FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

Real Exchange Rate Dynamics in Sticky-Price Models with Capital

Carlos Carvalho
PUC-Rio

Fernanda Nechio
Federal Reserve Bank of San Francisco

July 2012

Working Paper 2012-08
http://www.frbsf.org/publications/economics/papers/2012/wp12-08bk.pdf

Real Exchange Rate Dynamics in Sticky-Price Models with Capital*

Carlos Carvalho
PUC-Rio

Fernanda Nechio
Federal Reserve Bank of San Francisco

July, 2012

Abstract

The standard argument for abstracting from capital accumulation in sticky-price macro models is based on their short-run focus: over this horizon, capital does not move much. This argument is more problematic in the context of real exchange rate (RER) dynamics, which are very persistent. In this paper we study RER dynamics in sticky-price models with capital accumulation. We analyze both a model with an economy-wide rental market for homogeneous capital, and an economy in which capital is sector specific. We find that, in response to monetary shocks, capital increases the persistence and reduces the volatility of RERs. Nevertheless, versions of the multi-sector sticky-price model of Carvalho and Nechio (2011) augmented with capital accumulation can match the persistence and volatility of RERs seen in the data, irrespective of the type of capital. When comparing the implications of capital specificity, we find that, perhaps surprisingly, switching from economy-wide capital markets to sector-specific capital tends to *decrease* the persistence of RERs in response to monetary shocks. Finally, we study how RER dynamics are affected by monetary policy and find that the source of interest rate persistence – policy inertia or persistent policy shocks – is key.

JEL classification codes: F3, F41, E0

Keywords: real exchange rates, capital accumulation, factor mobility, multisector model, PPP puzzle

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of San Francisco or the Federal Reserve System. E-mails: cvianac@econ.puc-rio.br, fernanda.nechio@sf.frb.org.

1 Introduction

Most sticky-price macro models used for short-run analysis abstract from capital accumulation. This simplifying assumption is arguably reasonable, as in the short run capital does not move very much. When analyzing real exchange rate (RER) dynamics, however, this assumption is arguably less palatable. The "consensus view" in the empirical literature summarized by Rogoff (1996) is that departures of the RER from its average level have a half-life in the range of three to five years.[1] This is arguably not a short period of time when it comes to undertaking investments to adjust the capital stock. Meaningful changes in the capital stock might lead to important changes in the dynamic properties of the economy, and invalidate lessons based on models that abstract from capital accumulation.

In this paper we study RER dynamics in sticky-price models with capital accumulation by adding this feature to the two-country multisector model of Carvalho and Nechio (2011). The motivations for this choice are twofold. First, we want to start from a model that, in the absence of capital accumulation, produces empirically plausible RER dynamics. That paper shows that this is achieved when the model economy features multiple sectors, with heterogeneity in the degree of price rigidity that accords with the microeconomic evidence from the recent empirical literature on price setting in the U.S. economy. Second, the multisector nature of the framework allows us to introduce different types of capital in the model, and to study the implications of factor-market segmentation. In particular, we study both a model with an economy-wide rental market for homogeneous capital, and an economy in which capital is *sector specific*,[2] in line with the empirical evidence provided in the seminal contribution of Ramey and Shapiro (2001).

When comparing models with and without capital accumulation, we find that capital reduces the volatility of RER relative to Gross Domestic Product (GDP) and increases persistence in response to monetary shocks. These effects are intuitive and arise from the role of capital as an endogenous state variable that allows agents to better smooth consumption (Martinez-Garcia and Søndergaard 2010).

We find that adding homogeneous capital to the multisector sticky-price model of Carvalho and Nechio (2011) does not change its main findings. In particular, in response to monetary shocks the model can still match the persistence and volatility of RERs seen in the data. In contrast, versions of our model with only one sector (i.e., with homogeneous price rigidity) fail to produce enough RER persistence in response to nominal disturbances.

Turning to the implications of different types of capital, we find that switching from homogeneous

[1] Empirical research on RER dynamics devotes a large amount of effort to quantifying the uncertainty around estimates of persistence. Rogoff's consensus view itself is subject to criticism on these grounds. We refer readers interested in those questions to, e.g., Murray and Papell (2002), Kilian and Zha (2002), and Rossi (2005).

[2] Such specificity only matters when sectors differ in some dimensions (e.g., heterogeneity in price stickiness, sector-specific shocks, etc). If sectors are identical, this type of factor market segmentation becomes irrelevant for the purposes of our analysis, since there is no need for capital to move across sectors in response to aggregate shocks in order to equalize its marginal rates of return.

to sector-specific capital tends to *decrease* the persistence of RERs in response to monetary shocks. This effect might seem counterintuitive, in light of the results obtained by Woodford (2005) in a model with firm-specific capital. He finds that capital specificity *increases* the persistence of the real effects of nominal disturbances. Our results show that it matters a great deal whether capital is sector or firm specific. Woodford (2005) shows that firm-specificity strengthens the degree of strategic complementarities in firms' pricing decisions, and this is what leads to higher persistence in his model – when compared to an otherwise identical model with homogeneous capital. In contrast, specificity at the sectoral level induces *strategic substitutability* in pricing decisions within sectors. The reason is that, in this case, factor prices are determined by supply and demand in each sector, and marginal costs are equalized across all firms in the same sector. Thus, if prices in a given sector *increase*, all else equal, the demand for its varieties decreases, putting downward pressure on sectoral factor prices and lowering marginal costs for all firms in that sector. As a result, firms that get to change their prices will wish to *cut* prices – i.e., to move in the opposite direction of the initial sectoral price change. Despite the fact that sectoral capital specificity tends to lower RER persistence, we find that our multisector model with heterogeneous price stickiness can still match the persistence and volatility of RERs seen in the data.

The results discussed so far are based on models in which we postulate an exogenous stochastic process for nominal aggregate demand and leave monetary policy implicit, as in Carvalho and Nechio (2011). Our next step is to study the role of monetary policy. First, we analyze RER dynamics when policy is conducted according to an interest rate rule that is similar to the original Taylor (1993) rule and is subject to persistent shocks. In that case, our findings are essentially unchanged. Motivated by the debate about the source of interest-rate persistence – persistent shocks versus policy inertia (e.g., Rudebusch 2002, Coibion and Gorodnichenko 2012) – we then consider a policy rule with interest rate smoothing. We find that the source of interest rate persistence matters a great deal. If the policy rule followed by the monetary authorities has too strong an interest rate smoothing component, even our multisector sticky-price model fails to generate enough RER persistence in response to monetary shocks.

The reason why monetary policy matters so much can be traced back to Steinsson (2008). He argues that the ability of a model to produce hump-shaped RER dynamics is critical to matching the degree of persistence seen in the data. He then uses a standard sticky-price model to conclude that some shocks induce such hump-shaped dynamics, while monetary shocks do not. Our results show that, depending on the nature of policy, monetary shocks can also induce hump-shaped RER dynamics – this happens in our model when the policy rule resembles the original Taylor (1993) rule. With a large enough degree of policy inertia monetary shocks do not induce hump-shaped responses, which is why, in this case, the model fails to generate enough RER persistence.

Our paper is related to the literature on RER dynamics in sticky-price models; for a list of references, see Carvalho and Nechio (2011). Within this literature, our work is closely related to papers that focus on the importance of capital accumulation for RER dynamics, in particular, to Chari et al. (2002) and Martinez-Garcia and Søndergaard (2010). Owing to our analysis of the implications of sectoral capital specificity for aggregate dynamics, our paper contributes to the debate between Woodford (2005) and Chari et al. (2000) on how capital accumulation affects the degree of monetary non-neutrality. Our paper is also tangentially related to Morshed and Turnovsky (2004), who study the effects of sectoral capital adjustment costs on capital accumulation and RER dynamics in a (nonmonetary) model with a tradable and a nontradable sector. Finally, our results on the role of monetary policy highlight the importance of the debate about the sources of interest rate inertia (Rudebusch 2002, Coibion and Gorodnichenko 2012).

Section 2 presents the reference model of our multisector economy with an economy-wide rental market for homogeneous capital. It follows with a quantitative analysis of the effects of capital accumulation on RER dynamics. Section 3 presents the variant with sector-specific capital, and the corresponding quantitative analysis, focusing on the comparison between the two models of capital. Section 4 discusses our findings, with an emphasis on the effects of sector-specific capital on RER persistence. Section 5 discusses the importance of the nature of monetary policy. The last section concludes.

2 Baseline model with homogeneous capital

We depart from the benchmark model of Carvalho and Nechio (2011) by incorporating investment and capital accumulation. The world economy consists of two symmetric countries, *Home* and *Foreign*. In each country, identical consumers supply labor and capital to intermediate firms, that they own, invest in a complete set of state-contingent financial claims, and consume a nontraded final good. Nontraded final goods are produced by competitive firms that combine intermediate goods produced in the two countries. Intermediate goods are produced by monopolistically competitive firms that are divided into sectors that differ in their frequency of price changes. These firms can price-discriminate across countries and set prices in local currency.

2.1 Consumers

The Home representative consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint

$$P_t C_t + P_t I_t + E_t \left[\Theta_{t,t+1} B_{t+1} \right] \leq W_t N_t + Z_t K_t + B_t + T_t,$$

the law of motion for capital

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + \Phi(I_t, K_t) I_t, \\ I_t &\geq 0, \end{aligned}$$

and a standard "no-Ponzi" condition

$$B_{t+1} \geq -\sum_{l=t+1}^{\infty} E_{t+1} \left[\Theta_{t+1,l} \left(W_l N_l + T_l + Z_l K_l \right) \right] \geq -\infty,$$

where $\Theta_{t,l} \equiv \prod_{l'=t+1}^{l} \Theta_{l'-1,l'}$, E_t denotes the time-t expectations operator, C_t is consumption of the final good, N_t is total labor supply, W_t is the corresponding nominal wage rate, I_t denotes investment, K_t stands for physical capital, Z_t is the associated nominal return on capital, and T_t stands for net transfers from the government plus profits from Home intermediate firms. The final good can be used for either investment or consumption, and sells at the nominal price P_t. B_{t+1} accounts for the state-contingent value of the portfolio of financial securities held by the consumer at the beginning of $t+1$. Under complete financial markets, agents can choose the value of B_{t+1} for each possible state of the world at all times. A nonarbitrage condition requires the existence of a nominal stochastic discount factor $\Theta_{t,t+1}$ that prices in period t any financial asset portfolio with state-contingent payoff B_{t+1} at the beginning of period $t+1$.[3] Finally, β is the time-discount factor, σ^{-1} denotes the intertemporal elasticity of substitution, γ^{-1} is the Frisch elasticity of labor supply, δ is the rate of depreciation, and $\Phi(.)$ is the adjustment-cost function. We follow Chari et al. (2000, 2002), and assume $\Phi(I_t, K_t)$ takes the following form:

$$\Phi(I_t, K_t) = \Phi\left(\frac{I_t}{K_t}\right) = 1 - \frac{1}{2}\kappa \frac{\left(\frac{I_t}{K_t} - \delta\right)^2}{\frac{I_t}{K_t}},$$

which is convex and satisfies $\Phi(\delta) = 1$ and $\Phi'(\delta) = 0$ and $\Phi''(\delta) = -\frac{\kappa}{\delta}$.

The maximization problem yields as first-order conditions for consumption and labor:

$$\begin{aligned} \frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} &= \frac{\beta^l}{\Theta_{t,l}} \frac{P_t}{P_{t+l}}, \\ \frac{W_t}{P_t} &= N_t^{\gamma} C_t^{\sigma}. \end{aligned} \tag{1}$$

[3]To avoid cluttering the notation, we omit explicit reference to the different states of nature.

Under complete markets, we can price a one-period riskless nominal bond as:

$$\frac{1}{IR_t} = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right], \tag{2}$$

where IR_t is the short-term interest rate.

Taking the first-order conditions for capital and investment and simplifying them yields:

$$Q_t \left(\Phi' \left(\frac{I_t}{K_t} \right) \frac{I_t}{K_t} + \Phi \left(\frac{I_t}{K_t} \right) \right) = 1,$$

$$Q_t = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left(\frac{Z_{t+1}}{P_{t+1}} + Q_{t+1} \left[(1-\delta) - \Phi' \left(\frac{I_{t+1}}{K_{t+1}} \right) \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 \right] \right) \right\},$$

where Q_t is Tobin's q.

Finally, the solution must also satisfy a transversality condition:

$$\lim_{l \to \infty} E_t \left[\Theta_{t,t+l} B_{t+l} \right] = 0.$$

The Foreign consumer solves an analogous problem and maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{*1-\sigma} - 1}{1-\sigma} - \frac{N_t^{*1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t^* C_t^* + P_t^* I_t^* + E_t \left[\Theta_{t,t+1}^* \frac{B_{t+1}^*}{\mathcal{E}_t} \right] \leq W_t^* N_t^* + \frac{B_t^*}{\mathcal{E}_t} + T_t^* + Z_t^* K_t^*, \tag{3}$$

the law of motion for capital:

$$K_{t+1}^* = (1-\delta) K_t^* + \Phi \left(I_t^*, K_t^* \right) I_t^*$$

and an analogous "no-Ponzi" condition. A superscript "$*$" denotes the Foreign counterpart of the corresponding Home variable. Without loss of generality, we assume that the complete set of state-contingent assets are denominated in the Home currency. As a result, in the budget constraint (3), B_t^* appears divided by the nominal exchange rate, $\mathcal{E}_t$, to convert the value of the portfolio into Foreign currency. $\mathcal{E}_t$ is defined as the price of the Foreign currency in terms of the Home currency, hence, it is quoted in units of Home currency per unit of the Foreign currency.

The Foreign consumer's optimality conditions for consumption and labor, and transversality condition are:

$$\frac{C_t^{*-\sigma}}{C_{t+l}^{*-\sigma}} = \frac{\beta^s}{\Theta_{t,t+l}^*} \frac{\mathcal{E}_t P_t^*}{\mathcal{E}_{t+l} P_{t+l}^*}, \tag{4}$$

$$\frac{W_t^*}{P_t^*} = C_t^{*\sigma} N_t^{*\gamma},$$

$$\lim_{l\to\infty} E_t\left[\Theta_{t,t+l}^* B_{t+l}^*\right] = 0.$$

Assuming the same investment adjustment cost function for the Foreign consumer, her investment decision yields:

$$Q_t^*\left(\Phi'\left(\frac{I_t^*}{K_t^*}\right)\frac{I_t^*}{K_t^*} + \Phi\left(\frac{I_t^*}{K_t^*}\right)\right) = 1,$$

$$Q_t^* = \beta E_t\left\{\frac{\left(C_{t+1}^*\right)^{-\sigma}}{\left(C_t^*\right)^{-\sigma}}\left(\frac{Z_{t+1}^*}{P_{t+1}^*} + Q_{t+1}^*\left[(1-\delta) - \Phi'\left(\frac{I_{t+1}^*}{K_{t+1}^*}\right)\left(\frac{I_{t+1}^*}{K_{t+1}^*}\right)^2\right]\right)\right\},$$

where Q_t^* Tobin's q in Foreign.

Since there are no arbitrage opportunities and assets are freely traded, the stochastic discount factor has to be the same for both countries. Defining $RER_t \equiv \mathcal{E}_t \frac{P_t^*}{P_t}$ as the real exchange rate, from equations (1) and (4):

$$RER_{t+l} = RER_t \frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} \frac{C_{t+l}^{*-\sigma}}{C_t^{*-\sigma}}. \tag{5}$$

Iterating equation (5) backwards and assuming $RER_0 \frac{C_0^{-\sigma}}{C_0^{*-\sigma}} = 1$ yields:

$$RER_t = \frac{C_t^{*-\sigma}}{C_t^{-\sigma}}. \tag{6}$$

2.2 Final goods firms

A representative competitive firm produces the final good, which is a composite of varieties of intermediate goods from both countries. Monopolistically competitive firms produce each variety of intermediate goods. The latter firms are divided into sectors indexed by $s \in \{1, ..., S\}$, each featuring a continuum of firms. Sectors differ in the degree of price rigidity, as we detail below. Overall, firms are indexed by the country where they produce, by their sector, and are further indexed by $j \in [0, 1]$. The distribution of firms across sectors is given by sectoral weights $f_s > 0$, with $\sum_{s=1}^S f_s = 1$.

The final good is used for both consumption and investment and is produced by combining the intermediate varieties according to the technology:

$$Y_t = \left(\sum\nolimits_{s=1}^S f_s^{\frac{1}{\eta}} Y_{s,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{7}$$

$$Y_{s,t} = \left(\omega^{\frac{1}{\rho}} Y_{H,s,t}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} Y_{F,s,t}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{8}$$

$$Y_{H,s,t} = \left(f_s^{\frac{\theta-1}{\theta}} \int_0^1 Y_{H,s,j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \tag{9}$$

$$Y_{F,s,t} = \left(f_s^{\frac{\theta-1}{\theta}} \int_0^1 Y_{F,s,j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \tag{10}$$

where Y_t is the Home final good, $Y_{s,t}$ is the aggregation of sector-s Home and Foreign intermediate goods sold in Home, $Y_{H,s,t}$ and $Y_{F,s,t}$ are the composites of intermediate varieties produced by firms in sector s in Home and Foreign, respectively, to be sold in Home, and $Y_{H,s,j,t}$ and $Y_{F,s,j,t}$ are the varieties produced by firm j in sector s in Home and Foreign to be sold in Home. The parameters $\eta \geq 0$, $\rho \geq 0$, and $\theta > 1$ are, respectively, the elasticity of substitution across sectors, the elasticity of substitution between Home and Foreign goods, and the elasticity of substitution within sectors. Finally, $\omega \in [0,1]$ is the steady-state share of domestic inputs.

A representative Home final-good-producing firm solves:

$$\max \; P_t Y_t - \sum\nolimits_{s=1}^{S} f_s \int_0^1 \left(P_{H,s,j,t} Y_{H,s,j,t} + P_{F,s,j,t} Y_{F,s,j,t} \right) dj$$
$$s.t. \;\; (7)\text{-}(10),$$

which yields as first-order conditions, for $j \in [0,1]$ and $s = 1, ..., S$:

$$Y_{H,s,j,t} = \omega \left(\frac{P_{H,s,j,t}}{P_{H,s,t}} \right)^{-\theta} \left(\frac{P_{H,s,t}}{P_{s,t}} \right)^{-\rho} \left(\frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t,$$
$$Y_{F,s,j,t} = (1-\omega) \left(\frac{P_{F,s,j,t}}{P_{F,s,t}} \right)^{-\theta} \left(\frac{P_{F,s,t}}{P_{s,t}} \right)^{-\rho} \left(\frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t.$$

The price indices are given by:

$$P_t = \left(\sum\nolimits_{s=1}^{S} f_s P_{s,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{11}$$

$$P_{s,t} = \left(\omega P_{H,s,t}^{1-\rho} + (1-\omega) P_{F,s,t}^{1-\rho} \right)^{\frac{1}{1-\rho}}, \tag{12}$$

$$P_{H,s,t} = \left(\int_0^1 P_{H,s,j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \tag{13}$$

$$P_{F,s,t} = \left(\int_0^1 P_{F,s,j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \tag{14}$$

where P_t is the price of the Home final good, $P_{s,t}$ is the price index of sector-s intermediate goods sold in Home, $P_{H,s,t}$ is the price index for sector-s Home-produced intermediate goods sold in Home, and $P_{H,s,j,t}$ is the price charged in the Home market by Home firm j from sector s. $P_{F,s,t}$ is the price index for sector-s Foreign-produced intermediate goods sold in Home, and $P_{F,s,j,t}$ is the price charged in the Home market by Foreign firm j from sector s. Both $P_{H,s,j,t}$ and $P_{F,s,j,t}$ are set in the Home currency.

The Foreign final firm solves an analogous maximization problem and its demands for intermediate inputs from Foreign ($Y^*_{F,s,j,t}$) and Home ($Y^*_{H,s,j,t}$) producers are:

$$\begin{aligned}
Y^*_{F,s,j,t} &= \omega \left(\frac{P^*_{F,s,j,t}}{P^*_{F,s,t}}\right)^{-\theta} \left(\frac{P^*_{F,s,t}}{P^*_{F,t}}\right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_{t}}\right)^{-\eta} Y^*_t, \\
Y^*_{H,s,j,t} &= (1-\omega) \left(\frac{P^*_{H,s,j,t}}{P^*_{H,s,t}}\right)^{-\theta} \left(\frac{P^*_{H,s,t}}{P^*_{s,t}}\right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_{t}}\right)^{-\eta} Y^*_t.
\end{aligned}$$

In analogy to equations (11) to (14), Foreign price indices are given by:

$$\begin{aligned}
P^*_t &= \left(\sum\nolimits_{s=1}^{S} f_s P^{*1-\eta}_{s,t}\right)^{\frac{1}{1-\eta}}, \\
P^*_{s,t} &= \left(\omega P^{*1-\rho}_{F,s,t} + (1-\omega)\, P^{*1-\rho}_{H,s,t}\right)^{\frac{1}{1-\rho}}, \\
P^*_{H,s,t} &= \left(\int_0^1 P^{*1-\theta}_{H,s,j,t} dj\right)^{\frac{1}{1-\theta}}, \\
P^*_{F,s,t} &= \left(\int_0^1 P^{*1-\theta}_{F,s,j,t} dj\right)^{\frac{1}{1-\theta}}
\end{aligned}$$

where P^*_t is the price of the Foreign final good, $P^*_{s,t}$ is the price index of sector-s intermediate goods sold in Foreign, $P^*_{F,s,t}$ is the price index for sector-s Foreign-produced intermediate goods sold in Foreign, and $P^*_{F,s,j,t}$ is the price charged in the Foreign market by Foreign firm j from sector s. $P^*_{H,s,t}$ is the price index for sector-s Home-produced intermediate goods sold in Foreign, and $P^*_{H,s,j,t}$ is the price charged in the Foreign market by Home firm j from sector s. Both $P^*_{F,s,j,t}$ and $P^*_{H,s,j,t}$ are set in the Foreign currency.

2.3 Intermediate goods firms

Monopolistically competitive firms produce varieties of the intermediate good by employing capital and labor. As in Carvalho and Nechio (2011), these firms set prices as in Calvo (1983). The frequency of price changes varies across sectors, and in each period, each firm j in sector s changes its price independently with probability α_s. This is the only source of (ex-ante) heterogeneity.

At each time a Home-firm j from sector s adjusts its price, it chooses prices $X_{H,s,j,t}$, $X^*_{H,s,j,t}$ to be charged in the Home and Foreign markets, respectively, with each price being set in the corresponding local currency. The maximization problem is:

$$\max \; E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \begin{bmatrix} X_{H,s,j,t} Y_{H,s,j,t+l} + \mathcal{E}_{t+l} X^*_{H,s,j,t} Y^*_{H,s,j,t+l} \\ -W_{t+l} N_{s,j,t+l} - Z_{t+l} K_{s,j,t+l} \end{bmatrix}$$

$$st \qquad Y_{H,s,j,t} = \omega \left(\frac{P_{H,s,j,t}}{P_{H,s,t}}\right)^{-\theta} \left(\frac{P_{H,s,t}}{P_{s,t}}\right)^{-\rho} \left(\frac{P_{s,t}}{P_t}\right)^{-\eta} Y_t$$

$$Y^*_{H,s,j,t} = (1-\omega) \left(\frac{P^*_{H,s,j,t}}{P^*_{H,s,t}}\right)^{-\theta} \left(\frac{P^*_{H,s,t}}{P^*_{s,t}}\right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_t}\right)^{-\eta} Y^*_t$$

$$Y_{H,s,j,t} + Y^*_{H,s,j,t} = F(K_{s,j,t} N_{s,j,t}) = (K_{s,j,t})^{1-\chi} (N_{s,j,t})^{\chi}.$$

The first-order conditions for optimal price setting lead to:

$$X_{H,s,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda_{H,s,t+l} \left(\chi K^{1-\chi}_{s,j,t+l} N^{\chi-1}_{s,j,t+l}\right)^{-1} W_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda_{H,s,t+l}},$$

$$X^*_{H,s,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda^*_{H,s,t+l} \left(\chi K^{1-\chi}_{s,j,t+l} N^{\chi-1}_{s,j,t+l}\right)^{-1} W_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \mathcal{E}_{t+l} \Lambda^*_{H,s,t+l}},$$

where:

$$\Lambda_{H,s,t} = \omega \left(\frac{1}{P_{H,s,t}}\right)^{-\theta} \left(\frac{P_{H,s,t}}{P_{s,t}}\right)^{-\rho} \left(\frac{P_{s,t}}{P_t}\right)^{-\eta} Y_t,$$

$$\Lambda^*_{H,s,t} = (1-\omega) \left(\frac{1}{P^*_{H,s,t}}\right)^{-\theta} \left(\frac{P^*_{H,s,t}}{P^*_{s,t}}\right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_t}\right)^{-\eta} Y^*_t.$$

Given the optimality (cost-minimization) conditions for capital and labor, the real marginal cost can be expressed as:

$$MC_{s,j,t} = \frac{W_t/P_t}{\chi K^{1-\chi}_{s,j,t} N^{\chi-1}_{s,j,t}} = \frac{1}{\chi^{\chi} (1-\chi)^{1-\chi}} \left(\frac{W_t}{P_t}\right)^{\chi} \left(\frac{Z_t}{P_t}\right)^{(1-\chi)} = MC_t,$$

and hence marginal costs are the same for all firms in the economy. This is expected, given that the (homogeneous) production inputs are negotiated in economy-wide markets.

By analogy, the Foreign firm problem yields:

$$X^*_{F,s,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda^*_{F,s,t+l} \left(\chi \left(K^*_{s,j,t+l}\right)^{1-\chi} \left(N^*_{s,j,t+l}\right)^{\chi-1}\right)^{-1} W^*_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda^*_{F,s,t+l}},$$

$$X_{F,s,j,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda_{F,s,t+l} \left(\chi \left(K^*_{s,j,t+l}\right)^{1-\chi} \left(N^*_{s,j,t+l}\right)^{\chi-1}\right)^{-1} W^*_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda_{F,s,t+l}/\mathcal{E}_{t+l}},$$

where

$$\Lambda^*_{F,s,t} = \omega \left(\frac{1}{P^*_{F,s,t}}\right)^{-\theta} \left(\frac{P^*_{F,s,t}}{P^*_{s,t}}\right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_t}\right)^{-\eta} Y^*_t,$$

$$\Lambda_{F,s,t} = (1-\omega) \left(\frac{1}{P_{F,s,t}}\right)^{-\theta} \left(\frac{P_{F,s,t}}{P_{s,t}}\right)^{-\rho} \left(\frac{P_{s,t}}{P_t}\right)^{-\eta} Y_t.$$

Analogous steps in the derivation of the expression for marginal costs in Foreign, MC^*_t, show that they are also equalized across firms.

Finally, the market clearing conditions for Home include:

$$K_t = \sum_{s=1}^{S} f_s \int_0^1 K_{s,j,t} dj,$$

$$N_t = \sum_{s=1}^{S} f_s \int_0^1 N_{s,j,t} dj,$$

and likewise for Foreign.

2.4 Monetary policy

In our baseline specification we follow Carvalho and Nechio (2011) and assume that the growth rate of nominal aggregate demand in each country follows a first-order autoregressive (AR) process, thus leaving monetary policy implicit. This specification can be justified through a cash-in-advance constraint when money growth itself follows an $AR(1)$, or as the result of a monetary policy rule. Denoting nominal aggregate demand in Home and Foreign, respectively, by $M_t \equiv P_t Y_t$, $M^*_t \equiv P^*_t Y^*_t$, our assumption is:

$$\Delta m_t = \rho_m \Delta m_{t-1} + \sigma_{\varepsilon_m} \varepsilon_{m,t},$$

$$\Delta m^*_t = \rho_m \Delta m^*_{t-1} + \sigma_{\varepsilon_m} \varepsilon^*_{m,t},$$

where $m_t \equiv \log(M_t)$, ρ_m determines the autocorrelation in nominal aggregate demand growth, and $\varepsilon_{m,t}$ and $\varepsilon^*_{m,t}$ are purely monetary, uncorrelated, zero-mean, unit-variance *i.i.d.* shocks.

2.5 Quantitative analysis

In this section, we parameterize the model and analyze its quantitative predictions. To discipline our analysis, we follow an approach that is common in the real business cycle literature (e.g., Chari et al. 2002) and calibrate the intertemporal elasticity of substitution (σ^{-1}) and the investment adjustment-cost parameter (κ) to match the standard deviation of the RER and of investment in the data, both relative to the standard deviation of GDP. Whenever we analyze a different version of the model in subsequent sections, we redo the calibration.[4] Our exercise thus consists of fixing RER (and investment) volatility, and comparing the RER dynamics implied by different versions of the model. In particular we will compare models with and without capital accumulation, in multisector and in one-sector economies.

The parameterization of the cross-sectional distribution of price stickiness follows Carvalho and Nechio (2011). The 271 categories of goods and services reported by Nakamura and Steinsson (2008) are aggregated into 67 expenditure classes. The frequency of price changes for each expenditure class is obtained as a weighted average of the frequencies for the underlying categories, using the expenditure weights provided by those authors. The resulting average monthly frequency of price changes is $\bar{\alpha} = \Sigma_{s=1}^{S} f_s \alpha_s = 0.211$.

Solving and simulating the multisector model with 67 sectors is computationally costly. To sidestep this problem we work with a 3-sector approximation to the underlying 67-sector economy. We choose the frequencies of price changes and sectoral weights in the approximating model to match a suitably chosen set of moments of the cross-sectional distribution of price stickiness of the original 67-sector economy. In the Appendix, we show that this delivers an extremely good approximation to the RER dynamics of the original model.

The remaining parameters are also set as in Carvalho and Nechio (2011). The (Frisch) labor supply elasticity is set to unit, $\gamma = 1$, labor share χ is set at $2/3$, and the consumer's time preference rate is set at 2% per year. The elasticity of substitution between varieties within sectors is set to $\theta = 10$, the elasticity of substitution between Home and Foreign goods is set to $\rho = 1.5$, the elasticity of substitution between varieties of different sectors is set to unit ($\eta = 1$), and the share of domestic goods is set to $\omega = 0.9$. Finally, the autocorrelation of nominal aggregate demand growth is set to $\rho_m = 0.8$.[5]

We analyze the model using a loglinear approximation around the zero-inflation steady state. Details of the derivation of the steady state and of the loglinear equations are provided in an online appendix.

2.5.1 Results

Table 1 reports quantitative results produced by our model for the half-life ($\mathcal{HL}$) of RER deviations from parity (measured in months), in addition to the quarter-life ($\mathcal{QL}$) and the up-life ($\mathcal{UL}$) of those

[4] The calibration strategy of Chari et al. (2002) is slightly different. They choose the value of σ so that the standard deviation of the real exchange rate relative to GDP in their benchmark model approximates the data. Then, for each alternative model they consider, they only recalibrate the investment adjustment cost parameter. We opt to calibrate both parameters for each version of the model. The results of adopting the Chari et al. (2002) approach are essentially unchanged. As yet a third alternative, we also set the values for σ and κ in all versions of the model equal to those obtained in the calibration of the baseline framework. Our main results are unchanged and are available upon request.

[5] Carvalho and Nechio (2011) provide more details on the choices for these parameter values.

deviations. The last two measures are meant to provide a better picture of the shape of the impulse response function of the real exchange rate in response to a monetary shock. They correspond to, respectively, the time it takes for the impulse response function to drop below 1/4 of the initial impulse, and the time it takes for the real exchange rate to peak after the initial response. As an additional measure of persistence, we also report the first-order autocorrelation of the real exchange rate (ρ_1).[6]

Table 1: Model with economy-wide capital and labor markets

	Data	Multisector model: With capital	Multisector model: No capital	One-sector model: With capital	One-sector model: No capital
Persistence measures					
$\mathcal{HL}$	36-60	37	31	12	11
$\mathcal{QL}$	76	55	43	15	14
$\mathcal{UL}$	28	23	20	6	6
ρ_1	0.78	0.85	0.85	0.71	0.71

The first column shows empirical measures of persistence taken from the literature.[7] The second column provides results from the baseline multisector model with capital described in Section 2. The third column provides statistics for a version of the multisector model that abstracts from capital, obtained by fixing the returns to labor at unit, $\chi = 1$.[8] The last two columns report analogous results for versions of the model with only one sector (i.e., with homogeneous price rigidity).

As highlighted by Carvalho and Nechio (2011), the multisector models significantly increase RER persistence when compared to their one-sector counterparts. This result holds irrespective of the presence of capital accumulation. Moreover, the multisector nature of the models substantially improve their ability to match other statistics that provide useful information about RER dynamics ($\mathcal{QL}$ and $\mathcal{UL}$). This, again, holds irrespective of the presence of capital accumulation. The bottom line is that the presence of capital accumulation need not hinder the ability of multisector sticky-price models to generate enough RER persistence in response to monetary shocks. In contrast, the results for the one-sector economies confirm the fact – well-known in the theoretical PPP literature – that standard sticky-price models have difficulty generating enough RER persistence in response to monetary shocks, irrespective of the presence of capital accumulation (Rogoff 1996, Chari et al. 2002).

[6] Following Chari et al. (2002), the first-order autocorrelations are based on HP-filtered model-generated data. We simulate 100 replications of each economy with 2000 observations each. After dropping the first 500 observations, we average each series over three-month periods to obtain a quarterly series, to which we apply a Hodrick-Prescott filter with bandwidth 1600.

[7] The results for the half-life summarize Rogoff's "consensus view", while the other statistics are from Steinsson (2008).

[8] This is essentially the model in Carvalho and Nechio (2011) The only difference is that here we assume a constant-returns technology, to make the results more comparable to those of the models with capital accumulation.

3 Sector-specific capital

In this section, we further our study of the effects of allowing for capital accumulation in sticky-price models of the RER and consider a variant of the baseline model in which capital mobility across sectors is limited. Our main motivation for this assumption is the empirical evidence presented by Ramey and Shapiro (2001), which suggests that capital is, to a large extent, sector-specific. We thus assume that capital can be reallocated freely across firms in the same sector but cannot be moved across sectors. To isolate the role of capital specificity, we maintain the assumption of an economy-wide labor market. In the Appendix we consider a version of the model with limited mobility in both capital and labor markets.

The representative consumer still buys the final good for consumption and investment, but now she has to decide in which sectors to add to the capital stock. Once allocated to a particular sector, capital cannot be used elsewhere. These changes require that we reformulate the consumers' and intermediate firms' problems. The maximization problem of final goods firms remains the same as in the baseline specification.

3.1 Consumers

The Home representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t C_t + P_t I_t + E_t \left[\Theta_{t,t+1} B_{t+1} \right] \leq W_t N_t + B_t + T_t + \sum_{s=1}^{S} Z_{s,t} K_{s,t},$$

the law of motion for the stocks of sector-specific capital:

$$\begin{aligned} K_{s,t+1} &= (1-\delta) K_{s,t} + \Phi \left(I_{s,t}, K_{s,t} \right) I_{s,t}, \\ I_{s,t} &\geq 0, \end{aligned}$$

and a standard "no-Ponzi" condition:

$$B_{t+1} \geq - \sum_{l=t+1}^{\infty} E_{t+1} \left[\Theta_{t+1,l} \left(W_l N_l + \sum_{s=1}^{S} Z_{s,l} K_{s,l} + T_l \right) \right] \geq -\infty.$$

The notation is the same as before, except that now $K_{s,t}$ is physical capital supplied to firms in sector s, and $Z_{s,t}$ is the associated nominal return on capital, $I_{s,t}$ denotes investment in sector-s capital, and $I_t \equiv \sum_{s=1}^{S} I_{s,t}$.

The first-order conditions for consumption and labor are as before. For all investment ($I_{s,t}$) and capital ($K_{s,t+1}$) types:

$$Q_{s,t} = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left(\frac{Z_{s,t+1}}{P_{t+1}} + Q_{s,t+1} \left[(1-\delta) - \Phi' \left(\frac{I_{s,t+1}}{K_{s,t+1}} \right) \left(\frac{I_{s,t+1}}{K_{s,t+1}} \right)^2 \right] \right) \right\},$$

$$Q_{s,t} \left(\Phi' \left(\frac{I_{s,t}}{K_{s,t}} \right) \frac{I_{s,t}}{K_{s,t}} + \Phi \left(\frac{I_{s,t}}{K_{s,t}} \right) \right) = 1,$$

where $Q_{s,t}$ denotes Tobin's q for sector s.

The solution must also satisfy a transversality condition:

$$\lim_{l \to \infty} E_t \left[\Theta_{t,l} B_l \right] = 0.$$

The problem for the representative consumer in Foreign is analogous.

3.2 Intermediate goods firms

Once we introduce sectoral capital markets, the intermediate goods producer's problem also changes. It becomes:

$$\begin{aligned}
& \max \; E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \left[\begin{array}{c} X_{H,s,j,t} Y_{H,s,j,t+l} + \mathcal{E}_{t+l} X_{H,s,j,t}^* Y_{H,s,j,t+l}^* \\ -W_{t+l} N_{s,j,t+l} - Z_{s,t+l} K_{s,j,t+l} \end{array} \right] \\
st \quad & Y_{H,s,j,t} = \omega \left(\frac{P_{H,s,j,t}}{P_{H,s,t}} \right)^{-\theta} \left(\frac{P_{H,s,t}}{P_{s,t}} \right)^{-\rho} \left(\frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t \\
& Y_{H,s,j,t}^* = (1-\omega) \left(\frac{P_{H,s,j,t}^*}{P_{H,s,t}^*} \right)^{-\theta} \left(\frac{P_{H,s,t}^*}{P_{s,t}^*} \right)^{-\rho} \left(\frac{P_{s,t}^*}{P_t^*} \right)^{-\eta} Y_t^* \\
& Y_{H,s,j,t} + Y_{H,s,j,t}^* = F\left(K_{s,j,t} N_{s,j,t}\right) = \left(K_{s,j,t}\right)^{1-\chi} \left(N_{s,j,t}\right)^{\chi}.
\end{aligned}$$

Optimal price setting implies:

$$\begin{aligned}
X_{H,s,j,t} &= \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \Lambda_{H,s,t+l} \left(\chi K_{s,j,t+l}^{1-\chi} N_{s,j,t+l}^{\chi-1} \right)^{-1} W_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \Lambda_{H,s,t+l}}, \\
X_{H,s,j,t}^* &= \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \Lambda_{H,s,t+l}^* \left(\chi K_{s,j,t+l}^{1-\chi} N_{s,j,t+l}^{\chi-1} \right)^{-1} W_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} \left(1-\alpha_s\right)^l \mathcal{E}_{t+l} \Lambda_{H,s,t+l}^*},
\end{aligned}$$

where:

$$\Lambda_{H,s,t} = \omega \left(\frac{1}{P_{H,s,t}}\right)^{-\theta} \left(\frac{P_{H,s,t}}{P_{s,t}}\right)^{-\rho} \left(\frac{P_{s,t}}{P_t}\right)^{-\eta} Y_t,$$

$$\Lambda^*_{H,s,t} = (1-\omega) \left(\frac{1}{P^*_{H,s,t}}\right)^{-\theta} \left(\frac{P^*_{H,s,t}}{P^*_{s,t}}\right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_t}\right)^{-\eta} Y^*_t.$$

Given the optimality (cost-minimization) conditions for capital and labor, the real marginal cost can be expressed as:

$$MC_{s,j,t} = \frac{W_t/P_t}{\chi\left(\frac{(1-\chi)}{\chi}\frac{W_{s,t}}{Z_{s,t}}\right)^{1-\chi}} = \frac{1}{\chi^{\chi}(1-\chi)^{1-\chi}}\left(\frac{W_t}{P_t}\right)^{\chi}\left(\frac{Z_{s,t}}{P_t}\right)^{(1-\chi)} = MC_{s,t}.$$

Note that marginal costs are now equalized only within sectors. This is a direct implication of the assumption of sectoral capital markets. Finally, under this assumption, the market-clearing condition for capital becomes:

$$K_{s,t} = f_s \int_0^1 K_{s,j,t}dj,\ \forall\ s.$$

3.3 Quantitative analysis

In this section, we replicate the exercise of Section 2.5 using the model with sectoral capital markets. Recall that, for each model, we recalibrate the intertemporal elasticity of substitution (σ^{-1}) and the investment adjustment-cost parameter (κ) to match the standard deviation of the RER and of investment in the data, both relative to the standard deviation of GDP. The results are reported in Table 2.

Table 2: Model with sector-specific capital

	Data	With capital	No capital
Persistence measures:			
$\mathcal{HL}$	36-60	35	31
$\mathcal{QL}$	76	52	43
$\mathcal{UL}$	28	22	20
ρ_1	0.78	0.85	0.85

For ease of comparison, the first column of the table replicates the data column from Table 1. The second column reports the results for the model with sectoral capital markets, while the last column considers a version of the model without capital accumulation ($\chi = 1$). Comparing the last two columns, for the same level of RER volatility, the introduction of capital accumulation increases persistence of the real exchange rate, measured by its half-life, moving it closer to the empirical evidence. The same is true for the autocorrelation, the up-life, and the quarter-life. Hence, once again, the presence of capital accumulation does not alter the results significantly, and the multisector model is still able to generate a large degree of RER persistence.

Finally, a comparison of the results reported in Tables 1 and 2 shows that switching from economy-wide to sector-specific capital and labor markets tends to decrease RER persistence. We discuss this result in the next section.

4 The effects of different types of capital

The fact that switching from an economy-wide to a sector-specific capital market tends to decrease RER persistence may seem counterintuitive, in light of the results obtained by Woodford (2005). He develops a model with firm-specific capital,[9] and finds that specificity *increases* the persistence of the real effects of nominal disturbances relative to an economy with a rental market for homogeneous capital.

As shown by Woodford (2005), firm-specific capital is a source of real rigidities in the sense of Ball and Romer (1990), and tends to induce strategic complementarities in firms' pricing decisions. To understand the mechanism at work, consider first the case of a rental market for homogeneous capital. This market structure tends to make firms' pricing decisions be strategic substitutes. This is because firms that do not respond to an increase in aggregate demand with a price increase need to employ disproportionately more production inputs. This puts upward pressure on factor prices, and leads firms that do adjust prices in response to the demand increase to set relatively *higher* prices. This cost pressure transmitted through common factor markets tends to speed up the response of the aggregate price level to an increase in nominal demand, thus decreasing the persistence of its real effects on the RER and other real variables.

Now let us consider the case of firm-specific factor markets – capital in particular. As described by Woodford (2005), under the same circumstances, adjusting firms will have less of an incentive to increase their prices. The reason is that marginal costs no longer depend on common factor prices but are instead determined by firm-specific factor prices. Consider an increase in nominal aggregate demand as in our analysis of common factor markets. Let us assume that adjusting firms choose to increase prices by as much as they would in that case. With firm-specific factors, the relative decrease in demand induced by higher prices puts downward pressure on factor prices, relative to the case of common factor markets. This makes it suboptimal for firms to raise prices by as much as in

[9] Woodford sometimes refers to his assumption regarding factor markets as involving industry-specific (e.g., Woodford 2003, chapter 3) or sector-specific (e.g., Woodford 2005) markets. This is not the same as our assumption of sector-specificity. In Woodford's work, an industry (or sector) is characterized by fully synchronized price setting, so that his assumption is mathematically equivalent to firm-specific factor markets – as long as firms behave competitively and do not try to exploit their monopsony power. In contrast, in our model, firms in any given sector set prices independently of one another.

that case. In other words, firm-specific inputs lead firms that change prices to keep them closer to other firms' unchanged prices. This comparison shows that factor specificity at the firm level induces a complementarity (or weakens the degree of substitutability) in pricing decisions. The different implications of factor market structures for the interactions between firms' pricing decisions is what leads to higher persistence of the real effects of nominal disturbances in Woodford's model.

So why does this not happen when capital is sector specific as opposed to firm specific? Once again, the reason has to do with the implications of factor market structure for the interdependence between firms' pricing decisions. To understand the mechanism at work when capital is sector specific, consider again an increase in demand – but now in sectoral demand. Firms that do not respond with a price increase need to employ disproportionately more production inputs. This puts upward pressure on sectoral factor prices, and leads firms that do adjust prices in response to the demand increase to set relatively *higher* prices. This cost pressure transmitted through sectoral factor markets induces what we refer to as a *within-sector strategic substitutability* in pricing decisions. This is the mechanism pushing nominal shocks to have less persistent effects on the RER in our calibrated models with sectoral capital.

5 The role of monetary policy

In our previous analyses we left monetary policy unspecified and postulated an exogenous stochastic process for nominal aggregate demand. In this section we look at the role of monetary policy. In what follows, we consider a Taylor-type interest rate rule of the form:

$$IR_t = \beta^{-1} \left(IR_{t-1}\right)^{\phi_i} \left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{GDP_t}{GDP}\right)^{\phi_Y} e^{\upsilon_t},$$

where IR_t is the nominal interest rate on one-period riskless bonds at time t, $GDP_t \equiv Y_t + \sum_{s=1}^{S} f(s) \int_0^1 Y^*_{H,s,j,t} dj - \sum_{s=1}^{S} f(s) \int_0^1 Y_{F,s,j,t} dj$ is gross domestic product,[10] GDP denotes gross domestic product in steady state, ϕ_i, ϕ_π, and ϕ_Y are the parameters associated with the interest rate rule, and υ_t is a persistent shock with process $\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \sigma_{\varepsilon_\upsilon} \varepsilon_{\upsilon,t}$, where $\varepsilon_{\upsilon,t}$ is a zero mean, unit variance *i.i.d.* shock, and $\rho_\upsilon \in [0,1)$. We assume throughout that monetary policy in Foreign follows the same rule as in Home, and that monetary shocks are uncorrelated across the two countries.

Our motivation for considering two sources of interest-rate persistence – autoregressive shocks versus policy inertia – is the literature that tries to discriminate between these two alternatives in the data (e.g., Rudebusch 2002, Coibion and Gorodnichenko 2012). We study whether this makes a difference for the aggregate dynamics implied by the model.

We start by looking at a case with persistent shocks only ($\rho_\upsilon > 0$, $\phi_i = 0$). To fix values for the

[10] In defining GDP this way we follow Chari et al. (2002).

additional parameters in the policy rule (IR), we resort once more to Carvalho and Nechio (2011). We set $\phi_\pi = 1.5$, $\phi_Y = 0.5/12$, and $\rho_\upsilon = 0.965$. We then add interest rate smoothing ($\phi_i > 0$) by keeping the same parameter values as before and also setting $\phi_i = 0.965$. Table 3 reports the results analogous to those in Tables 1 and 2 for multisector economies.[11]

Table 3: Alternative monetary policy rules

		Homogeneous capital		Sector-specific capital	
	Data	No IR smoothing	IR smoothing	No IR smoothing	IR smoothing
Persistence measures					
$\mathcal{HL}$	36-60	63	6	56	7
$\mathcal{QL}$	76	95	22	86	22
$\mathcal{UL}$	28	34	0	27	0
ρ_1	0.78	0.87	0.62	0.86	0.63

For convenience, the first column of Table 3 reproduces the empirical moments reported in Tables 1 and 2. The second and third columns present the results for the baseline model with homogeneous capital with and without interest rate smoothing in the monetary policy rule. The last two columns present analogous results for economies in which capital is sector specific. It is clear that the nature of monetary policy matters a lot. While the versions of the multisector model in which there is no policy inertia manage to produce a large amount of RER persistence, models in which the policy rule features interest rate smoothing fail to do so.[12,13]

Why does the source of interest rate persistence make such a big difference? The reason can be traced back to Steinsson (2008). He argues that the ability of a model to produce hump-shaped RER dynamics is key to matching the degree of persistence seen in the data. He then uses a model to conclude that some shocks induce such hump-shaped dynamics (productivity shocks, labor supply shocks, government spending shocks, shocks to the world demand for home goods, and cost-push shocks), while monetary shocks do not.

Our results show that the real message in Steinsson's (2008) paper is not about the different types of shocks, per se, as in some of our models, monetary shocks do produce hump-shaped RER dynamics and enough persistence. Rather, the lesson from his paper seems to be that hump-shaped dynamics are key. Indeed, as can be seem from Table 3, the versions of the model that succeed in producing enough RER persistence in response to monetary shocks also produce pronounced hump-shaped RER dynamics – as can be seen from the nonzero up-lives ($\mathcal{UL}$). In contrast, the models with

[11] As before, for each model we recalibrate σ and κ to match the volatilities of the RER and investment relative to GDP in the data.

[12] We also analyzed one-sector models with interest rate policy and found that the conclusions we obtain here and in Carvalho and Nechio (2011) apply – i.e., one-sector models fail to generate enough RER persistence even without interest-rate smoothing in the policy rule.

[13] The comparison of the half-lives in the third and last columns shows that sectoral capital specificity need not always lead to less RER persistence when compared with homogeneous capital.

interest rate smoothing, which fail to produce enough persistence, also fail to generate hump-shaped RER dynamics.

But how can the models without policy inertia induce hump-shaped RER dynamics in response to monetary shocks? Let us revisit Steinsson's (2008) deconstruction of the mechanism that induces such dynamics. He departs from the well-known result that, in open economy models with complete markets and standard preferences, there is a close relationship between relative consumptions and the real exchange rate, as implied by equation (6). Thus, understanding RER dynamics in response to monetary shocks amounts to understating the response of consumption differentials across countries. Due to home bias in consumption, this response is well approximated by the response of consumption in the country where the monetary policy shock hit. The consumption Euler equation (2) implies a relationship between expected real interest rates and consumption. Solving (the loglinearized version of) equation (2) forward yields:

$$c_t = -\sigma^{-1} E_t \sum_{j=0}^{\infty} \left(i_{t+j} - E_{t+j} \pi_{t+1+j} \right).$$

Steinsson (2008) concludes that, for the response of consumption to a given shock to be hump-shaped, the response of nominal interest rates and expected inflation must be such that the real interest rate changes sign during the transition back to the steady state. How can this happen in response to, say, a contractionary monetary policy shock ($\upsilon_t > 0$)? The answer can be found in Galí's (2008, ch. 3) analysis of the basic New Keynesian model. He notes that "if the persistence of the monetary policy shock ρ_υ is sufficiently high, the nominal rate will decline in response to a rise in υ_t. This is the result of the downward adjustment in the nominal rate induced by the decline in inflation and output gap more than offsetting the direct effect of a higher υ_t." Thus, in response to a contractionary policy shock that is persistent enough, the real rate may at first decline and then rise above steady state before converging back. This is precisely the response that Steinsson (2008) concludes is necessary to induce hump-shaped consumption (and thus RER) dynamics.

With a strong enough interest rate smoothing component, the policy rate is prevented from making the movements that are necessary to induce the type of real rate response described above. This is what hinders the ability of the model with policy inertia to produce hump-shaped dynamics. Figure 1 illustrates these points using the versions of the multisector model with homogeneous capital.[14] The first column in the panel of figures shows the impulse response functions of real interest rates and aggregate RER to a Home monetary shock in the model without interest rate smoothing, while the figures in the second column do so for the model with policy inertia.

[14] The results based on the model with sectoral capital yield the same conclusion.

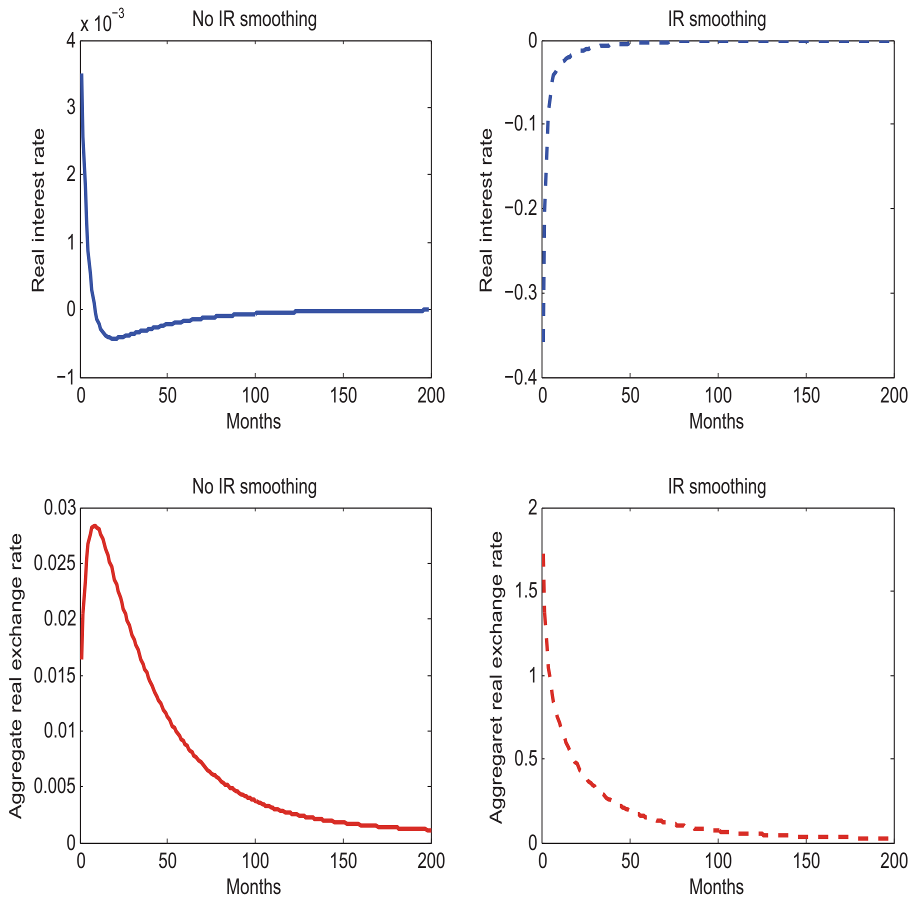

Figure 1: Impulse response functions of real interest rates and aggregate real exchange rates to an expansionary shock to Home policy rate in the multi-sector model with homogeneous capital

6 Conclusion

In this paper we study how capital accumulation affects RER dynamics in sticky-price models. We do so by introducing different types of capital in the two-country multisector model of Carvalho and Nechio (2011). We find that the multisector model with heterogeneous price stickiness can produce volatile and persistent RER in response to monetary shocks irrespective of the type of capital. One-sector versions of the economy with the same average frequency of price changes fail to do so.

Turning to the different types of capital, we find that the model with sector-specific capital tends to produce RERs that are less persistent than in the model with an economy-wide market for capital. While this result may at first seem surprising, in light of the effects of firm-level capital specificity (Woodford, 2005), we discuss how it can be understood through the implications that different assumptions about the nature of capital specificity have for whether firms' pricing decisions are strategic complements or strategic substitutes. Sectoral capital specificity induces within-sector pricing substitutability, and this tends to lower the persistence of the real effects of monetary shocks. Our results thus show that it matters a great deal whether capital is sector or firm specific. This highlights the importance of additional research into the nature of capital specificity, along the lines of the seminal contribution by Ramey and Shapiro (2001).

Finally, we study the role of monetary policy in determining RER persistence. Our main conclusions are essentially unchanged when we switch from models in which nominal aggregate demand is assumed to be exogenous and monetary policy is left unspecified, to models with an interest rate rule subject to persistent monetary shocks. However, we find that when the monetary policy rule displays a strong interest rate smoothing component, even our multisector sticky-price model fails to generate enough RER persistence in response to monetary shocks. This result highlights the importance of the empirical debate about the source of the high degree of interest rate persistence observed in the data – whether it stems from persistent shocks, or from policy inertia.

References

[1] Ball, L. and D. Romer (1990), "Real Rigidities and the Non-Neutrality of Money," *Review of Economic Studies* 57: 183-203.

[2] Calvo, G. (1983), "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* 12: 383-98.

[3] Carvalho, C. and F. Nechio (2011), "Aggregation and the PPP Puzzle in a Sticky Price Model," *American Economic Review* 101: 2391-2424

[4] Chari, V., P. Kehoe, and E. McGrattan (2000), "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?," *Econometrica* 68: 1151-1180.

[5] ____________ (2002), "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?," *Review of Economic Studies* 69*:* 533-63.

[6] Coibion, O. and Y. Gorodnichenko (2012), "Why Are Target Interest Rate Changes So Persistent?," forthcoming in *American Economic Journal: Macroeconomics.*

[7] Galí, J. (2008), *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press.

[8] Martinez-Garcia, E. and J. Søndergaard (2010), "Investment and Real Exchange Rates in Sticky Price Models," forthcoming *Macroeconomic Dynamics.*

[9] Kilian, L. and T. Zha (2002), "Quantifying the uncertainty about the half-life of deviations from PPP," *Journal of Applied Econometrics*, John Wiley & Sons, Ltd. 17: 107-125.

[10] Morshed, M. and S. Turnovsky, (2004), "Sectoral Adjustment Costs and Real Exchange Rate Dynamics in a Two-Sector Dependent Economy," *Journal of International Economics* 63: 147-177.

[11] Murray, C. and D. Papell (2002), "The Purchasing Power Parity Persistence Paradigm," *Journal of International Economics* 56: 1-19.

[12] Nakamura, E. and J. Steinsson (2008), "Five Facts About Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics* 123: 1415-1464.

[13] Ramey, V. and M. Shapiro (2001), "Displaced Capital: A Study of Aerospace Plant Closings," *Journal of Political Economy* 109: 958-992.

[14] Rogoff, K. (1996), "The Purchasing Power Parity Puzzle," *Journal of Economic Literature* 34: 647-668.

[15] Rossi, B. (2005), "Confidence Intervals for Half-Life Deviations From Purchasing Power Parity," *Journal of Business & Economic Statistics*, 23: 432-442.

[16] Rudebusch, G. (2002), "Term structure evidence on interest rate smoothing and monetary policy inertia," *Journal of Monetary Economics*, 49: 1161-1187.

[17] Steinsson, J. (2008), "The Dynamic Behavior of the Real Exchange Rate in Sticky-Price Models," *American Economic Review* 98: 519-533.

[18] Taylor, J. (1993), "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy* 39: 195-214.

[19] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

[20] ____________ (2005), "Firm-Specific Capital and the New Keynesian Phillips Curve," *International Journal of Central Banking* 1(2).

A The approximating 3-sector economy

Here we show that a model with three sectors, with suitably chosen degrees of price stickiness and sectoral weights, provides an extremely good approximation to the original 67-sector economy. We choose the sectoral weights and frequencies of price changes to match the following moments of the distribution of price stickiness from our baseline parametrization with 67 sectors: average frequency of price changes ($\overline{\alpha} = \sum_{s=1}^{S} f_s \alpha_s$), cross-sectional average of the expected durations of price spells ($\overline{d} \equiv \sum_{s=1}^{S} f_s \alpha_s^{-1}$), cross-sectional standard deviation of the expected durations of price spells ($\sigma_d = \sqrt{\sum_{s=1}^{S} f_s \left(\alpha_s^{-1} - \overline{d}\right)^2}$), skewness of the cross-sectional distribution of expected durations of price spells ($\mathcal{S}_d = \frac{1}{\sigma_d^3} \sum_{s=1}^{S} f_s \left(\alpha_s^{-1} - \overline{d}\right)^3$), and kurtosis of the cross-sectional distribution of expected durations of price spells ($\mathcal{K}_d = \frac{1}{\sigma_d^4} \sum_{s=1}^{S} f_s \left(\alpha_s^{-1} - \overline{d}\right)^4$).[15]

We present our findings in Figure 2. It shows the impulse response functions of the aggregate real exchange rate to a nominal shock in Home in our baseline multisector economy, and in the approximating three-sector economy obtained with the moment-matching exercise described above.[16] The three-sector economy provides a very good approximation to our multisector economy, which justifies our use of the approximating model to save on computational time.

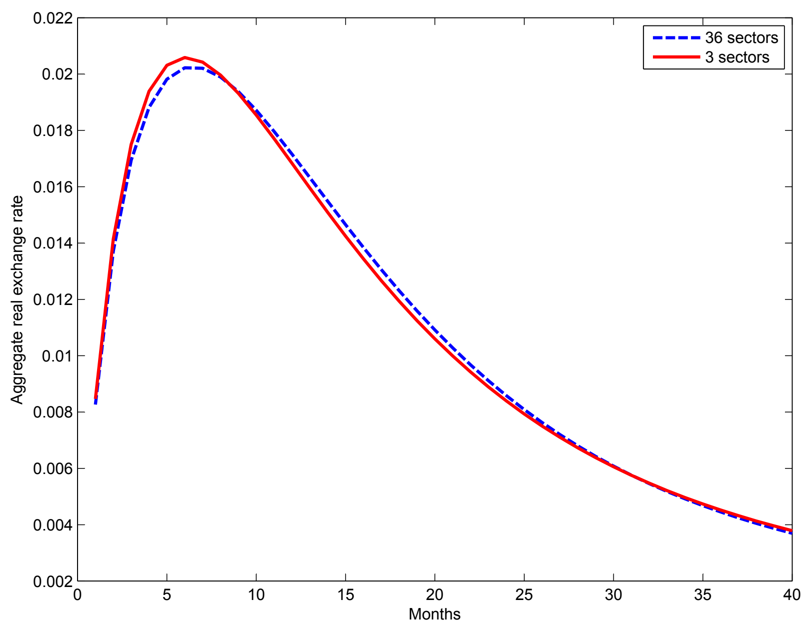

Figure 2: Impulse response functions of aggregate real exchange rate in baseline 3- and 67-sector economies

[15] We have 5 degrees of freedom (2 weights and 3 frequencies of price change) to match 5 moments from the distribution of price stickiness in the 67-sector economy.

[16] The values of all other parameters are the same as in the baseline model.

B Sector-specific capital and labor markets

In this section we present a model in which both capital and labor are sector specific.

B.1 Consumers

The Home representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{s=1}^{S} \omega_s \frac{N_{s,t}^{1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t C_t + P_t I_t + E_t \left[\Theta_{t,t+1} B_{t+1} \right] \leq \sum_{s=1}^{S} W_{s,t} N_{s,t} + B_t + T_t + \sum_{s=1}^{S} Z_{s,t} K_{s,t},$$

the law of motion for the stocks of sector-specific capital:

$$\begin{aligned} K_{s,t+1} &= (1-\delta) K_{s,t} + \Phi \left(I_{s,t}, I_{s,t-1}, K_{s,t} \right) I_{s,t}, \\ I_{s,t} &\geq 0, \end{aligned}$$

and a standard "no-Ponzi" condition:

$$B_{t+1} \geq -\sum_{l=t+1}^{\infty} E_{t+1} \left[\Theta_{t+1,l} \left(\sum_{s=1}^{S} \left(W_{s,l} N_{s,l} + Z_{s,l} K_{s,l} \right) + T_l \right) \right] \geq -\infty.$$

The notation is the same as before, except that now $N_{s,t}$ denotes total labor supplied to firms in sector s, $W_{s,t}$ is the associated nominal wage rate, and ω_s is the relative disutility of supplying labor to sector s.[17] $I_{s,t}$ denotes investment in sector-s capital, $I_t \equiv \sum_{s=1}^{S} I_{s,t}$, $K_{s,t}$ is physical capital supplied to firms in sector s, and $Z_{s,t}$ is the associated nominal return on capital.

The first-order conditions for consumption and labor are now:

$$\frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} = \frac{\beta^l}{\Theta_{t,l}} \frac{P_t}{P_{t+l}},$$

$$\frac{W_{s,t}}{P_t} = \omega_s N_{s,t}^{\gamma} C_t^{\sigma}, \ \forall \ s.$$

For all investment $I_{s,t}$ and capital $K_{s,t+1}$ types:

$$Q_{s,t} = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left(\frac{Z_{s,t+1}}{P_{t+1}} + Q_{s,t+1} \left[(1-\delta) + \Phi' \left(\frac{I_{s,t+1}}{K_{s,t+1}} \right) \left(\frac{I_{s,t+1}}{K_{s,t+1}} \right)^2 \right] \right) \right\}, \tag{15}$$

[17] This parameter is only used to obtain a symmetric steady state, and simplify the algebra. It does not play a role in any of our findings.

$$Q_{s,t}\left(\Phi'\left(\frac{I_{s,t}}{K_{s,t}}\right)\frac{I_{s,t}}{K_{s,t}}+\Phi\left(\frac{I_{s,t}}{K_{s,t}}\right)\right)=1, \tag{16}$$

where $Q_{s,t}$ denote Tobin's q for sector s.

The solution must also satisfy a transversality condition:

$$\lim_{l\to\infty} E_t\left[\Theta_{t,l}B_l\right]=0.$$

The problems for the representative consumer in Foreign are analogous.

B.2 Intermediate goods firms

Once we introduce sectoral capital and labor markets, the intermediate-goods producers problem also changes. It becomes:

$$\begin{aligned}
&\max\ E_t\sum_{l=0}^{\infty}\Theta_{t,t+l}\left(1-\alpha_s\right)^l\left[\begin{array}{c} X_{H,s,j,t}Y_{H,s,j,t+l}+\mathcal{E}_{t+l}X^*_{H,s,j,t}Y^*_{H,s,j,t+l} \\ -W_{s,t+l}N_{s,j,t+l}-Z_{s,t+l}K_{s,j,t+l}\end{array}\right] \\
st\qquad &Y_{H,s,j,t}=\omega\left(\frac{P_{H,s,j,t}}{P_{H,s,t}}\right)^{-\theta}\left(\frac{P_{H,s,t}}{P_{s,t}}\right)^{-\rho}\left(\frac{P_{s,t}}{P_t}\right)^{-\eta}Y_t \\
&Y^*_{H,s,j,t}=(1-\omega)\left(\frac{P^*_{H,s,j,t}}{P^*_{H,s,t}}\right)^{-\theta}\left(\frac{P^*_{H,s,t}}{P^*_{s,t}}\right)^{-\rho}\left(\frac{P^*_{s,t}}{P^*_t}\right)^{-\eta}Y^*_t \\
&Y_{H,s,j,t}+Y^*_{H,s,j,t}=F\left(K_{s,j,t}N_{s,j,t}\right)=\left(K_{s,j,t}\right)^{1-\chi}\left(N_{s,j,t}\right)^{\chi}.
\end{aligned}$$

Optimal price setting implies:

$$\begin{aligned}
X_{H,s,j,t} &= \frac{\theta}{\theta-1}\frac{E_t\sum_{l=0}^{\infty}\Theta_{t,t+l}\left(1-\alpha_s\right)^l\Lambda_{H,s,t+l}\left(\chi K^{1-\chi}_{s,j,t+l}N^{\chi-1}_{s,j,t+l}\right)^{-1}W_{s,t+l}}{E_t\sum_{l=0}^{\infty}\Theta_{t,t+l}\left(1-\alpha_s\right)^l\Lambda_{H,s,t+l}}, \\
X^*_{H,s,j,t} &= \frac{\theta}{\theta-1}\frac{E_t\sum_{l=0}^{\infty}\Theta_{t,t+l}\left(1-\alpha_s\right)^l\Lambda^*_{H,s,t+l}\left(\chi K^{1-\chi}_{s,j,t+l}N^{\chi-1}_{s,j,t+l}\right)^{-1}W_{s,t+l}}{E_t\sum_{l=0}^{\infty}\Theta_{t,t+l}\left(1-\alpha_s\right)^l\mathcal{E}_{t+l}\Lambda^*_{H,s,t+l}},
\end{aligned}$$

where:

$$\begin{aligned}
\Lambda_{H,s,t} &= \omega\left(\frac{1}{P_{H,s,t}}\right)^{-\theta}\left(\frac{P_{H,s,t}}{P_{s,t}}\right)^{-\rho}\left(\frac{P_{s,t}}{P_t}\right)^{-\eta}Y_t, \\
\Lambda^*_{H,s,t} &= (1-\omega)\left(\frac{1}{P^*_{H,s,t}}\right)^{-\theta}\left(\frac{P^*_{H,s,t}}{P^*_{s,t}}\right)^{-\rho}\left(\frac{P^*_{s,t}}{P^*_t}\right)^{-\eta}Y^*_t.
\end{aligned}$$

The real marginal cost can be written as:

$$MC_{s,j,t}=\frac{W_{s,t}/P_t}{\chi K^{1-\chi}_{s,j,t}N^{\chi-1}_{s,j,t}}.$$

Given the optimality (cost-minimization) conditions for capital and labor, the marginal cost above can also be expressed as:

$$MC_{s,j,t} = \frac{W_{s,t}/P_t}{\chi\left(\frac{(1-\chi)}{\chi}\frac{W_{s,t}}{Z_{s,t}}\right)^{1-\chi}} = \frac{1}{\chi^{\chi}(1-\chi)^{1-\chi}}\left(\frac{W_{s,t}}{P_t}\right)^{\chi}\left(\frac{Z_{s,t}}{P_t}\right)^{(1-\chi)} = MC_{s,t}.$$

Note that marginal costs are now equalized only within sectors. This is a direct implication of the assumption of sectoral factor markets. Finally, under this assumption, the market-clearing conditions for capital and labor become:

$$\begin{aligned} K_{s,t} &= f_s \int_0^1 K_{s,j,t}dj, \ \forall\ s, \\ N_{s,t} &= f_s \int_0^1 N_{s,j,t}dj, \ \forall\ s. \end{aligned}$$

B.3 Quantitative analysis

In this section we replicate the exercise of Section 2.5 using the model with sectoral capital and labor markets. As before, we recalibrate the intertemporal elasticity of substitution (σ^{-1}) and the investment adjustment-cost parameter (κ) to match the standard deviation of the RER and of investment in the data, both relative to the standard deviation of GDP. The results are reported in Table 4.

Table 4: Model with sector-specific capital and labor markets

		Sector-specific factors	
	Data	With capital	No capital
Persistence measures:			
$\mathcal{HL}$	36-60	32	29
$\mathcal{QL}$	76	46	39
$\mathcal{UL}$	28	20	19
ρ_1	0.78	0.84	0.71

For ease of comparison, the first column of the table replicates the data column from Table 1. The second column reports the results for the model with sectoral capital and labor markets, while the last column considers a version of the model without capital accumulation ($\chi = 1$). Comparing the last two columns, for the same level of RER volatility, the introduction of capital accumulation increases persistence of the real exchange rate, measured by its half-life, moving it closer to the empirical evidence. The same is true for the autocorrelation, the up-life, and the quarter-life. Hence, once again, the presence of capital accumulation does not alter the results significantly, and the multisector model is still able to generate a large degree of RER persistence.

Finally, a comparison of the results reported in Tables 1 and 4 shows that switching from economy-wide to sector-specific capital and labor markets decreases RER persistence.