of the central bank that believes that they are responding to a combination of demand and supply shocks in an environment where growth is exogenous. The fact that the interest rates are identical means that the perceived output gap by the central bank is exactly the same as the output gap that would prevail if the path of output was coming from the first environment with exogenous growth.

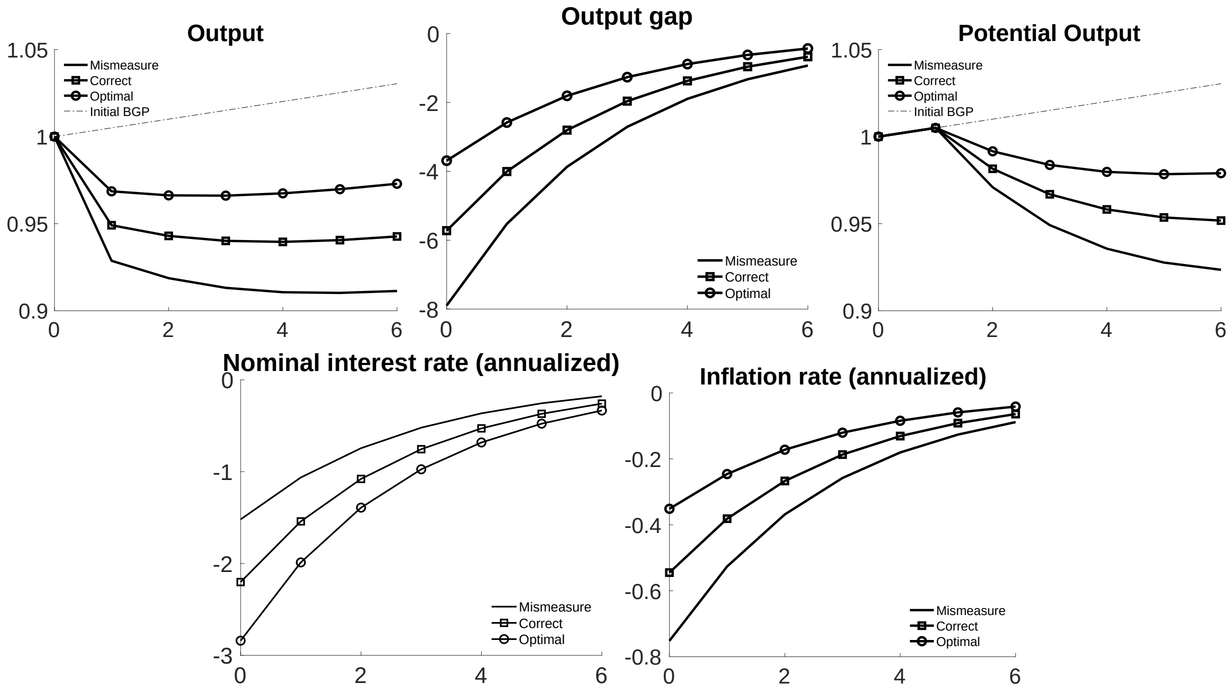
But the central bank is wrong in its assessment of the source of shocks and as a result there is divergence in path of output gap across the two models. The observed output path in the endogenous growth environment is the result of an economy where shocks are only demand driven, growth is endogenous and persistence is the outcome of hysteresis. The true output gap, depicted in the bottom left corner in the solid blue line is larger than what the central bank estimates it to be. Given their policy rule, this implies that if they had properly estimated the output gap, they would have been more aggressive in their reaction with lower interest rates in response to the shock. And the path of output would be different.[19]

We represent these alternative scenarios in Figure 2. We start with the solid line that comes from the previous exercise. This was the path followed in an environment where central banks were mistaken about the true model. We then introduce the actual output gap into their policy function, so we correct for their mistaken estimate. The solid line with square marks shows the new path. The central bank sets a more aggressive monetary policy with lower interest rates. As a result, the output gap ends up being smaller than in the previous scenario and output is less persistent.

Finally we produce an additional scenario (solid line with circles) where the central bank not only properly estimates the output gap but it also follows a rule that is optimal under the

[19] Note that the inflation rate is proportional to the output gap since we have not modeled shocks to the Phillips curve and only considered a sequence iid shock processes. In Section 5.2, we generalize the environment to include price markup shocks.

Figure 2: Model response of output, the output gap, the potential output, the nominal interest rate, and the inflation rate to a decline in r-star in the endogenous growth model: Optimal Policy



Notes: Graphs plot simulations of the economy to a natural rate shock in the endogenous growth economy presented in Section 4.1. Under the “Mismeasure" scenario, policy maker incorrectly believes output gap is same as the one generated by the exogeneous growth model of Section 3. In the “Correct" scenario, policy maker measures output gap correctly in the endogenous growth model, and sets interest rate to react to the correct measure of output gap but using the policy rule coefficient from Section 3. In the “Optimal" scenario (solid line with circles), policy maker measures output gap correctly in the endogenous growth model, and sets interest rate optimally for this environment.

endogenous growth environment. In this third instance, the central bank reacts even more than before to the change in the output gap leading to an even smaller gap and a less persistent output response.

5 Discussion and Extensions

5.1 Confusion with anticipated TFP growth shock

In the baseline exogenous growth model, we assumed iid unanticipated shocks to the TFP growth rate. An unanticipated one-time shock to current TFP growth rate does not affect the natural interest rate. Therefore, the central bank does not respond to the supply shock in the exogenous growth environment. We now generalize the baseline model to include an anticipated TFP growth rate shock. The confusion of the central banker persists and we thus show robustness of

Proposition 2 to this alternate shock characterization.

Specifically, we assume that there is an iid news shock ϵ_{t-1}^{news} that hits the economy at time $t-1$ about TFP growth rate at time t. The TFP growth process is given by: $z_t = \epsilon_{t-1}^{news}$. Other shock processes are iid as in Proposition 2. We obtain the following result

Proposition 5. *Assume all shock processes are iid (i.e. $\rho = \rho_e = 0$). There exists a sequence of iid shocks $\{\epsilon_t^b, \epsilon_t^{news}\}$ in the exogenous growth environment, and iid shocks $\{\epsilon_t^e\}$ in the endogenous growth environment such that the output path is identical.*

In the appendix H, we show that this equivalence in output path can also be obtained with persistent shock processes.

5.2 Confusion with an inflation targeting central banker

Another important shortcoming of our baseline result is that the inflation path differs across the two economies. A central bank by observing inflation and possibly including it in its policy rule could improve the equilibrium outcome. The improvements is not because an optimal rule in our case required the use of inflation, it is simply that inflation could provide an avenue for the central banker to learn about their mistaken view of the model.

To address this shortcoming we now extend the exogenous growth model to feature iid price-markup shocks denoted with ϵ_t^ν. In this extension, by introducing an inflation output tradeoff we will be able to preserve the confusion of the central banker, even if they are trying to learn from the path of inflation. The reason is that the equilibrium inflation path is now identical in both scenarios and therefore coincides with the mistaken beliefs of the policymaker, validating their priors that the exogenous growth is the correct environment.

The system of equilibrium equations under exogenous growth is now given by:

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^e \right) \tag{11}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + \epsilon_t^\nu \tag{12}$$

The optimal policy rule in this environment will have the central bank react to both the output gap and the inflation rate:

$$i_t = \psi_x x_t + \psi_\pi \pi_t \tag{13}$$

The endogenous growth environment is as described above. The central bank obtains the incorrect output gap from the exogenous growth environment and has the reaction function described in Equation 13.

We arrive at the following proposition

Proposition 6. *Assume all shock processes are iid (i.e. $\rho = \rho_e = 0$). There exists a sequence of shocks $\{\epsilon_t^b, \epsilon_t, \epsilon_t^\nu\}$ in the exogenous growth environment, and a corresponding sequence of shocks $\{\epsilon_t^e\}$ in the endogenous growth environment such that the paths of output and the inflation rate are identical.*

The central bank now observes the realized paths of inflation and output. The realizations of the inflation rate and the level of output are consistent with the mistaken belief that the correct framework is the exogenous growth framework.

As in Proposition 3, we can show that the true output gap and inflation in the endogenous growth model are actually lower than the counterfactual equilibrium when the central bank had correctly measured the output gap. We summarize this analytical result in Proposition 7:

Proposition 7. *Let $\epsilon_1^e < 0$. If the central bank targeted the correct output gap in the endogenous growth environment, the output gap, the potential output and the level of the long-run output are higher than the scenario in Proposition 6.*

5.3 The interest rate variability in the loss function

In the optimal policy, we assumed a welfare-loss term to accommodate concerns of interest rate variability. If interest rate variability term was not included in the loss function, the optimal policy without price-markup shocks follows strict output gap targeting. With the price-markup shocks, the optimal policy would not be the strict inflation or the strict output gap targeting, in this case a strict price level targeting rule would be optimal. Even with endogenous growth, away from ZLB, the strict price-level targeting policy can be optimal.

There are three reasons to consider an aversion to interest rate variability. First, concerns of zero lower bound may lead to such an aversion. Rotemberg and Woodford (1997, Sec 7, p. 337) find that the path of interest rates under optimal policy without interest variability concerns exhibits high variance that it could open up macro-stabilization issues when confronted with a zero lower bound on nominal rates. They characterize optimal rules when central banks are averse to interest rate variability. Woodford (2003a, Ch 6 Sec 4.1) derives microfounded loss function with an interest rate variability term reflecting both welfare costs of transactions frictions that account for the demand for the monetary base and an approximation to the zero lower bound on nominal interest rates (Woodford, 1999, 2003b; Giannoni, 2002, 2007). Second, Orphanides (2003) provides a case where mismeasurement of output gap necessitates "cautious policy".[20] And, third, the inclusion of an interest variability term provides a better empirical fit to gradualistic policies pursued by central banks that seem to be averse to moving interest rates in large steps over a short period of time (Brainard, 1967; Bernanke, 2004). Our exercises in this paper follow this spirit of a "gradualistic" policymaker. We believe the interest rate variability concerns are justified, both from the perspective of micro-founded models with money in utility function or the zero lower bound constraint, as well as the policymakers' beliefs in practice.

6 Conclusions

In this paper we study the case of a policymaker whose errors in understanding the true model and estimating potential output leads to hysteresis through suboptimal policies. While suboptimal policies in the presence of information gaps by policy makers should not be a surprise (Orphanides and Norden, 2002), our setting is unique because of the large costs and the fact that policy makers cannot learn from their mistakes. We present an environment where policy makers estimate potential output by using historical data and a filter to separate demand and supply shocks under the assumption that growth is exogenous. Their policy rule would be optimal under their assumed environment, but the true environment is one where there are no supply shocks and all the observed persistence of output is due to hysteresis in the presence

[20] Vice-Chair Blinder (1998) remarked that "a little stodginess at the central bank is entirely appropriate". Such a penalty on interest rate variability is often considered in the baseline welfare loss assumed in the robust monetary policy rules literature (Taylor and Williams, 2010, Equation 2). Williams (2003) discusses additional interest rate variability concerns.

of endogenous growth. The fact that their assumed mistaken model generates dynamics that are equivalent to the true model means that, ex-post, they find in the data a validation of the mistaken assumptions.

Output is persistent partly because of the lack of inaction of the central bank and the fact that they underestimate the true output gap. The hysteresis caused by the excessive output gap leads to permanent output losses that produce the level of output persistence that policy makers expected.

References

Acharya, S., J. Bengui, K. Dogra, and S. L. Wee (2022). Slow Recoveries and Unemployment Traps: Monetary Policy in a Time of Hysteresis. *The Economic Journal*.

Andrés, J., J. D. Lopez-Salido, and E. Nelson (2005). Sticky-price models and the natural rate hypothesis. *Journal of Monetary Economics 52*(5), 1025–1053.

Annicchiarico, B. and A. Pelloni (2021). Innovation, growth, and optimal monetary policy. *Macroeconomic Dynamics 25*(5), 1175–1198.

Anzoategui, D., D. Comín, M. Gertler, and J. Martínez (2019). Endogenous technology adoption and r&d as sources of business cycle persistence. *American Economic Journal: Macroeconomics 11*(3), 67–110.

Arnold, R. W. (2009). The challenges of estimating potential output in real time. *Federal Reserve Bank of St. Louis Review 91*(July/August 2009).

Barnett, R., S. Kozicki, and C. Petrinec (2009). Parsing shocks: real-time revisions to gap and growth projections for canada. *Federal Reserve Bank of St. Louis Review 91*(4), 247–65.

Basistha, A. and C. R. Nelson (2007). New measures of the output gap based on the forward-looking new keynesian phillips curve. *Journal of Monetary Economics 54*(2), 498–511.

Basu, S. and J. G. Fernald (2009). What do we know and not know about potential output? *Federal Reserve Bank of St. Louis Review 91*(4), 187–213.

Benigno, G. and L. Fornaro (2018). Stagnation traps. *Review of Economic Studies 85*(3), 1425–1470.

Bernanke, B. (2004). Gradualism. *Remarks at an economics luncheon co-sponsored by the Federal Reserve Bank of San Francisco (Seattle Branch) and the University of Washington, Seattle, Washington*.

Beveridge, S. and C. R. Nelson (1981). A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'. *Journal of Monetary Economics 7*(2), 151–174.

Bianchi, F., H. Kung, and G. Morales (2019). Growth, slowdowns, and recoveries. *Journal of Monetary Economics 101*(C), 47–63.

Blanchard, O. (2018). Should we reject the natural rate hypothesis? *Journal of Economic Perspectives 32*(1), 97–120.

Blanchard, O. and J. Galí (2007). Real wage rigidities and the new keynesian model. *Journal of Money, Credit and Banking 39*(s1), 35–65.

Blanchard, O. and J. Galí (2010). Labor markets and monetary policy: A new keynesian model with unemployment. *American Economic Journal: Macroeconomics 2*(2), 1–30.

Blanchard, O. J. and C. M. Kahn (1980). The solution of linear difference models under rational expectations. *Econometrica 48*(5), 1305–1311.

Blinder, A. S. (1998). *Central banking in theory and practice*. MIT Press.

Brainard, W. C. (1967). Uncertainty and the effectiveness of policy. *The American Economic Review 57*(2), 411–425.

Cerra, V., A. Fatás, and S. C. Saxena (2022). Hysteresis and business cycles. *Journal of Economic Perspectives forthcoming*.

Cerra, V. and S. C. Saxena (2008). Growth dynamics: the myth of economic recovery. *American Economic Review 98*(1), 439–457.

Chalaux, T. and Y. Guillemette (2019). The oecd potential output estimation methodology. OECD Working Paper 1563, OECD.

Cochrane, J. H. (2017). The new-keynesian liquidity trap. *Journal of Monetary Economics 92*, 47–63.

Coibion, O., Y. Gorodnichenko, and M. Ulate (2018). The cyclical sensitivity in estimates of potential output. *Brookings Papers on Economic Activity*, 343–442.

De Resende, C. (2014). An assessment of imf medium-term forecasts of gdp growth. *IEO Background Paper No. BP/14/01 (Washington: Independent Evaluation Office of the IMF)*.

Edge, R. M., M. T. Kiley, and J.-P. Laforte (2008). Natural rate measures in an estimated dsge model of the us economy. *Journal of Economic Dynamics and Control 32*(8), 2512–2535.

Eggertsson, G. B. and M. Woodford (2003). The zero bound on interest rates and optimal monetary policy. *Brookings Papers on Economic Activity 2003*(1), 139–211.

Ehrmann, M. and F. Smets (2003). Uncertain potential output: implications for monetary policy. *Journal of economic dynamics and control 27*(9), 1611–1638.

Fatás, A. (2019). Fiscal policy, potential output, and the shifting goalposts. *IMF Economic Review 67*(3), 684–702.

Fatás, A. and I. Mihov (2012). Fiscal policy as a stabilization tool. CEPR Discussion Papers 8749, C.E.P.R. Discussion Papers.

Fatás, A. and I. Mihov (2013). Recoveries. CEPR Discussion Papers 9551, C.E.P.R. Discussion Papers.

Fornaro, L. and M. Wolf (2023). The scars of supply shocks: Implications for monetary policy. *Journal of Monetary Economics*.

Friedman, M. (1969). The optimum quantity of money. In *The optimum quantity of money, and other essays*. Aldine, Chicago.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: An introduction to the new Keynesian framework and its applications*. Princeton, N.J.: Princeton University Press.

Garga, V. and S. R. Singh (2021). Output hysteresis and optimal monetary policy. *Journal of Monetary Economics 117*, 871–886.

Gaspar, V. (2020). Future of fiscal rules in the euro area. In *Workshop on Fiscal rules in Europe: Design and enforcement. Brussels: DG Ecfin, January*, Volume 28.

Giannoni, M. P. (2002). Does model uncertainty justify caution? robust optimal monetary policy in a forward-looking model. *Macroeconomic Dynamics 6*(1), 111–144.

Giannoni, M. P. (2007). Robust optimal monetary policy in a forward-looking model with parameter and shock uncertainty. *Journal of Applied Econometrics 22*(1), 179–213.

Giannoni, M. P. (2014). Optimal interest-rate rules and inflation stabilization versus price-level stabilization. *Journal of Economic Dynamics and Control 41*, 110–129.

Giorno, C., P. Richardson, D. Roseveare, and P. Van den Noord (1995). Estimating potential output, output gaps and structural budget balances.

Gorodnichenko, Y. and M. D. Shapiro (2007). Monetary policy when potential output is uncertain: Understanding the growth gamble of the 1990s. *Journal of Monetary Economics 54*(4), 1132–1162.

Guerron-Quintana, P. A. and R. Jinnai (2019). Financial frictions, trends, and the great recession. *Quantitative Economics 10*(2), 735–773.

Jordà, Ò., M. Schularick, and A. M. Taylor (2013). When credit bites back. *Journal of Money, Credit and Banking 45*(2), 3–28.

Jordà, Ò., S. R. Singh, and A. M. Taylor (2021). The long-run effects of monetary policy. Working Paper 26666, National Bureau of Economic Research.

Justiniano, A. and G. E. Primiceri (2008). Potential and natural output. *Manuscript, Northwestern University*, 12.

Kuusi, T. (2017). Does the structural budget balance guide fiscal policy pro-cyclically? evidence from the finnish great depression of the 1990s. *National Institute Economic Review 239*, R14–R31.

Mc Morrow, K., W. Roeger, and V. Vandermeulen (2017). Evaluating medium term forecasting methods and their implications for eu output gap calculations. Discussion paper, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Moran, P. and A. Queraltó (2018). Innovation, productivity, and monetary policy. *Journal of Monetary Economics 93*, 24–41.

Neiss, K. S. and E. Nelson (2003). The real-interest-rate gap as an inflation indicator. *Macroeconomic dynamics 7*(2), 239–262.

Nelson, C. R. and C. R. Plosser (1982). Trends and random walks in macroeconmic time series: some evidence and implications. *Journal of Monetary Economics 10*(2), 139–162.

Orphanides, A. (2003). Monetary policy evaluation with noisy information. *Journal of Monetary economics 50*(3), 605–631.

Orphanides, A. and S. v. Norden (2002). The unreliability of output-gap estimates in real time. *The Review of Economics and Statistics 84*(4), 569–583.

Orphanides, A., R. D. Porter, D. Reifschneider, R. Tetlow, and F. Finan (2000). Errors in the measurement of the output gap and the design of monetary policy. *Journal of Economics and Business 52*(1-2), 117–141.

Queralto, A. (2020). A model of slow recoveries from financial crises. *Journal of Monetary Economics 114*, 1–25.

Queralto, A. (2022). Monetary policy in a model of growth. International Finance Discussion Papers 1340, Federal Reserve Board of Governors.

Rotemberg, J. J. and M. Woodford (1997). An optimization-based econometric framework for the evaluation of monetary policy. *NBER macroeconomics annual 12*, 297–346.

Schmöller, M. E. and M. Spitzer (2021). Deep recessions, slowing productivity and missing (dis-) sinflation in the euro area. *European Economic Review 134*, 103708.

Shackleton, R. (2018). *Estimating and Projecting Potential Output Using CBO's Forecasting Growth Model*. Congressional Budget Office.

Stadler, G. W. (1990). Business cycle models with endogenous technology. *American Economic Review 80*(4), 763–78.

Taylor, J. B. and J. C. Williams (2010). Simple and robust rules for monetary policy. In *Handbook of monetary economics*, Volume 3, pp. 829–859. Elsevier.

Vetlov, I., T. Hlédik, M. Jonsson, K. Henrik, and M. Pisani (2011). Potential output in dsge models. Ecb working paper.

Vinci, F. and O. Licandro (2021). Switching-track after the great recession. CESifo Working Paper 9107.

Werning, I. (2011). Managing a Liquidity Trap: Monetary and Fiscal Policy. NBER Working Paper 17344.

Williams, J. C. (2003). Simple rules for monetary policy. *Economic Review (Federal Reserve Bank of San Francisco)*, 1–12.

Woodford, M. (1999). Optimal monetary policy inertia. *The Manchester School 67*(s1), 1–35.

Woodford, M. (2001). The taylor rule and optimal monetary policy. *American Economic Review 91*(2), 232–237.

Woodford, M. (2003a). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, N.J.: Princeton University Press.

Woodford, M. (2003b). Optimal interest-rate smoothing. *The Review of Economic Studies 70*(4), 861–886.

A Proof of Proposition 1

Proof. We only solve for optimal non-inertial plan, i.e. when policy maker optimizes every period and cannot commit to offsetting past variables. Let ϕ_{1t} and ϕ_{2t} be Lagrange multipliers on Euler equation and Phillips curve respectively. First order conditions wrt π_t, x_t and i_t are given by :

$$\pi_t + \phi_{2t} = 0$$
$$\lambda_x(x_t - x^*) + \phi_{1t} - \kappa\phi_{2t} = 0$$
$$\lambda_i(i_t - i^*) + \sigma^{-1}\phi_{1t} = 0$$

Since we only have shocks to the Euler equation, we obtain one linear restriction on coefficients ψ_x and ψ_π. This linear restriction is given by:

$$i_r = \psi_x x_r + \psi_\pi \pi_r$$

where

$$i_r = \lambda_x(1-\beta\rho)^2 + \kappa^2 > 0; \quad x_r = \lambda_i(\sigma(1-\rho)(1-\beta\rho) - \rho\kappa)(1-\beta\rho) > 0$$
$$\pi_r = \lambda_i(\sigma(1-\rho)(1-\beta\rho) - \rho\kappa)\kappa > 0$$

We make the assumption that $\sigma(1-\rho)(1-\beta\rho) > \rho\kappa$ to get conventional impulse responses.

If we impose $\psi_\pi = 0$, then we get that the optimal Taylor rule is given by:

$$i_t = \psi_x x_t$$

where $\psi_x = \frac{\lambda_x(1-\beta\rho)^2+\kappa^2}{\lambda_i(\sigma(1-\rho)(1-\beta\rho)-\rho\kappa)(1-\beta\rho)} > 0$. $\square$

B Proof of Proposition 2

Proof. To generate the two observationally equivalent scenarios, we first consider an isomorphic rule in the exogenous growth environment. Let $i_t = \psi_x x_t + \epsilon_t^i$, where x_t is now the actual gap and we can think of ϵ_t^i as the interest error that is introduce by the central bank because of their confusion about the economic model

We then choose the values for all four shocks (demand shock and TFP growth rate shock in the exogenous growth model and demand and monetary policy shocks in the endogenous growth model).

Assume that all shocks are iid. Then, the solution for the output gap under *exogenous growth* is:

$$x_t^{ex} = \frac{\epsilon_t^b}{\sigma + \psi_x}$$

Output under exogenous growth will be

$$Y_1 = \epsilon_1 + x_1^{ex}; \quad Y_t = \epsilon_1 \ \forall t > 1$$

Interest rate:

$$i_1 = \psi_x x_1^{ex} = \frac{\psi_x \epsilon_1^b}{\sigma + \psi_x}$$

In the second environment, where we have *endogenous growth*, the output gap is equal to

$$x_t = \frac{\epsilon_t^e - \epsilon_t^i}{\sigma(1-\eta) + \psi_x}$$

Output is equal to:

$$Y_1 = x_1; \quad Y_t = \eta x_1 \ \forall t > 1$$

And the interest rate

$$i_1 = \psi_x x_1 + \epsilon_1^i = \frac{\psi_x(\epsilon_1^e - \epsilon_1^i)}{\sigma(1-\eta) + \psi_x} + \epsilon_1^i = \frac{\psi_x \epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta) + \psi_x}$$

What we now need is:

- identical path of output (with identical long-term effects

$$Y_1: \quad \epsilon_1 + \frac{\epsilon_1^b}{\sigma + \psi_x} = \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x} \tag{14}$$

$$Y_t \ \forall t > 1: \quad \epsilon_1 = \eta \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x} \tag{15}$$

- identical path of nominal interest rate, consistent with the mistaken views of the central bank on the output gap

$$\frac{\psi_x \epsilon_1^b}{\sigma + \psi_x} = \frac{\psi_x \epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta) + \psi_x} \tag{16}$$

Fix ϵ_1^e. From equations 14 and 15, we get:

$$\frac{\epsilon_1^b}{\sigma + \psi_x} = \frac{(1-\eta)(\epsilon_1^e - \epsilon_1^i)}{\sigma(1-\eta) + \psi_x} \tag{17}$$

Equations 16 and 17 comprise a system of two equations in two unknowns ϵ_1^b, and ϵ_1^i, for a given fixed ϵ_1^e. We can solve:

$$\psi_x \frac{(1-\eta)(\epsilon_1^e - \epsilon_1^i)}{\sigma(1-\eta) + \psi_x} = \frac{\psi_x \epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta) + \psi_x}$$

$$\iff \psi_x(1-\eta)(\epsilon_1^e - \epsilon_1^i) = \psi_x \epsilon_1^e + \sigma(1-\eta)\epsilon_1^i$$
$$\iff -\eta \psi_x \epsilon_1^e = (\sigma + \psi_x)(1-\eta)\epsilon_1^i$$
$$\iff \epsilon_1^i = -\frac{\eta \psi_x}{(\sigma + \psi_x)(1-\eta)} \epsilon_1^e$$

From equations 15 and 16, we can solve for ϵ_1 and ϵ_1^b respectively. □

C Proof of Proposition 3

Proof. We build on the proof of Proposition 2 discussed above. The output gap under endogenous growth with mis-measured output gap is given by

$$x_1 = \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x}$$

The output gap under endogenous growth with correctly measured output gap is given by:

$$x_1^c = \frac{\epsilon_1^e}{\sigma(1-\eta) + \psi_x}$$

We need to show that, for shocks $\epsilon_1^e < 0$.

$$\begin{aligned} & x_1^c > x_1 \\ \iff & \frac{\epsilon_1^e}{\sigma(1-\eta) + \psi_x} > \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x} \\ \iff & 0 > \frac{-\epsilon_1^i}{\sigma(1-\eta) + \psi_x} \end{aligned}$$

From the proof of Proposition 2, we know that

$$\epsilon_1^i = -\frac{\eta\psi_x}{(\sigma + \psi_x)(1-\eta)}\epsilon_1^e$$

Since $\epsilon_1^e < 0$, we have that $\epsilon_1^i > 0$. Hence, $x_1^c > x_1$ is true.

Since $z_{t+1} = \eta x_t$, the output in the long-run is higher when policy maker uses the correct measure of output gap.

Potential output under endogenous growth is the level of output that would prevail if prices were set flexibly current period onwards, taking as given the evolution of the state variable. The state variable is the level of productivity. At time 1, potntial output is unaffected by the shock. TFP at time 2 relative to the counterfactual path of no-shock is given by ηx_1. This level of TFP is higher with correctly measured output gap than the mis-measured scenario. Hence potential output is higher at date 2 onwards under the correct-measured scenario.

□

D Proof of Proposition 4

Proof. We prove Proposition 4 in two steps. In the first step, we derive that optimal policy rule under endogenous growth has a higher coefficient on output gap than under exogenous growth. That is, $\psi_x^{en} > \psi_x$. In the second step, we use this insight in the calculation of equilibrium output gap and show that equilibrium output gap negatively depends on the policy rule coefficient.

Optimal plan under endogenous growth

$$\begin{aligned} x_t &= E_t x_{t+1} + E_t z_{t+1} - \sigma^{-1}\left(i_t - E_t\pi_{t+1} - \tilde{r}_t^e\right) \\ \pi_t &= \kappa x_t + \beta E_t \pi_{t+1} \end{aligned}$$

$$z_{t+1} = \eta x_t$$

where $\tilde{r}^e_t = \rho_e \tilde{r}^e_{t-1} + \epsilon^e_t$.

The optimal policy problem is then as follows:

$$\min\ (1-\beta)E\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + \lambda_x(x_t - x^*)^2 + \lambda_i(i_t - i^*)^2 + \lambda_z z_t^2\right]$$
$$(1-\eta)x_t = E_t x_{t+1} - \sigma^{-1}\left(i_t - E_t\pi_{t+1} - \tilde{r}^e_t\right)$$
$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}$$

When $\lambda_i > 0$, optimal policy does not fully offset changes in r^e_t. Policy maker trades off welfare loss from inflation variability with smoother changes in nominal interest rate. Since we only have shocks to the Euler equation, we get only one linear restriction on coefficients ψ^{en}_x and ψ^{en}_π. This linear restriction is given by:

$$i^{en}_r = \psi^{en}_x x^{en}_r + \psi^{en}_\pi \pi^{en}_r$$

where

$$i^{en}_r = \lambda_x(1-\beta\rho)^2 + \kappa^2 > 0; \quad x^{en}_r = \lambda_i(\sigma(1-\eta-\rho)(1-\beta\rho) - \rho\kappa)(1-\beta\rho) > 0$$

$$\pi^{en}_r = \lambda_i(\sigma(1-\eta-\rho)(1-\beta\rho) - \rho\kappa)\kappa > 0$$

We make the assumption that $\sigma(1-\rho)(1-\beta\rho) > \rho\kappa$ to get conventional impulse responses.

If we impose $\psi^{en}_\pi = 0$, then we get that the optimal Taylor rule is given by:

$$i_t = \psi^{en}_x x_t$$

where $\psi^{en}_x = \frac{\lambda_x(1-\beta\rho)^2+\kappa^2}{\lambda_i(\sigma(1-\eta-\rho)(1-\beta\rho)-\rho\kappa)(1-\beta\rho)} > 0$.

Comparing the value of ψ_x derived in A, it follows that

$$\psi^{en}_x > \psi_x.$$

Output gap and Taylor rule reaction coefficient Under endogenous growth environment, the output gap is given by

$$x^c = \frac{\epsilon^e_1}{\sigma(1-\eta) + \psi_x}$$

It then follows that output gap is decreasing in ψ_x. When $\epsilon^e_1 < 0$, output gap is higher when the policy maker uses the policy rule optimized for the endogenous growth environment.

Following the arguments presented in Section C, potential output and the long-run output are also higher when the the policy maker uses the policy rule optimized for the endogenous growth environment.

□

E Proof of Proposition 5

Proof. Output gap in the exogenous growth model is given by:

$$x_1^{ex} = \frac{\sigma \epsilon_1^{news} + \epsilon_1^b}{\sigma + \psi_x}$$

Output under exogenous growth will be

$$Y_1 = \frac{\sigma \epsilon_1^{news} + \epsilon_1^b}{\sigma + \psi_x}; Y_t = \epsilon_1^{news} \forall\ t > 1$$

Interest rate:

$$i_1 = \psi_x x_1^{ex}$$

The endogenous growth environment is same as in Proposition 2.
What we require is

- identical path of output (with identical long-term effects

$$Y_1 : \quad \frac{\sigma \epsilon_1^{news} + \epsilon_1^b}{\sigma + \psi_x} = \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x} \tag{18}$$

$$Y_t\ \forall t > 1 : \quad \epsilon_1^{news} = \eta \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x} \tag{19}$$

- identical path of nominal interest rate, consistent with the mistaken views of the central bank on the output gap

$$\psi_x \frac{\sigma \epsilon_1^{news} + \epsilon_1^b}{\sigma + \psi_x} = \frac{\psi_x \epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta) + \psi_x} \tag{20}$$

Fix ϵ_1^e. Then equations 18 – 20 solve for three unknown shocks $\epsilon_1^{news}, \epsilon_1^b, \epsilon_1^i$. □

F Proof of Proposition 6 and Graphical Illustration

Proof. Assume that all shocks are iid. Then, the solution for the output gap under *exogenous growth* is:

$$x_t^{ex} = \frac{\epsilon_1^b - \phi_\pi \epsilon_1^\nu}{\sigma + \psi_x + \phi_\pi \kappa}$$

Output under exogenous growth will be

$$Y_1 = \epsilon_1 + x_1^{ex}; \quad Y_t = \epsilon_1\ \forall t > 1$$

Inflation is:

$$\pi_1 = \frac{\kappa \epsilon_1^b + (\sigma + \psi_x)\epsilon_1^\nu}{\sigma + \psi_x + \phi_\pi \kappa}$$

Interest rate:

$$i_1 = \psi_x x_1^{ex} + \psi_\pi \pi_1^{ex} = \frac{(\psi_x + \phi_\pi \kappa)\, \epsilon_1^b + \sigma \phi_\pi \epsilon_1^\nu}{\sigma + \psi_x + \phi_\pi \kappa}$$

In the second environment, where we have *endogenous growth*, the output gap is equal to

$$x_t = \frac{\epsilon_t^e - \epsilon_t^i}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa}$$

Output is equal to:

$$Y_1 = x_1; \quad Y_t = \eta x_1 \; \forall t > 1$$

And the interest rate

$$i_1 = (\psi_x + \phi_\pi \kappa)\, x_1 + \epsilon_1^i = \frac{(\psi_x + \phi_\pi \kappa)\,(\epsilon_1^e - \epsilon_1^i)}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa} + \epsilon_1^i = \frac{(\psi_x + \phi_\pi \kappa)\,\epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa}$$

What we now need is:

- identical path of output (with identical long-term effects

$$Y_1: \quad \epsilon_1 + \frac{\epsilon_1^b - \phi_\pi \epsilon_1^\nu}{\sigma + \psi_x + \phi_\pi \kappa} = \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa} \tag{21}$$

$$Y_t \; \forall t > 1: \quad \epsilon_1 = \eta \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa} \tag{22}$$

- identical path of nominal interest rate, consistent with the mistaken views of the central bank on the output gap

$$\frac{(\psi_x + \phi_\pi \kappa)\,\epsilon_1^b + \sigma \phi_\pi \epsilon_1^\nu}{\sigma + \psi_x + \phi_\pi \kappa} = \frac{(\psi_x + \phi_\pi \kappa)\,\epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa} \tag{23}$$

- identical path of inflation rate, consistent with the mistaken views of the central bank on the output gap

$$\frac{\kappa \epsilon_1^b + (\sigma + \psi_x)\epsilon_1^\nu}{\sigma + \psi_x + \phi_\pi \kappa} = \kappa \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa} \tag{24}$$

Fix ϵ_1^e. From equations 21 and 22, we get:

$$\frac{\epsilon_1^b - \phi_\pi \epsilon_1^\nu}{\sigma + \psi_x + \phi_\pi \kappa} = \frac{(1-\eta)(\epsilon_1^e - \epsilon_1^i)}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa} \tag{25}$$

Equations 22, 23, 24, and 25 comprise a system of four equations in four unknowns ϵ_1, ϵ_1^b, ϵ_1^i, and ϵ_1^ν for a given fixed ϵ_1^e. We obtain the following solution

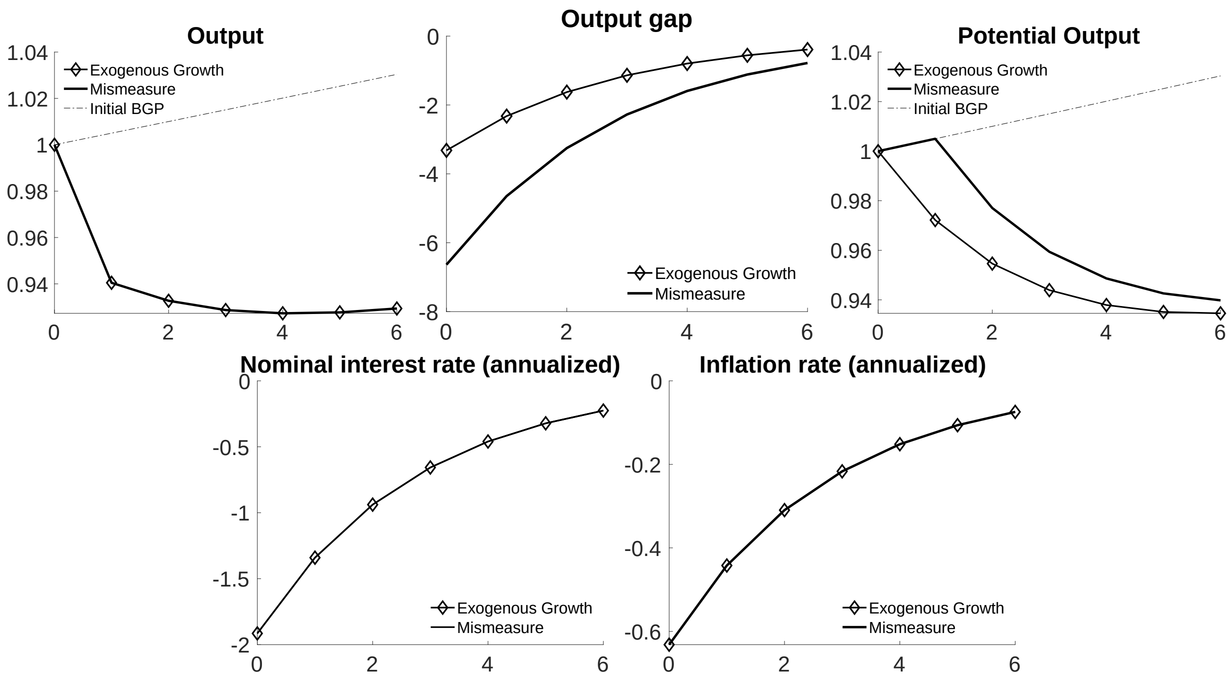
$$\epsilon_1^i = \frac{-\psi_x \eta}{(\sigma + \psi_x)\,(1-\eta) + \phi_\pi \kappa} \epsilon_1^e; \quad \epsilon_1 = \eta \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa}$$

$$\epsilon_1^b = \frac{(\sigma + \psi_x)\,(1-\eta) + \phi_\pi \kappa}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa}(\epsilon_1^e - \epsilon_1^i); \quad \epsilon_1^\nu = \frac{\eta \kappa}{(\sigma + \psi_x)\,(1-\eta) + \phi_\pi \kappa} \epsilon_1^b$$

□

In addition, we provide a graphical illustration in Figure 3. As in the baseline model, we shock the r-star in the endogenous growth model (true model) and we compare the responses of output, the output gap, the inflation rate and the interest rate in this scenario to our scenario where the central banks believe in the exogenous growth model (with price-markup shocks). We now have the same path for output, inflation rate and interest rate.

Figure 3: Model response of output, nominal interest rate, the output gap, the inflation rate, and the potential output to a decline in r-star in the endogenous growth model



Notes: "Exogenous Growth" refers to simulation of the economy under the new Keynesian model, with exogenous growth and iid price markup shocks, presented in Section 5.2. "Mismeasure" refers to simulation of the economy to a natural rate shock in the endogenous growth economy. Under the "Mismeasure" scenario, policy maker incorrectly believes output gap is same as the one generated by the exogeneous growth model.

G Proof of Proposition 7

Proof. We build on the proof of Proposition 6 discussed above. The output gap under endogenous growth with mis-measured output gap is given by

$$x_1 = \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa}$$

The output gap under endogenous growth with correctly measured output gap is given by:

$$x_1^c = \frac{\epsilon_1^e}{\sigma(1-\eta) + \psi_x + \phi_\pi \kappa}$$

We need to show that, for shocks $\epsilon_1^e < 0$.

$$x_1^c > x_1$$

$$\iff \frac{\epsilon_1^e}{\sigma(1-\eta)+\psi_x+\phi_\pi\kappa} > \frac{\epsilon_1^e-\epsilon_1^i}{\sigma(1-\eta)+\psi_x+\phi_\pi\kappa}$$
$$\iff 0 > \frac{-\epsilon_1^i}{\sigma(1-\eta)+\psi_x+\phi_\pi\kappa}$$

From the proof of Proposition 6, we know that

$$\epsilon_1^i = \frac{-\psi_x\eta}{(\sigma+\psi_x)(1-\eta)+\phi_\pi\kappa}\epsilon_1^e$$

Since $\epsilon_1^e < 0$, we have that $\epsilon_1^i > 0$. Hence, $x_1^c > x_1$ is true.

Following the arguments presented in Section C, the potential output and the long-run output are also higher when the the policy maker uses the correctly measured output gap. □

H Generalizing Proposition 2 with persistent shock processes

We can further generalize the environment to allow persistent shock processes in the the exogenous growth environment. That is $\rho \neq 0$ in equations 4 and 5. In that setting, we obtain the following result.

Proposition 8. *Assume shock processes in the exogenous growth model are persistent, that is $\rho \in (0,1)$. There exists a sequence of iid unanticipated shocks $\{\epsilon_t^b, \epsilon_t\}_{t=0}^T$ in the exogenous growth environment, and shocks $\{\epsilon_t^e\}_{t=0}^T$ in the endogenous growth environment such that the output path is identical.*

Proof. Without loss of generality, we assume the economy is in the deterministic steady state at date 0. The solution for output gap under exogenous growth, with persistent shock processes, is given by:

$$x_t^{ex} = \frac{(1-\beta\rho)r_t^e}{(1-\rho)(1-\beta\rho)\sigma+\psi_x(1-\beta\rho)-\rho\kappa}$$

The interest rate is given by:

$$i_t = \psi_x x_t^{ex}$$

Building on the proof of Proposition 2, the output gap under *endogenous growth*, is equal to

$$x_t = \frac{\epsilon_t^e-\epsilon_t^i}{\sigma(1-\eta)+\psi_x}$$

Output is equal to:

$$Y_1 = x_1; \quad Y_2 = \eta x_1 + x_2; \quad Y_3 = \eta^2 x_1 + \eta x_2 + x_3 \quad ...$$

And the interest rate

$$i_t = \psi_x x_t + \epsilon_t^i = \frac{\psi_x(\epsilon_t^e-\epsilon_t^i)}{\sigma(1-\eta)+\psi_x} + \epsilon_t^i = \frac{\psi_x\epsilon_t^e+\sigma(1-\eta)\epsilon_t^i}{\sigma(1-\eta)+\psi_x}$$

We can construct a sequence of shocks such that the output path and the interest rate path is identical upto horizon T for a given initial shock. We show the proof until horizon 2.

At date 1:

- Equalization of Output requires

$$\epsilon_1 + \frac{(1-\beta\rho)r_1^e}{(1-\rho)(1-\beta\rho)\sigma + \psi_x(1-\beta\rho) - \rho\kappa} = \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta)+\psi_x}$$

- Interest rate equalization:

$$\frac{\psi_x(1-\beta\rho)r_1^e}{(1-\rho)(1-\beta\rho)\sigma + \psi_x(1-\beta\rho) - \rho\kappa} = \frac{\psi_x\epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta)+\psi_x}$$

where $r_1^e = \rho\epsilon_1 + \epsilon_1^b$. For a fixed ϵ_1, and a fixed ϵ_1^e, we can solve for ϵ_1^i and ϵ_1^b, such that the output and interest rate at date 1 are equalized.

At date 2:

- Equalization of Output requires

$$(1+\rho)\epsilon_1 + \frac{(1-\beta\rho)r_2^e}{(1-\rho)(1-\beta\rho)\sigma + \psi_x(1-\beta\rho) - \rho\kappa} = \eta\frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta)+\psi_x} + \frac{\epsilon_2^e - \epsilon_2^i}{\sigma(1-\eta)+\psi_x}$$

- Interest rate equalization:

$$\frac{\psi_x(1-\beta\rho)r_2^e}{(1-\rho)(1-\beta\rho)\sigma + \psi_x(1-\beta\rho) - \rho\kappa} = \frac{\psi_x\epsilon_2^e + \sigma(1-\eta)\epsilon_2^i}{\sigma(1-\eta)+\psi_x}$$

where $r_2^e = \rho^2\epsilon_1 + \epsilon_2^b$. With pre-determined, ϵ_1, ϵ_1^e, and a given ϵ_1, we can solve for ϵ_2^i and ϵ_2^b, such that the output and interest rate at date 1 are equalized.

We can continue this process and find shocks ϵ_T^i and ϵ_T^b, such that path of output and interest rate are identical up to horizon T.

□