Labor Market Shocks and Monetary Policy*

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Abstract

We develop a heterogeneous-agent New Keynesian model featuring a frictional labor market with on-the-job search to quantitatively study the role of job mobility dynamics on inflation and monetary policy. Motivated by our empirical finding that the historical negative correlation between the unemployment rate and the employer-to-employer (EE) transition rate up to the Great Recession disappeared during the recovery, we use the model to quantify the effect of EE transitions on inflation in this period. We find that inflation would have been around 0.25 percentage points higher between 2016 and 2019 if the EE rate increased commensurately with the decline in unemployment. We then decompose the channels through which fluctuations in EE transitions affect inflation. We show that an increase in the EE rate leads to an increase in the real marginal cost, but this direct effect is partially mitigated by the equilibrium decline in market tightness that exerts downward pressure on the marginal cost. Finally, we show that responding to fluctuations in EE rate explicitly when conducting monetary policy substantially reduces the welfare loss due to the fluctuations in unemployment and output and yields heterogeneous welfare gains across subpopulations.

Keywords: Job mobility, monetary policy, HANK, job search
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1 Introduction

The Federal Reserve is tasked with a dual mandate of price stability and maximum employment. Its main tool for achieving these goals is the federal funds rate, which it sets based on a measure of inflation and a measure of slack in the labor market. Slack measures tend to focus on the *quantity* of employment (e.g. the unemployment rate) and ignore the *quality* dimension. There is growing interest in understanding how inflation is affected by employer-to-employer (EE) transitions, which affect wages by facilitating competition and the quality of jobs by facilitating labor reallocation toward productive jobs.[1]

In this paper, we analyze the positive and normative implications of EE rate fluctuations for inflation and monetary policy and answer two questions: First, motivated by the observation that the EE rate remained low relative to conventional slack measures after the Great Recession, we ask how much and through which mechanisms can the depressed EE rate account for the missing inflation during this episode. Second, we ask if and how much explicitly targeting the EE rate in the response function could help a dual mandate central bank achieve its goals.

To answer these questions, we make three contributions. First, we develop a model that combines the new insights from the Heterogeneous Agent New Keynesian (HANK) literature with a search model of the labor market with on-the-job search (OJS) and employer competition. Second, we use this model to quantify the macroeconomic implications of EE fluctuations over the business cycle. To do so, we focus on the "missing inflation" episode in the U.S., the recovery episode from the Great Recession between 2016 and 2019 during when inflation remained low even though the unemployment rate steadily declined around 25 percent below its trend. We document that, as opposed to the historical negative correlation between the unemployment and EE rates, the EE rate remained flat at this trend during this recovery period, and consequently the correlation between the two weakened dramatically. We simulate this recovery episode using our model with and without the weakening of the correlation between the unemployment and EE rates and show that inflation would have been around 0.25 percentage points higher between 2016 and 2019 if the EE rate increased with the decline in unemployment following the historical negative correlation between the two. A key feature of this exercise is that we also provide a full-decomposition of the channels through which fluctuations in EE transitions affect inflation. Third, on the normative side, we study optimal monetary policy within a class of Taylor rules. We consider an augmented Taylor rule that responds to both deviations of unemployment and EE rates from their steady state values as well as the inflation gap. We find that when we jointly optimize over the coefficients on unemployment and EE gaps, the optimal

[1] Seminal work by Moscarini and Postel-Vinay (2019) argues that the central bank should keep an eye on the EE flows as it shapes the tradeoff between wages and labor productivity. Moscarini and Postel-Vinay (2017) and Karahan, Michaels, Pugsley, Şahin, and Schuh (2017) show that EE flows are a better predictor of wage growth than the unemployment rate.

coefficient on the EE gap is positive and large in magnitude, around 70 percent of the optimal coefficient on the unemployment gap. This policy reduces the welfare loss due to the deviations of unemployment and output from their targets substantially and yields heterogeneous welfare gains across subpopulations.

Our starting point is to build a HANK model combined with a labor search model featuring OJS. The economy is hit by aggregate shocks to demand, labor productivity, and the efficiency of OJS. Individuals work, retire and die stochastically, and face idiosyncratic unemployment risk due to job destruction while working or due to search frictions that prevent them to find a job. Unemployed workers also face a risk of human capital depreciation. As in most HANK models, we allow individuals to save by investing in shares of a mutual fund for self insurance against these labor market shocks and life cycle reasons. Micro founding idiosyncratic income fluctuations using an OJS model allows us to endogenize income risk and how it correlates with aggregate shocks and marginal propensity to consume (MPC), which Acharya and Dogra (2020) show is a critical component of HANK models. Individuals work in firms that produce labor services. Their productivity depends on their human capital and the match-specific productivity of their job. Their wage is an endogenous piece-rate of their output, which is determined through Bertrand competition based on their flow output. Both unemployment and employed workers look for work in the frictional labor market, where job-finding and job-to-job transition probabilities are determined by two factors: the endogenous labor market tightness and exogenous job search efficiencies of the unemployed and the employed, the latter of which is called as the efficiency of OJS and is subject to aggregate shocks. Workers accumulate human capital stochastically while employed and also engage in OJS, both of which allows them to obtain higher wages. In particular, contacting outside employers may potentially result in higher wages for the employed even when they stay with their firm because such contacts may lead to rebargaining. The rest of the model follows the New Keynesian tradition. Monopolistically competitive intermediate firms buy labor services from the service firms to produce their differentiated goods, which are then sold to final-good producers. The government issues nominal bonds and implements progressive labor income taxation together with consumption tax to finance an exogenous stream of unproductive government expenditures, social security for retirees, and an unemployment insurance program.

Estimating shocks and solving for optimal monetary policy requires solving and simulating the model under aggregate shocks efficiently. We overcome this challenge by implementing the sequence-space Jacobian method of Auclert, Bardóczy, Rognlie, and Straub (2021). To this end, we cast the model as a Directed Acyclic Graph (DAG) that represents equilibrium conditions as separate blocks that are interconnected via model variables. This step requires us to extend their method to incorporate discretized endogenous worker distributions into the DAG, as these distributions directly enter to agents problems in our framework.

We calibrate the steady state of the model to match several aspects of the U.S. economy over the period 2004–2006 before the onset of the Great Recession. In light of recent work highlighting the importance of the wealth distribution for the transmission and effectiveness of monetary policy, we calibrate our model to match the fraction of hand-to-mouth individuals. To discipline the relative importance of human capital formation and the job ladder for the dynamics of idiosyncratic labor income, we target the wage growth of job switchers and the earnings loss associated with job loss. We match the average unemployment rate, separation rate into unemployment, and the EE rate over this period. The New Keynesian block of the economy is calibrated to match the average level of markups, the slope of the Philips curve, and the responsiveness of the nominal rate to inflation and unemployment gaps. We then jointly estimate processes for aggregate shocks to demand, labor productivity, and the efficiency of OJS by targeting empirical moments regarding the correlations of unemployment rate, EE rate, and inflation with output as well as their standard deviations. To understand the contribution of each shock to the cyclical movements of our target outcomes, we also provide a variance decomposition of these moments. Importantly, we show that shocks to OJS efficiency account for more than 40 percent of fluctuations in inflation.

Next, we use the calibrated model to study how inflation responds to fluctuations in worker mobility between employers, which we capture by shocks to the efficiency of OJS. We do so through a case study of the recovery episode following the Great Recession, more specifically over the period 2016–2019. During this time, although the unemployment rate steadily declined around 25 percent below its trend, inflation remained low. As such, researchers and policy makers were puzzled by the "missing inflation", given that historically low unemployment rate did not trigger a commensurate rise in inflation. Our main exercise offers an explanation to this phenomenon from the labor market perspective. We first document that the EE rate, which tends to comove negatively with the unemployment rate, remained flat around its long-run trend, well below the level implied by its historical relationship with the unemployment rate. Demand shocks alone cannot explain this episode as they predict a counterfactually rising EE rate because higher demand stimulates vacancy creation, which leads to an endogenous increase in EE rate in the absence of OJS efficiency shocks. We first back out the sequence of (positive) demand and (negative) OJS shocks that replicate the path of unemployment and EE rates over this period. We then compare the outcomes of two transitions starting from the same steady state: a counterfactual economy with just positive demand shocks that has the same path for the unemployment rate and an endogenously increasing EE rate and a post-Great Recession economy with positive demand and negative OJS shocks that replicate the path of unemployment and EE rates between 2016-2019. We find that the OJS shocks that our estimation infers lowered annual inflation by 0.23 percentage points at their peak.

Given the quantitative significance of OJS shocks for inflation dynamics, we then set out to decompose the various channels through which a positive OJS shock results in higher inflation.

Our decomposition leverages the DAG representation of the model and essentially applies the implicit function theorem to equilibrium conditions of choice in a particular order to express an outcome variable as a linear function of the shocks and other endogenous variables. The linear relationships allow for a simple decomposition of the different channels. An increase in OJS efficiency leads to a decline in match values of labor services firm because the rise in worker's probability to contact other firms leads to either rebargaining with the incumbent firm to extract a greater share of match surplus or a shorter match duration if the worker is poached. All else the same, the real price of labor services — which is the real marginal cost of production for intermediate firms, i.e., the main determinant of inflation — needs to increase for the free-entry condition to hold. This direct effect explains 139 percent of the total increase in the real marginal cost. Importantly, we show that this direct effect is partially mitigated by secondary effects through general equilibrium (GE) responses. In particular, higher job mobility leads to a decline in market tightness in equilibrium for two reasons. First, a higher OJS efficiency results in a higher labor productivity due to a better match distribution. For demand and supply of labor services to be equal in equilibrium, this effect requires a decline in tightness. Second, a higher OJS efficiency leads to a lower aggregate demand driven by lower job finding rate and higher unemployment rate, necessitating a lower labor market tightness in equilibrium. Overall, when the labor market is more slack, firms find it easier to fill vacancies and workers find it more difficult to contact other firms, both of which imply an increase in expected match values of labor services firm. Therefore, the price of labor services declines to preserve the free-entry condition. These two channels that reduces tightness explain -42 percent of the increase in inflation. Thus, counteracting labor market effects explain 97 percent of the total increase in inflation. The remaining 3 percent is accounted by the changes in the real rate due to the GE responses of inflation and unemployment. The rise in inflation leads the monetary authority to raise the nominal (and consequently the real) interest rate, which dominates the downward pressure from higher unemployment rate on interest rate. Higher interest rate leads to lower expected match values of labor services firm, necessitating an increase in the price of labor services.

Finally, we study the normative implications of job mobility for monetary policy. To do so, we consider an augmented Taylor rule that responds to both deviations of unemployment and EE rates from their steady state values as well as the inflation gap. We find that when we jointly optimize over the coefficients on unemployment and EE gaps, the optimal coefficient on the EE gap is positive and large in magnitude, around 70 percent of the optimal coefficient on the unemployment gap. This policy reduces the welfare loss due to the deviations of unemployment and output from their targets substantially and yields heterogeneous welfare gains across subpopulations.

Related literature. This paper contributes to several strands of the literature. On the modeling side, this paper contributions to the literature that bring together elements from search models together with those from New Keynesian models. Ravn and Sterk (2016) develop a tractable New Keynesian model with uninsurable risk and characterize the interactions between unemployment risk, aggregate demand and monetary policy. Gornemann, Kuester, and Nakajima (2021) develop a fully stochastic New Keynesian model with uninsurable idiosyncratic risk and search frictions. We add to this literature by allowing for OJS and heterogeneity across jobs, which allows us to endogenize income risk and how it varies with aggregate fluctuations and MPC, which are shown to be key elements of HANK models (Acharya and Dogra, 2020 and Patterson, 2022). We then use this model to uncover positive and normative implications of worker mobility on aggregate dynamics with a particular focus on inflation and the conduct of monetary policy. To accomplish these goals, on the computational side, we build on the sequence-space Jacobian method of Auclert, Bardóczy, Rognlie, and Straub (2021). One challenge in adapting this method to our setting is that the endogenous distribution of workers across jobs and human capital levels directly enters into equilibrium conditions. This is in contrast to settings where only scalars (such as aggregate capital and labor) enter equilibrium conditions. We show how their method can be generalized to a multi-stage model with search frictions, where one needs to keep track of worker distributions to ensure market clearing.

A growing literature studies inflation dynamics after the Great Recession. Earlier studies focused on why there was no disinflation in the earlier years following the Great Recession (2009-2011) (Coibion and Gorodnichenko, 2015; Ball and Mazumder, 2011) despite high unemployment rates. The ensuing recovery phase and the “missing inflation” that would have been implied by low unemployment rates in the years after the Great Recession also motivated several other important studies. Hazell, Herreno, Nakamura, and Steinsson (2020) argue that well-anchored inflation expectations weaken the link between measures of labor market tightness and inflation and reduce the volatility of inflation. An alternative view is that structural shifts in the economy have caused the Phillips curve to flatten over time. Del Negro, Lenza, Primiceri, and Tambalotti (2020) find that the disconnect between the labor market and inflation is due primarily to the muted reaction of inflation to cost pressures and rule out stories centered around changes in the structure of the labor market or in how one should measure its tightness. Hooper, Mishkin, and Sufi (2020) estimate the slopes of the price and wage Phillips curves over time and reach a similar conclusion. These findings are consistent with those in Heise, Karahan, and Şahin (2020), who use disaggregated data to find a declining pass-through of wage pressures to inflation. Carvalho, Eusepi, Moench, and Preston (2017) estimate a decline in the natural rate of unemployment, and articulate this as a reason for why historically low unemployment rates do not have to translate to wage pressures. We view our work as complementary to these papers in that we focus on a specific labor market friction, the dynamics of the EE rate, quantify its independent effect on inflation, decompose channel through which it affects inflation, and study its normative implications without taking a stance on the slope of the price Phillips curve or inflation expectations.

Our work is most closely related to Moscarini and Postel-Vinay (2019), Faccini and Melosi (2021), and Alves (2019), who focus on the role of the job ladder in inflation dynamics. Seminal work by Moscarini and Postel-Vinay (2019) is the first in this literature to establish the distribution of workers across matches as an important determinant of wage pressures on inflation. Faccini and Melosi (2021) build on their work and highlight the role of variations on worker mobility in explaining the missing inflation after the Great Recession. Relative to these papers, our model features imperfect insurance against labor market risk, and therefore changes in the job mobility are an important determinant of aggregate demand. We show that a complete-markets model would attribute a smaller role to changes in the job ladder in explaining the missing inflation after the Great Recession. On the normative side, we also show that accounting for heterogeneity in wealth holdings is important for correctly quantifying the welfare implications of a monetary policy that explicitly targets the EE rate. Finally, Alves (2019) embeds the key insights in Moscarini and Postel-Vinay (2019) in a HANK model and obtains sizable demand side effects from changes in the job mobility. Our work differs from his in three important ways. First, our model features richer labor-market heterogeneity by allowing for differences in human capital as well as match productivity. Second, we not only quantify the total effect of job mobility on inflation but also decompose channels through which a change in job mobility affects inflation, using the DAG representation of the model and relying on the sequence-space Jacobian method. Third, we study the normative implications of job mobility and show that responding to fluctuations in EE rate explicitly when conducting monetary policy substantially reduces the welfare loss due to the fluctuations in unemployment and output and yields heterogeneous gains across subpopulations.

Outline. Section 2 presents our model combining the HANK framework with a search model of the labor market. Section 3 discusses the calibration of model's parameters and estimation of aggregate shocks, and Section 4 explains how we solve and simulate the model. Section 5 quantifies the role of job mobility in inflation and Section 6 studies the normative implications of job mobility for monetary policy. Section 7 concludes.

2 Model

We now describe our model combining a New Keynesian framework with heterogeneous agents and a frictional labor market, where both employed and unemployed workers search for jobs.

2.1 Environment

Time is discrete and runs forever. The economy is populated by a measure one of ex-ante identical individuals, firms in three vertically integrated sectors (producing labor services, intermediate goods, and final goods), a mutual fund, a fiscal and a monetary authority.

Firms. Labor firms hire workers in a frictional labor market (to be described below) and produce labor services. These are sold in a competitive market to intermediate firms, who produce differentiated varieties of intermediate inputs using a linear technology with aggregate productivity z. As in the standard New Keynesian model, intermediate goods firms are monopolistically competitive and set prices subject to quadratic adjustment costs and a downward-sloping demand from final goods producers. Final goods firms produce the consumption good by combining the intermediate inputs using a constant elasticity of substitution (CES) technology.

Individuals. An individual's life consists of a working stage and a retirement stage. During their working lives, individuals are heterogeneous in their holdings of mutual fund shares $s \geq 0$, their employment status e (employed E or unemployed U), their general human capital (skill) $h \in \mathcal{H} = \{\underline{h}, \ldots, \overline{h}\}$, and—among the employed—in their match-specific productivity $x \in \mathcal{X} \equiv \{\underline{x}, \ldots, \overline{x}\}$ and their piece-rate $\alpha \in (0, 1]$ governing the share of output that they receive as wages. Individuals are born with skill h drawn from distribution Γ^h. During their working lives, they experience stochastic appreciation or depreciation of skills depending on their employment status, as in Ljungqvist and Sargent (1998). In particular, an employed individual's skill increases by Δh percent with probability π^E, while an unemployed individual's skill depreciates by Δh percent with probability π^U in each period. Formally,

$$h' = \begin{cases} h \times (1 + \Delta h) & \text{with probability } \pi^E \\ h & \text{with probability } 1 - \pi^E \end{cases}$$

when employed and,

$$h' = \begin{cases} h \times (1 - \Delta h) & \text{with probability } \pi^U \\ h & \text{with probability } 1 - \pi^U \end{cases}$$

when unemployed. Individuals trade shares of the mutual fund and make consumption decisions (bought at price P_t) in the face of idiosyncratic income risk due to stochastic human capital evolution and frictions in the labor market. Each period, working-age individuals retire with probability ψ^R. Retirees ($e = R$) finance consumption through their private savings and from pension income ϕ^R. They die with probability ψ^D, upon which they are replaced with unemployed individuals.[2]

Labor market. The labor market in the service sector is frictional and features random search. Unemployed and employed individuals search for jobs, and their probability of contacting a vacancy depends on their job search efficiency as well as the labor market tightness, θ_t. Upon

[2]When an individual dies, she is replaced by an offspring who inherits her mutual fund holdings and enters working stage as unemployed with the lowest skill level $\underline{h}$.

meeting, the worker-firm pair draws a match-specific productivity x from distribution Γ^x, which remains constant throughout the match. The match operates a production technology given by $F(h,x) = hx$. The individual supplies labor inelastically and is paid real wages according to a predetermined rule $w(h,x,\alpha)$ every period until the termination of the match (described below). The match can dissolve because of an exogenous job separation which occurs at rate δ, retirement, or endogenous job-to-job transitions by the worker. Unemployed individuals receive unemployment insurance (UI) benefits from the government according to the function $UI(h) = \phi^U F(h, \underline{x})$ (denoted in consumption units), where we assume that UI payments are designed as a replacement rate ϕ^U of output that the worker would have received when working at a job with the lowest match productivity $\underline{x}$. On the other side of this labor market, service sector firms pay a per-period fixed cost κ to post vacancies and sell their output to intermediate firms at nominal price P_t^l ($p_t^l = P_t^l/P_t$ in units of the final good).[3]

Wage determination. In each period, the wage paid to an employed worker is an endogenous piece-rate α of the flow output from the worker-firm match. We follow a static bargaining protocol —a simplified version of the dynamic bargaining protocol in Postel-Vinay and Robin (2002)—for the determination of α, where firms Bertrand compete based on current flow output (instead of present values). Figure 1 summarizes this bargaining protocol.

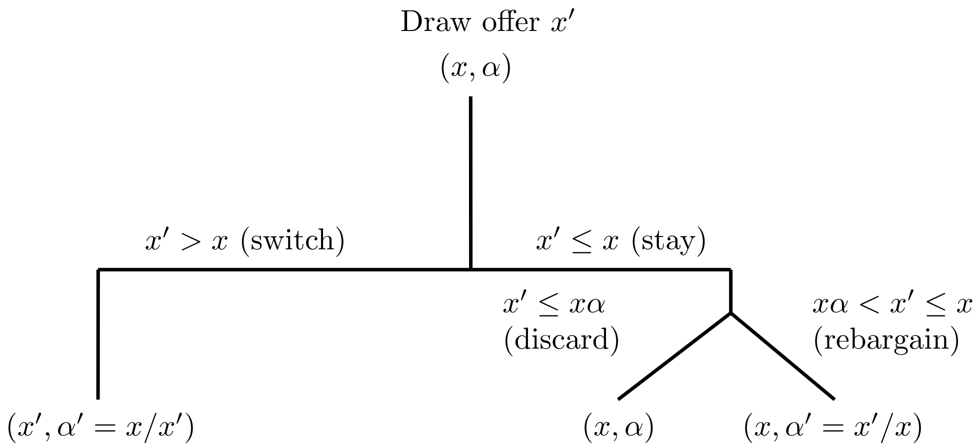
Consider a worker with human capital h employed in a match with productivity x and piece rate α, whose wage is given by $w(h,x,\alpha) = \alpha\phi^E F(h,x)$, where $\phi^E \in (0,1)$ represents the maximum share of output that a worker with maximum piece rate $\alpha = 1$ can capture as wage.[4] Suppose this worker meets a new firm with a higher productivity $x' > x$, in which case she switches jobs. This is because the most the incumbent firm can offer to the worker is $w(h,x,1) = \phi^E F(h,x)$. We assume that the new firm is willing to match this wage, i.e., $w(h,x',\alpha') = w(h,x,1)$, which implies a new piece rate $\alpha' = x/x'$ for this worker. While this new piece rate is $\alpha' < 1$, the worker is better off in switching to the more productive firm with x' given that the piece rate can only become (weakly) larger in the future once a new contact is made with and outside firm with sufficiently high productivity, as discussed below.

Now suppose the same worker receives an offer with a lower productivity $x' < x$, resulting in the worker staying with the incumbent firm. This case induces two scenarios. First, the new productivity x' could be so low that even the maximum potential wage from the new job cannot match the worker's current wage, i.e., $w(h,x',1) < w(h,x,\alpha)$, which happens when $x' < \alpha x$. In this case the worker simply discards the offer and continues with the same piece rate. Second, x' could be sufficiently high to serve as a credible threat for the worker to bid up her wage with the incumbent firm. This happens when $w(h,x,1) > w(h,x',1) > w(h,x,\alpha)$, i.e., $x > x' > \alpha x$,

[3] Unless otherwise stated, we use uppercase letters to denote nominal variables and lowercase letters for their real counterparts.

[4] This assumption guarantees that whenever $\phi^E < p^l$, the firm's flow profit is greater than zero. As a result, there are no firms with negative surplus.

Figure 1: Wage Determination



in which case the incumbent firm matches the maximum potential wage from the outside offer, $w(h, x, \alpha') = w(h, x', 1)$, implying an updated piece-rate $\alpha' = x'/x$. We note that such unrealized job switches that lead to rebargaining of the piece rate α are inflationary since they lead to an increase in the worker's wage without any change in her productivity.[5]

The piece rate for a worker out of unemployment follows the same logic. We assume that for a new match with productivity x', the piece rate is given by $\alpha' = \underline{x}/x'$. We also assume that all offers out of unemployment are accepted.[6]

Mutual fund. The mutual fund owns all the firms in the economy, as well as all nominal bonds B_t issued by the government, and sells shares in return. The fund pays a nominal dividend D_t per share and can be traded by individuals at price P_t^s.

Fiscal and monetary authorities. The government implements a linear consumption tax τ_c and a progressive income tax. For any gross income level ω, net income is given by $\tau_t \omega^{1-\Upsilon}$, where τ_t captures a potentially time-varying level of taxation and $\Upsilon \geq 0$ captures the rate of progressivity built into the tax system, as in Benabou (2002) and Heathcote, Storesletten, and Violante (2014).[7] Together with these taxes, the government issues nominal bonds B_t to finance UI benefits, retirement pensions, and an exogenous stream of nominal expenditures G_t. The central bank sets the short-term nominal interest rate i_t using a reaction function responding to the inflation rate and the unemployment rate.

[5]On the other hand, realized job switches are only inflationary when the increase in the piece rate is larger than the increase in productivity. This happens when $x < x' < x/\alpha$, implying a new piece rate of $\alpha' = x/x' > \alpha$ upon a job switch. That is, job-to-job transitions can be inflationary if the current match productivity is relatively high or the current piece-rate is relatively small.

[6]In equilibrium under our baseline calibration, we verify that all new matches out of unemployment indeed have positive surplus, even though there is an opportunity cost of accepting an offer as we ultimately estimate on-the-job search to be less efficient than searching while unemployed. This is because dynamic gains of being employed dominate the option value of waiting for another match with higher productivity.

[7]Note that τ is inversely related to the tax rate. Under a linear schedule with $\Upsilon = 0$, the tax rate is $1 - \tau$.

Timing of events. At the start of each period, (unanticipated) aggregate shocks realize, which we elaborate in subsequent sections. Then, the monetary authority decides on the nominal interest rate, the government sets taxes and government spending, and exogenous retirement, mortality and job destruction shocks realize. Next, worker skills evolve based on the beginning of period employment status and reborn workers replenish the dead starting as unemployed with the lowest skill level. Then, the job search stage opens. Firms post vacancies, and employed and unemployed workers search for jobs. Once new contacts are made, match productivities are observed, new matches are formed, and job-to-job transitions occur. Then in the production stage, each worker-firm pair produces labor services. Intermediate firms produce differentiated goods using these labor services and set their prices subject to nominal rigidities, and final goods are produced using the intermediate goods. Next, intermediate and service firms realize their profits, service firms pay wages to their workers, the mutual fund pays out dividends, and the government collects taxes, issues new bonds, pays out UI and retirement benefits, and spends an exogenous amount. In the final stage of the period, individuals decide on how much to consume and how many shares of the mutual fund to purchase.

2.2 Individuals

We turn to describing in detail the decision problem of individuals. They choose whether to accept a job offer (that are received while employed), how many shares of the mutual fund to buy, and how much to consume subject to a budget constraint and a short-selling constraint for the fund shares. We cast the problems recursively, where time subscripts encode all the relevant aggregate state variables. We now present the problem of unemployed, employed, and retired individuals in turn.

Unemployment. Let $V_t^U(s,h)$ denote the value of unemployed individuals with s shares of the mutual fund and skill h in period t. The problem of the unemployed worker is given by

$$\begin{aligned} V_t^U(s,h) = \max_{s' \geq 0,\, c} \; & u(c) + \beta(1-\psi^R)\mathbb{E}_{h'|h}\left[\Omega_{t+1}^U(s',h')\right] + \beta\psi^R V_{t+1}^R(s') \\ \text{s.t.} \quad & P_t c(1+\tau_c) + P_t^s s' = P_t \tau_t UI(h)^{1-\Upsilon} + (P_t^s + D_t)s, \end{aligned} \tag{1}$$

where we express the budget constraint in nominal terms. Here, $\Omega_{t+1}^U(s',h')$ is the value of job search for unemployed workers at the beginning of the next period that we describe below. Unemployed workers receive dividends D_t from the mutual fund in proportion to their share holdings s. They receive real after-tax UI benefits specified by $\tau_t UI(h)^{1-\Upsilon}$ and decide how much to consume and how many shares to buy for the next period subject to the budget constraint.

Employment. Let $V_t^E(s,h,x,\alpha)$ denote the value of employed individuals with s shares, skill h, match productivity x, and piece rate α. The employed individual's problem is given by

$$V_t^E(s,h,x,\alpha) = \max_{s'\geq 0,\, c} u(c) + \beta(1-\psi^R)\mathbb{E}_{h'|h}\left\{(1-\delta)\Omega_{t+1}^E(s',h',x,\alpha) + \delta\Omega_{t+1}^U(s',h')\right\} + \beta\psi^R V_{t+1}^R(s')$$
$$\text{s.t.} \quad P_t c(1+\tau_c) + P_t^s s' = P_t \tau_t w(h,x,\alpha)^{1-\Upsilon} + (P_t^s + D_t)s. \tag{2}$$

Similar to the unemployed, employed individuals collect dividends D_t from the mutual fund, a real after-tax wage of $\tau_t w(h,x,\alpha)^{1-\Upsilon}$, and choose consumption and share holdings before entering the next period. At the beginning of the next period, the job might dissolve exogenously, in which case the worker becomes unemployed and searches for a new job. If not, the worker can engage in on-the-job search, whose value is given by $\Omega_{t+1}^E(s',h',x,\alpha)$.

Retirement. Finally, the value of retirement is given by

$$\begin{aligned} V_t^R(s) &= \max_{s'\geq 0,\, c} u(c) + \beta(1-\psi^D)V_{t+1}^R(s') \\ \text{s.t.} &\quad P_t c(1+\tau_c) + P_t^s s' = P_t \tau_t (\phi^R)^{1-\Upsilon} + (P_t^s + D_t)s. \end{aligned} \tag{3}$$

The retirees are only subject to mortality risk and make consumption-saving decisions given their real after-tax pension income $\tau_t(\phi^R)^{1-\Upsilon}$.

Job search problems. Employed and unemployed individuals search for jobs in a frictional labor market with tightness θ_t that we formally define below. Let $f(\theta_t)$ be the workers' aggregate job-finding rate per unit of search effort. The value of job search for an unemployed worker is

$$\Omega_t^U(s,h) = \zeta f(\theta_t)\,\mathbb{E}_x V_t^E(s,h,x,\underline{x}/x) + (1-\zeta f(\theta_t))\,V_t^U(s,h), \tag{4}$$

where ζ is the job search efficiency among unemployed workers. On-the-job search value is

$$\begin{aligned} \Omega_t^E(s,h,x,\alpha) = {} & \nu f(\theta_t)\,\mathbb{E}_{\widetilde{x}}\left[\max\left\{V_t^E(s,h,\widetilde{x},x/\widetilde{x}),\, V_t^E(s,h,x,\max\{\alpha,\widetilde{x}/x\})\right\}\right] \\ & + (1-\nu f(\theta_t))\,V_t^E(s,h,x,\alpha), \end{aligned} \tag{5}$$

where ν is the search efficiency of the employed. Upon contact, the worker-firm pair draws match productivity x and the expectations are taken with respect to the sampling distribution Γ^x. The first term inside the expectation represents the worker's value when she switches to a new job with match productivity $\widetilde{x}$ and new piece rate $\alpha' = x/\widetilde{x}$. The second term represents the worker's value of staying with the incumbent firm, either with current piece rate α (if $\widetilde{x} < \alpha x$) or a higher piece rate $\widetilde{x}/x$ (if $\widetilde{x} > \alpha x$).

2.3 Production

The economy has three sectors that we now describe in more detail: final goods, intermediate goods, and labor services.

Final goods. The final-good producer purchases differentiated intermediate goods $y_t(j)$ at relative price $p_t(j) = P_t(j)/P_t$ and produces the final consumption good Y_t using the technology:

$$Y_t = \left(\int y_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}, \tag{6}$$

where η is the elasticity of substitution between varieties, and solves the following profit maximization problem:

$$\max_{\{y_t(j)\}} \; Y_t - \int p_t(j) y_t(j) dj. \tag{7}$$

This problem determines the demand for each intermediate good, $y_t(j) = p_t(j)^{-\eta} Y_t$ as a function of the relative price of variety $p_t(j)$ and aggregate demand conditions Y_t, and implies an ideal price index satisfying $1 = \left(\int p_t(j)^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$ that the intermediate-goods firms take as given.

Intermediate goods. Intermediate firms produce $y_t(j)$ using a linear technology with labor services as the only input: $y_t(j) = z_t l_t(j)$, where z_t is the aggregate productivity. They set the price for their differentiated good taking into account the demand from the final-good producer and price adjustment costs à la Rotemberg (1982). Pricing frictions render the last period's *relative* price $p_{t-1}(j)$ a state variable for the intermediate goods producers. They solve the following profit maximization problem:

$$\Theta(p_{t-1}(j)) = \max_{p_t(j)} \; p_t(j) y_t(p_t(j)) - p_t^l \frac{y_t(p_t(j))}{z_t} - Q(p_{t-1}(j), p_t(j)) Y_t + \frac{1}{1+r_{t+1}} \Theta(p_t(j)). \tag{8}$$

Price adjustment costs are given by

$$Q(p_{t-1}(j), p_t(j)) = \frac{\eta}{2\vartheta} \log \left(\frac{p_t(j)}{p_{t-1}(j)} (1+\pi_t) - \pi^* \right)^2,$$

where π^* is the inflation target of the monetary authority. In Appendix A.1, we show that this profit maximization problem implies the following New Keynesian Phillips curve (NKPC):

$$\frac{\log\left(1+\pi_t-\pi^*\right)\left(1+\pi_t\right)}{1+\pi_t-\pi^*} = \vartheta \left(\frac{p_t^l}{z_t} - \frac{\eta-1}{\eta} \right) + \frac{1}{1+r_{t+1}} \frac{\log\left(1+\pi_{t+1}-\pi^*\right)\left(1+\pi_{t+1}\right)}{1+\pi_{t+1}-\pi^*} \frac{Y_{t+1}}{Y_t}, \tag{9}$$

where $\pi_{t+1} = P_{t+1}/P_t - 1$ is the inflation rate between periods t and $t+1$, and $mc_t = p_t^l/z_t$ is the real marginal cost of production.

Labor services. A continuum of service-sector firms post vacancies incurring a cost of κ per vacancy. Labor market tightness, θ_t, is defined as the ratio of vacancies v_t to the aggregate measure of job search effort by both unemployed and employed workers $S_t = \zeta \int d\mu_t^U(s,h) + \nu \int d\mu_t^E(s,h,x,\alpha)$, where μ^U and μ^E are distributions of unemployed and employed workers over their relevant states at the search stage within a period, respectively. Let $M(v,S)$ be a constant-returns-to-scale (CRS) matching function that determines the number of worker-firm matches as a function of vacancies and search effort. We can then define $q(v,S) = \frac{M(v,S)}{v} = M\left(1,\frac{1}{\theta}\right)$ to be the firm's contact rate and $f(v,S) = \frac{M(v,S)}{S} = M(\theta,1)$ to be the worker's contact rate per unit search effort, where the CRS assumption implies that θ is sufficient to determine these rates.

We now turn to the problem of the service firms, which mirror those of the workers. Consider a firm that employs a worker with skill level h and piece rate α in a match with productivity x. The worker-firm pair produces labor services according to the production technology $F(h,x)$. The output is then sold to intermediate goods producers at real price p_t^l. Let $J_t(h,x,\alpha)$ denote the real value of this firm given by

$$\begin{aligned} J_t(h,x,\alpha) = p_t^l F(h,x) - w(h,x,\alpha) + \frac{1}{1+r_{t+1}}\left(1-\psi^R\right)(1-\delta) \\ \times \mathbb{E}_{h'|h}\Big\{ \left(1-\nu f(\theta_{t+1})\right) J_{t+1}(h',x,\alpha) + \nu f(\theta_{t+1}) \int_{\underline{x}}^{x} J\left(h',x,\max\{\alpha,\widetilde{x}/x\}\right) d\Gamma^x(\widetilde{x}) \Big\}, \end{aligned} \tag{10}$$

where the match survives if the worker does not retire, does not exogenously separate into unemployment, and does not find a new job through on-the-job search. As discussed above, the worker accepts the new job offer if $\widetilde{x} > x$, in which case the firm's value is 0. If the new match quality $\widetilde{x}$ is below current x, then the firm keeps the worker either at a higher piece rate $\widetilde{x}/x$ (if $\widetilde{x} > \alpha x$) or at the current piece rate α (if $\widetilde{x} < \alpha x$). As firms are risk-neutral and owned by the risk-neutral mutual fund, they discount the future at the real interest rate r_{t+1}.

The real value of a firm posting a vacancy is

$$\begin{aligned} V_t = -\kappa + q(\theta_t)\frac{1}{S_t}\Big[\zeta \int_{s,h}\int_{\widetilde{x}} J_t\left(h,\widetilde{x},\underline{x}/\widetilde{x}\right) d\Gamma^x(\widetilde{x})\, d\mu_t^U(s,h) \\ + \nu \int_{s,h,x,\alpha}\int_{x}^{\overline{x}} J_t\left(h,\widetilde{x},x/\widetilde{x}\right) d\Gamma^x(\widetilde{x})\, d\mu_t^E(s,h,x,\alpha)\Big], \end{aligned} \tag{11}$$

where the first term captures the value of filling a vacancy with workers originating from unemployment and the second term captures workers the firm can poach from other firms.

A free-entry condition implies that profits are just enough to cover the cost of filling a vacancy κ in expectation. Thus, we have $V_t = 0$, which together with Equation (11), pins down equilibrium market tightness θ.

Mutual fund. The mutual fund issues shares to raise funds and owns the intermediate and labor service firms, and all government bonds in the economy. The fund can issue shares at price P^s and short government bonds to earn a gross return of $1+i$. No arbitrage implies that the rate of return on shares must equal the rate of return on government bonds:

$$\frac{P^s_{t+1}+D_{t+1}}{P^s_t}=1+i_t. \tag{12}$$

The mutual fund is not allowed to retain any funds. All balances (positive or negative) are distributed to share owners in the form of dividends given by

$$D_t=B_{t-1}-\frac{B_t}{1+i_t}+P_t\Gamma^I_t+P_t\Gamma^S_t, \tag{13}$$

where the aggregate per-period real profits of intermediate and service firms are as follows:[8]

$$\Gamma^I_t=\left(1-\frac{p^l_t}{z_t}-\frac{\eta}{2\vartheta}\log(1+\pi_t-\pi^*)^2\right)Y_t, \tag{14}$$

and

$$\Gamma^S_t=\int\left(p^l_tF(h,x)-w(h,x,\alpha)\right)d\lambda^E_t(s,h,x,\alpha). \tag{15}$$

Here, $\lambda^E_t(s,h,x,\alpha)$ is the distribution of employed workers at the consumption stage, i.e., at the end of the period. Equation (13) implies that the mutual fund collects payments for the existing debt obligations B_{t-1}, profits of intermediate firms Γ^I_t, profits of service firms Γ^S_t and finances all the new debt purchases B_t. The remaining balance accrues to the individuals as dividends in proportion to their shareholdings.

Fiscal authority. The fiscal authority taxes individuals and issues bonds to finance an exogenous stream of expenditures G_t as well as UI benefits and retirement pensions. The government budget constraint is given by

$$\begin{aligned}B_{t-1}+G_t+P_t\int UI(h)d\lambda^U_t(s,h)+P_t\int\phi^Rd\lambda^R_t(s)=\frac{B_t}{1+i_t}&+P_t\tau_c\int c(l,s,h,x,\alpha)d\lambda_t(e,s,h,x,\alpha)\\&+P_t\int\left(UI(h)-\tau_tUI(h)^{1-\Upsilon}\right)d\lambda^U_t(s,h)\\&+P_t\int\left(w(h,x,\alpha)-\tau_tw(h,x,\alpha)^{1-\Upsilon}\right)d\lambda^E_t(s,h,x,\alpha)\\&+P_t\int\left(\phi^R-\tau_t(\phi^R)^{1-\Upsilon}\right)d\lambda^R_t(s),\end{aligned} \tag{16}$$

[8]We assume that vacancy creation costs are psychic in that they do not consume real resources and hence do not show up in the profits of service-sector firms.

where the left hand side is total government expenses and the right hand side is the total government revenues generated from issuing bonds and consumption and income taxation, respectively. Here, $\lambda_t(e, s, h, x, \alpha)$, $\lambda_t^E(s, h, x, \alpha)$ and $\lambda_t^U(s, h)$, $\lambda_t^R(s)$ are the distributions of all, employed, unemployed, and retired individuals, respectively, with given (relevant) state variables as of the consumption stage, i.e., at the end of the period.

Monetary authority. A monetary authority controls the short-term nominal interest rate and we assume that this nominal rate i_t is set according to the following reaction function

$$i_t = i^* + \Phi_\pi (\pi_t - \pi^*) + \Phi_u (u_t - u^*). \tag{17}$$

Here, i^* denotes the steady-state nominal interest rate, Φ_π governs the responsiveness of the central bank to deviations from its inflation target, and Φ_u controls how much the central bank responds to deviations of the unemployment rate from its steady state value.

Finally, real interest rate, r_t, satisfies the Fisher equation

$$1 + i_t = (1 + \pi_{t+1})(1 + r_{t+1}). \tag{18}$$

Timing conventions for these variables are as follows: The nominal interest rate i_t is indexed to the period in which it is set, and is the interest rate that applies between periods t and $t+1$. The inflation rate is denoted by the period in which it is measured, i.e., π_{t+1} is the realized inflation between periods t and $t+1$. The real rate has the same timing convention as inflation: r_{t+1} is the ex-post realized real interest rate from t to $t+1$.

2.4 Equilibrium

In this section, we present the conditions that characterize the equilibrium of our model.

Market clearing requires that labor services demanded by intermediate firms Y_t/z_t is equal to the aggregate supply of labor services and mutual fund shares demanded by all individuals aggregate to one. Formally, these conditions are given by:

$$Y_t/z_t = \int F(h, x)\, d\lambda_t^E(s, h, x, \alpha), \tag{19}$$

$$1 = \int g_t^{Us}(s, h) d\lambda_t^U(s, h) + \int g_t^{Es}(s, h, x, \alpha) d\lambda_t^E(s, h, x, \alpha) + \int g_t^{Rs}(s) d\lambda_t^R(s), \tag{20}$$

where g_t^{es} denotes the saving decision of workers with employment status $e \in \{E, U, R\}$.

Definition of equilibrium. Given fiscal policy instruments that determine UI replacement rate ϕ^U, retirement transfers ϕ^R, tax parameters $\{\tau_c, \tau_t, \Upsilon\}$, and government spending G_t, monetary policy rule in Equation (17), and paths of exogenous shocks to discount factor β_t, on-the-job-search efficiency ν_t, and productivity z_t, an equilibrium of the model is a sequence of individual decision rules for consumption $g_t^{Ec}, g_t^{Uc}, g_t^{Rc}$ and mutual fund share demand $g_t^{Es}, g_t^{Us}, g_t^{Rs}$, intermediate and service firm profits Γ_t^I and Γ_t^S, dividends D_t, unit labor cost p_t^l, share price P_t^s, labor market tightness θ_t, interest rates r_t, i_t and bond holdings B_t, and worker distributions $\left\{\lambda_t^E, \lambda_t^U, \lambda_t^R\right\}$ such that

- Given the path of inflation π_t, the nominal and real interest rates satisfy the Taylor rule (17) and the Fisher equation (18).
- Intermediate and service firm profits satisfy Equations (14) and (15), respectively.
- Share prices satisfy Equation (12) and dividends are given by Equation (13).
- Bonds are such that the government budget constraint in Equation (16) holds every period.
- Individual decisions g_t^{Ec}, g_t^{Uc}, g_t^{Rc}, g_t^{Es}, g_t^{Us} and g_t^{Rs} are optimal.
- θ_t is such that value of posting a vacancy expressed in Equation (11) is zero.
- Unit labor costs p_t^l satisfy the Philips curve in Equation (9).
- The labor and shares markets clear as specified in Equations (19) and (20).[9]
- The worker distribution evolves according to the laws of motion in Appendix A.2.1.

The stationary equilibrium of the model is obtained by setting all exogenous shocks to zero. In steady state, we assume that tax parameter τ^* clears the government budget constraint, and that outstanding bonds and government expenditures are a fraction of output $B^* = x_B Y^*$ and $G^* = x_G Y^*$, respectively. We provide details on the computation of the economy's transitional dynamics in Section 4 and further computational details in Appendices A.2 and A.3.

3 Calibration

In this section we discuss how we discipline our model. We assume the economy is in steady state and calibrate the model to match several targets of the U.S. economy prior to the Great Recession, specifically, over the period 2004–2006. Our model period is a quarter. We first discuss the parameters that are set outside the model and then explain how we discipline the remaining ones using our model. Next, we use our calibrated model and estimate shock processes for discount rate β, aggregate labor productivity z, and OJS efficiency ν.

Table 1: Externally calibrated parameters

Parameter	Explanation	Value	Reason
σ	Curvature in utility function	2	Standard
ψ^R	Retirement probability	0.00625	40 years of work stage
ψ^D	Death probability	0.0125	20 years of retirement stage
Δh	Skill appreciation/depreciation amount	0.275	Set
π^E	Skill appreciation probability	0.018	Wage growth for job stayers
ξ	Matching function elasticity	1.6	Set
ζ	Search efficiency of the unemployed	1	Normalization
η	Elasticity of substitution	6	20 percent markup
ϑ	Price adjustment cost parameter	0.021	Slope of Phillips curve, Galí and Gertler (1999)
x_G	Government spending/GDP ratio	0.19	Total net federal outlay/ GDP
x_B	Debt/GDP ratio	2.43	Total public debt/GDP
τ_c	Consumption tax rate	0.0312	Sales tax receipt/consumption exp.
Υ	Progressivity of income tax	0.151	Heathcote et al. (2014)
ρ_τ	Responsiveness of income tax parameter to debt level	0.10	Auclert et al. (2020)
π^*	Steady-state inflation rate	0.00496	2% annual inflation rate
Φ_π	Responsiveness of interest rate to deviations from inflation target	1.5	Taylor (1993) and Galí and Gertler (1999)
Φ_u	Responsiveness of interest rate to deviations from unemployment target	-0.25	Taylor (1993) and Galí and Gertler (1999)

Notes: This table summarizes externally calibrated parameters. See the main text for a detailed discussion.

3.1 Calibration of parameters

Functional forms and externally calibrated parameters. Table 1 summarizes the externally calibrated parameters. The utility function over consumption is of the CRRA form with $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. As is standard in the literature, we set the risk aversion parameter to $\sigma = 2$. As for the life cycle, workers spend 40 years in the labor force and 20 years in retirement in expectation, which require setting $\psi^R = 0.625\%$ and $\psi^D = 1.25\%$ on a quarterly basis.

Turning to the evolution of worker productivity, we use five equally-spaced (in logs) grid points between the lowest value $\underline{h} = 1$ and the highest value $\overline{h} = 3$ for human capital. These choices imply that worker skills change by a proportion $\Delta h = (ln(3) - ln(1))/4 = 0.275$ between grid points when they appreciate while working or depreciate during unemployment. We discipline the probability of skill appreciation for the employed π^E by the annual wage growth of job stayers. Karahan, Ozkan, and Song (2022) document that this is around 2% for a large share of the U.S. population, which implies that expected quarterly wage growth of job stayers should be around 0.5%, which requires setting $\pi^E = 0.005/0.275 \approx 0.018$. We further assume that the

[9] We do not check for goods market clearing due to Walras's Law.

match-specific productivity x is drawn from a log-normal distribution with standard deviation σ_x (to be discussed below). We discretize this process with 7 equally-spaced grid points (in logs) between the 1st and 99th percentiles of the log-normal distribution.

Following Menzio and Shi (2011) and Schaal (2017), we pick a CES matching function so that the worker and firm contact rates are given by $f(\theta) = \theta(1+\theta^{\xi})^{-1/\xi}$ and $q(\theta) = (1+\theta^{\xi})^{-1/\xi}$, respectively. Here, ξ controls the elasticity of contact rates with respect to market tightness, and we choose $\xi = 1.6$ following Schaal (2017). We also normalize the search efficiency of unemployed workers to $\zeta = 1$.

The elasticity of substitution across intermediate goods varieties η controls the markup of prices over the marginal cost—and therefore the profit share—at the steady state. We set this parameter to 6 so as to obtain a profit share of $\eta/(\eta-1) = 20\%$ (Auclert, Bardóczy, Rognlie, and Straub, 2021; Faccini and Melosi, 2021). Without loss of generality, we normalize the productivity of the intermediate sector to $z = 1$ at the steady state. Finally, as Equation (9) shows, the price adjustment cost parameter ϑ directly dictates the slope of the Phillips curve. We set ϑ to 0.021 to match the slope of Phillips curve as estimated by Galí and Gertler (1999).

Given that Ricardian equivalence does not hold in our model, fiscal policy matters for how the economy responds to shocks. We assume that government transfers are a fixed share of output, $G_t/Y_t = x_G$. Over the period 2004-2006, the ratio of government spending to GDP was around 19 percent, so we set $x_G = 0.19$. We calibrate the model to have a realistic amount of government debt. In the data, the ratio of debt to annual GDP averages to 60.8% over the same period. The quarterly frequency in the model dictates us to set this ratio to $B_t/Y_t = x_B = 4 \times 0.608 = 2.43$. We set the consumption tax rate to $\tau_c = 3.02\%$, which we obtain as the ratio of state and local sales tax receipts to personal consumption expenditures in the data for 2006. There are two parameters related to labor income taxes, one governing the average level of taxes, τ and the other one governing its progressivity, Υ. We follow Heathcote, Storesletten, and Violante (2014), and set Υ exogenously to 0.151. We explain below how we calibrate τ jointly with other parameters to match a set of targets.

As we discussed above, the government uses debt to balance its budget. Along a transition path—off the steady state that we discuss below—the level of debt can go above or below its steady state level of $x_B Y$. In these cases, the fiscal authority follows an exogenous rule that adjusts the level parameter of income taxes τ to eventually bring the level of real debt back to its steady state value. This response function is given by

$$\tau_t = \tau^* - \rho_\tau \left(b_{t-1} - b^*\right)/Y^*. \tag{21}$$

Here, τ^* is the steady state value of τ, which is inversely related to the level of income taxes. The second term in Equation (21) controls how strongly fiscal policy reacts to deviations of debt-to-GDP from its steady state value. A higher value for ρ_τ indicates that taxes go up more when debt-to-GDP rises above the steady state. Following Auclert, Rognlie, and Straub (2020), we set $\rho_\tau = 0.1$.

Turning to monetary policy, the central bank targets an annual inflation rate of 2%. Quarterly calibration requires us to set $\pi^* = 1.02^{1/4} - 1 \approx 0.496\%$. In disciplining the Taylor rule, we follow Taylor (1993) and Galí (2015). We set the coefficient on inflation to $\Phi_\pi = 1.5$ as in their work. One notable difference of our specification is that our Taylor rule reacts to the unemployment gap rather than to the output gap. Galí (2015) sets the the coefficient on the output gap in a quarterly model as 0.125. To map their coefficient on the output gap to the unemployment gap, we use Okun's law with a coefficient of -2, as in Okun (1962). This implies setting $\Phi_u = -0.25$.

Internal calibration. The remaining nine parameters are the discount factor β, vacancy creation cost κ, job separation probability δ, job search efficiency of the employed ν, skill depreciation probability when unemployed π^U, standard deviation parameter of match specific productivity distribution σ_x, maximum share of output potentially paid to worker as wages ϕ^E, UI replacement rate ϕ^U, and retirement benefit amount ϕ^R. These parameters are calibrated internally by matching a set of data moments that we now describe. Specifically, we use the simulated method of moments where we minimize the sum of squared percentage deviations of the model moments from their empirical counterparts. Table 2 summarizes the targeted moments and the calibrated parameter values. While all parameters are jointly calibrated, Table 2 presents each parameter next to the target its mostly informative about.

Given the recent work highlighting the role of the asset distribution in the transmission of monetary policy, we target the fraction of hand-to-mouth (HtM) households in the labor force to discipline discount factor β. We define HtM households as those with non-positive liquid wealth holdings. We use the 2004 panel of the Survey of Income and Program Participation (SIPP) and work with a sample of individuals aged 25–65, who do not own any business. 16 percent of our sample are HtM households according to our classification.

On the labor market side, we target a steady state unemployment rate of 5.1%, as well as worker flows. We obtain the targets for the flow rates from various sources. Using data from the Current Population Survey (CPS), we compute the average monthly employment-to-unemployment separation rate over the period 2004-2006. We convert this monthly job loss rate to a quarterly frequency and obtain our target of 3.8%. To compute the job-to-job transition rate, we make use of quarterly data from the Longitudinal Employer-Household Dynamics (LEHD). We find that the job-to-job transition rate (or EE rate), measured as the job switching rate of workers who do not have any intervening nonemployment spell, is around 2% over the same period. These moments are informative about the vacancy creation cost κ, job separation rate δ, and employed search efficiency ν, respectively.

The probability of skill depreciation when unemployed π^U is informative about the magnitude

Table 2: Internally calibrated parameters

Parameter	Explanation	Value	Target	Data	Model
β	Discount factor	0.981	Fraction of individuals with non-positive liquid wealth	0.16	0.11
κ	Vacancy creation cost	0.670	Unemployment rate	0.051	0.052
δ	Job separation probability	0.091	EU rate	0.038	0.033
ν	Search efficiency of employed	0.108	EE rate	0.02	0.02
π^U	Skill depreciation probability	0.022	Earnings drop upon job loss	-0.35	-0.36
σ_x	Standard deviation parameter of match productivity distribution	0.063	Wage growth of job switchers	0.09	0.09
ϕ^E	Maximum share of output as wages	0.823	Labor share	0.67	0.74
ϕ^U	UI replacement rate	0.385	UI replacement rate	0.40	0.44
ϕ^R	Retirement benefit amount	0.473	Retirement income/labor income	0.34	0.41

Notes: This table summarizes internally calibrated parameters. See the main text for a detailed discussion.

of earnings loss upon job displacement. Getting this moment right is not only important to discipline skill depreciation but also to get at the cost of job loss and the welfare effects of stimulating the economy. A large literature has estimated the magnitude of earnings losses upon job displacement using a variety of datasets and approaches (see, for example, Jacobson, LaLonde, and Sullivan, 1993; Stevens, 1997; Davis and von Wachter, 2011; Jarosch, 2021; Birinci, 2021, among others). Across these studies, the median estimate of the earnings loss in the year of job displacement is about 35%. To facilitate comparison with the literature, we generate a simulated panel of households in the model, aggregate quarterly simulations to an annual frequency, and estimate a distributed-lag regression on these model-generated data, analogously with empirical studies.

Another important aspect of the model is what happens to wages when workers change employers. This feature disciplines how important job-to-job transitions are for aggregate demand in the economy. Using the LEHD, we calculate the change in earnings for continuously employed workers upon a job change, which we find to be around 9%. This moment is informative about the dispersion parameter for match productivity σ_x, which governs the increase in wages upon a job-to-job switch in the model. Given σ_x, we pick the mean parameter of the match productivity distribution $\mu_x = -\sigma_x^2/2$ so that the mean of the distribution is normalized to one. Finally, we choose the maximum share of output that is paid to workers as wages ϕ^E to target an average labor share of 0.67.

Turning to the generosity of government programs, we calibrate the UI replacement rate ϕ^U to match an average replacement rate of 40%. To discipline pension benefits during retirement ϕ^R, we calculate the average retirement income to labor income ratio in the SIPP. Specifically, we add up Social Security Income and pension incomes from federal, state, and local governments for the sample of retirees and compute a per-person retirement income as an average of this measure in our sample. We then divide it by the average labor income among nonretirees to obtain a ratio of 0.34 in the data.

3.2 Estimation of shocks

Next, we estimate shock processes that will be used in our positive and normative analysis in Section 5 and 6.

We assume that the economy starts from steady state and is subject to demand, supply, and labor market shocks, which are modeled as innovations to the discount rate β, aggregate labor productivity z, and OJS efficiency ν. To do so, we consider AR(1) processes for β, z, and ν given by:

$$\beta_t = (1-\rho_\beta)\beta^* + \rho_\beta \beta_{t-1} + \sigma_\beta \epsilon_{\beta,t}, \quad z_t = (1-\rho_z)z^* + \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}, \quad \nu_t = (1-\rho_\nu)\nu^* + \rho_\nu \nu_{t-1} + \sigma_\nu \epsilon_{\nu,t},$$

where ρ_j denotes the persistence of the AR(1) process, $\epsilon_j \sim N(0,1)$ is i.i.d. and $\sigma_j > 0$ denotes the standard deviation of innovations for $j \in \{\beta, z, \nu\}$.

We estimate the parameters of these processes by matching moments between the model and the data. In particular, we jointly estimate the persistence and standard deviations of innovations to β, z, and ν by targeting the autocorrelation of output, correlations unemployment rate, EE rate, and inflation with output (separately) as well as the standard deviations of output, unemployment rate, EE rate, and inflation in the data.[10] We find that $\rho_\beta = 0.909$, $\rho_z = 0.332$, $\rho_\nu = 0.936$ and $\sigma_\beta = 0.001$, $\sigma_z = 0.002$, and $\sigma_\nu = 0.003$.

Table 3 compares the resulting model moments their data counterparts. The model generates nearly identical values for correlations of unemployment rate, EE rate, and inflation with output as well as the autocorrelation of output when compared to their empirical counterparts. In terms the standard deviations, the model is less successful. A well-known feature of the search and matching models is their inability of generating the observed empirical volatility of the unemployment rate when simulated under labor productivity shocks (Shimer 2005). Although

[10] We obtain monthly data on the unemployment rate and core CPI from the BLS which we convert to a quarterly frequency by taking averages; quarterly data on real GDP from U.S. Bureau of Economic Analysis; monthly data on the EE rate from Fujita, Moscarini, and Postel-Vinay (2020), which we convert to a quarterly frequency by compounding $EE_t^{\text{qrt}} = 1 - (1 - EE_t)^3$; and monthly data on number of vacancies from JOLTS which we convert to quarterly frequency by taking averages and divide it by the unemployment rate to obtain the labor market tightness θ in the data. All data cover the period between 1995:Q3 and 2008:Q4. Both in the model and the data, we take logs and detrend the time series of output (real GDP), unemployment rate, and labor market tightness using the HP filter with a smoothing parameter of 10^5 and calculate correlations and standard deviations of the cyclical components. Because inflation is negative in some periods in the data, we detrend the level of inflation. Finally, in the data, we calculate the percent deviation of EE rate from its sample average. To calculate model moments, we simulate aggregate time series many times and take averages of moments across these simulations.

Table 3: Estimation of shocks

	Data			Model		
	Std. Dev	Autocorr.	Corr. w/ Y	Std. Dev	Autocorr.	Corr. w/ Y
Y	0.024	0.963	1	0.005	0.924	1
u	0.148	0.953	-0.882	0.092	0.859	-0.882
EE	0.090	0.907	0.147	0.068	0.765	0.145
θ	0.275	0.930	0.809	0.062	0.105	0.626
π	0.245	0.388	0.538	0.270	0.825	0.543

Notes: This table compares model outcomes with their empirical counterparts using the estimated AR(1) processes for the discount rate β, aggregate labor productivity z, and OJS efficiency ν. Y u, EE, θ, and π denote the output, unemployment rate, EE rate, labor market tightness, and inflation, respectively.

the unemployment volatility is still lower in our model than the data, our model delivers a much higher unemployment volatility than what is implied by the standard model especially due to shocks to discount factor (see Table 4) and the presence of job-to-job transitions (Fujita and Ramey 2012). Our model also delivers volatilities of inflation and EE rate close to their empirical values.

To understand the contribution of each shock to the cyclical movements of our target outcomes, we provide a variance decomposition of these moments. Table 4 presents the fraction of variance of output, unemployment rate, EE rate, labor market tightness, and inflation explained by shocks to aggregate productivity z, OJS efficiency ν, and discount factor β alone. Shocks to β explain almost all the fluctuations in the output and 81.2 percent of fluctuations in the unemployment rate, while shocks to z and ν jointly account for the remaining 18.8 percent of fluctuations in the unemployment rate. We also find that variations in ν is an important driver behind fluctuations in the EE rate and inflation: shocks to ν account for 78.7 percent of fluctuations in the EE rate and 43.1 percent of fluctuations in inflation.

What are OJS efficiency shocks? We have shown that fluctuations in OJS efficiency have sizable affects on EE rate and inflation. The natural question to ask then is what can be the micro-foundation of these OJS efficiency shocks? Bilal, Engbom, Mongey, and Violante (2022) show that worsening of financial frictions led to a decline in new entry of young and highly productive firms into the labor market around the Great Recession. They find that the reason behind the failure of the job ladder during this episode was the decline in vacancy creation among such firms whose net poaching rate is typically high. As such, our framework can be extended to a framework with ex-ante heterogeneity of firms whose entry decision is subject to financial frictions in order to microfound OJS efficiency shocks. In this paper, given the complexity of our framework, we do not attempt to do so, and instead, we estimate them using the model as

Table 4: Variance decomposition of moments

	Share of variance explained by		
	z	ν	β
Y	0.008	0.031	0.961
u	0.111	0.077	0.812
EE	0.070	0.787	0.143
θ	0.337	0.046	0.618
π	0.049	0.431	0.520

Notes: This table presents a variance decomposition of output, unemployment rate, EE rate, labor market tightness, and inflation, respectively. The columns represent the fraction of each moment's variance explained by shocks to aggregate productivity z, OJS efficiency ν, and discount factor β alone.

discussed above and study their implications on aggregate dynamics.

4 Solving for transitional dynamics

In this section, we discuss our methodology to solve the model's transitional dynamics upon a shock with further details relegated to Appendix A.3.

We assume that the economy is in steady state at time $t = 0$ and people expect it to remain that way. Entering period $t = 1$, they observe an unexpected and transitory shock to the economy (e.g. productivity, discount rate, and labor market shocks). Because the shock is transitory, the economy returns to the same real allocations but potentially with different nominal price levels. We conjecture the transition is completed by period $t = T$ for some large enough T.

We use the sequence space Jacobian method developed by Auclert, Bardóczy, Rognlie, and Straub (2021), which allows us to efficiently solve for the impulse responses to shocks. To apply this method, we first recast key model equations in terms of real variables and relative prices so that the terminal value of all variables following a shock attain their initial steady state values.[11] We then cast the model as a directed acyclical graph (DAG), presented in Figure A.1, which expresses the model as various nodes and how they relate to one another. The nodes in the DAG can be classified into three groups: the initial node that contains potential exogenous shocks to the economy as well endogenous variables to be solved for, the intermediate (green) nodes that represent blocks that contain the model's various components (such as the conduct of monetary policy via the Taylor rule, fiscal policy via the tax rule, or the heterogeneous agent household problem), and the terminal nodes that represent equilibrium conditions. Importantly, the DAG relates each node by specifying variables which are used as inputs to and generated as outputs from these nodes. At each node, we calculate partial Jacobians of each output with respect to

[11]Note that we assume a trend inflation of 2% per year, and hence the nominal variables in the initial and terminal steady states are not necessarily the same.

each input. We then forward accumulate these partial Jacobians along a topological sort of the DAG and use the implicit function theorem to obtain the general equilibrium Jacobians of the model. These general equilibrium Jacobians can in turn can be used to compute the response of any endogenous variable to any exogenous shock. Furthermore, using the equivalence of the impulse response function and the moving average representation of the process generating that variable, we simulate the time path of aggregate variables as well as a large panel of individuals to obtain a rich set of aggregate and cross-sectional moments of the model under aggregate shocks.

Relative to a standard shooting algorithm to obtain general equilibrium impulse responses to a shock, the method of Auclert, Bardóczy, Rognlie, and Straub (2021) provides major computational efficiency improvements along two dimensions. The first improvement allows for the computation of policy function responses to any shock that may hit at any period by a single backward value function iteration. The second improvement introduces a fake-news algorithm to offer an efficient method of forward iteration of equilibrium distributions in a model with rich heterogeneity. We closely follow Auclert, Bardóczy, Rognlie, and Straub (2021) to implement both of these improvements when solving for transitional dynamics in our model.

Importantly, we generalize the sequence-space Jacobian method and allow for model blocks to directly interact not only via aggregate variables but also through the discretized distribution of individuals across state variables. This modification is crucial for our application because outcomes of the heterogeneous-agent (HA) block in the DAG include various distributions of employed and unemployed individuals, which are required as inputs for the labor-service firm and other equilibrium conditions. First, the distribution of employed individuals across human capital and match productivity levels and the distribution of unemployed individuals across human capital at the job search stage within a period, i.e., $\mu^E(h, x)$ and $\mu^U(h)$, respectively, affect the expected value from a match $\mathbb{E}J$ for firms deciding on vacancy creation. This is because (1) human capital affects the magnitude of output in a match and (2) employed workers' match productivity with their current employer affects their job acceptance decision and the piece rate that the poaching firm would offer to the worker (and thus their wage level) upon a new match. Second, the distribution of employed workers across human capital, match productivity, and piece rate levels at the consumption/production stage in a period, $\lambda^E(h, x, \alpha)$, affect service-firm profits Γ^S by determining the output and wage levels in a match, which in turn affect dividend payments as all profits are collected by the mutual fund and are distributed back to households in proportion to their share holdings.

To summarize, we generalize the sequence-space Jacobian method and incorporate discretized distributions across state variables as direct inputs and outputs along the DAG to solve our model which combines a New Keynesian framework with heterogeneous agents and a frictional labor market featuring on-the-job search. In this sense, our approach combines the sequence-space

Jacobian method of Auclert, Bardóczy, Rognlie, and Straub (2021) with Reiter (2009).

5 Positive implications of job mobility on inflation

In this section, we use the calibrated model to study how the economy responds to exogenous shifts in worker mobility between employers, which we capture by shocks to OJS efficiency ν. These shocks are motivated by a significant weakening of the negative correlation between the EE rate and the unemployment rate during the expansion following the Great Recession, which we document in Section 5.1. We mimic this episode of declining unemployment but flat EE rate in our model with positive demand and negative OJS shocks. We compare the outcomes of this to an economy with the same path of unemployment but an endogenously increasing EE rate.

We quantify the drag on inflation from the negative OJS shocks that keep EE flows suppressed and relate this to the "missing inflation" puzzle in this period. Finally, we use the model's DAG representation to decompose the inflation effect of OJS shocks into various channels.

5.1 Weaker EE relative to unemployment after the Great Recession

Panel (a) of Figure 2 presents a scatter plot of monthly EE rate (Fujita, Moscarini, and Postel-Vinay, 2020) and the civilian unemployment rate i) prior to the Great Recession (1995-2007), ii) during the Great Recession and the subsequent recovery (2008-2015), iii) post-Great Recession (2016-2019), and iv) the COVID-19 period (2020-2022). The correlation between the two series is negative and significant, except during 2016–2019, when this correlation turned slightly positive (but insignificant). To present a more continuous view and to separate trend from the cycle, panel (b) plots the rolling correlation between the cyclical components of log unemployment and EE rates using a five-year window where both time series are detrended using the HP filter with a smoothing parameter of 10^5. There was a strong negative comovement among the two series, which disappeared over the period 2016–2019. This breakdown can be traced to a flat EE rate during this episode despite an around 25 percent decline of unemployment rate from its trend, as shown in Figure B.1, where we also show that growth of unit labor cost in the data was also muted during this episode. Our subsequent exercises that study the role of worker mobility in inflation dynamics are motivated by this missing correlation between EE flows and unemployment during 2016–2019 and the so-called "missing inflation" in this episode.

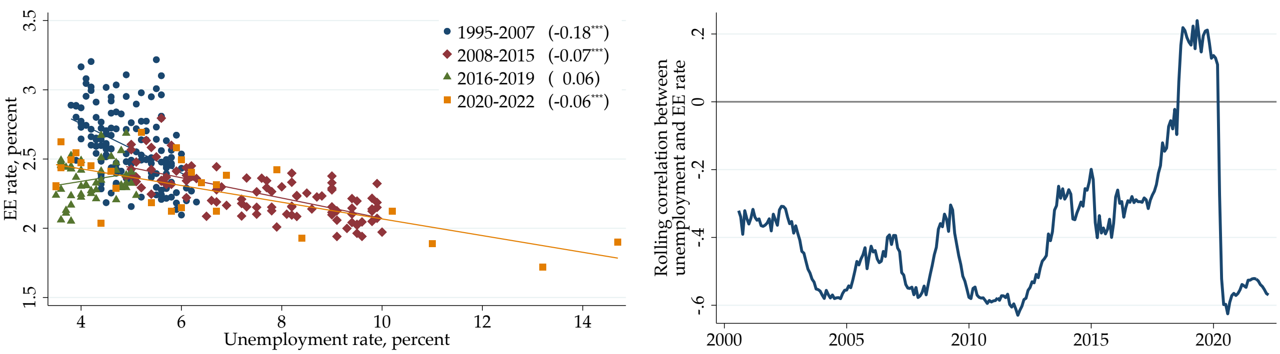
5.2 Missing inflation due to low EE during 2016–2019

We isolate the role of the low EE rate in missing inflation by comparing two economies that mimic the path of the unemployment rate over the 2016–2019 period but differ in their equilibrium EE rate. The first economy features an endogenously rising EE rate due to a tightening labor market, consistent with its negative historical correlation with the unemployment rate but inconsistent with the experience during this episode. The second economy, besides featuring the

Figure 2: Correlation between EE rate and unemployment rate over time

(a) Unemployment vs EE rate

(b) Rolling correlation between unemployment and EE



Notes: Panel (a) presents a scatter plot of monthly EE rate and unemployment rate across different episodes: prior to the Great Recession (1995–2007), during the Great Recession and the subsequent recovery (2008–2015), post-Great Recession (2016–2019), and the Covid-19 episode (2020–2022). Values in parenthesis report the coefficient from regressing the EE rate on the unemployment rate and *** denotes significance at the 1 percent level. Panel (b) presents the rolling correlation between the cyclical components of the logs of unemployment and EE rates using a five-year window. Both time series are detrended using the HP filter with a smoothing parameter of 10^5. Source: BLS and Fujita, Moscarini, and Postel-Vinay (2020).

same unemployment rate path as the first economy, also replicates the stable EE rate in the data using additional shocks as we discuss later.

Starting from the same steady state described in Section 3, we allow for two shocks to hit the economy starting in 2016.[12] We model demand shocks and shocks to OJS efficiency as innovations to the discount factor β and ν, respectively, following AR(1) processes:

$$\beta_t = (1-\rho_\beta)\beta^* + \rho_\beta \beta_{t-1} + \varepsilon_{\beta,t} \tag{22}$$

$$\nu_t = (1-\rho_\nu)\nu^* + \rho_\nu \nu_{t-1} + \varepsilon_{\nu,t}. \tag{23}$$

The first economy features only demand shocks ($\varepsilon_{\nu,t} = 0$). We back out the path of $\varepsilon_{\beta,t}$ that generates a gradual decline in the unemployment rate by 15% relative to its steady state level of 5.2%.[13] To approximate the steady decline in the unemployment rate over the 16 quarters, we assume that the decline is linear and is completed within $\overline{T} = 16$ quarters from the onset of the first shock. Upon reaching its trough, the unemployment rate is assumed to revert back to

[12] That the economy is in steady state in 2016 is a plausible assumption from the perspective of the labor market. The unemployment rate at that time was just below 5%, close to the estimate of the natural rate of unemployment at the time.

[13] This is consistent with the decline in the unemployment rate attributable to an increase in the job-finding rate observed between 2016 and 2019. In particular, holding the separation rate fixed at its January 2016 level, the rise in the job-finding rate between 2016 and 2019 alone leads to a 15% decline in the unemployment rate.

its steady state in accordance with the following law of motion for $t > \overline{T}$:

$$u_t = (1 - \rho_u)u^* + \rho_u u_{t-1}.$$

For the first economy, we estimate the path of demand innovations $\varepsilon_{\beta,t}$ to exactly match the path of the unemployment rate we posit above.[14] The second scenario allows for shocks to OJS efficiency to mimic the actual expansionary episode during 2016–2019. We jointly estimate the path of the two shocks, $\varepsilon_{\beta,t}$ and $\varepsilon_{\nu,t}$, such that the economy generates the same unemployment rate path described in the first scenario and, in addition, has the EE rate unchanged throughout.[15]

The results are presented in Figure 3. Positive demand shocks alone and the combination of positive demand and negative OJS shocks generate identical paths for the unemployment rate as intended (Panel (a)). The two economies differ in their EE rate by construction (Panel (b)): In the first economy, positive demand shocks increase vacancy posting by firms, resulting in a tighter labor market, and consequently, in an endogenous increase in the EE rate from its steady state level of 2% to a peak of 2.06%. In the second economy, negative OJS shocks keep the EE rate suppressed, in line with the empirical observations in Figure 2 and Figure B.1.

The first economy produces slightly more output (Panel (c)). The output difference is entirely attributable to the differences in average labor productivity, since the path of (un)employment is identical across the two economies. Labor productivity decreases at first in both economies as the increase in the job finding rate means more unemployed joining the ranks of the employed. Since the unemployed typically have lower human capital h than the employed and accept offers with any match productivity x, their entry to employment lowers average labor productivity. Labor productivity eventually increases in both economies because the higher job finding rate results in a higher level of human capital and match productivity (Panel (d)). The increase is slightly larger in the first economy, because the increase in EE transitions due to more frequent worker-firm contacts generates productivity-improving employment switches. However, the gap in labor productivities is quantitatively small because the distribution of match productivity is a slow moving object (Figure B.2).

If we were to solely focus on unemployment and output and ignored job mobility, we would have inferred that the two economies were hit by very similar shocks. Looking at inflation changes this view, however. In particular, (annualized) inflation is 0.23 percentage points smaller in the economy with OJS shocks (Panel (e)).[16] This is not a small quantitative effect.[17] Annual inflation

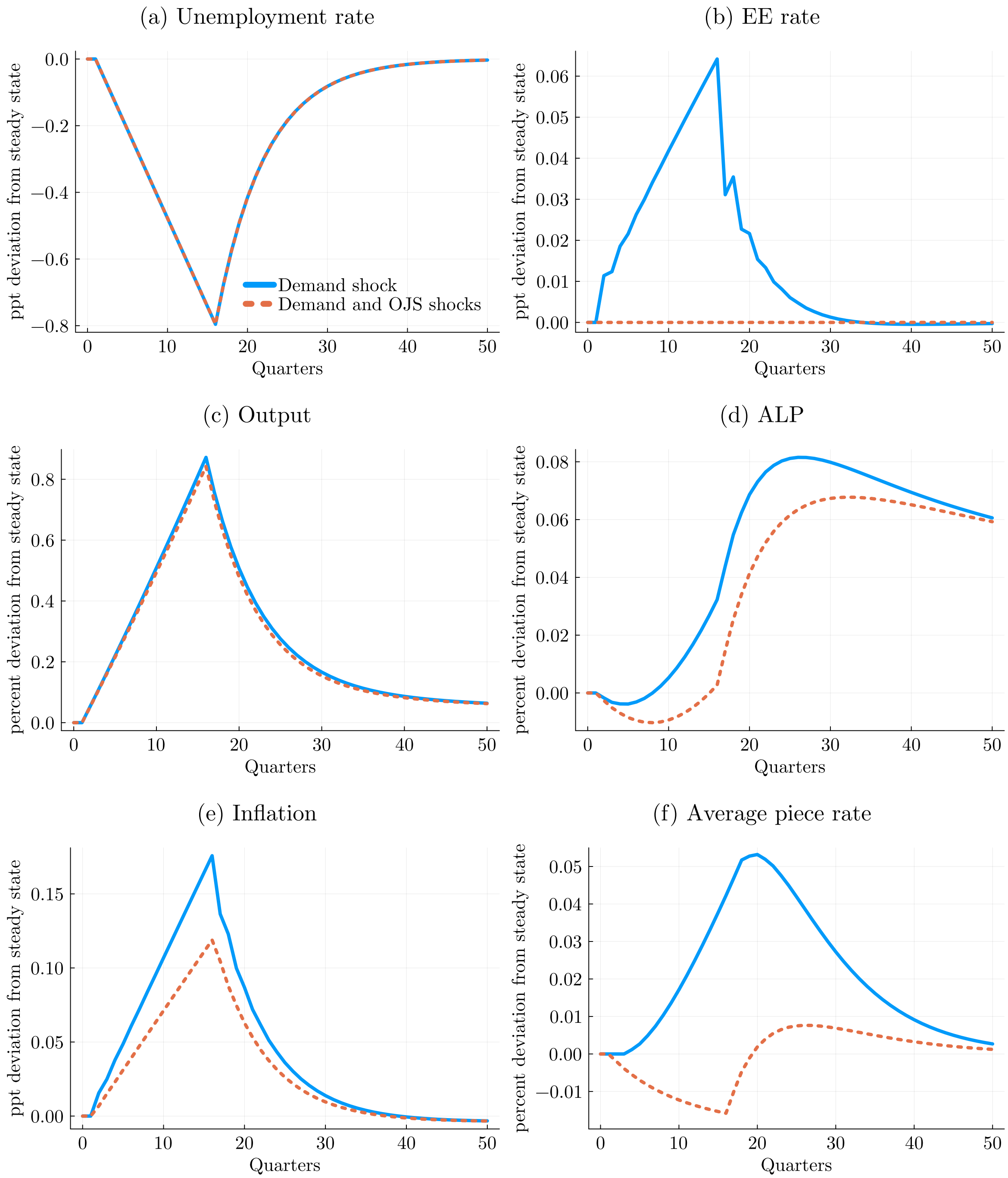
[14]Recall that in Section 3.2, we jointly estimated the parameters of the AR(1) processes governing discount factor β, productivity z, and OJS efficiency ν to match empirical moments. In Equations (22) and (23), we use the estimated persistence parameters ρ_β and ρ_ν for these AR(1) processes.

[15]Figure B.5 plots the estimated paths of these shocks for both economies.

[16]To obtain this number, we calculate the annualized inflation rate in each economy and report their maximum difference, which materializes 16 quarters after the shock, when unemployment is at its lowest level.

[17]Recently, Bostanci, Koru, and Villalvazo (2022) and Pilossoph and Ryngaert (2022) document in the data that inflationary shocks lead to higher EE rates. Because our model does not feature endogenous job search

Figure 3: Effects of negative OJS efficiency shocks on output and inflation



Notes: This figure presents the dynamics of unemployment rate, EE rate, output, average labor productivity (ALP), inflation, and average piece rate in an economy subject to (1) only a series of positive demand shocks (solid-blue lines) and (2) series of positive demand shocks and negative OJS efficiency shocks (dashed-orange lines). The shocks in the two economies are estimated to generate the same path of unemployment. The EE rate is untargeted in the first economy whereas the OJS efficiency shocks are such that the EE rate remains unchanged in the second economy.

effort, it does not allow for this margin. However, if we were to allow for this channel, the magnitude of missing inflation implied by our model would have been even larger because the increase in EE rate under the economy without OJS efficiency shocks would be even larger. As such, we view our estimate as a lower bound.

was around 1.8 percent in 2019 when the unemployment rate was at its lowest level after the Great Recession. The 0.23 percentage points drag on inflation implies that inflation would have been around 2 percent in 2019 if the EE rate had increased over the period 2016–2019 in line with its historical negative co-movement with unemployment.[18]

One reason behind lower inflation under negative OJS shocks is that contact between the employed and poaching firms occurs less frequently. This limits the incidence of incumbent firms making counteroffers that raise average piece rates (Panel (f)). The smaller increase in the piece rate means that wage growth exceeds productivity growth by less in this economy. This lower growth in the cost of producing labor services eventually translates to a lower (relative) price of labor services p^l as explained in Moscarini and Postel-Vinay (2019).[19] Quantifying the various channels through which job mobility affects inflation requires a separate investigation, which we undertake in the next section.

5.3 Decomposing the effect of OJS shocks on inflation

The preceding case study demonstrated that OJS shocks have sizable effects on inflation. We now quantify the channels through which a positive OJS shock translates to, on net, higher inflation by providing a decomposition of the impact response of inflation to a unit shock to ν. This decomposition leverages the DAG representation of the model in Figure A.1 and the system of sequence-space Jacobians we compute to solve and simulate our model.[20]

The NKPC in Equation (9) reveals that—to a first-order approximation—inflation is driven entirely by the relative price of labor services p^l, which, absent productivity shocks, determines the real marginal cost p^l/z for intermediate firms. Therefore, it is sufficient to provide a decomposition of ν's effects on p^l to fully understand inflation responses to an OJS shock.

The decomposition essentially applies the implicit function theorem to equilibrium conditions of choice in a particular order, to express one of the outcome variables as a (linear) function of the shocks and other endogenous variables. For instance, we can think of the free-entry condition as pinning down the price of labor services, p^l given labor market tightness.[21] Hence, we can decompose equilibrium changes in p^l to the contributions from the various inputs into that condition. In this case, they are labor market effects via search efficiency ν and labor market tightness θ, and discount rate effects via r.[22] Because r is determined in the monetary policy

[18] Inflation in 2019 was expected to be larger than 2 percent, because unemployment in 2019 ($\approx$3.7 percent) was deemed below the natural rate of unemployment u^*. Therefore, the muted EE rate explains a sizable portion but not all of the missing inflation.

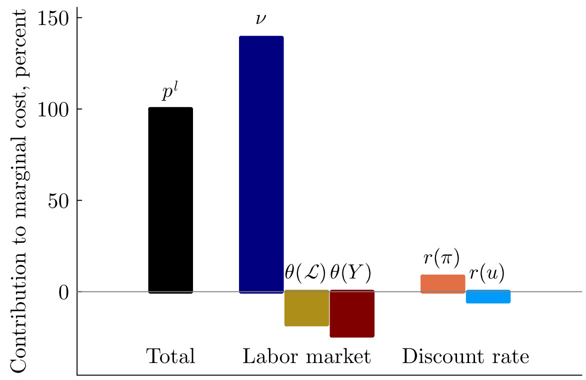
[19] Specifically, we compare the distribution of match productivities and piece rates between the two economies in Appendix Figure B.2. The piece rate distribution shifts to the right in the first economy with only expansionary demand shocks and to the left in the second when there are also negative OJS shocks.

[20] Our decomposition can be extended to the entire IRF; for brevity we focus on the impact effect.

[21] Of course, equilibrium prices and allocations are a result of complex interactions of all endogenous and exogenous variables. We acknowledge that the decomposition may depend on the particular sequence in which we choose to analyze equilibrium conditions.

[22] We note that there are also composition effects via the search-stage worker distributions μ^E and μ^U. However,

Figure 4: Decomposition of channels that OJS efficiency shock affects the marginal cost



Notes: This figure presents a decomposition on the share of overall impact increase in the marginal cost, i.e., the price of labor services p^l, explained by various channels in response to an increase in the OJS efficiency parameter ν. In particular, the fraction of the total change in p^l is accounted for by labor market effects and discount rate effects. ν refers to the direct effect of OJS efficiency on p^l; $\theta(\mathcal{L})$ refers to the effect of ν on market tightness θ through its effect on the total supply of labor services $\mathcal{L}$; $\theta(Y)$ denotes the effect of ν on θ through its effect on output Y; $r(\pi)$ denotes the effect of ν on real rate r through inflation π; and $r(u)$ refers to the effect of ν on real rate r through unemployment u.

block, taking π and u as inputs, the contribution of r to p^l can be further attributed to these variables, which we denote by $r(\pi)$ and $r(u)$. The contribution of θ too can be further dissected by recognizing θ as being pinned down by the labor services market clearing condition: A higher (lower) θ induces production of more (less) labor services in the household block, and allows the system to clear the labor services market. The variables that enter this block that have an effect on θ are OJS efficiency, output, and labor services. Therefore, θ and its contribution to p^l can be further broken down to a direct effect of ν, and contributions from output $\theta(Y)$ and labor services $\theta(\mathcal{L})$. Appendix A.4 provides further details.

A higher ν leads to a higher p^l, and hence inflation. We now turn to quantifying the channels through which this comes about. Figure 4 shows the percent contribution of each of these channels to higher inflation, which we group into two categories: the labor market and discount rate effects.

Labor market effects. A shock to ν directly affects the expected match value for the firm $\mathbb{E}J$ by raising a worker's probability of receiving an outside offer. The higher frequency of such contacts raise the likelihood of wage re-bargaining or quitting, both of which reduce firm surplus. Thus, an increase in ν leads to a decline in $\mathbb{E}J$.[23] All else equal, the decline in $\mathbb{E}J$ necessitates

as we discuss below, these have relatively small quantitative effects on p^l.

[23]A higher ν implies a higher weight for employed job-searchers in the aggregate measure of job searchers S, which has a distinct effect on $\mathbb{E}J$. We find this to be small due to offsetting forces. On the one hand, employed workers are typically more productive as they have higher skills than the unemployed. On the other hand, they are less likely to accept offers and, when starting a job, more likely to dictate higher wages than new hires from unemployment.

an increase in p^l for the free-entry condition to hold. Quantitatively, we find that the direct effect of ν on p^l, labeled as ν in Figure 4, explains 139% of the total (100%) increase in p^l upon impact.

There are further general equilibrium (GE) effects in addition to this direct effect. In the labor market, an increase in ν leads to a decline in labor market tightness θ (Panel (b) of Appendix Figure B.3). How does this decline in θ affect p^l? According to the DAG, θ enters the free-entry condition through its effects on the service firm.[24] For an unmatched service firm, a lower θ increases the probability of filling the vacancy $q(\theta)$. In addition, a lower θ reduces the worker's probability of contacting other firms in the future, implying less frequent wage re-bargaining and longer match durations. Thus, the matched firm's value J and hence $\mathbb{E}J$ increase. All else constant, a higher vacancy filling rate and expected match value require a decline in p^l for the free-entry condition to hold. Quantitatively, this GE effect of ν on p^l through θ accounts for -42% of the increase in p^l.

Because the indirect effect through θ is large and of the opposite sign with the direct effect, we use the market clearing condition for labor services in Equation (19) to further decompose the response of θ. From the DAG, the direct effect of ν on θ is through the aggregate supply of labor services in the HA block, $\mathcal{L} = \int F(h,x)\, d\lambda_t^E(s,h,x,\alpha)$. All else the same, a higher ν implies a more productive match distribution, and therefore increased labor services production. For the labor services market to clear, the productivity gains which raise $\mathcal{L}$ should be counteracted by a decline in θ. This effect of ν on θ through the supply of labor services, $\theta(\mathcal{L})$, explains -18% of the total increase in p^l.

In addition to $\mathcal{L}$, ν has a separate effect on θ through output Y, which declines in GE (Panel (c) of Figure B.3). The decline in output is driven by a lower aggregate demand (Panel d), which itself is a result of a higher unemployment rate (Panel e), a lower job finding rate, and other GE effects. Lower output implies less demand for labor services $L = Y/z$. All else the same, a commensurate decline in θ is required for the labor services market to clear. This effect of ν on θ through output, $\theta(Y)$, accounts for -24% of the total increase in p^l. We conclude that the GE effects of ν on θ through $\mathcal{L}$ and Y mitigate a much larger direct effect of ν on p^l.

To summarize the labor market effects, a higher ν implies lower expected match values as firms face more frequent wage re-bargaining and shorter match durations. This direct effect entails a compensatory increase in p^l to maintain the free-entry condition. However, a higher ν also reduces θ because of increased labor supply due to higher productivity and because of lower demand, both of which require a decrease in labor market tightness to clear the market for labor services. Lower tightness translates to higher expected match values since firms fill vacancies faster and are less susceptible to quits. To satisfy the free entry condition, this necessitates a

[24] Market tightness θ also affects the distribution of employed workers over time in the heterogeneous agent (HA) block. However, this change does not affect the distribution of the employed at the search stage μ^E in the first period of a positive ν shock. Therefore, it has no effect on p^l upon impact.

decline in p^l to reduce firm entry, partially mitigating the rise in p^l due to the direct effect.

Discount rate effects. We now turn to the GE effects of ν on p^l through the real interest rate, which we label as the discount rate effects. These effects arise because of the monetary authority, which adjusts the nominal interest rate—and hence the real interest rate—in response to inflation and unemployment. In response to an increase in ν, the unemployment rate and inflation both increase (panels (e) and (f) in Figure B.3). The higher equilibrium inflation induces a more than one-for-one increase in the nominal rate i as we assume that $\Phi_\pi > 1$ and therefore an increase in the real rate. A higher real rate reduces the valuation of service firms (see the third term in the right-hand side of Equation (10)), which in turn reduces the expected value from a match $\mathbb{E}J$. All else constant, a lower $\mathbb{E}J$ requires a higher p^l for the free-entry condition to hold. Quantitatively, we find that the inflation channel $r(\pi)$ accounts for 8% of the total increase in p^l. This is a smaller effect compared to the labor market effects.

Similar reasoning implies that the increase in unemployment induces a decline in the the real rate. This effect is also small in magnitude; $r(u)$ explains -5% of the total increase in p^l.

Taking stock. While an increase in ν increases p^l through its direct effect, GE forces through market tightness θ partially mitigate the increase in p^l. Overall, the labor market effects account for 97% of the total increase in p^l. The remaining 3% is accounted for by the changes in the real rate due to the GE effects of ν on inflation and unemployment.

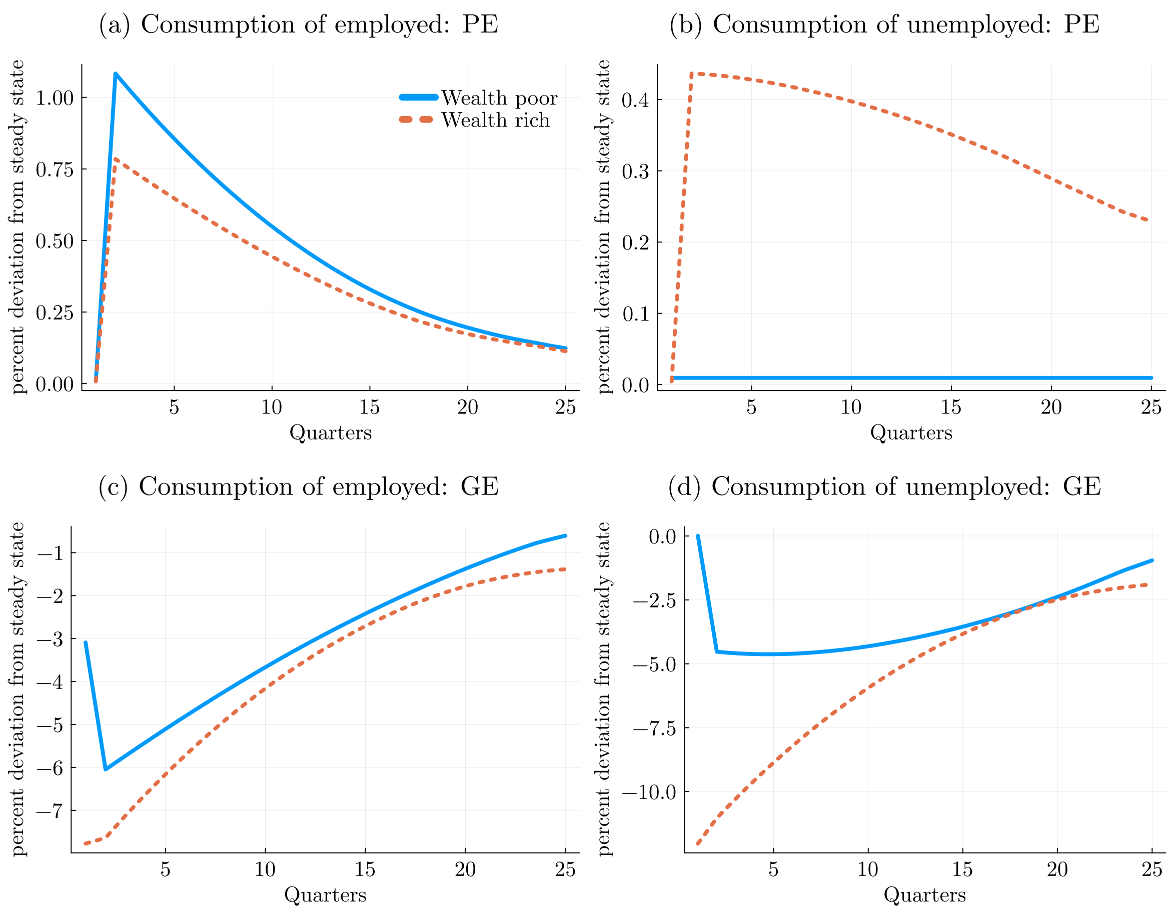
5.4 Role of wealth heterogeneity

In this section, we explore the role of incomplete markets in our quantitative results regarding the impact of OJS efficiency shocks on inflation. To do so, we study how the consumption response to a positive ν shock differs across wealth holdings and employment states in partial (PE) and general equilibrium (GE) settings.

We compare the consumption response of two individuals with the same human capital h, and when employed, the same match-specific productivity x and piece-rate α (set at their respective middle grid points), but differ only in their wealth. The wealth-poor worker has no wealth $s = 0$, and the wealth-rich holds shares at the middle grid point for shares, which corresponds roughly to the median of the wealth distribution. To obtain the consumption responses in PE, we fix all equilibrium objects relevant for the budget constraint (e.g., prices, dividends, taxes, etc.) at their steady-state values and only allow for the individual consumption choice to respond to the higher OJS efficiency. GE consumption responses are simply the optimal consumption decision for individuals that expect all budget-relevant variables to also evolve endogenously in response to the ν shock.

In PE, a positive (and persistent) OJS efficiency shock affects consumption decisions through several motives. First, employed individuals would like to consume more as they now anticipate

Figure 5: Impulse responses of consumption to OJS efficiency shock



Notes: This figure presents how the consumption response to a positive ν shock differs across wealth and employment status in partial (PE) and general equilibrium (GE). We compare the consumption response of two individuals who have the same the human capital h, and when employed, the same match-specific productivity x and the piece-rate α (set at their respective middle grid points) but differ only in their wealth. The wealth-poor individual has no wealth $s = 0$, and the wealth-rich holds shares at the middle grid point for shares, which corresponds to median of the wealth distribution.

higher future wage growth due to more frequent arrival of external offers. Second, more outside offers while employed reduces the wage scar of unemployment by making it easier for workers to climb up the job ladder coming out of unemployment. This decline in the cost of unemployment reduces the precautionary savings motive, which declines with wealth in a large class of incomplete markets models. Panel (a) of Figure 5 shows that, consistent with this logic, among the employed, consumption increases more for the wealth-poor individual. Turning to unemployed individuals in panel (b), it is the consumption of the *wealth-rich* that increases by more. Wealth-poor unemployed individuals cannot raise their consumption by much, as doing so requires borrowing against expected future income and the borrowing constraint precludes this possibility. This heterogeneity is different from that found in standard HANK models, where poorer individuals tend to be more responsive as they have a higher MPC. Because the direct effect of the shock in PE is largely on *future* income, increasing current consumption requires individuals to have either some wealth or a decent level of labor income.

Consumption responses in GE are different. As panels (c) and (d) of Figure 5 show, consumption declines for all. Recall that a higher ν leads to lower labor market tightness and higher inflation which, because of the monetary policy response, results in a higher real rate. These changes reduce demand for everyone. What explains the larger decline for the wealth-rich individual? We rule out changes in taxes as a potential explanation given that both workers have the same wage or UI income, and therefore pay the same taxes. The decline in labor market tightness cannot explain the heterogeneous responses either. If that was the primary driver, consumption would have fallen by more for the wealth-poor individual due to the precautionary motive. Moreover, labor income is a smaller share of total income for the wealth-rich individual and changing labor market prospects would be expected to have a smaller effect on her budget. It is the decline in the value of financial wealth driven by lower dividends d and share prices p^s that drive the large consumption response of the wealth-rich (Figure B.4). Dividends fall because the cost of labor relative to the final good rises (p^l), resulting in lower per-period profits for intermediate firms. Lower profits combined with a higher real interest rate result in a lower share price. Because wealth-rich individuals finance some of their consumption by decumulating financial wealth, the unexpected 8% decline in financial wealth leads to a large spending cut for them. In contrast, the wealth-poor (who in this example have no financial wealth) finance consumption through labor income, and therefore dividends and share prices have little first-order impacts on their consumption.

What do these observations imply for our quantitative results? In a complete-markets model, the aggregate consumption response would look like that of the wealth-rich in our model. Consequently, such a model would *overstate* the decline in aggregate demand upon a positive ν shock, resulting in a larger GE effect on market tightness. Based on the economic forces uncovered by our decomposition in Section 5.3, this translates to a more negative $\theta(Y)$, a smaller increase in marginal costs p^l, and consequently a lower increase in inflation upon a positive ν shock. Analogously, we would attribute a smaller role to changes in the job ladder in explaining the missing inflation after the Great Recession in Section 5.2.

6 Monetary policy with labor market dynamics

Thus far, we have established that shifts in EE transitions relative to the unemployment rate have important effects on the relationship between unemployment and inflation as well as on other labor market outcomes. A natural question is whether explicitly accounting for such worker transitions matters for the conduct of monetary policy.

6.1 Optimal monetary policy

In this section, we turn to a normative analysis and study the implications of ignoring job mobility dynamics when setting monetary policy. Specifically, we solve for the optimal monetary policy within a generalized Taylor rule—one where the nominal interest rate reacts to inflation and unemployment as well as the EE rate—under a dual-mandate central bank loss function. We then compare aggregate and worker level outcomes under the optimal monetary policy and the baseline policy which ignores EE dynamics and responds to inflation and unemployment only, which allows us to uncover the welfare consequences of considering job mobility dynamics in monetary policy. To understand our welfare results, we also analyze the implications of ignoring job mobility in monetary policy by comparing macroeconomic and distributional outcomes between the optimal and baseline policies. In these exercises, we assume that the economy is subject to the three aggregate shocks estimated in Section 3.2.

Central bank loss function. We start by positing that the central bank sets its monetary policy rule to minimize the following loss function

$$\mathcal{W} = \mathrm{var}(\pi_t - \pi^*) + \Psi \mathrm{var}(Y_t - Y^*), \tag{24}$$

which penalizes increases in the variance of quarterly inflation and output gaps. We choose the relative weight of the output gap in this loss function as $\Psi = 0.25$. This value is commonly used in literature (see, for example, Jensen 2002 and Walsh 2003). This choice of weight on the output gap can be also be interpreted as a central bank operating under a dual mandate.[25]

Central bank reaction function. We study monetary policy within a restricted class of monetary policy reaction functions. We consider monetary policy rules of the following form

$$i_t = i^* + \Phi_\pi (\pi_t - \pi^*) + \Phi_u (u_t - u^*) + \Phi_{EE} (EE_t - EE^*), \tag{25}$$

where Φ_{EE} governs the responsiveness of the central bank to the EE rate. This is a generalized version of the reaction function in Equation (17) that we use as our baseline, where we allow the central bank to explicitly condition on the EE rate while setting the nominal interest rate.

Evaluating alternative monetary policy rules. Naively, solving and simulating the model, and calculating the variances of inflation and unemployment to evaluate the objective function in Equation (24) under alternative Taylor-rule coefficients $\Phi_\pi, \Phi_u, \Phi_{EE}$ in Equation (25) requires the repeated computation of the entire sequence-space Jacobian system. As our model features

[25] According to Okun's law, changes in the output gap imply half the change in the unemployment gap. Using the relationship that $u_t - u_t^* = \frac{Y_t - Y_t^*}{2}$, the loss function in Equation (24) reduces to $\mathcal{W} = \mathrm{var}(\pi_t - \pi^*) + \mathrm{var}(u_t - u^*)$. This is also consistent with descriptions of how the Federal Reserve converts the unemployment gap into the output gap in Yellen (2012).

random search in the labor market, the worker distribution explicitly enters equilibrium conditions, as opposed to model blocks depending on one another only through aggregate variables.[26] This added layer of complication in our context renders the computation of derivatives costly. To overcome this challenge, we use the approach developed by McKay and Wolf (2022). The key insight is that firms and households do not care about the systematic component of monetary policy but what matters for their decisions is the time path of the interest rate in response to the structural shocks in the economy. One can then compute model IRFs under alternative Taylor rule parameters without having to recompute the system of Jacobians, but by only solving a linear system of equations in structural shocks as well as a series of monetary *policy* shocks and leveraging Jacobians computed once under the baseline parameterization. We delegate further implementation details to Appendix A.5.

Optimal policy and its macroeconomic implications. When we solve for the optimal coefficients of the generalized Taylor rule (25), we jointly optimize over Φ_u and Φ_{EE}. As our focus is on the response of monetary policy to the labor market, we keep the coefficient on inflation at its baseline value of $\Phi_\pi = 1.5$. We find that the optimal monetary policy prescribes $\Phi_u^* = -3.18$ and $\Phi_{EE}^* = 2.22$, implying that the optimal coefficient on the EE gap is large in magnitude, around 70 percent of the optimal coefficient on the unemployment gap. In this case, the central bank loss reduces by 78.7 percent relative to that under the calibrated (baseline) Taylor rule coefficients.

When we ignore job mobility dynamics in the Taylor rule, i.e., we set $\Phi_{EE} = 0$ in Equation (25), and optimize only over the coefficient on unemployment gap, we find that the optimal coefficient is $\Phi_u = -2.71$. However, this leads to a smaller reduction in central bank loss relative to the scenario where we jointly optimize over Φ_u and Φ_{EE}. Relative to $\Phi_u = -2.71$ and $\Phi_{EE} = 0$, the optimal monetary policy that also responds to job mobility dynamics $\Phi_u^* = -3.18$ and $\Phi_{EE}^* = 2.22$ reduces central bank loss by 12 percent. As a result, we conclude that explicitly accounting for worker transitions matters for the conduct of monetary policy.

Next, we compare volatilities of macroeconomic outcomes under the baseline and optimal monetary policies to understand how the optimal monetary policy changes magnitudes of fluctuations in aggregate variables. In doing so, for each variable ω, we report the standard deviation of $\omega - \overline{\omega}$, where $\overline{\omega}$ is the steady state value of ω. Because the optimal policy is obtained through minimizing the fluctuations in inflation and output gaps, magnitudes of fluctuations in inflation and output gaps are unsurprisingly smaller under the optimal policy, as shown by Table 5. As discussed above, the optimal policy feature coefficients on inflation and unemployment gaps that are significantly larger in magnitude than those under the baseline policy. As a result,

[26]Specifically and as summarized in Figure A.1, the free-entry condition—where firm entrants need to keep track of the cross sectional distribution of workers to form expectations—and the consistency condition for service firm profits require us to compute the sequence-space Jacobians of the worker distribution at the search and consumption stages, respectively.

Table 5: Volatility of macroeconomic outcomes under baseline and optimal monetary policies

	π	Y	r	θ	u	C	p^l	p^s
Baseline Taylor rule	0.0013	0.0059	0.0019	0.0600	0.0047	0.0059	0.0203	0.1975
Optimal Taylor rule	0.0011	0.0020	0.0033	0.0175	0.0013	0.0020	0.0081	0.3051

Notes: This table presents standard deviations of macroeconomic variables under the baseline and optimal monetary policies. In particular, for each variable ω, we report the standard deviation of $\omega - \overline{\omega}$, where $\overline{\omega}$ is the steady state value of ω.

more aggressive monetary policy response under the optimal policy leads to larger fluctuations in nominal and real rates as well as the labor market tightness. Stronger monetary policy response helps to achieve lower fluctuations in unemployment and consumption as well as the real marginal cost p^l, but at the same time, it leads to larger fluctuations in real price of shares p^s.

Heterogeneous welfare effects of the optimal policy Having obtained the loss-minimizing Taylor rule coefficients and evaluated their effects on magnitude of fluctuations in macroeconomic aggregates, we now study the magnitude and sources of (consumption-equivalent) welfare gains in aggregate and across heterogeneous groups. To compute for aggregate consumption-equivalent welfare, we solve for $\overline{\chi}$ as in Lucas (1987). Formally, we compute for $\overline{\chi}$ such that

$$\int E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u\left((1+\overline{\chi})\, \overline{c}_t\left(e,s,h,x,\alpha\right)\right) \lambda\left(e,s,h,x,\alpha\right) = \int E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u\left(\widetilde{c}_t\left(e,s,h,x,\alpha\right)\right) \lambda\left(e,s,h,x,\alpha\right),$$

where $\overline{c}_t\left(e,s,h,x,\alpha\right)$ and $\widetilde{c}_t\left(e,s,h,x,\alpha\right)$ denote the consumption of an individual with state (e,s,h,x,α) in date t under the baseline and optimal Taylor rules respectively, while λ denotes the steady state distribution of agents in an economy without aggregate shocks. Here, $\overline{\chi}$ is the percent additional lifetime consumption that must be endowed at all future dates and states to all agents under the stationary distribution of the economy where the baseline Taylor rule is implemented so that the average welfare will be equal to that of an economy populated with the same agents but where the optimal policy is implemented.

Given the functional form of the utility function u outlined in Section 3, $\overline{\chi}$ can be expressed as

$$\overline{\chi} = \left(\frac{\int \widetilde{V}^e\left(s,h,x,\alpha;p^*\right) \lambda\left(e,s,h,x,\alpha\right)}{\int \overline{V}^e\left(s,h,x,\alpha;\bar{p}\right) \lambda\left(e,s,h,x,\alpha\right)} \right)^{\frac{1}{1-\sigma}} - 1,$$

where $\overline{V}$ and $\widetilde{V}$ denote value functions under the baseline and optimal Taylor rules respectively.

Finally, in order to obtain group-specific measures of welfare, we divide the steady state distribution into groups of interest. Let group $o \in \mathcal{O}$ be a subset of individual states within the

Table 6: Heterogeneous welfare effects of optimal monetary policy

Match quality x			Share s			Human capital h			Employment e	
Bottom	Middle	Top	Bottom	Middle	Top	Bottom	Middle	Top	E	U
0.24	0.13	0.16	0.23	0.13	0.11	0.13	0.18	0.10	0.15	0.20

Notes: This table presents percent additional lifetime consumption gains from the optimal monetary policy relative to the baseline policy across different groups. We divide individuals based on their position in the distributions of match quality x, share s, and human capital h, as well as their employment status e (employed E and unemployed U). Bottom and top refer to bottom and top quintiles of respective distributions and middle refers to second, third, and fourth quintiles of these distributions.

set of all possible individual states $\mathcal{O}$. Then, group-specific welfare $\overline{\chi}_o$ is given by:

$$\int E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u\left(\left(1+\overline{\chi}_o\right) \overline{c}_t\left(e, s, h, x, \alpha\right)\right) \lambda^o\left(e, s, h, x, \alpha\right) = \int E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u\left(\widetilde{c}_t\left(e, s, h, x, \alpha\right)\right) \lambda^o\left(e, s, h, x, \alpha\right),$$

where λ^o represents the steady state distribution of agents, conditional on being in group o.

In the aggregate, we find that the optimal monetary policy yields 0.16 percent additional lifetime consumption relative to the baseline policy. Importantly, the optimal policy generates heterogeneous welfare gains across subpopulations, as shown by Table 6. When we group individuals based on their position in the match quality distribution, we find that those at the bottom quintile have welfare gains of 0.24 percent and those at the top quintile have welfare gains of 0.16 percent, while those who are in the middle (second, third, and fourth quintiles) have welfare gains of 0.13 percent. The optimal policy the most because the optimal policy yields smaller decline and faster recovery of the EE rate during economic fluctuations, relative to what would be experienced under the baseline policy. As a result, individuals in the bottom quintile gains from the optimal policy the most because their climb of the job ladder from low to high match qualities becomes much easier. On the other hand, individuals in the top quintile also experience substantial gains because their valuation of a smooth job ladder is high given that they have the most to lose in case they experience a job separation. In terms of heteregenous gains across the share distribution, because the optimal policy achieves lower fluctuations in unemployment, less volatile unemployment risk leads to larger welfare gains among wealth poor individuals. On the other hand, larger fluctuations in the price of shares caused by the more aggressive monetary policy response under the optimal policy is the main reason behind the smaller welfare gains of individuals at the top quintile of the share distribution. Finally, unemployed individuals experience larger welfare gains than employed individuals on average given that the former group benefits not only from a smoother job ladder when they are employed but also from labor markets that recovery faster from downturns.

7 Conclusions

In this paper, we build a heterogeneous-agent New Keynesian model featuring a frictional labor market with on-the-job search to quantitatively study the role of job mobility dynamics on inflation and monetary policy. Using this model, we quantitatively study the impact of the weakening correlation between the unemployment rate and EE rate during the recovery from the Great Recession on inflation dynamics. We compare two economies that have the same path of declining unemployment rates driven by positive demand shocks but different paths of EE rates: The first economy experiences an increase in the EE rate, while the second economy observes a flat EE rate caused by additional negative OJS efficiency shocks, mimicking the period between 2016 and 2019 in the U.S. A comparison of inflation dynamics between the two economies reveals that inflation would have been around 0.25 percentage points higher between 2016 and 2019 if the EE rate increased commensurately with the decline in unemployment.

We show that while the direct effect of an increase in OJS efficiency on match value leads to a significant increase in the real marginal cost pushing up inflation, this effect is partially mitigated by the equilibrium decline of market tightness through changes in aggregate demand and labor productivity distribution. Overall, these counteracting labor market effects explain 97 percent of the total increase in the real marginal cost upon impact and the remaining 3 percent is accounted for by changes in the real interest rate due to the GE effects of OJS efficiency on inflation and unemployment. Finally, on the normative side, we study optimal monetary policy within a class of Taylor rules. We consider an augmented Taylor rule that responds to both deviations of unemployment and EE rates from their steady state values as well as the inflation gap. We find that when we jointly optimize over the coefficients on unemployment and EE gaps, the optimal coefficient on the EE gap is positive and large in magnitude, around 70 percent of the optimal coefficient on the unemployment gap. This policy reduces the welfare loss due to the deviations of unemployment and output from their targets substantially.

Our model features a rich set of fiscal policy instruments such as a consumption tax, progressive labor income tax, unemployment and retirement benefits, and government debt. Therefore, it provides a framework to quantitatively study fiscal and monetary policy interactions, accounting for rich labor market dynamics. In addition, it is straightforward to introduce other exogenous shocks (such as shocks to monetary policy, markups, and other labor market parameters) into our model. Given our solution method, it is feasible to estimate a rich set of exogenous shocks jointly to evaluate the model's performance in matching time-series and cross-sectional empirical moments. We leave these considerations for future research.

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Online Appendix

A Model

A.1 Solving the intermediate firm's problem

The problem of the intermediate firm can be solved analytically. The solution is used to obtain an expression for profits in steady state—used to calculate dividends—and also to derive the New Keynesian Phillips curve. The pricing problem of an intermediate firm j whose last period *relative* price is $p_{t-1}(j)$ is given by

$$\Theta\left(p_{t-1}\left(j\right)\right)=\max_{p_t(j)} p_t\left(j\right)y_t\left(p_t\left(j\right)\right)-p_t^l\frac{y_t\left(p_t\left(j\right)\right)}{z_t}-\frac{\eta}{2\vartheta}\log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right)-\pi^*\right)^2Y_t+\frac{1}{1+r_{t+1}}\Theta\left(p_t\left(j\right)\right).$$

Substituting in the demand for each variety, $y_t(j)=p_t(j)^{-\eta}Y_t$, the problem can be written as

$$\Theta\left(p_{t-1}\left(j\right)\right)=\max_{p_t(j)} p_t\left(j\right)^{1-\eta}Y_t-p_t^lp_t\left(j\right)^{-\eta}\frac{Y_t}{z_t}-\frac{\eta}{2\vartheta}\log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right)-\pi^*\right)^2Y_t+\frac{1}{1+r_{t+1}}\Theta\left(p_t\left(j\right)\right).$$

The first-order condition with respect to relative price $p_t(j)$ is given by

$$\begin{aligned}0=&\left(1-\eta\right)p_t\left(j\right)^{-\eta}Y_t+\eta p_t^lp_t\left(j\right)^{-\eta-1}\frac{Y_t}{z_t}\\&-\frac{\eta}{\vartheta}\log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right)-\pi^*\right)\frac{1}{\frac{p_t(j)}{p_{t-1}(j)}\left(1+\pi_t\right)-\pi^*}\frac{1+\pi_t}{p_{t-1}\left(j\right)}Y_t+\frac{1}{1+r_{t+1}}\Theta'\left(p_t\left(j\right)\right),\end{aligned}$$

and the envelope condition is

$$\Theta'\left(p_{t-1}\left(j\right)\right)=\frac{\eta}{\vartheta}\log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right)-\pi^*\right)\frac{1}{\frac{p_t(j)}{p_{t-1}(j)}\left(1+\pi_t\right)-\pi^*}\frac{p_t\left(j\right)\left(1+\pi_t\right)}{p_{t-1}\left(j\right)^2}Y_t.$$

Iterating the envelope condition forward by one period yields

$$\Theta'\left(p_t\left(j\right)\right)=\frac{\eta}{\vartheta}\log\left(\frac{p_{t+1}\left(j\right)}{p_t\left(j\right)}\left(1+\pi_{t+1}\right)-\pi^*\right)\frac{1}{\frac{p_{t+1}(j)}{p_t(j)}\left(1+\pi_{t+1}\right)-\pi^*}\frac{p_{t+1}\left(j\right)\left(1+\pi_{t+1}\right)}{p_t\left(j\right)^2}Y_{t+1}.$$

Consolidating the envelope and the first-order conditions, we obtain:

$$\begin{aligned}0=&\left(1-\eta\right)p_t\left(j\right)^{-\eta}Y_t+\eta p_t^lp_t\left(j\right)^{-\eta-1}\frac{Y_t}{z_t}-\frac{\eta}{\vartheta}\log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right)-\pi^*\right)\frac{1}{\frac{p_t(j)}{p_{t-1}(j)}\left(1+\pi_t\right)-\pi^*}\frac{1+\pi_t}{p_{t-1}\left(j\right)}Y_t\\&+\frac{1}{1+r_{t+1}}\underbrace{\frac{\eta}{\vartheta}\log\left(\frac{p_{t+1}\left(j\right)}{p_t\left(j\right)}\left(1+\pi_{t+1}\right)-\pi^*\right)\frac{1}{\frac{p_{t+1}(j)}{p_t(j)}\left(1+\pi_{t+1}\right)-\pi^*}\frac{p_{t+1}\left(j\right)\left(1+\pi_{t+1}\right)}{p_t\left(j\right)^2}Y_{t+1}}_{\Theta'(p_t(j))}.\end{aligned}$$

All firms set the same price due to symmetry, $p_t(j) = 1 \; \forall \, t, \; j$, and the equation simplifies to

$$0 = (1-\eta) Y_t + \eta p_t^l \frac{Y_t}{z_t} - \frac{\eta}{\vartheta} \frac{\log(1+\pi_t-\pi^*)(1+\pi_t)}{1+\pi_t-\pi^*} Y_t + \frac{1}{1+r_{t+1}} \frac{\eta}{\vartheta} \frac{\log(1+\pi_{t+1}-\pi^*)(1+\pi_{t+1})}{1+\pi_{t+1}-\pi^*} Y_{t+1}.$$

Rearranging terms and using the definition of π_t, we obtain the Phillips curve in Equation (9):

$$\frac{\log(1+\pi_t-\pi^*)(1+\pi_t)}{1+\pi_t-\pi^*} = \vartheta \left(\frac{p_t^l}{z_t} - \frac{\eta-1}{\eta} \right) + \frac{1}{1+r_{t+1}} \frac{\log(1+\pi_{t+1}-\pi^*)(1+\pi_{t+1})}{1+\pi_{t+1}-\pi^*} \frac{Y_{t+1}}{Y_t}.$$

A.2 Solving for a steady state equilibrium

A.2.1 Laws of motion

In this section, we present the laws of motion that characterize the worker distribution measured at the consumption stage within a period. We denote by λ_t the distribution of agents across individual states (i.e., share holdings s, human capital h, match productivity x, and piece rate α) at time t. As the population is normalized to one and the dead are replenished with unemployed workers, we have

$$\sum_{s,h,x,\alpha} \lambda_t^E(s,h,x,\alpha) + \sum_{s,h} \lambda_t^U(s,h) + \sum_{s} \lambda_t^R(s) = 1,$$

where $\lambda_t^E(\cdot)$, $\lambda_t^U(\cdot)$ and $\lambda_t^R(\cdot)$ denote the mass of employed, unemployed and retired workers by individual state variables, respectively, and we omit states which are not relevant for the agents. Also for reference below, let $\boldsymbol{S}_t^E(s';h,x,\alpha) = \left\{ s \in \mathbf{S} : \; g_t^{Es}(s,h,x,\alpha) = s' \right\}$, $\boldsymbol{S}_t^U(s';h) = \left\{ s \in \mathbf{S} : \; g_t^{Us}(s,h) = s' \right\}$, and $\boldsymbol{S}_t^R(s') = \left\{ s \in \mathbf{S} : \; g_t^{Rs}(s) = s' \right\}$ denote the set of period t share holdings s that map into a given level of share holdings s' in $t+1$ by employment status.

We now turn to explicitly writing down the system of equations that determine worker flows. To reduce notational clutter, we define $f_{t+1} = f(\theta_{t+1})$ and suppress some of the function arguments.

Flows into employment. Conditional on not retiring, flows into employment include the following mutually exclusive events.

- Employed worker stays with the same employer, skill appreciates or skill does not appreciate.
    - The worker's piece rate can either (i) remain the same ($\alpha' = \alpha$) either because no meeting occurs or an offer is not met with a counteroffer or (ii) rise to due rebargaining induced by an external offer. When considering inflows into specific match productivity x' and piece rate α', it must be that the poaching firm's match productivity is $\tilde{x} = x'\alpha'$ in the latter case. Further, it must be that the poaching firm's match productivity is higher than the current output share: $x\alpha < x'\alpha' = \tilde{x}$.

- Employed worker accepts new offer, skill appreciates or skill does not appreciate.
  - The worker's piece rate changes (declines) due to a job-to-job transition. When considering inflows into specific match productivity x' and piece rate α', it must be that $\alpha' = \frac{x}{x'}$, where x is the productivity of the previous match. This implies that previous match productivity must have been $x = \alpha' x'$.
- Employed worker loses job but finds new one, skill appreciates or skill does not appreciate.
  - When considering inflows into specific match productivity x' and piece rate α' from unemployment, it must be that $\alpha' = \frac{x}{x'}$. Here, it does not matter what the previous job's x or α was.
- Unemployed worker accepts new offer, skill depreciates or skill does not depreciate.
  - The evolution of piece rate is similar to above.

We then have the following law of motion for the distribution of employed workers:

$$
\begin{aligned}
&\lambda_{t+1}^{E}\left(s',h',x',\alpha'\right)=\left(1-\psi^{R}\right)\times\\
&\Bigg(\sum_{s\in\boldsymbol{S}_t^E}\underbrace{\lambda_t^E\left(s,h'-1,x',\alpha'\right)}_{\substack{\text{no outside offer/discard offer;}\\ \alpha\text{ remains the same}}}\pi^E\left(1-\delta\right)\left[\left(1-\nu f_{t+1}\right)+\nu f_{t+1}\underbrace{\sum_{\tilde{x}<x'\alpha'}}_{\text{discard offers}}\Gamma^x\left(\tilde{x}\right)\left(1-g_{t+1}^{Ea}\left(\cdot\right)\right)\right]\\
&+\underbrace{\sum_{\alpha}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h'-1,x',\alpha\right)}_{\text{received offer from firm with productivity }\alpha'x'}\pi^E\left(1-\delta\right)\left[\left(1-\nu f_{t+1}\right)+\nu f_{t+1}\Gamma^x\left(x'\alpha'\right)\underbrace{\mathbf{1}_{x'\alpha'>x'\alpha}}_{\text{rebargain}}\left(1-g_{t+1}^{Ea}\left(\cdot\right)\right)\right]\\
&+\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h',x',\alpha'\right)\left(1-\pi^E\right)\left(1-\delta\right)\left[\left(1-\nu f_{t+1}\right)+\nu f_{t+1}\sum_{\tilde{x}<x'\alpha'}\Gamma^x\left(\tilde{x}\right)\left(1-g_{t+1}^{Ea}\left(\cdot\right)\right)\right]\\
&+\sum_{\alpha}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h',x',\alpha\right)\left(1-\pi^E\right)\left(1-\delta\right)\left[\left(1-\nu f_{t+1}\right)+\nu f_{t+1}\Gamma^x\left(x'\alpha'\right)\mathbf{1}_{x'\alpha'>x'\alpha}\left(1-g_{t+1}^{Ea}\left(\cdot\right)\right)\right]\\
&+\sum_{\alpha}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h'-1,\underbrace{\alpha'x'}_{x},\alpha\right)\pi^E\left[\left(1-\delta\right)\nu g_{t+1}^{Ea}\left(\cdot\right)\right]f_{t+1}\Gamma^x\left(x'\right)\\
&+\sum_{\alpha}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h',x',\alpha\right)\left(1-\pi^E\right)\left[\left(1-\delta\right)\nu g_{t+1}^{Ea}\left(\cdot\right)\right]f_{t+1}\Gamma^x\left(x'\right)\\
&+\sum_{\alpha}\sum_{x}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h'-1,x,\alpha\right)\pi^E\delta_{t+1}^{Ua}f_{t+1}\Gamma^x\left(x'\right)\underbrace{\mathbf{1}_{\alpha'=\frac{x}{x'}}}_{\alpha'\text{ must be }\frac{x}{x'}}\\
&+\sum_{\alpha}\sum_{x}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h',x,\alpha\right)\left(1-\pi^E\right)\delta_{t+1}^{Ua}f_{t+1}\Gamma^x\left(x'\right)\mathbf{1}_{\alpha'=\frac{x}{x'}}\\
&+\sum_{s\in\boldsymbol{S}_t^U}\lambda_t^U\left(s,h'+1\right)\pi^U\zeta f\left(\theta_{t+1}\right)\Gamma^x\left(x'\right)\mathbf{1}_{\alpha'=\frac{x}{x'}}g_{t+1}^{Ua}\left(\cdot\right)\\
&+\sum_{s\in\boldsymbol{S}_t^U}\lambda_t^U\left(s,h'\right)\left(1-\pi^U\right)\zeta f\left(\theta_{t+1}\right)\Gamma^x\left(x'\right)\mathbf{1}_{\alpha'=\frac{x}{x'}}g_{t+1}^{Ua}\left(\cdot\right)\Bigg).
\end{aligned}
\tag{A.1}
$$

Flows into unemployment. Conditional on not retiring, flows into unemployment include the following transitions.

- Employed worker loses job and does not find job or refuses offer, skill appreciates
- Employed worker loses job and does not find job or refuses offer, skill does not appreciate
- Unemployed worker does not find job or refuses offer, skill depreciates
- Unemployed worker does not find job or refuses offer, skill does not depreciate

- Dead retiree is reborn, inherits shares but draws new human capital

Hence, we have the following law of motion for the distribution of unemployed workers

$$
\begin{aligned}
&\lambda_{t+1}^{U}(s', h') = (1-\psi^R)\times \\
&\Bigg(\sum_{\alpha}\sum_{x}\sum_{s\in \boldsymbol{S}_t^E} \lambda_t^E(s, h'-1, x, \alpha)\pi^E\delta \left[1-\zeta f_{t+1} + \zeta f_{t+1}\sum_{\tilde{x}}\Gamma^x(\tilde{x})\left(1-g_{t+1}^{Ua}(\cdot)\right)\right] \\
&+\sum_{\alpha}\sum_{x}\sum_{s\in \boldsymbol{S}_t^E} \lambda_t^E(s, h', x, \alpha)(1-\pi^E)\delta \left[1-\zeta f_{t+1} + \zeta f_{t+1}\sum_{\tilde{x}}\Gamma^x(\tilde{x})\left(1-g_{t+1}^{Ua}(\cdot)\right)\right] \\
&+\pi^U \left[1-\zeta f_{t+1} + \zeta f_{t+1}\sum_{\tilde{x}\in\tilde{X}}\Gamma^x(\tilde{x})\left(1-g_{t+1}^{Ua}(\cdot)\right)\right]\sum_{s\in \boldsymbol{S}_t^U}\lambda_t^U(s, h'+1) \\
&+(1-\pi^U)\left[1-\zeta f(\theta_{t+1}) + \zeta f(\theta_{t+1})\sum_{\tilde{x}}\Gamma^x(\tilde{x})\left(1-g_{t+1}^{Ua}(\cdot)\right)\right]\sum_{s\in \boldsymbol{S}_t^U}\lambda_t^U(s, h')\Bigg) \\
&+\psi^D\Gamma^h(h')\sum_{s\in \boldsymbol{S}_t^R}\lambda_t^R(s).
\end{aligned} \tag{A.2}
$$

Flows into retirement. Flows into retirement include the following set of transitions.

- Employed worker retires
- Unemployed worker retires
- Retired worker does not die

These inflows imply, we have the following law of motion for the distribution of retirees:

$$
\lambda_{t+1}^R(s') = \psi^R \sum_{s\in \boldsymbol{S}_t^E, h, x, \alpha} \lambda_t^E(s, h, x, \alpha) + \psi^R \sum_{s\in \boldsymbol{S}_t^U, h} \lambda_t^U(s, h) + \left(1-\psi^D\right)\sum_{s\in \boldsymbol{S}_t^R}\lambda_t^R(s). \tag{A.3}
$$

A.2.2 Casting the model in relative prices and real variables

Nominal frictions are not relevant in the steady state, where prices rise by the rate of long-run inflation π^* and hence firms do not incur an adjustment cost while increasing their prices by that amount. For the same reason, the price level is indeterminate. Therefore, we solve for relative prices (relative to the price of output) and allocations. We start by deriving the equations governing these relative prices, real dividends, and real profits of intermediate firms in steady state.

- Evaluating the NKPC at the steady state, we obtain the real marginal cost $mc = p^l/z$:

$$mc = \frac{\eta - 1}{\eta}.$$

- The price of labor services is then given by

$$p^l = mc \times z = \frac{\eta - 1}{\eta} z. \tag{A.4}$$

- Per-period real profits of the intermediate firms are given by

$$\Gamma^I = (1 - mc)Y = \frac{Y}{\eta}. \tag{A.5}$$

- Real dividends in the steady state are given by

$$\begin{aligned} d &= x_B Y - \frac{x_B Y(1+\pi^*)}{(1+i)} + \Gamma^I + \Gamma^S \\ &= x_B Y \frac{r}{1+r} + \Gamma^I + \Gamma^S. \end{aligned} \tag{A.6}$$

- Dividing the no arbitrage condition by aggregate price level P, we solve for share price

$$\begin{aligned} \frac{(p^s + d)(1+\pi^*)}{p^s} &= 1 + i \\ (1+r)p^s &= p^s + d \\ p^s &= \frac{d}{r}. \end{aligned} \tag{A.7}$$

- Finally, we rewrite the government budget constraint in real terms as follows. Let $b_t = B_t/P_{t+1}$. Then, dividing both sides by P_t, multiplying the first term on the right hand side by $\frac{P_{t+1}}{P_{t+1}}$, and recognizing that $1 + i_t = (1 + r_{t+1})(1 + \pi_{t+1})$, we get

$$\begin{aligned} b_{t-1} + g_t + \int UI(h)\, d\lambda_t^U(s,h) + \int \phi^R d\lambda_t^R(s) = \frac{b_t}{1+r_{t+1}} &+ \tau_c \int c(e,s,h,x,\alpha)\, d\lambda_t(e,s,h,x,\alpha) \\ &+ \int \left(UI(h) - \tau_t (UI(h))^{1-\Upsilon}\right) d\lambda_t^U(s,h) \\ &+ \int \left(w(h,x,\alpha) - \tau_t w(h,x,\alpha)^{1-\Upsilon}\right) d\lambda_t^E(s,h,x,\alpha) \\ &+ \int \left(\phi^R - \tau_t \left(\phi^R\right)^{1-\Upsilon}\right) \lambda_t^R(s). \end{aligned}$$

Here, the lower case variables b_{t-1} and g_t represent the real values of government debt and government spending, respectively. It is useful to define the real net revenue of government (tax proceeds minus outlays for pensions and unemployment insurance), $\mathcal{R}_t$, as

$$\begin{aligned}\mathcal{R}_t = &-\int UI\left(h\right)d\lambda_t^U\left(s,h\right) - \int \phi^R \lambda_t^R\left(s\right) \\ &+ \tau_c \int c\left(e,s,h,x,\alpha\right)d\lambda_t\left(e,s,h,x,\alpha\right) + \int \left(UI\left(h\right) - \tau\left(UI\left(h\right)\right)^{1-\Upsilon}\right)d\lambda_t^U\left(s,h\right) \\ &+ \int \left(w(h,x,\alpha) - \tau w(h,x,\alpha)^{1-\Upsilon}\right)d\lambda_t^E\left(s,h,x,\alpha\right) + \int \left(\phi^R - \tau\left(\phi^R\right)^{1-\Upsilon}\right)\lambda_t^R\left(s\right).\end{aligned} \tag{A.8}$$

With these definitions, the government budget can be expressed in real terms as

$$\begin{aligned} b_{t-1} + g_t &= \frac{b_t}{1+r_{t+1}} + \mathcal{R}_t \\ \Rightarrow 0 &= (1+r_{t+1})(b_{t-1} + g_t - \mathcal{R}_t) - b_t. \end{aligned} \tag{A.9}$$

A.2.3 Solution algorithm for the steady state equilibrium

We solve for the steady state using the following algorithm by bisecting over a nominal interest rate i that clears the share market given by Equation (20).

1. For a given nominal interest rate i, given π^*, obtain r from the Fisher equation (18).

2. Outer loop: Guess a tax parameter τ, level of output Y, and service firm profits Γ^S.

   - Calculate the real bond holdings $b = x_B Y$, real government expenditures $g = x_G Y$, and real intermediate firm profits Γ^I using Equation (A.5).
   - Calculate real dividends d using Equation (A.6).
   - Calculate real share price p^s using Equation (A.7).

3. Inner loop: Guess a market tightness θ.

   - Calculate worker contact rate $f(\theta)$.
   - Solve the workers' problems given by Equations (1), (2), (3).
   - Compute the stationary worker distributions over state variables μ^E, μ^U, μ^R, λ, λ^E, λ^U, and λ^R.
   - Solve the matched firm problem in the labor services sector given by Equation (10).
   - Given the solution to the firm problem and worker distributions, calculate the implied market tightness $\widetilde{\theta}$ consistent with the free-entry condition $V = 0$, where V satisfies Equation (11).

- Iterate over the inner loop until $\widetilde{\theta}$ agrees with the guessed market tightness θ.

4. Using the worker distributions, calculate the implied output $\widetilde{Y}$ using market clearing for labor services in Equation (19) and real service firm profits $\widetilde{\Gamma}^S$ in Equation (15).

5. Calculate the implied tax parameter $\widetilde{\tau}$ that clears the government budget constraint, which can be obtained from Equations (A.8) and (A.9) as:

$$\widetilde{\tau} = \frac{-\frac{r}{1+r}x_B Y - x_g Y + \tau_c \int c d\lambda + \int w d\lambda^E + \int UI d\lambda^U + \int \phi^R d\lambda^R}{\int w^{1-\Upsilon} d\lambda^E + \int UI^{1-\Upsilon} d\lambda^U + \int \left(\phi^R\right)^{1-\Upsilon} d\lambda^R}.$$

6. Iterate over the outer loop until $\widetilde{\tau}$, $\widetilde{Y}$, and $\widetilde{\Gamma}^S$ agree with guesses for τ, Y, and Γ^S.

A.3 Solving for the transition path using DAGs

In Section 4, we discuss how we employ and expand the sequence-space Jacobian method detailed in Auclert, Bardóczy, Rognlie, and Straub (2021) to solve for the transitional dynamics in our model. In the following discussion, we provide additional details on this procedure.

To solve the model using sequence-space Jacobians, we first cast the model as a DAG, depicted in Figure A.1.[27] The leftmost red node contains exogenous variables which represent shocks the economy might be subject to as well as endogenous variables (unknowns) whose dynamics we are interested in. The intermediate (green) nodes represent various model components and importantly, demonstrate how each component relates with one another via their respective input and output variables. The intermediate nodes can be categorized into simple blocks and the heterogeneous agent block. An example of the former would be model components that relate various aggregate variables such as the fiscal policy rule (Equation 21), the Taylor rule (Equation 17) or the expression for dividends and no-arbitrage that relate to the mutual fund (Equations 12 and 13). The latter is the most complex model component wherein heterogeneous agents solve for decision rules that govern their consumption-saving choices and labor market outcomes, which play an important role in the dynamics of aggregates and distributions in the economy. Finally, the rightmost red node represents the target sequences that must equal zero in equilibrium (market clearing and consistency conditions).[28] This final node might take inputs directly from the initial node with exogenous and endogenous variables, as well as outputs from the intermediate nodes.

[27] For visual clarity, we consolidate the terminal "target" blocks that capture various equilibrium conditions into a single node. One should think of the last node as consisting of eight different ones representing each of the equilibrium conditions separately with inputs from the relevant intermediate blocks.

[28] Note that the number of unknown variables specified in the leftmost node must be equal to the number of target conditions in the rightmost node.

Formally, let $\upsilon = \left(\{\pi_t, Y_t, p^l_t, b_t, u_t, \theta_t, \Gamma^S_t, e2e_t\}_{t=0}^{T-1}\right)$ represent the path of unknown endogenous variables and $\Theta = \left(\{z, \beta, \nu\}_{t=0}^{T-1}\right)$ represent the path of exogenous variables.[29] The system of equations, labeled as "targets" in the rightmost node, that govern the transition path is:[30]

$$H(\upsilon;\Theta) = \begin{pmatrix} \frac{\log(1+\pi_t-\pi^*)(1+\pi_t)}{1+\pi_t-\pi^*} - \vartheta\left(\frac{p^l_t}{z_t} - \frac{\eta-1}{\eta}\right) - \frac{1}{1+r_{t+1}}\frac{\log(1+\pi_{t+1}-\pi^*)(1+\pi_{t+1})}{1+\pi_{t+1}-\pi^*}\frac{Y_{t+1}}{Y_t} \\ \mathcal{L}_t - L_t \\ \mathcal{S}_t - 1 \\ (1+r_{t+1})(b_{t-1}+g_t-\mathcal{R}_t) - b_t \\ \mathcal{U}_t - u_t \\ \theta_t - q^{-1}\left(\kappa/\mathbb{E}J_t\right) \\ \mathit{\Gamma}^S_t - \Gamma^S_t \\ \mathcal{E}2\mathcal{E}_t - e2e_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{A.10}$$

The main purpose of setting up the model as a DAG is for us to be able to systematically solve for Jacobians that summarize the partial equilibrium responses of each node's output (including targets in the rightmost node) with respect to each direct input to that node. We are then able to forward accumulate—that is apply the chain rule in a systematic fashion—the partial Jacobians along the DAG to obtain the total Jacobians of any output (again including targets) with respect to changes in any exogenous variable or unknown endogenous variable. Simply put, a total Jacobian combines the direct and indirect responses of an output with respect to an input. For example, the response of the value of posting a vacancy $\mathbb{E}J$ (service firm block output) is affected directly by the real rate r through discounting in the firm's match value but also indirectly through how the real rate affects share prices and dividends, which ultimately affect household decisions and thus the distribution of workers that vacancies contact with.

Having obtained the total Jacobians of targets $H(\upsilon;\Theta)$ with respect endogenous unknowns υ and with respect to exogenous variables Θ, we can apply the implicit function theorem to compute the response of any endogenous unknown $d\upsilon$ to a change in the exogenous variables $d\Theta$. Formally, let $H_\upsilon = \partial H/\partial\upsilon$ and $H_\Theta = \partial H/\partial\Theta$ be the total Jacobians of targets with respect to endogenous unknowns and exogenous variables, then, the impulse responses of unknowns is given by:

$$d\upsilon = \underbrace{-H_\upsilon^{-1}H_\Theta}_{G_\upsilon} d\Theta,$$

[29]Namely, the endogenous variables in the model are inflation, real output, real price of labor services, real debt, unemployment rate, labor market tightness, real profits of the labor-service sector and the mass of employer-to-employer transitions. The exogenous variables are the total factor productivity in intermediate goods production, the discount factor and the on-the-job search efficiency.

[30]These equations in order capture the New-Keynesian Phillips curve, market clearing for labor services, market clearing for mutual fund shares, government budget balance, consistency of the unemployment rate, the free-entry condition, consistency of labor-service profits and consistency of employer-to-employer transitions.

where G_v denotes the GE Jacobians of the endogenous variables.

Equipped with the partial Jacobians of the intermediate variables and GE Jacobians of the unknown variables, we compute the GE Jacobians of the intermediate variables too, which allow us to compute their IRFs with respect to exogenous variables as well. Firms and households do not care about the systematic component of monetary policy, but they only care about the current and future path of interest rates.

Finally, we use the equivalence of the impulse response function with the moving-average process representation of a time series. This allows us to flexibly simulate a time-path of aggregate variables and—given the path of these aggregate variables and policy responses in response to aggregate shocks—also simulate a large panel of workers. We in turn use this simulated micro worker panel to study a wide range of cross-sectional outcomes and to evaluate the welfare effects of monetary policy.

A.4 Decomposing inflation effects of OJS shocks using the DAG

To clarify how we operationalize the DAG and its associated input-output structure for this exercise, some discussion is warranted. We start from the total Jacobians of each block's outputs with respect to their inputs already computed when solving the model (Section 4). We then use the implicit function theorem (IFT) for each block to compute the derivative of the output of interest with respect to all the endogenous and exogenous model variables listed in the initial node in the DAG. In Section 5.3, where we use the free-entry condition for decomposing the changes in p^l, we obtain the derivative of p^l with respect to all model variables by applying the IFT to this equilibrium condition. We then multiply the total derivative of p^l with respect to these variables with the general equilibrium IRFs of these variables with respect to ν (also already computed while solving the model). As a result, we obtain the response of each component that makes up p^l with respect to the shock of interest ν. Specifically, we end up with the effect of ν on p^l through the shock's direct effect as well as its effect through equilibrium objects such as market tightness θ, inflation π, and unemployment u. As discussed below, we find that the effect of ν on p^l through changes in θ partially mitigates the direct effect of ν on p^l. To better understand this mitigating force, we follow the same steps to above to further decompose θ to its subcomponents based on the labor market clearing condition (the second equilibrium condition in the DAG), which yields a break down of the response of p^l through changes in θ into the effect of ν on θ through on supply of labor services $\mathcal{L}$ and the effect of ν on θ through output Y.

A.5 Evaluating alternative monetary policy rule coefficients

As discussed in Section 6.1, in our context with random search, the repeated computation of the sequence-space Jacobians for each policy rule parameter combination is costly due to the worker distributions entering equilibrium conditions directly. To this end, we follow the approach described in McKay and Wolf (2022) and use the Jacobians computed under the baseline parameterization of the monetary policy rule to evaluate the model under alternative coefficients.

The key idea is to utilize *policy* shocks to monetary policy to compute IRFs to *non-policy shocks* under alternative Taylor rule coefficients, using *only* the Jacobian system computed once under the baseline monetary policy rule. The reason this approach works is that firms and households do not care about the systematic component of monetary policy—i.e., how aggressively does the central bank react to inflation, unemployment and the EE rate separately—but they only care about the current and future path of interest rates.

Specifically, given the sequence-space truncation horizon T and alternative monetary policy coefficients $\widetilde{\Phi}_\pi, \widetilde{\Phi}_u, \widetilde{\Phi}_{EE}$, we solve the $T \times T$ linear system of equations for the path of policy news shocks $\boldsymbol{\mu} = \{\mu_t\}_{t=1}^T$ below:

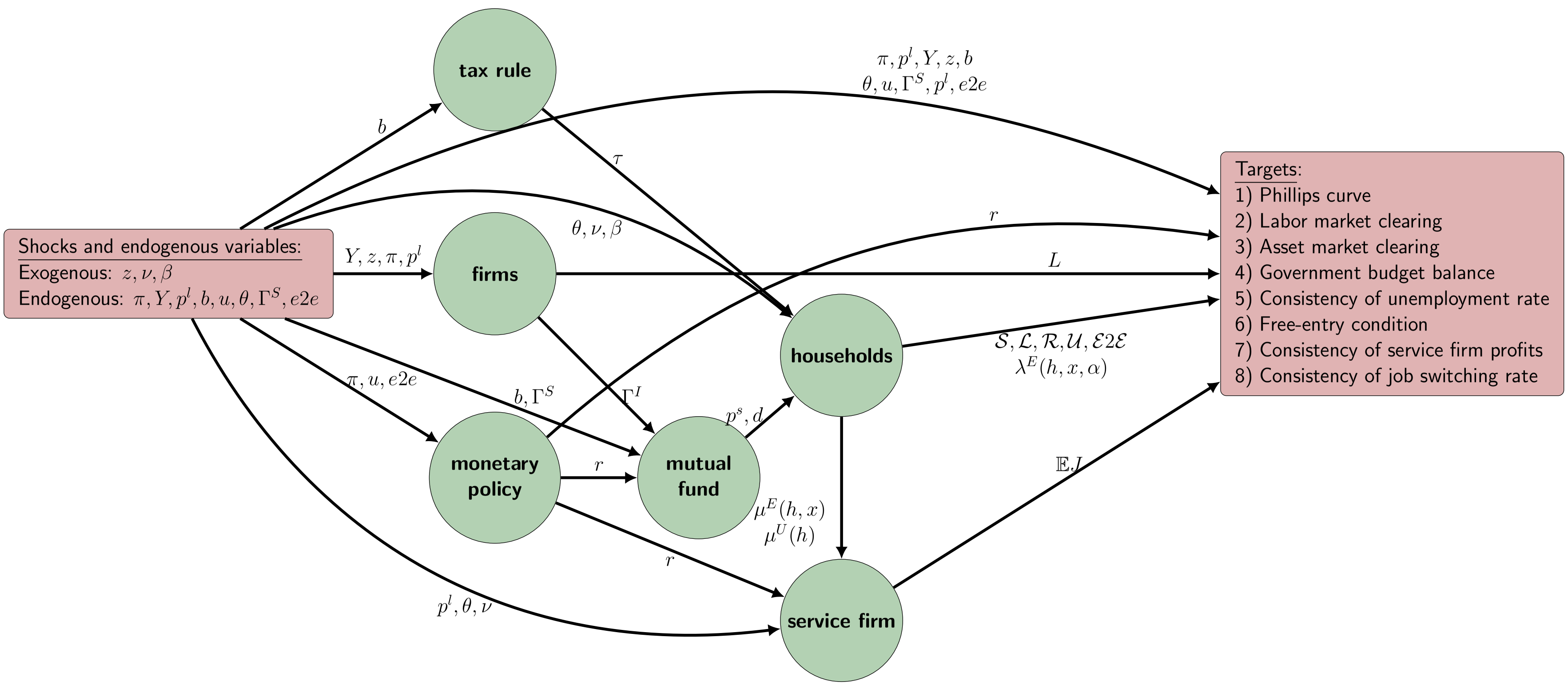
$$\begin{aligned}\underbrace{\boldsymbol{i}_{\Phi_\pi,\Phi_u}(\varepsilon) + \boldsymbol{\Theta}^{i,\mu}_{\Phi_\pi,\Phi_u}\boldsymbol{\mu}}_{\text{IRF of } i \text{ under baseline}} \;=\; & \widetilde{\Phi}_\pi \underbrace{\left(\boldsymbol{\pi}_{\Phi_\pi,\Phi_u}(\varepsilon) + \boldsymbol{\Theta}^{\pi,\mu}_{\Phi_\pi,\Phi_u}\boldsymbol{\mu}\right)}_{\text{IRF of } \pi \text{ under baseline}} + \widetilde{\Phi}_u \underbrace{\left(\boldsymbol{u}_{\Phi_\pi,\Phi_u}(\varepsilon) + \boldsymbol{\Theta}^{u,\mu}_{\Phi_\pi,\Phi_u}\boldsymbol{\mu}\right)}_{\text{IRF of } u \text{ under baseline}} \\ & + \widetilde{\Phi}_{EE} \underbrace{\left(\boldsymbol{EE}_{\Phi_\pi,\Phi_u}(\varepsilon) + \boldsymbol{\Theta}^{EE,\mu}_{\Phi_\pi,\Phi_u}\boldsymbol{\mu}\right)}_{\text{IRF of } EE \text{ under baseline}},\end{aligned} \tag{A.11}$$

where $\boldsymbol{\Theta}^{Y,X}_{\Phi_\pi,\Phi_u}$ denotes the $T \times T$ Jacobian matrix of variable Y with respect to X for various X, Y combinations under the baseline monetary policy rule and ε is a non-policy/structural shock, i.e., shocks to supply, on-the-job search efficiency and demand as estimated in Section 3.2.

The left-hand side of Equation (A.11) is the combined IRF of the nominal interest rate to the structural shock ε and the sequence of policy news shocks $\boldsymbol{\mu}$ under the baseline rule. The right-hand side scales each of the baseline IRFs of inflation, unemployment rate and EE rate to the structural and policy shocks by the alternative monetary policy coefficients to compute the IRF of nominal interest rate under the alternative Taylor rule subject to the same shocks. The IRF to the structural shock ε under the *alternative* monetary policy rule is then equal to the IRF to ε and policy shocks $\{\mu_t\}_{t=1}^T$ under the *baseline* rule.

Once we solve the system of equations in (A.11), we similarly compute the IRF of other model variables under the alternative Taylor rule that are relevant for the central bank's objective function, i.e., inflation and the unemployment rate. Using these IRFs, we can calculate variances and evaluate Equation (24) for any combination of $\widetilde{\Phi}_\pi, \widetilde{\Phi}_u, \widetilde{\Phi}_{EE}$.

Figure A.1: DAG representation of the HANK model with a frictional labor market and on the job search



B Additional results

This appendix provides additional results to complement results and discussions in Section 5 of the main text.

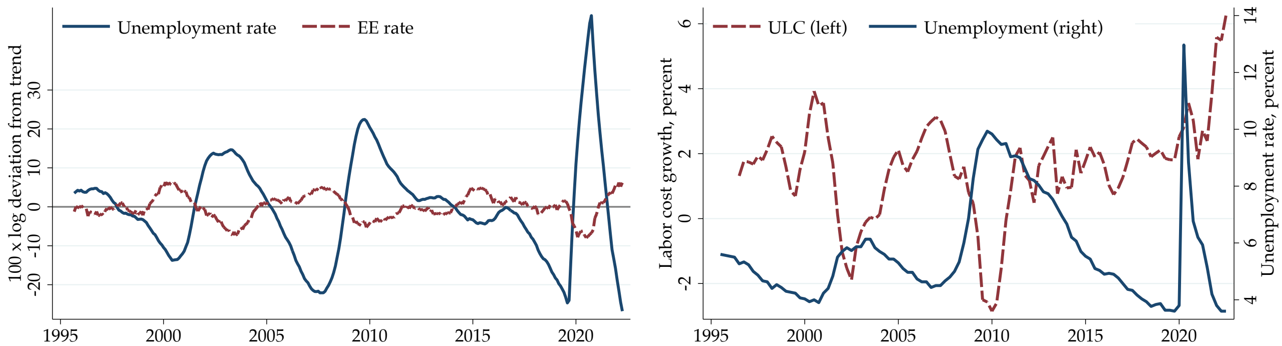
Labor market dynamics and unit labor cost Section 5.1 presents the historical relationship between EE and unemployment rates. In doing so, we use Figure 2 to document a scatter plot of raw monthly EE rate and unemployment rate across different episodes and a time-series of the rolling correlation between the cyclical components of the log unemployment and EE rates using a five-year window. Here, in Panel (a) of Figure B.1, we plot cyclical components of the log unemployment and EE rates over time, where both time series are detrended using the HP filter with a smoothing parameter of 10^5. It shows that during all recovery episodes except the recovery from the Great Recession between 2016 and 2019, when the unemployment rate declined below its trend, the EE rate increased above its trend. However, between 2016 and 2019, the unemployment rate around 25 percent below its trend, while the EE rate remained unchanged. This is different from the recovery from the COVID-19 recession when the unemployment rate declined by almost the same amount, but the EE rate increased by about 10 percent above its trend. To investigate the potential implications of flat EE rate between 2016 and 2019 on labor costs, Panel (b) of Figure B.1 presents a time-series of unit labor cost against the unemployment rate. We obtain the quarterly unit labor cost index for nonfarm business sector constructed by the BLS. Then, in each quarter, we calculate the four-quarter growth rate of this index and smooth this series by taking a four-quarter moving average. The figure shows that the unit labor cost typically increases when unemployment rate declines. For instance, during the recent recovery episode after the COVID-19 recession, when the unemployment rate was below 4 percent, the growth of unit labor cost reached to its historically highest levels of around 6 percent. On the other hand, despite the fact that the unemployment rate also reached below 4 percent between 2016 and 2019, the unit labor cost growth was only around 2 percent. The differential behavior of EE dynamics between the two episodes suggests that EE dynamics likely to play a role in driving the unit labor cost growth.

Effects of OJS efficiency shocks on productivity and piece rate Section 5.2 studies macroeconomic implications of negative OJS efficiency shocks by comparing outcomes between two different transitions starting from the same steady state. In doing so, in Figure 3, we compare average labor productivity (ALP) and average piece rate dynamics across the two economies to discuss their effects on output and inflation dynamics. Here, in panel (a) of Figure B.2, we document average match-specific productivity x dynamics between the two economies. We show that because negative OJS efficiency shocks limit employed workers' ability to change jobs, they mute the increase in the average match productivity when labor market is recovering. In fact, as unemployed workers accept the first job offer, they initially lower the average match quality,

Figure B.1: Labor market dynamics and unit labor cost

(a) Cyclical components of unemployment and EE rates

(b) Unit labor cost vs unemployment rate



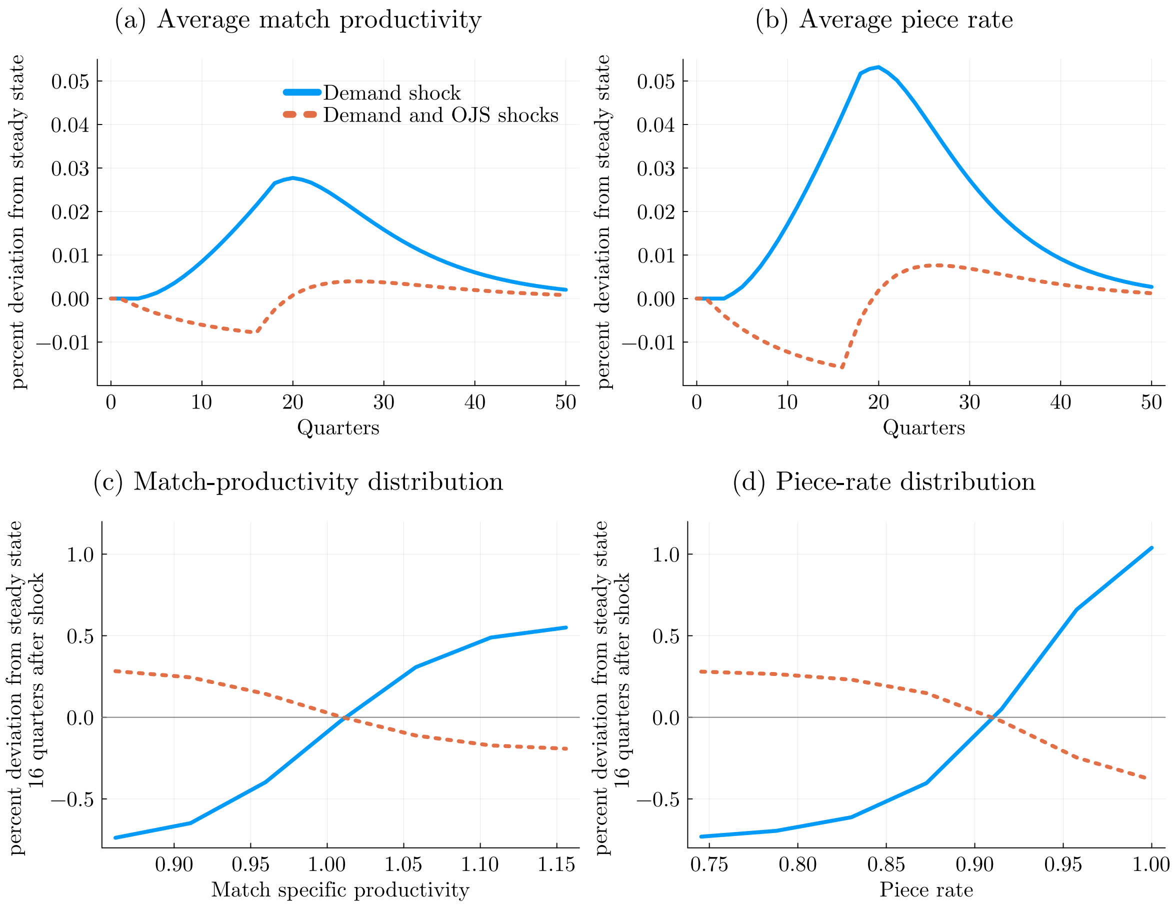
Notes: Panel (a) plots the time series of the cyclical components of log unemployment rate and EE rate. Both time series are detrended using the HP filter with a smoothing parameter of 10^5. Panel (b) plots the four-quarter growth rate of unit labor cost index for the nonfarm business sector constructed by the BLS against the unemployment rate. We smooth the unit labor cost growth by taking a four-quarter moving average.
Source: BLS and Fujita, Moscarini, and Postel-Vinay (2020).

which only recovers after 20 quarters as they climb up the ladder slowly due to the presence of negative OJS efficiency shocks. Importantly, panel (b) shows that the average piece-rate increases much more when there are only positive demand shocks. This implies that wages are higher in this economy than wages in the economy with negative OJS efficiency shocks, despite similar ALP dynamics between the two economies, as shown in panel (d) of Figure 3. As such, inflationary wage pressures are stronger when there are no OJS efficiency shocks.

We also compare match-specific productivity and piece-rate dynamics across the cross-section between the two economies, as we think that only looking at averages may mask interesting results across heterogeneous agents. The next two panels of Figure B.2 plot changes in the distribution of match-specific productivity (panel c) and the distribution of piece rate (panel d) 16 quarters after the shock relative to their respective across the two economies. Under positive demand shocks alone, we find that both the match productivity distribution and the piece-rate distribution shift rightward. On the other hand, when there are negative OJS efficiency shocks, both of these distributions shift leftward as negative OJS efficiency shocks slow down the job ladder.

Impulse responses to an OJS efficiency shock In Section 5.3, we use results on the impulse responses of model outcomes to a positive unit shock to the OJS efficiency parameter ν in our discussion on decomposing the effects of various channels on inflation upon a positive unit shock to ν. Here, we provide these impulse responses in Figure B.3. As discussed in Section 5.3, an increase in ν leads to a decline in the market tightness, output, and consumption, and an increase in the price of labor services, unemployment rate, and inflation.

Figure B.2: Effects of negative OJS efficiency shocks on productivity and piece rate distributions

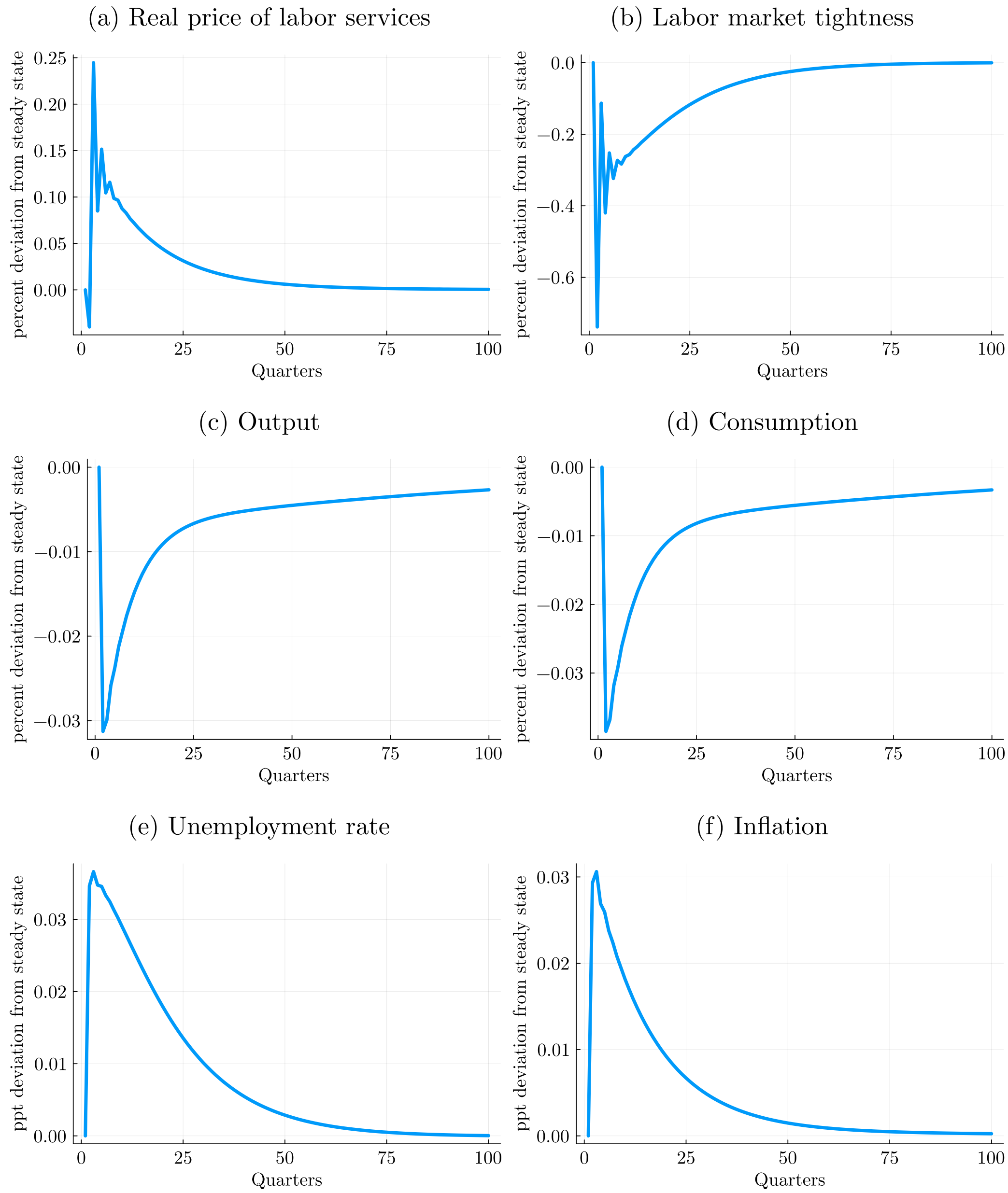


Notes: This figure presents dynamics of average match-specific productivity x (panel a) over time, average piece rate over time (Panel b), and changes in the distribution of match-specific productivity (panel c) and the distribution of piece rate (panel d) 16 quarters after the shock relative to their respective steady state in an economy with (1) only a series of positive demand shocks (solid-blue lines) and (2) series of positive demand shocks and negative OJS efficiency shocks (dashed-orange lines). These shocks in the two economies are calibrated such that they lead to the same path of unemployment rate. The additional negative OJS shocks in the second economy are estimated to keep the EE rate unchanged.

Next, in Section 5.4, we analyze the role of incomplete markets in our quantitative results regarding the impact of OJS efficiency shocks on inflation. In our discussions in that section we cite equilibrium changes in real dividends d, real share price p^s, real financial wealth — calculated as the summation of real dividends and real share price, — real interest rate r, real profits of intermediate firm Γ^I and real profits of the service firm Γ^S, and real total firm profits — calculated as the summation of real profits of intermediate firm and real profits of the service firm — in response to a positive OJS efficiency shock. Here, we provide these impulse responses in Figure B.4. An increase in ν leads to a decline in real dividends, real share price, and as a result, real financial wealth. Dividends fall because of the decline in real total firm profits, which is driven by the decline in intermediate firms' profits due to the rising price of labor services p^l.

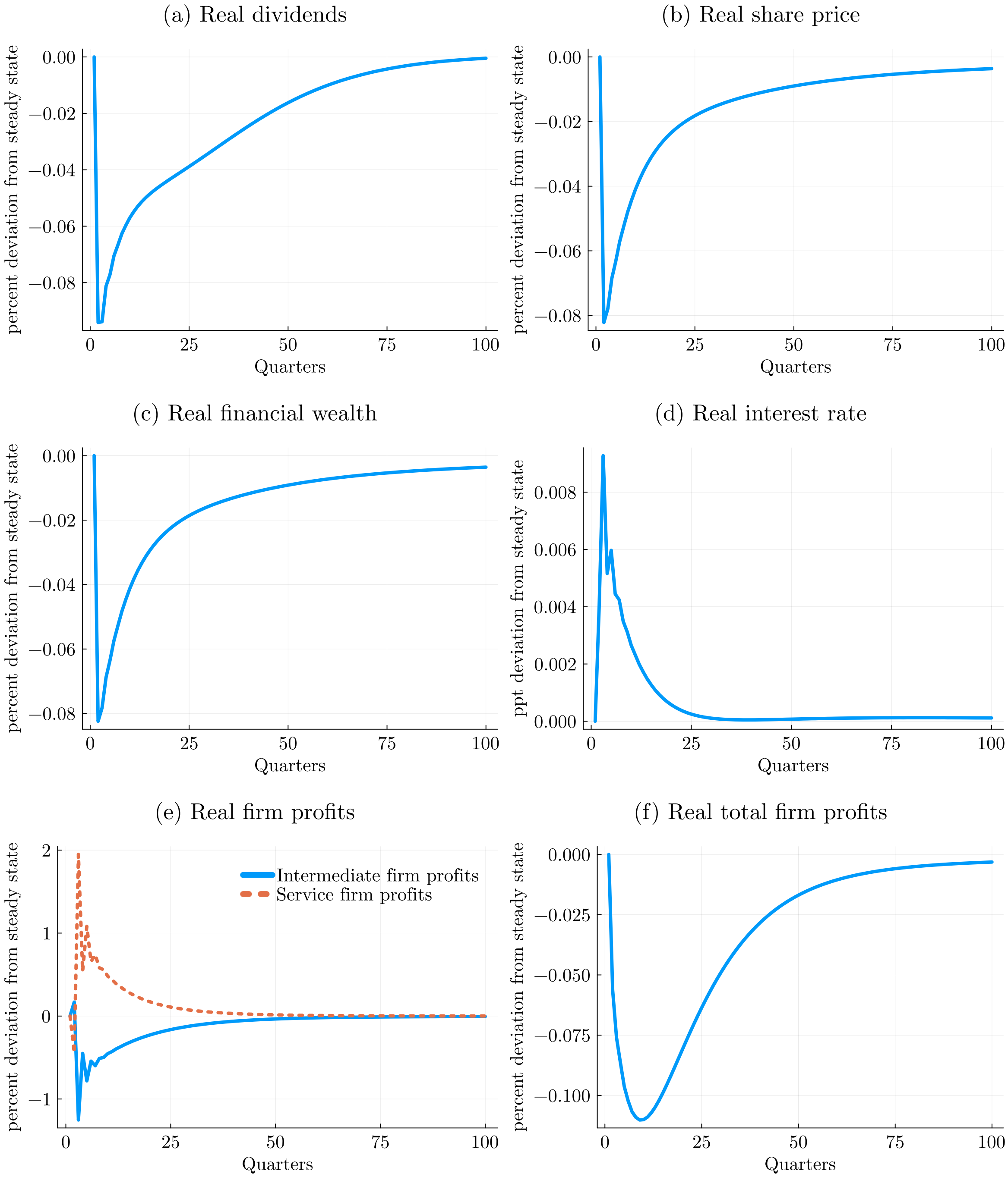
Real share price fall because of the decline in real dividends and the rise in the real interest rate.

Figure B.3: Impulse responses to an OJS efficiency shock



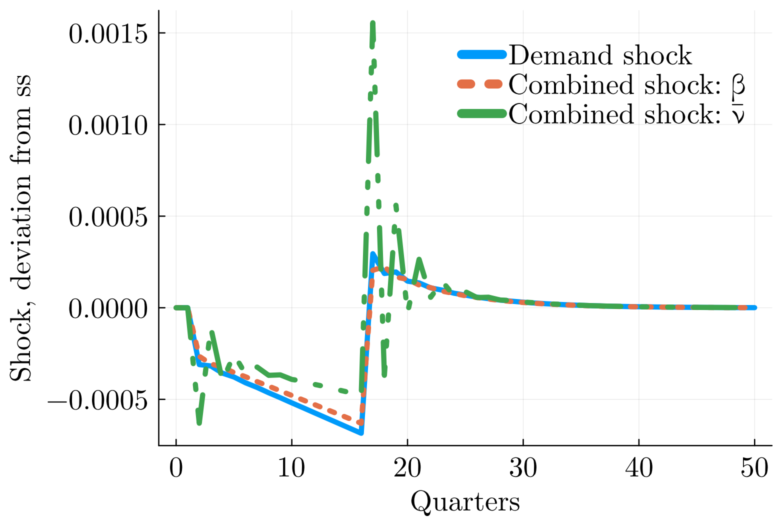
Notes: This figure presents the impulse responses of model outcomes to a positive unit shock to the OJS efficiency parameter ν.

Figure B.4: Impulse responses to an OJS efficiency shock: continued



Notes: This figure presents the impulse responses of model outcomes to a positive unit shock to the OJS efficiency parameter ν. Real financial wealth is the summation of real share price and real dividends. Real total firm profits represent the summation of real profits for the intermediate firm and real profits for the service firm.

Figure B.5: Estimated path of innovations to demand and on-the-job-search efficiency



Notes: This figure plots the estimated path of innovations to demand and on-the-job search efficiency as described in Section 5.2. The solid blue line is the sequence of innovations to demand to match the path of unemployment in the post-Great Recession episode without targeting the path of the EE rate. The red and green lines are the paths of demand and OJS efficiency shocks to jointly match the same unemployment rate as in the first exercise as well as the flat EE rate in the post-Great Recession period.