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# Interest Rates Under Falling Stars

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## Abstract

Macro-finance theory implies that trend inflation and the equilibrium real interest rate are fundamental determinants of the yield curve. However, empirical models of the term structure of interest rates generally assume that these fundamentals are constant. We show that accounting for time variation in these underlying long-run trends is crucial for understanding the dynamics of Treasury yields and predicting excess bond returns. We introduce a new arbitrage-free model that captures the key role that long-run trends play in determining interest rates. The model also provides new, more plausible estimates of the term premium and accurate out-of-sample yield forecasts.

*Keywords:* yield curve, macro-finance, inflation trend, equilibrium real interest rate, shifting endpoints, bond risk premia

*JEL Classifications:* E43, E44, E47

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# 1 Introduction

Researchers have made numerous attempts to connect macroeconomic variables to the term structure of interest rates using a variety of approaches ranging from reduced-form no-arbitrage models to fully fledged dynamic macro models.<sup>1</sup> Despite both theoretical and empirical progress, there is no clear consensus about how macroeconomic information should be incorporated into yield curve analysis. One important link between the macroeconomy and the yield curve that has been largely overlooked is the connection between the time-varying long-run means of macroeconomic series and interest rates, that is, between their stochastic trends. Specifically, macroeconomic data and models can shed light on estimates of the trend in inflation ( $\pi_t^*$ ) and the equilibrium real interest rate ( $r_t^*$ ), while finance theory—from the Fisher equation to modern no-arbitrage theory—implies that such macroeconomic trends must be reflected in interest rates. Of course, the key outstanding issue is whether there is empirically significant variation in these long-run trends over time. Almost all term structure analyses in finance assume constant long-run means. Instead, in this paper, we document that accounting for changes in  $\pi_t^*$  and  $r_t^*$  is essential for modeling the term structure, estimating bond risk premia, and forecasting yields. We quantify the importance of time-varying macroeconomic trends for interest rates in two steps. First, using a variety of empirical proxies for these shifting endpoints, we establish some stylized facts about macroeconomic and interest rate trends. One of these stylized facts is the importance of such trends for predicting excess bond returns. Second, to account for these stylized facts, we introduce a new dynamic term structure model that embeds a stochastic interest rate trend. This no-arbitrage model provides new estimates of the long-run stochastic trend in Treasury yields and the term premium and accurately predicts interest rates out of sample. This analysis bridges an important gap between macroeconomics and finance by incorporating shifting macro-finance trends.

A simple illustration of the potential importance of macro trends for U.S. interest rates is provided in Figure 1. The underlying drivers of the apparent secular decline in longer-term Treasury yields since the early 1980s remain contentious. Standard finance (no-arbitrage) models of the yield curve assume that interest rates are stationary, i.e., mean-reverting—see [Kim and Wright \(2005\)](#), [Joslin et al. \(2011\)](#), and [Adrian et al. \(2013\)](#), among many others. This mean reversion implies that expected short-term rates are invariably close to their historical average at horizons beyond a few years ahead. That is, the expectations component of long-term interest rates varies only modestly. Consequently, low-frequency variation in interest rates must be captured by the term premium, which is the residual difference between a long-

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<sup>1</sup>See, among many others, [Ang and Piazzesi \(2003\)](#), [Diebold et al. \(2006\)](#), [Bikbov and Chernov \(2010\)](#), and [Rudebusch and Swanson \(2012\)](#). A detailed survey is [Gürkaynak and Wright \(2012\)](#).

term interest rate and its expectations component (e.g., [Wright, 2011](#)). Instead, our analysis stresses the important role of the time-varying trends underlying interest rates. To illustrate this role, [Figure 1](#) displays observable proxies of these trends (described further in [Section 2](#)). Trend inflation,  $\pi_t^*$ , measured using long-horizon survey forecasts, declined almost 6 percentage points from the Volcker disinflation of the early 1980s to the late 1990s and then remained stable around 2 percent. In contrast, the trend or equilibrium real interest rate,  $r_t^*$ —shown here as the average of several estimates—changed little early on but declined notably over the past two decades, likely reflecting structural changes in the global economy. As a result, the equilibrium nominal short-term interest rate,  $i_t^* = \pi_t^* + r_t^*$ , which reflects movements in both underlying macro trends, accounts for much of the low-frequency movements in the ten-year Treasury yield through variation in the expectations component. This suggests that the early downward trend in  $\pi_t^*$  and the more recent decline in  $r_t^*$ —that is, an environment of falling stars—played a major role in the secular decline in nominal interest rates.

This link between macro trends and interest rate trends is at the core of our analysis. We first establish two stylized facts about macro-finance trends and yields using empirical proxies for these trends.<sup>2</sup> First, variation in both the inflation trend and the equilibrium real interest rate is responsible for the very high persistence of interest rates. It has long been recognized that nominal interest rates of different maturities contain a slow-moving common trend component and are best modelled as cointegrated ([Campbell and Shiller, 1987](#); [Hall et al., 1992](#)). We show that this common trend is  $i_t^*$  and that it is driven by quantitatively important fluctuations in both underlying macroeconomic trends,  $\pi_t^*$  and  $r_t^*$ . Accounting only for the inflation trend on its own, as in [Kozicki and Tinsley \(2001\)](#) and [Cieslak and Povala \(2015\)](#), leaves a highly persistent component of interest rates unexplained. We show that it is necessary to include movements in  $r_t^*$  to fully capture the trend component in interest rates.

Our second stylized fact is that macro trends are crucial for understanding bond risk premia. Relative to the canonical predictive regressions for excess bond returns that forecast future excess bond returns with current yields ([Fama and Bliss, 1987](#); [Campbell and Shiller, 1991](#); [Cochrane and Piazzesi, 2005](#)), we find that adding  $\pi_t^*$  and  $r_t^*$  as regressors provides notable gains in predictive power. Our findings provide evidence against the *spanning hypothesis* implied by most asset pricing models, which posits that the current yield curve contains all the relevant information for predicting future interest rates (e.g., [Duffee, 2013](#); [Bauer and Rudebusch, 2017](#); [Bauer and Hamilton, 2018](#)). The predictive gains are economically large,

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<sup>2</sup>Along with a survey-based  $\pi_t^*$ , we use existing estimates of  $r_t^*$  from [Laubach and Williams \(2003, 2016\)](#), [Kiley \(2015\)](#), [Holston et al. \(2017\)](#), [Del Negro et al. \(2017\)](#), and [Johannsen and Mertens \(2016, 2018\)](#). For robustness, we also consider four  $r_t^*$  estimates that we create, which range from a simple moving-average of the real rate to a multivariate estimate.

so, for example, when interest rates adjust to a lower trend, long-term bond holders benefit substantially, just as they did during the adjustment to a lower  $r_t^*$  following the 2008 financial crisis. Indeed, including the equilibrium real rate is especially important later in our sample when the inflation trend shows less variation.<sup>3</sup>

To account for these two stylized facts, we introduce a new dynamic term structure model (DTSM) with a shifting endpoint for interest rates. This novel no-arbitrage specification parsimoniously models the Treasury yield curve using four state variables: three “yield factors” which are simply linear combinations of yields, and one common stochastic trend,  $i_t^*$ , which captures movements in the underlying macroeconomic trends. The existence of a common trend in interest rates stands in stark contrast to the usual stationary DTSMs in which yield factors follow a stationary VAR(1). Furthermore, in our model the trend is *unspanned* or hidden, so it cannot be backed out from the cross section of yields. This specification is consistent with our evidence showing that adding  $i_t^*$  improves predictions of bond returns. In addition, an unspanned  $i_t^*$  need not follow Treasury yields in lockstep and can play a similar role as the trend component in an unobserved components model. For robustness, we employ two different estimation methodologies, which provide similar results. First, we estimate the model using an observed proxy for  $i_t^*$  in order to carry out a model-based analysis using the same real-time interest rate trend we employed in establishing the stylized facts about macro-finance trends. Second, we use Bayesian estimation of our model to infer  $i_t^*$  using only Treasury yields, imposing a certain degree of smoothness on our trend estimate, similar to [Del Negro et al. \(2017\)](#).

We then show that simulated data from these empirical specifications are able to match the stylized facts that we established about the role of trends for the yield curve. The model-based estimates of the trend and cycle components of interest rates are consistent with the cointegration regression results from the actual data. Also, the model accounts for the additional predictive power for future bond returns provided by the trend underlying interest rates as was evident in our historical data sample. Specifically, predictive regressions using simulated data from our model demonstrate notable increases in  $R^2$  when the interest rate trend is added as a predictor—just as in the actual data.

We also use our shifting-endpoint model to investigate two important applications of DTSMs: (1) decomposing long-term interest rates into an expectations component and a term premium and (2) forecasting interest rates. For the former, our model provides markedly

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<sup>3</sup>The improvement we document using trends to predict excess bond returns dovetails with recent research showing predictive gains from including slow-moving averages of past inflation, real output growth, and consumption growth as predictors (e.g., [Cieslak and Povala, 2015](#); [Brooks and Moskowitz, 2017](#); [Garg and Mazzoleni, 2017](#); [Jorgensen, 2017](#))—variables that are closely related to  $\pi_t^*$  and  $r_t^*$ .

different decompositions of long-term rates than the usual stationary DTSM. A stationary DTSM implies a quite stable expectations component and attributes most of the secular decline in interest rates to the residual term premium, as discussed in critiques by [Kim and Orphanides \(2012\)](#) and [Bauer et al. \(2014\)](#). Instead, our shifting-endpoint model decomposition attributes the majority of that secular decline to a fall in long-term expectations driven by  $i_t^*$ . Consequently, our term premium estimates exhibit only a modestly decreasing trend and more pronounced cyclical swings, similar to other risk premia in asset prices ([Fama and French, 1989](#)). As for out-of-sample interest rate forecasting, previous researchers have found it surprisingly difficult to consistently beat the simple random walk forecast, which predicts that a future yield will equal the current yield. But we find that forecasts implied by our shifting-endpoint DTSM are generally more accurate than this martingale benchmark and also better than long-range projections from the Blue Chip survey of professional forecasters. By contrast, the standard stationary DTSM (corresponding to the special case of our model with a constant  $i^*$ ) forecasts substantially worse than the random walk. The improvements in forecast accuracy from allowing a slow-moving shifting endpoint are both economically and statistically significant, and they are consistent with yields returning over time to their underlying macro trend.<sup>4</sup>

Our analysis is related to several different strands of the macro and finance literatures. Some studies have described a link between the inflation trend and nominal yields ([Kozicki and Tinsley, 2001](#); [van Dijk et al., 2014](#); [Cieslak and Povala, 2015](#)), but this leaves unexplained the continuing decline in yields over the past 20 years. Time variation in  $r_t^*$  has so far received little attention in finance, despite growing evidence of a sizable decline in the equilibrium real interest rate.<sup>5</sup> Our paper comprehensively documents the empirical importance of both relevant macro trends,  $\pi_t^*$  and  $r_t^*$ , for the dynamics of the yield curve.

No-arbitrage models of the yield curve typically rule out time-varying trends by specifying that the dynamics of yields are stationary. However, such models are ill-suited to capture the extreme persistence of interest rates, and a variety of remedies have been suggested.<sup>6</sup> Our

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<sup>4</sup>In related work, [van Dijk et al. \(2014\)](#) documented forecast improvements relative to a random walk by including shifting endpoints based on their proxy of  $\pi_t^*$ . We demonstrate how to achieve even greater forecast gains in an arbitrage-free DTSM framework that accounts for time-varying  $\pi_t^*$  and  $r_t^*$ .

<sup>5</sup>The economic forces reducing the equilibrium real interest rate likely include lower productivity growth, changing demographics, a decline in the price of capital goods, and strong precautionary saving flows from emerging market economies, which have tended to increase global savings, reduce desired investment, and push down the steady-state real interest rate. Discussions include [Summers \(2014\)](#), [Kiley \(2015\)](#), [Rachel and Smith \(2015\)](#), [Hamilton et al. \(2016\)](#), [Laubach and Williams \(2016\)](#), [Johannsen and Mertens \(2016, 2018\)](#), [Christensen and Rudebusch \(2019\)](#), [Del Negro et al. \(2017\)](#), [Holston et al. \(2017\)](#) and [Lunsford and West \(2019\)](#). In macroeconomics,  $r_t^*$  is often labeled the neutral or natural rate of interest although, as noted below, there can be subtle differences among various definitions.

<sup>6</sup>Proposed remedies include adding survey information ([Kim and Wright, 2005](#); [Kim and Orphanides, 2012](#)),

results suggest that allowing for shifting long-run trends is a better solution. The idea of a shifting-endpoint term structure model goes back to [Kozicki and Tinsley \(2001\)](#), who specified a restrictive two-factor DTSM that imposed the expectations hypothesis and identified  $i_t^*$  with a distant-horizon forward rate (with a unit root under both the risk-neutral and the real-world measure). Some macro-finance models of the yield curve allow for changes in the inflation trend but still assume a constant equilibrium real rate ([Hördahl et al., 2006](#); [Rudebusch and Wu, 2008](#)). The model of [Campbell et al. \(2017\)](#), which is designed to capture the changing comovement of stock and bond returns, is an asset pricing model with a time-varying  $\pi_t^*$  and constant  $r^*$ . A few no-arbitrage models allow for changes in  $r_t^*$ , but they also make strong assumptions about this trend by tightly linking it to  $\pi_t^*$  ([Dewachter and Lyrío, 2006](#)) or trend output growth ([Dewachter and Iania, 2011](#)). Our model allows both stochastic macroeconomic trends to affect term structure dynamics. We also depart from the existing literature by treating  $i_t^*$  as unspanned, which allows this trend to exhibit some variation independent of contemporaneous yields.

Three recent papers propose statistical models of interest rates with time-varying long-run means, and are thus closely related to ours. [Del Negro et al. \(2017\)](#) estimate a Bayesian common-trend VAR for interest rates and inflation that allows for time variation in  $\pi_t^*$  and  $r_t^*$ . We use their estimates of  $r_t^*$ , along with others, in our analysis establishing stylized facts. [Johannsen and Mertens \(2018\)](#) also propose a time series model for yields and a Bayesian estimation framework that allows for stochastic volatility, and we also include their real-time  $r_t^*$  among our trend proxies. Finally, [Crump et al. \(2018\)](#) allow for shifting macroeconomic trends in a common-trend VAR for interest rates and inflation with  $i_t^*$ ,  $r_t^*$ , and  $\pi_t^*$  pinned down from surveys of professional forecasters. These three papers provide complementary time series analyses to our approach. However, our arbitrage-free specification, which jointly captures the time series and cross section of Treasury yields, allows us to investigate the role of trends for the yield curve, the term premium and expected bond returns.

The paper is structured as follows. Section 2 describes key concepts and our initial proxies for the macroeconomic trends. Section 3 presents stylized facts about the macroeconomic trends and Treasury yields. Section 4 introduces our no-arbitrage model, shows how it captures the stylized facts, and presents novel estimates of the term premium as well as out-of-sample forecast results. Section 5 concludes. The Appendix contains further details, and a separate Online Appendix<sup>7</sup> contains supplemental material.

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restricting risk prices ([Joslin et al., 2014](#); [Bauer, 2018](#)), correcting for small-sample bias ([Bauer et al., 2012, 2014](#)), and using near-integration or long memory processes ([Jardet et al., 2013](#); [Goliński and Zaffaroni, 2016](#)).

<sup>7</sup>The Online Appendix is available at [https://www.michaeldbauer.com/files/br\\_trends\\_online\\_appendix.pdf](https://www.michaeldbauer.com/files/br_trends_online_appendix.pdf).

## 2 Interest rate trends: theory and estimates

The role of trends for interest rates can be illustrated with the usual decomposition of any  $n$ -period yield,  $y_t^{(n)}$ , into expectations of short-term rates and a term premium,  $TP_t^{(n)}$ :

$$y_t^{(n)} = \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j} + TP_t^{(n)} = i_t^* + \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^c + TP_t^{(n)}, \quad (1)$$

where  $i_t$  is the nominal short rate,  $y_t^{(1)}$ .<sup>8</sup> The term premium compensates bond investors for the duration risk in longer-term bonds, though in practice, it also captures liquidity and financial market frictions. The expectations theory of interest rates assumes that the term premium is zero (strong form) or constant (weak form), whereas modern finance theories of interest rates explicitly model the time variation in the term premium, as described in Section 4. The second equality in equation (1) introduces the trend or equilibrium short rate,  $i_t^*$ , and the short rate cycle,  $i_t^c = i_t - i_t^*$ . We define  $i_t^*$  as the Beveridge-Nelson trend in the short-term nominal interest rate,

$$i_t^* \equiv \lim_{j \rightarrow \infty} E_t i_{t+j}. \quad (2)$$

We employ this stochastic trend concept throughout this paper and assume no deterministic time trends. All nominal yields are affected by changes in  $i_t^*$ , and if this trend is time-varying, then yields are nonstationary with a common trend. In this case, [Kozicki and Tinsley \(2001\)](#) fittingly call  $i_t^*$  a “shifting endpoint.” The Fisher equation suggests that two macroeconomic trends determine this endpoint:

$$i_t = r_t + E_t \pi_{t+1} \quad \Rightarrow \quad i_t^* = r_t^* + \pi_t^*,$$

where  $r_t$  is the real short rate,  $\pi_t$  is inflation, and the trends in  $r_t$  and  $\pi_t$  are defined analogously to  $i_t^*$ . If inflation or the real interest rate contain long-run trend components, then nominal interest rates will also be driven by these same macroeconomic trends.

The remainder of this section describes the empirical proxies of these trends that we employ to establish stylized facts about macro-finance trends in interest rates.<sup>9</sup> For the long-run inflation trend, the prevailing consensus in empirical macroeconomics models inflation as an  $I(1)$  process—in order to produce accurate forecasts and correctly capture the evolution of

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<sup>8</sup>This intuitive definition of the term premium—the expected excess log return on the long-term bond held to maturity over a strategy of rolling over short-term bonds—is common in macroeconomics. It differs from the definition used in DTSMs by a Jensen inequality term, but in homoskedastic models such as the one proposed in this paper, this variance correction term is a constant.

<sup>9</sup>Appendix A provides an overview of all trend estimates.

inflation expectations (Stock and Watson, 2007; Faust and Wright, 2013). From a macroeconomic perspective, the resulting time-varying  $\pi_t^*$  can be viewed as the perceived inflation target of the central bank. Empirical proxies for trend inflation have been constructed from surveys, statistical models, or a combination of the two—see, for example, Stock and Watson (2016) and the references therein. We employ a well-known, widely used survey-based measure of long-run inflation expectations, the Federal Reserve’s series on the perceived target rate (PTR) for inflation (e.g., Clark and McCracken, 2013; Del Negro et al., 2017). Figure 1 shows that, from the beginning of our sample to the late 1990s, this estimate generally mirrored the rise and fall of the ten-year yield.<sup>10</sup> Since then, however, it has remained close to 2 percent, which has been the explicit longer-run inflation goal of the Federal Reserve since 2012. Given our focus on forecasting, real-time availability is an important issue, and for most of our sample (since 1979) the PTR estimate was available in real time from the Survey of Professional Forecasters.

As a proxy for  $r_t^*$ , we use a variety of estimates to avoid reliance on one specific model or estimation approach. All of these estimates are consistent with our definition of  $r_t^*$  as the stochastic trend in the real short rate, analogous to equation (2). The first two  $r_t^*$  estimates are the long-run real rate trends from time series models identified with Bayesian methods: Del Negro, Giannone, Giannoni, and Tambalotti (2017)(DGGT) estimate a linear state space model with common trends, and Johansen and Mertens (2016, 2018)(JM) employ a nonlinear state space model with a shadow rate. Three additional estimates are based on simple New Keynesian macro models and use the Kalman filter to infer the *neutral* real interest rate, that is, the level of the real rate consistent with real output at potential and inflation at target: Laubach and Williams (2003, 2016) (LW) implemented this concept empirically using the Rudebusch and Svensson (1999) model; Holston, Laubach, and Williams (2017) (HLW) use a slight simplification of the LW model; and Kiley (2015) augments the LW model to account for changes in financial conditions. The implementation of these macro models assumes that the neutral rate follows a martingale; thus, their  $r_t^*$  estimates are consistent with the long-run concept we employ. We also construct three model-based estimates of our own. The first is based on a simple univariate unobserved components model, and the second uses the same model augmented with moving averages of real GDP and labor force growth that appear to be correlated with the real rate trend (Lunsford and West, 2019). The third estimate uses the observed nominal short rate and inflation in a state-space model with a latent real rate and  $r_t^*$ . We estimate these models using Bayesian methods, and like Del Negro et al. (2017) we

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<sup>10</sup>Other estimates of the inflation trend, such as the long-range forecasts in the Blue Chip survey, the moving-average estimate of Cieslak and Povala (2015), or various model-based estimates (Clark and Doh, 2014; Stock and Watson, 2016) exhibit a similar pattern.

employ a prior with a low innovation variance for  $r_t^*$ . Finally, as a simple, model-free estimate of  $r_t^*$ , we include an exponentially-weighted moving average of past real rates.<sup>11</sup>

Figure 2 summarizes these  $r_t^*$  estimates. The left panel shows the average and range of the smoothed (two-sided) estimates, which use the full data sample for inference about the parameters and state variables. These show a relatively stable  $r_t^*$  until the late 1990s, but then exhibit a pronounced decline. To better capture what bond market investors could have known about macro trends contemporaneously, our analysis will focus on filtered (one-sided) and real-time estimates of  $r_t^*$ , which are shown in the right panel of Figure 2. Filtered estimates—from the LW, HLW, and Kiley models—infer latent state variables at time  $t$  using data available up to time  $t$ . However, they also use model parameter estimates from the full sample, which would not have been available to investors in real time.<sup>12</sup> Therefore, to avoid the pitfall of look-ahead bias in our analysis of predictive power, we consider (pseudo) real-time estimates that only use data through period  $t$  to construct  $r_t^*$ . Such real-time estimates are available for the JM and DGGT models, our own three models, and the moving-average of past real rates.<sup>13</sup> As shown in Figure 2, the average of these six real-time estimates increased from about one to two percent early in the sample, moved sideways for about 15 years, and then around 2000 started a substantial, steady decline until the end of the sample. This pattern closely follows the movements of the moving-average estimate of  $r_t^*$ , which has the added benefit of not relying on a model or estimation technique unavailable to investors historically. Finally, the average of all nine filtered and real-time  $r_t^*$  estimates, which is displayed in Figure 1, also shows a similar pattern.

Substantial estimation and model uncertainty underlie estimates of any long-run trends, as suggested by the range of estimates in Figure 2. To reduce the uncertainty inherent in individual estimates, we take averages of  $r^*$ -estimates across models. While this approach is in no way a cure-all for estimation uncertainty and measurement error, model-averaging is well-known to improve forecast accuracy (Timmermann, 2006).<sup>14</sup> Our baseline estimate of  $i_t^*$  for the analysis in Section 3 will be the sum of the PTR estimate of  $\pi_t^*$  and the average of all six real-time estimates of  $r_t^*$ .<sup>15</sup>

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<sup>11</sup>Our smoothing parameter of 0.98 follows other research estimating macro trends (Malmendier and Nagel, 2011; Orphanides and Williams, 2005).

<sup>12</sup>Still, Laubach and Williams (2016) argue that, for the past decade, truly real-time  $r_t^*$  estimates are similar to their filtered estimates.

<sup>13</sup>These estimated trends all rely on revised inflation data, and the multivariate model also uses revised output data. Although the use of real-time data vintages can be important for near-term forecasting, it is arguably less relevant for estimation of long-run trends.

<sup>14</sup>Because we find a close link between the yield curve and our noisy estimates of the macro trends, our results are likely somewhat conservative regarding the tightness of the connection between the yield curve and the *true* underlying trends (as discussed in the Online Appendix).

<sup>15</sup>An alternative proxy for  $i_t^*$  could be obtained from surveys, such as the long-range forecast for the three-

## 3 Macro trends and yields: stylized facts

### 3.1 The common trend in nominal yields

We first investigate whether macroeconomic trends account for the high persistence of interest rates. We then assess whether accounting for just the inflation trend is sufficient or if a real rate trend also materially contributes to movements in nominal bond yields. Our proxies for these macro trends are the PTR survey-based estimate of  $\pi_t^*$  and the three alternative estimates of  $r_t^*$ : the average of three filtered estimates, the average of the six real-time estimates, and a simple (exponentially weighted) moving average of past real rates. We proxy for the equilibrium nominal short rate  $i_t^*$  with the sum of  $\pi_t^*$  and the real-time average estimate of  $r_t^*$  (but obtain similar results with the filtered or moving-average measures).

We find that variation in the macro trends accounts for the persistence of interest rates, and the trend and cycle components of interest rates can be identified using our proxy  $i_t^*$ . This is illustrated in Figure 3, which shows the (demeaned) ten-year yield and two detrended versions of that yield.<sup>16</sup> The first detrended yield is the simple difference between the yield and  $i_t^*$ , which should be stationary if the yield is indeed driven by this trend and the loading on  $i_t^*$  is unity. In comparison to the ten-year yield, this series exhibits much less of a trend, for example, declining by less than half as much from the early 1980s to the end of our sample. Still, a notable trend remains in this simple difference, which indicates that the yield trend component moves more than one-for-one with  $i_t^*$ . By definition, the expectations component of yields moves one-for-one with  $i_t^*$ , so the ten-year term premium must contain a trend component positively related to  $i_t^*$ . To account for this relationship, the second detrended series is the residual from a (Dynamic OLS) cointegration regression of the yield on  $i_t^*$ , discussed below. This cointegration residual shows no discernible trend, indicating that changes in  $i_t^*$ , properly scaled, can fully capture the trend in the ten-year Treasury yield. The fitted value of the cointegration regression is the trend component of the ten-year yield, for which we also obtain a model-based estimate in Section 4.3, and the residual—shown in Figure 3—corresponds to the interest rate cycle.

It has long been known that nominal interest rates are highly persistent.<sup>17</sup> Summary statistics and unit root tests reported in the Online Appendix confirm that Treasury yields are best modeled as having a stochastic trend in our sample. We examine whether this persistence

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month T-bill rate from the Blue Chip Financial Forecasts. This series, which is only available semiannually starting in 1986, fluctuates around our baseline estimate of  $i_t^*$ . We consider survey forecasts in Section 4.6.

<sup>16</sup>We use end-of-quarter zero-coupon Treasury yields from [Gürkaynak et al. \(2007\)](#) with maturities from one to 15 years and three-month and six-month Treasury bill rates from the Federal Reserve’s H.15 release, over the period from 1971:Q4 to 2018:Q2.

<sup>17</sup>See, for example, [Nelson and Plosser \(1982\)](#), [Rose \(1988\)](#), and [King et al. \(1991\)](#).

is driven by a common macro trend. Figure 3 suggests a likely cointegration relationship with a loading on the trend larger than unity. To formally investigate this relationship, including the individual contributions of  $\pi_t^*$  and  $r_t^*$ , we use cointegration regressions of the form  $y_t^{(40)} = \beta_0 + \beta_1' X_t^* + u_t$ , where  $y_t^{(40)}$  is the ten-year yield, the vector  $X_t^*$  contains one or two macro trend proxies, and  $u_t$  is a stationary residual under cointegration. We estimate the coefficients  $\beta_0$  and  $\beta_1$  using the Dynamic OLS estimator of Stock and Watson (1993) by including four leads and lags of  $\Delta y_t^{(40)}$  and  $\Delta X_t^*$ .<sup>18</sup>

Table 1 shows the results of our cointegration analysis, including coefficient estimates, Newey-West standard errors (using six lags) and the regression  $R^2$ . The first regression specification includes only  $\pi_t^*$ , and the results echo those of Cieslak and Povala (2015), who used simple OLS regressions but a similar inflation trend proxy. The inflation trend coefficient is somewhat higher than one and strongly significant, and the  $R^2$  is very high. However, trend inflation does not fully account for the trend behavior of yields, as evident from the persistence of the cointegration residuals  $\hat{u}_t = y_t^{(40)} - \hat{\beta}_0 + \hat{\beta}_1 \pi_t^*$ . While  $\hat{u}_t$  is much less persistent than the ten-year yield, it is still quite persistent. As shown in the second column, for this series, unit root tests—the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests—do not reject the null, and the low-frequency stationary test of Müller and Watson (2013) rejects stationarity at the 5 percent level.<sup>19</sup> The inadequacy of detrending with only  $\pi_t^*$  is also supported by a Johansen (1991) trace test, which does not reject the null of absence of cointegration among the yield and the inflation trend.<sup>20</sup> Finally, an error-correction model, where  $\Delta y_t^{(40)}$  is regressed on  $\hat{u}_{t-1}$  and four lags each of the first-differenced yield and the inflation trend, indicates only modest equilibrating forces pushing yields toward the inflation trend.

The missing driver of the level of the yield curve is the real rate trend. The remaining columns of Table 1 show that there is strong evidence for cointegration once  $r_t^*$  is added. For all three  $r^*$  proxies, the estimated coefficients are at or above one and strongly significant. Adding the real rate trend increases the  $R^2$  by 8 to 11 percentage points, and the cointegration residual now appears to be stationary. For the real-time and moving-average estimates of  $r_t^*$ , the Johansen test indicates a cointegration rank of one, and the error-correction model shows that yields quickly remedy deviations from trend. The coefficients on the two macro trends are roughly equal, and the last column of Table 1 shows that  $i_t^*$  can fully capture the trend

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<sup>18</sup>In the Online Appendix, we report results for the level of the yield curve, which are similar to those for the ten-year yield. Also, standard OLS as well as the reduced-rank maximum likelihood VAR estimation of Johansen (1991) lead to the same conclusions as Dynamic OLS (not reported).

<sup>19</sup>For the unit root tests of cointegration residuals, we omit intercepts in the ADF and PP regressions and use the critical values provided by Phillips and Ouliaris (1990), which vary with the number of regressors (as detailed in the Online Appendix.)

<sup>20</sup>The test is based on a VAR representation with four lags, based on information criteria.

component in the ten-year yield, since the evidence for cointegration is as strong as when both macro trends are included individually. The cointegration coefficient is 1.67, suggesting a positive loading of the term premium on  $i_t^*$ , a result replicated by our model in Section 4.3. The error-correction coefficient is estimated as -0.45 and is strongly significant. Accordingly, when the ten-year yield is high relative to  $i_t^*$ , it tends to revert quickly toward this trend, with almost half of the equilibrium error on average reversed within one quarter. Therefore, knowledge of the macro trend appears quite useful when predicting future yields. In sum, these results indicate that the low-frequency behavior of Treasury yields is explained by movements in a common underlying macroeconomic trend,  $i_t^*$ , in which both  $\pi_t^*$  and  $r_t^*$  play important roles.

### 3.2 Predicting excess bond returns with macro trends

We now consider whether macroeconomic trends can help predict future excess bond returns. The one-quarter return on a bond with maturity  $n$  in excess of the risk-free rate is

$$rx_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)} = -(n-1)y_{t+1}^{(n-1)} + ny_t^{(n)} - y_t^{(1)},$$

where  $p_t^{(n)}$  denotes the log-price of a zero-coupon bond. Since Fama and Bliss (1987) and Campbell and Shiller (1991) it is well-known that the yield curve contains useful information for predicting excess bond returns. The key question is whether the current yield curve contains *all* of the information relevant for predicting future returns, that is, whether the *spanning hypothesis* holds. Cieslak and Povala (2015) showed that adding a proxy for the inflation trend significantly improved predictive power—an apparent violation of the spanning hypothesis. Bauer and Hamilton (2018) found that even after correcting for the small-sample econometric distortions associated with highly persistent regressors the trend inflation proxy remains a relatively robust predictor and thus is unspanned by the yield curve.<sup>21</sup>

To investigate this result more broadly, we examine the gains from including  $\pi_t^*$ ,  $r_t^*$ , and  $i_t^*$  in predictive regressions for excess bond returns. We predict the one-quarter excess return averaged across maturities from two to 15 years.<sup>22</sup> Table 2 reports results for six different predictive regressions. The first column is the usual baseline specification that includes only

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<sup>21</sup>Other empirical results contradict spanning in different ways. For example, Cochrane and Piazzesi (2005) find that lagged forward rates add predictive power to current forward rates.

<sup>22</sup>We use one-quarter holding periods to avoid the econometric problems resulting from overlapping observations. The one-quarter returns are calculated with the usual approximation  $y_{t+1}^{(n-1)} \approx y_t^{(n)}$ . We have obtained qualitatively similar results with annual returns (not reported). In addition, the Online Appendix provides estimates for individual bond maturities.

a constant and the first three principal components (PCs) of yields.<sup>23</sup> The specification in the second column adds  $\pi_t^*$ , and columns three through five include  $\pi_t^*$  as well as one of our three proxies of  $r_t^*$  in order to capture the effects of both trends. The final column adds only  $i_t^*$  instead of two separate macro trends. We report White standard errors as well as small-sample  $p$ -values for the macro trends using the parametric bootstrap of [Bauer and Hamilton \(2018\)](#) to obtain accurate small-sample tests even though regressors are persistent and correlated with lagged forecast errors.<sup>24</sup> The top panel uses the full sample of data, while the bottom panel considers a subsample from 1985 to 2018 for robustness. There is some evidence of a shift in the conduct of U.S. monetary policy in the mid-1980s, which would likely lead to a change in interest rate dynamics, so such a subsample is often considered for yield curve analysis (e.g., [Rudebusch and Wu, 2007](#); [Joslin et al., 2014](#)).

In the full sample, the inclusion of the inflation trend increases the predictive power substantially compared with using only yield curve information. Both the inflation trend and the level of yields (PC1) are highly significant (in addition to the slope, PC2), as in [Cieslak and Povala \(2015\)](#). However, adding  $r_t^*$  to the regression leads to further impressive gains in predictive power. Notably, for the moving-average estimate of  $r_t^*$ , the  $R^2$  jumps, the coefficients and significance for  $\pi_t^*$  and PC1 rise, and the coefficient on  $r_t^*$  is large and significant.<sup>25</sup> Not surprisingly given [Figure 1](#), the relative predictive power of the two macro trends changes in the more recent period as variation in  $r_t^*$  has become more pronounced. In the subsample starting in 1985, the inflation trend by itself is not statistically significant. Only with the addition of the equilibrium real rate do both trends matter for bond risk premia: the coefficients on  $\pi_t^*$  and PC1 more than double, the  $R^2$  increases substantially, and the coefficients on  $\pi_t^*$  and  $r_t^*$  are statistically significant. These results imply that the trend in the real interest rate is as important as, and recently more important than, the trend in inflation.

The inflation and real-rate trends have roughly equal coefficients (as for the cointegration regression). This suggests they play similar roles for yield dynamics and that their sum,  $i_t^*$ , can capture the relevant information. Indeed, as shown in the final column of [Table 2](#), including just  $i_t^*$  as a predictor provides the same strong predictive gains. The coefficient on  $i_t^*$  is of course negative: If the trend component increases, then interest rates also rise in

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<sup>23</sup>We scale the PCs such that they correspond to common measures of level, slope, and curvature, as in [Joslin et al. \(2014\)](#). For example, the loadings of yields on PC1 sum to one.

<sup>24</sup>We simulate 5000 bootstrap samples for yields and predictors under the spanning hypothesis, using separate (bias-corrected) VAR(1) models for yield factors and predictors. The small-sample  $p$ -values are the fractions of simulated samples in which the  $t$ -statistics of the macro trends are at least as large (in absolute value) as in the actual data.

<sup>25</sup>Changes in  $R^2$  can be difficult to interpret in the presence of highly persistent predictors, and the [Online Appendix](#) confirms these results using our bootstrap.

response, producing losses for bond holders.<sup>26</sup> The coefficient is also larger in magnitude than the coefficient on the yield curve level (PC1), consistent with the evidence in Sections 3.1 and 4 that the trend component of yields moves more than one-for-one with changes in  $i_t^*$ .

The earlier literature showed that a high yield curve *slope* predicts positive excess bond returns. Cochrane and Piazzesi (2005) also found additional predictive power in other aspects of the shape of the yield curve—captured in mid-curve forward rates or higher-order PCs. Our findings, by contrast, document the importance of the *level* of the yield curve as a powerful predictor. While in yields-only regressions, such as specification (1) in Table 2, the level appears irrelevant, it becomes a strong predictor once  $i_t^*$  is accounted for. This result implies that the level of interest rates matters for expected future bond returns, but that it needs to be viewed relative to its underlying yield trend. Furthermore, accounting for  $i_t^*$  makes the slope of the yield curve an even more powerful predictor.

In Table 3, we further investigate the importance of viewing interest rates relative to their stochastic trend by using stochastically detrended yields as predictors. The use of such predictors with low persistence has the added benefit that conventional, large-sample inference is more reliable (Bauer and Hamilton, 2018). We use the first three PCs of the 17 detrended yields as predictors, where each detrended yield is obtained as the cointegration residual from a regression of the yield on macro trends. As before, we consider three types of detrending: using only  $\pi_t^*$ , using both  $\pi_t^*$  and (real-time)  $r_t^*$ , as well as using  $i_t^*$ . In the full sample, all three detrending methods lead to substantial predictive gains compared to the baseline regression with just the first three PCs of the 17 original yields. Notably, the detrended level of the yield curve (PC1) becomes strongly significant. Detrending with  $i_t^*$  works particularly well, giving the largest gain of 11 percentage points. In the post-1985 sample, the  $R^2$  does not increase when detrending with only  $\pi_t^*$ , but it increases by 9 and 6 percentage points, respectively, when detrending with either both  $\pi_t^*$  and  $r_t^*$  or with only  $i_t^*$ . These estimates provide additional support for the importance of macro trends in predicting bond returns.

In sum, the key to predicting bond returns and estimating bond risk premia is to account for the long-run trend in interest rates. Both underlying macro trends have strong incremental predictive power for bond returns relative to the observed yield curve. That is, the long-run trend cannot be inferred from the cross-section of yields alone, so our results contradict the spanning hypothesis. An alternative method to uncover their predictive power is to detrend interest rates, but it is crucial to use  $i_t^*$  instead of  $\pi_t^*$  for that detrending.

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<sup>26</sup>Numerically, an increase in  $i_t^*$  by 0.1 percentage point predicts a decrease in the next quarter’s excess bond return by 0.45 percentage point. The Online Appendix shows that this coefficient rises with bond maturity.

## 4 A no-arbitrage model with a stochastic trend

To account for the stylized facts establishing an important low-frequency macro-finance interplay, we introduce and estimate a new DTSM. While existing DTSMs generally assume stationarity, our new formulation incorporates a shifting endpoint for interest rates,  $i_t^*$ , as a common stochastic trend for yields. This novel specification allows us to explain the stylized facts, provide new estimates of the term premium, and more accurately forecast interest rates.

### 4.1 Model specification

The state variables of our model of yield dynamics include  $N$  linear combinations of yields in the vector  $P_t$ , and we use  $N = 3$  such yield factors.<sup>27</sup> The key feature of our model is the presence of a single stochastic trend  $\tau_t$  common to the three factors. The common trends representation of the dynamic system is:

$$P_t = \bar{P} + \gamma\tau_t + \tilde{P}_t, \quad \tau_t = \tau_{t-1} + \eta_t, \quad \tilde{P}_t = \Phi\tilde{P}_{t-1} + \tilde{u}_t, \quad (3)$$

where  $\gamma$  is an  $N$ -vector with the loadings on the common trend,  $\eta_t$  is *iid* normal with variance  $\sigma_\eta^2$ ,  $\tilde{P}_t$  are the  $N$  cycle components that have *iid* normal innovations  $\tilde{u}_t$  with covariance matrix  $\tilde{\Omega}$ , and  $\Phi$  is a mean-reversion matrix that has eigenvalues with modulus less than one so that  $\tilde{P}_t$  is stationary. Accordingly, the state variables  $Z_t = (\tau, P_t)'$  are trivially cointegrated.<sup>28</sup> We assume that innovations to trend and cycle are orthogonal,  $E(\eta_t\tilde{u}_t) = 0$ , as is commonly done with unobserved components models (Watson, 1986; Del Negro et al., 2017). Our assumptions imply that the long-run trend components of  $P_t$  are

$$P_t^* \equiv \lim_{j \rightarrow \infty} E_t P_{t+j} = \bar{P} + \gamma\tau_t. \quad (4)$$

The short-term interest rate is taken to be an affine function of  $P_t$ :

$$i_t = \delta_0 + \delta_1' P_t. \quad (5)$$

Finally, we assume absence of arbitrage opportunities so that there exists a “risk-neutral” probability measure, denoted by  $\mathbb{Q}$ , which prices all financial assets (Harrison and Kreps,

<sup>27</sup>Let the vector  $Y_t$  contain  $J$  model-implied yields. Then  $P_t = WY_t$ , where  $W$  is an  $N \times J$  coefficient matrix.

<sup>28</sup>Appendix B gives the VAR representation of the dynamic system. Innovations to  $P_t$  are  $P_t - E_{t-1}P_t \equiv u_t = \gamma\eta_t + \tilde{u}_t$  with covariance matrix  $\Omega = \gamma\gamma'\sigma_\eta^2 + \tilde{\Omega}$ .

1979). The risk-neutral dynamics are specified as

$$P_t = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} P_{t-1} + u_t^{\mathbb{Q}}, \quad (6)$$

where the innovations  $u_t^{\mathbb{Q}}$  are *iid* normal (under  $\mathbb{Q}$ ) with covariance matrix  $\Omega$ . We assume that  $P_t$  is stationary under  $\mathbb{Q}$  by imposing that  $\Phi^{\mathbb{Q}}$  has eigenvalues less than one in absolute value, thus forward rates, yields, and bond convexity remain finite in the limit as maturity increases. Equations (5) and (6) imply that yields are affine functions of the yield factors  $P_t$ ,  $Y_t = A + BP_t$ . The loadings in  $A$  and  $B$  are determined by the risk-neutral parameters.<sup>29</sup>

Two types of normalizations are required for the state variables and parameters to be identified. The first set of normalizations, required in any affine term structure model, uniquely pins down the yield factors  $P_t$ . In their absence, an affine transformation of  $P_t$  would leave implied interest rates  $Y_t$  unchanged (Dai and Singleton, 2000). We use the normalization of Joslin et al. (2011), which imposes restrictions only on the  $\mathbb{Q}$ -dynamics and is ideally suited for the use of linear combinations of yields as yield factors. In this normalization, only four parameters determine the  $\mathbb{Q}$ -dynamics: a scalar  $k^{\mathbb{Q}}$  and an  $N$ -vector  $\lambda^{\mathbb{Q}}$  containing the eigenvalues of  $\Phi^{\mathbb{Q}}$  which we assume to be real, distinct, and less than one. These four parameters and  $\Omega$  fully determine the yield loadings  $A$  and  $B$ . Details are in the Online Appendix.

The second normalization is needed to identify the unobserved trend  $\tau_t$ . Without it, an affine transformation of  $\tau_t$  would leave the yield factors  $P_t$  unchanged. For identification, we require one linear constraint on each of the vectors  $\bar{P}$  (to fix the level) and  $\gamma$  (to fix the scale).<sup>30</sup> We identify  $\tau_t$  as the long-run mean of the short rate,  $\tau_t = i_t^* = \delta_0 + \delta_1' P_t^*$ , using the normalizations  $\delta_0 + \delta_1' \bar{P} = 0$  and  $\delta_1' \gamma = 1$ . In total, the parameters of the model are  $k^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \gamma, \bar{P}, \Phi, \Omega, \sigma_{\eta}^2$ , and  $\sigma_e^2$ , where  $\gamma$  and  $\bar{P}$  each have two degrees of freedom.

The key innovation of this model relative to existing affine DTSMs is that the equilibrium nominal interest rate  $i_t^*$  is allowed to be time-varying. This implies that bond yields have a common stochastic trend and are  $I(1)$ . Imposing a unit root in this fashion is a convenient way to account for a very persistent process in a dynamic model, and it crystalizes the arguments regarding slow-moving interest rate trends. Taken literally, a unit root specification for interest rates is unlikely because nominal interest rates appear to have remained bounded over the centuries. However, in finite samples, a persistent stationary process can always be

<sup>29</sup>Appendix B provides additional details, including the VAR representation of (3), the stochastic discount factor, expressions for excess returns and the term premium, and more details on the role of the long-run trend for yields and risk premia.

<sup>30</sup>For further discussion of identification requirements in multivariate unobserved components (common trends) models, see, for example, Harvey (1990).

approximated arbitrarily well by a unit root process, and it is well-known that doing so can often be beneficial for forecasting (e.g., [Campbell and Perron, 1991](#)).<sup>31</sup> To illustrate the importance of the time-varying trend in interest rates, we will compare our model, which we refer to as the “shifting endpoint” (*SE*) model, to a conventional DTSM with stationary dynamics, a “fixed endpoint” (*FE*) model. The latter specification corresponds to a restricted special case of our model where  $i_t^* = i^*$ , i.e.,  $\sigma_\eta^2 = 0$ , which is equivalent to the popular three-factor affine DTSM of [Joslin et al. \(2011\)](#).

Another important aspect of our model is that the long-run trend is *unspanned* by the yield curve, that is, the cross section of interest rates at time  $t$  is not deterministically related to  $i_t^*$ . Asset pricing models generally imply spanning, since in the absence of knife-edge restrictions the state variables can be backed out from the yield curve.<sup>32</sup> Such inference is not possible in our model because the trend does not *directly* affect yields: Conditional on  $P_t$ , yields do not load on  $i_t^*$ . Therefore, the model can match the evidence against the spanning hypothesis documented in [Section 3.2](#), as we demonstrate below.<sup>33</sup> Another important rationale for an unspanned trend is to allow it to be unobserved. To incorporate an unobserved components specification within a dynamic term structure model, it is necessary for the trend component to be unspanned; otherwise,  $i_t^*$  would be completely determined by the time- $t$  yield curve and thus observed. A trend hardwired to current interest rates would also result in excessively volatile trend estimates.<sup>34</sup>

Our parsimonious specification can accurately fit the cross section of yields, is consistent with absence of arbitrage and time-varying risk premia, and includes a stochastic trend in line with much evidence showing that nominal interest rates are effectively  $I(1)$ . Importantly, changes in  $i_t^*$  may affect expectations of future short rates as well as the term premium, and our estimates below will assess the empirical importance of a trend in the term premium. While our DTSM framework is quite flexible, it is kept deliberately simple by focusing on

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<sup>31</sup>For example, a unit root in inflation may not be literally true, but assuming so often improves forecast performance ([Faust and Wright, 2013](#)).

<sup>32</sup>See [Duffee \(2013\)](#). Starting with [Duffee \(2011\)](#), many papers have built no-arbitrage models of the yield curve that impose such knife-edge restrictions and therefore allow for unspanned factors.

<sup>33</sup>A regression of  $i_t^*$  on *contemporaneous* yields provides an alternative perspective on spanning. For our preferred  $i^*$  proxy, such a regression gives an  $R^2$  of 95%, which might seem to suggest that the trend is effectively spanned, but this high  $R^2$  is misleading because of the presence of stochastic trends. Furthermore, this regression does not reveal whether any unspanned information is relevant for risk premia, as noted by [Joslin et al. \(2014\)](#).

<sup>34</sup>While sufficiently flexible *spanned* DTSM can generally match the data equally well as the corresponding *unspanned* DTSM, such spanned models require many more parameters ([Bauer and Rudebusch, 2017](#)). Furthermore, this result applies only to models in which the unspanned factors are observed, as in [Joslin et al. \(2014\)](#), whereas we allow for the possibility that the trend is unobserved. We have estimated term structure models with spanned trend factors and obtained implausibly volatile trend estimates, as movements in long-term yields are tightly linked to movements in the trend factor.

the nominal yield curve instead of separately modeling inflation and real yields. Such a separation, in the spirit of [Christensen et al. \(2010\)](#), would be a useful future extension that could differentiate  $r_t^*$  and  $\pi_t^*$ , but it is unnecessary for our purposes of estimating nominal risk premia and forecasting nominal Treasury yields.

## 4.2 Model estimation

The shifting trend model can be cast in state-space form for estimation. The measurement equation is  $Y_t^o = A + BP_t + e_t$ , with  $e_t \stackrel{iid}{\sim} N(0, \sigma_e^2 I_J)$ . For our data set,  $Y_t^o$  includes  $J = 17$  yields. The state equation follows from (3).

The common trend in our DTSM is both unspanned and latent, unlike the observed unspanned factors in models like [Joslin et al. \(2014\)](#). Our specification is therefore effectively an unobserved components model. While this is desirable for matching the stylized facts of trends in interest rates, we face the usual estimation problem that empirically uncovering long-run trends using limited sample sizes is intrinsically difficult ([Watson, 1986](#)). For a very persistent time series with relatively infrequent business cycle fluctuations, even several decades of data typically contain limited information that can pin down the underlying trend. Simply estimating an unobserved components model with maximum likelihood and no external additional information is problematic and unreliable.<sup>35</sup> Therefore, in the literature, two broad approaches have been used to facilitate estimation of unobserved trends from time series data: the first includes additional data or proxies for the trend, the second incorporates prior information about properties of the trend such as its smoothness. For robustness, we consider each of these two methods for estimating our stochastic trend DTSM.

The first method adds data that can directly help pin down the trend estimate. For example, long-run survey expectations of inflation are often used to inform estimates of  $\pi_t^*$  ([Kozicki and Tinsley, 2012](#); [Del Negro et al., 2017](#)). Similarly, [Crump et al. \(2018\)](#) use surveys of professional forecasters to estimate  $i_t^*$  in a factor model for the yield curve. In this spirit, we use the empirical proxy for  $i_t^*$  equal to the sum of the PTR estimate of  $\pi_t^*$  and our real-time estimate of  $r_t^*$ . Section 3 showed that this trend proxy captures low-frequency movements of yields and predicts excess bond returns. Because we can now treat both  $i_t^*$  and  $P_t$  as observed, estimation is greatly simplified.<sup>36</sup> We denote the resulting estimates as the “observed shifting

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<sup>35</sup>The likelihood is essentially flat in some dimensions of the parameter space, and numerical optimization leads to local optima with very different trend estimates, depending on starting values.

<sup>36</sup>Details are in the Online Appendix. Taking  $i_t^*$  as observed is justified because maximum likelihood estimation of a state-space model with an additional measurement equation for  $i_t^*$  results in a noise variance equal to essentially zero. Taking the yield factors  $P_t$  as observed was proposed and justified by [Joslin et al. \(2011\)](#). If estimation with the state-space model is desired, excellent starting values for the model’s parameters can be obtained from a first-step estimation with observed state variables.

endpoint” (*OSE*) model. To guard against the problems that the zero lower bound (ZLB) on nominal interest rates poses for the estimation of affine yield curve models (Bauer and Rudebusch, 2016), we estimate the parameters of the *OSE* model using a sample that ends in 2007:Q4, before the height of the financial crisis and the ZLB period. Once the parameter estimates are obtained, we proceed to use the model over the entire sample period.<sup>37</sup> We find that the *OSE* model fits observed yields well, with a root mean-squared error of 8 basis points over the full sample. The  $\mathbb{Q}$ -dynamics are highly persistent, with the largest eigenvalue of  $\Phi^{\mathbb{Q}}$  equal to 0.997. For the real-world dynamics, the mean reversion to the common trend is relatively quick, with the largest eigenvalue of  $\Phi$  equal to 0.933.

Our second estimation method of the long-run trend model uses only Treasury yield data while making use of prior information in a Bayesian framework. In this regard, an effective approach to estimating unobserved trends imposes a tight prior distribution for the variance of the trend innovations. Such priors are commonly used in a variety of macroeconomic contexts to produce smooth trend estimates, consistent with the underlying macroeconomics that such trends are determined by gradual variation in structural factors such as demographic change. For interest rates, Del Negro et al. (2017) used such a smoothness prior for estimation of  $r_t^*$ . Similarly, we specify a tight inverse-gamma prior for  $\sigma_\eta^2$  with a mean of  $0.06^2/400$ , implying that the standard deviation of the change in  $i_t^*$  over a century is six percent. In light of the observed variation in interest rates and our trend proxies, we view this as a conservative choice in the sense that it limits the amount of yield variation that can be attributed to the long-run trend. Also, we follow Del Negro et al. (2017) by combining this prior with the stationary version of a Minnesota VAR-prior for the cycle components  $\tilde{P}_t$  with the distribution for the diagonal elements of  $\Phi$  centered around zero. We call this representation the “estimated shifting endpoint” (*ESE*) model. We use largely uninformative priors for the remaining parameters, and design a Markov chain Monte Carlo (MCMC) algorithm to simulate draws from the joint posterior distribution of the latent state variables and parameters (as described in detail in the Online Appendix). To account for the ZLB, we treat short-term yields as missing observations when they were constrained.<sup>38</sup>

Figure 4 plots the resulting *ESE* model estimate of  $i_t^*$ —specifically, the posterior mean and 95 percent credibility intervals. The *ESE* model  $i_t^*$  rose until the early 1980s to over eight

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<sup>37</sup>An extension of our model to explicitly account for the zero lower bound, for example with a shadow-rate specification as in Kim and Singleton (2012) and Bauer and Rudebusch (2016), is conceptually straightforward but beyond the scope of this paper.

<sup>38</sup>Specifically, three- and six-month yields are assumed to be missing from 2008:Q4 to 2015:Q3, the one-year yield is omitted from 2011:Q4 to 2014:Q3, and the two-year yield is omitted from 2012:Q4 to 2013:Q1. The start dates of these periods are based on results in Swanson and Williams (2014) while the end dates are when the yields lifted off from the ZLB.

percent and then gradually declined over the following decades to just below three percent at the end of our sample. For comparison, the ten-year Treasury yield and the real-time proxy estimate of  $i_t^*$  are also plotted. The *ESE* model estimate matches the proxy fairly closely and can account for much of the low-frequency variation in the ten-year yield. As usual for unobserved trend models, the uncertainty around the estimated trend is quite large. From 2008 to 2015, the estimation intervals widen a bit as uncertainty increases because the yield curve is partially constrained by the ZLB and is less informative about the underlying trend.

### 4.3 Implications for trend components of yields

Section 3.1 showed that proxies for  $i_t^*$  were important drivers of yields, particularly in terms of cointegration. Now we interpret this evidence in the context of our model.

First, note that our shifting-endpoint DTSM can decompose yields into trend and cycle components:

$$Y_t = Y_t^* + \tilde{Y}_t = A + B\bar{P} + B\gamma i_t^* + B\tilde{P}_t, \quad Y_t^* \equiv \lim_{j \rightarrow \infty} E_t Y_{t+j}. \quad (7)$$

Figure 5 shows the (actual and model-fitted) ten-year yield and its trend and cycle components. The trend component accounts for most of the low-frequency movements in the yield, while the cycle component—the model-based analog of the regression-based interest rate cycles discussed in Section 3.1—exhibits clear mean reversion around zero and little to no apparent trend. This confirms that our model accounts for the long-run trend in interest rates.

Second, we can compare the model-implied coefficients of yields on  $i_t^*$  to the empirical results we obtained from the cointegration regressions of yields on proxies of  $i_t^*$  in Section 3.1. In the model, the trend  $i_t^*$  is unspanned and only indirectly affects yields through its relationship with  $P_t$ . Equation (7) shows that the implied loadings of yields on  $i_t^*$  are  $B\gamma$ . Figure 6 plots these coefficients, which gradually rise from unity at the short end—since  $i^*$  is by definition the trend component of the short rate—to around 1.7. In the data, the coefficients (shown with their 95 percent confidence intervals) are relatively flat around 1.6-1.9, with a slight hump around two years and a gradual decline thereafter. Note that the estimated coefficient for the ten-year yield corresponds to the value reported in the last column of the top panel of Table 1. To gauge whether the model is consistent with the estimated cointegration coefficients we need to account for sampling uncertainty, hence we simulate 5,000 artificial samples of the trend and yields with the same size as the actual data, and estimate the cointegration regressions in each simulated sample. The 95 percent Monte Carlo intervals of the coefficient estimates, shown as dashed lines in Figure 6, comfortably contain the values

obtained with the actual data.<sup>39</sup>

Our results here and in Section 3.1 show that the trend component of long-term Treasury yields moves more than one-for-one with changes in  $i_t^*$ —the coefficients on  $i_t^*$  are larger than one. Because expectations of future short rates by definition load with a unit coefficient on  $i_t^*$ , coefficients above one indicate that the term premium positively responds to changes in  $i_t^*$ . (For a yield of maturity  $n$ , the term premium loading on  $i_t^*$  is  $B'_n\gamma - 1$ .) That is, our cointegration regressions and the model estimates suggest that the term premium responds positively to  $i_t^*$ . In Section 4.5, we will show the resulting term premium estimates and quantify the relative importance of the trend in short-rate expectations and in the term premium. Our model has similar implications for the trend behavior of the *slope of the yield curve*. The slope, if measured as the simple difference between a long-term yield and the short rate, in fact has the exact same trend component and loading on  $i_t^*$  as the term premium in that long-term yield. While some macroeconomic time series models, such as DGGT, assume that separate trends drive short rates and the slope, in our no-arbitrage model the slope contains a trend due to the risk in  $i_t^*$ .

#### 4.4 Model-based explanation of excess return predictability

A key stylized fact from Section 3 is that various measures of the long-run trends in interest rates have substantial predictive power for excess bond returns, and this additional information is not spanned by the first three principal components of yields. To explain this result with our shifting endpoint DTSMs, we first simulate 10,000 artificial samples from each of our three models: the *FE* model with stationary yields, the *OSE* model with an observed trend proxy, and the *ESE* model with an estimated trend.<sup>40</sup> We then run excess return regressions using the simulated data sets from the three models, and report the means and the 2.5th and 97.5th percentiles of the resulting Monte Carlo distributions.

Table 4 presents the  $R^2$  results from the actual and simulated data. The top row reports the  $R^2$  for the predictive regressions of excess bond returns using actual data with and without  $i_t^*$  as an additional predictor, as described in Section 3.2 (corresponding to the estimates in the first and last column of the top panel of Table 4). The reported  $\Delta R^2$  shows the substantial predictive gains from including the macro trend proxy.

The remaining three rows provide results using data simulated from the *FE*, *OSE*, and *ESE* models. For the *FE* model, there are essentially no predictive gains on average from

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<sup>39</sup>Because our model is stationary under  $\mathbb{Q}$ , the infinite-maturity yield is constant, so loadings of yields on all risk factors eventually decline to zero as maturity increases. For maturities of practical interest—say, less than 100 years—the loadings on  $i_t^*$  are notably larger than one (as discussed in the Online Appendix).

<sup>40</sup>For the *FE* model, we separately simulate  $i_t^*$  from a random walk process using the *OSE* parameters.

adding  $i_t^*$  to the yield principal components, with a mean  $\Delta R^2$  of 1 percentage point. This is not surprising because this model imposes the null hypothesis that  $i_t^*$  truly has no predictive power. The upper bound of the Monte Carlo interval (4 percentage points) is much lower than the actual improvement in  $R^2$  from adding trends in the data (12 percentage points), consistent with our rejections of the spanning hypothesis in Section 3.2 using the bootstrap, which also simulated data under the null hypothesis.

For the *OSE* model, the predictive gains from adding  $i_t^*$  are large: the mean gain of 9 percentage points is close to the  $\Delta R^2$  of 12 percentage points in the actual data. Importantly, the Monte Carlo interval comfortably contains the value found in the data. For the *ESE* model, the results are similar, with large model-implied predictive gains from including the trend as an additional predictor. Overall, the shifting-endpoint models accurately replicate the actual predictive gains from adding  $i_t^*$  in regressions for excess bond returns.

In our shifting-endpoint model, shifts in  $i_t^*$  are unspanned by the yield curve by construction. Why are changes in the trend unspanned in the data? The intuitive reason is the following: Shifts in the level of the yield curve can occur because of either changes in the underlying trend (see the loadings in Figure 6) or because of movements in interest rates relative to the trend. These two drivers have opposite implications for expectations of future bond returns, and investors cannot distinguish between them based on observed yields alone.<sup>41</sup> Therefore, yields and macro trends contain important separate pieces of information for predicting future interest rates.

To better understand the predictability of excess bond returns through the lens of our model, we examine model-implied expected excess bond returns analytically. Leaving aside constant terms that are irrelevant for predictability, bond risk premia are

$$E_t r x_{t+1}^{(n)} = \mathcal{B}'_{n-1} \left( E_t P_{t+1} - E_t^Q P_{t+1} \right) = \mathcal{B}'_{n-1} \left[ (I_N - \Phi) \gamma \tau_t + (\Phi - \Phi^Q) P_t \right], \quad (8)$$

where  $\mathcal{B}_n$  contains the affine loadings of the log bond price  $p_t^{(n)}$  on the risk factors  $P_t$ . The derivation is given in the Online Appendix. The first expression shows that time variation in bond risk premia is due to differences in expectations of future yields under the physical and risk-neutral probability measures. In the presence of an unspanned trend in yields, as in our model, physical-measure expectations depend not only on current yields ( $P_t$ ) but also on the shifting endpoint  $i_t^*$ . Thus, bond risk premia depend on both as well, as evident from the second expression. Ignoring the trend misses an important part of the variation in  $E_t P_{t+1}$  and

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<sup>41</sup>Even in a term structure model with a spanned trend, the cross-sectional loadings of yields on  $i_t^*$  would differ only slightly from the loadings of the level factor in detrended yields, and small measurement errors would make them effectively indistinguishable (Jorgensen, 2017).

understates the predictability of bond returns.

A final point concerns the role of detrended yields, which we have shown in Section 3.2 to capture most of the variation in bond risk premia. Our model implies that the term premium and excess returns also have a small trend component, meaning that detrended yields cannot *fully* explain movements in bond risk premia. Empirically, however, when predicting excess bond returns it is of first-order importance to account for the trend in yields, and only of second-order importance to separately account for the trend when conditioning on detrended yields (as described in Appendix B).

## 4.5 The term premium in long-term yields

Another perspective on the pricing of risk in Treasury bonds is given by the term premium in long-term interest rates.<sup>42</sup> As evident from the definition in equation (1), estimating the term premium requires long-horizon projections of future short-term rates, which will depend crucially on the nature of the long-run trend in interest rates. Furthermore, because the term premium is the average of expected future excess bond returns,  $TP_t^{(n)} = n^{-1} \sum_{j=0}^{n-2} E_t r x_{t+j+1}^{(n-j)}$ , our results about the importance of a trend in interest rates for predictions of excess bond returns suggest that this trend should matter for the estimation of term premia as well.

Our baseline for comparison is the *FE* model in which the state variables follow a stationary VAR. This special case of our no-arbitrage model with a constant endpoint  $i_t^*$  is representative of the conventional DTSM in the literature. The left panel of Figure 7 shows the five-to-ten-year forward rate as well as its expectations component, i.e., the “risk-neutral rate,” and the term premium as estimated from the *FE* model.<sup>43</sup> The expectations component is estimated to be very stable, and it remains near the unconditional mean of the short rate, the fixed endpoint  $i^*$ . Therefore, the term premium, the residual, has to account for the trend in the long-term interest rate since the 1980s. As argued by Kim and Orphanides (2012) and Bauer et al. (2012), among others, such behavior by the expectations component and the term premium appears at odds with observed trends in survey-based expectations (Kozicki and Tinsley, 2001) and with the cyclical behavior of risk premia in asset prices (Fama and French, 1989). By allowing for a long-run trend in interest rates, we avoid such counterfactual decompositions of long-term rates into expectations and term premium. The right panel of Figure 7 shows the estimates for the risk-neutral rate and the term premium from our no-arbitrage *OSE* and *ESE* models.<sup>44</sup> For these shifting-endpoint models, short-rate expectations are anchored at  $i_t^*$ , and

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<sup>42</sup>In a DTSM the term premium is defined as the difference between model-implied yields and risk-neutral yields, see Appendix B.

<sup>43</sup>Results for the ten-year yield are qualitatively similar.

<sup>44</sup>For the *OSE* model, the risk-neutral rate and term premium is calculated using point estimates of the

the cycle components  $\tilde{P}_t$  mean-revert relatively quickly; hence, the expectations component of the five-to-ten-year forward rate closely mirrors this trend. Movements in the expectations component account for the majority of the low-frequency variation in the observed interest rate. In particular, the estimates exhibit a pronounced downward trend since the 1980s due to the decline in  $i_t^*$ .

Our shifting-endpoint models attribute a majority of the secular decline in interest rates not to the term premium but instead to the decline in long-run expectations of future short-term rates. The drop in the forward rate from its average during 1980-1982 to its average during 2015-2017 was close to 10 percentage points. According to the stationary *FE* model, the estimated term premium accounts for 75 percent of this decline. In contrast, the shifting-endpoint models attribute only 35 percent (*OSE*) and 37 percent (*ESE*) to the term premium, while attributing the majority to a falling expectations component, in line with the substantial downward shift in the underlying macro trends.<sup>45</sup> Furthermore, the estimated term premium with a shifting endpoint exhibits more pronounced cyclical variation, in line with the notion that risk premia are countercyclical (Fama and French, 1993; Cochrane and Piazzesi, 2005). This stark difference demonstrates how accounting for the slow-moving trend component in interest rates fundamentally alters our understanding of the driving forces of long-term interest rates, and bridges the gap between the secular trends in macroeconomics and no-arbitrage models for the yield curve.

Although the shifting-endpoint models attribute most of the trend in the forward rate to the expectations component, the estimated forward term premium does exhibit some moderate low-frequency swings, declining from around 4-5 percent in the early 1980s to about 0-1 percent at the end of the sample. The reason is that long-term interest rates exhibited more pronounced secular movements than our estimates of  $i_t^*$ , and expectations move one-for-one with  $i_t^*$ , thus the estimated term premium also contains a trend component.<sup>46</sup> In other words, our estimates show that changes in the trend carry a positive risk premium. From a macro-finance perspective, some downward trend in the term premium since the 1980s is plausible. Notably, inflation risk declined substantially over this period as average inflation declined, and the term premium includes compensation for this risk (Wright, 2011).<sup>47</sup> In any

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parameters. For the *ESE* model, we calculate the means of the MCMC posterior distribution for these objects.

<sup>45</sup>Some researchers estimate stationary DTSMs with a sample beginning in, say, the mid-1980s to avoid the earlier shifts in inflation, but our comparison results were little changed using a post-1984 sample.

<sup>46</sup>As discussed above in Section 4.3, yields load on  $i_t^*$  with coefficients larger than unity, and the term premium consequently has a positive loading on this trend.

<sup>47</sup>Furthermore, in theory, a decline in the term premium is consistent with the fall in the covariance between nominal bond returns and stock returns since the 1980s, as emphasized by Campbell et al. (2017). A decline in real risk premia would also be implied by a change in business cycle dynamics that increased persistent risk relative to transitory risk, as described by Campbell (1986) and Beeler and Campbell (2012).

event, the decline in the term premium is much more moderate and more plausible with a shifting rather than a fixed endpoint.

Using a shifting-endpoint model with a slow-moving trend component solves the knife-edge problem of [Cochrane \(2007\)](#), who pointed out that assuming either stationarity or a random walk for the level of interest rates leads to drastically different implications for expectations and term premia, in both cases at odds with widely held views about interest rates. Imposing a random walk for the level is known to forecast well but leads to the implausible implication that the expectations component accounts for essentially all variation in yields. Imposing stationarity, as shown in the left panel of [Figure 7](#), leads to the implausible implication that the term premium component accounts for all of the low-frequency variation in yields. Our approach of assuming that interest rates contain a slow-moving common trend avoids both extremes. It also delivers term premium estimates that are in line with the common macro-finance priors and produces interest rate forecasts that outperform the random walk, as we will show below.

In recent work, [Crump et al. \(2018\)](#) estimate the term premium from a statistical model with shifting endpoints and find a somewhat more variable term premium than we do. Based on their estimates, the term premium accounts for just over half of the variance of long-term interest rates, whereas it accounts for about one-third according to our estimates.<sup>48</sup> While we use an arbitrage-free term structure specification, the different results largely reflect how the shifting endpoint is identified. Crump et al. use survey data, and their  $i_t^*$  closely mirrors long-run survey expectations of the three-month T-bill rate. This estimate of  $i_t^*$  is fairly stable and implies a somewhat more limited role for trends and long-run expectations and greater variability in the residual term premium.

Finally, we note that when interpreting model-based estimates of the term premium, it is important to keep in mind that they assume rational expectations, which may not represent the expectations of investors at each point in time. As shown by [Piazzesi et al. \(2015\)](#) and [Cieslak \(2018\)](#), statistical estimates of risk premia are affected by persistent forecast errors by investors, i.e., deviations from rational expectations.<sup>49</sup> While this caveat also applies to our estimates, models like ours that allow for shifting endpoints are better suited to capture changes in long-run investor expectations that can result from learning about macroeconomic

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<sup>48</sup>Specifically, we calculated the same decomposition of the sample variance of forward rates as in [Table 2](#) of [Crump et al. \(2018\)](#). For the variances of levels, they find that 53 to 56 percent is explained by the forward term premium at maturities of five years and beyond. By contrast, over their sample period, we find that only 35 percent of the variance is explained by the term premium.

<sup>49</sup>While some might conclude that estimates of term premia should therefore rely exclusively on survey expectations, there is no guarantee that expectations from surveys of professional forecasters are any closer to the true subjective expectations of bond traders than statistical models.

trends and policies. Estimates of such models are also much less sensitive to the choice of sample period than stationary DTSMs, which require inference about the unconditional mean of the short-term interest rate, assumed to be constant (Orphanides and Wei, 2012). For these reasons, our shifting-endpoint model is likely to be better able to recover true historical investor expectations and bond risk premia.

## 4.6 Out-of-sample forecasts of interest rates

We now turn to out-of-sample (OOS) forecasts of long-term interest rates using our new shifting-endpoint DTSM. Despite many advances in yield curve modeling, the random walk model has proven very hard to beat when forecasting bond yields, due to the extreme persistence of interest rates (Duffee, 2013). But our results so far suggest that one might be able to obtain more accurate forecasts by accounting for the interest rate trend.

We construct out-of-sample forecasts using the *OSE* model, which proxies for  $i_t^*$  with the sum of the PTR estimate for  $\pi_t^*$  and the real-time estimate of  $r_t^*$  described in Section 2. We will compare these forecasts to those from the *FE* model and from a driftless random walk.<sup>50</sup> The models are recursively estimated—that is, using an expanding estimation window using all data available up to each forecast date—starting in 1976:Q1 when five years of data are available.<sup>51</sup> We focus on forecasts of the ten-year yield; we have found results for other maturities to be qualitatively similar. We forecast at horizons of 4, 10, 20, 30, and 40 quarters, and because our data end in 2018:Q2 the last forecast date is 2008:Q2, for a total of 127 (overlapping) forecasts.<sup>52</sup> The top panel of Table 5 reports the root mean-squared errors in percentage points. We also calculate  $p$ -values for tests of equal finite-sample forecast accuracy using the approach of Diebold and Mariano (1995) (DM).<sup>53</sup> We calculate the DM  $p$ -values using standard normal critical values for one-sided tests of the null hypothesis that the *OSE* model does not improve upon the *RW* or *FE* forecasts. We find that the forecasts from our common-trend model *OSE* are substantially and statistically significantly more accurate

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<sup>50</sup>We do not include estimates from the *ESE* model because the estimation of this model using our MCMC sampler takes a significant amount of time, which renders recursive estimation with each additional available observation period as required for out-of-sample forecasting computationally too burdensome.

<sup>51</sup>A rolling scheme, which uses only a fixed number of observations for parameter estimation, allows for an easier asymptotic justification of tests for predictive ability (Giacomini and White, 2006) but requires a specific choice of the window length. We have also obtained forecasts with such a scheme, using a variety of different window lengths, and found qualitatively similar results as when using a recursive scheme.

<sup>52</sup>The last date for which we re-estimate the models is 2007:Q4, consistent with Section 4.2.

<sup>53</sup>Following common use, we construct the DM test with a rectangular window for the long-run variance and the small-sample adjustment of Harvey et al. (1997). Monte Carlo evidence in Clark and McCracken (2013) indicates that this test has good size in finite samples. However, for very long forecast horizons, the long-run variance is estimated with considerable uncertainty because there are only a few such non-overlapping observations in our sample.

than the *FE* model. For example, when forecasting 10 years ahead, model *OSE* lowers the RMSE by about 40 percent relative to the *FE* model. The improvements relative to the *RW* are smaller but still statistically significant.<sup>54</sup> These results document that, relative to conventional stationary models, more accurate Treasury yields forecasts can be obtained by allowing for an underlying common trend in interest rates. Gains are even possible relative to the random walk benchmark. In contrast to the random walk forecast, which simply assumes all changes are permanent, accounting for  $i_t^*$ , the underlying source of the highly persistent changes in interest rates, improves forecast accuracy.<sup>55</sup>

Finally, we compare the accuracy of our statistical models to that of professional forecasters. Since 1988, the Blue Chip Financial Forecasts (*BC*) survey has asked its respondents for long-range forecasts of interest rates twice a year. The respondents provide their average expectations of the target variable for each of the upcoming five calendar years and for the subsequent five-year period—we will focus on the five annual forecast horizons. We match the available information sets by using only data up to the quarter preceding the survey date for our model-based forecasts, and we exactly match the forecast horizons with the *BC* forecasts by taking averages of model-based forecasts over the relevant calendar years. The sample includes 48 forecast dates from March 1988 to December 2011. The bottom panel of Table 5 shows the RMSEs of the survey forecasts and the three model-based forecasts. Shifting-endpoint forecasts based on  $i_t^*$  improve over the *RW* forecasts in this sample as well, which are better than the *BC* forecasts. Because of the smaller sample size and substantial overlap, the gains of the *OSE* model are not as strongly significant, and for some horizons they are insignificant. The gains relative to the *FE* model are again strongly significant. The reason for the poor performance of the survey forecasts is that they consistently overpredict future yields, in particular at long horizons (results not shown). Other studies have documented the poor performance of survey forecasts of interest rates (e.g., [van Dijk et al., 2014](#)), which contrasts with the very good performance of survey-based inflation forecasts ([Ang et al., 2007](#); [Faust and Wright, 2013](#)). Our results suggest that the underlying reason for this poor performance is that professional forecasters have not sufficiently accounted for the shifting long-run trend component in interest rates.

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<sup>54</sup>Additional, unreported analysis—using plots of differences in cumulative sums of forecast errors over time—shows that the forecast gains of *OSE* are not driven by certain unusual subperiods but instead are a consistent pattern over most of our sample period.

<sup>55</sup>Earlier research by [van Dijk et al. \(2014\)](#) found that, when forecasting interest rates, it is beneficial to link long-run projections of interest rates to long-run expectations of inflation, but their forecasts do not account for time variation in  $r_t^*$  and are not based on an arbitrage-free model.

## 5 Conclusion

This paper bridges the gap between macroeconomic and finance views of long-run trends. We provide new evidence that interest rates and bond risk premia are determined over time by variation in the perceived trend in inflation and the equilibrium real rate of interest. Indeed, allowing for persistent shifts in  $\pi_t^*$  and  $r_t^*$  is crucial to account for yield dynamics. To understand and forecast long-term interest rates and bond returns requires a time-varying trend component in yields. In the recent period, variation in the equilibrium real rate is even more consequential than movements in the inflation trend. Our analysis started by establishing the links between macroeconomic trends and yields using trend estimates from surveys and macroeconomic and statistical models. However, to understand these results, we formulated and estimated a new no-arbitrage model that simultaneously models the long-run trend and term premium in yields—a key contribution of our analysis. In particular, when estimated with just yield curve data, this model, which embeds a non-stationary  $i_t^*$ , provides a new finance-based measure of the trend in nominal interest rates that complements estimates in the macroeconomics literature. It also provides for the first time an internally consistent formulation of that equilibrium trend and bond risk premia.

Instead of the ubiquitous stationary dynamic yield curve models with constant means, our results strongly support specifications that allow for slow-moving changes in the long-run means of nominal and real interest rates. Building upon our new common-trend dynamic term structure model, much work remains in jointly estimating macro trends and yield curve dynamics. Three future research avenues seem particularly promising. First, a joint model with shifting endpoints of real yields, nominal yields, and inflation expectations could provide separate estimates of  $\pi_t^*$  and  $r_t^*$ . Second, combining our shifting-endpoint DTSM specification with the shadow-rate paradigm ([Bauer and Rudebusch, 2016](#)) would allow for a sophisticated treatment of the effective lower bound on interest rates, which may provide additional insights into macro and yield-curve trends over post-crisis period. Third, one could consider time variation in the variability of trends by adding stochastic volatility (as in [Stock and Watson, 2007](#)) in order to assess how our (mostly unconditional) results about the relative importance of trends are affected by taking a conditional perspective.

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# Appendix

## A Trend estimates

Table A.1 provides an overview of all of the trend estimates used in our analysis. The first five estimates of  $r_t^*$  are obtained from models in published studies, and the following four estimates of  $r_t^*$  are new. In Section 3, we use a “filtered” estimate and a “real-time” estimate of  $r_t^*$ , which are averages of the three filtered and six real-time estimates, respectively. The real-time estimate of  $i_t^*$  is the sum of the PTR estimate of  $\pi_t^*$  and the real-time estimate of  $r_t^*$ . The *ESE* model estimate of  $i_t^*$  is described in Section 4.2.

Table A.1: Overview of trend estimates

Trend	Source	Real-time	Filtered	Smoothed
$r^*$	Del Negro et al. (2017) (DGGT)	✓		✓
	Johannsen and Mertens (2016) (JM)	✓		✓
	Laubach and Williams (2016) (LW)		✓	✓
	Holston et al. (2017) (HLW)		✓	
	Kiley (2015)		✓	✓
$r^*$	UC	✓		✓
	Proxies	✓		✓
	SSM	✓		✓
	MA	✓		
$\pi_t^*$	PTR	✓		
$i^*$	Real-time	✓		
	ESE			✓

Overview of trend estimates used in the paper: five external sources of  $r^*$ , our own four estimates of  $r^*$ , the PTR estimate of  $\pi^*$ , and two  $i^*$  estimates. “Real-time” indicates (pseudo) real-time estimation of the trend proxy, “filtered” indicates estimates from a (one-sided) Kalman filter using full-sample parameter estimates, and “smoothed” indicates (two-sided) Kalman smoother or full-sample Bayesian estimate of the trend. All trend estimates are quarterly from 1971:Q4 to 2018:Q1.

Our proxy estimate of the inflation trend  $\pi_t^*$  is the Fed’s PTR measure, the perceived target rate (PTR) of inflation. PTR measures expectations for inflation in the price index of personal consumption expenditures (PCE). Consistent with this definition, our estimates of  $r_t^*$  are based on real interest rates relative to PCE inflation. Since 1979, PTR corresponds to long-run inflation expectations from the Survey of Professional Forecasters (SPF). Before 1979, it is based on estimates from a learning model for expected inflation. For details see Brayton and Tinsley (1996). Data are available at <https://www.federalreserve.gov/econresdata/frbus/us-models-package.htm>.

The estimates of  $r_t^*$  are plotted and described in the Online Appendix. All of these estimates are based on the assumption that the real rate contains a stochastic trend, i.e., a stochastic endpoint.

## B A shifting-endpoint dynamic term structure model

Equation (3) specifies the evolution of the state variables  $Z_t = (\tau, P_t)'$  under the real-world/physical probability measure  $\mathbb{P}$ . It has the form of a common trends representation and shows that  $Z_t$  is trivially cointegrated with one common trend. The same holds for yields, see equation (7). The VAR representation is

$$Z_t = \mu_Z + \Phi_Z Z_{t-1} + v_t, \quad v_t = (\eta_t, u_t)'. \quad (\text{B.1})$$

with

$$\mu_Z = \begin{pmatrix} 0 \\ (I_N - \Phi)\bar{P} \end{pmatrix}, \quad \Phi_Z = \begin{pmatrix} 1 & 0_{1 \times N} \\ (I_N - \Phi)\gamma & \Phi \end{pmatrix},$$

$$\Omega_v = E(v_t v_t') = \begin{pmatrix} \sigma_\eta^2 & \gamma' \sigma_\eta^2 \\ \gamma \sigma_\eta^2 & \Omega \end{pmatrix}.$$

The innovations to  $P_t$ ,  $u_t = \gamma \eta_t + \tilde{u}_t$ , have covariance matrix  $\Omega = E(u_t u_t') = \gamma \gamma' \sigma_\eta^2 + \tilde{\Omega}$ .

Equation (6) specifies the evolution under the risk-neutral measure  $\mathbb{Q}$ . We assume that under  $\mathbb{Q}$  the yield factors follow a *stationary* VAR. This specification avoids the counterfactual implication of a unit root under  $\mathbb{Q}$  that yields and forward rates diverge to minus infinity with maturity.

From the short rate equation (5) and the  $\mathbb{Q}$ -dynamics (6), it follows that the trend  $\tau_t$  is *unspanned by yields*, meaning that there is no deterministic mapping from  $P_t$  (or yields) to  $\tau_t$ . Formally, the requirement for  $\tau_t$  to be an unspanned risk factor is

$$E^{\mathbb{Q}}(i_{t+h} | P_t, \tau_t) = E^{\mathbb{Q}}(i_{t+h} | P_t), \quad \text{for all } h,$$

see Joslin et al. (2014), which holds true because neither the short rate  $i_t$  nor expectations of future yield factors  $P_{t+h}$  depend on  $\tau_t$  once we condition on  $P_t$ . The key features of the risk-neutral distribution for  $P_t$  are its stationarity and its independence of  $\tau_t$ .

The dynamics under  $\mathbb{P}$  and  $\mathbb{Q}$  together imply a stochastic discount factor (SDF) and prices of risk. The SDF is the projection of the economy-wide SDF on the risk factors driving bond prices,  $P_t$ , as in Joslin et al. (2014). As usual in affine DTSMs, the SDF is exponentially affine:

$$M_{t+1} = \exp(-i_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1}) \quad (\text{B.2})$$

with  $\varepsilon_t = L^{-1} u_t$ , where  $LL' = \Omega$ . The risk prices  $\lambda_t$  are affine in  $Z_t$ :

$$\lambda_t = \Omega^{-1/2} (\lambda_0 + \lambda_1 Z_t), \quad (\text{B.3})$$

where  $\lambda_0$  is a  $N$ -vector and  $\lambda_1$  is a  $N \times (N + 1)$  matrix of risk sensitivities. The Online

Appendix shows that

$$\lambda_0 = (I_n - \Phi)\bar{P} - \mu^{\mathbb{Q}} \quad (\text{B.4})$$

and

$$\lambda_1 = [(I_N - \Phi)\gamma, \Phi - \Phi^{\mathbb{Q}}]. \quad (\text{B.5})$$

A “knife-edge” restriction is imposed so that  $\tau_t$  does not affect  $E^{\mathbb{Q}}(P_{t+1}|Z_t)$ . Namely, the risk prices  $\lambda_t$  respond exactly in the same way to changes in  $\tau_t$  as do the expectations  $E^{\mathbb{P}}(P_{t+1}|Z_t)$ , and the sensitivities in each case are  $(I_N - \Phi)\gamma$ . This restriction implies that movements in  $\tau_t$  lead to changes in term premia that exactly offset the changes in expectations, leaving yields unchanged.

Prices of zero-coupon bonds are exponentially affine, so that for the log-price of an  $n$ -period bond we have  $p_t^{(n)} = \mathcal{A}_n + \mathcal{B}'_n P_t$ , and the coefficients follow the usual recursions (e.g., [Ang and Piazzesi, 2003](#)):

$$\mathcal{A}_{n+1} = \mathcal{A}_n + \mathcal{B}'_n \mu^{\mathbb{Q}} + \frac{1}{2} \mathcal{B}'_n \Omega \mathcal{B}_n - \delta_0 \quad (\text{B.6})$$

$$\mathcal{B}_{n+1} = (\Phi^{\mathbb{Q}})' \mathcal{B}_n - \delta_1 \quad (\text{B.7})$$

with initial conditions  $\mathcal{A}_0 = 0$  and  $\mathcal{B}_0 = 0$ . Yields are affine functions of the factors:

$$y_t^{(n)} = A_n + B'_n P_t \quad A_n = -\frac{1}{n} \mathcal{A}_n, \quad B_n = -\frac{1}{n} \mathcal{B}_n.$$

If we denote the vector of  $J$  yields that are included in our model as  $Y_t$ , we can write  $Y_t = A + B P_t$  for  $J$ -vector  $A = (A_{n_1}, \dots, A_{n_J})'$  and  $J \times N$  matrix  $B = (B_{n_1}, \dots, B_{n_J})'$ . For forward rates  $f_t^{(n)} = p_t^{(n)} - p_t^{(n+1)}$  the Online Appendix shows that

$$f_t^{(n)} = -\frac{1}{2} \mathcal{B}'_n \Omega \mathcal{B}_n + \delta_0 - \mathcal{B}'_n \mu^{\mathbb{Q}} + \delta'_1 (\Phi^{\mathbb{Q}})^n P_0 + \delta'_1 (\Phi^{\mathbb{Q}})^n \gamma \tau_t + \delta'_1 (\Phi^{\mathbb{Q}})^n \tilde{P}_t. \quad (\text{B.8})$$

The convexity term in the expression for forward rates is  $-\frac{1}{2} \mathcal{B}'_n \Omega \mathcal{B}_n$ . Since the  $\mathbb{Q}$ -dynamics are stationary,  $\mathcal{B}_n$  converges to a finite limit as  $n$  tends to infinity, along with the convexity in yields and forward rates. If, by contrast,  $\Phi^{\mathbb{Q}}$  had an eigenvalue on the unit circle, convexity and therefore yields and forward rates would diverge to minus infinity. This fact, discussed in [Campbell et al. \(1997, p. 433\)](#), is an important reason why term structure models are usually specified with stationary risk-neutral dynamics, and we follow this tradition here.

Loadings of yields and forward rates on the trend  $\tau_t$  are hump-shaped with respect to maturity. Short rate loadings equal one since  $\tau_t = i_t^*$ . For longer maturities in our sample, the term premium responds positively to  $\tau_t$ , so yields move more than one-for-one with the trend. Eventually, loadings decline very gradually toward their limit of zero, and the infinite-maturity forward rate/bond yield is a constant. Further details are provided in the Online Appendix.

Model-implied excess bond returns are

$$rx_{t+1}^{(n)} = -\frac{1}{2} \mathcal{B}'_{n-1} \Omega \mathcal{B}_{n-1} + \mathcal{B}'_{n-1} \left( E_t P_{t+1} - E_t^{\mathbb{Q}} P_{t+1} \right) + \mathcal{B}'_{n-1} u_{t+1}. \quad (\text{B.9})$$

thus expected excess returns, adjusted for convexity, are

$$E_t r x_{t+1}^{(n)} + \frac{1}{2} \text{Var}_t r x_{t+1}^{(n)} = \mathcal{B}'_{n-1} [\lambda_0 + (I_N - \Phi)\gamma\tau_t + (\Phi - \Phi^Q)P_t] \quad (\text{B.10})$$

$$= \mathcal{B}'_{n-1} \left[ \lambda_0 + (\Phi - \Phi^Q)\bar{P} + (I_N - \Phi^Q)\gamma\tau_t + (\Phi - \Phi^Q)\tilde{P}_t \right]. \quad (\text{B.11})$$

Equation (B.10) shows that when holding the yield factors  $P_t$  constant, the loadings on  $\tau_t$  are  $\mathcal{B}'_{n-1}(I_N - \Phi)\gamma$ , and our model estimates imply substantially negative values for these loadings, across all  $n$ , in line with the regression estimates in Section 3.2. Conditional on current yields, an increase in the trend implies lower expected returns. Equation (B.11) shows the separate roles of trends and cycles for bond risk premia. If we hold the yield cycles  $\tilde{P}_t$  constant, the loadings on  $\tau_t$  are  $\mathcal{B}'_{n-1}(I_N - \Phi^Q)\gamma$ . These loadings tend to be an order of magnitude smaller (in absolute value) than the loadings on  $\tau_t$  in equation (B.10), consistent with the findings in Section 3.2 that predictive regressions with only detrended yields capture most of the predictability of excess bond returns. The loadings on  $\tau_t$  in (B.11) are positive because risk premia are positively related to movements in the trend. Unreported results show that predictive regressions with both detrended yields and trend proxies lead to coefficients on the trend that are insignificantly different from both zero and the model-based loadings—consistent with the implications of our model.

A term premium is the difference between a long-term interest rate and an expectations component of the same maturity. For simplicity, the expectations component is often specified as the average of expected future short rates, as in equation (1). In a no-arbitrage term structure model, it is advantageous to instead use “risk-neutral rates” which, like yields, account for the convexity of bond prices. Specifically, the risk-neutral yield on an  $n$ -period bond is

$$\tilde{y}_t^{(n)} = -\log E_t \exp \left( -\sum_{j=0}^{n-1} i_{t+j} \right) / n,$$

which differs from the average expected future short rate  $\sum_{j=0}^{n-1} E_t i_{t+j} / n$  by a Jensen’s inequality term that is constant in a homoskedastic model like ours. The risk-neutral yields are affine in  $Z_t$ , and the loadings are calculated using recursions similar to those in equations (B.6)- (B.7), with  $\mu_Z$  and  $\Phi_Z$  replacing  $\mu^Q$  and  $\Phi^Q$ ,  $\Omega_v$  replacing  $\Omega$ , and an  $N + 1$  vector  $(0, \delta_1)'$  replacing  $\delta_1$ . Crucially, risk-neutral yields depend on both  $\tau_t$  and  $P_t$ , since  $\tau_t$  affects real-world expectations of future short rates. The term premium on an  $n$ -period bond is then defined as

$$TP_t^{(n)} = y_t^{(n)} - \tilde{y}_t^{(n)}.$$

Because the real-world dynamics feature a unit root, risk-neutral rates diverge to minus infinity (and the term premium to infinity) as maturity increases. This issue is neither of theoretical nor practical concern: First, no tradeable securities have payoffs tied to these quantities. Second, at the maturities we focus on, due to the high persistence of the risk-neutral dynamics, the convexity in actual and risk-neutral yields is quite similar, and estimates of the term premium are not noticeably affected by convexity.

Risk-neutral forward rates are

$$\tilde{f}_t^{(n)} = -\frac{1}{2}\tilde{\mathcal{B}}_n'\Omega_v\tilde{\mathcal{B}}_n + \tau_t + \delta_1'\Phi^n\tilde{P}_t, \quad (\text{B.12})$$

consistent with  $\tilde{f}_t^{(n)} = E_t i_{t+n}$  + convexity and short-rate expectations that move one-for-one with the trend. Equations (B.8) and (B.12) imply that the forward term premium is

$$ftp_t^{(n)} = f_t^{(n)} - \tilde{f}_t^{(n)} = \text{constant} + \delta_1' [(\Phi^Q)^n - I_N] \gamma \tau_t + \delta_1' [(\Phi^Q)^n - \Phi^n] \tilde{P}_t. \quad (\text{B.13})$$

The loadings on  $\tau_t$  have a limit of -1 as  $n$  tends to infinity. The limiting-maturity forward rate is constant, as the limiting-maturity forward term premium exactly offsets movements in the limiting-maturity risk-neutral rate. However, for maturities relevant in the real world, forward rates and yields have loadings on  $\tau_t$  greater than one and positive term premia loadings—consistent with the cointegration regressions reported in Section 3.1.

Table 1: Cointegration regressions and tests

	Yield	(1)	(2)	(3)	(4)	(5)
constant	6.48 (0.55)	0.65 (0.54)	-0.22 (0.33)	-1.82 (0.43)	-0.64 (0.39)	-2.17 (0.30)
$\pi_t^*$		1.65 (0.11)	1.26 (0.08)	1.53 (0.07)	1.47 (0.07)	
$r_t^*$			0.99 (0.15)	1.76 (0.16)	1.18 (0.13)	
$i_t^*$						1.67 (0.06)
$R^2$		0.85	0.93	0.96	0.96	0.95
Memo: $r^*$			filtered	real-time	mov. avg.	real-time
SD	2.94	1.31	1.09	0.70	0.87	0.70
$\hat{\rho}$	0.97	0.88	0.85	0.65	0.75	0.64
Half-life	26.4	5.6	4.3	1.6	2.5	1.5
ADF	-1.13	-2.60	-3.94**	-5.32***	-4.33**	-5.37***
PP	-3.11	-18.32*	-26.73**	-68.47***	-46.30***	-70.30***
LFST	0.00	0.03	0.16	0.72	0.23	0.71
Johansen $r = 0$		13.34	33.08*	46.83***	45.49***	30.69***
Johansen $r = 1$		1.29	5.92	11.57	9.14	0.73
ECM $\hat{\alpha}$		-0.11 (0.03)	-0.18 (0.05)	-0.44 (0.08)	-0.49 (0.09)	-0.45 (0.08)

Dynamic OLS regressions of the ten-year yield,  $y_t^{(40)}$ , on macroeconomic trends, including four leads and lags of  $\Delta y_t^{(40)}$  and differenced trend variables. Newey-West standard errors using six lags are in parentheses. The  $r^*$  estimates are described in Section 2, and the long-run nominal short rate  $i_t^*$  is the sum of  $\pi_t^*$  and the real-time estimate of  $r_t^*$ . For the cointegration residuals (and, in the first column, for the ten-year yield itself), the second panel reports standard deviations (SD), first-order autocorrelation coefficients ( $\hat{\rho}$ ), half-lives ( $\ln(0.5)/\ln(\hat{\rho})$ ), Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root test statistics, and  $p$ -values for Mueller-Watson low-frequency stationary test (LFST). The table also reports the Johansen trace statistic which tests whether the cointegration rank ( $r$ ) among the yield and the macro trends is zero/one against the alternative that it exceeds zero/one, using four lags in the VAR. For ADF, PP and Johansen tests \*, \*\* and \*\*\* indicate significance at 10%, 5%, and 1% level. Estimates of the coefficient  $\alpha$  (with White standard errors) on the cointegration residual in the error-correction model (ECM) for  $\Delta y_t^{(40)}$  that also includes an intercept, four lags of  $\Delta y_t^{(40)}$ , and four lags of differenced macro trends. The data are quarterly from 1971:Q4 to 2018:Q1.

Table 2: Predictive regressions: yields and macro trends

	(1)	(2)	(3)	(4)	(5)	(6)
<i>Full sample: 1971:Q4–2018:Q1</i>						
PC1	0.08 (0.17)	0.98 (0.26)	1.39 (0.39)	2.38 (0.67)	2.04 (0.56)	2.47 (0.61)
PC2	0.43 (0.17)	0.47 (0.17)	0.43 (0.17)	0.67 (0.15)	0.68 (0.15)	0.70 (0.15)
PC3	-2.37 (1.34)	-1.79 (1.27)	-1.92 (1.22)	-0.92 (1.39)	-0.90 (1.43)	-0.86 (1.35)
$\pi_t^*$		-1.95 (0.44) [0.00]	-2.21 (0.47) [0.00]	-4.40 (1.10) [0.00]	-3.89 (0.92) [0.00]	
$r_t^*$			-1.19 (0.59) [0.14]	-3.89 (1.47) [0.07]	-2.71 (1.04) [0.04]	
$i_t^*$						-4.50 (1.05) [0.00]
$R^2$	0.09	0.16	0.18	0.21	0.20	0.21
Memo: $r^*$			filtered	real-time	mov. avg.	real-time
<i>Subsample: 1985:Q1–2018:Q1</i>						
PC1	0.25 (0.16)	0.59 (0.22)	1.67 (0.47)	2.65 (0.57)	2.38 (0.51)	1.93 (0.47)
PC2	0.41 (0.15)	0.50 (0.16)	0.49 (0.16)	0.53 (0.15)	0.65 (0.15)	0.58 (0.15)
PC3	-1.09 (1.14)	-0.97 (1.12)	0.14 (1.30)	1.74 (1.48)	2.11 (1.55)	0.56 (1.19)
$\pi_t^*$		-1.05 (0.73) [0.38]	-1.95 (0.75) [0.10]	-3.44 (0.87) [0.01]	-3.34 (0.83) [0.01]	
$r_t^*$			-2.03 (0.82) [0.07]	-5.80 (1.54) [0.01]	-4.11 (1.08) [0.01]	
$i_t^*$						-3.08 (0.91) [0.02]
$R^2$	0.08	0.10	0.14	0.19	0.18	0.16
Memo: $r^*$			filtered	real-time	mov. avg.	real-time

Predictive regressions for quarterly excess bond returns, averaged across two- to 15-year maturities. The predictors are the first three principal components of yields (PC1, PC2, PC3) and estimates of the inflation trend  $\pi_t^*$ , the real-rate trend  $r_t^*$ , and the long-run nominal short rate  $i_t^*$ . The long-run nominal short rate  $i_t^*$  is the sum of  $\pi_t^*$  and the real-time estimate of  $r_t^*$ . The macro trend estimates are described in Section 2. Numbers in parentheses are White standard errors and in squared brackets are small-sample  $p$ -values obtained with the bootstrap method of [Bauer and Hamilton \(2018\)](#). The data are quarterly from 1971:Q4 to 2018:Q1.

Table 3: Predictive regressions: detrended yields

	Full sample: 1971–2018				Subsample: 1985–2018			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
PC1	0.08 (0.17)	0.98 (0.25)	1.25 (0.51)	1.36 (0.50)	0.33 (0.16)	0.41 (0.22)	1.14 (0.39)	0.81 (0.38)
PC2	0.43 (0.17)	0.48 (0.17)	0.76 (0.16)	0.78 (0.16)	0.34 (0.14)	0.44 (0.14)	0.62 (0.14)	0.62 (0.14)
PC3	-2.37 (1.34)	-1.77 (1.26)	-0.79 (1.38)	-0.73 (1.33)	-1.04 (1.01)	-0.96 (1.01)	1.53 (1.32)	0.45 (1.05)
$R^2$	0.09	0.15	0.18	0.20	0.08	0.08	0.17	0.14

Predictive regressions for quarterly excess bond returns, averaged across two- to 15-year maturities. In specification (1) the predictors are the first three principal components of observed yields (PC1, PC2, PC3). In specifications (2), (3), and (4) the predictors are the first three principal components of detrended yields, that is, of residuals in regressions for yields on  $\pi_t^*$ ,  $\pi_t^*$  and  $r_t^*$ , or  $i_t^*$ , respectively. The long-run nominal short rate  $i_t^*$  is the sum of  $\pi_t^*$  and the real-time estimate of  $r_t^*$ . The macro trend estimates are described in Section 2. Numbers in parentheses are White standard errors with six lags. The data are quarterly from 1971:Q4 to 2018:Q1.

Table 4: Model-implied predictability of excess bond returns

	$R^2$ PCs only	$R^2$ with $i_t^*$	$\Delta R^2$
Data	0.09	0.21	0.12
<i>FE</i> model	0.09	0.10	0.01
	[0.04, 0.17]	[0.04, 0.17]	[0.00, 0.04]
<i>OSE</i> model	0.10	0.19	0.09
	[0.04, 0.17]	[0.13, 0.26]	[0.02, 0.18]
<i>ESE</i> model	0.07	0.14	0.08
	[0.02, 0.13]	[0.05, 0.27]	[0.00, 0.20]

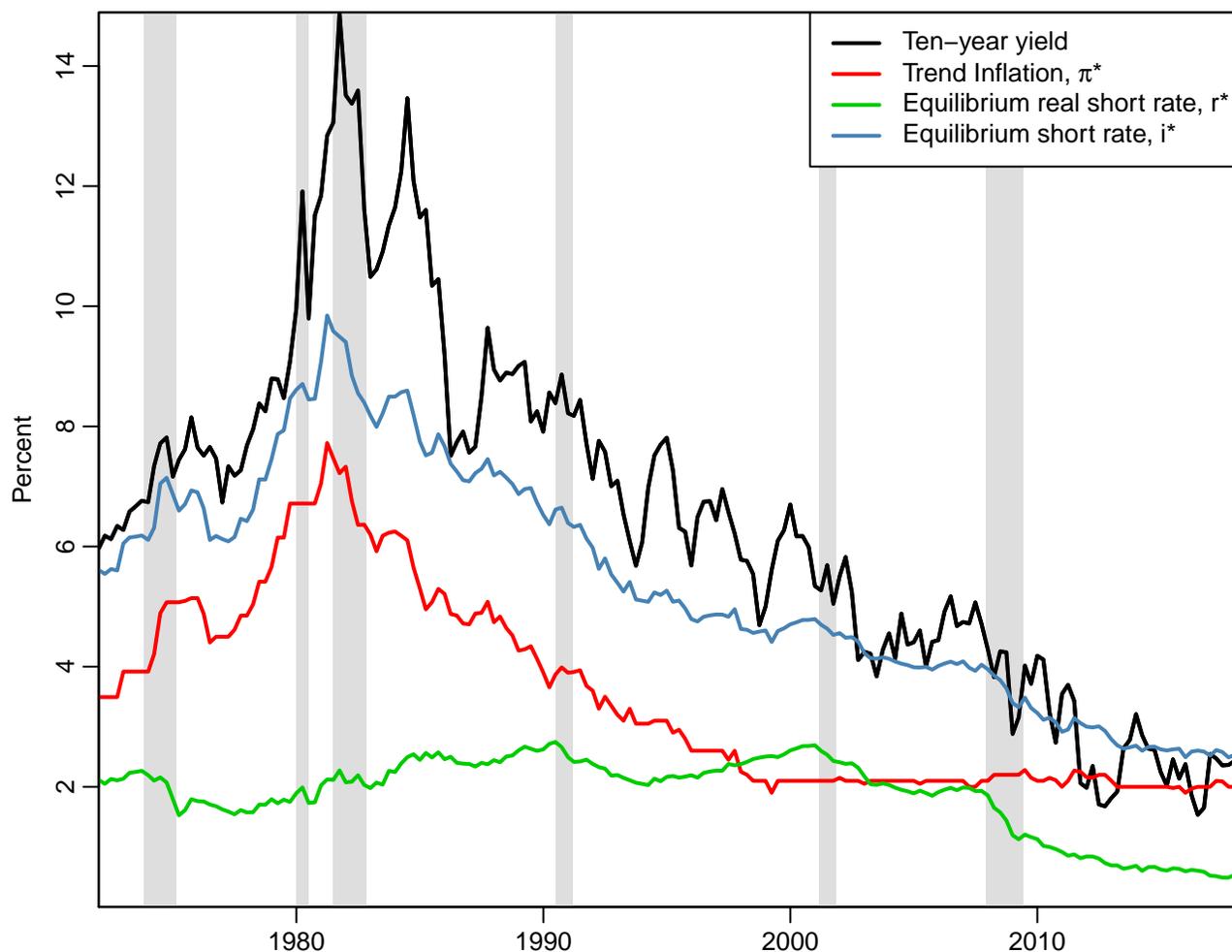
The  $R^2$  of predictive regressions for quarterly excess bond returns, averaged across maturities of 2 to 15 years. The  $R^2$  in the data correspond to the full-sample estimates in Table 2 (first and last columns of top panel). The model-implied  $R^2$  are based on 10,000 simulations of artificial data sets of the same size as the full sample. The table reports means and 95%-Monte Carlo intervals (in square brackets) of the  $R^2$  of predictive regressions estimated in these simulated data.

Table 5: Accuracy of out-of-sample forecasts for the ten-year yield

<i>Quarterly sample: 1976:Q3-2008:Q1</i> (127 quarters)	Horizon in quarters				
	4	10	20	30	40
Random walk ( <i>RW</i> )	1.33	1.85	2.52	2.60	2.88
Fixed endpoint ( <i>FE</i> )	1.42	2.25	3.28	3.72	4.19
Observed shifting endpoint ( <i>OSE</i> )	1.17	1.76	2.37	2.39	2.60
<i>p</i> -value: $OSE \geq RW$	0.05	0.00	0.00	0.03	0.04
<i>p</i> -value: $OSE \geq FE$	0.00	0.00	0.01	0.03	0.05
<i>Blue Chip sample: 1988:Q1-2011:Q4</i> (48 Blue Chip surveys)	Horizon in years				
	1	2	3	4	5
Blue Chip ( <i>BC</i> )	1.06	1.39	1.59	1.79	1.99
Random walk ( <i>RW</i> )	0.85	1.08	1.21	1.37	1.56
Fixed endpoint ( <i>FE</i> )	1.53	2.08	2.52	2.96	3.34
Observed shifting endpoint ( <i>OSE</i> )	0.87	0.95	1.04	1.18	1.37
<i>p</i> -value: $OSE \geq BC$	0.10	0.08	0.15	0.18	0.20
<i>p</i> -value: $OSE \geq RW$	0.58	0.05	0.01	0.04	0.08
<i>p</i> -value: $OSE \geq FE$	0.00	0.00	0.00	0.00	0.00

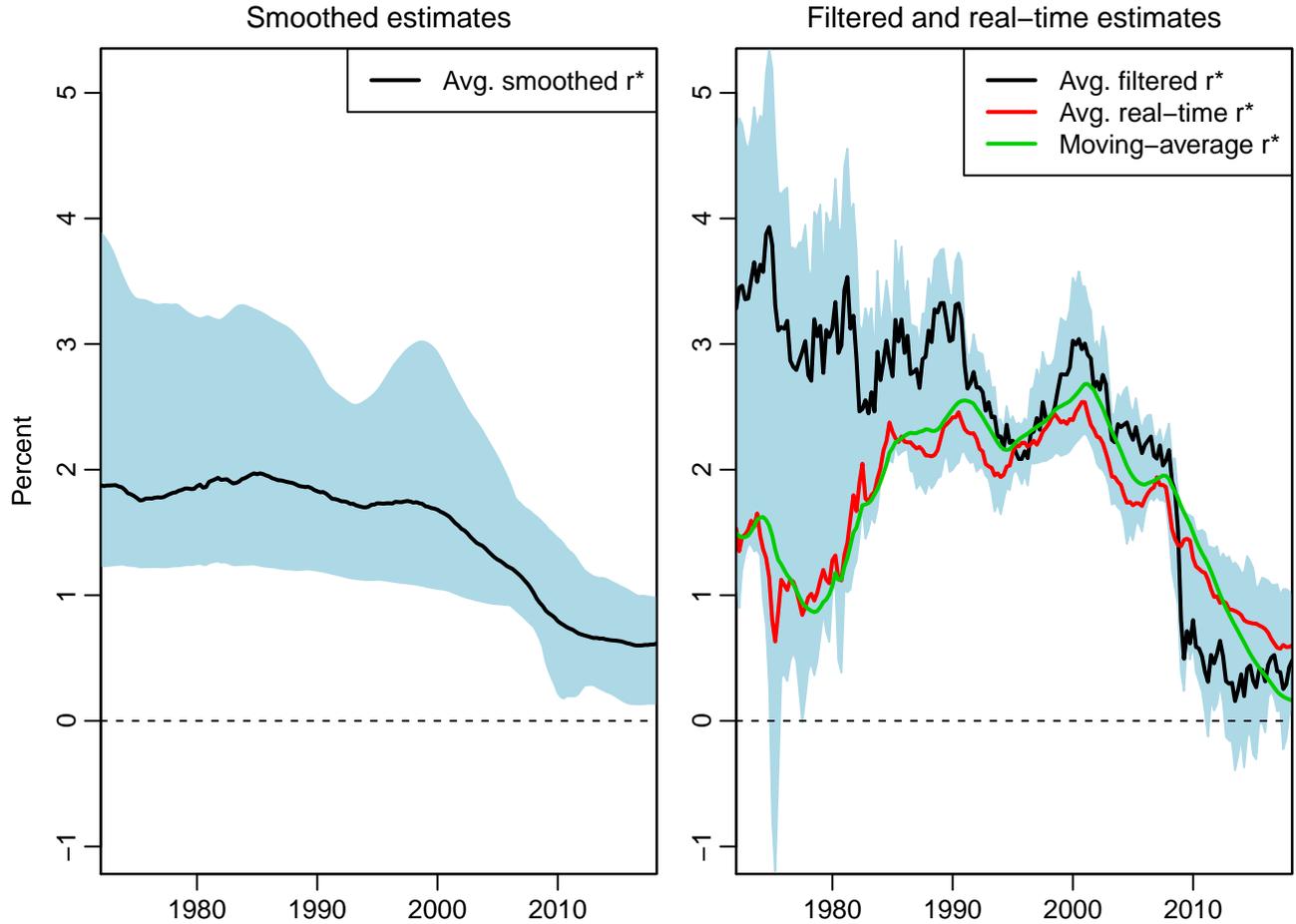
Root mean-squared errors for forecasts for the ten-year Treasury yield. Method *RW* is a driftless random walk, *FE* is a common stationary dynamic term structure model (DTSM), which has a fixed endpoint, and *OSE* is a model with an observed shifting endpoint that uses the proxy  $i_t^* = \pi_t^* + r_t^{*,RT}$ . The data are quarterly from 1971:Q4 to 2018:Q1. Top panel: forecasts for horizons from 4 to 40 quarters made each quarter from 1976:Q3 (once five years of data are available) to 2008:Q1 for a total of 127 (overlapping) observations. Bottom panel: forecasts for horizons from 1 to 5 years (yearly averages of quarterly forecasts) made each quarter preceding a Blue Chip survey (long-range forecasts in Blue Chip Financial Forecasts) from 1988:Q1 to 2011:Q4 (48 surveys). Models are re-estimated each quarter up to 2007:Q4 (expanding window). Last rows in each panel report one-sided *p*-values for testing the null hypothesis of equal forecast accuracy against the alternative that method *OSE* is more accurate, using the method of [Diebold and Mariano \(1995\)](#) with a small-sample correction.

Figure 1: Ten-year yield and macroeconomic trends



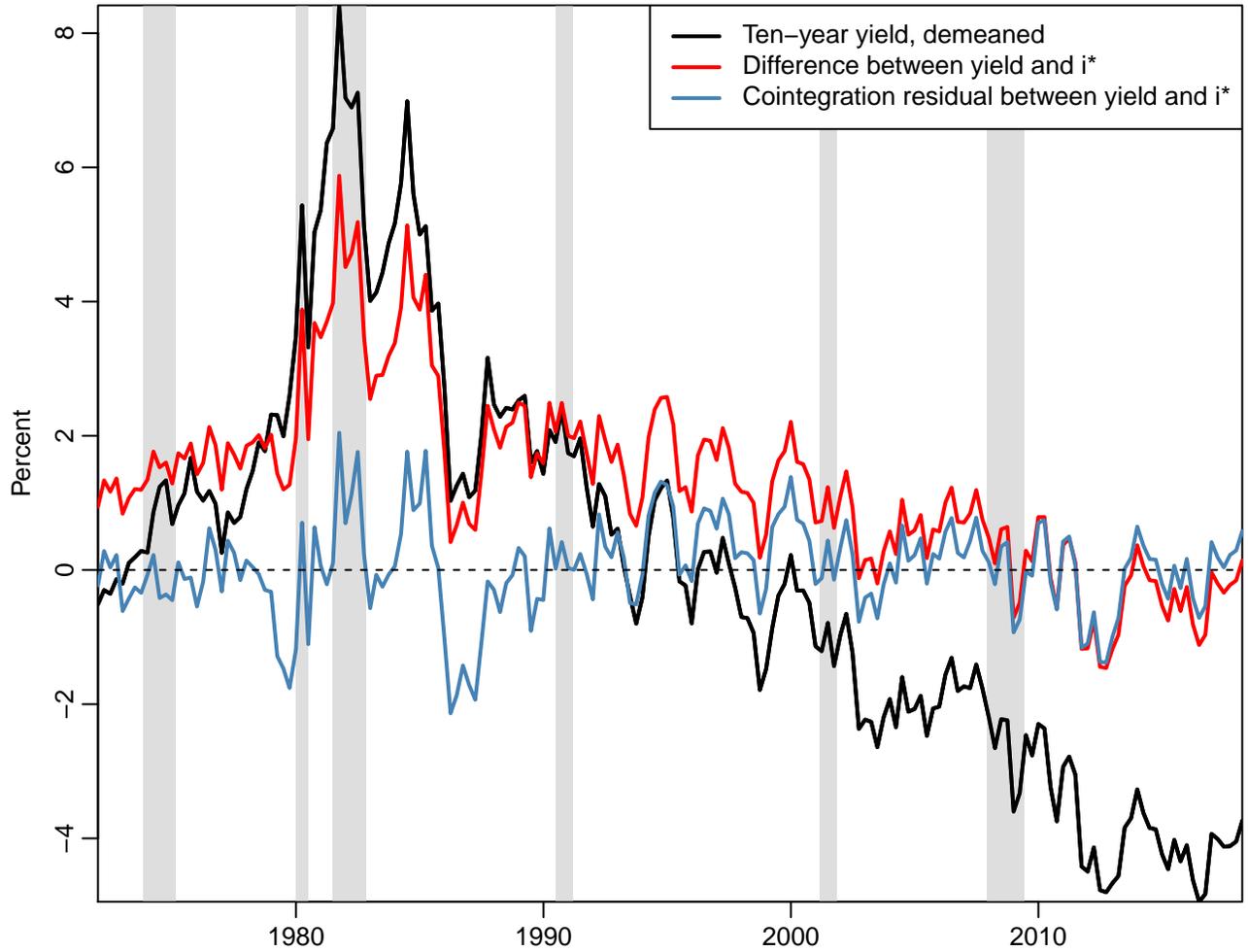
Ten-year Treasury yield and estimates of trend inflation,  $\pi_t^*$  (the mostly survey-based PTR measure from FRB/US), the equilibrium real rate,  $r_t^*$  (an average of all filtered and real-time estimates, see Section 2), and the equilibrium short rate,  $i_t^* = \pi_t^* + r_t^*$ . Shaded areas are NBER recessions. The data are quarterly from 1971:Q4 to 2018:Q1.

Figure 2: Measures of the equilibrium real interest rate



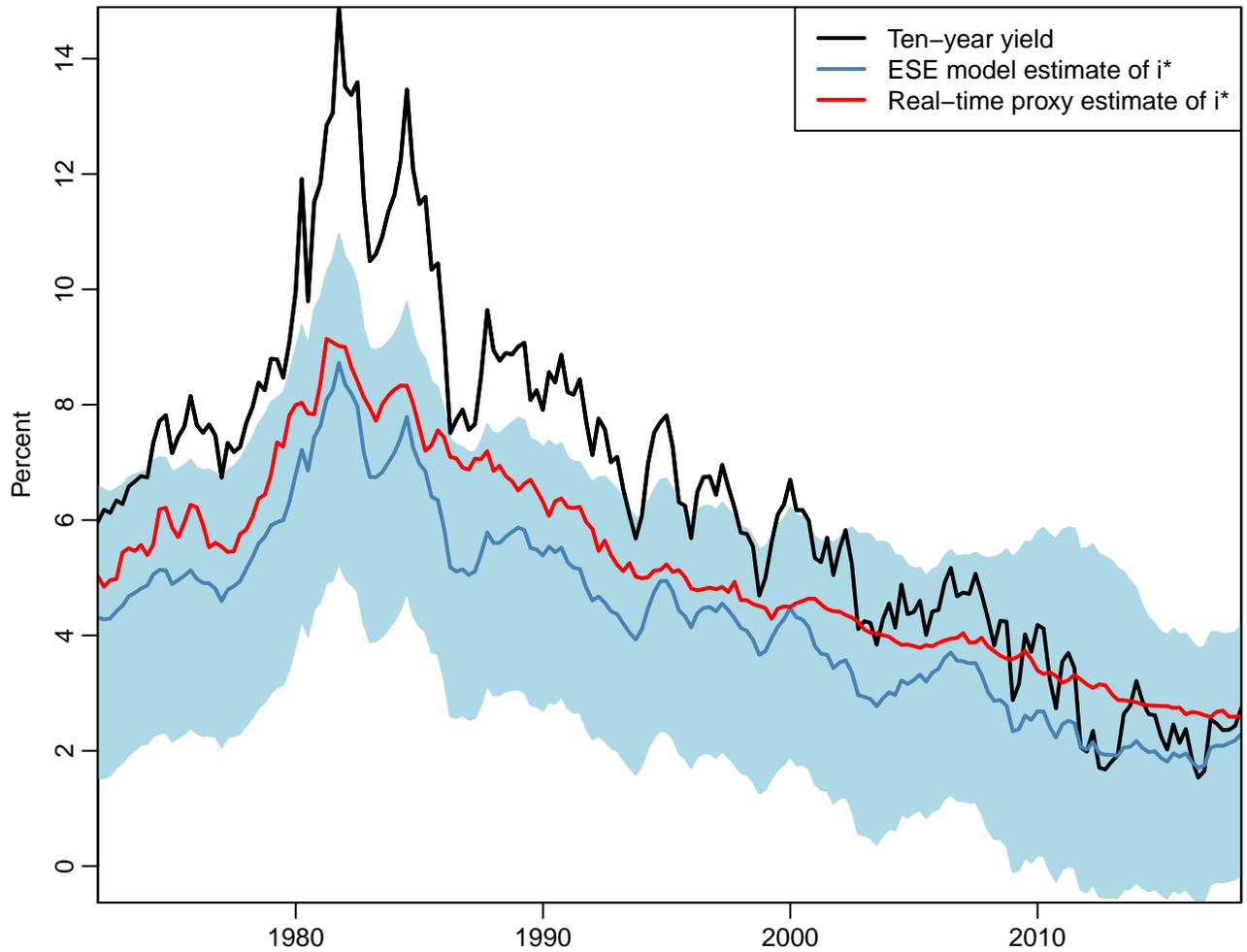
Smoothed (two-sided), filtered (one-sided), and real-time estimates of the equilibrium real interest rate, or long-run  $r_t^*$ . Left panel: range and average of seven smoothed estimates, including the four estimates by [Del Negro et al. \(2010\)](#) (DGGT), [Johannsen and Mertens \(2018\)](#) (JM), [Laubach and Williams \(2016\)](#) (LW), and [Kiley \(2015\)](#), as well as our own three model-based estimates. Right panel: range and average of filtered and real-time estimates. Filtered estimates are the estimates of LW, [Holston et al. \(2017\)](#), and Kiley. Real-time estimates are those of DGGT, JM, our own three real-time model-based estimates, and the model-free moving-average estimate, which is also shown separately. For details, see [Section 2](#) and [Appendix A](#). The data are quarterly from 1971:Q4 to 2018:Q1.

Figure 3: Detrending the ten-year yield



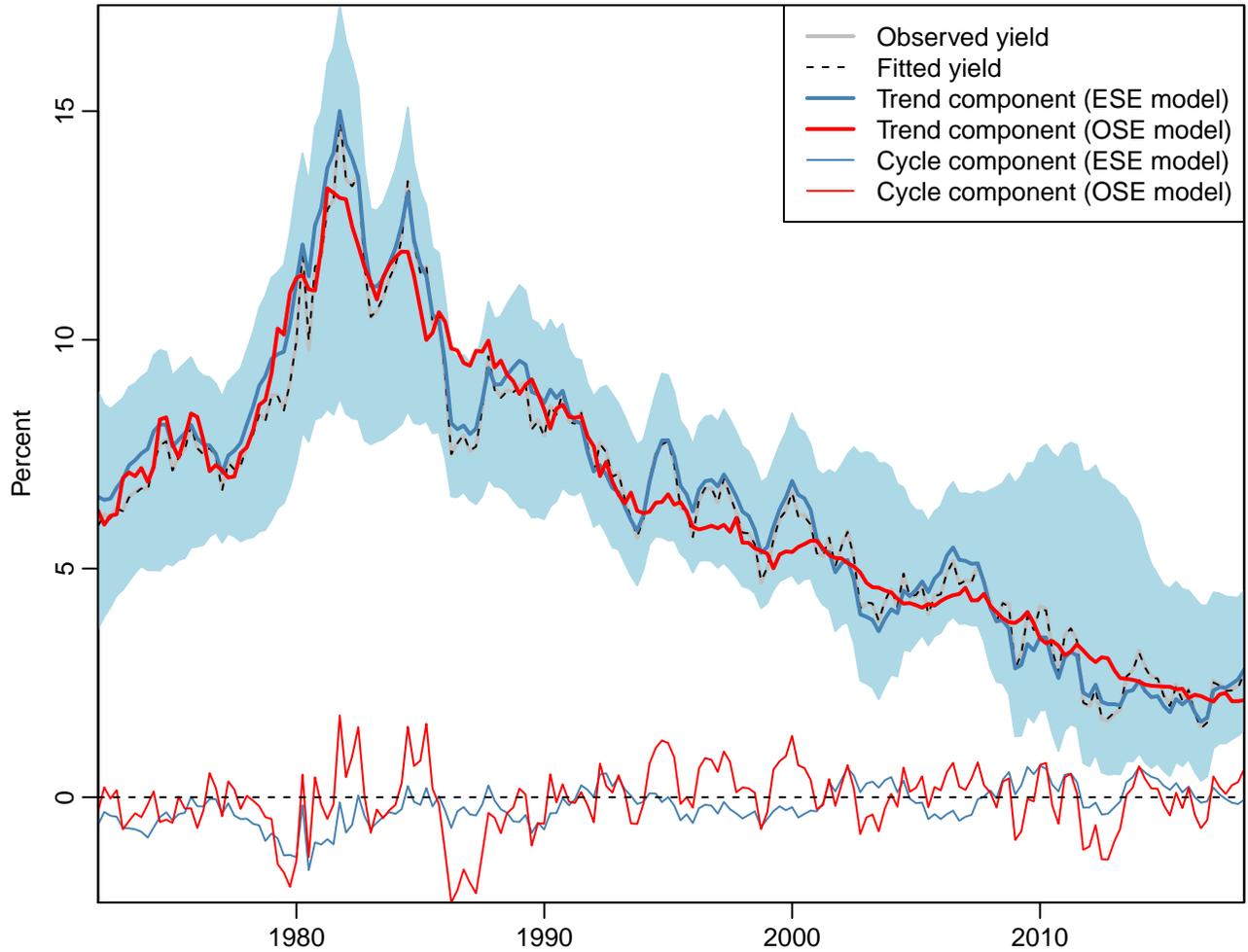
Ten-year Treasury yield and two detrended yield series using as the proxy for the equilibrium interest rate  $i_t^*$  the sum of the PTR estimate of  $\pi_t^*$  and the real-time estimate of  $r_t^*$  shown in Figure 2. The first detrended yield series is the difference between the yield and  $i_t^*$ . The second detrended series is the residual from the (Dynamic OLS) cointegration regression of the yield on  $i_t^*$ , estimates of which are reported in the last column of Table 1. Shaded areas are NBER recessions. The data are quarterly from 1971:Q4 to 2018:Q1.

Figure 4: Model-based estimate of equilibrium interest rate



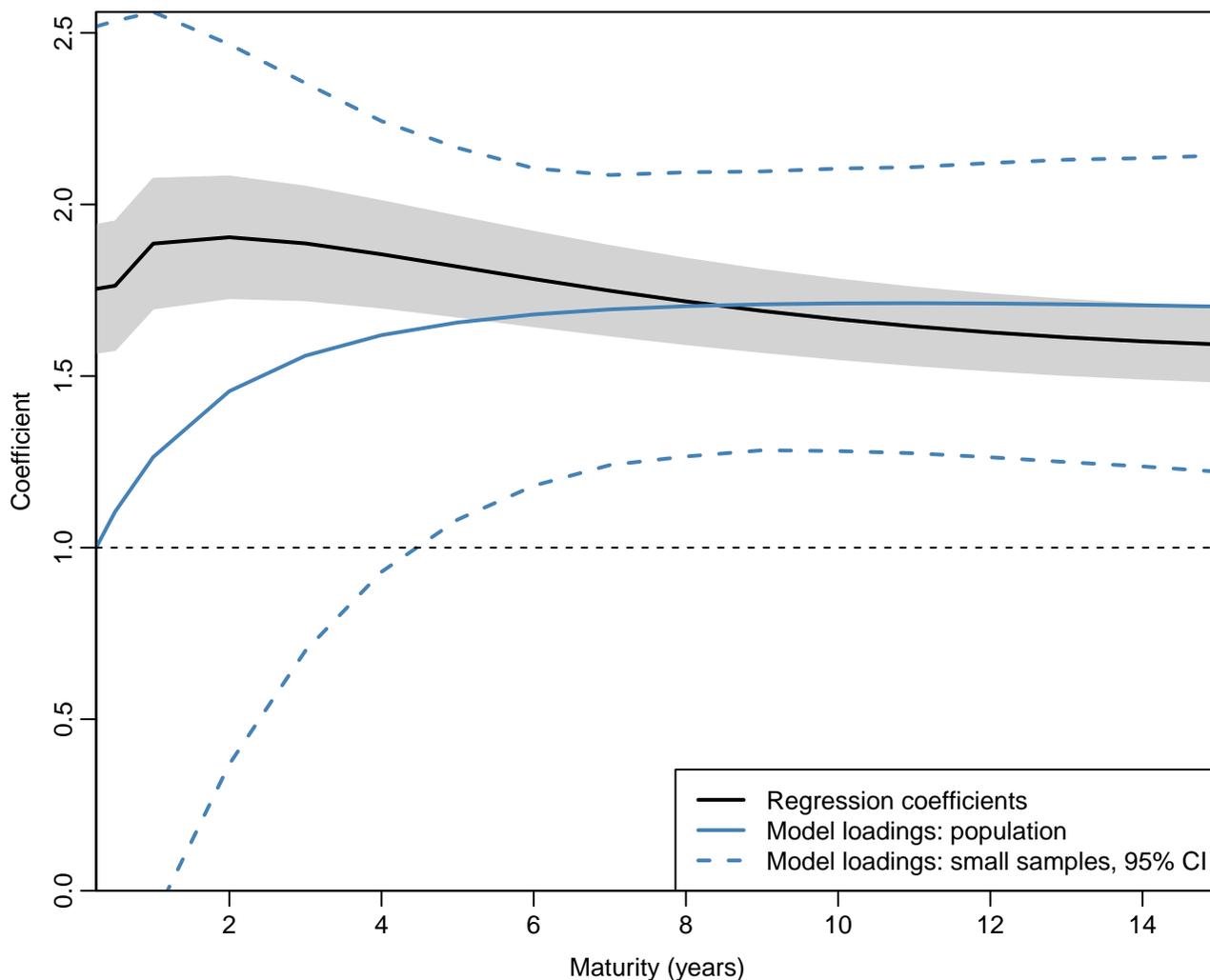
Bayesian (MCMC) estimate—posterior mean and 95%-posterior credibility intervals—of the equilibrium nominal short rate,  $i^*$ , from the estimated shifting endpoint (*ESE*) dynamic term structure model. Also shown is a proxy estimate of  $i^*$ , the sum of the PTR estimate of  $\pi^*$  and the real-time macro estimate of  $r^*$ . The data are quarterly from 1971:Q4 to 2018:Q1.

Figure 5: Trend and cycle components of ten-year yield



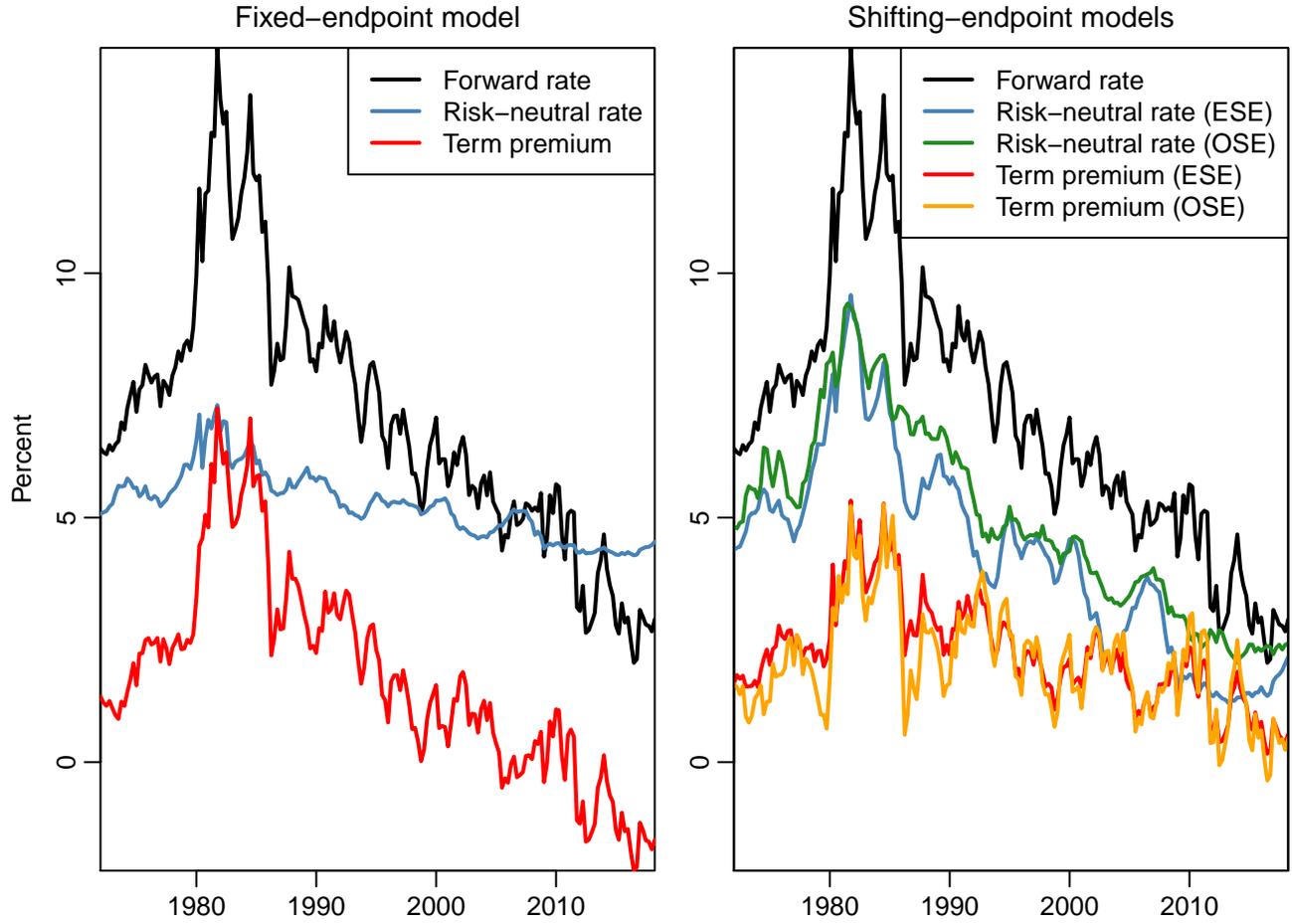
Model-based decomposition of ten-year yield into trend and cycle components. Blue lines show trend and cycle (posterior means) for the model with estimated shifting endpoint (*ESE*), shaded area shows 95%-posterior credibility intervals for trend component; red lines show trend and cycle for model with observed shifting endpoint (*OSE*). The data are quarterly from 1971:Q4 to 2018:Q1.

Figure 6: Loadings of yields on the equilibrium interest rate



Comparison of loadings of Treasury yields on the equilibrium nominal short rate  $i_t^*$  in the data and in the shifting-endpoint dynamic term structure model. Data: regression coefficients (with 95%-confidence intervals) for cointegration regressions of yields on the proxy-estimate of  $i_t^*$  (the PTR estimate of  $\pi^*$  plus the real-time macro estimate of  $r^*$ ). Model: loadings of yields on  $i_t^*$  in population and 95%-Monte Carlo intervals for regression coefficients in small samples simulated from the model with an observed shifting endpoint (*OSE*).

Figure 7: Expectations and term premium components in long-term interest rates



Five-to-ten-year forward rate with estimated expectations component (risk-neutral rate) and term premium. Left panel: conventional dynamic term structure model (DTSM) with a fixed endpoint  $i^*$  (Joslin et al., 2011). Right panel: DTSM with a common stochastic trend, where the shifting endpoint is either estimated (*ESE*, estimated shifting endpoint) or taken as observed using a proxy of  $i_t^*$  (*OSE*, observed shifting endpoint). The data are quarterly from 1971:Q4 to 2018:Q1.