Online Appendix:

Has U.S. Monetary Policy Tracked the Efficient Interest Rate?*

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*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Banks of New York or San Francisco, or the Federal Reserve System.

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A The Model

This Appendix presents the microfoundations of the baseline model discussed in Section 2 of the paper.

A.1 Households

A continuum of households of measure one populates the economy. All households, indexed by $j \in (0,1)$, discount the future at rate $\beta \in (0,1)$ and have the same instantaneous utility function, additively separable over consumption and labor, so that their objective is

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \prod_{s=0}^{t} e^{-\delta_s} \left[\log(C_t^j - \eta C_{t-1}^j) - \frac{(h_t^j)^{1+\omega}}{1+\omega} \right] \right\}. \tag{A.1}$$

The aggregate preference shock δ_t shifts the intertemporal allocation of consumption without affecting the intratemporal margin between labor and leisure. It follows a stationary AR(1) process

$$\delta_t = \rho_\delta \delta_{t-1} + \varepsilon_t^\delta. \tag{A.2}$$

The consumption index C_t^j is a constant elasticity of substitution aggregator over differentiated goods indexed by $i \in (0,1)$

$$C_t^j \equiv \left[\int_0^1 c_t^j(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{A.3}$$

Households supply their specialized labor input for the production of a specific final good. As a consequence of labor market segmentation, the wage w_t^j differs across households. However, household j can fully insure against idiosyncratic wage risk by buying at time t state-contingent securities D_{t+1}^j at price $Q_{t,t+1}$. Besides labor income, households earn profits Γ_t^j from ownership of the firms. The flow budget constraint for household j is

$$\int_0^1 p_t(i) c_t^j(i) di + E_t(Q_{t,t+1} D_{t+1}^j) = w_t^j h_t^j + D_t^j + \Gamma_t^j, \tag{A.4}$$

where $p_t(i)$ is the dollar price of the i^{th} good variety.

A.2 Firms

Firm i produces the differentiated consumption good $y_t(i)$ with a linear production function in labor

$$y_t(i) = A_t h_t(i). \tag{A.5}$$

We assume that productivity grows at rate $\gamma_t \equiv \Delta \log A_t$ and that growth rate shocks display some persistence

$$\gamma_t = (1 - \rho_\gamma)\,\gamma + \rho_\gamma \gamma_{t-1} + \varepsilon_t^\gamma. \tag{A.6}$$

Firms take wages as given and sell their products in monopolistically competitive goods markets, setting prices in a staggered fashion, as in Calvo (1983). Each firm faces a probability $(1-\alpha)$ of optimally choosing its price every period. The fraction α of firms that do not fully optimize in a given period adjust their price according to the indexation scheme

$$p_t(i) = p_{t-1}(i) \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\zeta} e^{(1-\zeta)\pi^*}, \tag{A.7}$$

where P_t is the aggregate price level consistent with the consumption aggregator (A.3) and we allow for partial indexation to the long run inflation target π^*.

In the event of a price change at time t, firm i chooses $p_t(i)$ to maximize the present discounted value of profits net of sales taxes τ_t

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{T-t} Q_{t,s} \left[(1-\tau_s) p_t(i) \left(\frac{P_{s-1}}{P_{t-1}}\right)^{\zeta} e^{(1-\zeta)\pi^*(s-t)} y_{t,s}(i) - w_s(i) h_s(i) \right] \right\}, \tag{A.8}$$

subject to its production function (A.5) and the demand for its own good conditional on no further price change after period t

$$y_{t,s}(i) = \left(\frac{p_t(i)}{P_s}\right)^{-\theta} Y_s, \tag{A.9}$$

where Y_t is an index of aggregate demand of the same form as (A.3).

A.3 Monetary Policy

The central bank sets the nominal interest rate R_t in gradual response to departures of inflation from target and of output from "potential". In the baseline W&T rule, this response takes the non-linear form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho} \left[e^{r_t^e} \left(\frac{P_t}{P_{t-1}e^{\pi^*}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^e}\right)^{\phi_x}\right]^{1-\rho} e^{\varepsilon_t^i}, \tag{A.10}$$

where the gross nominal interest rate is defined as

$$R_t \equiv \frac{1}{E_t Q_{t,t+1}}. \tag{A.11}$$

Its average can be decomposed via the Fisher equation as $R = e^{r+\pi^*}$, which defines the steady state real interest rate r.

The continuously compounded nominal interest rate in the text is defined as $i_t \equiv \log R_t$, so that the rule can be log-linearized as

$$i_t = \rho i_{t-1} + (1-\rho) i_t^* + \varepsilon_t^i \tag{A.12}$$

with

$$i_t^* = r_t^e + \phi_\pi \pi_t + \phi_x x_t^e \tag{A.13}$$

where i_t^* is the systematic response of the interest rate to the state of the economy. The functional form of i_t^* for each rule considered in the robustness exercises is shown in Table 3 of the paper and in Table D.1.

A.4 Efficient Equilibrium

In the efficient equilibrium, the marginal rate of substitution between hours and consumption equals their marginal rate of transformation (i.e. the marginal product of labor). With the linear production function in labor of (A.5), this equality is

$$\frac{\lambda_{A,t}^e}{(Y_{A,t}^e)^{\omega}} = 1. \tag{A.14}$$

where $\lambda_{A,t}^e \equiv \lambda_t^e A_t$ and $Y_{A,t}^e \equiv Y_t^e / A_t$ are the marginal utility of real consumption and the level of output in the efficient equilibrium, both normalized by the productivity level. Given the internal habit, the marginal utility of consumption is

$$\lambda_{A,t}^e = \frac{1}{Y_{A,t}^e - \eta e^{-\gamma_t} Y_{A,t-1}^e} - \eta \beta E_t \left[\frac{e^{-\delta_{t+1}}}{e^{\gamma_{t+1}} Y_{A,t+1}^e - \eta Y_{A,t}^e}\right] \tag{A.15}$$

which can be log-linearized to yield

$$\begin{aligned}\omega y_t^e = & -\varphi_\gamma \left[y_t^e - \eta_\gamma (y_{t-1}^e - \gamma_t) \right] \\ & + \varphi_\gamma \beta \eta_\gamma E_t \left[(1-\eta_\gamma)\delta_{t+1} + (y_{t+1}^e + \gamma_{t+1}) - \eta_\gamma y_t^e \right],\end{aligned} \tag{A.16}$$

where $y_t^e \equiv \ln(Y_{A,t}^e / Y_A^e)$, and η_γ and φ_γ are composite parameters defined as

$$\eta_\gamma \equiv \eta e^{-\gamma}, \tag{A.17}$$

$$\varphi_\gamma \equiv \frac{1}{(1-\beta\eta_\gamma)(1-\eta_\gamma)}. \tag{A.18}$$

This law of motion for efficient output can be manipulated to obtain the expression in the text

$$\omega y_t^e + \varphi_\gamma (y_t^e - \eta_\gamma y_{t-1}^e) - \varphi_\gamma \eta_\gamma \beta (E_t y_{t+1}^e - \eta_\gamma y_t^e) = \varphi_\gamma \eta_\gamma (\beta E_t \gamma_{t+1} - \gamma_t) + \frac{\beta\eta_\gamma}{1-\beta\eta_\gamma} E_t \delta_{t+1}. \tag{A.19}$$

The intertemporal Euler equation

$$\lambda_{A,t}^e = \beta E_t \left[\lambda_{A,t+1}^e e^{r_t^e - \delta_{t+1} - \gamma_{t+1}} \right], \tag{A.20}$$

holds in the efficient equilibrium and can be log-linearized to yield

$$r_t^e = E_t \gamma_{t+1} + E_t \delta_{t+1} - \omega \left(E_t y_{t+1}^e - y_t^e \right), \tag{A.21}$$

as shown in the paper.

B Marginal Posterior Distributions

This Appendix presents the marginal prior and posterior distributions for selected parameters in the baseline T and W specifications, as discussed in Section 4 of the text.

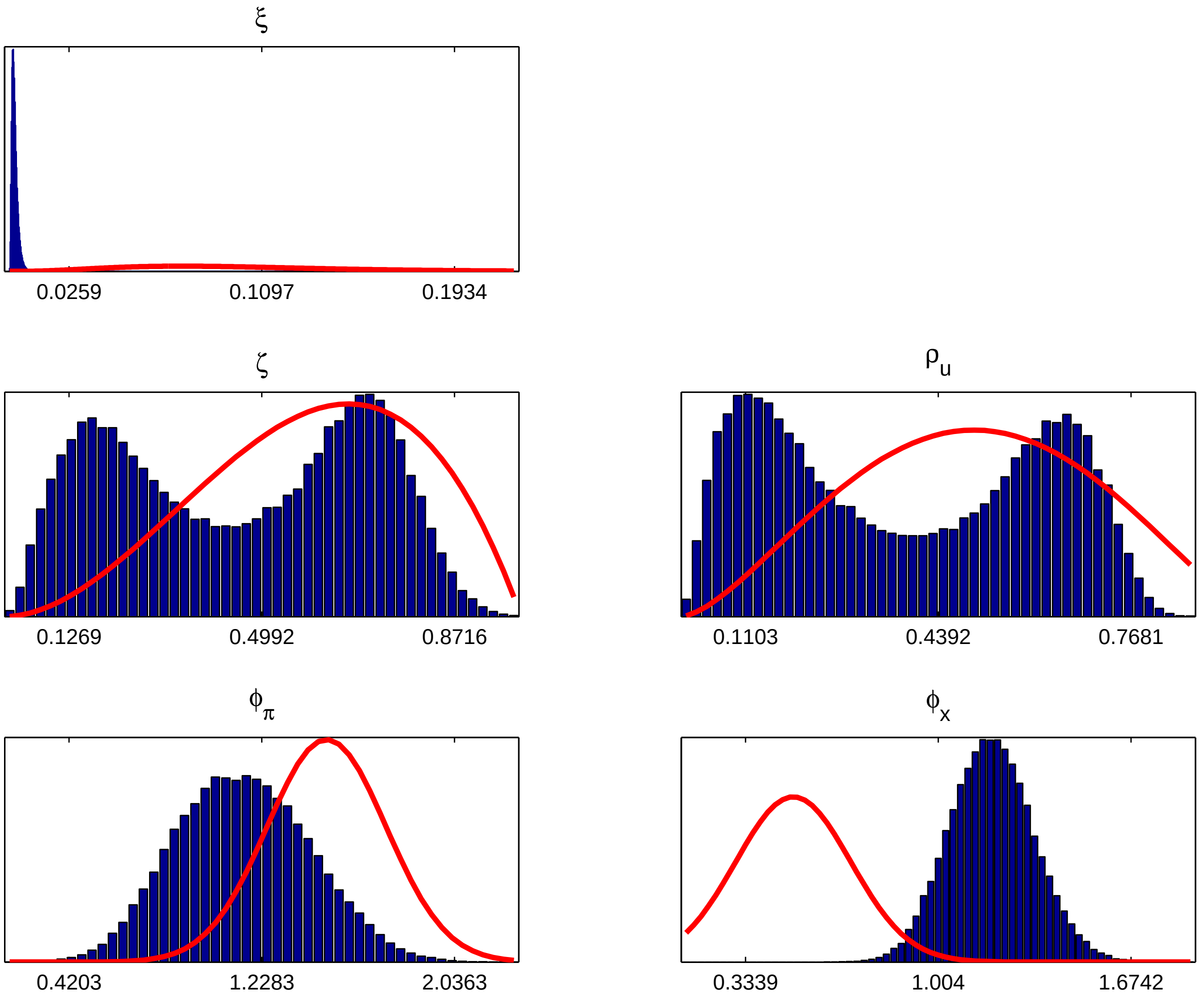

Figure B.1: Prior and posterior distributions for ξ, ζ, ρ_u, ϕ_π and ϕ_x under the baseline T rule. The solid red lines are the marginal priors for the parameters, while the blue histograms represent their posteriors from the MCMC simulations.

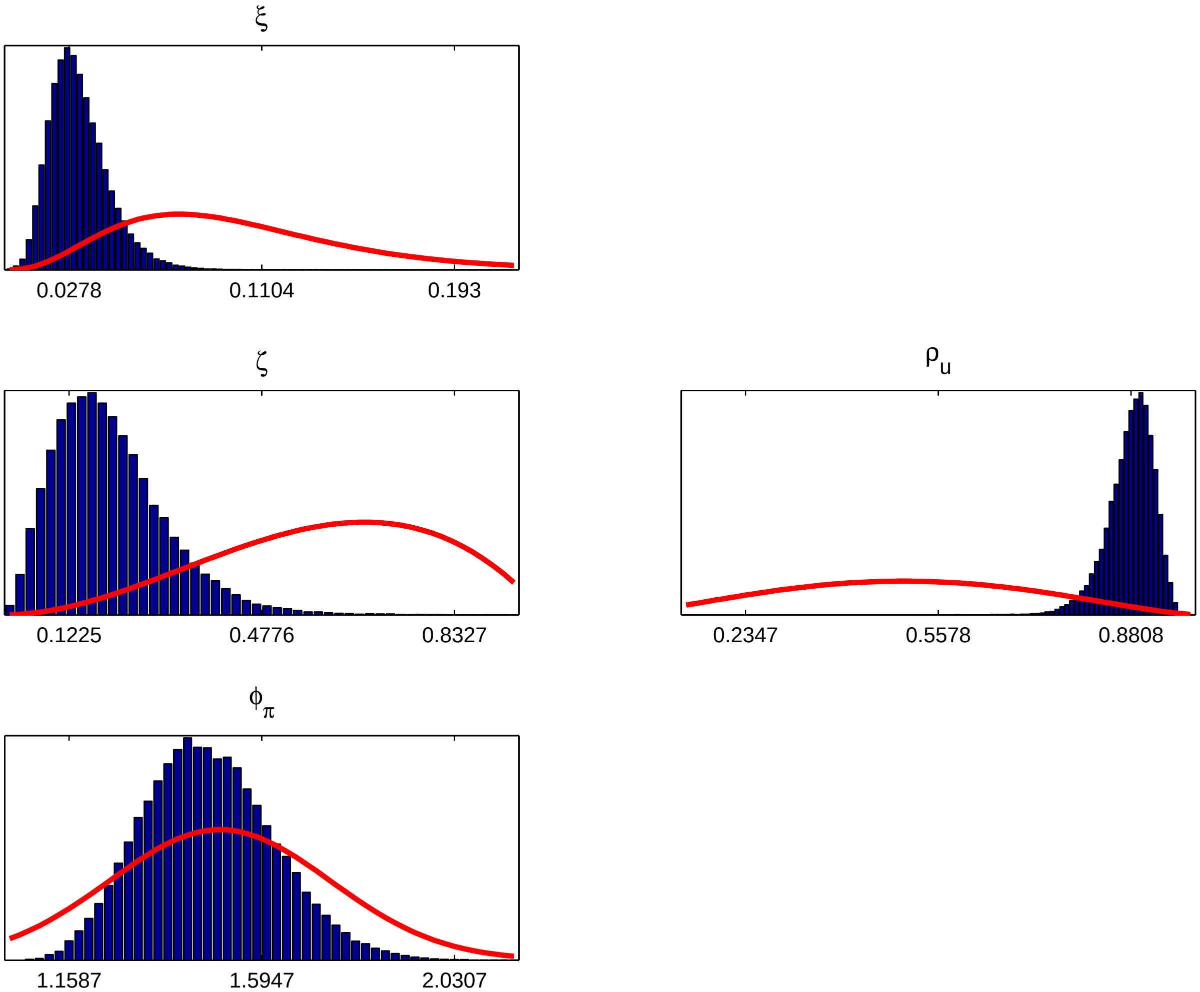

Figure B.2: Prior and posterior distributions for ξ, ζ, ρ_u and ϕ_π under the baseline W rule. The solid red lines are the marginal priors for the parameters, while the blue histograms represent their posteriors from the MCMC simulations.

C Statistical Filters in DSGE Models

This Appendix illustrates how to embed a linear filter into a dynamic rational expectations model. We begin with a brief general description of linear filtering problems. We then focus on the application to the Hodrick and Prescott (HP) filter (Hodrick and Prescott, 1997).

C.1 Linear Filters

The objective of "filtering" is to decompose the stochastic process x_t into two orthogonal components

$$x_t = y_t + \tilde{x}_t, \tag{C.1}$$

where the process y_t has power only in some frequency interval $\{(a,b) \cup (-a,-b)\} \in (-\pi,\pi)$. Then, we can represent y_t as

$$y_t = B(L)x_t, \tag{C.2}$$

where $B(L)$—the *ideal* band-pass filter—is of the form

$$B(L) = \sum_{j=-\infty}^{\infty} B_j L^j. \tag{C.3}$$

The previous formula shows that the implementation of the ideal filter requires an infinite dataset. We can think about approximating the ideal filter as a projection problem. Given a sample $x = [x_1, ..., x_T]$, the estimate of $y = [y_{1,} ..., y_T]$ is $\hat{y} = P[y|x]$, which is of the form

$$\hat{y}_t = \sum_{=-f}^{p} \hat{B}_j^{p,f} x_{t-j}, \tag{C.4}$$

where $f = T - t$ and $p = t - 1$. The main problem of this estimate is that the B coefficients require knowledge of $f_x(\omega)$, the spectral density of x.

Christiano and Fitzgerald (2003) show that, for most macro variables, the coefficients obtained by assuming that x_t is a random walk work well. One approach to the calculation of these coefficients is then to "expand" the available sample with the least squares optimal guesses of the missing data at the beginning and end of the sample. For the random walk, these data are just x_1 and x_T. Our proposal is to adopt the same philosophy (i.e. to expand the available dataset) in the context of our framework, using the rational expectation forecasts of the missing data obtained from the model.[1]

[1] Watson (2007) proposes a similar procedure using unrestricted ARIMA processes as forecasting tools.

C.2 Application to the HP Filter

In this section, we discuss the application of our methodology to the HP filter. We focus on the HP filter because of its wide use in macroeconomics as a flexible device to draw a smooth trend through the data. The HP filter provides a typical example of a "traditional" smooth measure of potential output and of the associated output gap. Its added advantage in our context is that the expression for the ideal filter is a simple function of lag polynomials. The result is a parsimonious (i.e. two leads and lags) recursive representation, which requires only a modest expansion of the model's state space.

The ideal HP filter is of the form (e.g. Baxter and King, 1999)

$$HP^g = \frac{\lambda(1-L)^2\,(1-F)^2}{1+\lambda(1-L)^2\,(1-F)^2} \tag{C.5}$$

$$HP^t = \frac{1}{1+\lambda(1-L)^2\,(1-F)^2} \tag{C.6}$$

where HP^g denotes the filter whose application results in the "gap", while HP^t denotes the filter whose application produces the trend.[2] The practical application of these filters requires an approximation, since they embed a two-sided, infinite moving average of the data. However, the application of Christiano and Fitzgerald (2003) insight to a rational expectations context allows us to use the ideal filter directly, where the approximation relies on the substitution of the infinite leads and lags implicit in $HP(L)$ with RE forecasts.

In particular, given observations on $\log GDP_t = y_t$, we define the HP gap with parameter λ as

$$\left[1+\lambda(1-L)^2(1-F)^2\right] x_t^{HP(\lambda)} = \lambda(1-L)^2(1-F)^2 y_t, \tag{C.7}$$

where now the forward and backward operators are defined by

$$Ly_t = y_{t-1} \tag{C.8}$$

$$Fy_t = E_t y_{t+1} \tag{C.9}$$

as it is standard in rational expectations models (e.g. Blanchard and Fischer, 1989).

Juillard, Kamenik, Kumhof, and Laxton (2006) is the only example we could find of an application to DSGEs models. The main objective of all these papers is to improve the end-of-sample performance of the filters in consideration.

[2] King and Rebelo (1993) originally derived these expressions as the solution of a "smoothing" problem. However, they also showed that this filter, with $\lambda = 1600$, approximates very well a high pass filter with cutoff frequency $\pi/16$ or 32 quarters.

D More Policy Rule Specifications

This Appendix reports results for many additional policy rule specifications beyond those considered in the text. These specifications include T rules with alternative measures of the output gap, as well as various permutations of the ingredients featured in the paper.

In all the cases reported here, as well as in several others we considered but are not reporting, W rules always fit the data better than the corresponding T rules, regardless of the measure of the output gap featured in the latter. In our numerous experiments, we could find no exception to this remarkable regularity. In many instances, as in the four cases reported in Table 3 in the text, W&T rules outperform W rules. However, this improvement in fit is always much smaller than that obtained when moving from the T rule to the W rule, confirming the superiority of r_t^e over the output gap as an indicator of the real economic developments that appear to be relevant for monetary policy.

These results are summarized in Table D.1. Panel I reports specifications of the T rule with alternative definitions of the output gap. These include the four-quarter growth rate of output, versions of the HP gap with different values of the smoothing parameter and two versions that use an exponential filter. HP($\hat{\lambda}$) gap estimates the smoothing coefficient along with the other parameters of the model. HP(λ^H) gap sets the smoothing parameter equal to 160,000, which produces an even smoother trend than $\lambda = 1,600$, close to a linear trend. The exponential filter x_t^{Exp} (Estrella, 2007) is similar to the HP filter, but simpler. It is defined by

$$[1 + \lambda(1 - L)]x_t^{Exp} = \lambda(1 - L)y_t,$$

where the smoothing parameter is set to $\lambda = 61.5$ or otherwise estimated ($\hat{\lambda}$).[3].

Panel II considers the same set of rules in the case of a time varying inflation target.

Panels III to VI consider the same set of rules as Panels I and II in Table 3 in the text (with and without a TVIT), but modified to include forward looking terms, as in Clarida, Galí, and Gertler (2000), or the four-quarter change in the price level

$$\pi_t^{4Q} \equiv (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4.$$

The latter is often used as a simple gauge of underlying inflationary pressures, since it is less influenced by high-frequency variation than the quarterly rate featured in the baseline rules.

[3]The value of 61.5 is chosen to match the gain of the HP filter at frequency $\omega = 2\pi/32$, which corresponds to an eight-year cycle (King and Rebelo, 1993)

Name	Policy Rule (i_t^*)	KR	ML
Panel I: Baseline, Additional Gaps			
W	$r_t^e + \phi_\pi \pi_t$		-360.7
T with 4Q growth	$\phi_\pi \pi_t + \phi_{\Delta y}(y_t - y_{t-4})/4$	-26.0	
T with HP($\hat{\lambda}$) gap	$\phi_\pi \pi_t + \phi_x x_t^{HP(\hat{\lambda})}$	-7.8	
T with HP(λ^H) gap	$\phi_\pi \pi_t + \phi_x x_t^{HP(\lambda^H)}$	-29.1	
T with Exp gap	$\phi_\pi \pi_t + \phi_x x_t^{Exp}$	-27.8	
T with Exp($\hat{\lambda}$) gap	$\phi_\pi \pi_t + \phi_x x_t^{Exp(\hat{\lambda})}$	-26.4	
Panel II: TVIT, Additional Gaps			
W	$r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*)$		-348.9
T with 4Q-Output growth rate	$\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_{\Delta y}(y_t - y_{t-4})/4$	-31.8	
T with HP($\hat{\lambda}$) gap	$\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{HP(\hat{\lambda})}$	-22.3	
T with HP(λ^H) gap	$\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{HP(\lambda^H)}$	-12.5	
T with Exp gap	$\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{Exp}$	-10.8	
T with Exp($\hat{\lambda}$) gap	$\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t^{Exp(\hat{\lambda})}$	-11.4	

Table D.1: Comparison of policy rules. Each panel shows the log-marginal likelihood (ML) for the relevant W rule, and the KR ratio for the other rules relative to the W rule. The second column contains the systematic component of the rule under consideration in the absence of interest rate smoothing (i_t^*), defined such that $i_t = \rho i_{t-1} + (1-\rho) i_t^* + \varepsilon_t^i$.

Name	Policy Rule (i_t^*)	KR	ML
Panel III: Forward-Looking			
W	$r_t^e + \phi_\pi E_t \pi_{t+1}$		-370.4
T	$\pi_t^* + \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1}^e$	-23.2	
W&T	$r_t^e + \pi_t^* + \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1}^e$	10.0	
T with Growth	$\pi_t^* + \phi_\pi E_t \pi_{t+1} + \phi_{\Delta y}(E_t y_{t+1} - y_t)$	-46.4	
T with HP Gap	$\pi_t^* + \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1}^{HP}$	-28.0	
Panel IV: Forward-Looking with TVIT			
W	$r_t^e + \pi_t^* + \phi_\pi E_t(\pi_{t+1} - \pi_{t+1}^*)$		-345.0
T	$\pi_t^* + \phi_\pi E_t(\pi_{t+1} - \pi_{t+1}^*) + \phi_x E_t x_{t+1}^e$	-16.4	
W&T	$r_t^e + \pi_t^* + \phi_\pi E_t(\pi_{t+1} - \pi_{t+1}^*) + \phi_x E_t x_{t+1}^e$	-13.0	
T with HP Output Gap	$\pi_t^* + \phi_\pi E_t(\pi_{t+1} - \pi_{t+1}^*) + \phi_x E_t x_{t+1}^{HP}$	-19.2	
T with Growth	$\pi_t^* + \phi_\pi E_t(\pi_{t+1} - \pi_{t+1}^*) + \phi_{\Delta y}(E_t y_{t+1} - y_t)$	-44.3	
Panel V: Four-Quarter Inflation			
W	$r_t^e + \phi_\pi \pi_t^{4Q}$		-359.5
T	$\phi_\pi \pi_t^{4Q} + \phi_x x_t^e$	-13.6	
W&T	$\phi_\pi \pi_t^{4Q} + \phi_x x_t^e$	6.8	
T with Growth	$\phi_\pi \pi_t^{4Q} + \phi_{\Delta y}(y_t - y_{t-1})$	-28.2	
T with HP Output Gap	$\phi_\pi \pi_t^{4Q} + \phi_x x_t^{HP}$	-10.0	
Panel VI: Four-Quarter Inflation with TVIT			
W	$r_t^e + \pi_t^* + \phi_\pi(\pi_t^{4Q} - \pi_t^*)$		-347.4
T	$\pi_t^* + \phi_\pi(\pi_t^{4Q} - \pi_t^*) + \phi_x x_t^e$	-20.1	
W&T	$r_t^e + \pi_t^* + \phi_\pi(\pi_t^{4Q} - \pi_t^*) + \phi_x x_t^e$	-0.1	
T with Growth	$\pi_t^* + \phi_\pi(\pi_t^{4Q} - \pi_t^*) + \phi_{\Delta y}(y_t - y_{t-1})$	-34.1	
T with HP Output Gap	$\pi_t^* + \phi_\pi(\pi_t^{4Q} - \pi_t^*) + \phi_x x_t^{HP}$	-9.3	

Table D.1: Comparison of policy rules (Continued).

E Estimates of the Efficient Real Rate

Figure E.1 complements Figures 1 and 3 in the text by reporting posterior medians of the time path of the efficient real rate estimated within a variety of models featured either in the text or in the Appendix. The consistency of the estimates of r_t^eacross these many disparate specifications, which include both the baseline and JPT model, along with several different policy specifications in the former, is remarkable.

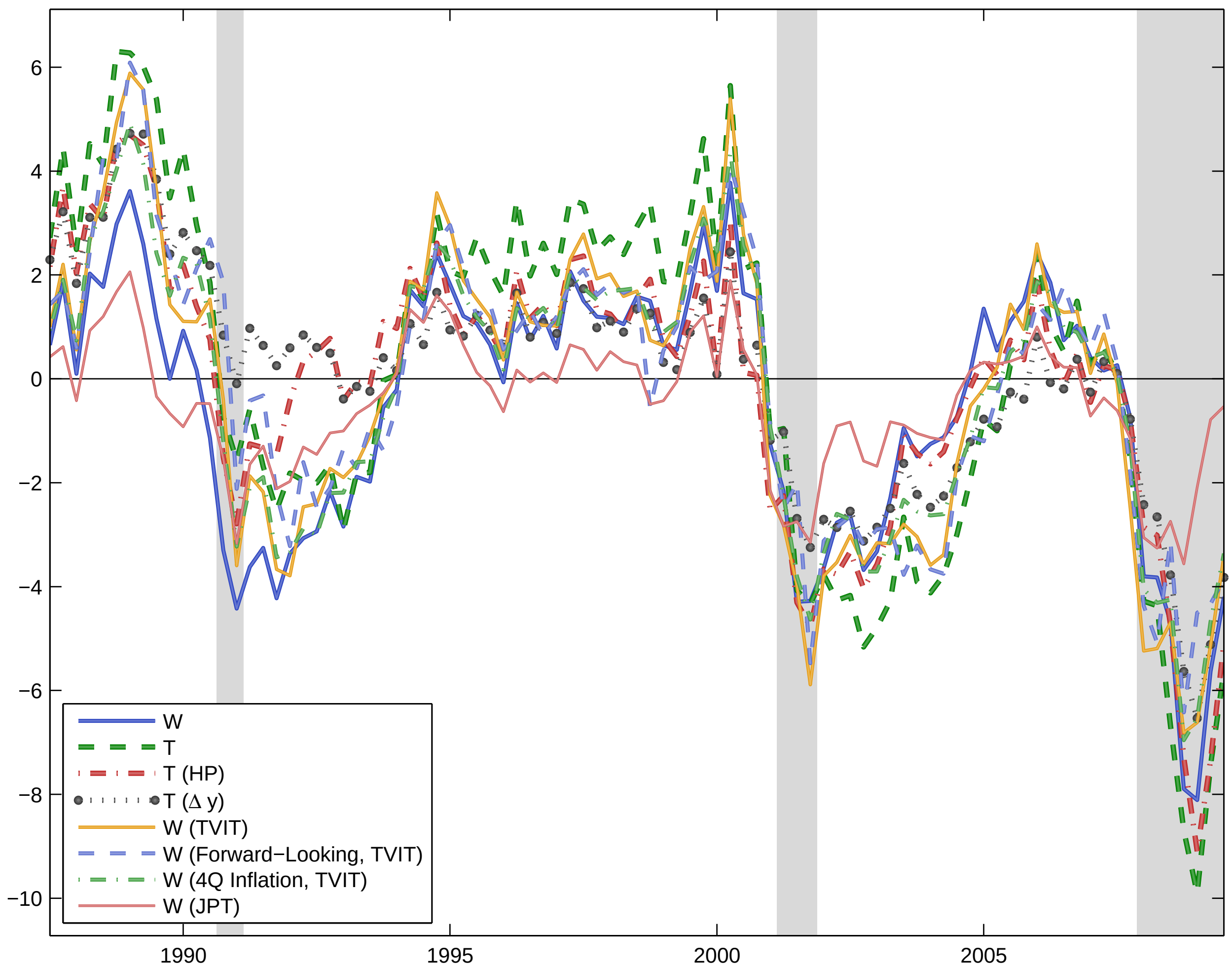

Figure E.1: Smoothed posterior median estimates of r_t^e across a variety of specifications.

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