Discussion of "Optimal Policy Rules in HANK" by Alisdair McKay and Christian Wolf

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Some history of the New Keynesian literature

- **1995-2008: New Keynesian model** developed into the leading paradigm for studying monetary and fiscal policy
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  - **positive** literature has converged on three key findings:
    1. redistribution and high MPCs matter [Auclert, Kaplan-Moll-Violante]
    2. agg. effects of MP often well captured by RA model [Werning]
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  - **normative** literature has typically found that distributional considerations are an important/dominant concern for optimal policy

[Bhandari et al, Davila-Schaab, Acharya et al, LeGrand et al, Nuno-Thomas, Smirnov..]

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- I do like their modeling assumptions better
- But I will argue that 2. is not natural and may make a big difference

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     - eg, sticky prices but flexible wages; now understood to make MP unrealistically redistribute between workers and capitalists
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   - The model under study is close to that in the Intertemporal Keynesian Cross (IKC). I think it's the right model for this purpose. I have made that case in the IKC paper and others.

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2. Optimal Ramsey policy in HANK is a hard problem
   - Different authors try to cut through this in different ways
   - This paper choses to **make the steady state exactly first-best**
   - This is very tractable, but also throwing out baby with the bathwater

Rest of discussion

Go over main results:

1. Ad-hoc objective
2. "Ramsey" problem

with comments on framing and methodology

Review of optimal commitment policy in NK model

- Textbook analysis (eg, Gali) derives objective of policymaker

$$\mathcal{L} = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(c_t^2 + \lambda\pi_t^2\right) \qquad \text{(NK-loss)}$$

for a particular λ, and Phillips curve with cost-push shock u_t,

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- The solution is well-known, involving the "target criterion"

$$\lambda \pi_t = -\frac{c_t - c_{t-1}}{\kappa} \tag{1}$$

and the path for the price level

$$p_t - \delta p_{t-1} = \delta \sum_{k=0}^{\infty} (\beta \delta)^k u_{t+k} \tag{2}$$

where $\delta \in (0,1)$ is the root of $X^2 - \left(\beta + 1 + \kappa^2 \lambda\right) X + \beta$.

From Gali: transitory shock

- For instance with transitory shock, $\mathbf{u} = (1, 0, \ldots)$

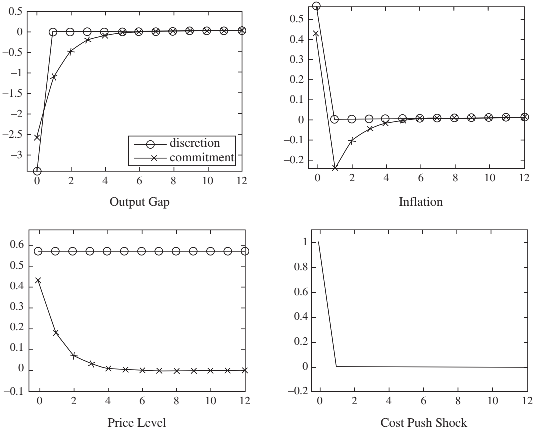

Figure 5.1 Optimal Responses to a Transitory Cost Push Shock

What about HANK?

- The solution makes no reference the way in which monetary policy implements the optimal $\{p_t, \pi_t, y_t\}$—the "IS curve" in RANK
- Suppose that we **assume** that the loss function in HANK is still (NK-loss) for *some* λ, and that the Phillips curve is still (NK-PC).
- Then the solution is still given by (1) and (2), provided that you can implement it with your instruments. This is proposition 1. Nice!

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- But:
    1. Not clear that (NK-loss) is the right loss function.
    2. Not clear that (NK-PC) is the right model of the PC in HANK
    3. Not obvious that implementation is just "mild regularity condition"

Next: address these three points, in reverse order.

On implementation

- Equilibrium in sequence space solves (in authors' notation)

$$\tilde{\mathbf{y}} \equiv \mathbf{y} - \mathbf{y}_0 - \mathcal{C}_y \mathbf{y} - \mathcal{C}_\pi \boldsymbol{\pi} = \mathcal{C}_i \mathbf{i}$$

  The implementability question is: can I find a path for $\mathbf{i}$ that implements my desired $\{\mathbf{y}_0, \mathbf{y}, \boldsymbol{\pi}\}$?

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  - Answer: check that the winding number of $\mathcal{C}_i$ is ≤ 0
  - See Proposition 2 in Auclert-Rognlie-Straub (2023)

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- The $\mathcal{C}$'s and the Θ's in the paper are referred to as "linear maps". This is strange. These maps are called sequence-space Jacobians.

Sufficient statistics for general equilibrium?

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SEQUENCE-SPACE JACOBIAN 2391

Jacobians as Sufficient Statistics for the Heterogeneous-Agent Problem. Since the Jacobians $\mathcal{J}$ locally describe the mapping $\mathbf{Y} = h(\mathbf{X})$, they are all that is needed to capture the local behavior of the heterogeneous-agent problem. This observation implies that all of the complexity introduced by heterogeneity in any given model boils down entirely to the Jacobian of the resulting heterogeneous-agent problem. This facilitates the analysis of the importance of heterogeneity for general equilibrium, and the connection of models to the data. For example, in simple general equilibrium models, the Jacobian of aggregate consumption with respect to income $\mathcal{J}^{C,y}$ is all that is needed for general equilibrium (Auclert, Rognlie, and Straub (2018)).

- I think that being clear about contribution would help the paper

On how HANK changes the Phillips curve

1. In the IKC model, heterogeneity does enter the Phillips curve through heterogeneous wealth effects, as $c_t \neq \left(\int c_{it}^{-\gamma} di\right)^{-\frac{1}{\gamma}} \equiv c_t^*$
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2. Here the Phillips curve is from sticky firm prices, not wages. Then, firm should be to discounting cash flows using $\frac{1}{1+r_t}$, not β.
   - Linearizing, Phillips curve will be $\pi_t = \kappa c_t + \frac{1}{1+r}\pi_{t+1} + u_t$
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3. Also, time-varying distortionary taxes show up in the Phillips curve
   - Here distortionary taxes constant, plus "lump-sum taxes that depend on who you are". Robustness to making those distortionary?

What about the general case?

- With more general PC, can still formulate as a QP problem: write $\mathbf{W} = \mathrm{diag}\left(1, \beta, \beta^2, \cdots\right)$ then (NK-loss) is

$$\mathcal{L} = \mathbf{y}'\mathbf{W}\mathbf{y} + \lambda \boldsymbol{\pi}'\mathbf{W}\boldsymbol{\pi}$$

  subject to the constraints from the linear sequence space solution

$$\mathbf{y} = \mathbf{y}_0 - \mathbf{B} \cdot \mathbf{r}; \qquad \boldsymbol{\pi} = \boldsymbol{\pi}_0 - \mathbf{D} \cdot \mathbf{r}$$

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  - "the instrument $\mathbf{r}$ is set to offset as well as possible—in a weighted least-squares sense—the perturbation to the policy targets $\boldsymbol{\pi}, \mathbf{y}$ caused by the exogenous shocks" (ie my $\mathbf{y}_0$, $\boldsymbol{\pi}_0$)

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- **Great takeaway**: can use the sequence-space Jacobian toolkit to solve ad-hoc optimal policy!

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- This is the purview of normative public finance
- So, I find the second part of the paper much more interesting

The Ramsey problem

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  - Formulate a welfare objective (eg, utilitarian)
  - Look for the steady state of that Ramsey plan (the RSS)
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  - "Planner hates the poor" vs "has exhausted its instruments to help"
- The paper is not clear about this. It talks about "our formulation of the problem", and says the difference to lit. is "methodological"
- Reads like it's the same problem as others, but a different solution.
- It's actually a different problem: a different objective function!

How different is the policy objective vs usual utilitarian?

- Using ζ for histories, McKay-Wolf planner (with log) maximizes

$$\int \varphi(\zeta) \log(\omega_t(\zeta) c_t) \, d\Gamma(\zeta)$$

where ω_t are consumption shares and $\varphi(\zeta)$ weights. To 2nd order:

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- Utilitarian planner places much higher weight on poor, $\frac{1}{\overline{\omega}(\zeta)^2} \gg \frac{1}{\overline{\omega}(\zeta)}$.

Concluding thoughts

- Nice paper, thought provoking conclusion
- As literature gears up to solve the usual Ramsey problem, we'll see if the main irrelevance-of-distribution result holds up
- It will for environments close to the Werning neutrality result
- But it's an open question for other, realistic ones where it fails
  - eg: add investment to this model! [Auclert-Rognlie-Straub 2020]
- Difference bw utilitarian vs "hate-the-poor" planner will likely matter