Optimal Policy Rules in HANK
by
McKay and Wolf

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The views expressed herein are those of the author and not necessarily those of the Bank of Canada
should household inequality affect the conduct of cyclical stabilization policy?
methodology

McKay and Wolf presents a sequence-space Jacobian based technique to

- derive a “welfare-based” quadratic loss function incorporating the planner’s concern for inequality
- derive a solution to the optimal policy problem in the form of a targeting rule
- analyze optimal policy which minimizes some ad-hoc loss functions

main result

concern for inequality only has a moderate effect on optimal interest rate policy
what does “optimal” mean?

- typically *Pareto optimal*
what does “optimal” mean?

- typically Pareto optimal
- allocations which minimize the McKay-Wolf “welfare based” loss function are not Pareto optimal
what does the McKay-Wolf loss function deliver?
insights from a simple risk sharing problem

- **2 agents**: \( i \in \{1, 2\} \)

- **stochastic endowments**: \( y_{i,t} = y_t \zeta_{i,t} \) for \( i \in \{1, 2\} \)
  - **idiosyncratic risk**: \( \zeta \in \{\zeta_l, \zeta_h\} \) where \( \zeta_l = 1 - \Delta, \zeta_h = 1 + \Delta \) for \( \Delta \in (0, 1) \)
  - **idiosyncratic risk perfectly negatively correlated**
    \[
    P [(\zeta_{1,t}, \zeta_{2,t}) = (\zeta_l, \zeta_h)] = P [(\zeta_{1,t}, \zeta_{2,t}) = (\zeta_h, \zeta_l)] = \frac{1}{2}
    \]

- **aggregate risk**: \( \ln y_t \sim N(0, \sigma_y^2) \)
  \[ y_{1,t} + y_{2,t} = y_t \]
Pareto problem

- **Pareto problem:**

  \[
  \max \bar{\varphi}_1 \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_1^t, y_t} \beta^t p(\zeta_1^t, y^t) \ln \left( c_1(\zeta_1^t, y^t) \right) \right\} + \bar{\varphi}_2 \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_2^t, y_t} \beta^t p(\zeta_2^t, y^t) \ln \left( c_2(\zeta_2^t, y^t) \right) \right\}
  \]

  s.t. \quad c_1(\zeta_1^t, y^t) + c_2(\zeta_2^t, y^t) = y_t

- **Pareto weights** \( \bar{\varphi}_i = \varphi_1(\zeta_i^0, \zeta_j^0, y^0) \) can depend on histories up to date 0
Pareto problem

□ Pareto problem:

\[
\max \bar{\varphi}_1 \left\{ \sum_{t=0}^{\infty} \sum_{\zeta^t_1, y^t} \beta^t p(\zeta^t_1, y^t) \ln (c_1(\zeta^t_1, y^t)) \right\} + \bar{\varphi}_2 \left\{ \sum_{t=0}^{\infty} \sum_{\zeta^t_2, y^t} \beta^t p(\zeta^t_2, y^t) \ln (c_2(\zeta^t_2, y^t)) \right\}
\]

s.t. \[c_1(\zeta^t_1, y^t) + c_2(\zeta^t_2, y^t) = y^t\]

□ Pareto weights \( \bar{\varphi}_i = \varphi_1(\zeta^0_i, \zeta^0_j, y^0) \) can depend on histories up to date 0

□ solution:

\[
\frac{\bar{\varphi}_1}{c_1(\zeta^t_1, y^t)} = \frac{\bar{\varphi}_2}{c_2(\zeta^t_2, y^t)} \quad \Rightarrow \quad c_1(\zeta^t_1, y^t) = \frac{\bar{\varphi}_1}{\bar{\varphi}_1 + \bar{\varphi}_2} y^t \quad \text{and} \quad c_2(\zeta^t_2, y^t) = \frac{\bar{\varphi}_2}{\bar{\varphi}_1 + \bar{\varphi}_2} y^t
\]

full insurance: \( c_i(\zeta^t_i, y^t) \) does not depend on realization of \( \zeta^t_i \)
McKay-Wolf problem

☐ **MW problem:**

\[
\max \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_1^t, y^t} \beta^t p(\zeta_1^t, y^t) \varphi_1(\zeta_1^t) \ln (c_1(\zeta_1^t, y^t)) \right\} + \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_2^t, y^t} \beta^t p(\zeta_2^t, y^t) \varphi_2(\zeta_2^t) \ln (c_2(\zeta_2^t, y^t)) \right\}
\]

s.t. \[ c_1(\zeta_1^t, y^t) + c_2(\zeta_2^t, y^t) = y^t \]

☐ **MW weights on flow utilities** at date \( t \) \( \varphi_1(\zeta_1^t), \varphi_2(\zeta_2^t) \) can depend on history of idiosyncratic shocks *up to date* \( t \)

- same as Pareto weights only if \( \varphi_1(\zeta_1^t) \) and \( \varphi_2(\zeta_1^t) \) are constant functions
- else, planner is maximizing *distorted individual preferences*
McKay-Wolf problem

- **MW problem**:
  \[
  \max \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_{1}, y^t} \beta^t p(\zeta_{1}^t, y^t) \varphi_1(\zeta_{1}^t) \ln \left( c_1(\zeta_{1}^t, y^t) \right) \right\} + \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_{2}, y^t} \beta^t p(\zeta_{2}^t, y^t) \varphi_2(\zeta_{2}^t) \ln \left( c_2(\zeta_{2}^t, y^t) \right) \right\}
  \]

  s.t. \[ c_1(\zeta_{1}^t, y^t) + c_2(\zeta_{2}^t, y^t) = y^t \]

- MW weights on flow utilities at date \( t \) \( \varphi_1(\zeta_{1}^t) \), \( \varphi_2(\zeta_{2}^t) \) can depend on history of idiosyncratic shocks up to date \( t \)
  - same as Pareto weights **only if** \( \varphi_1(\zeta_{1}^t) \) and \( \varphi_2(\zeta_{2}^t) \) are constant functions
  - else, planner is maximizing distorted individual preferences

- **solution**:
  \[
  \frac{\varphi_1(\zeta_{1}^t)}{c_1(\zeta_{1}^t, y^t)} = \frac{\varphi_2(\zeta_{2}^t)}{c_2(\zeta_{2}^t, y^t)}
  \]
McKay-Wolf problem

MW calibrate to US data, which generates steady state inequality/under-insurance

\[ \varphi_i(\zeta^t_i) \] to rationalize this level of steady state under-insurance as optimal

\[ \zeta^{1, t}_{1, c}(\zeta^{t}_{1, 1}) = \zeta^{2, t}_{2, c}(\zeta^{t}_{2, 1}) \Rightarrow c_i(\zeta^{t}_i, 1) = \zeta^{t}_{i, 1} \]

also alters planner's desire to provide insurance out of steady state

\[ \zeta^{1, t}_{1, c}(\zeta^{t}_{1, y, t}) = \zeta^{2, t}_{2, c}(\zeta^{t}_{2, y, t}) \Rightarrow c_i(\zeta^{t}_i, 1) = \zeta^{t}_{i, y, t} \]

can always find feasible Pareto improvement relative to MW's optimal allocation
McKay-Wolf problem

MW calibrate to US data, which generates steady state inequality/under-insurance

\[ \varphi_i(\zeta^t_i) \] to rationalize this level of steady state under-insurance as optimal

\[ \varphi_i(\zeta^t_i) = \frac{1}{u'(\zeta_{i,t})} = \zeta_{i,t} \] rationalizes no redistribution as optimal in steady state

\[ \frac{\zeta_{1,t}}{c_1(\zeta^t_1, 1)} = \frac{\zeta_{2,t}}{c_2(\zeta^t_2, 1)} \Rightarrow c_i(\zeta^t_i, 1) = \zeta_{i,t} \]
**McKay-Wolf problem**

MW calibrate to US data, which generates steady state inequality/under-insurance

- choose $\varphi_i(\zeta^t_i)$ to rationalize this level of steady state under-insurance as optimal
  
  - $\varphi_i(\zeta^t_i) = \frac{1}{u'(\zeta_i,t)} = \zeta_i,t$ rationalizes no redistribution as optimal in steady state
    
    $$\frac{\zeta_1,t}{c_1(\zeta^t_1, 1)} = \frac{\zeta_2,t}{c_2(\zeta^t_2, 1)} \implies c_i(\zeta_i, 1) = \zeta_i,t$$

- also alters planner’s desire to provide insurance out of steady state
  
  $$\frac{\zeta_1,t}{c_1(\zeta^t_1, y^t)} = \frac{\zeta_2,t}{c_2(\zeta^t_2, y^t)} \implies c_i(\zeta_i, 1) = \zeta_i,t y^t$$

MW weights reduce planner’s incentive to provide insurance
McKay-Wolf problem

MW calibrate to US data, which generates steady state inequality/under-insurance

□ choose \( \varphi_i(\zeta_i^t) \) to rationalize this level of steady state under-insurance as optimal

\( \varphi_i(\zeta_i^t) = \frac{1}{u'(\zeta_{i,t})} = \zeta_{i,t} \) rationalizes no redistribution as optimal in steady state

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\frac{\zeta_{1,t}}{c_1(\zeta_{1,t}^t, 1)} = \frac{\zeta_{2,t}}{c_2(\zeta_{2,t}^t, 1)} \implies c_i(\zeta_i^t, 1) = \zeta_{i,t}
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□ also alters planner’s desire to provide insurance out of steady state

\[
\frac{\zeta_{1,t}}{c_1(\zeta_{1,t}^t, y_t)} = \frac{\zeta_{2,t}}{c_2(\zeta_{2,t}^t, y_t)} \implies c_i(\zeta_i^t, 1) = \zeta_{i,t} y_t
\]

□ can always find feasible Pareto improvement relative to MW’s optimal allocation
implications for optimal monetary policy
simpler version of MW model

- households $i \in [0, 1]$ with preferences $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_{i,t} - n_t - \frac{\psi}{2} (\ln \Pi_t)^2 \right\}$

- stochastic income $y_{i,t} = \omega_{i,t} y_t$ where $\omega_{i,t} \in \{ \omega_{h,t}, \omega_{l,t} \}$
  
  - $\omega_{l,t} < \omega_{h,t}$, $\Pr(\omega_{j',t} | \omega_{j,t-1}) = \frac{1}{2}$ for any $(j,j')$ and $\frac{\omega_{h,t} + \omega_{l,t}}{2} = 1$

  - $\omega_{i,t}$ can vary with GDP $y_t$: $\omega_{h,t} = \omega_h y_t^\gamma$
simpler version of MW model

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  - $\omega_{i,t}$ can vary with GDP $y_t$: $\omega_{h,t} = \omega_{h} y_t^\gamma$

- 0 liquidity, borrowing limit: $c_{h,t} = y_{h,t}$, $c_{l,t} = y_{l,t}$ and $\frac{1}{2} c_{h,t} + \frac{1}{2} c_{l,t} = y_t$

$$y_{h,t}^{-1} = \beta R_t \mathbb{E}_t \left\{ 0.5y_{h,t+1}^{-1} + 0.5y_{l,t}^{-1} \right\} \quad \text{monetary policy controls } R_t$$
simpler version of MW model

- households $i \in [0, 1]$ with preferences $E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_{i,t} - n_t - \frac{\psi}{2} (\ln \Pi_t)^2 \right\}$

- stochastic income $y_{i,t} = \omega_{i,t} y_t$ where $\omega_{i,t} \in \{\omega_{h,t}, \omega_{l,t}\}$
  - $\omega_{l,t} < \omega_{h,t}$, $Pr(\omega_{j',t} | \omega_{j,t-1}) = \frac{1}{2}$ for any $(j, j')$ and $\frac{\omega_{h,t} + \omega_{l,t}}{2} = 1$
  - $\omega_{i,t}$ can vary with GDP $y_t$: $\omega_{h,t} = \omega_{h} \gamma_t$

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  $$ y_{h,t}^{-1} = \beta R_t E_t \left\{ 0.5 y_{h,t+1}^{-1} + 0.5 y_{l,t}^{-1} \right\} \quad \text{monetary policy controls } R_t $$

- Phillips curve

  $$ \ln \Pi_t = \beta \ln \Pi_{t+1} + \kappa (\log y_t - \log z_t) + \epsilon_t $$
do MW weights lead to meaningfully different answer?

□ RANK \((\omega_{h,t} = \omega_{l,t} = 1)\)

\[
\left(\hat{y}_t - \hat{z}_t\right) + \lambda \hat{p}_t = 0
\]

output-gap \hspace{1cm} price-stability

□ TANK (any Pareto weights)

\[
\left(\hat{y}_t - \hat{z}_t\right) + \lambda \hat{p}_t + \delta \times \hat{y}_t = 0
\]

□ TANK (MW weights)

\[
\left(\hat{y}_t - \hat{z}_t\right) + \lambda \hat{p}_t + \delta^{\star} \times \hat{y}_t = 0
\]

□ MW solution

○ puts less weight on output stabilization than any Pareto problem

\[0 < \delta^{\star} < \delta\]

○ relative magnitude proportional to steady state inequality

\[
\delta^{\star} \propto \frac{\omega_h}{\omega_l}
\]

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do MW weights lead to meaningfully different answer?

- RANK \((\omega_{h,t} = \omega_{l,t} = 1)\)
  \[
  (\hat{y}_t - \hat{z}_t) + \lambda \hat{p}_t = 0
  \]
  output-gap + price-stability

- TANK (any Pareto weights)
  \[
  (\hat{y}_t - \hat{z}_t) + \lambda \hat{p}_t + \delta \times \hat{y}_t = 0
  \]
  distributional concerns

\(\delta \propto \omega_h \omega_l \)
do MW weights lead to meaningfully different answer?

□ RANK \((\omega_{h,t} = \omega_{l,t} = 1)\)

\[
\begin{aligned}
\left( \hat{y}_t - \hat{z}_t \right) + \lambda \hat{p}_t &= 0 \\
&\quad \text{output-gap} \quad \text{price-stability}
\end{aligned}
\]

□ TANK (any Pareto weights)

\[
\left( \hat{y}_t - \hat{z}_t \right) + \lambda \hat{p}_t + \delta \times \hat{y}_t = 0
\]

□ TANK (MW weights)

\[
(\hat{y}_t - \hat{z}_t) + \lambda \hat{p}_t + \delta^* \times \hat{y}_t = 0
\]

□ MW solution

- puts less weight on output stabilization than any Pareto problem \(0 < \delta^* < \delta\)
- relative magnitude proportional to steady state inequality

\[
\frac{\delta}{\delta^*} \propto \frac{\omega_h}{\omega_l}
\]
how did we get here?

- first-order accurate optimal policy from naive LQ requires efficient steady state
- If constrained-efficient steady state, then need to solve QQ (Benigno-Woodford)
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- HANK: steady state consumption risk is inefficient for any Pareto planner
  - cannot use naive LQ unless fiscal policy provides full insurance in steady state
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☐ MW weights rationalize steady state with idiosyncratic risk as “optimal”
  ○ naive LQ gives “accurate solution” but not to the “correct” problem
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- MW weights rationalize steady state with idiosyncratic risk as “optimal”
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- ideally, extend methodology to solve QQ problem
  - not trivial since constrained efficient steady state may not exist (Bhandari et al.)
final thoughts

- solution to policy problem using MW weights does not satisfy Pareto optimality
  - can trivially always find alternative allocation which makes all agents better off

- using MW weights ⇒ optimal monetary policy biased to be closer to RANK
  - assumptions reduce the planner’s motives to provide insurance/ reduce inequality