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Working Paper 2010-06
http://www.frbsf.org/publications/economics/papers/2010/wp10-06bk.pdf

Aggregation and the PPP Puzzle in a Sticky-Price Model*

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Abstract

We study the purchasing power parity (PPP) puzzle in a multi-sector, two-country, sticky-price model. Across sectors, firms differ in the extent of price stickiness, in accordance with recent microeconomic evidence on price setting in various countries. Combined with local currency pricing, this leads sectoral real exchange rates to have heterogeneous dynamics. We show analytically that in this economy, deviations of the real exchange rate from PPP are more volatile and persistent than in a counterfactual one-sector world economy that features the same average frequency of price changes, and is otherwise identical to the multi-sector world economy. When simulated with a sectoral distribution of price stickiness that matches the microeconomic evidence for the U.S. economy, the model produces a half-life of deviations from PPP of 39 months. In contrast, the half-life of such deviations in the counterfactual one-sector economy is only slightly above one year. As a by-product, our model provides a decomposition of this difference in persistence that allows a structural interpretation of the different approaches found in the empirical literature on aggregation and the real exchange rate. In particular, we reconcile the apparently conflicting findings that gave rise to the "PPP Strikes Back debate" (Imbs et al. 2005a,b and Chen and Engel 2005).

JEL classification codes: F30, F41, E00
Keywords: heterogeneity, aggregation, price stickiness, real exchange rates, PPP puzzle

*An earlier version of this paper is available as Federal Reserve Bank of New York Staff Report No. 351. For comments and suggestions we thank Richard Rogerson, three anonymous referees, and seminar participants at UC Berkeley, Harvard, EEA 2009, the SCIIE 2009 Conference, ESEM 2009, XI Annual Inflation Targeting Seminar of the Central Bank of Brazil, PUC-Rio, IMF, San Francisco Fed, Geneva Graduate Institute, AEA 2009, NBER Summer Institute IFM 2008, SOEGW Conference 2008, NY Area Workshop on Monetary Economics - Spring 2008, Johns Hopkins, Princeton and New York Fed. We are also grateful to Gianluca Benigno, Roberto Chang, Michael Devereux, Gauti Eggertsson, Charles Engel, Doireann Fitzgerald, Pierre-Olivier Gourinchas, Jean Imbs, Nobu Kiyotaki, Anthony Landry, Paolo Pesenti, Ricardo Reis, Hélène Rey, Chris Sims, and Thomas Wu for comments and suggestions. Chris Candelaria provided outstanding research assistance. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York, the Federal Reserve Bank of San Francisco, or the Federal Reserve System. E-mails: carlos.carvalho@ny.frb.org, fernanda.nechio@sf.frb.org.

1 Introduction

Purchasing power parity (PPP) states that, once converted to the same currency, price levels across countries should be equal. As a result, the real exchange rate between any two countries – the ratio of their price levels in a common currency – should be constant and equal to unity. A more flexible version of PPP postulates that real exchange rates should be constant, but not necessarily equal to one. In contrast with the tight predictions of either version of PPP, in the data real exchange rates display large and long-lived fluctuations around their average levels. Rogoff's (1996) survey of the empirical literature on the subject reports a "consensus view" that places estimates of the half-life of deviations from PPP in the range of 3 to 5 years. While he suggests that the high volatility of the real exchange rate could be explained by a model with monetary shocks and nominal rigidities, so far models of this type with plausible nominal frictions have failed to produce the large persistence found in the data; hence, the puzzle.[1]

In this paper we study the PPP puzzle in a multi-sector, two-country, sticky-price model. We depart from the existing literature by introducing heterogeneity in the frequency of price changes across sectors, in accordance with recent microeconomic evidence on price setting for various countries (e.g. Bils and Klenow 2004; Dhyne et al. 2006 for the Euro area). Combined with local currency pricing, these differences in the extent of price stickiness lead sectoral real exchange rates to have heterogeneous dynamics, which are also evident in the data (Imbs et al. 2005a).

We isolate the role of heterogeneity by comparing the dynamic behavior of the aggregate real exchange rate in such a multi-sector economy with the behavior of the real exchange rate in an otherwise identical one-sector world economy with the same average frequency of price changes. We refer to this auxiliary economy as the *counterfactual one-sector world economy*. We show that, in response to nominal shocks, the aggregate real exchange rate in the heterogeneous economy is more volatile and persistent than in the counterfactual one-sector world economy, and that the difference can be arbitrarily large.

We then investigate whether quantitatively our multi-sector model can produce highly volatile and persistent real exchange rates in response to nominal disturbances, under a plausible parameterization. In particular, to discipline our analysis we use a cross-sectional distribution of the frequency of price changes that matches the recent microeconomic evidence for the U.S. economy. We ask the same question in the counterfactual one-sector world economy. Our multi-sector model produces a half-life of deviations from PPP of 39 months, well within the consensus view of 3 to 5 years. In contrast, such deviations in the one-sector world economy are relatively short-lived, with a half-life just above one year. In order to produce deviations from PPP with a half-life of 39

[1] For a subsequent survey of the PPP literature see, e.g., Taylor and Taylor (2004).

months, the one-sector model requires firms to change prices much less frequently – roughly once every 15 months, as opposed to once every 4.7 months in the micro data. The volatility of the real exchange rate is also much higher in the heterogeneous economy (by a factor that ranges from 2.5 to 7, depending on the specification of the model).

Our quantitative findings reveal that the counterfactual one-sector model is a poor representation of the multi-sector world economy. As a result of cross-sectional aggregation of sectoral exchange rates with heterogeneous dynamics, the aggregate real exchange rate in the multi-sector model displays much richer dynamics than the real exchange rate in the counterfactual one-sector world economy. As our analytical results show, the volatility and persistence of real exchange rates are convex functions of the frequency of price adjustments, which leads the counterfactual one-sector model to understate both quantities relative to the underlying heterogeneous economy.

We present our multi-sector general equilibrium model in Section 2. It features two countries trading intermediate goods produced by monopolistically-competitive firms, which are divided into sectors that differ in the frequency of price changes. Firms can price-discriminate across the two countries, and set prices in the currency of the market in which the good is sold. Consumers supply labor to these intermediate firms and consume the non-traded final good, which is produced by competitive firms that bundle the intermediate goods from the two countries.

Using common assumptions about preferences, technologies, and nominal shocks, in Section 3 we show analytically that the volatility and persistence of the aggregate real exchange rate in the multi-sector economy are higher than in the counterfactual one-sector model. Section 4 then presents quantitative results from parameterized versions of the model. It shows that in response to nominal shocks our multi-sector model generates much higher volatility and persistence than the counterfactual one-sector model. We also present several robustness exercises, and find that our results can survive important departures from the baseline specification.

In Section 5 we use our structural model to revisit the empirical literature on heterogeneity, aggregation and real exchange rate persistence. We decompose the effects of heterogeneity into two terms: an *aggregation effect* – defined as the difference between the persistence of the aggregate real exchange rate of the heterogeneous economy and the (weighted) average persistence of the underlying sectoral exchange rates; and a *counterfactuality effect* – defined as the difference between the latter weighted average and the persistence of the real exchange rate in the counterfactual one-sector world economy. Using the Eurostat data from Imbs et al. (2005a), we estimate the decomposition implied by our theory. Our results reconcile the apparently conflicting findings of the empirical literature that gave rise to the so-called “PPP Strikes Back debate” (Imbs et al. 2005a,b and Chen and Engel 2005). The bottom line is that different papers have measured different objects. In particular, Chen and Engel (2005) measure what we term the aggregation effect, which is indeed small.[2] In contrast, Imbs et al. (2005a) find the effects of heterogeneity to be large. The reason is that their measure corresponds to the *sum* of the aggregation and counterfactuality effects.

While most of our results focus on the effects of heterogeneity in price stickiness on aggregate real exchange rate dynamics, our multi-sector model also yields a series of cross-sectional implications for sectoral real exchange rates. Section 6 assesses these implications using the same Eurostat data from Imbs et al. (2005a). We first look at qualitative predictions of a simplified version of our model. Then, in the spirit of Kehoe and Midrigan (2007), we use this model to infer the degree of sectoral price rigidity from reduced-form coefficients of time-series models estimated on the sectoral real exchange rate data. We compare the results with measures of sectoral price rigidity derived from micro price data. The results lead naturally to a discussion of endogenous and exogenous sources of real exchange rate persistence, which we undertake in Section 7. We explore real rigidities in the sense of Ball and Romer (1990) as a source of endogenous persistence, and uncover important interactions between real rigidities, heterogeneity in price stickiness, and exogenous persistence.

Our paper is naturally related to the growing literature that focuses on the aggregate implications of heterogeneity in price setting.[3] It contributes to the body of work that uses dynamic sticky-price models to study the persistence of real exchange rates, such as Bergin and Feenstra (2001), Kollman (2001), Chari et al. (2002), Benigno (2004), Steinsson (2008), Johri and Lahiri (2008), and Martinez-Garcia and Søndergaard (2008). There is also a connection between the results from our multi-sector model, and the findings of the literature on cross-sectional aggregation of time-series processes (e.g. Granger and Morris 1976; Granger 1980; Zaffaroni 2004). Our focus on economic implications as opposed to purely statistical aspects of aggregation links our work with Abadir and Talmain (2002). Our paper is also related to Kehoe and Midrigan (2007) and Crucini et al. (2008), who analyze sectoral real exchange rate dynamics in multi-sector sticky-price models. Finally, our paper shares with Ghironi and Melitz (2005) and Atkeson and Burstein (2008) the themes of heterogeneity and real exchange rate dynamics. However, while we focus on the PPP puzzle in a sticky-price model, they emphasize productivity shocks in flexible-price models.

2 The model

The world economy consists of two symmetric countries, *Home* and *Foreign*. In each country, identical infinitely-lived consumers supply labor to intermediate firms that they own, invest in a

[2] Crucini and Shintani (2008) and Broda and Weinstein (2008) reach the same conclusion using different datasets.
[3] Carvalho and Schwartzman (2008) provide detailed references.

complete set of state-contingent financial assets, and consume a non-traded final good. The latter is produced by competitive firms that bundle varieties of intermediate goods produced in the two countries. The monopolistically competitive firms that produce these varieties are divided into sectors that differ in their frequency of price changes. Labor is the variable input in the production of intermediate goods, which are the only goods that are traded. Intermediate producers can price-discriminate across countries, and set prices in local currency.

The Home representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t C_t + E_t \left[\Theta_{t,t+1} B_{t+1} \right] \leq W_t N_t + B_t + T_t,$$

and a standard "no-Ponzi" condition. E_t denotes the time-t expectations operator, C_t is consumption of the final good, N_t is labor, P_t is the price of the final good, W_t is the nominal wage, T_t stands for profits received from Home intermediate firms, and β is the time-discount factor. B_{t+1} stands for the state-contingent value of the portfolio of financial securities held by the consumer at the beginning of $t+1$. Complete financial markets allow agents to choose the value of B_{t+1} for each possible state of the world at all times, and a no-arbitrage condition requires the existence of a nominal stochastic discount factor $\Theta_{t,t+1}$ that prices in period t any financial asset portfolio with value B_{t+1} at the beginning of period $t+1$. To avoid cluttering the notation we omit explicit reference to the different states of nature. Finally, σ^{-1} denotes the intertemporal elasticity of substitution and γ^{-1} is the Frisch elasticity of labor supply.

The first-order conditions for the consumer's problem are:

$$\frac{C_t^{-\sigma}}{C_{t+s}^{-\sigma}} = \frac{\beta^s}{\Theta_{t,t+s}} \frac{P_t}{P_{t+s}}, \tag{1}$$

$$C_t^{\sigma} N_t^{\gamma} = \frac{W_t}{P_t},$$

where $\Theta_{t,t} = 1$, $\Theta_{t,t+s} \equiv \prod_{r=t+1}^{t+s} \Theta_{r-1,r}$ for $s > 0$, and (1) holds for each future state of nature. The solution must also satisfy a transversality condition:

$$\lim_{s \to \infty} E_t \left[\Theta_{t,t+s} B_{t+s} \right] = 0. \tag{2}$$

The Foreign consumer solves an analogous problem. She maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{*1-\sigma} - 1}{1-\sigma} - \frac{N_t^{*1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t^* C_t^* + E_t \left[\Theta_{t,t+1}^* \frac{B_{t+1}^*}{\mathcal{E}_t} \right] \leq W_t^* N_t^* + \frac{B_t^*}{\mathcal{E}_t} + T_t^*, \tag{3}$$

and a "no-Ponzi" condition. A "$*$" superscript denotes the Foreign counterpart of the corresponding Home variable, and $\mathcal{E}_t$ is the nominal exchange rate, defined as the price of the Foreign currency in terms of the Home currency. $\mathcal{E}_t$ is thus quoted in units of Home currency per unit of the Foreign currency. Without loss of generality and for simplicity, we assume that the complete set of state-contingent assets are denominated in the Home currency. As a result, in the budget constraint (3) B_t^* appears divided by the nominal exchange rate, to convert the value of the portfolio into Foreign currency.

The optimality conditions for the Foreign consumer's problem are:

$$\frac{C_t^{*-\sigma}}{C_{t+s}^{*-\sigma}} = \frac{\beta^s}{\Theta_{t,t+s}^*} \frac{\mathcal{E}_t P_t^*}{\mathcal{E}_{t+s} P_{t+s}^*}, \tag{4}$$

$$C_t^{*\sigma} N_t^{*\gamma} = \frac{W_t^*}{P_t^*},$$

and a transversality condition analogous to (2), where, again, (4) holds for each future state of nature.

The stochastic discount factor has to be the same for both countries, since assets are freely traded and there are no arbitrage opportunities. Letting $Q_t \equiv \mathcal{E}_t \frac{P_t^*}{P_t}$ denote the real exchange rate, from equations (1) and (4):

$$Q_{t+s} = Q_t \frac{C_t^{-\sigma}}{C_{t+s}^{-\sigma}} \frac{C_{t+s}^{*-\sigma}}{C_t^{*-\sigma}}. \tag{5}$$

Iterating equation (5) backwards and assuming $Q_0 \frac{C_0^{-\sigma}}{C_0^{*-\sigma}} = 1$, yields:[4]

$$Q_t = \frac{C_t^{*-\sigma}}{C_t^{-\sigma}}.$$

The Home final good is produced by a representative competitive firm that bundles varieties of intermediate goods from both countries. Each variety is produced by a monopolistically competitive

[4] As is well-known, this full-risk-sharing condition implies that real exchange rates and relative consumptions should be (almost) perfectly correlated. It is also well known that this prediction is at odds with the data. Thus, our model is silent on this "puzzle."

firm. Intermediate firms are divided into sectors indexed by $k \in \{1, ..., K\}$, each featuring a continuum of firms. To highlight the role of heterogeneity in price stickiness, across sectors these intermediate firms differ only in their pricing practices, as we detail below. Overall, firms are indexed by the country where they produce, by their sector, and are further indexed by $j \in [0, 1]$. The distribution of firms across sectors is given by sectoral weights $f_k > 0$, with $\sum_{k=1}^{K} f_k = 1$.

The final good is produced by combining the intermediate varieties, as follows:

$$Y_t = \left(\sum\nolimits_{k=1}^{K} f_k^{\frac{1}{\eta}} Y_{k,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{6}$$

$$Y_{k,t} = \left(\omega^{\frac{1}{\rho}} Y_{H,k,t}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} Y_{F,k,t}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{7}$$

$$Y_{H,k,t} = \left(f_k^{\frac{\theta-1}{\theta}} \int_0^1 Y_{H,k,j,t}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \tag{8}$$

$$Y_{F,k,t} = \left(f_k^{\frac{\theta-1}{\theta}} \int_0^1 Y_{F,k,j,t}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \tag{9}$$

where Y_t denotes the Home final good, $Y_{k,t}$ is the aggregation of sector-k Home and Foreign intermediate goods sold in Home, $Y_{H,k,t}$ and $Y_{F,k,t}$ are the composites of intermediate varieties produced by firms in sector k in Home and Foreign, respectively, to be sold in Home, and $Y_{H,k,j,t}$ and $Y_{F,k,j,t}$ are the varieties produced by firm j in sector k in Home and Foreign to be sold in Home. Finally, $\eta \geq 0$ is the elasticity of substitution across sectors, $\rho \geq 0$ is the elasticity of substitution between Home and Foreign goods, $\theta > 1$ is the elasticity of substitution within sectors, and $\omega \in [0, 1]$ is the steady-state share of domestic inputs used in production.

The problem of a representative Home final-good-producing firm is to maximize:

$$P_t Y_t - \sum\nolimits_{k=1}^{K} f_k \int_0^1 \left(P_{H,k,j,t} Y_{H,k,j,t} + P_{F,k,j,t} Y_{F,k,j,t}\right) dj$$
$$s.t. \ \ (6)\text{-}(9).$$

The first-order conditions, for $j \in [0, 1]$ and $k = 1, ..., K$, are given by:

$$Y_{H,k,j,t} = \omega \left(\frac{P_{H,k,j,t}}{P_{H,k,t}}\right)^{-\theta} \left(\frac{P_{H,k,t}}{P_{k,t}}\right)^{-\rho} \left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t, \tag{10}$$

$$Y_{F,k,j,t} = (1-\omega) \left(\frac{P_{F,k,j,t}}{P_{F,k,t}}\right)^{-\theta} \left(\frac{P_{F,k,t}}{P_{k,t}}\right)^{-\rho} \left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t. \tag{11}$$

The underlying price indices are:

$$P_t = \left(\sum_{k=1}^{K} f_k P_{k,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \quad (12)$$

$$P_{k,t} = \left(\omega P_{H,k,t}^{1-\rho} + (1-\omega) P_{F,k,t}^{1-\rho}\right)^{\frac{1}{1-\rho}}, \quad (13)$$

$$P_{H,k,t} = \left(\int_0^1 P_{H,k,j,t}^{1-\theta} dj\right)^{\frac{1}{1-\theta}}, \quad (14)$$

$$P_{F,k,t} = \left(\int_0^1 P_{F,k,j,t}^{1-\theta} dj\right)^{\frac{1}{1-\theta}}, \quad (15)$$

where P_t is the price of the Home final good, $P_{k,t}$ is the price index of sector-k intermediate goods sold in Home, $P_{H,k,t}$ is the price index for sector-k, Home-produced intermediate goods sold in Home, and $P_{H,k,j,t}$ is the price charged in the Home market by Home firm j from sector k. $P_{F,k,t}$ is the price index for sector-k, Foreign-produced intermediate goods sold in Home, and $P_{F,k,j,t}$ is the price charged in the Home market by Foreign firm j from sector k. Both $P_{H,k,j,t}$ and $P_{F,k,j,t}$ are set in the Home currency.

With an analogous maximization problem, the Foreign final firm chooses its demands for intermediate inputs from Foreign ($Y_{F,k,j,t}^*$) and Home ($Y_{H,k,j,t}^*$) producers:

$$Y_{F,k,j,t}^* = \omega \left(\frac{P_{F,k,j,t}^*}{P_{F,k,t}^*}\right)^{-\theta} \left(\frac{P_{F,k,t}^*}{P_{F,t}^*}\right)^{-\rho} \left(\frac{P_{k,t}^*}{P_t^*}\right)^{-\eta} Y_t^*, \quad (16)$$

$$Y_{H,k,j,t}^* = (1-\omega) \left(\frac{P_{H,k,j,t}^*}{P_{H,k,t}^*}\right)^{-\theta} \left(\frac{P_{H,k,t}^*}{P_{k,t}^*}\right)^{-\rho} \left(\frac{P_{k,t}^*}{P_t^*}\right)^{-\eta} Y_t^*. \quad (17)$$

Foreign price indices are analogous to the Home ones (equations (12)-(15)):

$$P_t^* = \left(\sum_{k=1}^{K} f_k P_{k,t}^{*1-\eta}\right)^{\frac{1}{1-\eta}}, \quad (18)$$

$$P_{k,t}^* = \left(\omega P_{F,k,t}^{*1-\rho} + (1-\omega) P_{H,k,t}^{*1-\rho}\right)^{\frac{1}{1-\rho}}, \quad (19)$$

$$P_{H,k,t}^* = \left(\int_0^1 P_{H,k,j,t}^{*1-\theta} dj\right)^{\frac{1}{1-\theta}}, \quad (20)$$

$$P_{F,k,t}^* = \left(\int_0^1 P_{F,k,j,t}^{*1-\theta} dj\right)^{\frac{1}{1-\theta}} \quad (21)$$

where P_t^* is the price of the Foreign final good, $P_{k,t}^*$ is the price index of sector-k intermediate goods sold in Foreign, $P_{F,k,t}^*$ is the price index for sector-k Foreign-produced intermediate goods sold in Foreign, and $P_{F,k,j,t}^*$ is the price charged in the Foreign market by Foreign firm j from sector k. $P_{H,k,t}^*$ is the price index for sector-k Home-produced intermediate goods sold in Foreign, and

$P^*_{H,k,j,t}$ is the price charged in the Foreign market by Home firm j from sector k. Both $P^*_{F,k,j,t}$ and $P^*_{H,k,j,t}$ are set in the Foreign currency.

For ease of reference, we refer to $P_{H,k,t}, P_{F,k,t}, P^*_{H,k,t}$, and $P^*_{F,k,t}$ as *country-sector* price indices, and to $P_{k,t}$, and $P^*_{k,t}$ as *sectoral price indices*. We can then define the *sectoral real exchange rate* for sector k as the ratio of sectoral price indices in a common currency:

$$Q_{k,t} \equiv \mathcal{E}_t \frac{P^*_{k,t}}{P_{k,t}}.$$

For analytical tractability, we assume that intermediate firms set prices as in Calvo (1983). The frequency of price changes varies across sectors, and it is the only source of (ex-ante) heterogeneity. Thus, sectors in the model are naturally identified with their frequency of price changes. In each period, each firm j in sector k changes its price independently with probability α_k.[5] To keep track of the sectors, we order them in terms of increasing price stickiness, so that $\alpha_1 > ... > \alpha_K$.

Each time Home firm j from sector k adjusts, it chooses prices $X_{H,k,j,t}$, $X^*_{H,k,j,t}$ to be charged in the Home and Foreign markets, respectively, with each price being set in the corresponding local currency. Thus, its problem is to maximize:

$$E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1-\alpha_k)^s \left(X_{H,k,j,t} Y_{H,k,j,t+s} + \mathcal{E}_{t+s} X^*_{H,k,j,t} Y^*_{H,k,j,t+s} - W_{t+s} N_{k,j,t+s} \right)$$

$$\begin{aligned} s.t. \quad & (10),\ (17),\ \text{and} \\ & Y_{H,k,j,t} + Y^*_{H,k,j,t} = N^{\chi}_{k,j,t}, \end{aligned} \tag{22}$$

where $N_{k,j,t}$ is the amount of labor it employs, and χ determines returns to labor.

The first-order conditions are:

$$X_{H,k,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1-\alpha_k)^s \Lambda_{H,k,t+s} \left(\chi N^{\chi-1}_{k,j,t+s}\right)^{-1} W_{t+s}}{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1-\alpha_k)^s \Lambda_{H,k,t+s}},$$

$$X^*_{H,k,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1-\alpha_k)^s \Lambda^*_{H,k,t+s} \left(\chi N^{\chi-1}_{k,j,t+s}\right)^{-1} W_{t+s}}{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1-\alpha_k)^s \mathcal{E}_{t+s} \Lambda^*_{H,k,t+s}},$$

[5] Woodford (2009) shows that this model is a good approximation to a carefully microfounded model in which firms set prices subject to information frictions. Furthermore, in closed economies heterogeneity in price setting has similar implications in a large class of models that includes various sticky-price and sticky-information specifications. For a detailed analysis of such models, and additional references, see Carvalho and Schwartzman (2008). Nakamura and Steinsson (2010) find that heterogeneity in price stickiness also has similar implications in state-dependent pricing models.

where:

$$\Lambda_{H,k,t} = \omega \left(\frac{1}{P_{H,k,t}}\right)^{-\theta} \left(\frac{P_{H,k,t}}{P_{k,t}}\right)^{-\rho} \left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t,$$

$$\Lambda^*_{H,k,t} = (1-\omega) \left(\frac{1}{P^*_{H,k,t}}\right)^{-\theta} \left(\frac{P^*_{H,k,t}}{P^*_{k,t}}\right)^{-\rho} \left(\frac{P^*_{k,t}}{P^*_t}\right)^{-\eta} Y^*_t.$$

An analogous maximization problem for Foreign firms yields:

$$X^*_{F,k,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \Theta^*_{t,t+s} (1-\alpha_k)^s \Lambda^*_{F,k,t+s} \left(\chi N^{*\chi-1}_{k,j,t+s}\right)^{-1} W^*_{t+s}}{E_t \sum_{s=0}^{\infty} \Theta^*_{t,t+s} (1-\alpha_k)^s \Lambda^*_{F,k,t+s}},$$

$$X_{F,k,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \Theta^*_{t,t+s} (1-\alpha_k)^s \Lambda_{F,k,t+s} \left(\chi N^{*\chi-1}_{k,j,t+s}\right)^{-1} W^*_{t+s}}{E_t \sum_{s=0}^{\infty} \Theta^*_{t,t+s} (1-\alpha_k)^s \mathcal{E}^{-1}_{t+s} \Lambda_{F,k,t+s}},$$

where:

$$\Lambda^*_{F,k,t} = \omega \left(\frac{1}{P^*_{F,k,t}}\right)^{-\theta} \left(\frac{P^*_{F,k,t}}{P^*_{k,t}}\right)^{-\rho} \left(\frac{P^*_{k,t}}{P^*_t}\right)^{-\eta} Y^*_t,$$

$$\Lambda_{F,k,t} = (1-\omega) \left(\frac{1}{P_{F,k,t}}\right)^{-\theta} \left(\frac{P_{F,k,t}}{P_{k,t}}\right)^{-\rho} \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} Y_t.$$

We focus on a symmetric equilibrium in which, conditional on time-t information, the joint distribution of future variables that matter for price setting is the same for all firms in sector k in a given country that change prices in period t. Therefore, they choose the same prices, which we denote by $X_{H,k,t}$, $X^*_{H,k,t}$, and $X_{F,k,t}$, $X^*_{F,k,t}$. The country-sector price indices can thus be written as:

$$P_{H,k,t} = \left(\alpha_k X^{1-\theta}_{H,k,t} + (1-\alpha_k) P^{1-\theta}_{H,k,t-1}\right)^{\frac{1}{1-\theta}},$$

$$P^*_{H,k,t} = \left(\alpha_k X^{*1-\theta}_{H,k,t} + (1-\alpha_k) P^{*1-\theta}_{H,k,t-1}\right)^{\frac{1}{1-\theta}},$$

and likewise for $P_{F,k,t}$ and $P^*_{F,k,t}$.

Finally, the model is closed with a monetary policy specification that ensures existence and uniqueness of the rational-expectations equilibrium. We consider different specifications in subsequent sections. Equilibrium is characterized by the optimality conditions of the consumers' utility-maximization problem and of every firm's profit-maximization problem, and by market clearing in assets, goods, and labor markets.

We solve the model by log-linearizing around a zero-inflation steady state. Due to symmetry, in this steady state prices of all intermediate firms, levels of employment, and allocations of consumption, imports and exports are the same for both countries. Additionally, the common-preferences assumption implies that, in steady state, the real exchange rate Q equals unity. The derivations of the steady state and the log-linear approximation are in a supplementary appendix available upon request. Throughout the rest of the paper, lowercase variables denote log-deviations from the steady state.

2.1 The counterfactual one-sector world economy

We also build a counterfactual world economy with only one sector of intermediate firms in each country. The model is exactly the same as the one in the previous subsection, except that the frequency of price changes, $\overline{\alpha}$, is set equal to the average frequency of adjustments in the multi-sector world economy: $\overline{\alpha} = \sum_{k=1}^{K} f_k \alpha_k$. In terms of notation, we differentiate the variables in this one-sector economy from the corresponding variables in the heterogeneous economy by adding a "1 sec" superscript. We refer to this economy as the *counterfactual one-sector world economy.*

3 Analytical results

In this section we make a set of simplifying assumptions to deliver analytical results. This allows us to characterize the dynamic properties of aggregate and sectoral real exchange rates, and to compute different measures of persistence and volatility explicitly. We relax these assumptions in our quantitative analysis in Section 4.

We leave the specification of monetary policy implicit, and assume that the growth rate of nominal aggregate demand in each country follows an exogenous first-order autoregressive (AR) process. This specification, common in the Monetary Economics literature, fits the data well. Denoting nominal aggregate demand in Home and Foreign by, respectively, $Z_t \equiv P_t Y_t$ and $Z_t^* \equiv P_t^* Y_t^*$, our assumption is:

$$
\begin{aligned}
\Delta z_t &= \rho_z \Delta z_{t-1} + \sigma_{\varepsilon_z} \varepsilon_{z,t}, \\
\Delta z_t^* &= \rho_z \Delta z_{t-1}^* + \sigma_{\varepsilon_z} \varepsilon_{z,t}^*,
\end{aligned}
$$

where ρ_z denotes the autocorrelation in nominal aggregate demand growth, and $\varepsilon_{z,t}$ and $\varepsilon_{z,t}^*$ are uncorrelated, zero-mean, unit-variance *i.i.d.* shocks. For expositional simplicity, we assume that $\rho_z \in (1-\alpha_1, 1-\alpha_K)$.[6]

[6] This restriction is consistent with estimates of ρ_z and microeconomic evidence on the frequency of price changes. Generalizing our results to the case in which $\rho_z \in [0,1)$ is straightforward.

In addition, we impose restrictions on some parameters, as follows. We assume logarithmic consumption utility ($\sigma = 1$), linear disutility of labor ($\gamma = 0$), and linear production function ($\chi = 1$). These assumptions give rise to no strategic complementarity nor substitutability in price setting – i.e., to a Ball and Romer (1990) index of real rigidities equal to unity. We refer to this case as the one of *strategic neutrality* in price setting.

Under these assumptions, in the Appendix we derive explicit expressions for the processes followed by the aggregate and sectoral real exchange rates, and obtain the following:

Proposition 1 *Under the assumptions of this section, sectoral real exchange rates follow $AR(2)$ processes:*

$$(1-\rho_z L)(1-\lambda_k L)\, q_{k,t} = \varphi_k u_t,\ \ k = 1,...,K,$$

where $\lambda_k \equiv 1-\alpha_k$ is the per-period probability of no price adjustment for a firm in sector k, $u_t \equiv \sigma_{\varepsilon_z}\left(\varepsilon_{z,t}-\varepsilon^*_{z,t}\right)$ is white noise, $\varphi_k \equiv \lambda_k - (1-\lambda_k)\frac{\rho_z\beta\lambda_k}{1-\rho_z\beta\lambda_k}$, and L is the lag operator.

The dynamic properties of sectoral real exchange rates depend on the frequency of price adjustments in the sector, as well as on the persistence of shocks hitting the two economies.[7] Note that in this simplified version of the model the remaining structural parameters do not affect sectoral real exchange rate dynamics.

We highlight that the simplicity of the equilibrium processes followed by sectoral exchange rates, derived in **Proposition 1**, depends crucially on the simplifying assumptions used in this section. More generally, the solution of the (log-linear approximate) model laid out in the previous section can be written as a first-order vector-autoregression (VAR). Due to general-equilibrium effects, the dynamics of all variables depend on the whole cross-sectional distribution of price stickiness – as well as on all other structural parameters – and cannot be solved for explicitly.

Aggregating the sectoral exchange rates, we obtain the following well-known result from the work of Granger and Morris (1976):

Corollary 1 *The aggregate real exchange rate follows an $ARMA\,(K+1, K-1)$ process:*

$$(1-\rho_z L)\prod_{k=1}^{K}(1-\lambda_k L)\, q_t = \left(\sum\nolimits_{k=1}^{K}\prod_{j\neq k}^{K}(1-\lambda_j L)\, f_k\varphi_k\right) u_t.$$

The aggregate real exchange rate naturally depends on the whole distribution of the frequency of price changes across sectors, as well as on the shocks hitting the two countries. Because it

[7] This result was independently derived by Crucini et al. (2010). Kehoe and Midrigan (2007) first derived the (nested) result that when nominal aggregate demand in each country evolves as a random walk ($\rho_z = 0$), sectoral real exchange rates follow $AR(1)$ processes.

follows a possibly high-order ARMA, the dynamics of the aggregate real exchange rate can be quite different from those of the underlying sectoral real exchange rates.

Finally, the real exchange rate in the counterfactual one-sector world economy can be obtained as a degenerate case in which all firms belong to a single sector, with frequency of price adjustments equal to the average frequency of the heterogeneous economy:

Corollary 2 *The real exchange rate of the counterfactual one-sector world economy follows an* $AR(2)$ *process:*

$$\left(1-\rho_z L\right)\left(1-\overline{\lambda} L\right) q_t^{1\,\mathrm{sec}} = \varphi u_t,$$

where $\overline{\lambda} \equiv \sum_{k=1}^{K} f_k \lambda_k$ *and* $\varphi \equiv \overline{\lambda} - \left(1-\overline{\lambda}\right) \frac{\rho_z \beta \overline{\lambda}}{1-\rho_z \beta \overline{\lambda}}$.

3.1 Persistence and volatility

We are interested in analyzing the persistence and volatility of deviations of the real exchange rate from PPP. We start with persistence, and focus on measures used in the literature for which we can obtain analytical results. In particular, we focus on the cumulative impulse response, the largest autoregressive root, and the sum of autoregressive coefficients. The cumulative impulse response ($\mathcal{CIR}$) is defined as follows. Let $IRF_t(q)$, $t = 0, 1, ...$ denote the impulse response function of the q_t process to a *unit* impulse.[8] Then, $\mathcal{CIR}(q) \equiv \sum_{t=0}^{\infty} IRF_t(q)$. The largest autoregressive root ($\mathcal{LAR}$) for a process q_t with representation $\widetilde{A}(L)\, q_t = \widetilde{B}(L)\, u_t$, $\mathcal{LAR}(q)$, is simply the largest root of the $\widetilde{A}(L)$ polynomial. Finally, the sum of autoregressive coefficients ($\mathcal{SAC}$) of such a process is $\mathcal{SAC}(q) \equiv 1 - \widetilde{A}(1)$.

Let $\mathcal{P}$ denote a measure of persistence. We prove the following:

Proposition 2 *For the measures of persistence* $\mathcal{P} = \mathcal{CIR}, \mathcal{LAR}, \mathcal{SAC}$*:*

$$\mathcal{P}(q) > \mathcal{P}\left(q^{1\,\mathrm{sec}}\right).$$

Turning to volatility, we obtain the following result:

Proposition 3 *Let* $\mathcal{V}(q)$ *denote the variance of the* q_t *process. Then:*

$$\mathcal{V}(q) > \mathcal{V}\left(q^{1\,\mathrm{sec}}\right).$$

Propositions 2 and **3** show that a simple model with sectoral heterogeneity stemming solely from differences in price rigidity can generate an aggregate real exchange rate that is more volatile

[8] This removes the scale of the shock, making $\mathcal{CIR}$ a useful measure of persistence. The impulse response function to a one-standard-deviation shock is what we refer to as the "scaled impulse response function" (for more details see the Appendix).

and persistent than the real exchange rate in a one-sector version of the world economy with the same average frequency of price changes. The main determinant of this result is the fact that the counterfactual one-sector model is a poor representation of the multi-sector world economy. **Corollary 1** shows that, as a result of cross-sectional aggregation of sectoral exchange rates with heterogeneous dynamics, the aggregate real exchange rate in the multi-sector economy follows a richer stochastic process than the real exchange rate in the counterfactual one-sector model. Moreover, the persistence of real exchange rates under commonly used measures and its volatility are convex functions of the frequency of price adjustments. Thus, the counterfactual one-sector model understates the persistence and volatility of the real exchange rate relative to the underlying heterogeneous economy. In what follows we refer to the difference in persistence $\mathcal{P}(q)-\mathcal{P}\left(q^{1\,\text{sec}}\right)>0$ as the *total heterogeneity effect* (under $\mathcal{P}$).

In the Appendix we provide a limiting result showing that, under suitable conditions on the cross-sectional distribution of price stickiness, the real exchange rate in the multi-sector economy becomes arbitrarily more volatile and persistent than in the counterfactual one-sector world economy as the number of sectors increases. In the next section we turn to the more relevant question of whether a version of the model parameterized to match the microeconomic evidence on the frequency of price changes can produce significantly more persistence and volatility in real exchange rates.

4 Quantitative analysis

In this section we analyze the quantitative implications of our model. We describe our parameterization, starting with how we use the recent microeconomic evidence on price setting to specify the cross-sectional distribution of price stickiness. We then present the quantitative results for our baseline specification, and consider alternative configurations in our robustness analysis.

4.1 Parameterization

4.1.1 Cross-sectional distribution of price stickiness

In our model, whenever a firm changes its prices it sets one price for the domestic market and another price for exports, and for simplicity we impose the same frequency of price adjustments in both cases.[9] In addition, we also assume the same cross-sectional distribution of the frequency of

[9] Benigno (2004) studies a one-sector model in which he allows the frequency of price changes for those two pricing decisions to differ and also incorporates asymmetry in the frequency of price changes across countries. He shows that when this leads to different frequencies of price changes within a same country (due to differences in frequencies for varieties produced by local versus foreign firms), the real exchange rate becomes more persistent.

price changes in both countries. As a result, we must choose a single distribution to parameterize the model.

We analyze our model having in mind a two-country world economy with the U.S. and the rest of the world. Since the domestic market is relatively more important for firms' decisions (due to a small import share), we favor a distribution for the frequency of price changes across sectors that reflects mainly domestic rather than export pricing decisions. Due to our assumption of symmetric countries, we also favor distributions that are representative of price-setting behavior in different developed economies. Finally, and perhaps most importantly, we want to relate our results to the empirical PPP literature, which most often focuses on real exchange rates based on consumer price indices (CPIs).

We choose to use the statistics on the frequency of price changes reported by Nakamura and Steinsson (2008). We build from the statistics on the frequency of regular price changes – those that are not due to sales or product substitutions – for 271 categories of goods and services.[10] To make the model computationally manageable, in our benchmark specification we aggregate those 271 categories into 67 expenditure classes. Each class is identified with a sector in the model. As an example of what this aggregation entails, the resulting "New and Used Motor Vehicles" class consists of the categories "Subcompact Cars", "New Motorcycles", "Used Cars", "Vehicle Leasing" and "Automobile Rental"; the "Fresh Fruits" class comprises four categories: "Apples", "Bananas", "Oranges, Mandarins etc." and "Other Fresh Fruits." To perform this aggregation we rely on BLS's ELI codes reported by Nakamura and Steinsson (2008).[11] The frequency of price changes for each expenditure class is obtained as the weighted average of the frequencies for the underlying categories, using the expenditure weights provided by Nakamura and Steinsson (2008). Finally, expenditure-class weights are given by the sum of the expenditure weights for those categories. The resulting average monthly frequency of price changes is $\overline{\alpha} = \sum_{k=1}^{K} f_k \alpha_k = 0.211$, which implies that prices change on average once every 4.7 months.

4.1.2 Remaining parameters

In our baseline specification we fix the remaining structural parameters as follows. We set the intertemporal elasticity of substitution σ^{-1} to 1/3, unit labor supply elasticity ($\gamma = 1$), and the usual extent of decreasing returns to labor ($\chi = 2/3$). The consumer discount factor β implies a time-discount rate of 2% per year.

[10] Nakamura and Steinsson (2008) report statistics for 272 categories. We discard the category "Girls' Outerwear", for which the reported frequency of regular price changes is zero. We renormalize the expenditure weights to sum to unity.

[11] Each code comprises two letters and three numbers. Aggregating according to the two letters yields our 67 expenditure classes.

For the final-good aggregator, we set the elasticity of substitution between varieties of the same sector to $\theta = 10$. We set the elasticity of substitution between Home and Foreign goods to $\rho = 1.5$, and the share of domestic goods to $\omega = 0.9$. The elasticity of substitution between varieties of different sectors should arguably be smaller than within sectors, and so we assume a unit elasticity of substitution across sectors, $\eta = 1$ (i.e. the aggregator that converts sectoral into final output is Cobb-Douglas).

Finally, to specify the process for nominal aggregate demand, the literature usually relies on estimates based on nominal GDP, or on monetary aggregates such as M1 or M2. With quarterly data, estimates of ρ_z typically fall in the range of 0.4 to 0.7,[12] which maps into a range of roughly $0.75 - 0.90$ at a monthly frequency. We set $\rho_z = 0.8$ in our baseline parameterization, and discuss the implications of different parameter values in Section 7. The standard deviation of the shocks is set to $\sigma_{\varepsilon_z} = 0.6\%$ (roughly 1% at a quarterly frequency), in line with the same estimation results.[13]

4.2 Quantitative results

Table 1 presents the quantitative results of our parameterized model. The first column shows statistics computed for the aggregate real exchange rate in our benchmark multi-sector world economy, and the second column contains the same statistics for the real exchange rate in the counterfactual one-sector economy. We present results for the measures of persistence $\mathcal{CIR}$, $\mathcal{SAC}$, and $\mathcal{LAR}$, and also for the half-life ($\mathcal{HL}$) – reported in months – and the first-order autocorrelation (ρ_1), which are common measures used in the empirical literature. For expositional reasons we follow a large part of the empirical literature on PPP and focus mainly on the half-life. We also present results for a measure of volatility – the standard deviation – of the real exchange rate.[14]

Table 1 shows that the model with heterogeneity can generate a significantly more volatile and persistent real exchange rate. In particular, at 39 months the half-life of deviations from PPP falls well within the "consensus view" of 3 to 5 years reported by Rogoff (1996). In contrast, the counterfactual one-sector economy produces a half-life just above one year. In short, the total heterogeneity effect is quite large.

Table 1 also reports the up-lives ($\mathcal{UL}$) and quarter-lives ($\mathcal{QL}$) of the real exchange rates, following Steinsson (2008). These measures are defined as, respectively, the time it takes for the real exchange

[12] See, for instance, Mankiw and Reis (2002).

[13] All results for volatilities scale-up proportionately with σ_{ε_z}.

[14] Since under our baseline specification the real exchange rates no longer follow the exact processes derived in Section 3, we compute $\mathcal{SAC}$, $\mathcal{LAR}$, ρ_1, and $\mathcal{V}$ through simulation. Specifically, we simulate 150 replications of our economy and construct time series for the real exchange rates with 1500 observations each. After dropping the first 100 observations to eliminate possible effects from the initial steady-state conditions, we compute the statistics for each replication and then average across the 150 replications. While ρ_1 and $\mathcal{V}$ are computed directly from the simulated time series, for computing $\mathcal{SAC}$ and $\mathcal{LAR}$ we fit an $AR(30)$ process to the aggregate real exchange rates. The reported results are quite robust to varying the number of lags. Finally, $\mathcal{CIR}$, $\mathcal{HL}$, $\mathcal{UL}$ and $\mathcal{QL}$ are computed directly from the impulse response functions implied by the solution of the models.

Table 1: Results from baseline specification

Persistence measures:	$\mathcal{P}(q)$	$\mathcal{P}\left(q^{1\,\text{sec}}\right)$
$\mathcal{CIR}$	67.2	20.4
$\mathcal{SAC}$	0.98	0.95
$\mathcal{LAR}$	0.95	0.91
ρ_1	0.98	0.96
$\mathcal{HL}$	39	14
$\mathcal{UL}$	23	9
$\mathcal{QL}$	57	18
Volatility measure:	$\mathcal{V}(q)^{1/2}$	$\mathcal{V}\left(q^{1\,\text{sec}}\right)^{1/2}$
	0.05	0.02

rate to peak after the initial impulse, and the time it takes for the impulse response function to drop below 1/4 of the initial impulse. They are meant to provide a more nuanced picture of the underlying impulse response functions. The results from our multi-sector model are in line with the hump-shaped pattern of impulse responses that Steinsson (2008) emphasizes as an important feature of real exchange rate dynamics. He estimates the up-life of the trade-weighted U.S. real exchange rate to be 28 months, and the quarter-life to be 76 months. Our multi-sector model produces an up-life of 23 months and a quarter-life of 57 months. In contrast, the real exchange rate in the counterfactual one-sector world economy has an up-life of only 9 months, and a quarter-life of 18 months. These results suggest that the more sluggish response of the multi-sector economy relative to the counterfactual one-sector economy does not hinge on a particular segment of the impulse response function. This conjecture is confirmed by inspection of Figure 1, which shows the scaled impulse response functions of the real exchange rate to a (one-standard-deviation) shock to Home nominal aggregate demand in the two models.

4.3 Robustness

We consider several important departures from our baseline parameterization. For brevity, here we provide a summary of our findings and leave the details to the Appendix. We analyze versions of the model with strategic neutrality in price setting, with the assumption of exogenous nominal aggregate demand replaced by an interest-rate rule subject to shocks, and with additional shocks. We also check the robustness of our results to changes in various parameter values. In particular, we allow for a wide range of values for the three elasticities of substitution between varieties of intermediate goods in the model, and for the share of imported goods.[15] We also entertain models with different numbers of sectors, corresponding to different aggregations of the statistics on the frequency of price changes for the various goods and services categories reported in Nakamura and

[15] The literature on real exchange rate dynamics often emphasizes the roles of the elasticity of substitution between domestic and foreign goods and the share of imported goods. For the former parameter we consider values close to the low end of the range used in the literature (unity), and above the relatively high value of 7 estimated by Imbs and Mejean (2009) using disaggregated multilateral trade data. For the share of imported goods we consider alternatives from the relatively low value of 1.6% used by Chari et al. (2002) and Steinsson (2008) to values above the 16.5% used in Atkeson and Burstein (2008).

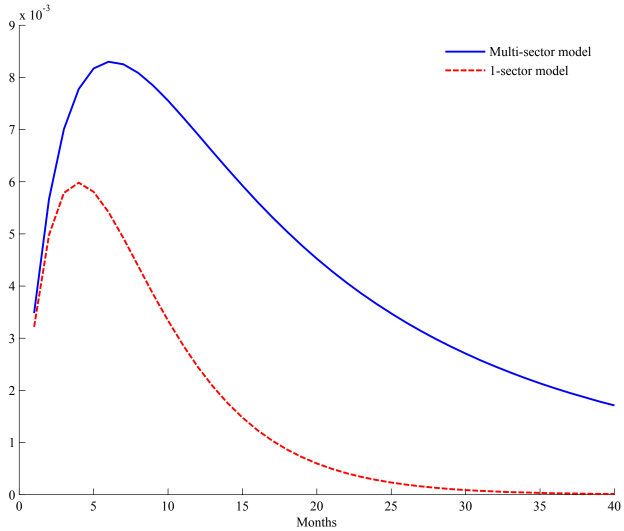

Figure 1: Scaled impulse response functions of aggregate real exchange rates to a (one-standard-deviation) shock to Home nominal aggregate demand

Steinsson (2008). While there are some quantitative differences in the results across specifications, our substantive conclusions are unchanged.

Finally, we explore the role of nominal aggregate demand persistence. Fixing all parameter values as in our baseline parameterization and varying the value of ρ_z, we compare the level of real exchange rate persistence in the multi-sector and counterfactual one-sector models. The difference between the two isolates the effects of heterogeneity. In both models, the level of real exchange rate persistence increases with the persistence of nominal aggregate demand. For small values of ρ_z, real exchange rate persistence is somewhat low in both models. However, for all values of ρ_z we find that heterogeneity adds to real exchange rate persistence (i.e. the total heterogeneity effect in terms of the half-life is positive), and that the degree of amplification tends to increase with ρ_z. In Section 7 we come back to these findings, and to the role of nominal aggregate demand persistence more generally.

5 Revisiting the empirical literature on heterogeneity and aggregation

The empirical relevance of heterogeneity and aggregation in accounting for the persistence of aggregate real exchange rates has been the subject of intense debate. While some studies find that they play at most a small role (e.g. Chen and Engel 2005, Crucini and Shintani 2008), Imbs et al. (2005a) conclude that heterogeneity can explain why the aggregate real exchange rate is so persistent. In this section we use the simplified version of our structural model from Section 3 to interpret the apparently conflicting findings in this empirical literature.

Recall that, for any measure of persistence $\mathcal{P}$, we define the total heterogeneity effect under $\mathcal{P}$ to be the difference between the persistence of the aggregate real exchange rate in the heterogeneous economy, q_t, and the persistence of the real exchange rate in the counterfactual one-sector world economy, $q_t^{1\,\mathrm{sec}}$:

$$\textit{total heterogeneity effect under } \mathcal{P} \equiv \mathcal{P}(q) - \mathcal{P}\left(q^{1\,\mathrm{sec}}\right).$$

We can rewrite the total heterogeneity effect by adding and subtracting the weighted average of the persistence of the sectoral exchange rates, $\sum_{k=1}^{K} f_k \mathcal{P}(q_k)$, to obtain the following decomposition:

$$\begin{array}{c}\textit{total heterogeneity} \\ \textit{effect under } \mathcal{P}\end{array} = \left(\mathcal{P}(q) - \sum\nolimits_{k=1}^{K} f_k \mathcal{P}(q_k)\right) + \left(\sum\nolimits_{k=1}^{K} f_k \mathcal{P}(q_k) - \mathcal{P}\left(q^{1\,\mathrm{sec}}\right)\right). \tag{23}$$

In (23), the first term in parentheses is what we define as the *aggregation effect*: the difference between the "persistence of the average" and the "average of the persistences":

$$\textit{aggregation effect under } \mathcal{P} \equiv \mathcal{P}(q) - \sum\nolimits_{k=1}^{K} f_k \mathcal{P}(q_k). \tag{24}$$

Note that $q_t = \sum_{k=1}^{K} f_k q_{k,t}$ and, as a result, the scaled impulse response function of the aggregate real exchange rate to a given shock is simply the weighted average of the sectoral scaled impulse response functions to that *same* shock.[16] Thus, the effect in (24) is indeed purely due to the impact of aggregation on the given measure of persistence $\mathcal{P}$.

The second term in the decomposition (23) is the difference between the weighted average of the persistence of sectoral real exchange rates in the heterogeneous economy, and the persistence of the real exchange rate in the counterfactual one-sector world economy. We refer to it as the *counterfactuality effect*:

$$\textit{counterfactuality effect under } \mathcal{P} \equiv \sum\nolimits_{k=1}^{K} f_k \mathcal{P}(q_k) - \mathcal{P}\left(q^{1\,\mathrm{sec}}\right). \tag{25}$$

Our next result gives substance to the decomposition in (23). It shows that, under the simplified model of Section 3, both the aggregation and the counterfactuality effects are positive (for commonly used measures of persistence):

Proposition 4 *Under the simplified model of Section 3, for the measures of persistence $\mathcal{P} = \mathcal{CIR}$, $\mathcal{LAR}$:*

$$\begin{aligned} \textit{aggregation effect under } \mathcal{P} &> 0, \\ \textit{counterfactuality effect under } \mathcal{P} &> 0. \end{aligned}$$

As seen in the previous section, our structural model uncovers a potentially large role for

[16] It is important to emphasize that this result holds when the impulse response functions refer to the same shock. For example, this applies if one identifies a given shock in a VAR that includes all underlying sectoral real exchange rates. In contrast, if one fits univariate time-series models to the aggregate and sectoral real exchange rates, the impulse response functions represent the dynamic response of each variable to its own *reduced-form shock*, and the aggregation result for impulse response functions need not apply. See, for instance, Mayoral (2008), and Mayoral and Gadea (2009).

heterogeneity in explaining the persistence of aggregate real exchange rates (i.e. a large total heterogeneity effect). Moreover, the decomposition above shows that the increase in persistence when moving from the counterfactual one-sector model to the multi-sector world economy can be due to two distinct effects.[17] Armed with that decomposition we now interpret the apparently conflicting findings of the empirical literature.

The bottom line is that different papers measure different objects. The strand of the literature that finds a small role for heterogeneity and aggregation measures their effects through the difference between the persistence of the aggregate real exchange rate and the average persistence across its underlying components (Chen and Engel 2005, Crucini and Shintani 2008). This corresponds to what we define above as the aggregation effect. In turn, Imbs et al. (2005a) measure the effects of heterogeneity and aggregation through econometric methods that, as we show below, can be interpreted as estimating the total heterogeneity effect.

Imbs et al. (2005a) focus on the comparison between estimates of persistence of aggregate real exchange rates and estimates of persistence based on a Mean Group (MG) estimator for heterogeneous dynamic panels (Pesaran and Smith 1995) applied to sectoral real exchange rates. To be more precise in our description of their empirical implementation, assume that sectoral real exchange rates follow $AR(p)$ processes with sector-specific coefficients:

$$q_{k,t} = \phi_{k,1} q_{k,t-1} + \phi_{k,2} q_{k,t-2} + ... + \phi_{k,p} q_{k,t-p} + \varepsilon_{k,t},$$

where $\varepsilon_{k,t}$ is an *i.i.d.* shock. The $AR(p)$ real exchange rate process constructed on the basis of the MG estimator, denoted q_t^{MG}, is given by:

$$q_t^{MG} = \phi_1^{MG} q_{t-1} + \phi_2^{MG} q_{t-2} + ... + \phi_p^{MG} q_{t-p} + \varepsilon_t^{MG},$$

where ε_t^{MG} is an *i.i.d.* shock, and $\phi_i^{MG} = \frac{1}{K} \sum_{k \in K} \widehat{\phi}_{k,i}$, with $\widehat{\phi}_{k,i}$ denoting the OLS estimate of the i^{th} autoregressive coefficient for the k^{th} cross-sectional unit of the panel of sectoral real exchange rates.[18] In words, q_t^{MG} is an $AR(p)$ process with autoregressive coefficients given by the cross-sectional averages of the estimated autoregressive coefficients of the sectoral real exchange rates, where the averages are taken for each of the p lags. The comparison made by Imbs et al. (2005a) is between the estimated persistence of the aggregate real exchange rate,[19] and the persistence of the MG-based real exchange rate.

An interpretation of the MG-based real exchange rate follows under our structural model and

[17] The implication of our analytical results that the aggregation effect is necessarily positive differs from the typical result in the literature on cross-sectional aggregation of autoregressive processes. In the latter case this effect can be positive or negative, depending on the relationships between the parameters of the time-series processes being aggregated. The unequivocal prediction from our structural model is due to the cross-equation restrictions that it imposes on the autoregressive processes for sectoral real exchange rates. This is clear from the proof of **Proposition 4**, which is available in the Appendix.

[18] For simplicity and consistency with the empirical implementation of Imbs et al. (2005), we assume equal sectoral weights.

[19] Or, alternatively, the persistence estimated from a panel of sectoral exchange rates with methods that impose homogeneous dynamics across all units of the panel.

its counterfactual one-sector world economy, in the case of equal sectoral weights. In that case, under the simplifying assumptions of Section 3, we prove the following:

Proposition 5 *Under the assumptions of Section 3 and equal sectoral weights, application of the Mean Group estimator to the sectoral real exchange rates from the multi-sector world economy yields the dynamics of the real exchange rate in the corresponding counterfactual one-sector world economy.*

We conclude that the comparison between the persistence of the aggregate real exchange rate in the heterogeneous world economy and the persistence implied by the MG estimator uncovers the *sum* of the aggregation and counterfactuality effects. In the next subsection we apply this insight to obtain an empirical decomposition of the total heterogeneity effect into those two components.

5.1 Estimation results

We revisit the empirical literature on heterogeneity and aggregation, having as a guide the results of the previous subsection. We use the Eurostat data underlying the paper by Imbs et al. (2005a). It consists of nominal exchange rates, and sectoral and aggregate price indices for 11 European countries versus the U.S., with up to 19 goods and services categories per country. The countries are Germany, France, Denmark, Spain, Belgium, Italy, Portugal, Greece, U.K., Netherlands, and Finland. The categories are bread, meat, dairy, fruits, tobacco, alcohol, clothing, footwear, rents, fuel, furniture, domestic appliances, vehicles, public transportation, communications, sound, leisure, books, and hotels. The frequency is monthly, and the sample runs from January 1981 through December 1995.[20]

Table 2 presents our replication of some of the results of Imbs et al. (2005a) in the first and last columns. The first column shows the results obtained with application of a standard fixed-effects estimator to an autoregressive panel of aggregate real exchange rates, while the last column presents our results for the MG estimator of Pesaran and Smith (1995) applied to the panel of underlying sectoral real exchange rates.[21] The middle column, in turn, presents the estimates for the cross-sectional averages across units of the sectoral panel. To construct this column we calculate the relevant persistence statistics for each series on the basis of the estimated autoregressive coefficients used to construct the MG estimator, and then take a cross-sectional average.

We focus on the half-life of deviations from PPP, which is central to the exposition of Imbs et al. (2005a). Our results confirm that the total heterogeneity effect is indeed large. In a world

[20] The authors make the data available on their websites. A few country-good pairs have shorter samples. Portugal and Finland have fewer goods and services categories (respectively, 17 and 16).

[21] The estimation follows Imbs et al. (2005a) and assumes 18 lags for the fixed-effects estimation, and 19 lags for the MG estimation. These results match those in Imbs et al. (2005a) exactly - refer to their Table II, first line, and Table III line 4. We also find very similar results for some of the other estimators that they report. Given the analytical results of Section 3, we also did the analysis assuming AR(2) processes (in Subsection 6.2 we show that this specification fits the data well). Our substantive conclusions are robust to - in fact strenghtened by - this change of specification.

Table 2: Decomposition of the total heterogeneity effect - Eurostat data

Data	Panel, aggregate	Panel, sectoral	Panel, sectoral
Estimation method	Fixed Effects	OLS	MG
Equal-weight model:	$\mathcal{P}(q)$	$\frac{1}{K}\sum\mathcal{P}(q_k)$	$\mathcal{P}\left(q^{1\,\mathrm{sec}}\right)$
Persistence measures:			
$\mathcal{CIR}$	64.4	59.5	33.2
$\mathcal{SAC}$	0.98	0.97	0.97
$\mathcal{LAR}$	0.97	0.94	0.95
$\mathcal{HL}$	46	43.2	26
$\mathcal{UL}$	24	18.6	16
$\mathcal{QL}$	72	68.9	37

without heterogeneity the $\mathcal{HL}$ would drop from 46 months to 26 months.[22] The aggregation effect is, however, only a small part of this difference. Indeed, the counterfactuality effect accounts for $\frac{43.2-26}{46-26} \approx 86\%$ of the total heterogeneity effect. We find similar results when we consider the "preferred" specification of Imbs et al. (2005a), based on Mean Group estimators with correction for common correlated effects (MG-CCE). Specifically, we find that the counterfactuality effect explains 92% of the total heterogeneity effect for the half-life.

Table 2 also reports our estimates of the up-lives and quarter-lives of deviations from PPP. These measures provide a more detailed picture of the underlying impulse response functions. Our results for aggregate real exchange rates are in accordance with, and similar in magnitude to the estimates reported by Steinsson (2008).

Our parameterized model produces very similar results for all persistence measures, and in particular for the decomposition of the total heterogeneity effect into its two components. In addition, when we apply the estimation methods used in this section to artificial data generated by the model, we find very similar estimates. Finally, while **Proposition 5** is derived under the restrictions of the simplified model of Section 3, we find that the MG-CCE estimator also does a good job of recovering the persistence of the counterfactual one-sector world economy when applied to data generated by a multi-sector model with increased real rigidities (see Section 7). For brevity we do not present these results here.

5.2 Bottom line

Imbs et al. (2005a) conclude that their empirical results show a large role for what they term a "dynamic aggregation bias" or "dynamic heterogeneity bias" in accounting for the PPP puzzle. In contrast, Chen and Engel (2005) and Crucini and Shintani (2008) find that the "aggregation bias" defined as the difference between the persistence of the aggregate real exchange rate and the

[22] To replicate the results in Imbs et al. (2005a), in Table 2 we use the actual aggregate series available in Eurostat. To be consistent with the model, we also analyze aggregate real exchange rates for each country constructed by equally weighting the percentage change of the real exchange rates for the goods that comprise the underlying sectoral panel. Applying a fixed-effects estimator to the resulting panel of country real exchange rates, we estimate a half-life of 39 months. Alternatively, when we estimate separate AR specifications for each country, compute each half-life and then take a simple average, we obtain an average half-life of 43 months.

average of the persistences of the underlying sectoral real exchange rates is small.

As Chen and Engel (2005) and Crucini and Shintani (2008), we find the difference between the persistence of the aggregate real exchange rate and the average of the persistences of the underlying sectoral real exchange rates – what we define as the aggregation effect – to be small, both in the quantitative results of our multi-sector model and in the data. At the same time, as Imbs et al. (2005a), we find that the difference between the persistence of the aggregate real exchange rate and the persistence of the counterfactual real exchange rate constructed with the MG estimators – what we refer to as the total heterogeneity effect – is large, both in the comparison of the quantitative results of our multi-sector and one-sector models, and in the data.

Our structural model provides an interpretation of the different measures of the effects of heterogeneity and aggregation used in the empirical literature. It shows that they serve to estimate conceptually different objects. Moreover, we find that both empirical measures accord well with the quantitative predictions of our model. We conclude that the different findings of the existing empirical literature are not in conflict.[23,24]

6 Cross-sectional implications

The previous sections show that our multi-sector model improves significantly on its one-sector counterpart in terms of producing empirically plausible aggregate real exchange rate dynamics, while remaining consistent with the empirical evidence on nominal price rigidity. It is natural to ask how the model fares in terms of its cross-sectional implications for the dynamics of sectoral real exchange rates.

As highlighted previously, our model produces a cross-section of sectoral real exchange rate dynamics that potentially depend on the whole distribution of price stickiness, due to general-equilibrium effects. This makes it difficult to derive clear-cut, testable cross-sectional implications of the theory that hold for any distribution of price rigidity. To sidestep this difficulty, we explore the simplified model of Section 3, for which we can derive such clear-cut implications.

The first result is the one given in **Proposition 1**: sectoral real exchange rates follow $AR(2)$ processes – with parameters that depend on the persistence of nominal aggregate demand growth and the degree of sectoral price rigidity. Comparative statics of the dynamic properties implied by those $AR(2)$ processes with respect to the degree of price rigidity yield the following set of cross-sectional implications:

Lemma 1 *The measures of persistence $\mathcal{P} = \mathcal{CIR}, \mathcal{LAR}, \mathcal{SAC}$ are (weakly) increasing in the degree*

[23] The debate around the role of aggregation in explaining aggregate real exchange rate persistence involved other methodological issues that we do not address. A summary of the issues involved is provided by Imbs et al. (2005b).

[24] Empirical research on PPP devotes a large amount of effort to quantifying the uncertainty around estimates of real exchange rate persistence. The "consensus view" itself is subject to criticism on these grounds. For brevity, and given our focus on pointing out a *conceptual distinction* between two prominent empirical approaches found in the literature on aggregation and PPP, we do not address these issues in this paper. We refer readers interested in those questions to, e.g., Murray and Papell (2002), Killian and Zha (2002), and Rossi (2004).

of sectoral price rigidity:

$$\frac{\partial \mathcal{CIR}\left(q_k\right)}{\partial \lambda_k} > 0, \frac{\partial \mathcal{LAR}\left(q_k\right)}{\partial \lambda_k} \geq 0, \frac{\partial \mathcal{SAC}\left(q_k\right)}{\partial \lambda_k} > 0,$$

*where λ_k is the "infrequency" of price changes in sector k, defined in **Proposition 1**. Moreover, the variance of sectoral real exchange rates is increasing in the degree of sectoral price rigidity:*

$$\frac{\partial \mathcal{V}\left(q_k\right)}{\partial \lambda_k} > 0.$$

Similar cross-sectional implications were first derived by Kehoe and Midrigan (2007). While the general version of our model differs from theirs, if we impose the additional restriction of random walk nominal aggregate demands ($\rho_z = 0$) to the simplified model of Section 3, the two models have the same implications for the dynamics of sectoral real exchange rates. We come back to the role of nominal aggregate demand persistence in Section 7. We now turn to an empirical assessment of the implications derived in this section, keeping in mind that they correspond to the simplified model of Section 3.

6.1 Data

We continue to use the Eurostat data from Imbs et al. (2005a). Before constructing the sectoral real exchange rate series we seasonally-adjust each price index using the Census Bureau X-12 procedure.[25] Besides maintaining consistency with our analysis of the recent debate on the empirical relevance of heterogeneity and aggregation for the dynamics of real exchange rates (Section 5), using this dataset has the advantage of making our results comparable to those of Kehoe and Midrigan (2007) – who perform similar analysis and also use Eurostat data.

Following Kehoe and Midrigan (2007), we match each of the goods and services categories in the Imbs et al. (2005a) dataset to the statistics on price stickiness for the U.S. economy. In line with our parameterized model, we use the statistics from Nakamura and Steinsson (2008) for regular price changes. Some of the categories can be matched directly to one of the 67 expenditure classes that we use in the quantitative analysis of our model. Other categories are better matched with a subset of the goods and services categories underlying one expenditure class. In these cases we calculate a specific average frequency of price changes for that subset. The only unmatched category is "rents", for which Nakamura and Steinsson (2008) do not report pricing statistics. The result of this matching procedure is summarized in Table 3. In what follows we refer to the resulting average durations of price rigidity as the *empirical durations*, and denote them by D_k.

6.2 Results

We start by assessing the prediction that sectoral real exchange rates follow $AR(2)$ processes. For that purpose we fit univariate autoregressive processes with up to 20 lags to each sectoral real

[25] Our conclusions are robust to using non-seasonally-adjusted data.

Table 3: Price stickiness for the Imbs et al. (2005a) goods and services categories

Category	$D_k^{a)}$	Category	$D_k^{a)}$
Bread	10.2	*Furniture*	18.0
Meat	3.8	*Dom.Appliances*	8.9
Dairy	4.7	*Vehicles*	5.3
Fruits	2.9	*Public Transp.*	2.3
Tobacco	4.4	*Communications*	2.5
Alcohol	10.8	*Sound*	15.6
Clothing	29.1	*Leisure*	13.1
Footwear	28.6	*Books*	5.8
Fuel	1.1	*Hotels*	2.4

a) Expected duration of price spells, in months: $D_k = \alpha_k^{-1}$.

exchange rate series, and select the number of lags based on the Schwarz information criterion. From the 204 country-good pairs, 176 result in the choice of two lags, 27 result in one lag, and one series indicates the presence of 4 lags.[26] Under an $AR(2)$ specification, 201 out of the 204 series produce a positive first-order and negative second-order autoregressive coefficient, as implied by the simplified theory of Section 3. Moreover, the fit of the regressions is extremely tight, with a minimum R^2 of 0.82, and an average R^2 of 0.97 across the 204 series. We conclude that an $AR(2)$ process is a good (reduced-form) approximation for the dynamics of sectoral real exchange rates in this Eurostat dataset.

We then compute various measures of persistence based on the estimated $AR(2)$ processes, and also compute the standard deviation of each sectoral real exchange rate. **Lemma 1** implies that, in the cross-section, both persistence and volatility of sectoral real exchange rates should increase with the degree of price stickiness. Figure 2 illustrates these cross-sectional relationships for some of the countries (we omit the remaining five countries for brevity). In all cases, the x-axis measures the empirical durations (D_k). The left y-axis measures the cumulative impulse responses ($\mathcal{CIR}$, blue squares) and the right y-axis measures the standard deviation of the sectoral real exchange rates (STD, red crosses).[27] We also include the least-squares regression lines for these two measures.

Overall, for 8 out of the 11 countries the cross-sectional relationships are in line with the predictions of the model, and highly statistically significant. The R^2s of the regressions of persistence and volatility on price stickiness range from 0.17 (STD for Greece) to 0.80 ($\mathcal{CIR}$ for Belgium). For two countries there is no evidence of a relationship between price stickiness and persistence and volatility of real exchange rates (U.K. and Netherlands), and for one country there is evidence of an inverse relationship for persistence, but no statistically significant evidence for volatility (Finland).

6.3 Inferring price stickiness from sectoral real exchange rate dynamics

The results of the previous subsection show that, in a qualitative sense, the cross-sectional predictions of the simplified model of Section 3 hold for a vast majority of countries in our sample.

[26] When we use the Akaike information criterion, the results are (numbers of country-good pairs/number of lags): 113/2, 64/4, 13/1, 1/3, 13/$\geq$5.

[27] The results for other measures of persistence are very similar.

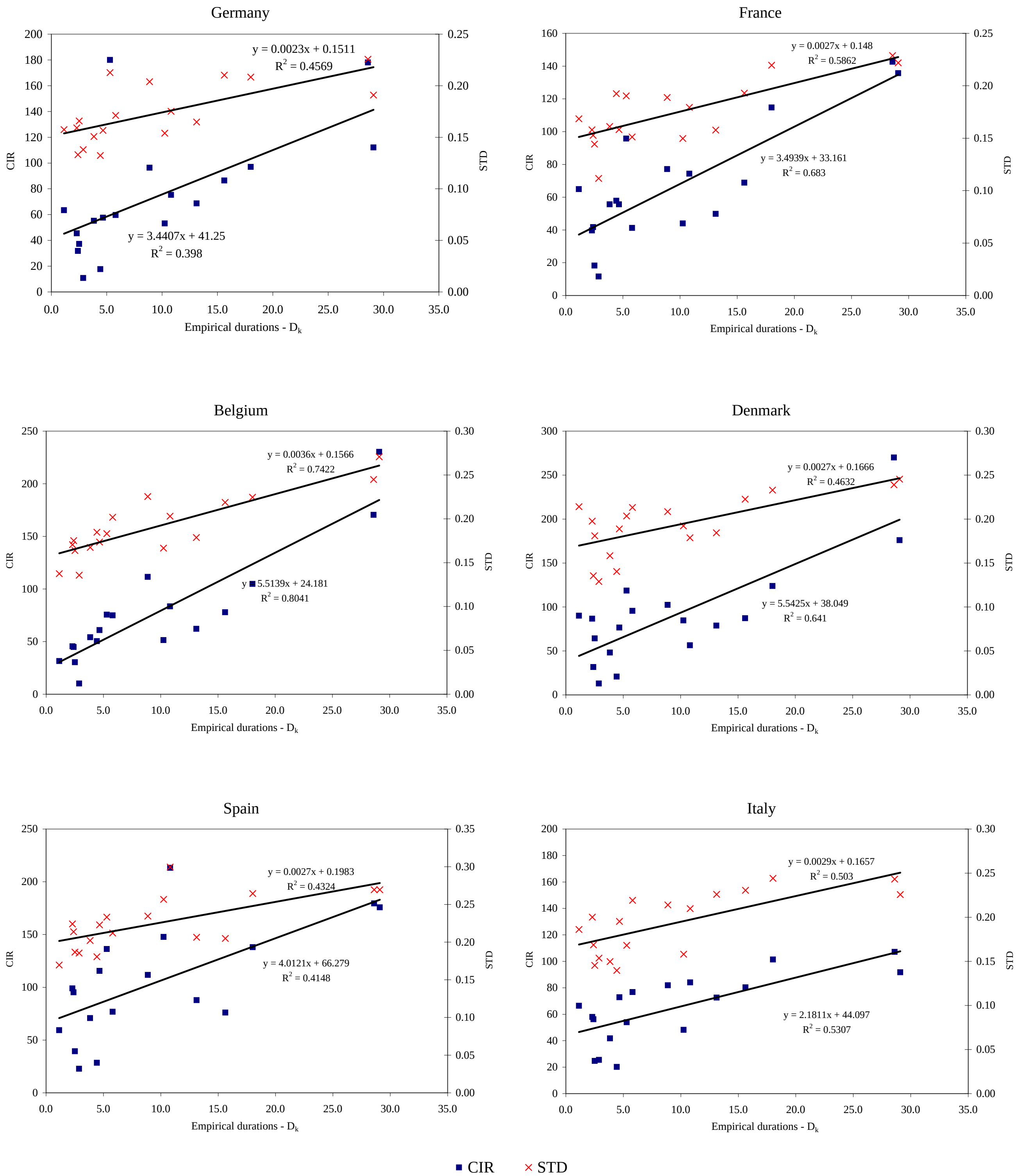

Figure 2: Empirical relationship between price stickiness and persistence and volatility of sectoral real exchange rates

However, those results are silent on whether the model can, in a quantitative sense, reproduce the cross-sectional relationships between price rigidity and the dynamics of sectoral real exchange rates observed in the data. That is the focus of Kehoe and Midrigan (2007). In this and the next subsections we analyze such cross-sectional relationships through the lens of our model, and relate our findings to theirs.

In the spirit of Kehoe and Midrigan (2007), we use the simplified model of Section 3 to infer the degree of price rigidity implied by the estimated parameter values for the $AR(2)$ processes that we fit to the sectoral real exchange rate data. We then compare the inferred degree of price rigidity to the empirical durations from Table 3.

We start from the prediction of the simplified model that sectoral real exchange rates follow $AR(2)$ processes (**Proposition 1**):

$$q_{k,t} = (\rho_z + \lambda_k)\, q_{k,t-1} - (\rho_z \lambda_k)\, q_{k,t-2} + \varphi_k u_t.$$

As noted previously, in this simplified model the autoregressive coefficients are known functions of the sectoral degree of price rigidity and the persistence of nominal aggregate demand. We denote the estimated $AR(2)$ processes underlying the results of the previous subsection by:

$$q_{k,t} = \widehat{\phi}_{k,1} q_{k,t-1} + \widehat{\phi}_{k,2} q_{k,t-2} + \varepsilon_{k,t},$$

where $\widehat{\phi}_{k,1}$ and $\widehat{\phi}_{k,2}$ are the least-squares estimates of the autoregressive coefficients. Our goal is to extract sectoral estimates of price rigidity from these estimated reduced-form coefficients to compare to the statistics in Table 3. This requires a value for ρ_z. We consider two alternatives: $\rho_z = 0$, as in Kehoe and Midrigan (2007), and $\rho_z = 0.8$, as in the baseline parameterization of our structural model. We denote the inferred infrequencies of price changes as a function of ρ_z by $\widehat{\lambda}_k(\rho_z)$, and the corresponding inferred durations of price rigidity by $\widehat{D}_k(\rho_z) \equiv \left(1 - \widehat{\lambda}_k(\rho_z)\right)^{-1}$.

Given the estimates $\widehat{\phi}_{k,1}$,$\widehat{\phi}_{k,2}$, we derive the implied infrequency of price changes from the estimated sum of autoregressive coefficients, as follows:[28]

$$\widehat{\lambda}_k(\rho_z) = \left\{\lambda | \rho_z + \lambda - \rho_z \lambda = \widehat{\phi}_{k,1} + \widehat{\phi}_{k,2}\right\} = \frac{\widehat{\phi}_{k,1} + \widehat{\phi}_{k,2} - \rho_z}{1 - \rho_z}. \tag{26}$$

When setting $\rho_z = 0$, we reestimate the processes for sectoral real exchange rates imposing an AR(1) process. Each sectoral autoregressive coefficient $\widehat{\phi}_{k,1}$ then automatically delivers the estimate for the infrequency of price changes $\widehat{\lambda}_k(0)$.

Finally, given each value for ρ_z, we can also assess the ability of the model to match the cross-sectional relationship between price rigidity and volatility of sectoral real exchange rates. To that

[28] We choose the sum of autoregressive coefficients for analytical tractability. Our conclusions are robust to using other measures of persistence.

end we compare the model-implied volatility of sectoral real exchange rates, given by:

$$STD_{\rho_z}(q_k) = \sqrt{\frac{(1+\rho_z\lambda_k)\left(\lambda_k - (1-\lambda_k)\frac{\rho_z\beta\lambda_k}{1-\rho_z\beta\lambda_k}\right)^2}{(1-\rho_z\lambda_k)\left((1+\rho_z\lambda_k)^2 - (\rho_z+\lambda_k)^2\right)}}\sigma_{\varepsilon_z},$$

with the data, fixing values for β and σ_{ε_z}.

6.3.1 Results

Table 4 summarizes the results by country. We report the cross-sectional average ($E\left[\widehat{D}_k\right]$) and standard deviation ($STD\left[\widehat{D}_k\right]$) of the inferred durations of price rigidity, which are measured in months.[29] For comparison purposes, note that the cross-sectional average ($E[D_k]$) and standard deviation ($STD[D_k]$) of the empirical durations of price rigidity from Table 3 are, respectively, 9.4 and 8.6 months.

Table 4: Inferred durations of price rigidity by country

	$\rho_z = 0$		$\rho_z = 0.8$	
	$E\left[\widehat{D}_k\right]$	$STD\left[\widehat{D}_k\right]$	$E\left[\widehat{D}_k\right]$	$STD\left[\widehat{D}_k\right]$
Germany	310.3	659.3	14.7	9.4
France	94.7	76.7	13.2	7.3
Belgium	197.5	411.3	15.2	10.6
Denmark	335.8	916.6	18.1	11.9
Spain	163.1	121.0	20.9	10.7
Italy	89.0	42.7	12.9	5.1
Greece	89.8	80.8	14.8	11.2
Netherlands	61.7	46.3	9.3	4.2
Portugal	591.5	1369.7	22.3	20.9
Finland	29.9	8.9	5.1	1.2
U.K.	27.9	11.1	5.3	1.6
Average	181.0	340.4	13.8	8.6

The first noteworthy finding is the fact that when $\rho_z = 0$ the inferred durations of price rigidity are, on average, an order of magnitude larger than the empirical durations. This is in line with the results in Kehoe and Midrigan (2007), who conclude that the degree of price stickiness inferred from their AR(1) estimates of sectoral real exchange rates is uniformly much higher than the corresponding sectoral stickiness obtained directly from micro price data. The same applies to the cross-country average of the standard deviations of price rigidity. In sharp contrast, setting

[29] Formally, for each country, $E\left[\widehat{D}_k\right] = \frac{1}{\#\mathcal{K}}\sum_{k\in\mathcal{K}}\widehat{D}_k(\rho_z)$, and $STD\left[\widehat{D}_k\right] = \sqrt{\frac{1}{\#\mathcal{K}}\sum_{k\in\mathcal{K}}\left(\widehat{D}_k(\rho_z) - E\left[\widehat{D}_k\right]\right)^2}$, where $\#\mathcal{K}$ denotes the number of goods and services categories available for that country. We drop the categories alcohol and clothing for Portugal, since their AR(1) estimates imply explosive dynamics (this is not the case for their AR(2) estimates).

$\rho_z = 0.8$ produces inferred durations of price rigidity that are much more in line with the empirical durations. The same holds for the cross-country average of the standard deviations of price rigidity.

Figure 3 shows a scatter plot of inferred durations obtained under the two values for ρ_z as a function of the empirical durations. For brevity, we only report results for France, which are illustrative of the pattern in the eight countries for which the cross-sectional predictions of the model hold in qualitative terms. It is clear that there is a positive correlation between inferred and empirical durations under both values for ρ_z. In contrast with the results obtained with $\rho_z = 0$, the inferred durations when $\rho_z = 0.8$ line up reasonably well along the 45$^{\text{o}}$ line.

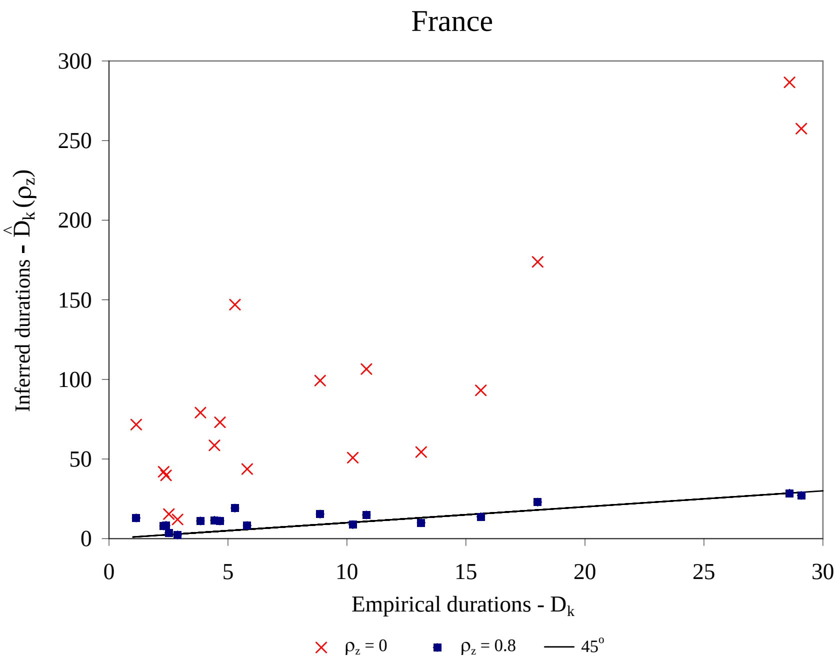

Figure 3: Relationship between inferred and empirical durations of price rigidity

Turning to sectoral real exchange rate volatility, for the eight countries for which the cross-sectional predictions of the model hold qualitatively, we find a strong positive correlation between model-implied and observed volatilities, for any value of ρ_z. However, the model implies too steep a relationship between sectoral price rigidity and real exchange rate volatility. In particular, it fails to produce enough real exchange rate volatility in sectors where prices are relatively more flexible, and produces too much volatility for the sectors with relatively more sticky prices. These findings are consistent with the results in Kehoe and Midrigan (2007).[30]

[30] Kehoe and Midrigan (2007) use a different measure of real exchange rate volatility, based only on the residuals of the AR(1) models that they fit to sectoral real exchange rate data - rather than using the unconditional variance of sectoral real exchange rates. They find evidence of a negative correlation between their measure of volatility and observed price rigidity. Nevertheless, our results accord with theirs, in that the model produces too little volatility at the flexible end of the price rigidity spectrum, and too much volatility at the other end.

6.3.2 Discussion

Taking the model of Section 3 to sectoral real exchange rate data under the assumption of $\rho_z = 0$ leads to an incredible extent of inferred price stickiness, echoing the findings of Kehoe and Midrigan (2007).[31] This result arises from the fact that, under that assumption, sectoral price stickiness is the only source of real exchange rate persistence.

Assuming $\rho_z = 0$ has important shortcomings. As argued by Crucini et al. (2010), it is inconsistent with direct estimates available in the literature.[32] Moreover, its implication that sectoral real exchange rates follow AR(1) processes is clearly rejected by the data (see Subsection 6.2).

In contrast, when $\rho_z > 0$, nominal aggregate demand persistence also contributes to real exchange rate persistence. As a result, the implied durations of price rigidity required to match the dynamic properties of sectoral real exchange rates are reduced. This link is clear from equation (26). The fact that, when $\rho_z = 0.8$, the inferred durations of price rigidity are much more in line with the micro evidence reported in Table 3 should not come as a surprise: it is fully consistent with the fact that our baseline model, which is parameterized to match the empirical distribution of price stickiness for the U.S. economy, is successful in reproducing the extent of aggregate real exchange rate persistence observed in the data. The observed degrees of sectoral price rigidity are by themselves insufficient to produce as much sectoral real exchange rate persistence as in the data, but the gap is "filled in" by the persistence of nominal aggregate demand.

7 Exogenous and endogenous persistence

As detailed in Subsection 4.1, our basis for setting $\rho_z = 0.8$ in the baseline parameterization of our model is the direct empirical evidence on the dynamic properties of nominal aggregate demand. However, the discussion in the last subsection leads to the natural question of whether this extent of (exogenous) nominal aggregate demand persistence has counterfactual implications for other variables in the model. It turns out that it does. As shown in the Appendix, in the simplified model of Section 3 the parameter ρ_z also determines the first-order autocorrelation of changes in the nominal exchange rate. If we were to calibrate ρ_z to match the properties of nominal exchange rates in the Imbs et al. (2005a) dataset along that dimension, the result would be $\rho_z \approx 0.35$. The obvious follow-up question is whether our model still produces empirically plausible real exchange dynamics with lower values for ρ_z.

To address that question we perform a sensitivity analysis of the results of the model with respect to variation in nominal aggregate demand persistence (ρ_z). Based on the baseline parameterization, Figure 4 shows the half-lives of deviations of the aggregate real exchange rate from PPP in the multi-sector and counterfactual one-sector models as a function of ρ_z. Three patterns are clear from the plot. First, real exchange rate persistence increases with ρ_z. Second, for all values of ρ_z persistence is higher in the multi-sector model (i.e. the total heterogeneity effect is positive).

[31] Crucini et al. (2008) find similar results using disaggregated data from the Economist Intelligence Unit.

[32] For example, Mankiw and Reis (2002), and Chari et al. (2002).

Finally, the degree of amplification induced by heterogeneity (given by the ratio $\mathcal{HL}(q)/\mathcal{HL}\left(q^{1\,\mathrm{sec}}\right)$) is strongly increasing in ρ_z. For $\rho_z \approx 0.35$, the model falls short of generating as much persistence in real exchange rates as in the data, even with heterogeneity in price stickiness.

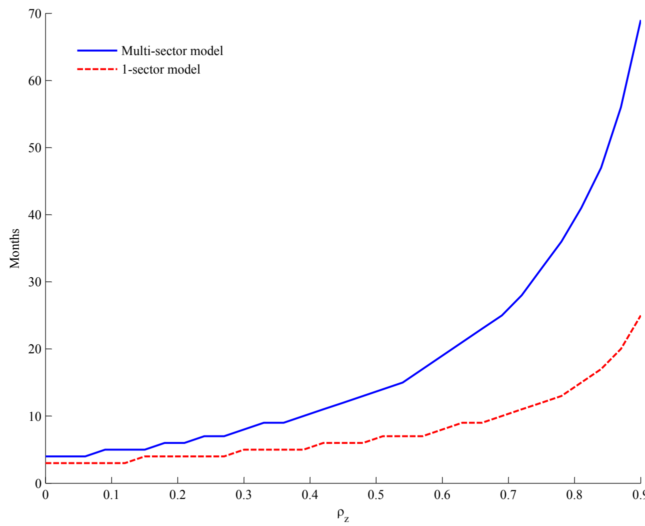

Figure 4: Half-life as a function of nominal aggregate demand persistence - baseline parameterization

The result of this sensitivity analysis should not come as a surprise. Since Ball and Romer (1990), it is well known that simple sticky-price models with empirically plausible degrees of nominal price rigidity struggle to produce realistic amounts of persistence in real variables in response to nominal disturbances. In the last two decades, much of the sticky-price literature has evolved around mechanisms that help reconcile empirically plausible amounts of micro price rigidity with the sluggish behavior of aggregate price measures. One important such mechanism are the so-called "real rigidities" – a term due to Ball and Romer (1990). Large real rigidities reduce the sensitivity of individual prices to aggregate demand conditions, and thus serve as a source of *endogenous persistence*: for a given degree of nominal price rigidity, they make the response of the aggregate price level to shocks more sluggish.[33] A natural question is whether real rigidities can help our model generate realistic real exchange rate dynamics with an amount of exogenous persistence that produces empirically plausible dynamics for nominal exchange rates.

To keep our model simple, in this paper we deliberately abstract from well-known sources of real rigidity, such as factor-market segmentation and technological input-output linkages among price-setting firms. However, the existence of decreasing returns in the production function is a potential source of real rigidity in the model. As long as there are decreasing returns, real rigidities can be strengthened by increasing the elasticity of substitution between same-country varieties of

[33] The closed-economy literature has also explored the role of information frictions as a way to generate more sluggish price dynamics. Crucini et al. (2010) apply this idea to study the dynamics of sectoral real exchange rates in a model with both sticky prices and sticky information.

intermediate goods.[34] Thus, a simple way to assess the implications of explicitly modeling other sources of real rigidity is to explore this mechanism in our model.

To that end, we do the following exercise. Starting from the simple model of Section 3, we set $\rho_z = 0.35$, fix the degree of decreasing returns to scale as in our baseline parameterization ($\chi = 2/3$), and then strengthen real rigidities in the model by increasing the elasticity of substitution between same-country varieties of intermediate goods. We target the same level of aggregate real exchange rate persistence as in our baseline parameterization, as measured by the half-life of deviations from PPP. We then use the same parameterization to produce comparable results for the counterfactual one-sector world economy. We stress that this exercise is meant to illustrate the likely effects of introducing other sources of real rigidity in the model, and should be interpreted accordingly.

The results are summarized in Table 5.[35] Although there are some quantitative differences relative to our baseline results, the main conclusions of our paper remain intact.[36] Our findings contrast with those of Kehoe and Midrigan (2007), who also explore the role of real rigidities in an extension to their baseline model. They argue that even "extreme" real rigidities fail to bring the model close to the data. However, they continue to impose the $\rho_z = 0$ restriction in their extended model. When we set $\rho_z = 0$ in our exercise to assess the effects of real rigidities, we also find that the model performs relatively poorly. In particular, the half-life of the aggregate real exchange rate in the multi-sector economy drops to 21 months, while the half-life in the counterfactual one-sector world economy drops to only 7 months. Moreover, setting $\rho_z = 0$ produces counterfactually low persistence for changes in the (log) nominal exchange rate.

Table 5: Results with real rigidities

Persistence measures:	$\mathcal{P}(q)$	$\mathcal{P}(q^{1\,\mathrm{sec}})$
$\mathcal{CIR}$	56.3	17.1
$\mathcal{SAC}$	0.98	0.94
$\mathcal{LAR}$	0.97	0.92
ρ_1	0.98	0.95
$\mathcal{HL}$	39	12
$\mathcal{UL}$	12	4
$\mathcal{QL}$	70	21
Volatility measure:	$\mathcal{V}(q)^{1/2}$	$\mathcal{V}(q^{1\,\mathrm{sec}})^{1/2}$
	0.022	0.012

To illustrate the sensitivity of the results in the economy with increased real rigidities to the level of exogenous nominal aggregate demand persistence, in Figure 5 we present results analogous

[34] For a detailed discussion of sources of real rigidities, and in particular of the role of decreasing returns to scale, see Woodford (2003, chapter 3).

[35] These results obtain with a value of the elasticities of substitution between varieties of $\theta = \eta = 30$. In the standard closed-economy model (e.g. Woodford, chapter 3), this parameter configuration implies a Ball and Romer (1990) index of real rigidities of roughly 0.063. The statistics presented in the table are calculated as detailed in footnote 14.

[36] As a consistency check, we compute the first-order autocorrelation of the log-change in the nominal exchange rate in each simulation of the model, and average across the 150 simulations. This results in 0.33, which compares with a cross-country average of 0.325 in the Imbs et al. (2005a) data used in our empirical analysis.

to Figure 4, and plot the level of aggregate real exchange rate persistence as a function of ρ_z. The results from the exercise of this section uncover an important interaction between heterogeneity in price stickiness, exogenous persistence, and endogenous persistence due to real rigidities. This follows from three observations, which are clear in the comparison of Figures 4 and 5: i) for a given amount of exogenous persistence and real rigidities, the counterfactual one-sector model produces less real exchange rate persistence than the multi-sector world economy; ii) despite the presence of real rigidities, both the multi-sector and the one-sector models perform poorly in the absence of *some* exogenous persistence; finally, iii) in the absence of real rigidities the multi-sector model can produce realistic aggregate real exchange rate dynamics under a higher level of exogenous persistence – but at the expense of producing counterfactual implications for the dynamic properties of the nominal exchange rate.

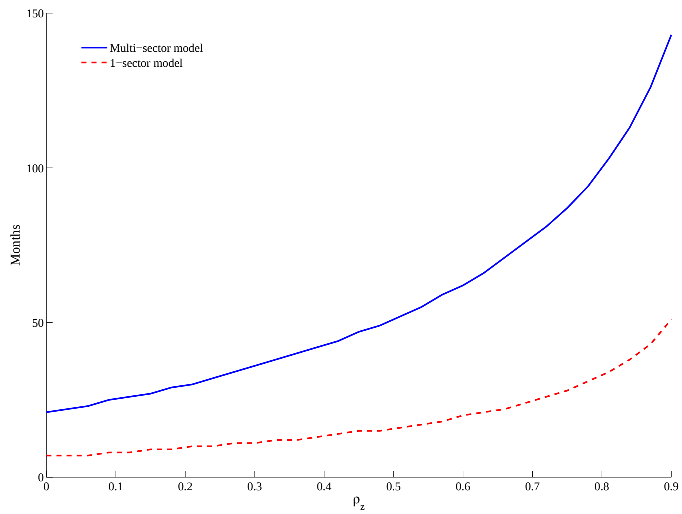

Figure 5: Half-life as a function of nominal aggregate demand persistence - parameterization with increased real rigidities

In principle we could also look at the sectoral implications of our real rigidity exercise. However, this cannot be done as simply as in Subsection 6.3, because in the parameterization of the current section, sectoral real exchange rates no longer follow the simple processes derived in **Proposition 1**. As we mentioned previously, equilibrium real exchange rate dynamics depend on the whole cross-sectional distribution of price stickiness and cannot be written explicitly as a function of model parameters. Thus, inferring price rigidity from the estimates of reduced-form autoregressive coefficients requires considering all sectoral real exchange rates jointly, and taking a stance on the values of all remaining structural parameters.[37] Nevertheless, we note incidentally that adding real rigidities to the simple model of Section 3 increases the persistence and volatility of sectoral real exchange rates in sectors where prices are more flexible relative to sectors in which prices are more

[37] We believe this endeavor is worth undertaking. However, it brings the exercise close to a structural estimation of the full model using sectoral real exchange rate data. This is beyond the scope of this paper, which focuses on aggregate real exchange rate dynamics.

sticky. Thus, adding real rigidities should definitely bring the model closer to the data along these dimensions.

We conclude by stressing that the exercise of this section is not meant to provide a definitive assessment of the effects of introducing other sources of real rigidity in the model. Rather, we use it to show that real rigidities allow our multi-sector model to match the degree of real exchange rate persistence seen in the data with less exogenous persistence. Thus, from the point of view of aggregate real exchange rate dynamics, the extra amount of exogenous persistence assumed in our baseline parameterization can be seen as a reduced-form for sources of endogenous persistence that were deliberately abstracted from in order to keep the model simple.

8 Conclusion

We show that a multi-sector model with heterogeneity in price stickiness parameterized to match the microeconomic evidence on price setting in the U.S. economy can produce much more volatile and persistent aggregate real exchange rates than a counterfactual one-sector version of the model that features the same average frequency of price changes. Nevertheless, despite the success in producing empirically plausible aggregate real exchange rate dynamics, our results still leave open a series of important research questions.

In our baseline parameterization of the model, as in the data, aggregate and sectoral real exchange rates are quite persistent, even for sectors in which prices change somewhat frequently. This uniformity in persistence is partly due to nominal aggregate demand persistence. However, our results suggest that the latter is only part of the story. This highlights the importance of investigating further the reasons for persistence being somewhat uniformly high across sectors. The exercise of adding real rigidities to our simple model suggests that this is a direction worth pursuing.

For analytical tractability, in this paper we model price stickiness as in Calvo (1983), and assume that the sectoral frequencies of price adjustment are constant. In closed economies, heterogeneity in price setting has similar aggregate effects in a much larger class of sticky-price (and sticky-information) models (Carvalho and Schwartzman 2008). While these results suggest that the nature of nominal frictions is not a crucial determinant of the effects of heterogeneity, it seems worthwhile to assess whether our results for real exchange rates do in fact hold in models with different nominal frictions. In particular, one such class of models involves endogenous, optimal pricing strategies, chosen in the face of explicit information and/or adjustment costs.[38] The importance of our assumption of local-currency pricing and, more generally, the stability of our findings across different policy regimes can also be assessed with models that feature fully endogenous pricing decisions, along the lines of Gopinath et al. (2010).

Another important line of investigation refers to the source of heterogeneity in sectoral exchange

[38] More specifically, menu-cost models, models with information frictions as in, e.g., Reis (2006), and models with both adjustment and information frictions as in, e.g., Bonomo and Carvalho (2004, 2010), Bonomo et al. (2010), and Gorodnichenko (2008).

rate dynamics. While we emphasize heterogeneity in price stickiness, an additional, potentially important source of heterogeneity is variation in the dynamic properties of sectoral shocks. It has been emphasized in recent work on the dynamics of international relative prices (e.g. Ghironi and Melitz 2005, and Atkeson and Burstein 2008), but, to our knowledge, a quantitative analysis in the context of the PPP puzzle has yet to be undertaken.

Finally, while it is a strength that our model can produce significant volatility and persistence in real exchange rates in response to different types of nominal disturbances, it would be interesting to introduce a richer set of shocks into the model, and analyze in more detail the differences between unconditional results and those conditional on particular shocks.[39] Combined with an empirical strategy that allows one to estimate the dynamic response of real exchange rates to identified shocks in the data, this richer model would likely deepen our understanding of real exchange rate dynamics.

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A Appendix

A.1 Proofs of propositions, corollaries, and lemmas

Proposition 1 *Under the assumptions of Section 3, sectoral real exchange rates follow* $AR(2)$ *processes:*

$$(1-\rho_z L)(1-\lambda_k L)\, q_{k,t} = \varphi_k u_t,$$

where $\lambda_k \equiv 1-\alpha_k$ *is the per-period probability of no price adjustment for a firm in sector* k*,* $u_t \equiv \sigma_{\varepsilon_z}\left(\varepsilon_{z,t} - \varepsilon^*_{z,t}\right)$ *is a white noise process,* $\varphi_k \equiv \lambda_k - (1-\lambda_k)\frac{\rho_z \beta \lambda_k}{1-\rho_z\beta\lambda_k}$*, and* L *is the lag operator.*

Proof. From the optimal-price equations:

$$\begin{aligned}
x_{H,k,t} &= (1-\beta(1-\alpha_k))\, E_t \sum\nolimits_{s=0}^{\infty} \beta^s (1-\alpha_k)^s \left[c_{t+s} + p_{t+s}\right] \\
&= (1-\beta(1-\alpha_k))\, E_t \sum\nolimits_{s=0}^{\infty} \beta^s (1-\alpha_k)^s z_{t+s} \\
&= z_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}(z_t - z_{t-1}),
\end{aligned}$$

and analogously:

$$\begin{aligned}
x_{F,k,t} &= z_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}(z_t - z_{t-1}), \\
x^*_{H,k,t} &= z^*_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}\left(z^*_t - z^*_{t-1}\right), \\
x^*_{F,k,t} &= z^*_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}\left(z^*_t - z^*_{t-1}\right).
\end{aligned}$$

This implies that the country-sector price indices follow:

$$\begin{aligned}
p_{H,k,t} &= (1-\alpha_k)\, p_{H,k,t-1} + \alpha_k \left(z_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}(z_t - z_{t-1})\right), \\
p_{F,k,t} &= (1-\alpha_k)\, p_{F,k,t-1} + \alpha_k \left(z_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}(z_t - z_{t-1})\right),
\end{aligned}$$

$$\begin{aligned}
p^*_{H,k,t} &= (1-\alpha_k)\, p^*_{H,k,t-1} + \alpha_k \left(z^*_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}\left(z^*_t - z^*_{t-1}\right)\right), \\
p^*_{F,k,t} &= (1-\alpha_k)\, p^*_{F,k,t-1} + \alpha_k \left(z^*_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}\left(z^*_t - z^*_{t-1}\right)\right),
\end{aligned}$$

and that sectoral price indices evolve according to:

$$\begin{aligned}
p_{k,t} &= (1-\alpha_k)\, p_{k,t-1} + \alpha_k \left[z_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}(z_t - z_{t-1})\right], \\
p^*_{k,t} &= (1-\alpha_k)\, p^*_{k,t-1} + \alpha_k \left[z^*_t + \frac{\rho_z \beta (1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z}\left(z^*_t - z^*_{t-1}\right)\right].
\end{aligned}$$

Therefore, sectoral real exchange rates follow:

$$\begin{aligned} q_{k,t} &= e_t + p^*_{k,t} - p_{k,t} \\ &= e_t + \alpha_k \begin{pmatrix} z^*_t - z_t \\ +\frac{\rho_z \beta(1-\alpha_k)}{1-\beta(1-\alpha_k)\rho_z} \left(\Delta z^*_t - \Delta z_t\right) \end{pmatrix} + (1-\alpha_k)\, q_{k,t-1} - (1-\alpha_k)\, e_{t-1}. \end{aligned} \tag{27}$$

In turn, the nominal exchange rate can be written as:

$$e_t = q_t + p_t - p^*_t = c_t - c^*_t + p_t - p^*_t = z_t - z^*_t. \tag{28}$$

Substituting (28) into (27) and simplifying yields:

$$q_{k,t} = (1-\alpha_k)\, q_{k,t-1} + \left(1 - \alpha_k - \alpha_k \frac{\rho_z \beta (1-\alpha_k)}{1 - \beta (1-\alpha_k) \rho_z}\right) \Delta e_t.$$

Finally, note that the nominal exchange rate evolves according to:

$$\begin{aligned} e_t &= z_t - z^*_t \\ &= (1+\rho_z)\left(z_{t-1} - z^*_{t-1}\right) - \rho_z \left(z_{t-2} - z^*_{t-2}\right) + \sigma_{\varepsilon_z}\left(\varepsilon_{z,t} - \varepsilon^*_{z,t}\right) \\ &= (1+\rho_z)\, e_{t-1} - \rho_z e_{t-2} + \sigma_{\varepsilon_z}\left(\varepsilon_{z,t} - \varepsilon^*_{z,t}\right), \end{aligned}$$

so that:

$$\Delta e_t = \rho_z \Delta e_{t-1} + u_t,$$

where $u_t \equiv \sigma_{\varepsilon_z}\left(\varepsilon_{z,t} - \varepsilon^*_{z,t}\right)$ is a white noise process. As a result, we can write:

$$(1-\rho_z L)(1-\lambda_k L)\, q_{k,t} = \varphi_k u_t,$$

where $\lambda_k \equiv 1 - \alpha_k$, and $\varphi_k \equiv \lambda_k - (1-\lambda_k)\frac{\rho_z \beta \lambda_k}{1-\rho_z \beta \lambda_k}$. ■

Corollary 1 *The aggregate real exchange rate follows an* $ARMA\,(K+1, K-1)$ *process:*

$$(1-\rho_z L) \prod_{k=1}^{K} (1-\lambda_k L)\, q_t = \left[\sum\nolimits_{k=1}^{K} \prod_{j \neq k}^{K} (1-\lambda_j L)\, f_k \varphi_k\right] u_t.$$

Proof. This is a standard result in aggregation of time-series processes (Granger and Morris 1976). The aggregate real exchange rate is given by:

$$q_t = \sum\nolimits_{k=1}^{K} f_k q_{k,t}.$$

From the result of **Proposition 1**, multiply each sectoral real exchange rate equation by its respective sectoral weight to obtain:

$$f_k (1-\rho_z L)(1-\lambda_k L)\, q_{k,t} = f_k \varphi_k u_t.$$

Multiplying each such equation by all $(K-1)$ L-polynomials of the form $(1-\lambda_m L)$, $m \neq k$ and adding them up yields:

$$(1-\rho_z L)\prod_{k=1}^{K}(1-\lambda_k L)\, q_t = \left[\sum\nolimits_{k=1}^{K}\prod_{m\neq k}^{K}(1-\lambda_m L)\, f_k\varphi_k\right] u_t,$$

so that q_t follows an $ARMA\,(K+1, K-1)$. ■

Corollary 2 *The aggregate real exchange rate of the counterfactual one-sector world economy follows an $AR\,(2)$ process:*

$$(1-\rho_z L)\left(1-\overline{\lambda}L\right) q_t^{1\,\mathrm{sec}} = \varphi u_t,$$

where $\overline{\lambda} \equiv \sum_{k=1}^{K} f_k\lambda_k$ *and* $\varphi \equiv \overline{\lambda} - \left(1-\overline{\lambda}\right)\frac{\rho_z\beta\overline{\lambda}}{1-\rho_z\beta\overline{\lambda}}$.

Proof. From **Corollary 1**, the real exchange rate in a one-sector world economy with frequency of price changes equal to $\overline{\alpha}$ – probability of no-adjustment equal to $\overline{\lambda} = 1-\overline{\alpha}$ – follows:

$$(1-\rho_z L)\left(1-\overline{\lambda}L\right) q_t = \left(1-\overline{\lambda}L\right)\left(\overline{\lambda} - \left(1-\overline{\lambda}\right)\frac{\rho_z\beta\overline{\lambda}}{1-\rho_z\beta\overline{\lambda}}\right) u_t.$$

■

Proposition 2 *For the measures of persistence $\mathcal{P} = \mathcal{CIR}$,$\mathcal{LAR}$,$\mathcal{SAC}$:*

$$\mathcal{P}\,(q) > \mathcal{P}\left(q^{1\,\mathrm{sec}}\right).$$

Proof. We prove separate results for each measure of persistence.

$\mathcal{CIR}$:

Recall that we denote the impulse response function of the q_t process to a unit impulse by $IRF_t\,(q)$. In turn, let $SIRF_t\,(q)$ denote the "scaled impulse response function," i.e. the impulse response function to one-standard-deviation shock. Since $q_t = \sum_{k=1}^{K} f_k q_{k,t}$, $SIRF_t\,(q) = \sum_{k=1}^{K} f_k SIRF_t\,(q_k)$. So, the impulse response function of the q_t process to a unit impulse, which is simply the scaled impulse response function normalized by the initial impact of the shock, can be written as:

$$IRF_t\,(q) = \frac{\sum_{k=1}^{K} f_k SIRF_t\,(q_k)}{\sum_{k=1}^{K} f_k SIRF_0\,(q_k)}. \tag{29}$$

From (29), the cumulative impulse response for q_t is:

$$\mathcal{CIR}\,(q) = \sum\nolimits_{t=0}^{\infty} IRF_t\,(q) = \frac{\sum_{k=1}^{K} f_k \sum_{t=0}^{\infty} SIRF_t\,(q_k)}{\sum_{k=1}^{K} f_k SIRF_0\,(q_k)}. \tag{30}$$

From the processes in **Proposition 1** we can compute $\sum_{t=0}^{\infty} SIRF_t\left(q_k\right)$, and $SIRF_0\left(q_k\right)$:

$$\sum\nolimits_{t=0}^{\infty} SIRF_t\left(q_k\right) = \frac{\lambda_k\left(1-\rho_z\beta\right)}{1-\rho_z\beta\lambda_k}\frac{1}{\left(1-\lambda_k\right)\left(1-\rho_z\right)}, \tag{31}$$

$$SIRF_0\left(q_k\right) = \frac{\lambda_k\left(1-\rho_z\beta\right)}{1-\rho_z\beta\lambda_k}. \tag{32}$$

Substituting (31) and (32) into (30) yields:

$$\mathcal{CIR}\left(q\right) = \frac{\sum_{k=1}^{K} f_k \frac{\lambda_k(1-\rho_z\beta)}{1-\rho_z\beta\lambda_k}\frac{1}{(1-\lambda_k)(1-\rho_z)}}{\sum_{k=1}^{K} f_k \frac{\lambda_k(1-\rho_z\beta)}{1-\rho_z\beta\lambda_k}}.$$

Note that $\frac{\lambda_k(1-\rho_z\beta)}{1-\rho_z\beta\lambda_k}$ is increasing in λ_k, so that $\widetilde{f}_k \equiv \frac{f_k \frac{\lambda_k(1-\rho_z\beta)}{1-\rho_z\beta\lambda_k}}{\sum_{k=1}^{K} f_k \frac{\lambda_k(1-\rho_z\beta)}{1-\rho_z\beta\lambda_k}}$ are sectoral weights obtained through a transformation of f_k, which attaches higher weight to higher λ_ks. The fact that $\frac{1}{(1-\lambda_k)(1-\rho_z)}$ is also increasing, and moreover convex, in λ_k, thus implies the following inequalities:

$$\underbrace{\sum\nolimits_{k=1}^{K} \widetilde{f}_k \frac{1}{\left(1-\lambda_k\right)\left(1-\rho_z\right)}}_{\mathcal{CIR}(q)} > \underbrace{\sum\nolimits_{k=1}^{K} f_k \frac{1}{\left(1-\lambda_k\right)\left(1-\rho_z\right)}}_{\sum_{k=1}^{K} f_k \mathcal{CIR}(q_k)} > \underbrace{\frac{1}{\sum_{k=1}^{K} f_k\left(1-\lambda_k\right)\left(1-\rho_z\right)}}_{\mathcal{CIR}(q^{1\,\mathrm{sec}})}. \tag{33}$$

This proves that $\mathcal{CIR}\left(q\right) > \mathcal{CIR}\left(q^{1\,\mathrm{sec}}\right)$.

$\mathcal{LAR}$:

We order the sectors in terms of price stickiness, starting from the most flexible: $\alpha_k > \alpha_{k+1}$ $\left(\lambda_k < \lambda_{k+1}\right)$. Moreover, recall that we assume $\rho_z \in \left(1-\alpha_1, 1-\alpha_K\right)$. Thus, based on **Proposition 1** and **Corollaries 1** and **2**, we obtain directly the following results:

$$\begin{aligned}
\mathcal{LAR}\left(q\right) &= \lambda_K, \\
\mathcal{LAR}\left(q_k\right) &= \max\left\{\lambda_k, \rho_z\right\}, \\
\mathcal{LAR}\left(q^{1\,\mathrm{sec}}\right) &= \max\left\{\overline{\lambda}, \rho_z\right\} = \max\left\{\sum\nolimits_{k=1}^{K} f_k\lambda_k, \rho_z\right\}.
\end{aligned}$$

Therefore:

$$\mathcal{LAR}\left(q\right) > \sum\nolimits_{k=1}^{K} f_k \mathcal{LAR}\left(q_k\right) > \mathcal{LAR}\left(q^{1\,\mathrm{sec}}\right). \tag{34}$$

$\mathcal{SAC}$:

From **Corollary 1**:

$$\begin{aligned}
\mathcal{SAC}(q) &= 1-(1-\rho_z)\prod_{k=1}^{K}(1-\lambda_k) \\
&> 1-(1-\rho_z)\prod_{k=1}^{K}(1-\lambda_k)^{f_k} \\
&> 1-(1-\rho_z)\sum_{k=1}^{K} f_k(1-\lambda_k) \\
&= \sum_{k=1}^{K} f_k(1-(1-\rho_z)(1-\lambda_k)) = \sum_{k=1}^{K} f_k\mathcal{SAC}(q_k) \\
&= 1-(1-\rho_z)\left(1-\sum_{k=1}^{K} f_k\lambda_k\right) \\
&= 1-(1-\rho_z)\left(1-\overline{\lambda}\right) = \mathcal{SAC}\left(q^{1\,\mathrm{sec}}\right).
\end{aligned} \tag{35}$$

■

Proposition 3 *Let $\mathcal{V}(q)$ denote the variance of the q_t process. Then:*

$$\mathcal{V}(q) > \mathcal{V}\left(q^{1\,\mathrm{sec}}\right).$$

Proof. We first construct an auxiliary $AR(K+1)$ process, which we denote by $\widetilde{\widetilde{q}}_t$, by dropping the moving average component of the process for the aggregate real exchange rate in the multi-sector economy (**Corollary 1):**

$$(1-\rho_z L)\prod_{k=1}^{K}(1-\lambda_k L)\,\widetilde{\widetilde{q}}_t = \sum_{k=1}^{K} f_k\varphi_k u_t.$$

It is clear that $\mathcal{V}(q) > \mathcal{V}\left(\widetilde{\widetilde{q}}\right)$. The next steps will show that $\mathcal{V}\left(\widetilde{\widetilde{q}}\right) > \mathcal{V}\left(q^{1\,\mathrm{sec}}\right)$, and thus establish the result. From **Corollary 2**, recall that the process for $q^{1\,\mathrm{sec}}$ is:

$$(1-\rho_z L)\left(1-\overline{\lambda}L\right) q_t^{1\,\mathrm{sec}} = \varphi u_t, \tag{36}$$

with $\overline{\lambda} \equiv \sum_{k=1}^{K} f_k\lambda_k$ and $\varphi \equiv \overline{\lambda} - \left(1-\overline{\lambda}\right)\frac{\rho_z\beta\overline{\lambda}}{1-\rho_z\beta\overline{\lambda}}$. Since φ_k is convex in λ_k ($\frac{\partial^2\varphi_k}{\partial\lambda_k^2} = \frac{2\beta\rho(\beta\rho-1)}{(\beta\lambda\rho-1)^3} > 0$), $\sum_{k=1}^{K} f_k\varphi_k > \varphi$. Thus, defining another auxiliary process $\widetilde{q}$ such that:

$$(1-\rho_z L)\prod_{k=1}^{K}(1-\lambda_k L)\,\widetilde{q}_t = \varphi u_t, \tag{37}$$

it suffices to show that $\mathcal{V}(\widetilde{q}) > \mathcal{V}\left(q^{1\,\mathrm{sec}}\right)$. We consider two cases:

i) $\exists\, k' \in \{1,...,K\} \mid \lambda_{k'} = \overline{\lambda}$. Since $\lambda_k \geq 0$ for all k, it is easy to check that $\mathcal{V}(\widetilde{q}) > \mathcal{V}\left(q^{1\,\mathrm{sec}}\right)$.[40]

[40] The strict inequality comes from the fact that, as long as prices are sticky in at least one sector, for the average frequency of price changes to be equal to the frequency in one of the sectors, there must be at least one more sector in which prices are sticky.

This follows directly from the fact that in the $MA(\infty)$ representation of $\widetilde{q}$, each coefficient is equal to the corresponding coefficient in the $MA(\infty)$ representation of $q^{1\,\text{sec}}$ *plus positive terms* that originate from all the additional λ_k roots, $k \neq k'$.

ii) $\forall\, k \in \{1, ..., K\}$, $\lambda_k \neq \overline{\lambda}$. In that case $\exists\, k'' \in \{1, ..., K-1\} \mid \lambda_{k''} < \overline{\lambda} < \lambda_{k''+1}$. We construct an auxiliary process $\widetilde{q}^{1\,\text{sec}}$ such that:

$$(1-\rho_z L)(1-\lambda_{k''+1} L)\,\widetilde{q}_t^{1\,\text{sec}} = \varphi u_t,$$

and note that $\mathcal{V}\left(\widetilde{q}^{1\,\text{sec}}\right) = \frac{1+\rho\lambda_{k''+1}}{(1-\rho^2)(1-\rho\lambda_{k''+1})\left(1-\lambda_{k''+1}^2\right)} > \frac{1+\rho\overline{\lambda}}{(1-\rho^2)(1-\rho\overline{\lambda})\left(1-\overline{\lambda}^2\right)} = \mathcal{V}\left(q^{1\,\text{sec}}\right)$. Thus, the same argument as in case i) shows that $\mathcal{V}(\widetilde{q}) > \mathcal{V}\left(\widetilde{q}^{1\,\text{sec}}\right)$. This completes the proof. ■

Proposition 4 *Under the simplified model of Section 3, for the measures of persistence* $\mathcal{P} = \mathcal{CIR}$, $\mathcal{LAR}$*:*

$$\begin{aligned} \textit{aggregation effect under } \mathcal{P} &> 0, \\ \textit{counterfactuality effect under } \mathcal{P} &> 0. \end{aligned}$$

Proof. The proof is a by-product of the proof of **Proposition 2**, equations (33) and (34). ■

Proposition 5 *Under the assumptions of Section 3 and equal sectoral weights, application of the Mean Group estimator to the sectoral real exchange rates from the multi-sector world economy yields the dynamics of the real exchange rate in the corresponding counterfactual one-sector world economy.*

Proof. From **Proposition 1** sectoral exchange rates follow $AR(2)$ processes:

$$q_{k,t} = (\rho_z + \lambda_k)\, q_{k,t-1} - \rho_z \lambda_k q_{k,t-2} + \varphi_k u_t.$$

Applying the MG estimator to these processes yields $\rho_z + \frac{1}{K}\sum_{k=1}^{K} \lambda_k$ as the cross-sectional average of the first autoregressive coefficients, and $-\rho_z \frac{1}{K}\sum_{k=1}^{K} \lambda_k$ as the cross-sectional average of the second autoregressive coefficients. An application of **Corollary 2** to the case of equal sectoral weights shows that these are exactly the autoregressive coefficients of the $AR(2)$ process followed by the aggregate real exchange rate in the corresponding counterfactual one-sector world economy.
■

Lemma 1 *The measures of persistence* $\mathcal{P} = \mathcal{CIR}, \mathcal{LAR}, \mathcal{SAC}$ *are (weakly) increasing in the degree of sectoral price rigidity:*

$$\frac{\partial \mathcal{CIR}(q_k)}{\partial \lambda_k} > 0, \frac{\partial \mathcal{LAR}(q_k)}{\partial \lambda_k} \geq 0, \frac{\partial \mathcal{SAC}(q_k)}{\partial \lambda_k} > 0,$$

Moreover, the variance of sectoral real exchange rates is increasing in the degree of sectoral price rigidity:

$$\frac{\partial \mathcal{V}(q_k)}{\partial \lambda_k} > 0.$$

Proof. From the proof of **Proposition 2**, $\mathcal{CIR}(q_k) = \frac{1}{(1-\lambda_k)(1-\rho_z)}$, and $\mathcal{LAR}(q_k) = \max\{\lambda_k, \rho_z\}$.

From **Proposition 1**, $\mathcal{SAC}(q_k) = \rho_z + \lambda_k - \rho_z\lambda_k$. Direct differentiation of these three expressions with respect to λ_k proves the first part of the lemma. As for the variance of sectoral real exchange rates, standard time-series calculations yield:

$$\mathcal{V}(q_k) = \underbrace{\frac{1+\rho_z\lambda_k}{(1-\rho_z\lambda_k)\left((1+\rho_z\lambda_k)^2 - (\rho_z+\lambda_k)^2\right)}}_{\mathcal{V}_1(q_k)} \underbrace{\left(\lambda_k - (1-\lambda_k)\frac{\rho_z\beta\lambda_k}{1-\rho_z\beta\lambda_k}\right)^2}_{\mathcal{V}_2(q_k)} \sigma^2_{\varepsilon_z}.$$

Differentiating each of $\mathcal{V}_1(q_k)$ and $\mathcal{V}_2(q_k)$ with respect to λ_k yields:

$$\begin{aligned}
\frac{\partial \mathcal{V}_1(q_k)}{\partial \lambda_k} &= 2\frac{\rho_z\lambda_k^2 + \rho_z^2\lambda_k^3 - \rho_z - \lambda_k}{\left(\lambda_k^2-1\right)^2(\rho_z\lambda_k-1)^2(\rho_z^2-1)} > 0 \\
\frac{\partial \mathcal{V}_2(q_k)}{\partial \lambda_k} &= -2\frac{\lambda_k(\rho_z-1)^2}{(\rho_z\lambda_k-1)^3} > 0.
\end{aligned}$$

Since $\mathcal{V}_1(q_k)$, $\mathcal{V}_2(q_k) > 0$, application of the product rule yields:

$$\frac{\partial \mathcal{V}(q_k)}{\partial \lambda_k} = \left(\frac{\partial \mathcal{V}_1(q_k)}{\partial \lambda_k}\mathcal{V}_2(q_k) + \frac{\partial \mathcal{V}_2(q_k)}{\partial \lambda_k}\mathcal{V}_1(q_k)\right)\sigma^2_{\varepsilon_z} > 0.$$

■

A.2 A limiting result

We show that a "suitably heterogeneous" multi-sector world economy can generate an aggregate real exchange rate that is arbitrarily more volatile and persistent than the real exchange rate in the counterfactual one-sector world economy.[41] We consider the effects of progressively adding more sectors, and assume that the frequency of price changes for each new sector is drawn from $(0, 1-\delta)$ for arbitrarily small $\delta > 0$, according to some distribution with density $g(\alpha|b)$, where α is the frequency of price changes and $b > 0$ is a parameter. For $\alpha \approx 0$ such density is assumed to be approximately proportional to α^{-b}, with $b \in \left(\frac{1}{2}, 1\right)$.[42] The shape of this distribution away from zero need not be specified. It yields a strictly positive average frequency of price changes: $\overline{\alpha} = \int_0^{1-\delta} g(\alpha|b)\,\alpha d\alpha > 0$. We prove the following:

Proposition 6 *Under the assumptions above:*

$$\begin{aligned}
\mathcal{V}\left(\frac{1}{K}\sum\nolimits_{k=1}^{K} q_{k,t}\right) &\underset{K\to\infty}{\longrightarrow} \infty, \\
CIR\left(\frac{1}{K}\sum\nolimits_{k=1}^{K} q_{k,t}\right) &\underset{K\to\infty}{\longrightarrow} \infty, \\
\mathcal{V}\left(q^{1\,\mathrm{sec}}\right), CIR\left(q^{1\,\mathrm{sec}}\right) &< \infty.
\end{aligned}$$

[41] We build on the work of Granger (1980), Granger and Joyeux (1980), and Zaffaroni (2004).

[42] Thus, we approximate a large number of potential new sectors by a continuum, and replace the general f_k distribution by this semi-parametric specification for $g(\alpha|b)$, based on Zaffaroni (2004). An example of a parametric distribution that satisfies this restriction is a Beta distribution with suitably chosen support and parameters.

Proof. We start with the case of $\rho_z = 0$. For each $q_{k,t}$ process $(1-\lambda_k L)\, q_{k,t} = \varphi_k u_t$ with $\varphi_k \equiv \lambda_k$ and $\alpha_k = 1-\lambda_k$ drawn from $g(\alpha|b)$, define an auxiliary $\widetilde{q}_{k,t}$ process satisfying:

$$(1-\lambda_k L)\, \widetilde{q}_{k,t} = \widetilde{\varphi} u_t,$$

where $\widetilde{\varphi} < \delta$ is a constant. Since $\widetilde{\varphi}$ is independent of λ_k, these $\widetilde{q}_{k,t}$ processes satisfy the assumptions in Zaffaroni (2004), and application of his Theorem 4 yields:

$$\mathcal{V}\left(\frac{1}{K}\sum\nolimits_{k=1}^{K} \widetilde{q}_{k,t}\right) \underset{K\to\infty}{\longrightarrow} \infty.$$

Since the α_k's have support $(0, 1-\delta)$ for small $\delta > 0$, $\mathcal{V}\left(\frac{1}{K}\sum_{k=1}^{K} q_{k,t}\right) > \mathcal{V}\left(\frac{1}{K}\sum_{k=1}^{K} \widetilde{q}_{k,t}\right)$, which proves that $\mathcal{V}\left(\frac{1}{K}\sum_{k=1}^{K} q_{k,t}\right) \underset{K\to\infty}{\longrightarrow} \infty$. Analogously, application of Zaffaroni's (2004) result to the spectral density of the limiting process at frequency zero shows that it is unbounded. In turn, the fact that the spectral density at frequency zero for $AR(p)$ processes is an increasing monotonic transformation of the cumulative impulse response (e.g., Andrews and Chen 1994) implies that $CIR\left(\frac{1}{K}\sum_{k=1}^{K} q_{k,t}\right) \underset{K\to\infty}{\longrightarrow} \infty$. The results for the real exchange rate in the limiting counterfactual one-sector world economy follow directly from the fact that $\overline{\alpha} = \int_0^{1-\delta} g(\alpha|b)\, \alpha d\alpha > 0$, so that it follows a stationary $AR(1)$ process. Finally, Zaffaroni's (2004) extension of his results to $ARMA(p,q)$ processes implies that **Proposition 6** also holds for $\rho_z > 0$. ■

The results in **Proposition 6** follow from the fact that, under suitable assumptions, the aggregate real exchange rate converges to a non-stationary process. It inherits some features of unit-root processes, such as infinite variance and persistence, due to the relatively high density of very persistent sectoral real exchange rates embedded in the distributional assumption for the frequencies of price changes. However, the process does not have a unit root, since none of the sectoral exchange rates actually has one. Moreover, the limiting process remains mean reverting in the sense that its impulse response function converges to zero as $t \longrightarrow \infty$.[43] In contrast, since $\overline{\alpha} > 0$, the limiting process for the real exchange rate in the counterfactual one-sector world economy remains stationary, and as such it has both finite variance and persistence.

B Robustness

B.1 Strategic neutrality in price setting

As a first robustness check of our parameterization, we redo the quantitative analysis imposing the restrictions on parameter values that underscore our analytical results from Section 3.[44] That is, we look at the quantitative implications of our model in the case of strategic neutrality in price

[43] Such properties characterize the so-called *fractionally integrated processes*. See, for example, Granger and Joyeux (1980).

[44] Recall that these are $\sigma = 1$, $\gamma = 0$, and $\chi = 1$. Under these assumptions, the additional structural parameters have no effect on the dynamics of real exchange rates.

setting.

The outcomes of the models are summarized in Table 6. Note that in this case the results are exact, since we know the processes followed by each of the variables from **Proposition 1,** and **Corollaries 1** and **2**.[45] Despite the change in the parameterization, the essence of our results is not affected: the aggregate real exchange rate in the heterogeneous economy is still more volatile and persistent than in the counterfactual one-sector world economy.

Table 6: Results under Strategic Neutrality in Price Setting

Persistence measures:	$\mathcal{P}(q)$	$\mathcal{P}\left(q^{1\,\mathrm{sec}}\right)$
$\mathcal{CIR}$	79.8	23.7
$\mathcal{SAC}$	$\simeq 1$	0.96
$\mathcal{LAR}$	.98	0.80
ρ_1	0.99	0.97
$\mathcal{HL}$	44	16
$\mathcal{UL}$	29	10
$\mathcal{QL}$	59	20
Volatility measure:	$\mathcal{V}(q)^{1/2}$	$\mathcal{V}\left(q^{1\,\mathrm{sec}}\right)^{1/2}$
	0.03	0.01

B.2 Interest-rate rule and different shocks

We consider a specification with an explicit description of monetary policy, and later also add productivity shocks. We assume that in each country monetary policy is conducted according to an interest-rate rule subject to persistent shocks:

$$I_t = \beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi}\left(\frac{GDP_t}{GDP_t^n}\right)^{\phi_Y} e^{\upsilon_t},$$

where I_t is the short-term nominal interest rate in Home, GDP_t is gross domestic product, GDP_t^n denotes gross domestic product when all prices are flexible, ϕ_π and ϕ_Y are the parameters associated with Taylor-type interest-rate rules, and υ_t is a persistent shock with process $\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \sigma_{\varepsilon_\upsilon}\varepsilon_{\upsilon,t}$, where $\varepsilon_{\upsilon,t}$ is a zero-mean, unit-variance *i.i.d.* shock, and $\rho_\upsilon \in [0,1)$. The policy rule in Foreign is analogous, and we assume that the shocks are uncorrelated across countries. We set $\phi_\pi = 1.5$, $\phi_y = .5/12$, and $\rho_\upsilon = 0.965$.[46] The remaining parameter values are unchanged from the baseline specification.

The results are presented in Table 7.[47] The model with heterogeneity still produces a significantly more volatile and persistent real exchange rate than the counterfactual one-sector world economy.

[45] The only exceptions are the first autocorrelation for the aggregate real exchange rate and the volatilities, which for simplicity are calculated through simulations, as outlined in footnote 14.

[46] Recall that the parameters are calibrated to the monthly frequency, and so this value for ρ_υ corresponds to an autoregressive coefficient of roughly 0.9 at a quarterly frequency. We specify the size of the shocks to be consistent with the estimates of Justiniano et al. (2010), and thus set the standard deviation to 0.2% at a quarterly frequency.

[47] We compute these statistics based on simulations, following the methodology outlined in footnote 14.

Table 7: Results under interest-rate rule

Persistence measures:	$\mathcal{P}(q)$	$\mathcal{P}\left(q^{1\,\text{sec}}\right)$
$\mathcal{CIR}$	49.6	28.4
$\mathcal{SAC}$	0.98	0.96
$\mathcal{LAR}$	0.96	0.91
ρ_1	0.98	0.96
$\mathcal{HL}$	39	20
$\mathcal{UL}$	14	0
$\mathcal{QL}$	60	39
Volatility measure:	$\mathcal{V}(q)^{1/2}$	$\mathcal{V}\left(q^{1\,\text{sec}}\right)^{1/2}$
	0.07	0.01

We also consider a version of the model with interest-rate and productivity shocks. We introduce the latter by changing the production function in (22) to:

$$Y_{H,k,j,t} + Y^*_{H,k,j,t} = A_t N^{\chi}_{k,j,t},$$

where A_t is a productivity shock. It evolves according to:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_{\varepsilon_A} \varepsilon_{A,t},$$

where $\rho_A \in [0,1)$ and $\varepsilon_{A,t}$ is a zero-mean, unit-variance *i.i.d.* shock. An analogous process applies to A^*_t, and once more we assume that the shocks are uncorrelated across countries.

We keep the same specification for the monetary policy rule, and set $\rho_A = 0.965$. To determine the relative size of the shocks we rely on the estimates obtained by Justiniano et al. (2010), and set $\sigma_{\varepsilon_\upsilon} = 0.12\%$, and $\sigma_{\varepsilon_A} = 0.52\%$. The remaining parameter values are unchanged from the baseline parameterization. The heterogeneous world economy still produces a significantly more volatile and persistent real exchange rate than the counterfactual one-sector world economy. The half-life of the aggregate real exchange rate in the multi-sector world economy is around 33.5 months, while in the counterfactual one-sector world economy it is around 19 months.

We also considered additional parameterizations. We found that the results with shocks to the interest-rate rule and productivity shocks are somewhat more sensitive to the details of the specification than under nominal aggregate demand shocks. On the one hand, they still hold under strategic neutrality in price setting. On the other, they are more sensitive to the source of persistence in the interest-rate rule – persistent shocks versus interest-rate smoothing.[48] Uncovering the reasons for such differences in results is an interesting endeavor for future research. In particular, it would be valuable to investigate the "demand block" of the model further. The reason is that the forward-looking "IS curve" that enters the demand side of the model has only weak empirical support (e.g., Fuhrer and Rudebusch 2004). Thus, in circumstances in which the model struggles to produce realistic real exchange rate dynamics, this should help us assess whether the problem

[48] Chari et al.(2002) find that their sticky-price model fails to generate reasonable business cycle behavior under a policy rule with interest-rate smoothing, in particular in terms of real exchange rate persistence.

originates in the nature of price setting – which is the focus of the paper – or in other parts of the model.

B.3 Additional sensitivity analysis

Our findings are robust to changes in the values of the elasticities of substitution between varieties of the intermediate goods, and in the share of imported goods. In particular, departing from our baseline parameterization we analyze the effects of increasing the value of the elasticity of substitution between Home and Foreign goods to as much as 10 (equal to the baseline value for the elasticity of substitution between varieties of the same sector in a given country), and the share of imported inputs to as much as 50%. Despite these extreme values, the half-life of deviations from PPP in the multi-sector model drops only modestly, to 31 months. We also analyze the sensitivity of our findings to changes in the time-discount factor β, and find only negligible effects. Finally, our conclusions are also robust to alternative aggregation schemes leading to different numbers of sectors in the heterogeneous economy, as in Carvalho and Nechio (2008).